

Hybrid Multiobjective Evolutionary Algorithms: A Survey of the State-of-the-art

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Abstract

This paper reviews some state-of-the-art hybrid multiobjective evolutionary algorithms (MOEAs) dealing with multiobjective optimization problem (MOP). The mathematical formulation of a MOP and some basic definition for tackling MOPs, including Pareto optimality, Pareto optimal set (PS), Pareto front (PF) are provided in Section 1. Section 2 presents a brief introduction to hybrid MOEAs.

Keywords: Multiobjective optimization, MOP, Hybrid MOEAs.

1. Introduction

A multiobjective optimization problem (MOP) can be stated as follow:¹

$$\begin{aligned} & \text{Minimize } F(x) = (f_1(x), \dots, f_m(x))^T \quad (1) \\ & \text{subject to } x \in \Omega \end{aligned}$$

Where Ω is the decision variable space, $x = (x_1, x_2, \dots, x_n)^T$ is a decision variable vector and $x_i, i = 1, \dots, n$ are called decision variables, $F(x): \Omega \rightarrow R^m$ consist of m real valued objective functions and R^m is called the objective space.

If Ω is closed and connected region in R^n and all the objectives are continuous of x , we call a problem 1 is a continuous MOP.

Very often, the objectives of the problem (1) are in conflict with one another or are incommensurable. There doesn't exist a single solution in the search space Ω that can minimize all the objectives functions simultaneously. Instead, one has to find the best trade-offs among the objectives. These trade-offs can be better defined in terms of Pareto optimality. The Pareto optimality concept

was 1st introduced by eminent economists Pareto and Edgeworth [1]. A formal definition of the Pareto optimality is given as follows [2, 3, 4, 5].

Definition: Let $u = (u_1, u_2, \dots, u_m)^T$ and $v = (v_1, v_2, \dots, v_m)^T$ be any two given vectors in R^m . Then u is said to dominate v , denoted as $u < v$, if and only if the following two conditions are satisfied.

1. $u_i \leq v_i$ for every $i \in \{1, 2, \dots, m\}$
2. $u_j < v_j$ for at least one index $j \in \{1, 2, \dots, m\}$.

Remarks: For any two given vectors, u and v , there are two possibilities:

1. Either u dominates v or v dominates u
2. Neither u dominates v nor v dominates u .

Definition: A solution $x \in \Omega$ is said to be a Pareto optimal to the problem (1) if there is no other solution $x \in \Omega$ such that $F(x)$ dominates $F(x^*)$. $F(x^*)$ is then called Pareto optimal (objective) vector.

Remarks: Any improvement in a Pareto optimal point in one objective must lead

¹ The minimization problem can easily convert into maximization problem by multiplying each objective with -1 and vice versa.

to deterioration in at least one other objective.

Definition: The set of all the Pareto optimal solutions is called Pareto set (PS):

$$PS = \{x \in \Omega | \nexists y \in \Omega, F(y) < F(x)\}$$

Definition: The image of the Pareto optimal set (PS) in the objective space is called *Pareto front (PF)*, $PF = \{F(x) | x \in PS\}$.

Recent years have witnessed significant development in MOEAs for dealing MOPs. In last two decades, a variety of MOEAs have been proposed. The success of most MOEAs depends on the careful balance of two conflicting goals, exploration (i.e., searching new Pareto-optimal solution) and exploitation (i.e., refining the obtained PS). To achieve these two goals, hybridization is good strategy [6]. The following section introduces hybrid algorithms.

2. Hybrid Multiobjective Evolutionary Algorithms

Hybrid MOEAs or combination of MOEAs with efficient techniques have been investigated for more than one decade [7]. Hybridization uses desirable properties of different techniques for better algorithmic improvements. Hybridization can be done in several ways, 1) to use one algorithm to generate a population and then apply another technique to improve it, 2) to use multiple operators in an evolutionary algorithm, and 3) to apply local search to improve the solutions obtained by MOEAs [8].

