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Choice Shifts in Groups: A Decision-Theoretic Basis

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ABSTRACT

The phenomenon of *choice shifts* in group decision-making has received much attention in the social psychology literature. Faced with a choice between a “safe” and “risky” decision, group members appear to move to one extreme or the other, relative to the choices each member might have made on her own. Both risky and cautious shifts have been identified in different situations. This paper demonstrates that from an individual decision-making perspective, choice shifts may be viewed as a systematic violation of expected utility theory. We propose a model in which a well-known failure of expected utility — captured by the Allais paradox — is equivalent to a particular configuration of choice shifts. Thus, our results imply a connection between two well-known behavioral regularities, one in individual decision theory and another in the social psychology of groups.

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1 Introduction

How do groups confront choices involving risk? Despite the fact that group decision-making is ubiquitous in social, economic and political life, economists haven't had much to say on the subject.¹ Before 1961, conventional wisdom on the subject (largely from social psychology) was fairly unambiguous: relative to the attitudes of group members, the group itself is likely to favor compromise and caution. But a series of experiments by James A. F. Stoner (1961) identified "risky shifts": when faced with the same decision problem, individuals within a group adopt a *riskier* course of action, compared with the decisions they would make outside the group.² Later, Nordhøy (1962), Stoner (1968) and others provided some evidence for *cautious* shifts: a group tendency to exhibit greater restraint in risk-taking relative to the proclivities of individuals in that group. To accommodate both directions of change, the general phenomenon was ultimately referred to as a *choice shift*. Today, choice shifts in group decision-making are universally viewed as a consistent and robust phenomenon (Davis *et al.* (1992)).³

At the broadest level, group decisions embody two functions: the aggregation of *information* and the aggregation of *preferences*. In this paper, we focus on the latter function (see below for comments on the former). That is, assuming group members are faced with all the information relevant to the decision at hand, we ask, can the mere fact that the ultimate decision is taken by the group as a whole distort the individual expression of preferences? Might an individual express support or vote for an outcome that he would not have chosen in isolation?

From the perspective of economic theory, the standard paradigm of group decision-making emphasizes *pivotal events*, special situations in which a particular individual's "vote" affects the final outcome. But in such an event, the individual must act as he would in isolation, controlling for informational differences.

This argument makes it obvious that the pivotality logic is akin to an independence axiom for decision-making, and so to some extent on the axiomatic foundations of expected utility. It is therefore not surprising that we propose to view the phenomenon of choice shifts as a systematic violation of expected utility theory. But more than this, we propose a model in which a well-known failure of expected utility — captured by the Allais paradox — is *equivalent to a particular configuration* of choice shifts (which include both risky and cautious shifts, but in a specific pattern).

A traditional explanation for the risky shift emphasizes the "diffusion of responsibility" (DOR)

created by a group decision (e.g., see Wallach *et al.* (1962, 1964)). The idea is simple: when an individual makes a risky choice which fails to generate a successful outcome, she might feel responsible for (or guilty about, or disappointed by) her failure. Similarly, success should bring on a sense of “elation” over and above the “direct” utility of the outcome. DOR can be interpreted as saying that individuals tend to place higher weight on events with low outcomes relative to events with high outcomes, but that this tendency is attenuated when they participate in a group decision; hence, they are willing to take greater risks within the group. Now, DOR appears to explain too much, in that it seemingly cannot account for cautious shifts (see, e.g., Dean G. Pruitt (1971a, b)). But we are going to argue that this assessment may have been too premature. In Theorem 1, we show that the tendency to place higher weight on low outcomes is actually related to a *family* of shifts, each contingent on the ambient group environment. Some of these shifts are cautious; some risky.

To make these points, we adopt the rank-dependent generalization of expected utility theory. This class of preferences, originally due to John Quiggin (1982) and Menahem E. Yaari (1987), extends the class of expected utility preferences to account for many experimental observations on decision-making under risk (see Chris Starmer (2000)). Endowed with these preferences, an individual faces a choice between a risky and a safe option, either alone or in a group. Say that she exhibits a *risky shift* if she is indifferent between the risky and safe lotteries when making the decision herself, yet strictly prefers to “vote” for the risky lottery in the group situation. Likewise define a cautious shift.⁴

