

Data and Formulae for Mechanical Engineering Students

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A General information

Table A.1: SI Units and abbreviations

Quantity	Unit	Unit symbol
Basic units		
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Derived units		
Acceleration, linear	metre/second ²	m s^{-2}
Acceleration, angular	radian/second ²	rad s^{-2}
Area	metre ²	m^2
Density	kilogram/metre ³	kg m^{-3}
Force	newton	$\text{N} (= \text{kg m s}^{-2})$
Frequency	hertz	$(\text{Hz} = \text{s}^{-1})$
Impulse, linear	newton-second	N s
Impulse, angular	newton-metre-second	N ms
Moment of force	newton-metre	N m
Second moment of area	metre ⁴	m^4
Moment of inertia	kilogram-metre ²	kg m^2
Momentum, linear	kilogram-metre/second	kg m s^{-1}
Momentum, angular	kilogram-metre ² /second	$\text{kg m}^2 \text{s}^{-1}$
Power	watt	$\text{W} (= \text{J s}^{-1} = \text{N m s}^{-1})$
Pressure, stress	pascal	$\text{Pa} (= \text{N m}^{-2})$
Stiffness (linear), spring constant	newton/metre	N m^{-1}
Velocity, linear	metre/second	m s^{-1}
Velocity, angular	radian/second	rad s^{-1}
Volume	metre ³	m^3
Work, energy	joule	J (= N m)
Electrical units		
Potential	volt	$\text{V} (= \text{W A}^{-1})$
Resistance	ohm	$\Omega (= \text{V A}^{-1})$
Charge	coulomb	C (= As)
Capacitance	farad	$\text{F} (= \text{As V}^{-1})$
Electric field strength	volt/metre	Vm^{-1}
Electric flux density	coulomb/metre ²	C m^{-2}
Magnetic units		
Magnetic flux	weber	$\text{Wb} (= \text{Vs})$
Inductance	henry	$\text{H} (= \text{Vs A}^{-1})$
Magnetic field strength	—	Am^{-1}
Magnetic flux density	—	Wb m^{-2}

Table A.2: Conversion factors from Imperial to SI units

To convert	from	to	multiply by
Acceleration	foot/second ² (ft/sec ²)	metre/second ² (m s ⁻²)	0.3048
	inch/second ² (in/sec ²)	metre/second ² (m s ⁻²)	0.0254
Area	foot ² (ft ²)	metre ² (m ²)	0.092903
	inch ² (in. ²)	metre ² (m ²)	6.4516 × 10 ⁻⁴
Density	pound mass/inch ³ (lbm/in ³)	kilogram/metre ³ (kg m ⁻³)	2.7680 × 10 ⁴
	pound mass/foot ³ (lbm/ft ³)	kilogram/metre ³ (kg m ⁻³)	16.018
Force	kip (1000 lb)	newton (N)	4.4482 × 10 ³
	pound force (lb)	newton (N)	4.4482
Length	foot (ft)	metre (m)	0.3048
	inch (in)	metre (m)	0.0254
	mile (mi), U.S. statute	metre (m)	1.6093 × 10 ³
	mile (mi), international nautical	metre (m)	1.852 × 10 ³
Mass	pound mass (lbm)	kilogram (kg)	0.45359
	slug (lb·sec ² /ft)	kilogram (kg)	14.594
	ton (2000 lbm)	kilogram (kg)	907.18
Moment of force	pound-foot (lb-ft)	newton-metre (N m)	1.3558
	pound-inch (lb-in.)	newton-metre (N m)	0.11298
Moment of inertia	pound-foot-second ² (lb-ft-sec ²)	kilogram-metre ² (kg m ²)	1.3558
Momentum, linear	pound-second (lb-sec)	kilogram-metre/second (kg m s ⁻¹)	4.4482
Momentum, angular	pound-foot-second (lb-ft-sec)	newton-metre-second (kg m ² s ⁻¹)	1.3558
Power	foot-pound/minute (ft-lb/min)	watt (W)	0.022597
	horsepower (550 ft-lb/sec)	watt (W)	745.70
Pressure, stress	atmosphere (std) (14.7 lb/in ²)	newton/metre ² (N m ⁻² or Pa)	1.0133 × 10 ⁵
	pound/foot ² (lb/ft ²)	newton/metre ² (N m ⁻² or Pa)	47.880
	pound/inch ² (lb/in. ² or psi)	newton/metre ² (N m ⁻² or Pa)	6.8948 × 10 ³
Second moment of area	inch ⁴	metre ⁴ (m ⁴)	41.623 × 10 ⁻⁸
Stiffness (linear)	pound/inch (lb/in.)	newton/metre (N m ⁻¹)	175.13
Velocity	foot/second (ft/sec)	metre/second (m s ⁻¹)	0.3048
	knot (nautical mi/hr)	metre/second (m s ⁻¹)	0.51444
	mile/hour (mi/hr)	metre/second (m s ⁻¹)	0.44704
	mile/hour (mi/hr)	kilometre/hour (km h ⁻¹)	1.6093
Volume	foot ³ (ft ³)	metre ³ (m ³)	0.028317
	inch ³ (in. ³)	metre ³ (m ³)	1.6387 × 10 ⁻⁵
	UK gallon	metre ³ (m ³)	4.546 × 10 ⁻³
Work, Energy	British thermal unit (BTU)	joule (J)	1.0551 × 10 ³
	foot-pound force (ft-lb)	joule (J)	1.3558
	kilowatt-hour (kw-h)	joule (J)	3.60 × 10 ⁶

Table A.3: Decimal prefixes

Multiplication factor ^a		Prefix	Symbol
1 000 000 000 000	=	10^{12}	tera
1 000 000 000	=	10^9	giga
1 000 000	=	10^6	mega
1 000	=	10^3	kilo
100	=	10^2	hecto ^a
10	=	10	deka ^a
			da
0.1	=	10^{-1}	deci ^b
0.01	=	10^{-2}	centi
0.001	=	10^{-3}	milli
0.000 001	=	10^{-6}	micro
0.000 000 001	=	10^{-9}	nano
0.000 000 000 001	=	10^{-12}	pico
			p

^aUse prefixes to keep numerical values generally between 0.1 and 1000^bThe use of prefixes hecto, deka, deci and centi should be avoided except for certain areas or volumes where the numbers would otherwise become awkward.

Table A.4: Physical constants

Avogadro's number ^a	N	$6.022 \times 10^{23} \text{ mol}^{-1}$
Absolute zero of temperature	—	$0 \text{ K} = -273.2 \text{ }^\circ\text{C}$
Boltzmann's constant	k	$1.380 \times 10^{-23} \text{ J K}^{-1}$
Characteristic impedance of vacuum	Z_0	$= \left(\frac{\mu_0}{\epsilon_0} \right)^{1/2} = 120\pi \Omega$
Electron volt	eV	$1.602 \times 10^{-19} \text{ J}$
Electronic charge	e	$1.602 \times 10^{-19} \text{ C}$
Electronic rest mass	m_e	$9.109 \times 10^{-31} \text{ kg}$
Electronic charge to mass ratio	$\left(\frac{e}{m_e} \right)$	$1.759 \times 10^{11} \text{ C kg}^{-1}$
Faraday's constant ^a	F	$9.65 \times 10^4 \text{ C mol}^{-1}$
Gas constant ^a	R	$8.314 \text{ J mol}^{-1} \text{ K}^{-1}$
Permeability of free space	μ_0	$4\pi \times 10^{-7} \text{ H m}^{-1}$
Permittivity of free space	ϵ_0	$\frac{1}{36\pi} \times 10^{-9} \text{ F m}^{-1}$
Planck's constant	h	$6.626 \times 10^{-34} \text{ Js}$
Standard gravitational acceleration	g	9.807 m s^{-2}
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8} \text{ J m}^{-2} \text{ s}^{-1} \text{ K}^{-4}$
Velocity of light in vacuum	c	$2.9979 \times 10^8 \text{ m s}^{-1}$
Volume of perfect gas at S.T.P. ^b	—	$22.42 \times 10^{-3} \text{ m}^3$

^aThese are conventional definitions in gram mol units. For SI calculations in kg mol units multiply the values given by 10^3 ^bAt Standard Temperature ($0 \text{ }^\circ\text{C}$) and Pressure (one atmosphere pressure or $1.013 \times 10^5 \text{ N m}^{-2}$)

B Mathematics and computing

Data and formulae for core course examinations in:

- Mathematics
- Computing

and in other, related, optional courses.

B.1 Algebra

B.1.1 Logarithms

If $b^y = x$, $y = \log_b(x)$ and:

$$\log(x_1x_2) = \log x_1 + \log x_2$$

$$\log\left(\frac{x_1}{x_2}\right) = \log x_1 - \log x_2$$

$$\log\left(\frac{1}{x}\right) = -\log x$$

$$\log x^n = n \log x$$

$$\log 1 = 0$$

For natural logarithms $b = e = 2.718282$ and if $e^y = x$,

$$y = \log_e(x) = \ln(x)$$

Hence

$$\log_{10} x = 0.4343 \ln x.$$

B.1.2 Quadratic equations

If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

and ($b^2 > 4ac$) for real roots.

B.1.3 Determinants

2nd order:

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

3rd order:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = +a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2 - a_3b_2c_1 - a_2b_1c_3 - a_1b_3c_2$$

B.1.4 Vector algebra

$$\mathbf{a} = (a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}) = (a_1, a_2, a_3) \text{ etc.}$$

Scalar (dot) product:

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Vector (cross) product:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Scalar triple product:

$$[\mathbf{a}, \mathbf{b}, \mathbf{c}] = \mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = \mathbf{b} \cdot \mathbf{c} \times \mathbf{a} = \mathbf{c} \cdot \mathbf{a} \times \mathbf{b} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Vector triple product:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b})$$

B.1.5 Series

Binomial series:

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!} x^3 + \dots \quad (\alpha \text{ arbitrary, } |x| < 1).$$

$$e^x = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^n}{n!} + \dots \quad (|x| < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots \quad (|x| < \infty)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots \quad (|x| < \infty)$$

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots \quad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$$

$$\sinh x = \frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots \quad (|x| < \infty)$$

$$\cosh x = \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \quad (|x| < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^n \frac{x^{n+1}}{(n+1)} + \dots \quad (-1 < x < 1)$$

Stirling's formula for $n!$ when n is large:

$$n! \cong \left(\frac{n}{e}\right)^n \sqrt{2\pi n}$$

or

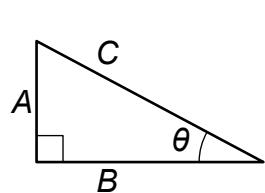
$$\ln(n!) \cong \left(n + \frac{1}{2}\right) \ln n - n + \frac{1}{2} \ln(2\pi)$$

or

$$\log_{10}(n!) \cong 0.39909 + \left(n + \frac{1}{2}\right) \log_{10} n - 0.43429n$$

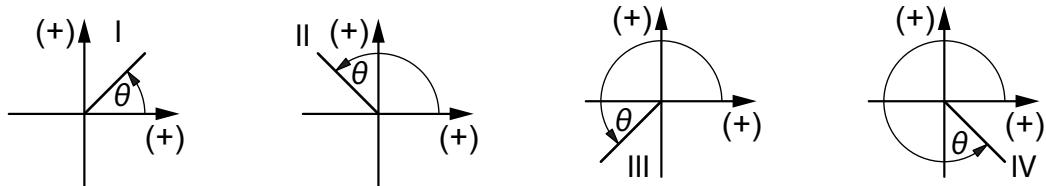
B.1.6 Trigonometry

Definitions:



$$\begin{aligned} \sin \theta &= \frac{A}{C} & \cos \theta &= \frac{B}{C} & \tan \theta &= \frac{A}{B} \\ \csc \theta &= \frac{C}{A} & \sec \theta &= \frac{C}{B} & \cot \theta &= \frac{B}{A} \end{aligned}$$

Signs of trigonometric functions in the four quadrants:



Quadrant:	I	II	III	IV
$\sin \theta$	+	+	-	-
$\cos \theta$	+	-	-	+
$\tan \theta$	+	-	+	-
$\csc \theta$	+	+	-	-
$\sec \theta$	+	-	-	+
$\cot \theta$	+	-	+	-

Trigonometrical identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

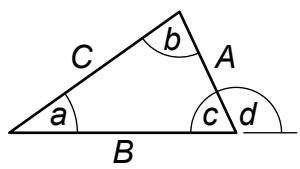
$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\sin \frac{\theta}{2} = \sqrt{\frac{1}{2} (1 - \cos \theta)}$$

$$\cos \frac{\theta}{2} = \sqrt{\frac{1}{2} (1 + \cos \theta)}$$



Sine rule: $\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$

Cosine rule: $C^2 = A^2 + B^2 - 2AB \cos c$
 $C^2 = A^2 + B^2 + 2AB \cos d$

$$\begin{aligned}\sin(a+b) &= \sin a \cos b + \cos a \sin b \\ \cos(a+b) &= \cos a \cos b - \sin a \sin b\end{aligned}$$

$$\begin{aligned}\sin a + \sin b &= 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) \\ \cos a + \cos b &= 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right)\end{aligned}$$

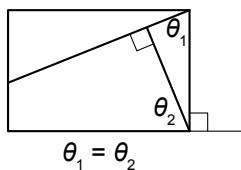
$$\begin{aligned}\sin iz &= i \sinh z \\ \cos iz &= \cosh z\end{aligned}$$

$$\begin{aligned}\sin(a-b) &= \sin a \cos b - \cos a \sin b \\ \cos(a+b) &= \cos a \cos b + \sin a \sin b\end{aligned}$$

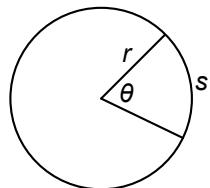
$$\begin{aligned}\sin a - \sin b &= 2 \cos\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right) \\ \cos a - \cos b &= -2 \sin\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right)\end{aligned}$$

$$\begin{aligned}\sinh iz &= i \sin z \\ \cosh iz &= \cos z\end{aligned}$$

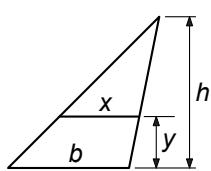
B.1.7 Geometry



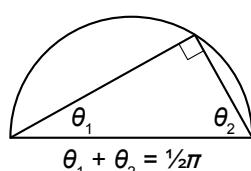
When the two intersecting lines are, respectively, perpendicular to two other lines, the angles formed by each pair are equal.



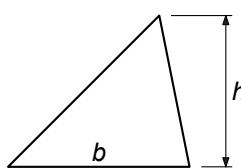
Circle:
circumference = $2\pi r$
Arc length $s = r\theta$
Sector area = $\frac{1}{2}r^2\theta$



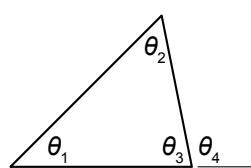
Similar triangles:
 $\frac{x}{b} = \frac{h-y}{h}$



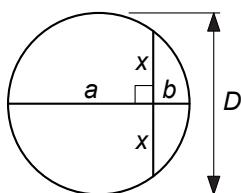
Every triangle inscribed within a semicircle is a right triangle.



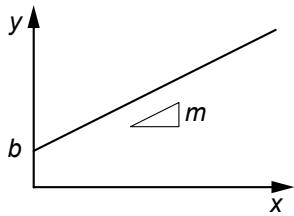
Any triangle:
Area = $\frac{1}{2}bh$



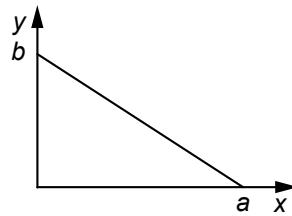
Angles of a triangle:
 $\theta_1 + \theta_2 + \theta_3 = 180^\circ$
 $\theta_4 + \theta_1 + \theta_2$



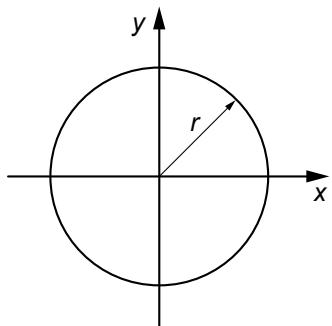
Intersecting chords:
 $x^2 = ab$
 $x^2 \approx Db$ when $b \ll D$

B.1.8 Analytic geometry


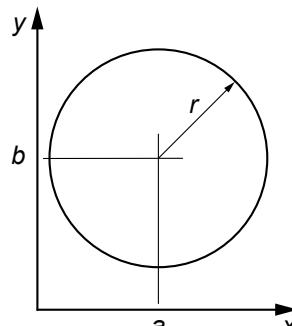
Straight line:
 $y = b + mx$



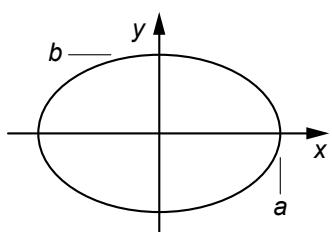
$$\frac{x}{a} + \frac{y}{b} = 1$$



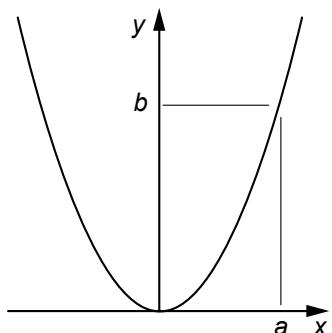
Circle:
 $x^2 + y^2 = r^2$



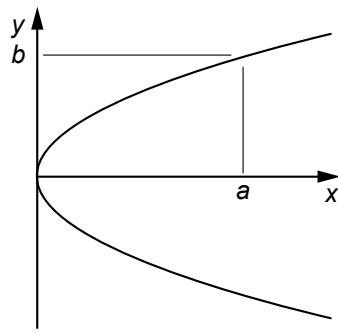
$$(x - a)^2 + (y - b)^2 = r^2$$



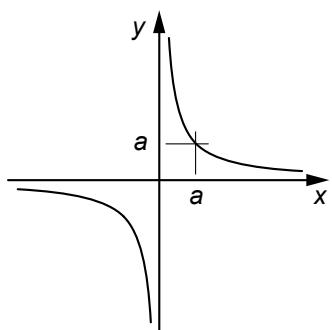
Ellipse:
 $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$



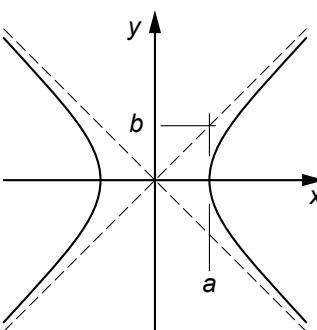
Parabola:
 $y = b \left(\frac{x}{a}\right)^2$



$$x = a \left(\frac{y}{b}\right)^2$$

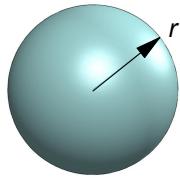


Hyperbola:
 $xy = a^2$

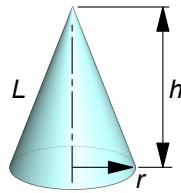


$$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$$

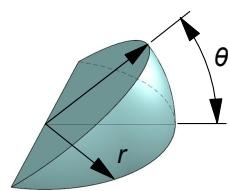
B.1.9 Solid geometry



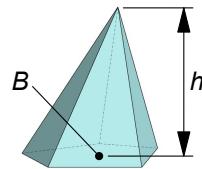
Sphere:
volume = $\frac{4}{3}\pi r^3$
surface area = $4\pi r^2$



Right-circular cone:
volume = $\frac{1}{3}\pi r^2 h$
lateral area = $\pi r L$
 $L = \sqrt{r^2 + h^2}$



Spherical wedge:
volume = $\frac{2}{3}r^3\theta$



Any pyramid or cone:
volume = $\frac{1}{3}Bh$
where B = area of base.

B.1.10 Differential calculus

Leibnitz's rule:

$$D^n(fg) = f(D^n g) + n(Df)(D^{n-1}g) + \frac{n(n-1)}{2!}(D^2f)(D^{n-2}g) + \cdots + (D^n f)g$$

where $D = \frac{d}{dx}$, $f = f(x)$ and $g = g(x)$

Taylor's expansion of $f(x)$ about $x = a$:

$$f(x) = f(a) + (x - a)f'(a) + \frac{(x - a)^2}{2!}f''(a) + \cdots + \frac{(x - a)^n}{n!}f^{(n)}(a) + \frac{(x - a)^{n+1}}{(n+1)!}f^{(n+1)}(\bar{x})$$

where $a < \bar{x} < x$. Substituting $h = x - a$ gives the following form:

$$f(a + h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \cdots + \frac{h^n}{n!}f^{(n)}(a) + R_n(h)$$

where $R_n(h) = \frac{h^{n+1}}{(n+1)!}f^{(n+1)}(a + \theta h)$, $(0 < \theta < 1)$.

Taylor's expansion of $f(x, y)$ about the point (a, b) :

$$\begin{aligned} f(x, y) &= f(a, b) + [(x - a)f_x + (y - b)f_y]_{a,b} \\ &\quad + \frac{1}{2!} [(x - a)^2 f_{xx} + 2(x - a)(y - b)f_{xy} + (y - b)^2 f_{yy}]_{a,b} + \dots \end{aligned}$$

Substituting $h = x - a$ and $k = y - b$ gives the following form:

$$f(a + h, b + k) = f(a, b) + [hf_x + kf_y]_{a,b} + \frac{1}{2!} [h^2 f_{xx} + 2hk f_{xy} + k^2 f_{yy}]_{a,b} + \dots$$

Partial differentiation:

If $y = Y(x)$, then $f(x, y) = f[x, Y(x)] \equiv F(x)$ and

$$\frac{dF}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \frac{dy}{dx}$$

If $x = X(t)$ and $y = Y(t)$, then $f(x, y) = F(t)$ and

$$\frac{dF}{dt} = \frac{\partial f}{\partial x} \frac{dX}{dt} + \frac{\partial f}{\partial y} \frac{dY}{dt}$$

If $x = X(u, v)$ and $y = Y(u, v)$ then $f(x, y) = F(u, v)$ and

$$\frac{\partial F}{\partial u} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial u}$$

$$\frac{\partial F}{\partial v} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial v}$$

Stationary points of $f(x, y)$:

These occur where $f_x = 0$, $f_y = 0$ simultaneously. Let (a, b) be a stationary point: examine

$$K = \left[f_{xx}f_{yy} - (f_{xy})^2 \right]_{a,b}$$

If:

- $K < 0$, then (a, b) is a *saddle point*;
- $K > 0$ and $f_{xx}(a, b) < 0$, then (a, b) is a *maximum*;
- $K > 0$ and $f_{xx}(a, b) > 0$, then (a, b) is a *minimum*.