Multiobjective memetic algorithms (MOMAs) are a special type of hybrid MOEAs. MOMAs are population-based algorithms inspired by the Darwinian principles of natural evolution and Dawkins

notion of a meme (i.e., defined as a unit of cultural evolution which is capable of local refinements). They are well-known algorithms for their fast convergence speed and for finding more accurate solutions to different search and optimization problems. In the following subsections, we present some state-of-the-art MOMAS.

1. Local Search Based Multiobjective Evolutionary Algorithms

Ishibuchi and Murata 1st proposed multi-objective genetic local search algorithm (MOGLS) for solving combinatorial multiobjective optimization problems [9, 10]. MOGLS applied a local search method after classical variation operators. In MOGLS, a scalar fitness function is used to select a pair of parent solutions to generate new solutions with crossover and mutation operator.

An improved version of MOGLS [9, 10] is proposed in [11]. It applies hill climbing local search optimizer on some best individuals in its current population. Its performance was tested on combinatorial multiobjective optimization in comparison with MOGLS [9, 10], strength Pareto evolutionary algorithm (SPEA) [12, 13], NSGA-II [14] and Hybrid NSGA-II [14].

Another version of MOGLS was proposed by Jaskiewicz in [15]. The basic idea of his MOGLS is to reformulate a MOP as simultaneous optimization of all the aggregation constructed by weight sum approach or Tchebycheff approach. At each generation, it optimizes a randomly generated aggregation objective.

Pareto memetic algorithm (PMA) is suggested in [16]. It uses unbounded" current set of solutions" and from it selects a small"temporary population (TP)" that compromises the best solutions with respect to scalarizing functions. Then TP is used to generate offspring by crossover operators. Jaskiewicz suggests that scalar functions are

very good to promote diversity than dominance ranking methods [17].

In [18], a biased neighborhood structure based local search is proposed. The algorithm assigns large probabilities to the neighbors of the current solution located in the promising region of the search space. The proposed algorithm perform very well on both multiobjective 0/1 knapsack and flowshop scheduling problems.

Memetic Pareto archive evolution strategy (M-PAES) is developed in [19]. It utilizes Pareto ranking based selection and grid-type partition of the objective space instead of scalarizing functions. This modified selection scheme is much faster than the scalarizing functions which are used in Ishibush's MOGLS [9] and Jaszkeiwicz's MOGLS [20, 21]. Furthermore, M-PAES maintains two archives, one stores global nondominated solutions and the other is used as the comparison set for the local search phase. M-PAES is tested against the local search optimizer, (1+1)-PAES [22] and SPEA [12, 13] on the multiobjective 0/1 knapsack problems. M-PAES has shown better experimental results than its competitors.

In [23], a memetic algorithm is suggested for dynamic multiobjective optimization. This algorithm has incorporated two adaptive hill climbing local search methods, greedy crossover-based hill climbing local search method and steepest mutation-based hill climbing local search method.

In [24], two fitness function schemes, the weighted sum fitness function and the NSGA-II fitness evaluation, are used probabilistically. The authors used the probability to specify how often the scalarizing function is used for parent selection. When the probability becomes very low, then the proposed algorithm is almost the same as NSGA-II.

[25] Proposed a local search method which uses quadratic approximations. The solutions

produced in the evolutionary process of the multiobjective genetic algorithm (MOGA) [26, 27] are utilized to fit these quadratic approximations around the point selected for local search. The proposed algorithm has shown more accurate experimental results than pure MOGA [26, 27]. The same local search is also used in [28, 29, 30]. A novel agent-based memetic algorithm (AMA) algorithm based on multi-agent concepts is suggested in [31]. AMA used different life span learning processes (LSLPs) based on several local and directed search procedures strategies such as totally random, random restricted, search directions based. In AMA, an agent chooses a LSLP as a search operator adaptively and improves its algorithmic performance. Same ideas are used in [31, 32, 33, 34].

In [35], a novel iterative search procedure, called the hill climber with sidestep (HCS) is designed. HCS is capable of moving toward and along the local Pareto set depending on the distance of the current iterate toward this set. HCS utilizes the geometry of the directional cones and works with or without gradient information. HCS used as a typical mutation operator in SPEA2

[36] and developed a MOMA denoted by SPEA2HCS. SPEA 2HCS is more effective and efficient in dealing with continuous MOPs.