We describe a group decision problem by a pair (a, b) , where $a \in (0, 1)$ is the probability that our individual is *pivotal* (i.e., decides the outcome) and $b \in [0, 1]$ is the probability that the group chooses safe over risky, conditional on our individual not being pivotal.⁵ Theorem 1 proves that an individual exhibits the Allais paradox if and only if he exhibits the following pattern of choice shifts: for every likelihood a of being pivotal, there exists a unique threshold b^* — the likelihood of the safe outcome in the non-pivotal case — such that our individual exhibits a risky shift when $b < b^*$, and a cautious shift when $b > b^*$. This result establishes an intimate connection between two well-known behavioral regularities, one in individual decision theory and another in the social psychology of groups.⁶

While we take a first step towards a formal theory of choice shifts, our emphasis on a “pure” preference-based theory is admittedly special. In particular, it is not meant to signal that an informational approach to choice shifts is any less important, just that the two are potentially complementary. Indeed, economic theory does offer a set of models that produce a shift in an individual’s vote, because the votes of others may embody information that a particular individual may not possess (see

Feddersen and Pesendorfer (1998)). Now, this particular route is of interest but it does necessitate that information *not* be revealed explicitly, requiring it to be indirectly transmitted through the final decision-making process instead. For a large class of situations, and particularly in group situations with a commonality of objectives, we do not find this argument very convincing.⁷ On the other hand, this is not to deny that group members bring new information to the table. They certainly do, ranging all the way from the provision of verifiable facts to different ways of reasoning about commonly available data.⁸ And it is certainly possible that such group interaction may lead to significant changes in the behavior or attitudes of each member. Whether these interactions are capable of generating predictable shifts in attitudes towards risk and caution is a different matter that we do not address here.⁹ Thus our focus is a more primitive one: we study the basic logic of individual decision-making in group situations.

Further discussion follows the statement of the theorem, and potential applications are described in Section 3. The formal proof of the theorem is relegated to an appendix.

2 Model

2.1 Preferences

Let \mathbf{P} denote the set of *simple* lotteries (i.e., lotteries with finite outcomes) over some ambient connected space of outcomes. The individual in question is presumed to have *rank-dependent preferences* (RDP) defined on \mathbf{P} . Such preferences are represented by a functional called a *rank-dependent utility*, which is similar to expected utility except that it is not linear in the probabilities.

To describe the representation, fix some preference ordering \succeq . Order the support outcomes of any simple lottery — call them $1, \dots, n$ — in weakly increasing order of preference. The lottery is then $\mathbf{p} = (p_1, \dots, p_n)$. For $k = 1, \dots, n + 1$, let $w_k(\mathbf{p})$ be the sum of all probability weights over outcomes that are worse than k ; $w_k(\mathbf{p}) = \sum_{i < k} p_i$. Of course, $w_1(\mathbf{p})$ is always 0.

Well-known axiomatizations¹⁰ assert that \succeq is an RDP ordering if and only if there exists a continuous, strictly increasing “probability transformation function” $f : [0, 1] \rightarrow [0, 1]$ with $f(0) = 0$ and $f(1) = 1$ and a continuous, nondegenerate utility function u defined on outcomes¹¹ such that $\mathbf{p} \succeq \mathbf{q}$ if and only if

$$\sum_{k=1}^n [f(w_{k+1}(\mathbf{p})) - f(w_k(\mathbf{p}))] u(k) \geq \sum_{k=1}^n [f(w_{k+1}(\mathbf{q})) - f(w_k(\mathbf{q}))] u(k). \quad (1)$$

In what follows, we employ the minor additional restriction that f is differentiable.

2.2 The Allais Paradox

As mentioned in the Introduction, the RDP class was introduced in response to a large body of evidence indicating a violation of the independence axiom. Perhaps the best known violation is the Allais Paradox, first discovered by Maurice Allais (1953). To describe Allais' original paradox consider the ordered set of outcomes $\{0, 1M, 5M\}$ (M is one hundred million francs in Allais' 1953 formulation). Let p, q, p' and q' be the following lotteries,

$$p = \left(\frac{89}{100}, \frac{11}{100}, \frac{0}{100}\right) \quad q = \left(\frac{90}{100}, \frac{0}{100}, \frac{10}{100}\right)$$

$$p' = (0, 1, 0) \quad q' = \left(\frac{1}{100}, \frac{89}{100}, \frac{10}{100}\right)$$

Allais found that for most subjects, $q \succeq p$ but $p' \succ q'$.