Radius of curvature in Cartesian coordinates:

$$\rho_{xy} = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\frac{d^2y}{dx^2}}$$

B.1.11 Standard Differentials

$f(x)$	$\frac{df(x)}{dx}$
x^n	nx^{n-1}
uv	$u \frac{dv}{dx} + v \frac{du}{dx}$
$\frac{u}{v}$	$v \frac{du}{dx} - u \frac{dv}{dx}$ v^2
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\log_e x = \ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}

B.1.12 Differential equations

The first-order linear equation

$$\frac{dy}{dx} + R(x)y = S(x)$$

has an integrating factor

$$\lambda(x) = \exp \left[\int R(x) dx \right],$$

so that

$$\frac{d}{dx}(y\lambda) = S\lambda.$$

$$P(x, y)dx + Q(x, y)dy = 0$$

is an exact equation if

$$\frac{dP}{dy} = \frac{dQ}{dx}.$$

B.2 Integral calculus

An important substitution:

$$\tan \frac{\theta}{2} = t.$$

Then

$$\begin{aligned}\sin \theta &= \frac{2t}{(1+t^2)} \\ \cos \theta &= \frac{(1-t^2)}{(1+t^2)}\end{aligned}$$

and

$$d\theta = \frac{2}{(1+t^2)} dt.$$

Table B.1: Some indefinite integrals

$f(x)$	$\int f(x) dx$
$\sec x$	$\ln(\sec x + \tan x) = \ln \tan\left(\frac{x}{2} + \frac{\pi}{4}\right)$
cosec x	$\ln(\cosec x - \cot x) = \ln \tan\left(\frac{x}{2}\right)$
$(a^2 - x^2)^{-1/2}$	$\sin^{-1}\left(\frac{x}{a}\right), (x < a)$
$(a^2 + x^2)^{-1/2}$	$\sinh^{-1}\left(\frac{x}{a}\right) = \ln\left[x + (a^2 + x^2)^{1/2}\right] - \ln a = \ln\left[\frac{x}{a} + \left(1 + \left(\frac{x}{a}\right)^2\right)^{1/2}\right]$
$(x^2 - a^2)^{-1/2}$	$\cosh^{-1}\left(\frac{x}{a}\right) = \ln\left[x + (x^2 - a^2)^{1/2}\right] - \ln a = \ln\left[\frac{x}{a} + \left(\left(\frac{x}{a}\right)^2 - 1\right)^{1/2}\right], (x \geq a)$
$(a^2 + x^2)^{-1}$	$\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$
$(a^2 - x^2)^{-1}$	$\frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) = \frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right), (x < a)$
$(x^2 - a^2)^{-1}$	$\frac{1}{2a} \ln\left(\frac{x-a}{x+a}\right), (x > a)$

Table B.2: Some definite integrals

$I_n \equiv \int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx = \frac{n-1}{n} I_{n-2}$, where $I_0 = \frac{\pi}{2}$ and $I_1 = 1$
$I_{m,n} \equiv \int_0^{\pi/2} \sin^m x \cos^n x dx = \frac{m-1}{m+n} I_{m-2,n} = \frac{n-1}{m+n} I_{m,n-2}, (m > 1, n > 1)$
$\int_0^\infty e^{-ax} \sin bx dx = \frac{b}{(a^2 + b^2)}, (a > 0)$
$\int_0^\infty e^{-ax} \cos bx dx = \frac{a}{(a^2 + b^2)}, (a > 0)$
$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$

B.3 Laplace transforms

Function	Transform
Definition: $f(t)$	$\bar{f}(s) = \int_0^\infty e^{-st} f(t) dt$
$af(t) + bg(t)$	$a\bar{f}(s) + b\bar{g}(s)$
$\frac{df}{dt}$	$s\bar{f}(s) - f(0)$
$\frac{d^2f}{dt^2}$	$s^2\bar{f}(s) - sf(0) - f'(0)$
$e^{-at}f(t)$	$\bar{f}(s-a)$
$tf(t)$	$-\frac{d\bar{f}(s)}{ds}$
$\frac{\partial f(t, a)}{\partial a}$	$\frac{\partial \bar{f}(s, a)}{\partial a}$
$\int_0^t f(t) dt$	$\frac{\bar{f}(s)}{s}$
$\int_0^t f(u)g(t-u) du$	$\bar{f}(s)\bar{g}(s)$
$\delta(t_0)$, unit impulse at $t = t_0$	1
1, unit step	$\frac{1}{s}$ $(s > 0)$
t^n , $n = 1, 2, \dots$	$\frac{n!}{s^{n+1}}$ $(s > 0)$
e^{at}	$\frac{1}{s-a}$ $(s > a)$
e^{-at}	$\frac{1}{s+a}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$
$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$\frac{1}{(b-a)}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
$\frac{1}{(b-a)}[(c-a)e^{-at} - (c-b)e^{-bt}]$	$\frac{s+c}{(s+a)(s+b)}$
$1 - \frac{b}{(b-a)}e^{-at} + \frac{a}{(b-a)}e^{-bt}$	$\frac{ab}{s(s+a)(s+b)}$
$\frac{e^{-at}}{(b-a)(c-a)} + \frac{e^{-bt}}{(c-a)(a-b)} + \frac{e^{-ct}}{(a-c)(b-c)}$	$\frac{1}{(s+a)(s+b)(s+c)}$
$c - \frac{b(c-a)}{(b-a)}e^{-at} + \frac{a(c-b)}{(b-a)}e^{-bt}$	$\frac{ab(s+c)}{s(s+a)(s+b)}$

Function	Transform
$\sin \omega t$	$\frac{\omega}{(s^2 + \omega^2)}$ ($s > 0$)
$\cos \omega t$	$\frac{s}{(s^2 + \omega^2)}$ ($s > 0$)
$\frac{\sqrt{(a^2 + \omega^2)}}{\omega} \sin(\omega t + \phi), \phi = \tan^{-1}\left(\frac{\omega}{a}\right)$	$\frac{s+a}{(s^2 + \omega^2)}$ ($s > 0$)
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
$\frac{1}{\omega} \sqrt{(c-a)^2 + \omega^2} e^{-at} \sin(\omega t + \phi), \phi = \tan^{-1}\left(\frac{\omega}{c-a}\right)$	$\frac{(s+c)}{(s+a)^2 + \omega^2}$
$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin \omega_n \sqrt{1-\zeta^2} t, (\zeta < 1)$	$\frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2}$
$\frac{1}{a^2 + \omega^2} + \frac{1}{\omega \sqrt{a^2 + \omega^2}} e^{-at} \sin(\omega t - \phi), \phi = \tan^{-1} \frac{-\omega}{a}$	$\frac{1}{s[(s+a)^2 + \omega^2]}$
$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin \left(\omega_n \sqrt{1-\zeta^2} t + \phi \right), \phi = \cos^{-1} \zeta, \zeta < 1$	$\frac{\omega_n^2}{s(s^2 + 2\zeta \omega_n s + \omega_n^2)}$
$H(t-T) \quad (= 0, t < T; = 1, t > T)$	$\frac{1}{s} e^{-sT} \quad (s, T > 0)$

B.4 Numerical analysis

B.4.1 Approximate solution of an algebraic equation

- An iterative method for $x = \phi(x)$ converges when $|\phi'(x)| < 1$ near the root: if a root occurs near to $x = a$ take $x_0 = a$ and

$$x_{n+1} = \phi(x_n), n = 0, 1, 2, \dots$$

- If a root of $f(x) = 0$ occurs near to $x = a$, take $x_0 = a$ and:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, n = 0, 1, 2, \dots$$

(the *Newton-Raphson method*).

B.4.2 Numerical integration

Write $x_n = x_0 + nh$, $y_n = y(x_n)$. Then:

- Trapezium Rule (1 strip):

$$\int_{x_0}^{x_1} y(x) dx \approx \frac{h}{2} [y_0 + y_1]$$

- Simpson's Rule (2 strips):

$$\int_{x_0}^{x_1} y(x) dx \approx \frac{h}{3} [y_0 + 4y_1 + y_2]$$

B.4.3 Richardson's error estimation formula for use with Simpson's rule

Let

$$I = \int_a^b f(x) dx$$

and let I_1, I_2 be two estimates of I obtained using Simpson's rule with intervals h_1 and h_2 , where $h_1 < h_2$ (i.e. $h_1 = \frac{b-a}{n_1}$, $h_2 = \frac{b-a}{n_2}$, where n_1, n_2 are even). Then a better estimate of I is given by:

$$I = I_2 + \frac{(I_2 - I_1)}{\left[\left(\frac{h_1}{h_2} \right)^4 - 1 \right]}.$$

If $h_2 = \frac{1}{2}h_1$ then $I = I_2 + \frac{1}{15}(I_2 - I_1)$.

B.4.4 Fourier series

If $f(x)$ is periodic of period $2L$, i.e. $f(x + 2L) = f(x)$, then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$$

where

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx, n = 0, 1, 2, \dots$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi x}{L} dx, n = 1, 2, 3, \dots$$

If $f(x)$ is an *even* function of x , i.e. $f(-x) = f(x)$, then

$$a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi x}{L} dx, b_n = 0, n = 0, 1, 2, \dots$$

If $f(x)$ is an *odd* function of x , i.e. $f(-x) = -f(x)$, then

$$b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi x}{L} dx, a_n = 0, n = 1, 2, 3, \dots$$

B.5 Statistics

B.6 Probabilities for events

For events A , B and C :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The *odds* in favour of A are:

$$\frac{P(A)}{P(\bar{A})}$$

Conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (\text{if } P(B) > 0)$$

The *chain rule*:

$$P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B)$$

Bayes' rule:

$$P(A|B) = \frac{P(A)P(B|A)}{P(A)P(B|A) + P(\bar{A})P(B|\bar{A})}$$

A and B are *independent* if

$$P(B|A) = P(B)$$

A , B and C are independent if

$$P(A \cap B \cap C) = P(A)P(B)P(C),$$

and $P(A \cap B) = P(A)P(B)$, $P(B \cap C) = P(B)P(C)$ and $P(C \cap A) = P(C)P(A)$.

B.6.1 Distribution, expectation and variance

The *probability distribution* for a *discrete* random variable X is the set $\{p_x\}$, where

$$p_x = P(X = x).$$

The *expectation* is

$$E(X) = \mu = \sum_x x p_x$$

From independent observations x_1, x_2, \dots, x_n , the *sample mean*

$$\bar{x} = \frac{1}{n} \sum_k x_k$$

estimates μ .

The *variance* is

$$\text{var}(X) = \sigma^2 = E \left\{ (X - \mu)^2 \right\} = E(X^2) - \mu^2,$$

where

$$E(X^2) = \sum_x x^2 p_x.$$

The *sample variance*:

$$s^2 = \frac{1}{n-1} \left\{ \sum_k x_k^2 - \frac{1}{n} \left(\sum_j x_j \right)^2 \right\}$$

estimates σ^2 .

The *standard deviation* is:

$$\text{sd}(X) = \sigma.$$

If the value y is observed with frequency n_y , then

$$n = \sum_y n_y, \quad \sum_k x_k = \sum_y y n_y, \quad \sum_k x_k^2 = \sum_y y^2 n_y.$$

For a function $g(x)$ of x ,

$$E \{g(x)\} = \sum_x g(x)p_x.$$

B.6.2 Probability distributions for a continuous random variable

The *cumulative distribution function* (cdf) is

$$F(x) = P(X \leq x) = \int_{x_0=-\infty}^x f(x_0) dx_0$$

The *probability density function* (pdf) is

$$f(x) = \frac{dF(x)}{dx}$$

$$\mu = \int_{-\infty}^{\infty} x f(x) dx, \quad \sigma^2 = E(X^2) - \mu^2$$

where

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

B.6.3 Discrete probability distributions

Binomial distribution *Binomial* (n, θ)

$$p_x = \binom{n}{x} \theta^x (1-\theta)^{n-x} \quad (x = 0, 1, 2, \dots, n)$$

$$\mu = n\theta, \sigma^2 = n\theta(1-\theta).$$

Poisson distribution *Poisson* (λ)

$$p_x = \frac{\lambda^x e^{-\lambda}}{x!} \quad (x = 0, 1, 2, \dots) \quad (\text{with } \lambda > 0)$$

$$\mu = \lambda, \sigma^2 = \lambda.$$

B.6.4 Continuous probability distributions

Uniform distribution *Uniform* (α, β)

$$f(x) = \begin{cases} \frac{1}{\beta - \alpha} & (\alpha < x < \beta) \\ 0 & (\text{otherwise}). \end{cases} \quad \mu = \frac{\alpha + \beta}{2}, \sigma^2 = \frac{(\beta - \alpha)^2}{12}$$

Exponential distribution *Exponential* (λ)

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & (0 < x < \infty), \\ 0 & (-\infty < x \leq 0). \end{cases} \quad \mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}.$$

Normal distribution *N*(μ, σ^2)

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} \quad (-\infty < x < \infty), \quad E(X) = \mu, \quad \text{var}(X) = \sigma^2$$

Standard normal distribution is *N*(0, 1)

If X is *N*(μ, σ^2), then $Y = \frac{X-\mu}{\sigma}$ is *N*(0, 1).

B.6.5 System reliability

For a system of k devices, which operate independently, let

$$R_i = P(D_i) = P(\text{"device } i \text{ operates"}).$$

The *system reliability*, R , is the probability of a path of operating devices.

A system of devices *in series* fails if any device fails:

$$R = P(D_1 \cap D_2 \cap \dots \cap D_k) = R_1 R_2 \dots R_k$$

A system of devices *in parallel* operates if any device operates:

$$R = P(D_1 \cup D_2 \cup \dots \cup D_k) = 1 - (1 - R_1)(1 - R_2) \dots (1 - R_k)$$

B.7 Bias, standard error and mean square error

If t estimates θ and comes from a distribution having random variable T :

- *Bias* of t : $\text{bias}(t) = E(T) - \theta$
- *Standard error* of t : $\text{se}(t) = \text{sd}(t)$
- *Mean square error* of t : $\text{MSE}(t) = E\{(t - \theta)^2\} = \{\text{se}(t)\}^2 + \{\text{bias}(t)\}^2$

if \bar{x} estimates μ , then $\text{bias}(\bar{x}) = 0$, $\text{se}(\bar{x}) = \frac{\sigma}{\sqrt{n}}$, $\text{MSE}(\bar{x}) = \frac{\sigma^2}{n}$, $\widehat{\text{se}}(\bar{x}) = \frac{s}{\sqrt{n}}$

B.7.1 Central limit property

If n is fairly large, \bar{x} is approximately from $N\left(\mu, \frac{\sigma^2}{n}\right)$.

B.7.2 Confidence intervals

If x_1, x_2, \dots, x_n are independent observations from $N(\mu, \sigma^2)$ and σ^2 is known, then the 95% confidence interval for μ is $\left(\bar{x} - 1.96\frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}\right)$.

If σ^2 is estimated then, from the table of $t_{(n-1)}$, we find $t_0 = t_{(n-1), 0.05}$. Then the 95% CI for μ is $\left(\bar{x} - t_0\frac{s}{\sqrt{n}}, \bar{x} + t_0\frac{s}{\sqrt{n}}\right)$.

y	$\phi(y)$	$\Phi(y)$	y	$\phi(y)$	$\Phi(y)$	y	$\phi(y)$	$\Phi(y)$	y	$\phi(y)$
0	0.399	0.5	0.9	0.266	0.816	1.8	0.079	0.964	2.8	0.997
0.1	0.397	0.540	1.0	0.242	0.841	1.9	0.066	0.971	3.0	0.998
0.2	0.391	0.579	1.1	0.218	0.864	2.0	0.054	0.977	0.841	0.8
0.3	0.381	0.618	1.2	0.194	0.885	2.1	0.044	0.982	1.282	0.9
0.4	0.368	0.655	1.3	0.171	0.903	2.2	0.035	0.986	1.645	0.95
0.5	0.352	0.691	1.4	0.150	0.919	2.3	0.028	0.989	1.96	0.975
0.6	0.333	0.726	1.5	0.130	0.933	2.4	0.022	0.992	2.326	0.99
0.7	0.312	0.758	1.6	0.111	0.945	2.5	0.018	0.994	2.576	0.995
0.8	0.290	0.788	1.7	0.094	0.955	2.6	0.014	0.995	3.09	0.999

Table B.3: Standard normal table: values of pdf $\phi(y) = f(y)$ and cdf $\Phi(y) = F(y)$.

p	0.10	0.05	0.02	0.01	p	0.10	0.05	0.02	0.01		
m	1	6.31	12.71	31.82	63.66	m	9	1.83	2.26	2.82	3.25
	2	2.92	4.30	6.96	9.92		10	1.81	2.23	2.76	3.17
	3	2.35	3.18	4.54	5.84		12	1.78	2.18	2.68	3.05
	4	2.13	2.78	3.75	4.60		15	1.75	2.13	2.60	2.95
	5	2.02	2.57	3.36	4.03		20	1.72	2.09	2.53	2.85
	6	1.94	2.45	3.14	3.71		25	1.71	2.06	2.48	2.78
	7	1.89	2.36	3.00	3.50		40	1.68	2.02	2.42	2.70
	8	1.86	2.31	2.90	3.36		∞	1.645	1.96	2.326	2.576

Table B.4: Student t table: values $t_{m,p}$ of x for which $P(|X| > x) = p$, when X is t_m .

C Mechatronics and control

Data and formulae for core course examinations in:

- Mechatronics
- Dynamics
- Machine System Dynamics

and in other, related, optional courses.

C.1 Charge, current, voltage and power

q = charge

i = current = $\frac{dq}{dt}$

v = electrical potential (*voltage*)

P = power leaving network

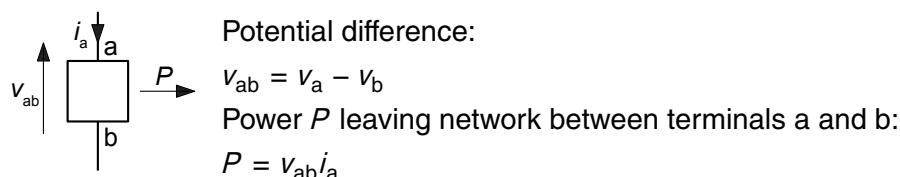
U = energy stored

R = resistance

C = capacitance

L = inductance

Subscript and arrow notations



Passive components

Resistor: $v = iR$ (Ohm's law) Power dissipated: $P = i^2R = \frac{v^2}{R}$

Inductor: $v = L \frac{di}{dt}$ Energy stored $U = \frac{1}{2}Li^2$

Capacitor: $i = C \frac{dv}{dt}$, $q = Cv$ Energy stored $U = \frac{1}{2}Cv^2$

Table C.1: Colour codes for resistors etc.

Colour	Digit	Colour	Digit
Black	0	Green	5
Brown	1	Blue	6
Red	2	Violet	7
Orange	3	Grey	8
Yellow	4	White	9

Table C.2: Standard values for components

E3 series	E6 series	E12 series	E24 series
10	10	10	10
			11
		12	12
			13
	15	15	15
			16
		18	18
			20
22	22	22	22
			24
		27	27
			30
	33	33	33
			36
		39	39
			43
47	47	47	47
			51
		56	56
			63
	68	68	68
			75
		82	82
			91

C.2 Networks

Kirchhoff's voltage law (KVL):

$$\sum (\text{p.d.s around loop}) = 0$$

Kirchhoff's current law (KCL):

$$\sum (\text{currents into node}) = 0$$

Resistors in series:

$$R_{\text{ser}} = R_1 + R_2 + \dots$$

Resistors in parallel:

$$\frac{1}{R_{\text{par}}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots$$

i.e. for two resistors

$$R_{\text{par}} = \frac{R_1 R_2}{R_1 + R_2}$$

Potential divider (with R_2 as output resistor):

$$V_{\text{out}} = \frac{R_2}{R_1 + R_2} V_{\text{in}}$$

C.3 Transients

- $x(t)$ = instantaneous voltage v or current i
- X_0 = initial value $x(0)$
- X_f = final (steady state) value $x(\infty)$.
- τ = time constant.

At time $t = 0$ a switch operates so that the network of resistors and d.c. voltage sources connected to a capacitor or inductor changes, instantaneously. Then for $t \geq 0$:

$$x(t) = X_f - (X_f - X_0) \exp\left(-\frac{t}{\tau}\right)$$

For a capacitor:

- v remains unchanged through $t = 0$;
- $i \rightarrow 0$ as $t \rightarrow \infty$;
- $\tau = R_s C$

For an inductor:

- i remains unchanged through $t = 0$;
- $v \rightarrow 0$ as $t \rightarrow \infty$;
- $\tau = \frac{L}{R}$

C.4 AC networks

- X_m = peak amplitude (or semi-amplitude)
- X_{av} = mean value $x(\infty)$.
- X_{pp} = $2X_m$ is peak-to-peak amplitude
- f = frequency (Hz), and $\omega = 2\pi f$ (rad s $^{-1}$)
- T = $\frac{1}{f}$ is period

C.4.1 Average and root mean square values

General definitions for any periodic waveform:

$$X_{\text{av}} = \frac{1}{T} \int_0^T x \, dt$$

$$X_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T x^2 \, dt}$$

For a waveform consisting of N samples of equal duration:

$$X_{\text{rms}} = \sqrt{\frac{1}{N} \sum_{n=1}^N x_n^2 dt}$$

For a sinusoidal waveform $x = X_m \sin(\omega t + \phi)$:

$$X_{\text{rms}} = \frac{1}{\sqrt{2}} X_m$$

and for a sinusoidal positive half-cycle:

$$X_{\text{av}} = \frac{2}{\pi} X_m$$

C.4.2 Phasors and complex impedance

CIVIL: current leads voltage for a capacitor, voltage leads current for an inductor.

Current is common phasor for series circuits, voltage is common phasor for parallel circuits.

Inductive reactance: $X_L = \omega L$;

Capacitative reactance $X_C = \frac{1}{\omega C}$.

Complex impedance: $\bar{V} = \bar{I} \cdot \bar{Z}$ where \bar{V} , \bar{I} , and \bar{Z} are complex quantities, and

$$\bar{Z} = R \pm jX$$

where \bar{X} is impedance.

C.4.3 Balanced 3 phase a.c supply

Relationships between line voltage V_L and current i_L , phase voltage V_P and current i_P for star connected load

$$V_L = \sqrt{3}V_P, \quad I_L = i_P$$

and for delta connected load:

$$V_L = V_P, \quad I_L = \sqrt{3}i_P$$

C.4.4 Electromagnetism

N	=	number of coil turns
H	=	magnetic field strength
l	=	magnetic flux path length
ϕ	=	magnetic flux
B	=	magnetic flux density
μ_r	=	relative permeability
μ_0	=	permittivity of free space
A	=	cross-sectional area of magnetic flux path.
L	=	length of conductor in magnetic field.
U	=	velocity of conductor

$$\text{Magnetomotive force (m.m.f.)} = iN, H = \frac{\text{m.m.f.}}{l}$$

$$B = \frac{\phi}{A} = \mu_r \mu_0 H \text{(tesla)}$$

Reluctance of magnetic path:

$$S = \frac{\text{m.m.f.}}{\phi} = \frac{l}{\mu_r \mu_0 A}$$

Magnetic force of attraction:

$$F = \frac{B^2 A}{2\mu_0}$$

Force acting on a conductor:

$$F = BiL$$

Induced e.m.f.:

$$E = BLU = \frac{d\phi}{dt}$$

C.4.5 DC machines

- T_0 = shaft torque
- K_e = e.m.f. constant
- R_a = armature resistance
- v_a = armature voltage
- i_a = armature current
- ω = angular velocity

Torque-speed relationship for a permanent-magnet or shunt-wound d.c. machine:

$$\omega = \frac{1}{K_e} v_a - \frac{R_a}{K_e^2} T_0$$

and $T_0 = K_e i_a$.

C.4.6 Transformers

- ϕ = peak flux
- f = frequency
- N_1, N_2 = primary and secondary turns

Ideal transformer:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$$

RMS value of induced e.m.f.:

$$E = 4.44 N f \phi$$

C.5 Communications

Information (in bits) communicated by each of N equally probable messages:

$$I = \frac{1}{\log_{10} 2} \log N$$

Information (in bits) communicated by a message of probability P :

$$I = \frac{1}{\log_{10} 2} \log \left(\frac{1}{P} \right)$$

C.6 Step function response and frequency response

$\theta_{\text{in}}, \theta_{\text{out}}$	= input and output variables
τ	= time constant
ω_n	= natural frequency
ζ	= damping factor
H	= gain
ϕ	= phase shift

The *transfer function* for any linear system is generally expressed as a linear function of the Laplace variable s .

C.6.1 First-order systems

Transfer function of first order low pass (lag):

$$\frac{\theta_{\text{out}}}{\theta_{\text{in}}} = \frac{1}{1 + \tau s}$$

Figure C.1 shows the time plot for response to a unit step input:

$$\theta_{\text{in}} = \begin{cases} 0 & (t < 0) \\ 1 & (t > 0) \end{cases}$$

Gain (power ratio) in decibels (dB): $|H| = 20 \log_{10} \left(\frac{V_{\text{out}}}{V_{\text{in}}} \right)$.

Figure C.2 shows the Bode plots for sinusoidal input $\theta_{\text{in}} = \hat{\theta}_{\text{in}} \sin(\omega t - \phi)$ to first-order low-pass and high-pass filters. For *active* filters (see Table C.3):

	Low-pass filter	High-pass filter
Passive	$ H = \frac{1}{\sqrt{1 + (\omega RC)^2}}$ $\phi = -\tan^{-1}(\omega RC)$	$ H = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$ $\phi = 90^\circ - \tan^{-1}(\omega RC)$
Active	$ H = \frac{R_2}{R_1} \frac{1}{\sqrt{1 + (\omega R_2 C)^2}}$ $\phi = 180^\circ - \tan^{-1}(\omega R_2 C)$	$ H = \frac{C_1}{C_2} \frac{\omega RC_2}{\sqrt{1 + (\omega RC_2)^2}}$ $\phi = -90^\circ - \tan^{-1}(\omega RC)$

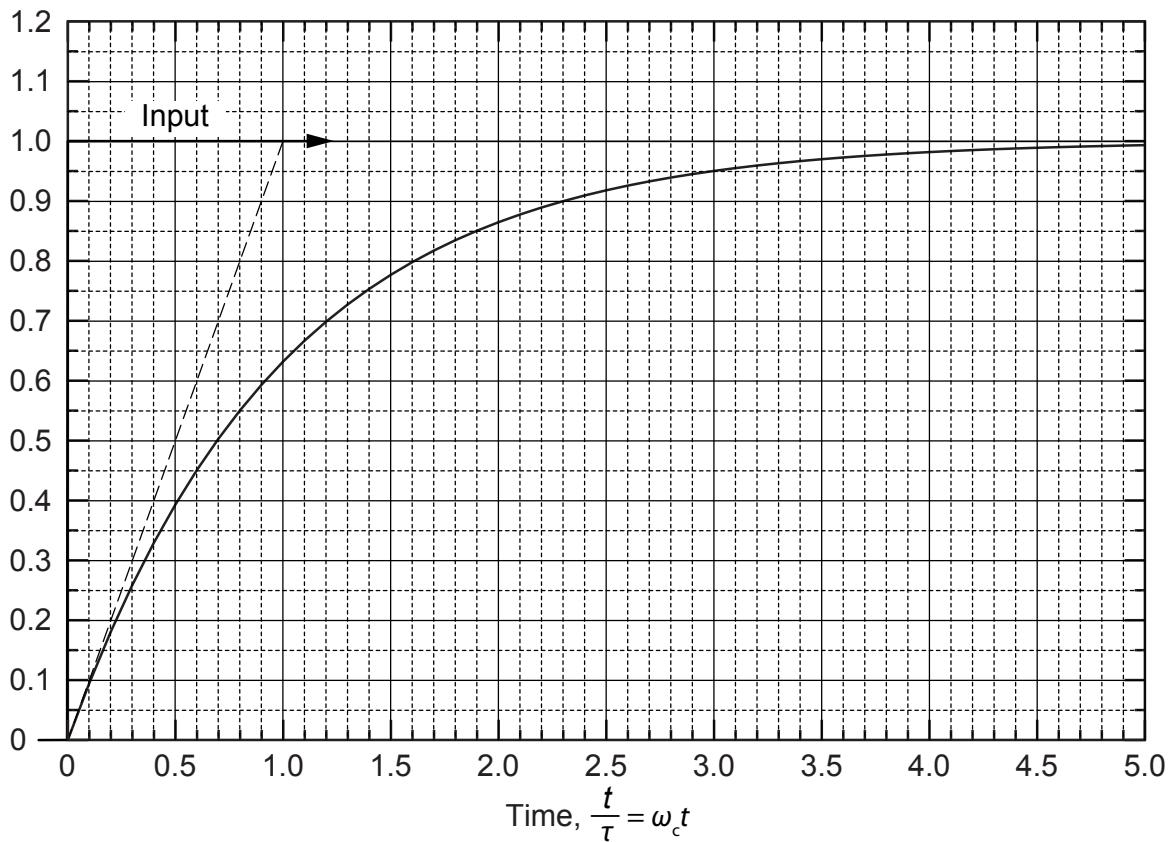


Figure C.1: Step response of a first-order low pass filter

C.6.2 Second-order systems

Transfer function of a second-order low-pass system:

$$\frac{\theta_{out}}{\theta_{in}} = \frac{1}{\left(1 + 2\frac{\zeta}{\omega_n}s + \frac{1}{\omega_n^2}s^2\right)}$$

Unit step and frequency response are shown in Figs. C.3 and C.4.