Two Local search methods, Hooke and Jeeves method [37, 38, 39] and steepest descent method [40, 41], are combined with S-Metric Selection Evolutionary Multiobjective Algorithm (SMS-EMOA) [42] and its two hybrid versions, Relay SMS-EMOA hybrid and Concurrent SMS-EMOA hybrid are developed in [43]. Steepest descent method used in Relay SMS-EMOA hybrid and Hooke and Jeeves method used in Concurrent SMS-EMOA hybrid. Experimental analysis on academic test functions [44] show increased convergence speed as well as improved accuracy of the

solution set of these new hybridizations.

A novel searching algorithm called the multiple trajectory search (MTS) is developed in [45]. The MTS uses multiple agents to search the solution space concurrently. Each agent does an iterated local search using one of four candidate local search methods. By choosing a local search method that best fits the landscape of a solution's neighborhood, an agent may find its way to a local optimum or the global optimum. MTS is tested on multiobjective optimization test problems designed for CEC'09 special session and competition on performance assessment of multiobjective optimization algorithms [46]. In [47], MTS is tested on CEC'09 test instances [48]. In [49], a novel Lamarckian learning strategy is designed and hybrid version of nondominated neighbor immune algorithm [50] called multi-objective lamarckian immune algorithm (MLIA) is proposed. The Lamarckian learning performs a greedy search which proceeds towards the goal along the direction obtained by Tchebycheff approach and generates the improved progenies or improved decision vectors, so single individual will be optimized locally and the newcomers yield an enhanced exploitation around the nondominated individuals in less-crowded regions of the current trade-off front. Simulation results based on twelve benchmark problems show that MLIA outperforms the original immune algorithm and NSGA-II in approximating Pareto-optimal front in most of the test problems. When compared with the state of the art algorithm MOEA/D, MLIA shows better performance in terms of the coverage of two sets metric, although it is laggard in the hyper volume metric.

A new hybrid line search approach called the Line search generator of Pareto frontier (LGP) is developed in [51]. The framework of the LGP consist of two phases, Convergence phase and spreading phase. It

has been tested on OKA1 and OKA2 test problems [52], DTLZa and DTLZb test problems [53] and VLMOP2 and VLMP3 test problems [54].

2.2. Hybrid MOEAs Based on Pareto Dominance

In [55], two well-known Pareto dominance based algorithms, SPEA2 [36] and NSGA-II [14], combined with probabilistic local search and developed its hybrid versions for dealing combinatorial multiobjective optimization. In both hybrid algorithms, the use of the Local search is terminated when no better solution to current solution is found in its k neighbors. One potential advantage of proposed hybrid algorithms over its pure versions is the decrease in the CPU time.

T. Murata et al. generalized the replacement rules based on dominance relation for multiobjective objective optimization in [56]. Ordinary two-replacement rules based on dominance are usually employed in the local search for multiobjective optimization. One rule replaces a current solution with a solution which dominated it. The other rule replaces the solution with a solution which is not dominated by it. The movable area with 1st rule is very small when the number of objectives is large. On the other hand, it is too huge to move efficiently with second rule. The authors generalized these extreme rules by counting the number of improved objective values. Proposed local search based on generalized replacement rules is incorporated in SPEA [12, 13] and developed its hybrid SPEA.

In [57], two hybrid MOEAs, hybrid NSGA-II and hybrid SPEA2 are developed. In both proposed hybrid algorithms, a convergence acceleration operator (CAO) is used as an additional operator for improving the search capability and convergence speed. CAO is applied in the objective space for improved solutions. The improved objective vectors

are then mapped back to the decision space to predict their corresponding decision variables. In [58], three local search methods: simulating annealing (SA), tabu search (TS), and hill climbing local search method, are combined with multi-objective genetic algorithm [26, 27]. MOGA with hill climbing local search method has found much better approximated set of solutions on ZDT test problems [44] than pure MOGA [26, 27] and others hybrid versions of MOGA.

A sequential quadratic programming (SQP) coupled with NSGA-II [14] in [59, 60] for solving continuous MOPs [46]. The same idea is used in [61]. In [62], hybrid version of NSGA-II is suggested which combines a local search method with NSGA-II [14] for estimating the nadir point.