Notice that in the paradox, we "transform" p to p' by moving weight (equal to $89/100$ in the specific example) from the low outcome to the middle outcome. Exactly the same shift of weight is applied to "transform" q to q' . One may view the Allais paradox as stating that such common shifts of weight raise the preference for p over q . Here is a formal definition based on that idea: an individual *exhibits the Allais Paradox* if for every pair of lotteries $(1 - \alpha, \alpha, 0)$ and $(1 - \beta, 0, \beta)$ with $\alpha > \beta$ and $(1 - \alpha, \alpha, 0) \succ (1 - \beta, 0, \beta)$, we have

$$(1 - \alpha - \gamma, \alpha + \gamma, 0) \succ (1 - \beta - \gamma, \gamma, \beta)$$

for all $\gamma \in (0, 1 - \alpha]$.

This definition turns out to be a special case of a more extensive class of situations that Segal (1987) refers to as the "generalized" Allais paradox (for variants of the Allais paradox, see the survey by Starmer (2000)). We return briefly to a discussion of our definition in the remarks following Theorem 1.

2.3 Choice Shifts

Now we develop the notion of choice shifts. As in the case of the Allais paradox, we are interested in choices over a risky lottery \mathbf{r} and a safe lottery \mathbf{s} . However, owing to the nature of group interaction, the individual must confront more complex (compound) lotteries.

Specifically, group decision-making introduces strategic uncertainty. An individual will generally cast her vote or express an opinion on the choice to be made between \mathbf{r} and \mathbf{s} ,¹² while often remaining uncertain of the final outcome. In its most abstract and general form, a group decision problem (from the point of view of a given individual) may be represented by a pair $\mathbf{g} \equiv (a, b)$, where $a \in (0, 1)$ is the probability that our individual is *pivotal* (i.e., decides the outcome) and $b \in [0, 1]$ is the probability with which the group decides on \mathbf{s} , conditional on our individual not being pivotal. The great advantage of this description is, of course, that it admits a large class of aggregation rules within the group. [A possible disadvantage is that a and b don't simply depend on the nature of the group problem but on the behavior of other group members, an "equilibrium issue" which we don't address.] Note that the restriction $a > 0$ means that our individual must have *some* say within the group, and the restriction $a < 1$ means that she cannot be a dictator.

We will say that an individual exhibits a *risky shift* over \mathbf{r} and \mathbf{s} within the group problem \mathbf{g} if she is indifferent between \mathbf{r} and \mathbf{s} , yet *strictly* prefers to "vote" for \mathbf{r} in the context of that group problem. Likewise, she exhibits a *cautious shift* over \mathbf{r} and \mathbf{s} (within the group problem \mathbf{g}) if she is indifferent between \mathbf{r} and \mathbf{s} , yet *strictly* prefers to "vote" for \mathbf{s} in the context of that group problem. A shift — risky or cautious — is generally referred to as a *choice shift*.

To see this more formally, construct the "effective lotteries" when the individual participates in a group decision with parameters (a, b) . Let \mathbf{p} and \mathbf{q} be a pair of simple lotteries and let x be some value in $[0, 1]$. We denote by $x[\mathbf{p}] + (1 - x)[\mathbf{q}]$ the compound lottery that yields the lottery \mathbf{p} with probability x and the lottery \mathbf{q} with probability $1 - x$. Thus, if a group member "votes" for \mathbf{r} , the effective compound lottery \mathbf{r}^* is given by

$$\mathbf{r}^* \equiv a[\mathbf{r}] + (1 - a)[b[\mathbf{s}] + (1 - b)[\mathbf{r}]]. \quad (2)$$

Likewise, when she votes for \mathbf{s} , the compound lottery thus generated is

$$\mathbf{s}^* \equiv a[\mathbf{s}] + (1 - a)[b[\mathbf{s}] + (1 - b)[\mathbf{r}]]. \quad (3)$$

A *risky shift* is the joint statement $\mathbf{r} \sim \mathbf{s}$ and $\mathbf{r}^* \succ \mathbf{s}^*$, while a *cautious shift* is the joint statement $\mathbf{r} \sim \mathbf{s}$ and $\mathbf{s}^* \succ \mathbf{r}^*$. A *choice shift* is just the lack of indifference between \mathbf{r}^* and \mathbf{s}^* , assuming that there is indifference between \mathbf{r} and \mathbf{s} to begin with.