C.7 Operational amplifier stages

Table C.3 shows op-amp networks which implement various signal processing operations.

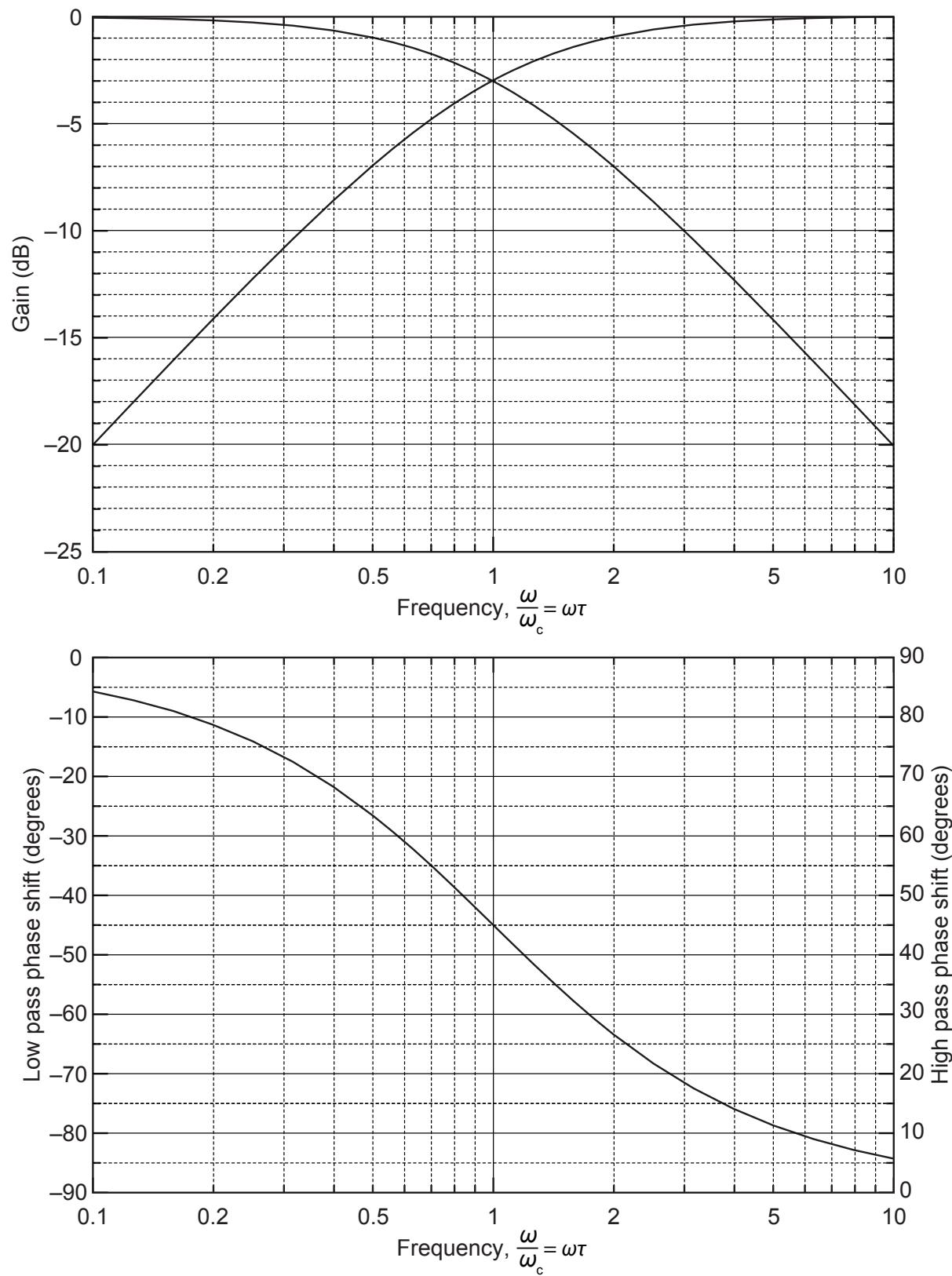


Figure C.2: Bode plot for first-order low and high pass filters

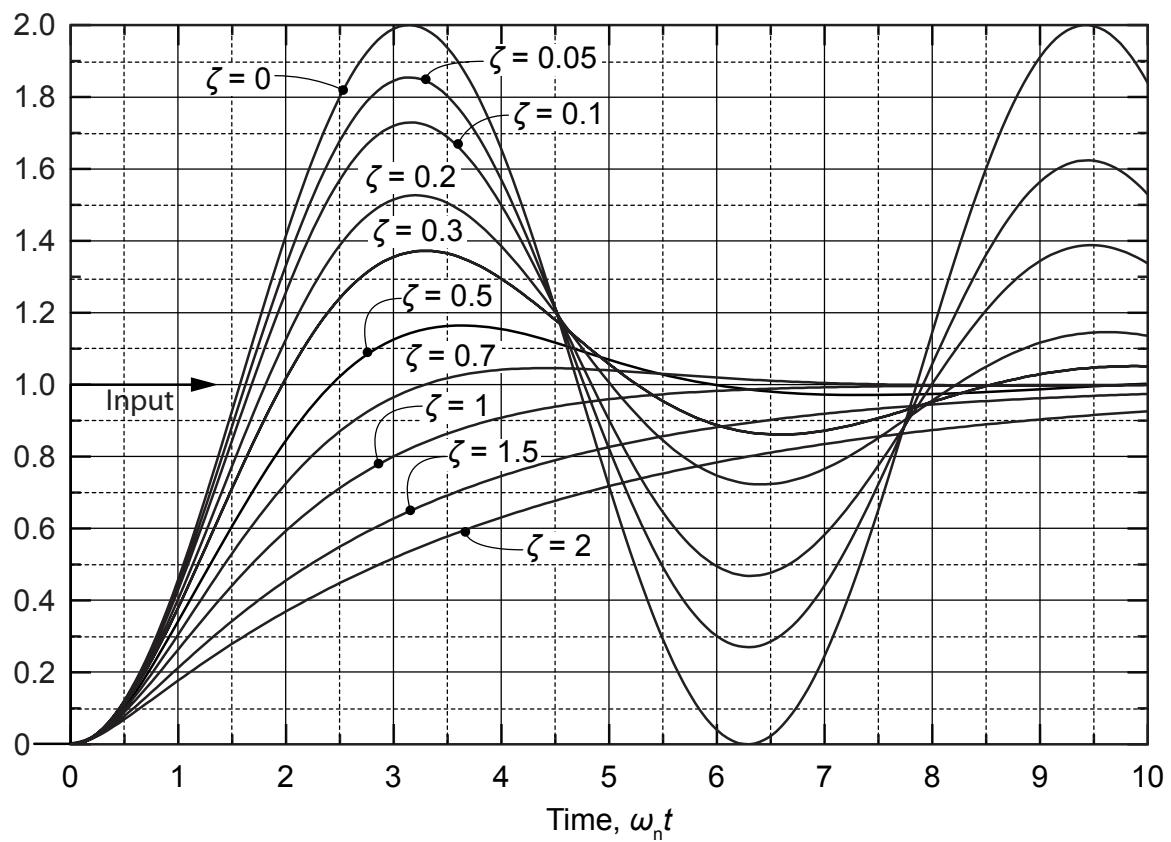


Figure C.3: Step response of a second-order low pass filter

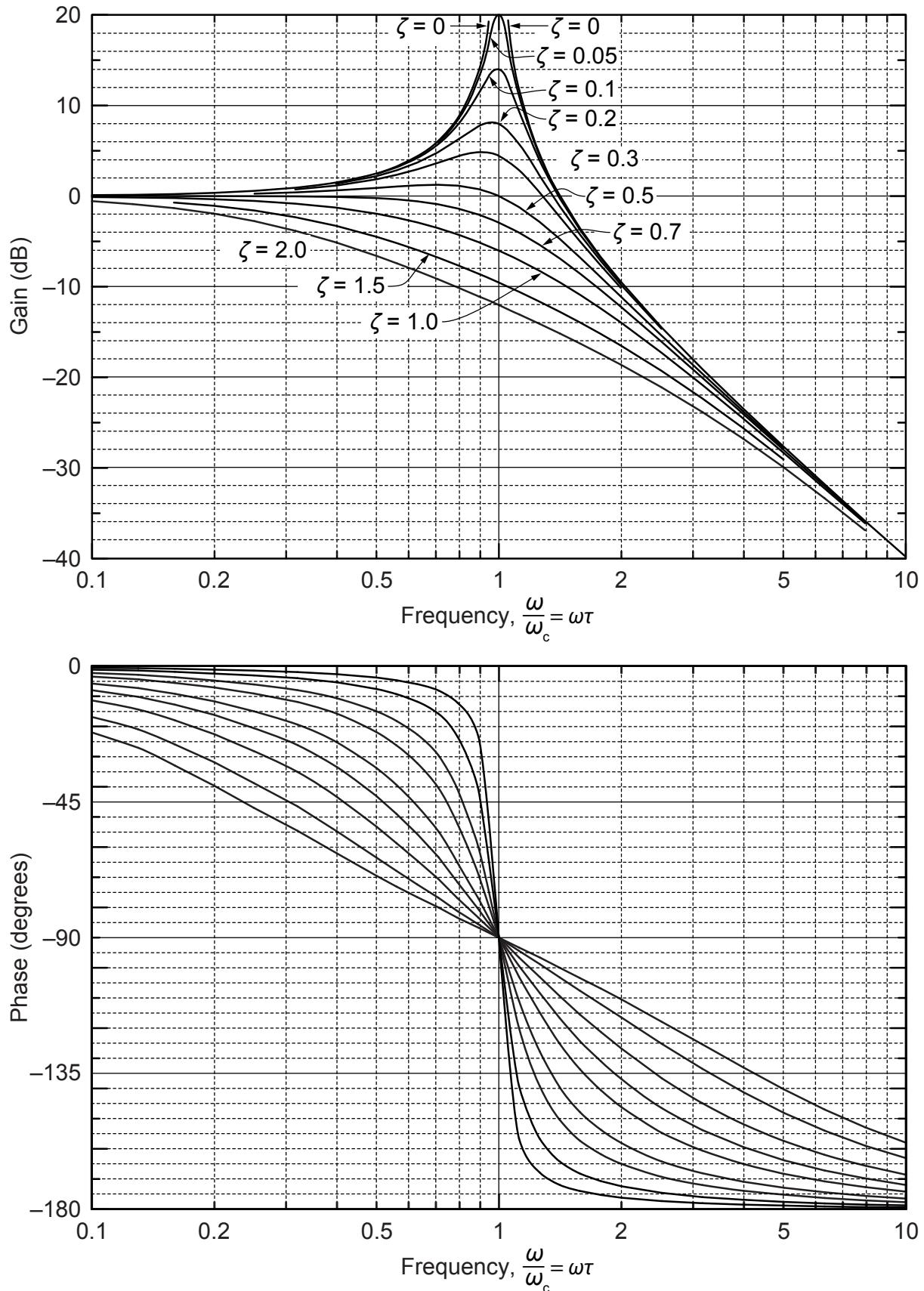
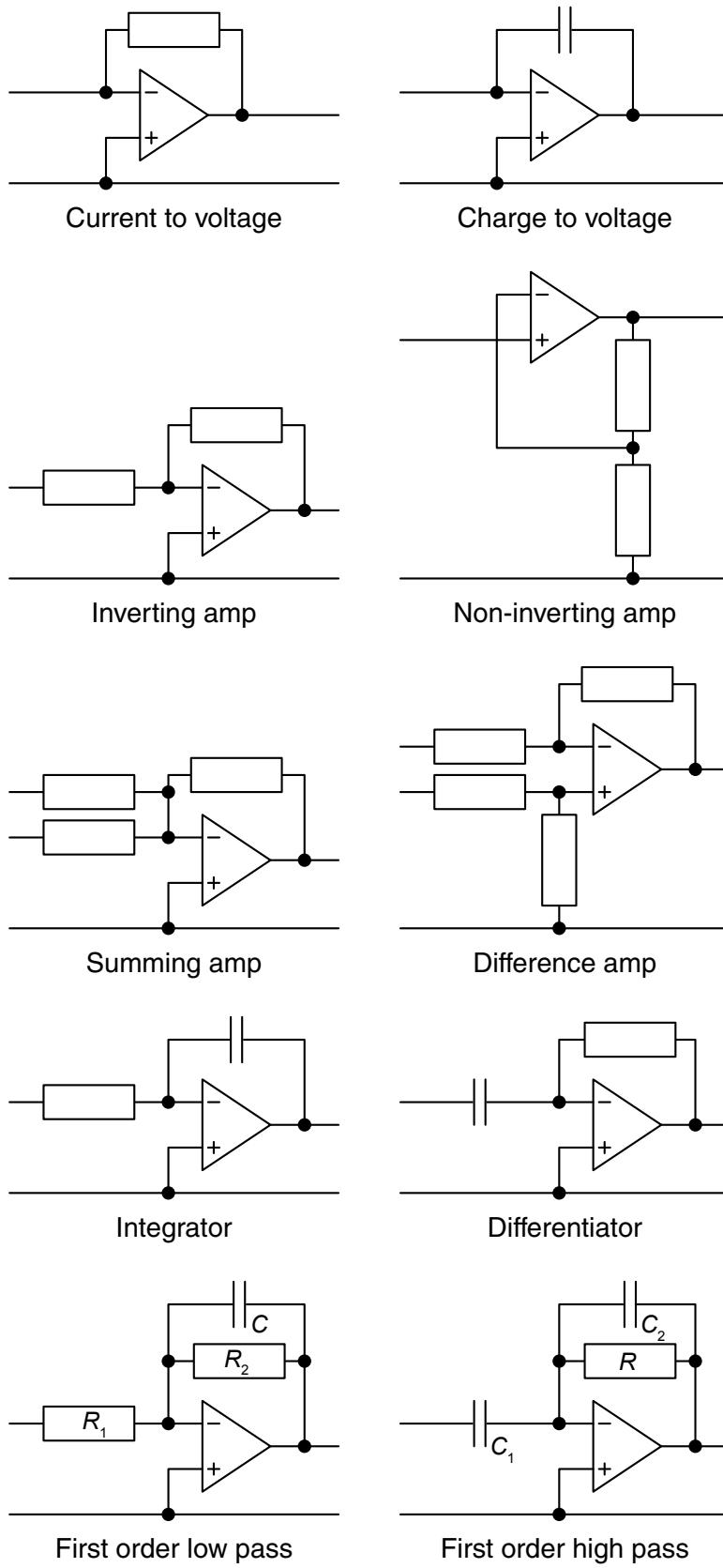


Figure C.4: Bode plot for a second-order low pass filter

Table C.3: Operational amplifier signal processing stages



D Solid Mechanics

Data and formulae for core course examinations in:

- Mechanics
- Stress Analysis
- Materials

and in other, related, optional courses.

D.1 Mechanics

D.1.1 Square screw threads

M = moment required to raise an axially loaded nut

W = axial load on nut

ϕ = helix angle of thread

μ_s = static coefficient of friction

d_m = mean thread diameter

$$M = \left(\frac{\tan \phi + \mu_s}{1 - \mu_s \tan \phi} \right) W \frac{d_m}{2}$$

D.1.2 Flat clutches

T = maximum torque transmitted

F = thrust

R_1, R_2 = outer and inner radii for annular clutch

μ_s = static coefficient of friction

For uniform pressure conditions:

$$T = \frac{2}{3} \mu_s F \left(\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2} \right)$$

For uniform wear conditions:

$$T = \mu_s F \left(\frac{R_1 + R_2}{2} \right)$$

D.1.3 Kinematics of particle

a = acceleration vector

s = distance travelled

v = tangential velocity

t = time

e_n, e_t = unit vectors in $n-t$ coordinates

e_r, e_θ = unit vectors in $r-\theta$ coordinates

ρ = instantaneous radius of path curvature

For normal and tangential components:

$$\mathbf{a} = \frac{d^2 s}{dt^2} \mathbf{e}_t + \frac{v^2}{\rho} \mathbf{e}_n$$

For polar components:

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2) \mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \mathbf{e}_\theta$$

D.1.4 Mass flow problems

- F = internal force vector exerted from the emitted mass
- a = acceleration vector
- m = mass of object
- m_f = emitted mass
- v_f = velocity vector of emitted mass relative to object

$$ma = -\frac{dm_f}{dt} v_f = F$$

D.1.5 Kinematics of rigid bodies with sliding contacts

- v = velocity vector
- a = acceleration vector
- v_{rel} = velocity vector relative to rotating body (sliding velocity)
- a_{rel} = acceleration vector relative to rotating body (sliding acceleration)
- ω = angular velocity vector
- α = angular acceleration vector
- r = position vector

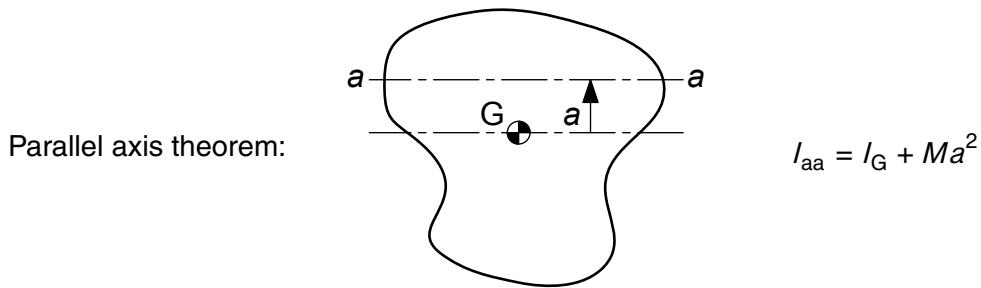
$$v = v_{rel} + \omega \times r$$

$$a = a_{rel} + 2\omega \times v_{rel} + \alpha \times r + \omega \times (\omega \times r)$$

D.1.6 Mass moments of inertia

- M = total mass of body
- G = centre of mass (centre of gravity)
- I_G = Mass moment of inertia about G
- I_{aa} = Mass moment of inertia about an axis $a - a$

Body	Mass moment of inertia
Rectangular lamina, $b \times h$	$I_G = \frac{1}{12}M(b^2 + h^2)$
Circular lamina, radius r	$I_G = \frac{1}{2}Mr^2$
Uniform slender rod, total length L	$I_G = \frac{1}{12}ML^2$
Sphere, radius r	$I_G = \frac{2}{5}Mr^2$



D.2 Stress analysis

D.2.1 Elastic constants of materials

- ρ = Density
- E = Young's modulus, modulus of elasticity
- G = Shear modulus, modulus of rigidity
- K = Bulk modulus
- ν = Poisson's ratio
- α = Coefficient of linear thermal expansion

Relationships between elastic constants:

$$G = \frac{E}{2(1+\nu)} \quad K = \frac{E}{3(1-2\nu)}$$

Some typical values:

	ρ kg m ⁻³	E GPa	G GPa	K GPa	ν	α $\times 10^6$ K ⁻¹
Mild steel	7850	207	79.6	175	0.3	11
Aluminium alloy	2720	68.9	26.5	69	0.3	23
Brass	8410	103	38.3	117	0.35	19
Titanium alloy	5000	110	42		0.31	11
Softwood along grain		9				
Water	1000			2.2		
Concrete	2400	13.8			0.1	

D.2.2 Beam theory

- σ = axial stress at axial position z and vertical distance y from neutral axis
- τ = shear stress in vertical \times axial plane
- d = total depth of beam
- M = bending moment about neutral axis at z
- S = shear force at z
- I = second moment of area about neutral axis
- R = radius of curvature at z
- v = vertical deflection at z

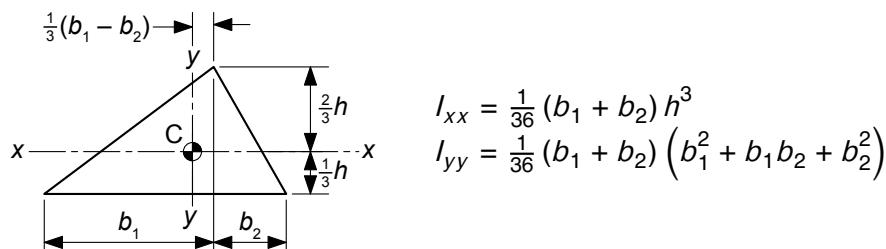
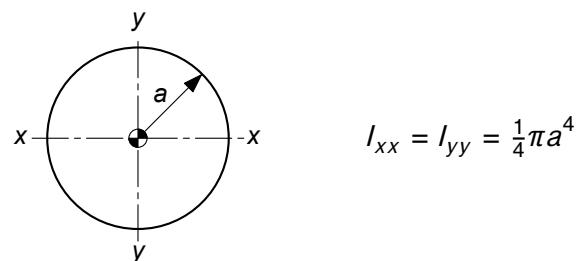
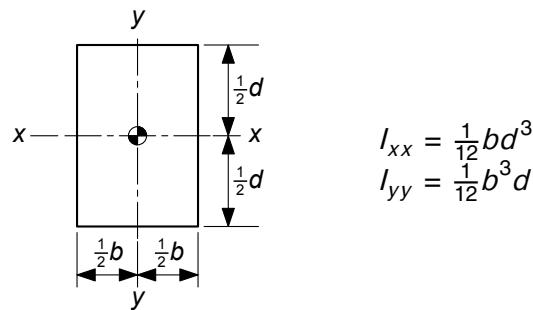
Bending about a principal axis :

$$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R} = E \frac{d^2v}{dz^2}$$

and

$$\tau = \frac{S}{2I} \left(\frac{d^2}{4} - y^2 \right)$$

Table D.1: Second moments of area for simple cross-sections



Parallel axis theorem:

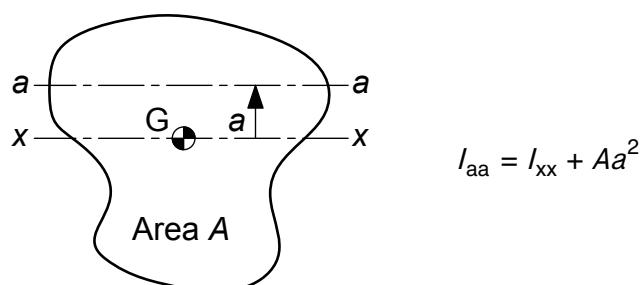
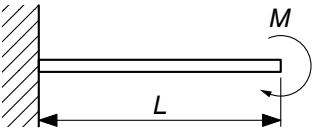
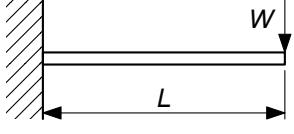
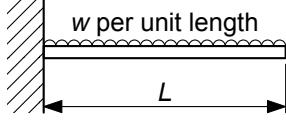
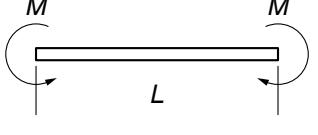
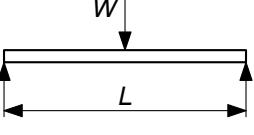
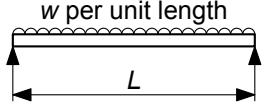
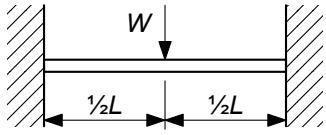
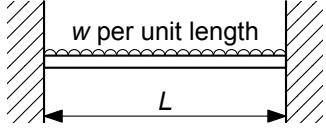


Table D.2: Beams bent about principal axis

	End slope	End deflection	Central deflection
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	
		$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$
		$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$
		$\frac{ML}{2EI}$	$\frac{ML^2}{8EI}$
		$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$
		$\frac{wL^3}{24EI}$	$\frac{5wL^4}{384EI}$
	End moment	Central deflection	
		$\frac{WL}{8}$	$\frac{WL^3}{192EI}$
		$\frac{WL^2}{12}$	$\frac{WL^4}{384EI}$

D.2.3 Elastic torsion

Circular solid and hollow shafts

- τ = shear stress at radius r
 T = applied torque
 J = polar second moment of area
 d = diameter of circular section
 θ = angle of twist over length L

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{L}$$

For a solid circular section:

$$J = 2I_{xx} = \frac{\pi d^4}{32}$$

Table D.3: Torsion of solid non-circular sections

Shape of cross section	Maximum shear stress, τ_{\max}	Angle of twist, θ
Square	$4.81 \frac{T}{a^3}$	$7.10 \frac{TL}{a^4 G}$
Equilateral triangle	$20 \frac{T}{a^3}$	$46 \frac{TL}{a^4 G}$
Ellipse	$\frac{2}{\pi} \frac{T}{ab^2}$	$(a^2 + b^2) \frac{TL}{\pi a^3 b^3 G}$

Thin walled tubes of arbitrary cross-section

A = enclosed area to mid-thickness

t = wall thickness

s = distance around perimeter

$$\tau = \frac{T}{2At}$$

Torsional stiffness:

$$\frac{T}{\theta/I} = \frac{4A^2G}{\oint \left(\frac{1}{t}\right) ds}$$

Springs

d = wire diameter

D = helix diameter

δ = deflection

F = force

End deflection of a closed-helix, round wire spring:

$$\delta = \frac{8FD^3N}{Gd^4}$$

Maximum shear stress (torsion only):

$$\tau = \frac{8FD}{\pi d^3}$$

D.2.4 Thin walled pressure vessels

R = mean radius

t = wall thickness

p = internal pressure

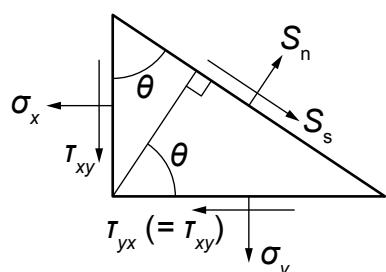
Hoop stress in hollow, pressurised cylinder:

$$\sigma_\theta = p \left(\frac{R}{t} \right)$$

Stress in hollow, pressurised sphere :

$$\sigma = p \left(\frac{R}{2t} \right)$$

D.3 Two-dimensional stress transformation



$$S_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$S_s = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

Principal stresses:

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

The direction of the principal stresses (and of the normal to the principal planes) to the x axis is θ_p where:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Maximum shear stress: The maximum shear stress is half the difference of the principal stresses and acts on planes at 45° to the principal planes.