In [63], SQP as a local search method based on augmented achievement scalarizing function (ASF) [64] is used in the framework of NSGA-II [14] for solving ZDT benchmarks [44, 53]. SQP is also used as local search method with NSGA-II [14] as global search method in [65] and solved the CEC'9 test instances [48] in effective ways.

Hybrid constrained optimization evolutionary algorithm (HCOEA) is proposed for constrained optimization in [66]. HCOEA used niching genetic algorithm (NGA) based on tournament selection as a global search method and the best infeasible individual replacement scheme as local search operator. NGA effectively promotes the diversity in its population and local search model remarkably accelerates the convergence speed of the HCOEA.

In [67], a fuzzy simplex genetic algorithm (FSGA) is developed. The proposed method uses fuzzy dominance concept and simplex-based local search method [68] for solving continuous MOPs. The performance of the FSGA is more effective than NSGA-II [14] and SPEA2 [36] on ZDT test problems.

A Pareto Following Variation Operator (PFVO) is used in NSGA-II [14] as an additional operator and designed hybrid NSGA-II in [69]. PFVO takes the available objectives values in the current nondominated front as inputs and generates approximated design variables for the next front as the output. The Proposed algorithm has obtained much better set of optimal solutions to ZDT test problems [44]. PFVO is also used in SPEA2

[36] and in regularity model-based multiobjective estimation of distribution algorithm (RM-MEDA) [70] and suggested its hybrid algorithms in [71]. Experimental analysis revealed that both hybrid algorithms PFVO has enhanced the convergence ability of SPEA2 [36] and RM-MEDA [70] on ZDT test problems [44, 53].

Recently, hybrid version of Archive-based Micro Genetic Algorithm (AMGA) [72] is developed in [73]. In this algorithm, SQP is used as a mutation operator genetic mutation. The inclusion of SQP speeds-up the search process of the proposed hybrid AMGA. Hybrid AMGA has found global Pareto-optimal front and the extreme solutions on most CEC'09 test instances [48]. In [74], the functional-specialization multi-objective real-coded genetic algorithm (FS-MOGA) is proposed. FS-MOGA adaptively switched two search strategies specialized for global and local search. This algorithm chooses an individual from the current population at random. If the chosen individual is a non-dominated solution, then it executes the local search procedure. Otherwise, it performs the global search procedure.

In [75], a hybrid NSGA-II is developed to deal with engineering shape design problems with two conflicting objectives: weight of the structure and maximum deflection of the structure. This algorithm used hill climbing local search method.

In [76, 77], hybrid strategy based on two-stage search process is developed for solving

many-objective optimization. The first stage of the search is directed by a scalarization function and the second stage by Pareto selection enhanced with adaptive Q-Ranking. In [78, 79], a hybrid version of NSGA-II [14] called NSS-GA is proposed for solving ZDT test problems

[44] and DTLZ [53] test problems. NSS-GA used two direct search methods, Nelder and Mead's method [68] and golden section algorithm, for improving.

A new hybrid MOEA, the niched Pareto tabu search combined with genetic algorithm (NPTSGA) is presented dealing with multi-objective optimization problems [80]. The NPTSGA is developed on the thoughts of integrating genetic algorithm (GA) with the improved tabu search (TS) based MOEA, niched Pareto tabu search (NPTS). The proposed NPTSGA is then tested through a simple test example and compared with other two techniques, NPTS and niched Pareto genetic algorithm (NPGA). Computational results indicate that the proposed NPTSGA is an efficient and effective method for solving multi-objective problems.

A hybrid algorithm with on-line landscapes approximation for expensive MOPs, called, ParEGO is developed in [81, 82]. ParEGO is an extension of the single-objective efficient global optimization (EGO) [83]. It uses a design-of-experiment inspired initialization procedure and learn a Gaussian processes model of the search landscape, which is updated after every function evaluation. ParEGO generally outperformed NSGA-II [14] on the used test problems.

2.3. Enhanced Versions of MOEA/D

Recently, an efficient framework known as MOEA/D: multiobjective evolutionary algorithm based on decomposition, is developed in [84]. This generic algorithm bridges decomposition techniques and evolutionary algorithms. MOEA/D

decomposes a MOP into many different single-objective sub problems (SOPs) and defines neighborhood relations among these sub problems. The objective of each sub problem is a weighted aggregation of the original objective functions. Each SOP is optimized by using information, mainly from its neighborhood sub problems. The SOPs in one neighborhood are assumed to have similar fitness landscapes and their respective optimal solutions are most probable be close to each others. This section provides some latest versions of MOEA/D [84].

In [85], 2-opt local search method is combined with MOEA/D [84] and tested on multiobjective traveling salesman problems (m-TSPs).

Two neighbourhoods are used and a new solution is allowed to replace a very small number of old solutions in [86]. The proposed algorithm denoted by MOEA/D-DE and tested on continuous test MOPs with complicated PS shapes [86]. MOEA/D-DE has shown much better algorithmic improvement than NSGA-II [14].

Recently, another important version of MOEA/D [84], called massively multi-topology sizing of analog integrated circuits is developed in [87]. In this version, each sub problem records more than one solution to maintain diversity.

In [88], an idea of simultaneously using different types of scalarizing functions in MOEA/D is proposed aimed to overcome the difficulty in choosing an appropriate scalarizing function for particular multiobjective problem. Weighted sum and the weighted Tchebycheff are used as scalarizing functions. Two implementation schemes of the proposed idea are examined in this paper. One is to use multiple grids of weight vectors where each grid is used by a single scalarizing function. The other is to use different scalarizing functions in a single grid of weight vectors where a different scalarizing function is alternately assigned to

each weight vector. The effectiveness of these implementation schemes was examined through experiments on multiobjective 0/1 knapsack problems with two, four and six objectives. Experimental results showed that the simultaneous use of the weighted sum and the weighted Tchebycheff outperformed their individual use in MOEA/D.

Another important extension of MOEA/D called MOEA/D-EGO, the Gaussian stochastic process model for expensive multiobjective optimization is proposed in [89]. At each iteration, in MOEA/D-EGO, a Gaussian stochastic process model for each subproblem is built based on data obtained from the previous search, and the expected improvements of these subproblems are optimized simultaneously by using MOEA/D for generating a set of candidate test points. Further, MOEA/D assisted by metamodel-Gaussian random field metamodel (GRFM) was proposed in [90].

Competition and adaptation of search directions are incorporated in MOEA/D and its effective hybrid version called EMOSA is developed in [91]. In EMOSA, the current solution of each sub problem is improved by simulated annealing with different temperature levels. After certain low temperature levels, to approximate various parts of the PF, a new method to tune the weight vectors of these aggregation functions is suggested. Contrary to the original MOEA/D, no crossover is performed in this hybrid approach. Instead, diversity is promoted by allowing uphill moves following the simulated annealing rationale. In [92], MOEA/D with NBI-style Tchebycheff approach is developed. The new style Tchebycheff approach replaces the already used weighted sum approach and Tchebycheff approach. The proposed algorithm deals with disparately scaled objectives of constrained portfolio optimization problems effectively.

In [93], an enhance version of MOEA/D [84]

is established. In this algorithm, 1) DE operator replaced with a guided mutation operator for reproduction, 2) a new update mechanism with a priority order is proposed. The update mechanism can improve MOEA/D's performance when the SOPs obtained by decomposition are not uniformly distributed on the Pareto front. Finally, the set of test instances for the CEC'09 competition is used for evaluating the performance of the various combinations of these mechanisms in developed approach.

A novel multiobjective particle swarm optimization based on decomposition algorithm developed in [94]. In algorithm, PSO coupled with MOEA/D [84] for solving continuous problems.

An adaptive mating selection mechanism (AMS) is introduced in MOEA/D and the resultant version is called MOEA/D-AMS. AMS consist of controlled subproblems selection scheme (CSS) and matting pool adjustment (MPA). The CSS assigns the computational efforts to different subproblems. The MPA mates individuals with those who are close on the decision space so that small change of gene values can be achieved, which are required at the late stage of evolutionary process.