D.4 Yield criteria

Y = yield stress in uniaxial tension
 t = wall thickness

In a three dimensional stress system having principal stresses σ_1, σ_2 and σ_3 where

$$\sigma_1 \geq \sigma_2 \geq \sigma_3.$$

Tresca yield criterion:

$$|\sigma_1 - \sigma_3| = Y$$

Von Mises yield criterion:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2$$

D.5 Two-dimensional strain transformation

$$e_n = \frac{e_x + e_y}{2} + \frac{e_x - e_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\frac{e_s}{2} = \frac{e_x - e_y}{2} \sin 2\theta - \frac{\gamma_{xy}}{2} \cos 2\theta$$

where:

- e_x, e_y and e_n are the direct strains acting in the same directions as, respectively, the stresses s_x, s_y and S_n above;
- γ_{xy} and e_s are the shear strains associated with the stresses τ_{xy} and S_s

NOTE: the relevant strain relationships may be obtained from the stress relationships by substituting the appropriate direct stresses by the associated direct strain and shear stresses by *one half* of the associated shear strain.

D.6 Elastic stress-strain relationships

$$e_x = \frac{1}{E} (\sigma_x - \nu (\sigma_y + \sigma_z)) \text{ etc}$$

$$\gamma_{xy} = \frac{1}{G} \tau_{xy} \text{ etc}$$

D.7 Thick-walled cylinders

For axi-symmetric systems, the circumferential and radial stresses at radius r are, respectively:

$$\sigma_{\theta\theta} = A + \frac{B}{r^2}$$

$$\sigma_{rr} = A - \frac{B}{r^2}$$

E Thermofluids

Data and formulae for core course examinations in:

- Fluid Mechanics
- Thermodynamics
- Heat Transfer
- Thermodynamics and Energy

and in other, related, optional courses.

E.1 Cross-references to table numbers

Some Tables in this handbook are referred to by different numbers in lecture notes and problem sheets.

External reference	Table number in this handbook
Table E1	Table E.1
Table E2	Table E.2
Table E3	Section E.4
Table E4	Section E.5
Table E5	Table E.3
Table E6	Table E.4
Table E7a	Table E.5
Table E7b	Table E.6
Table E7c	Table E.7
Table E8	Tables E.8 and E.9
Table E9a	Table E.16
Table E9b	Table E.17
Table E10	Table E.18
Table R1	Table E.10
Table R2	Table E.11
Table R3 Part 1	Table E.12
Table R3 Part 2	Table E.13
Table R3 Part 3	Table E.14
Table R3 Part 4	Table E.15
Table S1	Table E.19
Table S2	Tables E.20 to E.22
Table S3	Tables E.23 to E.29

E.2 Dimensionless groups

A	=	surface area
C_p	=	specific heat at constant pressure
D	=	pipe diameter
F_D, F_L	=	drag force, lift force
g	=	gravitational acceleration
h	=	surface heat-transfer coefficient
k_f, k_s	=	thermal conductivity of fluid, of solid
L	=	reference length
ΔP	=	pressure drop
T_∞, T_s	=	temperature at infinity, at surface
U	=	characteristic velocity
α	=	thermal diffusivity
β	=	coefficient of volumetric thermal expansion
ε	=	roughness height
μ	=	absolute viscosity
ρ	=	density

Table E.1: Dimensionless groups for Thermofluids

Parameter	Definition
Biot number (Bi)	$\frac{hL}{k_s}$
Coefficient of lift (C_L)	$\frac{F_L}{\frac{1}{2}\rho U^2 A}$
Coefficient of drag (C_D)	$\frac{F_D}{\frac{1}{2}\rho U^2 A}$
Fourier number (Fo)	$\frac{\alpha t}{L^2}$
Friction factor (f)	$\frac{\Delta P}{\left(\frac{L}{D}\right) \frac{1}{2}\rho U^2}$
Grashof number (Gr)	$\frac{\beta g (T_\infty - T_s) L^3 \rho^2}{\mu^2}$
Nusselt number (Nu)	$\frac{hL}{k_f}$
Prandtl number (Pr)	$\frac{\mu C_p}{k_f}$
Rayleigh number (Ra)	$Gr \cdot Pr$
Reynolds number (Re)	$\frac{UL\rho}{\mu}$
Roughness ratio	$\frac{\varepsilon}{L}$
Stanton number (St)	$\frac{h}{\rho U C_p} = \frac{Nu}{Re \cdot Pr}$

E.3 Heat transfer

Table E.2: Empirical correlations for forced convection

Correlation	Conditions
Laminar flow over a flat plate:	
$C_f(x) = \frac{0.664}{Re_x^{1/2}}$	$Pr \geq 0.6$
$Nu(x) = \frac{h(x)x}{k} = 0.332 Re_x^{1/2} Pr^{1/3}$	$Pr \geq 0.6$
$C_f = \frac{1.328}{Re_L^{1/2}}$	$Pr \geq 0.6$
$Nu = 0.664 Re_L^{1/2} Pr^{1/3}$	$Pr \geq 0.6$
Turbulent flow over a flat plate:	
$C_f(x) = \frac{0.0592}{Re_x^{1/5}}$	$5 \times 10^5 \leq Re_x \leq 10^7$
$Nu(x) = 0.0296 Re_x^{4/5} Pr_x^{1/3}$	$\begin{cases} 0.6 \leq Pr \leq 60 \\ 5 \times 10^5 \leq Re_x \leq 10^7 \end{cases}$
$C_f = \frac{0.074}{Re_L^{1/5}}$	$5 \times 10^5 \leq Re_L \leq 10^7$
$Nu = 0.037 Re^{4/5} Pr^{1/3}$	$\begin{cases} 0.6 \leq Pr \leq 60 \\ 5 \times 10^5 \leq Re_x \leq 10^7 \end{cases}$
Mixed flow over a flat plate:	
$C_f(x) = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L}$	$5 \times 10^5 \leq Re_L \leq 10^7$
$Nu = (0.037 Re_L^{4/5} - 871) Pr^{1/3}$	$\begin{cases} 0.6 \leq Pr \leq 60 \\ 5 \times 10^5 \leq Re_x \leq 10^7 \end{cases}$
Flat plate with uniform heat flux:	
$Nu(x) = 0.453 Re_x^{1/2} Pr^{1/3}$	Laminar flow
$Nu(x) = 0.0308 Re_x^{0.8} Pr^{1/3}$	Turbulent flow
Fully developed laminar flow in a pipe:	
$Nu = 3.66$	Constant surface temperature
$Nu = 4.36$	Constant heat flux
Fully developed turbulent flow in a pipe:	
$Nu_D = 0.023 Re_D^{4/5} Pr^n$ ($n = 0.4$ for heating, $n = 0.3$ for cooling)	$\begin{cases} 0.7 \leq Pr \leq 160 \\ Re > 10000 \end{cases}$

E.4 Continuity and equation of motion

E.4.1 Cylindrical polar coordinates

Equation of continuity for unsteady flow, variable density:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(r\rho v_r)}{\partial r} + \frac{1}{r} \frac{\partial(\rho v_\theta)}{\partial \theta} + \frac{\partial(\rho v_x)}{\partial x} = 0$$

Equations of Motion for unsteady flow, variable density: Cauchy form

$$\begin{aligned} \frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_r \frac{\partial v_x}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_x}{\partial \theta} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + F_{x,int} + F_{x,ext} \\ \frac{\partial v_r}{\partial t} + v_x \frac{\partial v_r}{\partial x} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} &= -\frac{1}{\rho} \frac{\partial p}{\partial r} + F_{r,int} + F_{r,ext} \\ \frac{\partial v_\theta}{\partial t} + v_x \frac{\partial v_\theta}{\partial x} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} &= -\frac{1}{\rho r} \frac{\partial p}{\partial \theta} + F_{\theta,int} + F_{\theta,ext} \end{aligned}$$

where $F_i, \{ \begin{smallmatrix} \text{int} \\ \text{ext} \end{smallmatrix} \}$ are the internal (viscous) or external (body) forces per unit mass, as appropriate, acting in the direction of coordinate i . For example,

$$F_{x,int} = \nu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_x}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 v_x}{\partial \theta^2} + \frac{\partial^2 v_x}{\partial x^2} \right]$$

E.4.2 Rectangular Cartesian coordinates

Equation of continuity for unsteady flow, variable density

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Equations of motion for unsteady flow, variable density: Cauchy form

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + F_{x,int} + F_{x,ext} \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + F_{y,int} + F_{y,ext} \\ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + F_{z,int} + F_{z,ext} \end{aligned}$$

Equations of motion for unsteady uniform property flow in two dimensions (the xy plane) only:

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \underbrace{\frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)}_{F_{x,int}} + F_{x,ext} \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \underbrace{\frac{\mu}{\rho} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)}_{F_{y,int}} + F_{y,ext} \end{aligned}$$

The boundary layer (“approximately Couette flow”) form of the equations of motion for strain confined to the xy plane with uniform properties:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dp}{dx} + \frac{\mu}{\rho} \left(\frac{\partial^2 u}{\partial y^2} \right)$$

E.4.3 Vector form

Equation of continuity for unsteady flow, variable density, in vector form:

$$\frac{\partial p}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

where ∇ is the vector-gradient operator expressing the divergence of a vector, in this case \mathbf{V} , the velocity field.

Equations of Motion in vector form for unsteady flow: Cauchy form

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \frac{\partial \mathbf{V}}{\partial x} = -\frac{1}{\rho} \nabla p + \mathbf{F}_{\text{int}} + \mathbf{F}_{\text{ext}}$$

E.5 Equations for compressible flows

ISENTROPIC COMPRESSIBLE FLOW RELATIONS

$$\rho_0 = \rho_1 \left(1 + \frac{\gamma - 1}{2} Ma_1^2 \right)^{\frac{1}{\gamma-1}}$$

$$p_0 = p_1 \left(1 + \frac{\gamma - 1}{2} Ma_1^2 \right)^{\frac{\gamma}{\gamma-1}}$$

$$T_0 = T_1 \left(1 + \frac{\gamma - 1}{2} Ma_1^2 \right)$$

$$a_0 = a_1 \left(1 + \frac{\gamma - 1}{2} Ma_1^2 \right)^{1/2}$$

Prandtl-Meyer function

$$v(Ma) = \left(\frac{\gamma + 1}{\gamma - 1} \right)^{1/2} \tan^{-1} \left\{ \left(\frac{(\gamma + 1)(Ma^2 - 1)}{\gamma - 1} \right)^{1/2} \right\} - \tan^{-1} \left\{ (Ma^2 - 1)^{1/2} \right\}$$

Normal Shock Relations

$$Ma_2^2 = \frac{1 + \frac{1}{2}(\gamma - 1)Ma_1^2}{\gamma Ma_1^2 - \frac{1}{2}(\gamma - 1)}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)Ma_1^2}{(\gamma - 1)Ma_1^2 + 2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{(\gamma + 1)} (\text{Ma}_1^2 - 1)$$

$$\frac{T_2}{T_1} = \frac{[2\gamma \text{Ma}_1^2 - (\gamma - 1)]}{(\gamma + 1)^2 \text{Ma}_1^2} [(\gamma - 1) \text{Ma}_1^2 + 2]$$

E.6 Friction factor for flow in circular pipes (Moody diagram)

- d = pipe diameter
 f = Darcy friction factor = $\frac{4\tau_w}{\frac{1}{2}\rho\bar{V}^2} = \frac{1}{(L/d)} \frac{\Delta P}{\frac{1}{2}\rho\bar{V}^2}$
 L = pipe length
 Re = Reynolds Number = $\frac{\rho\bar{V}d}{\mu}$
 \bar{V} = fluid bulk mean velocity
 ΔP = frictional pressure drop in length L
 ε = roughness height
 μ = absolute or dynamic viscosity
 ρ = fluid density
 τ_w = shear stress at pipe wall

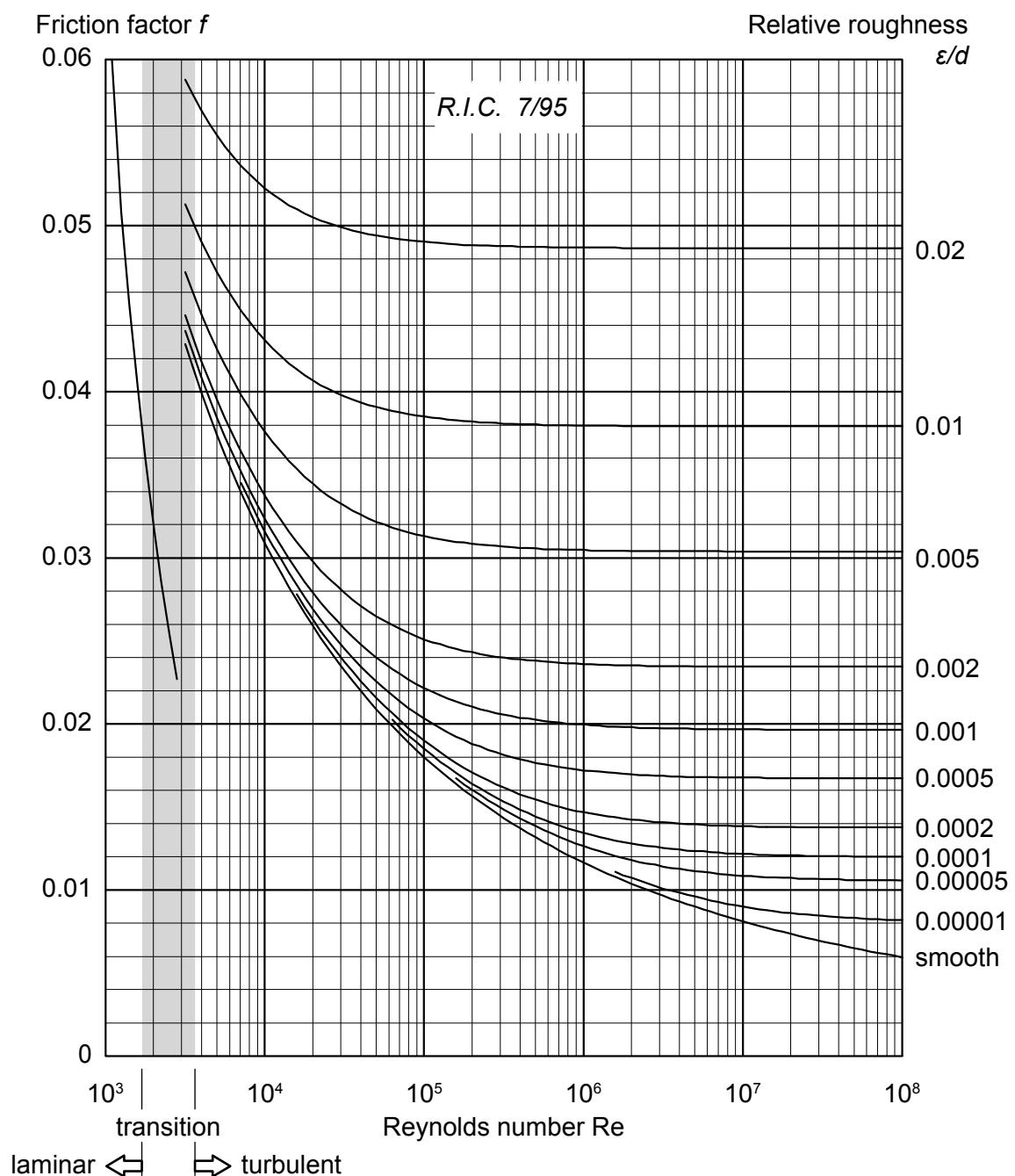


Figure E.1: Moody Diagram

E.7 Perfect gases

Ma = Mach number

P, P_0 = absolute pressure, stagnation pressure

T, T_0 = absolute temperature, stagnation temperature

v = specific volume

ρ, ρ_0 = density, stagnation density

Over a limited range of temperatures and pressures close to ambient values, the following substances can be assumed to behave as *perfect* gases with properties given by the following relationships:

equation of state

$$Pv = RT$$

specific heat at constant volume $C_v = \frac{du}{dT} = \text{constant for the particular gas}$

specific heat at constant pressure $C_p = \frac{dh}{dT} = \text{constant for the particular gas}$

ratio of principal specific heats $\gamma = \frac{C_p}{C_v} = \text{constant for the particular gas}$

gas constant $R = C_p - C_v = \text{constant for the particular gas}$

and $R = \frac{\bar{R}}{M}$ where \bar{R} = universal gas constant = $8.314 \text{ kJ kmol}^{-1} \text{ K}^{-1}$.

Table E.3: Perfect gases (ideal gases with constant specific heats)

Gas	Chemical formula	Molar mass M kg kmol^{-1}	Gas constant R $\text{kJ kg}^{-1} \text{ K}^{-1}$	\bar{C}_p	\bar{C}_v	γ
air ^a	—	28.96	0.287	1.01	0.72	1.40
oxygen	O ₂	32.00	0.260	0.92	0.66	1.40
nitrogen	N ₂	28.01	0.297	1.04	0.74	1.40
atmospheric nitrogen ^b	(AN)	28.17	0.295	1.03	0.74	1.40
carbon dioxide	CO ₂	44.01	0.189	0.84	0.65	1.29
carbon monoxide	CO	28.01	0.297	1.04	0.74	1.40
hydrogen	H ₂	2.016	4.12	14.31	10.18	1.41
methane	CH ₄	16.04	0.518	2.23	1.71	1.30
ethane	C ₂ H ₆	30.07	0.277	1.75	1.47	1.19
helium	He	4.00	2.08	5.20	3.12	1.67

^aComposition of dry air: 21.0% oxygen, 79.0% atmospheric nitrogen by no. of kmol or by volume; 23.2% oxygen, 76.8% atmospheric nitrogen by mass.

^bAtmospheric nitrogen contains approx. 1% (by no. of kmol or volume) argon and traces of carbon dioxide and other gases, in addition to nitrogen

Table E.4: Isentropic compressible flow functions for perfect gas with $\gamma = 1.40$

Ma	$\frac{P}{P_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$	Ma	$\frac{P}{P_0}$	$\frac{\rho}{\rho_0}$	$\frac{T}{T_0}$
0	1	1	1	1	0.5283	0.6339	0.8333
0.05	0.9983	0.9988	0.9995	1.02	0.5160	0.6234	0.8278
0.10	0.9930	0.9950	0.9980	1.04	0.5039	0.6129	0.8222
0.15	0.9844	0.9888	0.9955	1.06	0.4919	0.6024	0.8165
				1.08	0.4800	0.5920	0.8108
0.20	0.9725	0.9803	0.9921				
0.22	0.9668	0.9762	0.9904	1.10	0.4684	0.5817	0.8052
0.24	0.9607	0.9718	0.9886	1.15	0.4398	0.5562	0.7908
0.26	0.9541	0.9670	0.9867	1.20	0.4124	0.5311	0.7764
0.28	0.9470	0.9619	0.9846	1.25	0.3861	0.5067	0.7619
				1.30	0.3609	0.4829	0.7474
0.30	0.9395	0.9564	0.9823	1.35	0.3370	0.4598	0.7329
0.32	0.9315	0.9506	0.9799				
0.34	0.9231	0.9445	0.9774	1.40	0.3142	0.4374	0.7184
0.36	0.9143	0.9380	0.9747	1.45	0.2927	0.4158	0.7040
0.38	0.9052	0.9313	0.9719	1.50	0.2724	0.3950	0.6897
				1.55	0.2533	0.3750	0.6754
0.40	0.8956	0.9243	0.9690	1.60	0.2353	0.3557	0.6614
0.42	0.8857	0.9170	0.9659	1.65	0.2184	0.3373	0.6475
0.44	0.8755	0.9094	0.9627				
0.46	0.8650	0.9016	0.9594	1.70	0.2026	0.3197	0.6337
0.48	0.8541	0.8935	0.9559	1.75	0.1878	0.3029	0.6202
				1.80	0.1740	0.2868	0.6068
0.50	0.8430	0.8852	0.9524	1.85	0.1612	0.2715	0.5936
0.52	0.8317	0.8766	0.9487	1.90	0.1492	0.2570	0.5807
0.54	0.8201	0.8679	0.9449	1.95	0.1381	0.2432	0.5680
0.56	0.8082	0.8589	0.9410				
0.58	0.7962	0.8498	0.9370	2.00	0.1278	0.2300	0.5556
				2.10	0.1094	0.2058	0.5313
0.60	0.7840	0.8405	0.9328	2.20	0.09352	0.1841	0.5081
0.62	0.7716	0.8310	0.9286	2.30	0.07997	0.1646	0.4859
0.64	0.7591	0.8213	0.9243	2.40	0.06840	0.1472	0.4647
0.66	0.7465	0.8115	0.9199				
0.68	0.7338	0.8016	0.9153	2.50	0.05853	0.1317	0.4444
				2.60	0.05012	0.1179	0.4252
0.70	0.7209	0.7916	0.9107	2.70	0.04295	0.1056	0.4068
0.72	0.7080	0.7814	0.9061	2.80	0.03685	0.09463	0.3894
0.74	0.6951	0.7712	0.9013	2.90	0.03165	0.08489	0.3729
0.76	0.6821	0.7609	0.8964				
0.78	0.6691	0.7505	0.8915	3.00	0.02722	0.07623	0.3571
				3.20	0.02023	0.06165	0.3281
0.80	0.6560	0.7400	0.8865	3.40	0.01512	0.05009	0.3019
0.82	0.6430	0.7295	0.8815	3.60	0.01138	0.04089	0.2784
0.84	0.6300	0.7189	0.8763	3.80	0.008629	0.03355	0.2572
0.86	0.6170	0.7083	0.8711				
0.88	0.6041	0.6977	0.8659	4.00	0.006586	0.02766	0.2381
				4.50	0.003455	0.01745	0.1980
0.90	0.5913	0.6870	0.8606	5.00	0.001890	0.01134	0.1667
0.92	0.5785	0.6764	0.8552				
0.94	0.5658	0.6658	0.8498	6.00	0.000633	0.005194	0.1220
0.96	0.5532	0.6551	0.8444	8.00	0.000102	0.001414	0.07246
0.98	0.5407	0.6445	0.8389	10.00	0.000024	0.000495	0.04762
1	0.5283	0.6339	0.8333	∞	0	0	0

Table E.5: Ideal (semi-perfect) gas specific enthalpy h (kJ kg⁻¹, 25 °C datum)
 for combustion calculations on a mass basis