A new improved version of MOEA/D [84] called, TMOEA/D is developed. TMOEA/D utilizes a monotonic increasing function to transform each individual objective function into the one so that the curve shape of the non-dominant solutions of the transformed multi-objective problem get close to the hyper-plane whose intercept of coordinate axes is equal to one in the original objective function space. In [95], two mechanisms are introduced. Firstly, a new replacement mechanism to maintain a balance between the diversity of the population and the employment of good information from neighbors; secondly, a randomized scaling factor of DE is adopted in order to enhance the search ability of MOEA/D-DE [86] on

real-world problem, the sizing of a folded-cascade amplifier with four performance objectives.

In [96], a new version of MOEA/D [84], called (MOEA/DFD) is developed. The proposed algorithm introduced a fuzzy dominance concept for comparing two solutions and used scalar decomposition method when one of the solutions fails to dominate the other in terms of a fuzzy dominance level. MOEA/DFD outperforms other MOEAs.

In [97], an interactive version of the decomposition based multiobjective evolutionary algorithm (IMOEA/D) is proposed for interaction between the decision maker (DM) and the algorithm. During the stage of interaction, IMOEA/D presents preferred sub problems to DM to choose their most favorite one, and then guided the search to the neighborhood of selected sub problems. IMOEA/D used the utility function which is modeled in [98]. The used utility function simulates the responses of the DM in IMOEA/D implementation. IMOEA/D has been handled the preference informations very well and successfully converged to the expected preferred regions.

Very recently, the behavior of MOEA/D is examined on multiobjective problems with highly correlated objectives in [99]. The performance of MOEA/D is severely degraded while SPEA 2 [36] and NSGA-II [14] had offered good behaviors on highly correlated objectives.

In [100], a novel method called Pareto-adaptive weight vectors ($pa\lambda$) is proposed. This method automatically adjusts the weight vectors in MOEA/D [86] which are associated with each subproblem. The algorithm, called, multiobjective optimization by decomposition with ($pa\lambda$) is tested on continuous test problems [44, 53] in comparison with simple MOEA/D [84] and NSGA-II [14] on each test problem.

The paper in [101], studies the effects of the use of two crossover operators in multiobjective evolutionary algorithm based on decomposition with dynamical resource allocation (MOEA/D-DRA) [102] for multi-objective optimization. The two crossover operators used are, simplex crossover operator (SPX) and center of mass crossover operator (CMX). The use probability of each operator is updated dynamically based on its corresponding successful reward. The experimental results showed that the use of two crossover operators in MOEA/D-DRA [102] can improve its performance on most of the CEC'09 test instances [48].

A combination of MOEA/D and NSGA-II for dealing with multiobjective CARP (MO-CARP) is proposed in [103]. The MO-CARP is a challenging combinatorial optimization problem with many real-world applications, e.g., salting route optimization and fleet management. The proposed memetic algorithm (MA) called decomposition-based MA with extended neighborhood search (DMAENS) has shown better performances than NSGA-II [84] and the state-of-the-art multiobjective algorithm for MO-CARP (LMOGA) [104].

In [105], a hybrid evolutionary metaheuristics (HEMH) is presented. It combines different metaheuristics integrated with each other to enhance the search capabilities. In the proposed HEMH, the search process is divided into two phases. In the first one, the hybridization of greedy randomized adaptive search procedure (GRASP) with data mining (DM-GRASP) [106, 107] is applied to obtain an initial set of high quality solutions dispersed along the Pareto front within the framework of MOEA/D [84]. Then, the search efforts are intensified on the promising regions around these solutions through the second phase. The greedy randomized pathrelinking with local search or reproduction operators are applied to improve the quality and to guide the search

to explore the non discovered regions in the search space. The two phases are combined with a suitable evolutionary framework supporting the integration and cooperation. Moreover, the efficient solutions explored over the search are collected in an external archive. The HEMH is verified and tested against some of the state of the art MOEAs [84, 36, 14, 108] using a set of MOKSP instances used in [88] and in [84]. The experimental results indicated that the HEMH is highly competitive and can be considered as a viable alternative.

A new evolutionary clustering approach called k-mean algorithm based on multi-objective evolutionary algorithm based on decomposition (MOEA/D) [84] is developed in [109]. It optimizes two conflicting functions of data mining in its recent literature. One is snapshot quality function and the other is the history cost function. The experimental results demonstrated significantly better results than evolutionary k-mean (EKM) method.

In [110], a framework for continuous many-objective test problems with arbitrarily prescribed PS shapes is presented. Then the behavior of two popular MOEAs namely NSGA-II [14] and MOEA/D [84] are studied on the designed continuous test problems. The authors are hoped that it will promote an integrated investigation of MOEAs for their scalability with objectives and their ability to handle complicated PS shapes with varying nature of the PF.

2.4. Multimethod Search Approaches

A multialgorithm genetically adaptive for single objective optimization (AMALGAM-SO) is developed in [145]. This algorithm simultaneously combines the strengths of the covariance matrix adaptation (CMA) evolution strategy [146], genetic algorithm (GA) and particle swarm optimizer (PSO). It implements a self-adaptive learning strategy

and automatically tune the number of offspring allowed to be produced by each individual algorithm based on their reproductive success at each generation. AMALGAM-SO has been tested on CEC'05 test bed of single objective optimization problems [147].

In [148], an improved version of the AMALGAMSO is developed for dealing multiobjective optimization called AMALGAM-MO. It blends the attributes of the best available individual search algorithms, NSGAI [14], PSO [111], DE [128], adaptation Metropolis search (AMS) [149]. AMALGAM tested on 2objectives ZDT test problems [44].

A novel multi-objective memetic algorithm, called multi-strategy ensemble multi-objective algorithm (MS-MOEA) is proposed in [150]. In MS-MOEA, the convergence speed is accelerated by new offspring creating operator called adaptive genetic and differential mechanism (GDM). A Gaussian mutation operator is employed to cope with premature convergence. A memory strategy is proposed for achieving better starting population when a change taken place in dynamic environment. MS-MOEA has been tested on dynamic multiobjective optimization problems.

To deal with dynamic multiobjective optimization, a new co-evolutionary algorithm (COEA) is proposed in [151]. It hybridizes competitive and cooperative mechanisms observed in universe to track the Pareto front in a dynamic environment. The main idea of the competitive-cooperative co-evolution is to allow the decomposition process of the optimization problem to adapt and emerge rather than being hand-designed and fixed at the start of the evolutionary optimization process. COEA is tested in comparison with CCEA [152], NSGA-II [14], and SPEA2 [36] on real valued test problems.

A multi-objective hybrid optimizer

denoted by MOHO is presented in [153]. MOHO combines three MOEAs, SPEA 2 [36], a multi-objective particle swarm (MOPSO) [154], and NSGA-II [14] for dealing MOPs. MOHO favors automatically the individual search algorithm that quickly improves the Pareto approximation of the MOP. MOHO grades each algorithm based on five suggested improvements criteria during its course of evolution.

In [155], the feasibility study for integration of two methods: MOEA/D [7] and NSGA-II [4] in the proposed multimethod search approach (MMTD) is performed. MMTD allocated population dynamically to both its constituent algorithms, MOEA/D [84] and NSGA-II [14], based on their individual performance during its evolutionary process. MMTD is tested on two different test suites problems, the ZDT test problems [44] and the CEC'09 test instances [48]. The final best approximated results illustrates the usefulness of MMTD dealing with multiobjective optimization (MO).

In [156], the author combined two different types MOEAs and developed a hybrid method, called MMTD. In MMTD, the whole search is divided into a number of phases. At each phase, MOEA/D and NSGA-II are run simultaneously with different computational resources based on their respective performances at the current phase of MMTD and the computational resources of the next phase are allocated to MOEA/D and NSGA-II. The effectiveness of MMTD is tested on two test suites of continuous multi-objective optimization test problems.

3. Summary

Firstly, this paper provided a general mathematical formulation to MOP and some important basic definition.

Secondly, this paper presented the literature review of some state-of-the-art hybrid evolutionary algorithms. Our literature review is organized as follows: Subsection 2.1 local search based MOEAs; Subsection 2.2 provides some hybrid versions of well-known MOEAs Based on Pareto Dominance; Subsection 2.3 includes the enhanced Versions of MOEA/D paradigm; Subsection 2.4 multi-method search approaches.

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