T (°C)	carbon dioxide CO ₂	water vapour H ₂ O	nitrogen N ₂	atmos. nitrogen (AN)	oxygen O ₂	air —	carbon monoxide CO	hydrogen H ₂
0	-22.5	-45.7	-25.6	-25.4	-23.1	-24.8	-25.7	-356.8
25	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
100	69.2	139.5	77.5	76.8	70.1	75.2	77.9	1074.5
200	165.3	331.0	182.4	180.9	165.5	177.2	183.5	2517.0
300	266.0	528.9	289.3	286.8	263.1	281.2	291.2	3970.5
400	371.1	733.1	398.0	394.6	362.9	387.1	400.9	5435.3
500	480.7	943.6	508.6	504.3	465.0	494.9	512.7	6911.1
600	594.8	1160.5	621.1	615.8	569.2	604.7	626.5	8398.1
700	713.3	1383.8	735.4	729.2	675.6	716.4	742.5	9896.3
800	836.4	1613.3	851.6	844.4	784.2	830.0	860.4	11405.5
900	963.9	1849.2	969.8	961.5	895.0	945.6	980.5	12925.9
1000	1095.9	2091.5	1089.8	1080.4	1008.0	1063.2	1102.6	14457.5
1100	1232.3	2340.1	1211.6	1201.2	1123.2	1182.6	1226.7	16000.2
1200	1373.3	2595.0	1335.4	1323.9	1240.6	1304.0	1352.9	17554.0
1300	1518.7	2856.3	1461.0	1448.4	1360.2	1427.4	1481.2	19119.0
1400	1668.7	3123.9	1588.5	1574.8	1482.0	1552.7	1611.6	20695.1
1500	1823.1	3397.8	1717.9	1703.1	1606.0	1679.9	1744.0	22282.4

Enthalpy values in Table E.5 have been computed using the approximation

$$C_p = a + bT$$

so that

$$h - h_D = \int C_p dT = (T - T_D) \left[a + \frac{1}{2} b (T + T_D) \right]$$

where datum temperature $T_D = 298.15$ K and datum enthalpy h_D (at $T = T_D$) = 0 (this is equivalent to using a mean C_p between 298.15 K (25 °C) and T). The values of the constants are tabulated below; T must be in K, giving h in kJ kg⁻¹. The magnitudes of the maximum and mean errors in h refer to the range 0 to 1500 °C.

	carbon dioxide CO ₂	water vapour H ₂ O	nitrogen N ₂	atmos. nitrogen (AN)	oxygen O ₂	air —	carbon monoxide CO	hydrogen H ₂
a	0.772	1.647	0.970	0.962	0.861	0.938	0.969	13.95
$b \times 10^6$ (kJ kg ⁻¹ K ⁻²)	448	634	188	186	220	194	206	1114
max. error	4.0	0.7	0.5	0.6	1.3	0.4	0.5	0.7
mean. error (%)	1.4	0.2	0.2	0.2	0.5	0.2	0.2	0.3

Table E.6: Molar Enthalpy of Formation h_f^0 (kJ kmol $^{-1}$ at 25 °C and 1 atmosphere) as gas or vapour (g), except where indicated as solid (s) or liquid (l).

carbon as graphite	C(s)	0
hydrogen	H ₂	0
methane	CH ₄	-74 850
ethane	C ₂ H ₆	-84 680
propane	C ₃ H ₈	-103 850
n-octane	C ₈ H ₁₈ (l)	-249 950
ethanol	C ₂ H ₅ OH(l)	-277 690
hydrogen peroxide	H ₂ O ₂	-136 310
carbon dioxide	CO ₂	-393 520
water vapour	H ₂ O(g)	-241 820
liquid water	H ₂ O(l)	-285 830
nitrogen	N ₂	0
atmospheric nitrogen	(AN)	0
oxygen	O ₂	0
air	—	0
carbon monoxide	CO	-110 530

Table E.7: Ideal gas molar enthalpy h (kJ kmol $^{-1}$, 25 °C datum)

T (°C)	carbon dioxide	water vapour	nitrogen	atmos. nitrogen (AN)	oxygen	air ^a	carbon monoxide	hydrogen
	CO ₂	H ₂ O	N ₂		O ₂	—	CO	H ₂
0	-990	-823	-717	-715	-739	-719	-720	-719
25	0	0	0	0	0	0	0	0
100	3 045	2 513	2 170	2 164	2 244	2 179	2 181	2 166
200	7 276	5 964	5 110	5 096	5 297	5 133	5 139	5 074
300	11 705	9 528	8 103	8 080	8 420	8 143	8 155	8 005
400	16 332	13 207	11 148	11 117	11 614	11 210	11 229	10 957
500	21 155	17 001	14 245	14 205	14 878	14 333	14 360	13 933
600	26 176	20 908	17 396	17 347	18 213	17 512	17 550	16 931
700	31 393	24 930	20 599	20 540	21 618	20 747	20 796	19 951
800	36 808	29 066	23 855	23 786	25 094	24 038	24 101	22 994
900	42 420	33 316	27 163	27 085	28 640	27 385	27 463	26 059
1000	48 229	37 680	30 524	30 436	32 256	30 789	30 883	29 146
1100	54 236	42 159	33 938	33 839	35 943	34 249	34 361	32 256
1200	60 439	46 751	37 404	37 295	39 700	37 765	37 896	35 389
1300	66 840	51 458	40 923	40 803	43 527	41 337	41 489	38 544
1400	73 438	56 280	44 495	44 363	47 425	44 965	45 140	41 721
1500	80 233	61 215	48 119	47 976	51 393	48 650	48 848	44 921

^aN.B. In a reaction equation, $n_{ox}(O_2 + 3.762N_2)$ represents 4.762n_{ox} equivalent kmol of air

E.8 Heating (or calorific) values of fuels

M	= molar mass
\overline{C}_p	= mean constant-pressure specific heat for use near 25 °C Abbreviations:
$\overline{\rho}$	= approximate density
GCV	gross calorific value (= HHV = HCV)
HHV	higher heating value (= GCV = HCV = negative of enthalpy of combustion with liquid H ₂ O in products)
LHV	lower heating value (= NCV = LCV = negative of enthalpy of combustion with vapour H ₂ O in products)
NCV	net calorific value (= LHV = LCV)

Table E.8: Heating (or calorific) values of gas fuels at 25 °C.

Gas	M kg kmol ⁻¹	HHV or GCV kJ kg ⁻¹	LHV or NCV kJ kg ⁻¹	\overline{C}_p kJ kg ⁻¹ K ⁻¹
hydrogen	2.016	141 800	119 980	14.31
methane	16.04	55 500	50 020	2.23
ethane	30.07	51 880	47 490	1.75
propane	44.10	50 350	46 360	1.67
n-butane	58.12	49 500	45 720	1.68
n-pentane	72.15	49 020	45 360	1.67
n-hexane	86.18	48 680	45 110	1.66
carbon monoxide	28.01	10 100	10 100	1.04
typical North Sea gas ^a	17.05	53 510	48 290	2.15

^amolar composition: CH₄ 94.4%, C₂H₆ 3.0%, N₂ 1.5%, other gases 1.1%. Elemental composition by mass: C 73.26%, H 23.90%, O 0.38%, N 2.46%.

Table E.9: Heating (or calorific) values of liquid fuels at 25 °C.

Liquid	Approx. elemental composition by mass (%)	$\overline{\rho}$ kg m ⁻³	HHV or GCV kJ kg ⁻¹	LHV or NCV kJ kg ⁻¹	\overline{C}_p kJ kg ⁻¹ K ⁻¹
n-octane C ₈ H ₁₈	C84.1 H15.9	703	47 890	44 420	2.11
methanol CH ₃ OH	C37.5 H12.6 O49.9	790	22 690	19 960	2.51
petrol (gasoline)	C85.0 H15.0	740	46 900	43 630	2.06
kerosine	C86.1 H13.9	770	46 140	43 100	2.02
distillate fuel oil	C86.8 H13.2	820	45 600	42 720	1.95

E.9 Properties of R134a refrigerant

R134a or HFC134a is a hydrofluorocarbon refrigerant (1,1,1,2-tetrafluoroethane, CH_2FCF_3) with zero ozone-depleting potential, although it has some global warming potential. It is a substitute for the chlorofluorocarbon refrigerant R12 (banned under the Montreal Protocol 1984) in domestic refrigeration and freezing applications and in the coolers of air conditioning plant.

T, T_{sat}	=	temperature, at saturation
P	=	absolute pressure
v	=	specific volume, $\text{m}^3 \text{kg}^{-1}$
h	=	specific enthalpy, kJ kg^{-1}
s	=	specific entropy, $\text{kJ kg}^{-1} \text{K}^{-1}$

Table E.10: Saturated Refrigerant 134a — Temperature (−60°C to critical point)

T (°C)	P bar (abs)	v_f $\text{m}^3 \text{kg}^{-1}$	v_g $\text{m}^3 \text{kg}^{-1}$	h_f kJ kg^{-1}	h_g kJ kg^{-1}	s_f $\text{kJ kg}^{-1} \text{K}^{-1}$	s_g $\text{kJ kg}^{-1} \text{K}^{-1}$
-60	0.1587	0.0006795	1.0808	22.92	261.17	0.6828	1.8005
-50	0.2942	0.0006923	0.6064	35.33	267.44	0.7396	1.7798
-40	0.5118	0.0007060	0.3609	47.88	273.71	0.7946	1.7632
-35	0.6612	0.0007132	0.2838	54.22	276.84	0.8214	1.7562
-30	0.8436	0.0007206	0.2257	60.60	279.95	0.8479	1.7500
-25	1.064	0.0007284	0.1814	67.03	283.05	0.8740	1.7445
-20	1.327	0.0007364	0.1473	73.51	286.13	0.8998	1.7397
-15	1.639	0.0007448	0.1206	80.05	289.18	0.9252	1.7354
-10	2.005	0.0007536	0.09954	88.64	292.20	0.9504	1.7316
-5	2.432	0.0007627	0.08279	93.29	295.19	0.9753	1.7283
0	2.925	0.0007723	0.06933	100.00	298.10	1.0000	1.7254
5	3.492	0.0007823	0.05842	106.78	301.02	1.0244	1.7228
10	4.139	0.0007929	0.04951	113.62	303.86	1.0486	1.7205
15	4.873	0.0008041	0.04218	120.54	306.64	1.0727	1.7185
20	5.702	0.0008160	0.03610	127.54	309.35	1.0965	1.7167
25	6.634	0.0008285	0.03102	134.61	311.97	1.1202	1.7150
30	7.675	0.0008419	0.02676	141.77	314.49	1.1437	1.7134
35	8.835	0.0008563	0.02315	149.02	316.91	1.1671	1.7119
40	10.12	0.0008718	0.02008	156.37	319.20	1.1904	1.7104
45	11.55	0.0008886	0.01746	163.83	321.34	1.2136	1.7087
50	13.11	0.0009068	0.01520	171.41	323.31	1.2368	1.7069
60	16.73	0.0009493	0.01154	187.00	326.60	1.2833	1.7024
80	26.21	0.001077	0.006516	221.07	328.95	1.3798	1.6852
101	40.55	0.001964	0.001964	289.40	289.40	1.5609	1.5609

Table E.11: Saturated Refrigerant 134a — Pressure (0.2 bar to critical point)

P bar (abs)	T °C	v_f $\text{m}^3 \text{kg}^{-1}$	v_g $\text{m}^3 \text{kg}^{-1}$	h_f kJ kg^{-1}	h_g kJ kg^{-1}	s_f $\text{kJ kg}^{-1} \text{K}^{-1}$	s_g $\text{kJ kg}^{-1} \text{K}^{-1}$
0.2	-56.38	0.0006841	0.8703	27.40	263.44	0.7035	1.7925
0.4	-44.58	0.0006996	0.4547	42.11	270.84	0.7696	1.7703
0.6	-36.93	0.0007104	0.3109	51.77	275.64	0.8111	1.7588
0.8	-31.11	0.0007189	0.2373	59.18	279.26	0.8420	1.7513
1.0	-26.36	0.0007262	0.1923	65.28	282.21	0.8669	1.7460
1.2	-22.31	0.0007327	0.1620	70.51	284.71	0.8879	1.7418
1.4	-18.75	0.0007385	0.1400	75.14	286.89	0.9062	1.7385
1.6	-15.58	0.0007438	0.1234	79.29	288.83	0.9223	1.7358
2.0	-10.06	0.0007534	0.09978	86.53	292.16	0.9500	1.7316
2.5	-4.26	0.0007641	0.08062	94.28	295.62	0.9790	1.7278
3.0	0.70	0.0007737	0.06766	100.95	298.54	1.0034	1.7250
3.5	5.07	0.0007825	0.05829	106.87	301.06	1.0247	1.7228
4.0	8.98	0.0007907	0.05119	112.22	303.29	1.0437	1.7210
5.0	15.80	0.0008060	0.04113	121.66	307.08	1.0765	1.7182
6.0	21.66	0.0008201	0.03431	129.88	310.23	1.1044	1.7161
8.0	31.45	0.0008460	0.02565	143.86	315.21	1.1505	1.7130
10.0	39.55	0.0008703	0.02034	155.70	319.00	1.1883	1.7105
12.0	46.50	0.0008939	0.01675	166.09	321.95	1.2206	1.7082
15.0	55.45	0.0009289	0.01308	179.83	325.23	1.2621	1.7046
20.0	67.72	0.0009896	0.009318	199.58	328.37	1.3197	1.6975
30.0	86.38	0.001144	0.005306	233.52	327.42	1.4135	1.6747
40.55	101.00	0.001964	0.001964	289.35	289.35	1.5609	1.5609

Table E.12: Superheated Refrigerant 134a (0.2 bar to 1 bar)

T(°C)	0.2 bar abs ($T_{\text{sat}} = -56.38^{\circ}\text{C}$)			0.4 bar abs ($T_{\text{sat}} = -44.58^{\circ}\text{C}$)			0.6 bar abs ($T_{\text{sat}} = -36.93^{\circ}\text{C}$)			0.8 bar abs ($T_{\text{sat}} = -31.11^{\circ}\text{C}$)			1.0 bar abs ($T_{\text{sat}} = -26.36^{\circ}\text{C}$)			T(°C)
	v	\dot{h}	s													
Sat.	0.8703	263.44	1.7925	0.4547	270.84	1.7703	0.3109	275.64	1.7588	0.2373	279.26	1.7513	0.1923	282.21	1.7460	Sat.
-50	0.8975	267.92	1.8129	0.4647	274.21	1.7849	0.3212	280.90	1.7808	0.2386	280.13	1.7549	0.1982	287.27	1.7662	-50
-40	0.9398	275.08	1.8443	0.4864	281.67	1.8162	0.3359	288.61	1.8119	0.2499	287.95	1.7864	0.1982	287.27	1.7662	-40
-30	0.9818	282.41	1.8750	0.5079	289.27	1.8469	0.3504	296.46	1.8422	0.2610	295.88	1.8171	0.2073	295.30	1.7973	-30
-20	1.0236	289.91	1.9053	0.5291	297.03	1.8769	0.3647	304.45	1.8720	0.2719	303.94	1.8472	0.2162	303.43	1.8276	-20
-10	1.0652	297.59	1.9350	0.5502	304.94	1.9064	0.3887	312.58	1.9013	0.2827	312.14	1.8767	0.2250	311.69	1.8573	-10
0	1.107	305.43	1.9643	0.5712	313.02	1.9355	0.4127	320.87	1.9300	0.2935	320.47	1.9056	0.2337	320.08	1.8864	0
10	1.148	313.45	1.9931	0.5921	321.26	1.9641	0.4390	329.31	1.9584	0.3041	328.96	1.9341	0.2423	328.60	1.9150	10
20	1.189	321.64	2.0215	0.6129	329.66	1.9922	0.4671	342.10	1.9863	0.3147	337.59	1.9621	0.2509	337.27	1.9432	20
30	1.231	330.01	2.0496	0.6337	338.22	2.0200	0.4950	346.67	2.0138	0.3252	346.38	1.9897	0.2594	346.09	1.9709	30
40	1.272	338.54	2.0773	0.6544	346.95	2.0475	0.5247	355.58	2.0410	0.3357	355.32	2.0169	0.2679	355.05	1.9982	40
50				0.6751	355.84	2.0746	0.4489	364.66	2.0678	0.3462	364.41	2.0438	0.2763	364.17	2.0251	50
60													0.2847	373.44	2.0518	60
70																70
80																80

Table E.13: Superheated Refrigerant 134a (1.5 bar to 4 bar)

T(°C)	1.5 bar abs ($T_{\text{sat}} = -17.13^{\circ}\text{C}$)			2.0 bar abs ($T_{\text{sat}} = -10.06^{\circ}\text{C}$)			2.5 bar abs ($T_{\text{sat}} = -4.26^{\circ}\text{C}$)			3.0 bar abs ($T_{\text{sat}} = 0.70^{\circ}\text{C}$)			3.5 bar abs ($T_{\text{sat}} = 8.98^{\circ}\text{C}$)			T(°C)
	v	h	s	v	h	s	v	h	s	v	h	s	v	h	s	
Sat.	0.1311	287.89	1.7371	0.0998	292.16	1.7316	0.0806	295.62	1.7278	0.0677	298.54	1.7250	0.0512	303.29	1.7210	Sat.
-10	0.1357	293.79	1.7599	0.0998	292.22	1.7318	0.0824	299.37	1.7416							-10
0	0.1419	302.13	1.7910	0.1047	300.77	1.7638	0.0977	334.80	1.8627	0.0806	333.95	1.8458	0.0593	332.18	1.8181	0
10	0.1481	310.55	1.8212	0.1095	309.37	1.7947	0.0864	308.15	1.7732	0.0709	306.90	1.7550	0.0515	304.25	1.7244	10
20	0.1541	319.07	1.8508	0.1142	318.03	1.8247	0.0902	316.97	1.8038	0.0742	315.88	1.7862	0.0542	313.59	1.7568	20
30	0.1600	327.70	1.8798	0.1187	326.78	1.8541	0.0940	325.84	1.8336	0.0775	324.88	1.8164	0.0568	322.89	1.7880	30
40	0.1658	336.46	1.9082	0.1232	335.64	1.8828	0.0977	334.80	1.8627	0.0837	333.95	1.8458	0.0593	332.18	1.8181	40
50	0.1716	345.36	1.9362	0.1277	344.61	1.9110	0.1013	343.86	1.8911	0.0837	343.09	1.8745	0.0617	341.51	1.8475	50
60	0.1773	354.39	1.9637	0.1321	353.71	1.9388	0.1049	353.03	1.9191	0.0868	352.33	1.9027	0.0641	350.91	1.8761	60
70	0.1831	363.56	1.9908	0.1364	362.94	1.9661	0.1085	362.31	1.9465	0.0898	361.68	1.9303	0.0665	360.38	1.9041	70
80	0.1887	372.87	2.0176	0.1408	372.30	1.9930	0.1120	371.73	1.9736	0.0928	371.14	1.9575	0.0688	369.96	1.9316	80
90	0.1944	382.34	2.0440	0.1451	381.81	2.0195	0.1155	381.27	2.0002	0.0957	380.73	1.9843	0.0710	379.64	1.9586	90
100	0.2000	391.95	2.0701	0.1493	391.45	2.0457	0.1189	390.95	2.0265	0.0986	390.45	2.0107	0.0733	389.44	1.9853	100
110	0.1536	401.24	2.0716	0.1224	400.77	2.0525	0.1015	400.31	2.0368	0.0755	399.36	2.0115	0.0799	419.59	2.0630	110
120							0.1044	410.30	2.0625	0.0777	409.41	2.0374				120
130										0.0799						130

Table E.14: Superheated Refrigerant 134a (5 bar to 12 bar)

T(°C)	5 bar abs ($T_{\text{sat}} = 15.80^\circ\text{C}$)			6 bar abs ($T_{\text{sat}} = 21.66^\circ\text{C}$)			8 bar abs ($T_{\text{sat}} = 31.45^\circ\text{C}$)			10 bar abs ($T_{\text{sat}} = 39.55^\circ\text{C}$)			12 bar abs ($T_{\text{sat}} = 46.50^\circ\text{C}$)			T(°C)	
	v	T_h	s	v	T_h	s	v	T_h	s	v	T_h	s	v	T_h	s	T(°C)	
Sat.	0.0411	307.08	1.7182	0.0343	310.23	1.7161	0.0256	315.21	1.7130	0.0203	319.00	1.7105	0.0167	321.95	1.7082	Sat.	
20	0.0421	311.16	1.7322	0.0360	318.56	1.7440	0.0270	324.24	1.7422	0.0204	319.51	1.7121	0.0172	326.10	1.7211	20	
30	0.0443	320.79	1.7645	0.0379	328.41	1.7760	0.0286	334.56	1.7747	0.0218	330.58	1.7469	0.0195	337.53	1.7559	30	
40	0.0465	330.34	1.7955	0.0397	338.18	1.8067	0.0300	344.74	1.8057	0.0231	341.30	1.7796	0.0184	348.56	1.7886	40	
50	0.0485	339.88	1.8255	0.0397	338.18	1.8067	0.0314	354.85	1.8356	0.0243	351.82	1.8107	0.0195	348.56	1.7886	50	
60	0.0505	349.44	1.8546	0.0414	347.93	1.8364	0.0327	364.94	1.8646	0.0254	362.23	1.8406	0.0205	359.36	1.8196	60	
70	0.0524	359.06	1.8831	0.0431	357.69	1.8653	0.0332	375.05	1.8928	0.0265	372.60	1.8696	0.0215	370.04	1.8494	70	
80	0.0543	368.75	1.9109	0.0447	367.51	1.8935	0.0339	385.21	1.9204	0.0276	382.97	1.8978	0.0224	380.65	1.8782	80	
90	0.0562	378.53	1.9382	0.0463	377.39	1.9210	0.0352	397.42	1.9475	0.0286	393.38	1.9253	0.0234	391.26	1.9063	90	
100	0.0581	388.41	1.9651	0.0479	387.36	1.9481	0.0352	407.60	2.0010	0.0376	405.75	1.9740	0.0296	403.84	1.9337	100	
110	0.0599	398.40	1.9915	0.0495	397.42	1.9747	0.0364	417.89	2.0268	0.0388	416.15	2.0002	0.0306	414.38	1.9787	110	
120	0.0617	408.51	2.0175	0.0510	407.60	2.0040	0.0400	426.67	2.0259	0.0316	425.01	2.0048	0.0259	423.32	1.9868	120	
130	0.0635	418.74	2.0432	0.0525	417.89	2.0268	0.0412	428.30	2.0523	0.0412	437.30	2.0514	0.0325	435.73	2.0304	130	
140	0.0653	429.11	2.0686	0.0540	428.30	2.0523	0.0412	447.42	2.0771	0.0432	446.67	2.0669	0.0346	445.01	2.0251	140	
150																150	

Table E.15: Superheated Refrigerant 134a (16 bar to 30 bar)

T(°C)	15 bar abs ($T_{\text{sat}} = 55.45^{\circ}\text{C}$)			18 bar abs ($T_{\text{sat}} = 63.13^{\circ}\text{C}$)			22 bar abs ($T_{\text{sat}} = 71.96^{\circ}\text{C}$)			26 bar abs ($T_{\text{sat}} = 79.62^{\circ}\text{C}$)			30 bar abs ($T_{\text{sat}} = 86.38^{\circ}\text{C}$)			T(°C)
	v	h	s													
Sat.	0.0131	325.23	1.7046	0.0106	327.42	1.7006	0.00826	328.95	1.6941	0.00659	328.99	1.6858	0.00531	327.42	1.6747	Sat.
60	0.0136	331.03	1.7222	0.0113	336.80	1.7282				0.00664	329.72	1.6878				60
70	0.0147	343.14	1.7580	0.0123	349.46	1.7646	0.00909	341.13	1.7290	0.00763	346.45	1.7346	0.00575	335.43	1.6969	70
80	0.0156	354.70	1.7912	0.0131	361.45	1.7981	0.00993	354.66	1.7668	0.00840	360.68	1.7732	0.00665	352.83	1.7442	80
90	0.0165	365.94	1.8225	0.0139	373.05	1.8296	0.0107	367.28	1.8011							90
100	0.0173	376.99	1.8526													100
110	0.0181	387.94	1.8815	0.0146	384.42	1.8596	0.0113	379.37	1.8331	0.00905	373.80	1.8079	0.00734	367.52	1.7830	110
120	0.0189	398.85	1.9096	0.0153	395.67	1.8886	0.0120	391.17	1.8634	0.00965	386.31	1.8402	0.00793	381.00	1.8178	120
130	0.0196	409.76	1.9371	0.0159	406.85	1.9167	0.0125	402.78	1.8926	0.0102	398.45	1.8707	0.00846	393.83	1.8500	130
140	0.0203	420.71	1.9639	0.0165	418.02	1.9441	0.0131	414.30	1.9208	0.0107	410.38	1.8999	0.00894	406.27	1.8805	140
150	0.0210	431.71	1.9902	0.0171	429.21	1.9708	0.0136	425.77	1.9483	0.0112	422.19	1.9281	0.00940	418.47	1.9096	150

E.10 Transport properties of air, water and steam

C_p	= specific heat at constant pressure	$\text{kJ kg}^{-1} \text{K}^{-1}$
C_v	= specific heat at constant volume	$\text{kJ kg}^{-1} \text{K}^{-1}$
k	= thermal conductivity	$\text{W m}^{-1} \text{K}^{-1}$
T	= temperature	K or $^{\circ}\text{C}$
x	= definition	$\text{kJ kg}^{-1} \text{K}^{-1}$
x	= definition	$\text{kJ kg}^{-1} \text{K}^{-1}$
α	= thermal diffusivity	$\text{m}^2 \text{s}^{-1}$
μ	= absolute viscosity	$\text{kg m}^{-1} \text{s}^{-1}$ (= Ns m^{-2})
ν	= kinematic viscosity	$\text{m}^2 \text{s}^{-1}$
ρ	= density	kg m^{-3}

Table E.16: Transport properties of dry air at atmospheric pressure

Temperature T (°C)	Density ρ (kg m ⁻³)	Specific heat at constant pressure C_p	Absolute (or dynamic) viscosity μ	Kinematic viscosity $\nu = \frac{\mu}{\rho}$	Thermal conductivity κ	Thermal diffusivity α	Prandtl number $\text{Pr} = \frac{\mu C_p}{k}$	$\frac{g\beta}{\alpha\nu} = \frac{\text{GrPr}}{L^3 \Delta T}$	Temperature T (°C)
-180	3.72	1035	6.50×10^{-6}	1.75×10^{-6}	0.0076	1.9×10^{-6}	0.92	3.2×10^{10}	-180
-100	2.04	1010	1.16×10^{-5}	5.69×10^{-6}	0.016	7.6×10^{-6}	0.75	1.3×10^9	-100
-50	1.582	1006	1.45×10^{-5}	9.17×10^{-6}	0.020	1.30×10^{-5}	0.72	3.67×10^8	-50
0	1.293	1006	1.71×10^{-5}	1.32×10^{-5}	0.024	1.84×10^{-5}	0.72	1.48×10^8	0
10	1.247	1006	1.76×10^{-5}	1.41×10^{-5}	0.025	1.96×10^{-5}	0.72	1.25×10^8	10
20	1.205	1006	1.81×10^{-5}	1.50×10^{-5}	0.025	2.08×10^{-5}	0.72	1.07×10^8	20
30	1.165	1006	1.86×10^{-5}	1.60×10^{-5}	0.026	2.23×10^{-5}	0.72	9.07×10^7	30
60	1.060	1008	2.00×10^{-5}	1.89×10^{-5}	0.028	2.74×10^{-5}	0.70	5.71×10^7	60
100	0.946	1011	2.18×10^{-5}	2.30×10^{-5}	0.032	3.28×10^{-5}	0.70	3.48×10^7	100
200	0.746	1025	2.58×10^{-5}	3.46×10^{-5}	0.039	5.19×10^{-5}	0.68	9.53×10^6	200
300	0.616	1045	2.95×10^{-5}	4.79×10^{-5}	0.045	7.17×10^{-5}	0.68	4.96×10^6	300
500	0.456	1093	3.58×10^{-5}	7.85×10^{-5}	0.056	1.14×10^{-4}	0.70	1.42×10^6	500
1000	0.277	1185	4.82×10^{-5}	1.74×10^{-4}	0.076	2.42×10^{-4}	0.72	1.8×10^5	1000

Table E.17: Transport properties of saturated water and steam

Temp. T (°C)	Specific volume v_f (m ³ kg ⁻¹)	Specific volume v_g (m ³ kg ⁻¹)	Specific heat at constant pressure			Thermal conductivity			Absolute (or dynamic) viscosity			Prandtl number $\frac{\mu C_p}{k}$		Temp. T (°C)
			C_{pf} (J kg ⁻¹ K ⁻¹)	C_{pg} (J kg ⁻¹ K ⁻¹)	$(\text{J kg}^{-1}\text{K}^{-1})$	k_f (W m ⁻¹ K ⁻¹)	k_g (W m ⁻¹ K ⁻¹)	μ_f (kg m ⁻¹ s ⁻¹)	μ_g (kg m ⁻¹ s ⁻¹)	P_{rf}	P_{rg}			
0.01	0.001000	206.0	4217	1854	0.569	0.0173	1.755×10^{-3}	8.8×10^{-6}	13.02	0.942	0.01			
10	0.001000	106.3	4193	1860	0.587	0.0185	1.301×10^{-3}	9.1×10^{-6}	9.29	0.915	10			
20	0.001002	57.78	4182	1866	0.603	0.0191	1.002×10^{-3}	9.4×10^{-6}	6.95	0.918	20			
30	0.001004	32.90	4179	1875	0.618	0.0198	7.97×10^{-4}	9.7×10^{-6}	5.39	0.923	30			
40	0.001008	19.53	4179	1885	0.632	0.0204	6.51×10^{-4}	1.01×10^{-5}	4.31	0.930	40			
50	0.001012	12.04	4181	1899	0.643	0.0210	5.44×10^{-4}	1.04×10^{-5}	3.53	0.939	50			
60	0.001017	7.674	4185	1915	0.653	0.0217	4.62×10^{-4}	1.07×10^{-5}	2.96	0.947	60			
70	0.001023	5.045	4190	1936	0.662	0.0224	4.00×10^{-4}	1.11×10^{-5}	2.53	0.956	70			
80	0.001029	3.409	4197	1962	0.670	0.0231	3.50×10^{-4}	1.14×10^{-5}	2.19	0.966	80			
90	0.001036	2.362	4205	1992	0.676	0.0240	3.11×10^{-4}	1.17×10^{-5}	1.93	0.976	90			
100	0.001043	1.674	4216	2028	0.681	0.0249	2.78×10^{-4}	1.21×10^{-5}	1.723	0.986	100			
125	0.001065	0.7709	4254	2147	0.687	0.0272	2.19×10^{-4}	1.33×10^{-5}	1.358	1.047	125			
150	0.001090	0.3929	4310	2314	0.687	0.0300	1.80×10^{-4}	1.44×10^{-5}	1.133	1.110	150			
175	0.001121	0.2168	4389	2542	0.679	0.0334	1.53×10^{-4}	1.56×10^{-5}	0.990	1.185	175			
200	0.001156	0.1273	4497	2843	0.665	0.0375	1.33×10^{-4}	1.67×10^{-5}	0.902	1.270	200			
225	0.001199	0.07846	4648	3238	0.644	0.0427	1.182×10^{-4}	1.79×10^{-5}	0.853	1.36	225			
250	0.001251	0.05011	4867	3772	0.616	0.0495	1.065×10^{-4}	1.91×10^{-5}	0.841	1.45	250			
275	0.001317	0.03278	5202	4561	0.582	0.0587	9.72×10^{-5}	2.02×10^{-5}	0.869	1.56	275			
300	0.001404	0.02167	5762	5863	0.541	0.0719	8.97×10^{-5}	2.14×10^{-5}	0.955	1.74	300			
325	0.001528	0.01419	6861	8440	0.493	0.0929	7.90×10^{-5}	2.30×10^{-5}	1.100	2.09	325			
350	0.001740	0.008812	10100	17150	0.437	0.1343	6.48×10^{-5}	2.58×10^{-5}	1.50	3.29	350			
360	0.001894	0.006962	14600	25100	0.400	0.168	5.82×10^{-5}	2.75×10^{-5}	2.11	3.89	360			
374	0.003106	0.003106	∞	∞	0.24	0.24	4.5×10^{-5}	4.5×10^{-5}	∞	∞	374			

E.11 Approximate physical properties

ρ	= density	kg m^{-3}
C_p	= specific heat at constant pressure	$\text{J kg}^{-1} \text{K}^{-1}$
k	= thermal conductivity	$\text{W m}^{-1} \text{K}^{-1}$
μ	= absolute or dynamic viscosity	$\text{kg m}^{-1} \text{m}^{-1}$ or N s m^{-2}
σ	= surface tension	N m^{-1}

Table E.18: Approximate physical properties at 20 °C, 1 bar.

Gases ^a	ρ	C_p	k	μ	
air	1.19	1 010	0.025	1.81×10^{-5}	
oxygen	1.31	910	0.026	2.03×10^{-5}	
nitrogen	1.15	1 040	0.026	1.76×10^{-5}	
carbon dioxide	1.80	8 40	0.017	1.47×10^{-5}	
hydrogen	0.083	14 300	0.18	0.88×10^{-5}	
helium	0.164	5 230	0.14	1.96×10^{-5}	
Liquids	ρ	C_p	k	μ	σ
water	1000	4 190	0.60	1.00×10^{-3}	0.073
mercury	13 600	140	8.7	1.55×10^{-3}	0.51
ethanol	790	2 860	0.19	1.20×10^{-3}	0.022
R134a (25 °C)	1 210	1430	0.080	0.21×10^{-3}	
Solids	ρ	C_p	k	Notes	
mild steel	7 850	460	52		
stainless steel	7810	460	16	18% Ni & 8% Cr	
aluminium alloy	2720	880	170	Duralumin	
copper	8 950	380	400		
brass	8 410	380	120	30% Zn	
Polyethylene	1 000	2 100	0.5	moderately high density	
expanded polystyrene	25		0.04	board	
concrete	2 400	900	1.0	moderately dense	
brick	1 800	750	0.6	common building brick	
wood	500	2 500	0.15	pine & dry	
glass	2 500	800	1.0	window	

^aNB. Constant C_p values in Table E.3 are averages over a range of temperature. $C_p(T)$ relationships used in Table E.5 are fits over a different temperature range. Neither will necessarily agree well with the 20 °C values tabulated here.

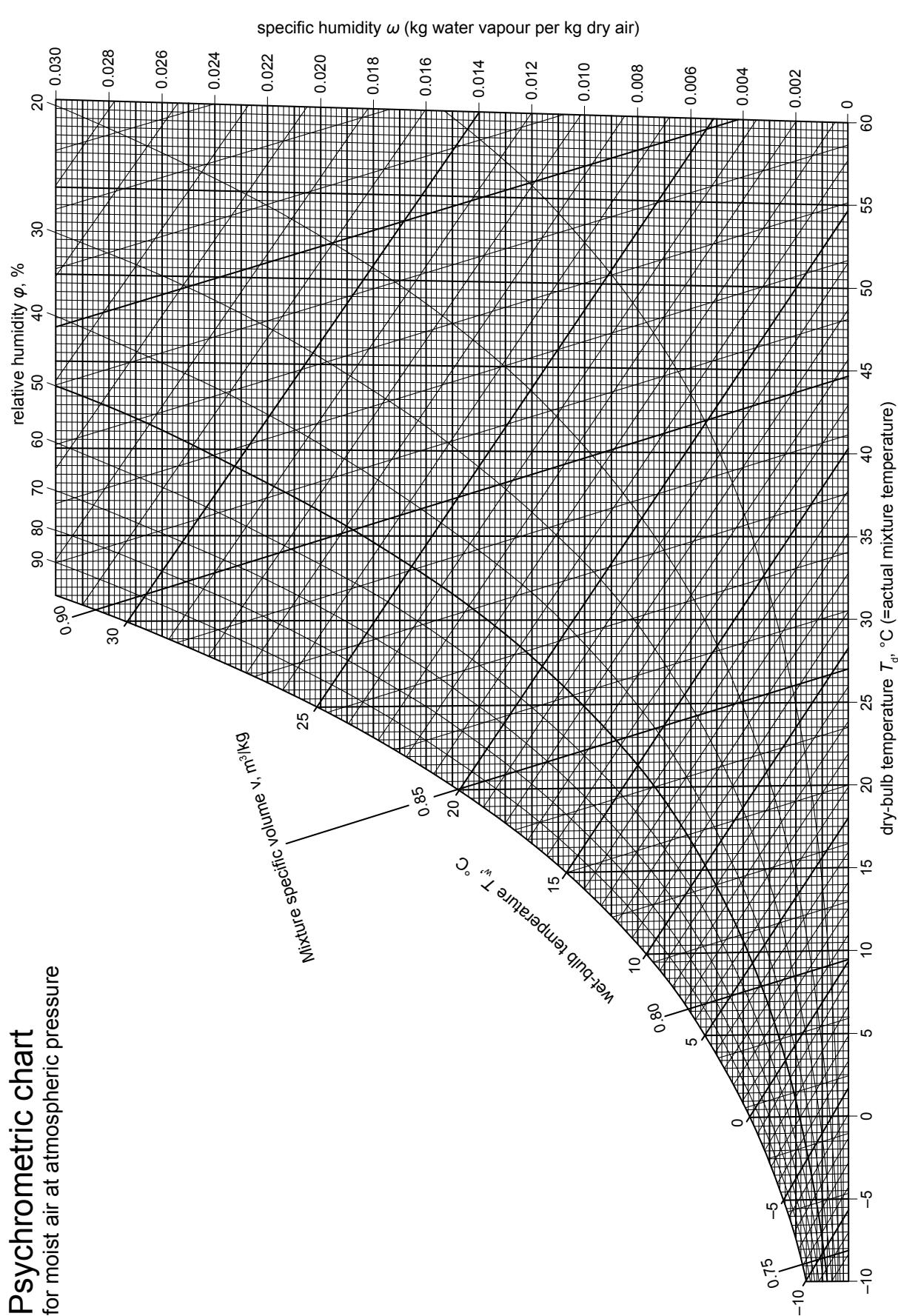


Figure E.2: Psychrometric Chart

E.12 Thermodynamic property tables for water/steam (IAPWS-IF97 formulation)

h	=	specific enthalpy	kJ kg^{-1}
P	=	absolute pressure	bar ($1 \text{ bar} = 10^5 \text{ Pa} = 10^5 \text{ N m}^{-2}$)
s	=	specific entropy	$\text{kJ kg}^{-1} \text{ K}^{-1}$
T	=	temperature	$^{\circ}\text{C}$
u	=	specific internal energy	kJ kg^{-1}
v	=	specific volume	$\text{m}^{-3} \text{ kg}^{-1}$

Subscripts:

f	=	saturated liquid state
fg	=	change between saturated liquid and saturated vapour states at constant pressure ($h_{fg} = h_g - h_f$)
g	=	saturated vapour state
sat	=	saturation state

Properties in these tables were evaluated from the Industrial Formulation 1997 of the International Association for the Properties of Water and Steam (IAPWS). There are small differences between these values and those in use in the Department of Mechanical Engineering up to 2003 (evaluated from the IAPWS 1984 Formulation for Scientific and General Use and 1986 Supplementary Release on Saturation Properties).

Triple point: $T = 0.01 \text{ }^{\circ}\text{C}$ $P = 0.006117 \text{ bar}$

Critical point: $T = 373.95 \text{ }^{\circ}\text{C}$ $P = 220.64 \text{ bar}$

u and s are chosen to be zero for saturated liquid at the triple point.

Linear interpolation in the tables is not advisable near the critical point.

Table E.19: Saturated water and steam — Temperature (triple point to 100 °C)

T °C	P bar (abs)	v_f $\text{m}^3 \text{kg}^{-1}$	v_g $\text{m}^3 \text{kg}^{-1}$	u_f kJ kg^{-1}	u_g kJ kg^{-1}	h_f kJ kg^{-1}	h_{fg} kJ kg^{-1}	h_g kJ kg^{-1}	s_f $\text{kJ kg}^{-1} \text{K}^{-1}$	s_g $\text{kJ kg}^{-1} \text{K}^{-1}$
0.01	0.006117	0.001000	206.0	0	2374.9	0.000612	2500.9	2500.9	0	9.155
1	0.006571	0.001000	192.4	4.18	2376.3	4.177	2498.6	2502.7	0.0153	9.129
2	0.007060	0.001000	179.8	8.39	2377.7	8.392	2496.2	2504.6	0.0306	9.103
3	0.007581	0.001000	168.0	12.60	2379.0	12.60	2493.8	2506.4	0.0459	9.076
4	0.008135	0.001000	157.1	16.81	2380.4	16.81	2491.4	2508.2	0.0611	9.051
5	0.008726	0.001000	147.0	21.02	2381.8	21.02	2489.1	2510.1	0.0763	9.025
6	0.009354	0.001000	137.6	25.22	2383.2	25.22	2486.7	2511.9	0.0913	8.999
7	0.01002	0.001000	128.9	29.42	2384.5	29.43	2484.3	2513.7	0.1064	8.974
8	0.01073	0.001000	120.8	33.62	2385.9	33.63	2481.9	2515.6	0.1213	8.949
9	0.01148	0.001000	113.3	37.82	2387.3	37.82	2479.6	2517.4	0.1362	8.924
10	0.01228	0.001000	106.3	42.02	2388.7	42.02	2477.2	2519.2	0.1511	8.900
11	0.01313	0.001000	99.79	46.21	2390.0	46.22	2474.8	2521.1	0.1659	8.876
12	0.01403	0.001001	93.72	50.41	2391.4	50.41	2472.5	2522.9	0.1806	8.851
13	0.01498	0.001001	88.07	54.60	2392.8	54.60	2470.1	2524.7	0.1953	8.828
14	0.01599	0.001001	82.80	58.79	2394.1	58.79	2467.7	2526.5	0.2099	8.804
15	0.01706	0.001001	77.88	62.98	2395.5	62.98	2465.4	2528.4	0.2245	8.780
16	0.01819	0.001001	73.29	67.17	2396.9	67.17	2463.0	2530.2	0.2390	8.757
17	0.01938	0.001001	69.01	71.36	2398.3	71.36	2460.6	2532.0	0.2534	8.734
18	0.02065	0.001001	65.00	75.55	2399.6	75.55	2458.3	2533.8	0.2678	8.711
19	0.02198	0.001002	61.26	79.73	2401.0	79.73	2455.9	2535.7	0.2822	8.689
20	0.02339	0.001002	57.76	83.92	2402.4	83.92	2453.5	2537.5	0.2965	8.666
21	0.02488	0.001002	54.49	88.10	2403.7	88.10	2451.2	2539.3	0.3108	8.644
22	0.02645	0.001002	51.42	92.29	2405.1	92.29	2448.8	2541.1	0.3250	8.622
23	0.02811	0.001003	48.55	96.47	2406.4	96.47	2446.4	2542.9	0.3391	8.600
24	0.02986	0.001003	45.86	100.7	2407.8	100.7	2444.1	2544.7	0.3532	8.578
25	0.03170	0.001003	43.34	104.8	2409.2	104.8	2441.7	2546.5	0.3673	8.557
26	0.03364	0.001003	40.98	109.0	2410.5	109.0	2439.3	2548.4	0.3813	8.535
27	0.03568	0.001004	38.76	113.2	2411.9	113.2	2437.0	2550.2	0.3952	8.514
28	0.03783	0.001004	36.68	117.4	2413.2	117.4	2434.6	2552.0	0.4091	8.493
29	0.04009	0.001004	34.72	121.6	2414.6	121.6	2432.2	2553.8	0.4230	8.473
30	0.04247	0.001004	32.88	125.7	2415.9	125.7	2429.8	2555.6	0.4368	8.452
32	0.04759	0.001005	29.53	134.1	2418.7	134.1	2425.1	2559.2	0.4643	8.411
34	0.05325	0.001006	26.56	142.5	2421.4	142.5	2420.3	2562.8	0.4916	8.372
36	0.05948	0.001006	23.93	150.8	2424.0	150.8	2415.6	2566.4	0.5187	8.332
38	0.06632	0.001007	21.60	159.2	2426.7	159.2	2410.8	2570.0	0.5457	8.294
40	0.07384	0.001008	19.52	167.5	2429.4	167.5	2406.0	2573.5	0.5724	8.256
42	0.08209	0.001009	17.67	175.9	2432.1	175.9	2401.2	2577.1	0.5990	8.218
44	0.09112	0.001009	16.01	184.2	2434.8	184.3	2396.4	2580.7	0.6255	8.182
46	0.1010	0.001010	14.54	192.6	2437.4	192.6	2391.6	2584.2	0.6517	8.145
48	0.1118	0.001011	13.21	201.0	2440.1	201.0	2386.8	2587.8	0.6778	8.110
50	0.1235	0.001012	12.03	209.3	2442.8	209.3	2382.0	2591.3	0.7038	8.075
52	0.1363	0.001013	10.96	217.7	2445.4	217.7	2377.1	2594.8	0.7296	8.040
54	0.1502	0.001014	10.01	226.0	2448.0	226.1	2372.3	2598.4	0.7552	8.007
56	0.1653	0.001015	9.145	234.4	2450.7	234.4	2367.4	2601.9	0.7807	7.973
58	0.1817	0.001016	8.369	242.8	2453.3	242.8	2362.6	2605.4	0.8060	7.940
60	0.1995	0.001017	7.668	251.1	2455.9	251.2	2357.7	2608.8	0.8312	7.908
62	0.2187	0.001018	7.034	259.5	2458.5	259.5	2352.8	2612.3	0.8563	7.876
64	0.2394	0.001019	6.460	267.9	2461.1	267.9	2347.9	2615.8	0.8811	7.845
66	0.2618	0.001020	5.940	276.2	2463.7	276.3	2343.0	2619.2	0.9059	7.814
68	0.2860	0.001022	5.468	284.6	2466.3	284.6	2338.0	2622.7	0.9305	7.784
70	0.3120	0.001023	5.040	293.0	2468.9	293.0	2333.1	2626.1	0.9550	7.754
72	0.3400	0.001024	4.650	301.4	2471.4	301.4	2328.1	2629.5	0.9793	7.725
74	0.3701	0.001025	4.295	309.7	2474.0	309.8	2323.1	2632.9	1.004	7.696
76	0.4024	0.001026	3.971	318.1	2476.5	318.2	2318.1	2636.3	1.028	7.667
78	0.4370	0.001028	3.675	326.5	2479.0	326.6	2313.1	2639.7	1.052	7.639
80	0.4741	0.001029	3.405	334.9	2481.6	334.9	2308.1	2643.0	1.075	7.611
82	0.5139	0.001030	3.158	343.3	2484.1	343.3	2303.0	2646.4	1.099	7.584
84	0.5564	0.001032	2.932	351.7	2486.6	351.7	2297.9	2649.7	1.123	7.557
86	0.6017	0.001033	2.724	360.1	2489.0	360.1	2292.8	2653.0	1.146	7.530
88	0.6502	0.001035	2.534	368.5	2491.5	368.6	2287.7	2656.3	1.169	7.504
90	0.7018	0.001036	2.359	376.9	2494.0	377.0	2282.6	2659.5	1.193	7.478
92	0.7568	0.001037	2.198	385.3	2496.4	385.4	2277.4	2662.8	1.216	7.453
94	0.8154	0.001039	2.050	393.7	2498.8	393.8	2272.2	2666.0	1.239	7.427
96	0.8777	0.001040	1.914	402.1	2501.2	402.2	2267.0	2669.2	1.262	7.403
98	0.9439	0.001042	1.788	410.6	2503.6	410.7	2261.7	2672.4	1.284	7.378
100	1.0142	0.001043	1.672	419.0	2506.0	419.1	2256.5	2675.6	1.307	7.354

Table E.23: Subcooled water and Superheated Steam (triple point to 0.1 bar)

T(°C)	0.006117 bar ($T_{\text{sat}} = 0.01^{\circ}\text{C}$)			0.01 bar ($T_{\text{sat}} = 7.0^{\circ}\text{C}$)			0.05 bar ($T_{\text{sat}} = 32.9^{\circ}\text{C}$)			0.1 bar ($T_{\text{sat}} = 45.8^{\circ}\text{C}$)		
	v	u	h	s	v	u	h	s	v	u	h	s
0.01	206.0	2374.9	2500.9	9.155	0.001000	0.000007	0.000007	0.00000	0.001000	0.000081	0.005082	0.00000
20	221.1	2403.1	2538.4	9.288	135.2	2403.0	2538.2	9.060	0.001002	83.92	83.92	0.2965
40	236.2	2431.3	2575.8	9.411	144.5	2431.2	2575.7	9.184	28.85	2454.4	257.8	0.001008
60	251.3	2459.5	2613.3	9.527	153.7	2459.5	2613.2	9.300	30.71	2458.7	2612.3	0.555
80	266.4	2487.9	2650.8	9.637	163.0	2487.8	2650.8	9.410	32.57	2487.3	2650.1	0.627
100	281.5	2516.4	2688.6	9.741	172.2	2516.3	2688.5	9.514	34.42	2516.0	2688.0	0.724
120	296.6	2545.0	2726.5	9.840	181.4	2545.0	2726.4	9.613	36.27	2544.7	2726.1	0.834
140	311.7	2573.9	2764.6	9.934	190.7	2573.9	2764.5	9.707	38.12	2573.6	2764.2	0.949
160	326.8	2602.9	2802.8	10.02	199.9	2602.9	2802.8	9.798	39.97	2602.7	2802.6	1.063
180	341.9	2632.2	2841.3	10.11	209.1	2632.2	2841.3	9.885	41.82	2632.0	2841.0	1.174
200	357.0	2661.6	2880.0	10.20	218.4	2661.6	2880.0	9.968	43.66	2661.5	2879.8	1.285
220	372.1	2691.3	2918.9	10.28	227.6	2691.3	2918.9	10.05	45.51	2691.2	2918.8	1.395
240	387.2	2721.3	2958.1	10.35	236.8	2721.2	2958.1	10.13	47.36	2721.1	2957.9	1.496
260	402.3	2751.4	2987.5	10.43	246.1	2751.4	2987.5	10.20	49.20	2751.3	2987.3	1.593
280	417.4	2781.8	3037.1	10.50	255.3	2781.8	3037.1	10.27	51.05	2781.7	3036.8	1.690
300	432.5	2812.4	3077.0	10.57	264.5	2812.4	3077.0	10.35	52.90	2812.4	3076.9	1.783
320	447.6	2843.3	3117.1	10.64	273.7	2843.3	3117.1	10.41	54.75	2843.3	3117.0	1.876
340	462.6	2874.5	3157.4	10.71	283.0	2874.5	3157.4	10.48	56.59	2874.4	3157.4	1.963
360	477.7	2905.9	3198.1	10.77	292.2	2905.8	3198.1	10.55	58.44	2905.8	3198.0	2.040
380	492.8	2937.5	3238.9	10.84	301.4	2937.5	3238.9	10.61	60.28	2937.4	3238.8	2.112
400	507.9	2969.4	3280.1	10.90	310.7	2969.4	3280.1	10.67	62.13	2969.4	3280.0	2.190
420	523.0	3001.6	3321.5	10.96	319.9	3001.6	3321.5	10.73	63.98	3001.5	3321.4	2.267
440	538.1	3034.0	3363.2	11.02	329.1	3034.0	3363.1	10.79	65.82	3034.0	3363.1	2.351
460	553.2	3068.7	3405.1	11.08	338.4	3068.7	3405.1	10.85	67.67	3068.7	3405.0	2.440
480	568.3	3099.7	3447.3	11.13	347.6	3099.7	3447.3	10.91	69.52	3099.7	3447.2	2.530
500	583.4	3132.9	3489.8	11.19	356.8	3132.9	3489.8	10.96	71.36	3132.9	3489.7	2.619
520	598.5	3166.5	3532.5	11.24	366.1	3166.5	3532.5	11.02	73.21	3166.4	3532.5	2.707
540	613.6	3200.3	3575.6	11.30	375.3	3200.3	3575.6	11.07	75.06	3200.2	3575.5	2.799
560	628.7	3234.4	3618.9	11.35	384.5	3234.4	3618.9	11.12	76.90	3234.3	3618.8	2.887
580	643.7	3268.7	3662.5	11.40	393.7	3268.7	3662.5	11.18	78.75	3268.7	3662.4	2.974
600	658.8	3303.4	3706.3	11.45	403.0	3303.4	3706.3	11.23	80.59	3303.3	3706.3	3.060
640	689.0	3373.5	3794.9	11.55	421.4	3373.5	3794.9	11.33	84.29	3373.5	3794.9	3.154
680	719.2	3444.8	3884.7	11.65	439.9	3444.8	3884.7	11.42	87.98	3444.7	3884.6	4.264
720	749.4	3517.2	3975.5	11.74	458.4	3517.2	3975.5	11.51	91.67	3517.2	3975.5	5.354
760	779.6	3560.7	4067.5	11.83	476.8	3560.7	4067.5	11.61	95.36	3560.7	4067.5	6.454
800	809.7	3665.4	4160.7	11.92	495.3	3665.4	4160.7	11.69	99.06	3665.4	4160.6	7.563

Table E.24: Subcooled water and Superheated Steam (0.1 bar to 1 atmosphere)

T(°C)	0.2 bar ($T_{\text{sat}} = 60.1^{\circ}\text{C}$)			0.5 bar ($T_{\text{sat}} = 81.3^{\circ}\text{C}$)			1 bar ($T_{\text{sat}} = 99.6^{\circ}\text{C}$)			1.01325 bar ($T_{\text{sat}} = 100.0^{\circ}\text{C}$)		
	v	u	h	s	v	u	h	s	v	u	h	s
0.01	0.001000	0.000360	0.02036	0.00000	0.001000	0.000915	0.05092	0.00000	0.001000	0.001841	0.1019	0.00001
20	0.001002	83.92	83.94	0.296	0.001002	83.91	83.96	0.2965	0.001002	83.91	84.01	0.2965
40	0.001008	167.5	167.6	0.572	0.001008	167.5	167.6	0.5724	0.001008	167.5	167.6	0.5724
60	0.001017	251.1	251.2	0.831	0.001017	251.1	251.2	0.8312	0.001017	251.1	251.2	0.8312
80	8.118	2485.3	2647.7	8.020	0.001029	334.9	335.0	1.075	0.001029	334.9	335.0	1.075
100	8.586	2514.5	2686.2	8.126	3.419	2511.5	2682.4	7.695	1.636	2506.2	2675.8	7.355
120	9.052	2543.6	2724.6	8.227	3.608	2541.3	2721.7	7.798	1.793	2537.3	2716.6	7.461
140	9.517	2572.7	2763.1	8.322	3.796	2570.9	2760.7	7.895	1.889	2567.8	2756.6	7.561
160	9.98	2602.0	2801.6	8.413	3.983	2600.5	2799.7	7.987	1.984	2598.0	2786.4	7.855
180	10.44	2631.4	2840.3	8.500	4.170	2630.2	2838.7	8.075	2.079	2628.1	2836.0	7.744
200	10.91	2661.0	2879.1	8.584	4.356	2660.0	2877.8	8.159	2.172	2658.2	2875.4	7.829
220	11.37	2690.8	2918.2	8.665	4.542	2689.9	2917.0	8.240	2.266	2688.4	2915.0	7.911
240	11.83	2720.8	2957.4	8.743	4.728	2720.0	2956.4	8.319	2.360	2718.7	2954.7	7.980
260	12.30	2751.0	2996.9	8.818	4.913	2750.3	2996.0	8.394	2.453	2749.2	2994.4	8.066
280	12.76	2781.4	3036.6	8.892	5.099	2780.8	3035.8	8.468	2.546	2779.8	3034.4	8.140
300	13.22	2812.1	3076.5	8.962	5.284	2811.6	3075.8	8.539	2.639	2810.7	3074.5	8.211
320	13.68	2843.0	3116.7	9.031	5.469	2842.5	3116.0	8.608	2.732	2841.7	3114.9	8.280
340	14.14	2874.2	3157.1	9.098	5.654	2873.7	3156.5	8.675	2.825	2873.0	3155.5	8.347
360	14.61	2905.6	3197.7	9.164	5.840	2905.2	3197.2	8.740	2.917	2904.5	3196.2	8.413
380	15.07	2937.3	3238.6	9.227	6.025	2936.9	3238.1	8.804	3.010	2936.3	3237.3	8.477
400	15.53	2969.2	3279.8	9.289	6.209	2968.9	3279.3	8.866	3.103	2968.3	3278.5	8.539
420	15.99	3001.4	3321.2	9.350	6.394	3001.1	3320.8	8.927	3.195	3000.5	3320.1	8.600
440	16.45	3033.8	3362.9	9.409	6.579	3033.5	3362.5	8.986	3.288	3033.0	3361.8	8.659
460	16.92	3065.5	3404.8	9.467	6.764	3064.5	3404.5	9.044	3.381	3065.8	3403.9	8.717
480	17.38	3099.5	3447.1	9.524	6.949	3099.3	3446.7	9.101	3.473	3098.8	3446.2	8.774
500	17.84	3132.8	3489.6	9.580	7.134	3132.6	3489.2	9.156	3.566	3132.2	3488.7	8.830
520	18.30	3166.3	3532.3	9.634	7.319	3166.1	3532.0	9.211	3.658	3165.7	3531.5	8.885
540	18.76	3200.1	3575.4	9.688	7.503	3199.9	3575.1	9.265	3.751	3199.6	3574.6	8.938
560	19.22	3234.2	3618.7	7.688	7.688	3234.0	3618.4	9.317	3.843	3233.7	3618.0	8.991
580	19.69	3268.6	3662.3	9.792	7.873	3268.4	3662.1	9.369	3.936	3268.1	3661.6	9.043
600	20.15	3303.2	3706.2	9.843	8.058	3303.1	3706.0	9.420	4.028	3302.8	3705.6	9.094
640	21.07	3373.4	3794.8	9.942	8.427	3373.2	3794.6	9.519	4.213	3373.0	3794.3	9.193
680	21.99	3444.7	3884.6	10.04	8.797	3444.5	3884.4	9.615	4.398	3444.3	3884.1	9.289
720	22.92	3517.1	3975.4	10.13	9.166	3517.0	3975.3	9.709	4.582	3516.7	3975.0	9.383
760	23.84	3590.6	4067.4	10.22	9.535	3590.5	4067.3	9.800	4.767	3590.3	4067.0	9.474
800	24.76	3665.3	4160.6	10.31	9.905	3665.2	4160.4	9.888	4.952	3665.0	4160.2	9.562

Table E.25: Subcooled water and Superheated Steam (2 bar to 8 bar)

T(°C)	v	2 bar ($T_{\text{sat}} = 120.2^\circ\text{C}$)		4 bar ($T_{\text{sat}} = 143.6^\circ\text{C}$)		6 bar ($T_{\text{sat}} = 158.8^\circ\text{C}$)		8 bar ($T_{\text{sat}} = 170.4^\circ\text{C}$)		s	T(°C)
		u	h	s	v	u	h	s	v	u	
0.01	0.001000	0.003688	0.2037	0.00001	0.001000	0.007365	0.4074	0.00003	0.001000	0.0110	0.00004
20	0.001002	83.91	84.11	0.2965	0.001002	83.89	84.29	0.2964	0.001002	83.88	84.48
40	0.001008	167.5	167.7	0.5724	0.001008	167.5	167.9	0.5723	0.001008	167.5	168.1
60	0.001017	251.3	0.8311	0.001017	251.1	0.8315	0.5310	0.001017	251.0	0.6542	0.5721
80	0.001029	334.9	335.1	1.075	0.001029	334.8	335.2	1.075	0.001029	334.8	335.4
100	0.001043	419.0	419.2	1.307	0.001043	418.9	419.3	1.307	0.001043	418.8	419.6
120	0.001060	503.6	503.8	1.528	0.001060	503.5	503.9	1.528	0.001060	503.4	504.1
140	0.9353	2561.3	2748.3	7.231	0.001080	588.8	589.2	1.739	0.001080	588.7	589.4
160	0.9843	2592.8	2789.0	7.329	0.4839	2581.6	2775.2	6.983	0.3167	2569.0	2759.0
180	1.033	2823.9	2830.4	7.421	0.5094	2614.9	2818.6	7.081	0.3347	2606.0	2806.0
200	1.081	2654.7	2870.8	7.508	0.5343	2647.3	2861.0	7.172	0.3521	2639.4	2850.7
220	1.128	2685.4	2911.0	7.591	0.5589	2679.1	2902.7	7.259	0.3690	2672.6	2894.0
240	1.175	2716.1	2951.2	7.671	0.5831	2710.7	2944.0	7.341	0.3857	2705.2	2936.6
260	1.222	2746.9	2991.4	7.748	0.6072	2742.2	2986.1	7.419	0.4021	2737.4	2978.6
280	1.269	2777.8	3031.7	7.822	0.6311	2773.7	3026.1	7.495	0.4183	2769.4	3020.4
300	1.316	2808.8	3072.1	7.894	0.6549	2805.2	3067.1	7.568	0.4344	2801.4	3062.1
320	1.363	2840.1	3112.7	7.964	0.6786	2836.8	3108.2	7.638	0.4504	2833.4	3103.7
340	1.410	2871.5	3153.4	8.031	0.7022	2868.5	3149.4	7.706	0.4663	2865.5	3145.3
360	1.456	2903.1	3194.4	8.097	0.7257	2900.4	3190.7	7.773	0.4822	2897.6	3187.0
380	1.503	2935.0	3225.6	8.161	0.7492	2925.2	3232.2	7.837	0.4980	2930.0	3228.8
400	1.549	2967.1	3277.0	8.223	0.7726	2964.8	3273.9	7.900	0.5137	2962.5	3270.7
420	1.596	2999.5	3318.6	8.284	0.7960	2997.3	3315.7	7.961	0.5294	2995.2	3312.8
440	1.642	3032.0	3360.5	8.344	0.8194	3030.1	3357.8	8.021	0.5451	3028.1	3355.1
460	1.689	3064.9	3402.6	8.402	0.8426	3063.0	3400.1	8.080	0.5608	3059.1	3407.7
480	1.735	3098.0	3445.0	8.459	0.8661	3096.3	3442.7	8.137	0.5764	3094.5	3440.4
500	1.781	3131.3	3487.6	8.515	0.8894	3129.7	3485.5	8.193	0.5920	3128.1	3483.3
520	1.828	3165.0	3530.5	8.570	0.9126	3163.5	3528.5	8.248	0.6076	3162.0	3526.5
540	1.874	3198.9	3573.7	8.624	0.9359	3197.5	3571.8	8.302	0.6222	3196.0	3569.9
560	1.920	3233.0	3617.1	8.676	0.9591	3231.7	3615.4	8.355	0.6387	3230.4	3613.6
580	1.967	3267.5	3660.8	8.728	0.9824	3266.2	3659.2	8.407	0.6542	3265.0	3657.5
600	2.013	3302.2	3704.8	8.779	1.006	3301.0	3703.2	8.458	0.6698	3299.8	3701.7
640	2.106	3372.4	3793.6	8.879	1.052	3371.4	3792.2	8.558	0.7008	3370.3	3790.8
680	2.198	3443.8	3883.4	8.975	1.098	3442.9	3882.2	8.654	0.7318	3441.9	3881.0
720	2.291	3516.3	3974.4	9.068	1.145	3515.4	3973.3	8.748	0.7627	3514.6	3972.2
760	2.383	3589.9	4066.5	9.159	1.191	3589.1	4065.5	8.839	0.7937	3588.3	4064.5
800	2.476	3664.7	4159.8	9.248	1.237	3663.9	4158.9	8.927	0.8246	3663.2	4157.9

Table E.26: Subcooled water and Superheated Steam (10 bar to 40 bar)

T(°C)	10 bar ($T_{\text{sat}} = 179.9^{\circ}\text{C}$)			20 bar ($T_{\text{sat}} = 212.4^{\circ}\text{C}$)			30 bar ($T_{\text{sat}} = 233.9^{\circ}\text{C}$)			40 bar ($T_{\text{sat}} = 250.4^{\circ}\text{C}$)							
	v	u	s	v	u	s	v	u	s	v	u	s					
0.01	0.001000	0.01827	1.018	0.00007	0.00099	0.03601	0.0034	0.00013	0.00099	0.05321	3.049	0.00019	0.00098	0.06989	4.063	0.00024	0.01
20	0.001001	83.86	84.86	0.2963	0.001001	83.80	85.80	0.2961	0.001000	83.74	86.74	0.2959	0.001000	83.68	87.68	0.2957	20
40	0.001007	167.4	168.4	0.5720	0.001007	167.3	169.3	0.5717	0.001007	167.2	170.2	0.5713	0.001006	167.1	171.1	0.5709	40
60	0.001017	251.0	252.0	0.8307	0.001016	250.8	252.8	0.8302	0.001016	250.6	253.7	0.8296	0.001015	250.4	254.5	0.8291	60
80	0.001029	334.7	335.7	1.075	0.001028	334.4	336.5	1.074	0.001028	334.2	337.3	1.073	0.001027	334.0	338.1	1.073	80
100	0.001043	418.7	419.8	1.306	0.001042	418.4	420.5	1.306	0.001042	418.2	421.3	1.305	0.001041	417.9	422.0	1.304	100
120	0.001060	503.3	504.3	1.527	0.001059	502.9	505.1	1.526	0.001059	502.6	505.8	1.525	0.001058	502.2	506.5	1.524	120
140	0.001079	588.5	589.6	1.739	0.001079	588.1	590.3	1.738	0.001078	587.7	590.9	1.737	0.001077	587.3	591.6	1.736	140
160	0.001102	674.7	675.8	1.942	0.001101	674.2	676.4	1.941	0.001101	673.7	677.0	1.940	0.001100	673.2	677.6	1.939	160
180	0.1944	2583.0	2777.4	6.586	0.001127	761.4	763.7	2.138	0.001126	760.8	764.2	2.137	0.001125	760.2	764.7	2.135	180
200	0.2060	2622.3	2828.3	6.695	0.001156	850.3	852.6	2.330	0.001155	849.5	853.0	2.329	0.001154	848.8	853.4	2.327	200
220	0.2170	2658.6	2875.6	6.793	0.1022	2617.3	2821.7	6.387	0.001189	940.3	943.8	2.517	0.001188	939.3	944.1	2.515	220
240	0.2276	2693.4	2921.0	6.884	0.1085	2600.2	2877.2	6.497	0.06823	2619.9	2824.6	6.228	0.001228	1032.7	1037.6	2.700	240
260	0.2379	2727.4	2965.2	6.968	0.1144	2699.7	2928.5	6.595	0.07289	2667.8	2864.6	6.346	0.05778	2837.2	2873.0	2.700	260
280	0.2480	2760.7	3008.7	7.048	0.1200	2737.1	2977.2	6.885	0.07716	2710.7	2942.2	6.449	0.05549	2802.9	2902.9	2.700	280
300	0.2580	2793.7	3051.7	7.125	0.1255	2773.2	3024.3	6.769	0.08118	2750.8	2994.3	6.541	0.05987	2726.2	2961.7	2.700	300
320	0.2678	2826.5	3094.4	7.198	0.1308	2808.5	3070.2	6.847	0.08502	2789.1	3044.2	6.627	0.06202	2768.2	3016.3	6.458	320
340	0.2776	2859.3	3136.9	7.268	0.1360	2843.2	3115.3	6.922	0.08874	2826.2	3092.4	6.707	0.06502	2808.1	3068.1	6.544	340
360	0.2873	2892.1	3179.4	7.337	0.1411	2877.6	3159.9	6.964	0.09235	2862.4	3139.5	6.782	0.06790	2846.5	3118.1	6.624	360
380	0.2970	2924.9	3221.9	7.403	0.1462	2911.8	3204.2	7.063	0.09590	2988.1	3186.8	6.854	0.07070	2883.9	3166.7	6.699	380
400	0.3066	2957.8	3264.4	7.467	0.1512	2945.8	3248.2	7.129	0.09938	2933.4	3231.6	6.923	0.07343	2920.6	3214.4	6.771	400
420	0.3162	2990.8	3307.0	7.529	0.1562	2979.8	3292.2	7.193	0.1028	2968.5	3277.0	6.990	0.07611	2956.9	3261.4	6.840	420
440	0.3257	3024.1	3349.8	7.590	0.1611	3013.9	3336.1	7.256	0.1062	3003.5	3322.1	7.054	0.07874	2992.9	3307.9	6.906	440
460	0.3352	3057.5	3392.7	7.649	0.1660	3048.1	3380.0	7.317	0.1086	3038.5	3367.2	7.116	0.08134	3028.7	3354.0	6.970	460
480	0.3447	3091.4	3435.7	7.707	0.1708	3032.3	3424.0	7.376	0.1129	3073.4	3412.1	7.177	0.08380	3064.4	3400.0	7.032	480
500	0.3541	3124.9	3479.0	7.764	0.1757	3116.7	3468.1	7.433	0.1162	3108.5	3457.0	7.236	0.08644	3100.1	3445.8	7.092	500
520	0.3636	3158.9	3522.5	7.819	0.1805	3151.3	3512.3	7.490	0.11195	3143.6	3502.0	7.293	0.08896	3135.8	3491.6	7.150	520
540	0.3730	3193.2	3566.2	7.874	0.1853	3186.0	3566.6	7.545	0.1227	3178.8	3547.0	7.349	0.09146	3171.5	3537.3	7.207	540
560	0.3824	3227.7	3610.1	7.927	0.1901	3221.0	3601.2	7.599	0.1280	3214.2	3592.2	7.404	0.09394	3207.4	3583.1	7.263	560
580	0.3917	3262.4	3654.2	7.980	0.1949	3266.1	3645.8	7.652	0.1292	3249.7	3637.4	7.458	0.09640	3243.3	3628.9	7.317	580
600	0.4011	3297.4	3698.6	8.031	0.200	3291.5	3680.7	7.704	0.1324	3285.5	3682.8	7.510	0.09886	3279.4	3674.8	7.370	600
640	0.4198	3368.2	3788.0	8.131	0.209	3362.9	3781.1	7.805	0.1389	3357.5	3774.1	7.612	0.1037	3352.1	3767.0	7.474	640
680	0.4385	3440.0	3878.5	8.228	0.219	3435.2	3872.3	7.903	0.1452	3430.4	3866.1	7.711	0.1086	3425.5	3856.8	7.573	680
720	0.4572	3512.8	3970.0	8.322	0.228	3508.5	3964.4	7.998	0.1516	3504.1	3958.8	8.090	0.1134	3495.7	3953.2	7.869	720
760	0.4758	3586.8	4062.5	8.413	0.237	3562.8	4057.5	8.090	0.1579	3578.8	4052.5	8.179	0.1182	3574.8	4047.4	7.762	760
800	0.4944	3661.8	4156.1	8.502	0.247	3658.1	4151.6	8.179	0.1642	3654.5	4147.0	7.989	0.1229	3650.8	4142.5	7.852	800

Table E.27: Subcooled water and Superheated Steam (50 bar to 80 bar)

T(°C)	50 bar ($T_{\text{sat}} = 263.9^{\circ}\text{C}$)			60 bar ($T_{\text{sat}} = 275.6^{\circ}\text{C}$)			70 bar ($T_{\text{sat}} = 285.8^{\circ}\text{C}$)			80 bar ($T_{\text{sat}} = 295.0^{\circ}\text{C}$)		
	v	u	h	s	v	u	h	s	v	u	h	s
0.01	0.000998	0.08604	5.074	0.00029	0.000997	0.10167	6.085	0.00034	0.000997	0.11679	7.094	0.00038
20	0.001000	83.62	88.61	0.2955	0.000999	83.55	89.55	0.2952	0.000999	83.49	90.48	0.2950
40	0.001006	166.9	172.0	0.5705	0.001005	166.8	172.8	0.5701	0.001005	166.7	173.7	0.5697
60	0.001015	250.3	255.3	0.8286	0.001014	250.1	256.2	0.8280	0.001014	249.9	257.0	0.8275
80	0.001027	333.8	338.9	1.072	0.001026	333.5	339.7	1.071	0.001026	333.3	340.5	1.071
100	0.001041	417.6	422.8	1.303	0.001040	417.3	423.5	1.302	0.001040	417.0	424.3	1.302
120	0.001058	501.9	507.2	1.523	0.001057	501.5	507.9	1.523	0.001057	501.2	508.6	1.522
140	0.001077	586.8	592.2	1.734	0.001076	586.4	592.9	1.733	0.001076	586.0	593.5	1.732
160	0.001099	672.7	678.1	1.938	0.001098	672.1	678.7	1.936	0.001097	671.6	679.3	1.935
180	0.001124	759.6	765.2	2.134	0.001123	759.0	765.7	2.133	0.001122	758.4	766.2	2.131
200	0.001153	848.0	853.8	2.325	0.001152	847.3	854.2	2.324	0.001151	846.6	854.6	2.322
220	0.001187	938.4	944.4	2.513	0.001186	937.6	944.7	2.511	0.001184	936.7	945.0	2.509
240	0.001227	1031.5	1037.7	2.698	0.001225	1030.4	1037.8	2.696	0.001224	1029.3	1037.9	2.695
260	0.001275	1128.4	1134.8	2.884	0.001273	1127.0	1134.6	2.881	0.001271	1125.6	1134.5	2.879
280	0.004227	2646.7	2858.1	6.091	0.03220	2606.0	2805.2	5.928	0.001331	1227.0	1236.3	3.066
300	0.04535	2698.9	2925.6	6.211	0.03619	2668.3	2885.5	6.070	0.02949	2633.4	2839.8	5.934
320	0.04813	2745.5	2986.2	6.315	0.03878	2720.9	2983.5	6.187	0.03201	2693.8	2917.9	6.067
340	0.05073	2778.7	3042.4	6.408	0.04114	2768.1	3014.9	6.289	0.03423	2745.9	2985.5	6.180
360	0.05319	3095.6	3289.7	6.493	0.04534	3011.9	3070.0	6.380	0.03626	2793.2	3007.0	6.278
380	0.05555	3286.1	3416.8	6.573	0.04542	2833.6	3126.1	6.465	0.03816	2837.3	3104.4	6.368
400	0.05784	2907.4	3196.6	6.648	0.04742	2883.6	3178.2	6.543	0.03996	2879.4	3159.1	6.450
420	0.06007	2945.0	3245.3	6.719	0.04936	2932.6	3228.8	6.617	0.04169	2919.9	3211.8	6.527
440	0.06225	2982.0	3293.3	6.788	0.05124	2970.9	3278.3	6.688	0.04337	2959.4	3263.0	6.600
460	0.06439	3018.7	3340.7	6.853	0.05304	3008.5	3327.0	6.755	0.04650	2998.1	3313.1	6.669
480	0.06650	3055.2	3387.7	6.917	0.05489	3045.9	3375.2	6.820	0.04816	3036.3	3362.5	6.736
500	0.06858	3091.6	3434.5	6.978	0.05667	3082.9	3422.9	6.882	0.04816	3074.1	3411.3	6.800
520	0.07064	3127.8	3481.1	7.037	0.05843	3119.8	3470.4	6.943	0.04970	3111.7	3459.6	6.861
540	0.07268	3164.1	3527.5	7.095	0.06016	3156.7	3517.6	7.002	0.05121	3149.1	3607.6	6.921
560	0.07470	3200.4	3574.0	7.152	0.06188	3193.5	3564.7	7.059	0.05271	3186.4	3555.4	6.979
580	0.07671	3236.8	3620.4	7.207	0.06358	3203.3	3611.8	7.115	0.05420	3223.7	3603.1	7.036
600	0.07870	3273.3	3666.8	7.260	0.06526	3267.2	3658.8	7.169	0.05566	3261.0	3650.6	7.091
640	0.08265	3346.7	3759.9	7.365	0.06860	3341.2	3752.8	7.275	0.05857	3335.7	3746.6	7.197
680	0.08657	3420.6	3853.5	7.465	0.07190	3415.7	3847.1	7.376	0.06143	3410.8	3840.8	7.299
720	0.09045	3495.3	3947.6	7.562	0.07517	3490.9	3941.9	7.473	0.06426	3486.4	3936.2	7.397
760	0.09431	3570.8	4042.4	7.655	0.07842	3566.8	4037.3	7.567	0.06706	3562.7	4032.2	7.492
800	0.09815	3647.1	4137.9	7.746	0.08164	3633.4	4133.3	7.658	0.06985	3639.7	4128.7	7.584

Table E.28: Subcooled water and Superheated Steam (90 bar to 140 bar)

T (°C)	v	90 bar ($T_{\text{sat}} = 303.3^{\circ}\text{C}$)	s	100 bar ($T_{\text{sat}} = 311.0^{\circ}\text{C}$)			120 bar ($T_{\text{sat}} = 324.7^{\circ}\text{C}$)			140 bar ($T_{\text{sat}} = 336.7^{\circ}\text{C}$)		
				v	h	s	v	h	s	v	h	s
0.01	0.000996	0.1455	9.107	0.00046	0.000995	0.1591	10.11	0.00049	0.000994	0.1847	12.12	0.00055
20	0.000998	83.37	92.35	0.2946	0.000997	83.31	93.29	0.2944	0.000996	83.19	95.15	0.2939
40	0.001004	166.5	175.5	0.5689	0.001003	166.3	176.4	0.5685	0.001003	166.1	178.1	0.5678
60	0.001013	249.6	258.7	0.8265	0.001013	249.4	259.5	0.8259	0.001012	249.1	261.2	0.8249
80	0.001025	332.9	342.1	1.070	0.001024	332.6	342.9	1.069	0.001023	332.2	344.5	1.068
100	0.001039	416.4	425.8	1.300	0.001038	416.2	426.5	1.299	0.001038	415.6	428.1	1.298
120	0.001055	500.5	510.0	1.520	0.001055	500.2	510.7	1.519	0.001054	499.5	512.1	1.517
140	0.001074	585.2	594.8	1.730	0.001074	584.8	595.5	1.729	0.001073	583.9	596.8	1.727
160	0.001096	670.7	680.5	1.933	0.001095	670.2	681.1	1.932	0.001094	669.2	682.3	1.930
180	0.001121	757.2	767.3	2.129	0.001120	756.6	767.8	2.127	0.001118	755.4	768.9	2.125
200	0.001149	845.1	855.5	2.319	0.001148	844.4	855.9	2.318	0.001146	843.0	856.8	2.315
220	0.001182	934.9	945.6	2.506	0.001181	934.1	945.9	2.504	0.001179	932.4	946.5	2.500
240	0.001221	1027.2	1038.2	2.690	0.001219	1026.1	1038.3	2.688	0.001216	1024.0	1038.6	2.683
260	0.001267	1122.8	1134.2	2.873	0.001265	1121.5	1134.1	2.871	0.001262	1118.2	1134.0	2.866
280	0.001325	1223.4	1235.3	3.059	0.001323	1221.6	1234.8	3.056	0.001317	1218.1	1233.9	3.050
300	0.001402	1331.6	1344.3	3.253	0.001398	1329.1	1343.1	3.248	0.001390	1324.3	1340.9	3.240
320	0.02271	2629.5	2833.9	5.835	0.01927	2589.9	2782.7	5.713	0.001494	1442.4	1460.3	3.444
340	0.02486	2695.8	2919.6	5.977	0.02149	2667.2	2882.1	5.878	0.01621	2598.9	2793.5	5.672
360	0.02672	2752.0	2992.5	6.094	0.02333	2729.3	2962.6	6.007	0.01812	2878.4	2895.9	5.837
380	0.02840	2802.4	3038.0	6.196	0.02495	2783.6	3033.1	6.117	0.01971	2742.6	2979.1	5.866
400	0.02996	2849.1	3118.8	6.287	0.02644	2833.0	3097.4	6.214	0.02111	2798.6	3051.9	6.076
420	0.03144	2893.2	3176.1	6.371	0.02783	2879.2	3157.5	6.302	0.02239	2849.6	3118.2	6.173
440	0.03284	2935.5	3231.1	6.450	0.02915	2923.1	3214.6	6.383	0.02358	2897.1	3180.1	6.261
460	0.03420	2976.6	3284.4	6.523	0.03041	2965.4	3295.5	6.459	0.02471	2942.3	3238.8	6.343
480	0.03551	3016.7	3336.3	6.593	0.03163	3006.6	3322.9	6.531	0.02579	2985.8	3295.2	6.418
500	0.03680	3056.2	3387.3	6.660	0.03281	3046.9	3375.1	6.599	0.02683	3028.0	3350.0	6.490
520	0.03805	3095.1	3437.6	6.724	0.03397	3086.6	3426.3	6.665	0.02784	3069.3	3403.4	6.558
540	0.03928	3133.7	3487.2	6.786	0.03510	3125.9	3476.9	6.728	0.02882	3108.9	3455.8	6.624
560	0.04049	3172.1	3536.5	6.846	0.03621	3164.8	3526.9	6.789	0.02978	3150.0	3507.4	6.686
580	0.04168	3210.3	3585.4	6.904	0.03730	3203.5	3576.5	6.847	0.03072	3189.8	3558.4	6.747
600	0.04286	3248.4	3634.2	6.960	0.03838	3242.1	3625.8	6.905	0.03165	3229.2	3609.0	6.806
640	0.04518	3324.6	3731.2	7.069	0.04049	3319.0	3723.9	7.014	0.03346	3307.6	3709.2	6.918
680	0.04746	3400.8	3828.0	7.173	0.04257	3395.8	3821.5	7.119	0.03523	3385.7	3808.5	7.024
720	0.04971	3477.4	3924.8	7.272	0.04461	3472.9	3919.0	7.219	0.03697	3463.8	3907.5	7.126
760	0.05193	3554.5	4021.9	7.368	0.04663	3550.4	4016.7	7.316	0.03868	3542.2	4006.4	7.223
800	0.05413	3632.2	4119.4	7.461	0.04862	3628.5	4114.7	7.409	0.04037	3621.0	4105.4	7.318

Table E.29: Subcooled water and Superheated Steam (160 bar to 220 bar)

T°C	160 bar ($T_{\text{sat}} = 347.4^{\circ}\text{C}$)			180 bar ($T_{\text{sat}} = 357.0^{\circ}\text{C}$)			200 bar ($T_{\text{sat}} = 365.7^{\circ}\text{C}$)			220 bar ($T_{\text{sat}} = 373.7^{\circ}\text{C}$)		
	v	u	h	s	v	u	h	s	v	u	h	s
0.01	0.000992	0.2302	16.11	0.00061	0.000991	0.2501	18.09	0.00063	0.000990	0.2680	20.08	0.00062
20	0.000995	82.95	98.87	0.2930	0.000994	82.83	100.7	0.2926	0.000993	82.71	102.6	0.2921
40	0.001001	165.6	181.7	0.5682	0.001000	165.4	183.4	0.5654	0.000999	165.2	185.2	0.5646
60	0.001010	248.4	264.5	0.8228	0.001009	248.1	266.2	0.8218	0.001008	247.7	267.9	0.8207
80	0.001022	331.3	347.6	1.065	0.001021	330.9	349.2	1.064	0.001020	330.4	350.8	1.062
100	0.001036	414.5	431.1	1.295	0.001035	414.0	432.6	1.293	0.001034	413.4	434.1	1.292
120	0.001052	498.1	515.0	1.514	0.001051	497.5	516.4	1.512	0.001050	496.8	517.8	1.510
140	0.001070	582.3	599.5	1.723	0.001069	581.5	600.8	1.721	0.001068	580.7	602.1	1.719
160	0.001091	667.3	684.7	1.925	0.001090	666.3	685.9	1.923	0.001089	665.4	687.2	1.920
180	0.001115	753.2	771.0	2.120	0.001114	752.0	772.1	2.117	0.001112	750.9	773.2	2.115
200	0.001143	840.3	858.6	2.309	0.001141	838.9	859.5	2.306	0.001139	837.6	860.4	2.303
220	0.001174	929.0	947.8	2.494	0.001172	927.4	948.5	2.490	0.001170	925.8	949.2	2.487
240	0.001211	1019.9	1039.3	2.675	0.001208	1018.0	1039.7	2.671	0.001205	1016.0	1040.1	2.668
260	0.001254	1113.7	1133.8	2.856	0.001251	1111.3	1133.8	2.851	0.001247	1108.9	1133.8	2.847
280	0.001307	1211.5	1232.5	3.038	0.001302	1208.4	1231.8	3.032	0.001298	1205.3	1231.3	3.026
300	0.001375	1315.2	1337.2	3.224	0.001368	1311.0	1335.6	3.216	0.001361	1306.9	1334.1	3.209
320	0.001467	1428.5	1451.9	3.420	0.001456	1422.2	1448.4	3.409	0.001445	1416.4	1445.3	3.399
340	0.001616	1561.4	1587.3	3.645	0.001591	1550.1	1578.7	3.625	0.001569	1540.1	1571.5	3.608
360	0.001766	2538.7	2715.6	5.462	0.001624	2415.8	2233.9	4.658	0.001565	2186.4	1876.1	4.101
380	0.01288	2642.2	2848.3	5.668	0.01042	2577.4	2764.9	5.505	0.0025258	2494.0	2659.2	5.314
400	0.01428	2719.0	2947.5	5.818	0.01191	2671.8	2886.3	5.688	0.009950	2617.8	2816.8	5.552
420	0.01548	2783.2	3030.9	5.940	0.01312	2745.7	2981.9	5.828	0.01120	2704.5	2928.6	5.716
440	0.01655	2840.2	3105.0	6.045	0.01417	2808.9	3064.0	5.945	0.01225	2775.3	3020.3	5.847
460	0.01753	2892.5	3173.0	6.139	0.01512	2865.6	3120.7	6.047	0.01317	2837.2	3101.6	5.958
480	0.01845	2941.5	3226.7	6.225	0.01599	2917.9	3205.7	6.138	0.01401	2893.2	3173.4	6.056
500	0.01932	2988.1	3297.3	6.305	0.01681	2967.1	3269.7	6.222	0.01479	2945.3	3241.2	6.145
520	0.02016	3038.1	3355.6	6.379	0.01759	3014.1	3330.7	6.300	0.01553	2994.6	3305.2	6.226
540	0.02096	3076.7	3412.1	6.449	0.01833	3059.5	3389.5	6.373	0.01623	3041.8	3366.4	6.303
560	0.02174	3119.4	3467.3	6.516	0.01905	3103.6	3446.6	6.443	0.01690	3087.5	3425.6	6.374
580	0.02250	3161.4	3521.4	6.581	0.01975	3146.8	3502.4	6.509	0.01755	3132.0	3483.0	6.443
600	0.02324	3202.8	3574.6	6.642	0.02043	3189.3	3557.0	6.572	0.01818	3175.5	3539.2	6.508
640	0.02467	3284.5	3679.2	6.759	0.02174	3272.7	3664.0	6.692	0.01940	3260.7	3648.7	6.630
680	0.02607	3365.2	3782.2	6.870	0.02301	3354.7	3768.9	6.804	0.02056	3344.2	3755.5	6.745
720	0.02742	3445.4	3864.1	6.975	0.02424	3436.0	3872.3	6.911	0.02169	3426.6	3865.3	6.853
760	0.02875	3525.5	3985.5	7.075	0.02544	3517.1	3975.0	7.012	0.02279	3508.6	3964.4	6.955
800	0.03006	3605.7	4086.6	7.171	0.02662	3598.1	4077.2	7.109	0.02387	3590.4	4067.7	7.053

Table E.30: Supercritical steam (250 bar to 500 bar)

T (°C)	250 bar			300 bar			400 bar			500 bar			T (°C)
	v	u	h	s	v	u	h	s	v	u	h	s	
0.01	0.000988	0.3049	25	0.00056	0.000986	0.3308	29.9	0.00043	0.000981	0.3511	39.6	-0.00008	0.000977
20	0.000991	82.41	107.18	0.2909	0.000989	82.12	111.8	0.2897	0.000985	81.52	120.9	0.2872	0.00098
40	0.000997	164.6	189.5	0.5627	0.000995	164.1	193.9	0.5607	0.000991	163	202.6	0.5568	0.000987
60	0.001006	246.9	272.1	0.8181	0.001004	246.1	276.2	0.8156	0.001001	246.3	284.6	0.8105	0.000996
80	0.001018	329.4	354.8	1.059	0.001016	328.3	358.8	1.056	0.001011	326.3	366.8	1.05	0.001007
100	0.001031	412.1	437.9	1.288	0.001029	410.8	441.7	1.284	0.001024	408.3	449.3	1.277	0.001012
120	0.001047	495.2	521.4	1.506	0.001045	493.6	525	1.502	0.00104	490.6	532.2	1.494	0.001035
140	0.001065	578.8	605.4	1.715	0.001062	576.9	608.8	1.71	0.001057	573.3	615.6	1.701	0.001052
160	0.001085	663.1	690.2	1.915	0.001082	660.8	693.3	1.91	0.001078	656.5	695.6	1.899	0.001057
180	0.001108	748.2	775.9	2.108	0.001105	745.5	778.7	2.102	0.001098	740.5	784.4	2.091	0.001091
200	0.001135	834.4	862.7	2.296	0.00113	831.2	865.1	2.289	0.001122	825.2	870.1	2.276	0.001115
220	0.001164	921.9	951.1	2.479	0.001159	918.2	953	2.471	0.001115	911.1	957.1	2.456	0.001141
240	0.001199	1011.3	1041.3	2.658	0.001193	1006.8	1042.6	2.649	0.001181	998.4	1045.6	2.632	0.001171
260	0.001239	1103.1	1134.1	2.835	0.001231	1097.6	1134.6	2.825	0.001217	1087.4	1136.1	2.805	0.001204
280	0.001287	1198.1	1230.2	3.012	0.001277	1191.3	1229.6	3	0.001259	1178.1	1229.1	2.976	0.001243
300	0.001346	1287.4	1331.1	3.192	0.001332	1288.7	1328.7	3.176	0.001308	1273.1	1325.4	3.147	0.001288
320	0.001421	1403.2	1438.7	3.376	0.001401	1391.5	1433.5	3.355	0.001368	1371.3	1426	3.319	0.001341
340	0.001526	1519.3	1557.5	3.573	0.001493	1502.3	1547.1	3.544	0.001443	1474.8	1532.5	3.496	0.001405
360	0.001658	1714.6	1750.4	3.897	0.001715	1683.3	1704.5	3.811	0.001581	1605.1	1686.1	3.713	0.001507
380	0.0020749	1987.1	2052.8	4.35	0.002165	1887.2	1922	4.145	0.001805	1757.9	1816.6	3.962	0.001656
400	0.005284	2372.9	2510.3	5.03	0.002937	2083.3	2180.3	4.518	0.002102	1913.8	1987.5	4.213	0.001838
420	0.007579	2580	2769.4	5.42	0.004566	2371.7	2512.3	4.999	0.002518	2077.6	2174.1	4.475	0.002065
440	0.008697	2679.6	2897.1	5.601	0.006228	2562	2748.9	5.342	0.003138	2254.3	2381.2	4.761	0.002355
460	0.009617	2758.8	2991.2	5.743	0.007193	2668	2883.8	5.528	0.004149	2447.4	2613.3	5.084	0.002741
480	0.01042	2826.6	3087.1	5.861	0.007992	2752.2	2992	5.674	0.00495	2579.2	2777.2	5.305	0.003277
500	0.01114	2887.4	3165.9	5.964	0.00869	2824.1	3084.8	5.796	0.005625	2681.7	2906.7	5.475	0.003889
520	0.01181	2943.2	3238.5	6.057	0.00932	2888.1	3167.7	5.901	0.006213	2766.9	3015.4	5.613	0.004417
540	0.01244	2985.7	3306.6	6.142	0.009899	2946.7	3243.7	5.996	0.00674	2841.1	3110.7	5.732	0.004896
560	0.01303	3045.6	3371.3	6.22	0.01044	3001.6	3314.8	6.083	0.007221	2907.8	3196.7	5.837	0.005332
580	0.01359	3093.6	3433.5	6.294	0.01095	3053.6	3382.3	6.163	0.007669	2969.3	3276	5.931	0.005734
600	0.01414	3140.2	3493.7	6.364	0.01144	3103.5	3446.9	6.237	0.008089	3026.9	3350.4	6.017	0.006109
640	0.01518	3230.2	3609.7	6.494	0.01237	3198.9	3569.9	6.375	0.008869	3134.1	3488.8	6.172	0.006796
680	0.01617	3317.4	3721.5	6.614	0.01324	3280.1	3687.2	6.501	0.009589	3234	3617.6	6.31	0.007422
720	0.01711	3402.8	3830.6	6.726	0.01406	3378.6	3800.5	6.617	0.01026	3294.9	3740	6.436	0.00804
760	0.01803	3487.2	3937.9	6.832	0.01486	3465.5	3911.3	6.727	0.01091	3421.7	3857.9	6.552	0.008552
800	0.01892	3570.9	4044	6.932	0.01563	3551.4	4020.2	6.83	0.01152	3611.9	3972.8	6.661	0.009074

Table E.31: Supercritical steam (600 bar to 1000 bar)

$T(^{\circ}\text{C})$	600 bar			700 bar			800 bar			1000 bar			$T(^{\circ}\text{C})$
	v	u	h	s	v	u	h	s	v	u	h	s	
0.01	0.000972	0.27777	58.63	-0.00193	0.000968	0.1902	67.97	-0.00323	0.000964	0.0725	77.22	-0.00476	0.000957
20	0.000977	80.35	138.9	0.2818	0.000973	79.76	147.9	0.279	0.000969	79.19	156.7	0.2761	0.000962
40	0.000983	160.9	219.9	0.5489	0.00098	159.9	228.4	0.5449	0.000976	158.9	237	0.5409	0.000969
60	0.000992	241.6	301.1	0.8004	0.000969	240.2	309.4	0.7956	0.000985	238.8	317.6	0.7905	0.000978
80	0.001003	322.5	382.7	1.038	0.000999	320.7	390.6	1.032	0.000996	319.8	398.5	1.026	0.000988
100	0.001016	403.5	464.5	1.263	0.001012	401.3	472.1	1.257	0.001008	399.1	479.8	1.25	0.001
120	0.00103	484.8	546.7	1.478	0.001026	482.1	554	1.47	0.001022	479.5	561.3	1.463	0.001014
140	0.001047	566.5	629.3	1.683	0.001042	563.3	636.2	1.674	0.001037	560.2	643.2	1.666	0.001028
160	0.001065	648.5	712.4	1.879	0.001079	644.8	718.9	1.87	0.001054	641.2	725.5	1.861	0.001045
180	0.001085	731	796.2	2.068	0.001074	726.7	802.2	2.058	0.001074	722.5	808.4	2.048	0.001063
200	0.001108	814.2	880.7	2.251	0.001101	809.1	886.2	2.239	0.001095	804.3	891.8	2.228	0.001083
220	0.001133	898.2	966.2	2.428	0.001125	892.3	971	2.415	0.001118	886.7	976.1	2.402	0.001104
240	0.001161	983.2	1052.8	2.6	0.001152	976.3	1056.9	2.586	0.001143	969.8	1061.3	2.572	0.001128
260	0.001193	1069.4	1141	2.769	0.001182	1061.4	1144.1	2.752	0.001172	1053.9	1147.7	2.737	0.001154
280	0.001229	1157.3	1231	2.934	0.001216	1147.8	1232.9	2.916	0.001204	1139.1	1235.4	2.898	0.001183
300	0.00127	1247	1323.3	3.098	0.001254	1235.9	1323.7	3.077	0.00124	1225.7	1324.9	3.057	0.001215
320	0.001318	1339.2	1418.3	3.261	0.001298	1325.9	1416.7	3.237	0.00128	1313.8	1416.3	3.214	0.00125
340	0.001374	1434.2	1516.7	3.424	0.001349	1418.1	1512.5	3.395	0.001327	1403.8	1509.9	3.369	0.00129
360	0.001457	1541.1	1623.1	3.604	0.001419	1519.1	1613.7	3.565	0.001388	1500.4	1607.4	3.532	0.001259
380	0.001569	1657.8	1740.6	3.794	0.001509	1627.2	1722.8	3.742	0.001464	1602.5	1710.7	3.7	0.001238
400	0.001701	1775.1	1864.6	3.982	0.001613	1735.6	1836.8	3.916	0.001549	1704.7	1818	3.863	0.001216
420	0.001856	1893.4	1994.1	4.167	0.001731	1844.5	1955	4.086	0.001645	1807.1	1928.5	4.023	0.001532
440	0.002042	2013.3	2128.8	4.352	0.001868	1954.2	2076.9	4.255	0.001754	1909.9	2041.9	4.181	0.001609
460	0.00227	2135.4	2268.6	4.539	0.002028	2055	2202.2	4.422	0.001877	2013.2	2158	4.337	0.001539
480	0.00255	2260.1	2413.7	4.73	0.002219	2177	2330.9	4.591	0.00202	2117.2	2276.4	4.492	0.001598
500	0.002922	2388	2564.6	4.926	0.002449	2290.5	2462.9	4.761	0.002186	2222	2397.1	4.647	0.001462
520	0.003361	2512.3	2713.9	5.119	0.002732	2405.5	2598.3	4.933	0.002382	2327.5	2519.8	4.801	0.002015
540	0.003762	2617.2	2842.9	5.28	0.003067	2518.7	2733.4	5.103	0.002616	2433.9	2644.8	4.957	0.00215
560	0.004142	2709.6	2958.1	5.42	0.003378	2617.6	2854.1	5.25	0.002885	2538.2	2769	5.11	0.002305
580	0.004499	2792.1	3062	5.543	0.003683	2707.3	2865.1	5.382	0.003135	2631.2	2882	5.244	0.002484
600	0.004834	2866.9	3157	5.663	0.003975	2789.3	3067.5	5.5	0.003384	2717.4	2988.1	5.367	0.002672
640	0.005447	3000.4	3327.3	5.844	0.00452	2934.8	3251.2	5.706	0.003861	2872.1	3181	5.583	0.003026
680	0.006003	3119.6	3479.8	6.007	0.005018	3063.1	3414.4	5.881	0.004306	3008.5	3352.9	5.768	0.003376
720	0.006518	3229.5	3620.6	6.152	0.005478	3180.2	3563.6	6.034	0.00472	3152.1	3509.7	5.929	0.003712
760	0.007007	3333.2	3753.2	6.283	0.005909	3289.5	3703.1	6.172	0.005108	3246.7	3655.3	6.073	0.004031
800	0.007457	3432.7	3880.2	6.403	0.006317	3393.6	3835.8	6.298	0.005476	3355.2	3793.3	6.204	0.004336

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The Editor wishes to express his sincere gratitude to all of the many Mechanical Engineering Department staff, past and present, who have helped to compile this work.

Perfect gas properties in Table E.3 are based on data from *Technical Data on Fuel* (7th edition), ed. J.W. Rose and J.R. Cooper, Scottish Academic Press, 1977.

Enthalpies of ideal (but not perfect) gases in Table E.5 are approximate values obtained from fitting quadratic relationships to enthalpy-temperature data from Rose and Cooper, *Technical Data on Fuel*; the coefficients of the corresponding linear specific heat relationships and the mean and maximum errors are also given. This allows a choice between tabular and analytical methods for solving combustion problems, with consistent results. The particular form of functional relationship is one, which enables combustion product temperature to be found without iteration. Tables E.8 and E.9, for heating (or calorific) values (negative of enthalpies of combustion) of simple compounds and some typical fuels, is based on data from the same source.

The Moody diagram of Figure E.1 was plotted using the equation of Colebrook and White for turbulent flow in rough pipes and Prandtl's equation for turbulent flow in smooth pipes, as quoted in Engineering Sciences Data Unit document ESDU 66027.

The psychrometric chart in Figure E.2, for standard atmospheric pressure, is based on that published in 1970 by the Institution of Heating and Ventilating Engineers.

The R134a data of Tables E.10 to E.15 have been condensed from much more detailed tables in the ICI Chemicals & Polymers Ltd publication *Thermodynamic Properties of KLEA134a*. Other physical properties of R134a, together with the equations from which the thermodynamic properties were computed, are in *Physical Property Data KLEA134a*, ICI Chemicals & Polymers Ltd, Runcorn, 1993. The refrigerants business ICI Klea was acquired in 2001 by INEOS Fluor.

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