Notice that our formulation of the choice shift phenomenon has nothing to do with attitudes to risk. For instance, if $b = 0$ then $\mathbf{r} = \mathbf{r}^*$ and $\mathbf{r}^* = a[\mathbf{s}] + (1 - a)[\mathbf{r}]$. Hence, $\mathbf{r} \sim \mathbf{s}$ must imply $\mathbf{r}^* \sim \mathbf{s}^*$ under

expected utility theory (or even under the weaker assumption of betweenness), regardless of attitudes to risk.

2.4 The Main Result

We are now in a position to state our equivalence result.

Theorem 1 *In the rank-dependent class, the following statements are equivalent:*

1. *An individual exhibits the Allais Paradox.*
2. *Given any pair \mathbf{r} and \mathbf{s} and any $a \in (0, 1)$, there exists $b^* \in [0, 1]$ such that for any group decision problem $\mathbf{g} = (a, b)$ with $b < b^*$, she prefers to support the risky option, while if $b > b^*$, she prefers to support the safe option. Moreover, if the agent is initially indifferent between \mathbf{r} and \mathbf{s} , then b^* lies strictly between 0 and 1. The individual exhibits a risky shift if $b < b^*$, and a cautious shift if $b > b^*$.*

The theorem shows that the phenomenon of choice shifts may be viewed as a preference reversal that is caused by the same failure of independence that triggers the Allais paradox.

In the formal proof, we show that both the Allais paradox and the phenomenon of choice shifts are separately equivalent to the strict concavity of the transformation function f .¹³ We use this connection here to provide some intuition for the theorem.

Suppose that a risky lottery \mathbf{r} places probability p on winning a high prize and probability $1 - p$ on winning a low prize. Suppose that \mathbf{s} is a safe lottery that promises some prize intermediate between the first two. Imagine “transforming” \mathbf{s} into \mathbf{r} by removing probability mass from the intermediate prize and transferring it in the ratio $p : 1 - p$ to the high and low prizes. The two transfers create a tradeoff, and an individual’s preferences across \mathbf{r} and \mathbf{s} may be viewed as an evaluation of this tradeoff.¹⁴

In a group decision problem our individual generates a choice between the “derived” lotteries \mathbf{r}^* (by voting risky) and \mathbf{s}^* (by voting safe). The tradeoff between these two lotteries is exactly the same as it was before: to “transform” \mathbf{s}^* into \mathbf{r}^* , remove probability mass from the intermediate prize and transfer it in the ratio $p : 1 - p$ to the high and low prizes. However, when f is nonlinear, an individual views the same marginal tradeoffs differently, depending on the “initial lottery” (for instance, the lotteries \mathbf{s} and \mathbf{s}^* in our discussion). In particular, the probability with which the low prize is realized in this “initial lottery” becomes important.

If f is strictly concave *and* the initial low-prize probability is small to start with, the transfer of mass to the low outcome is likely to have a large (negative) impact on utility. It follows that in this case, the individual is less likely to “accept” the implied tradeoff; he will vote for the safe outcome. Now, the probability of the “initial” low outcome being small is linked closely to the safe outcome being adopted with high probability when our agent votes safe. Because b is the probability that the safe outcome will emerge when our agent is not decisive, all this is connected with b being high. To summarize: high b assists an individual’s preference for caution in a group problem.

By exactly the same logic, a low b increases the chances of r being adopted even when our agent votes safe, so that the probability of the low outcome under s^* is thereby increased. By strict concavity, the same probability transfer has a smaller negative impact, raising the chances of voting risky. This explains, or at least indicates, how the strict concavity of f can be related to a class of choice shifts as described in part (2) of the theorem.

The strict concavity of f also implies the Allais paradox. Recall that we start with two lotteries $(1 - \alpha, \alpha, 0)$ and $(1 - \beta, 0, \beta)$ over which the individual is indifferent. The only way this can happen is if α exceeds β to begin with, or equivalently, if the probability of the low outcome in the former lottery is smaller than in the latter. But now a probability transfer of γ from the low outcome to the middle outcome will have a stronger positive impact in the former lottery, resulting in the comparison $(1 - \alpha - \gamma, \alpha + \gamma, 0) \succ (1 - \beta - \gamma, \gamma, \beta)$. This is precisely our condition.

We have therefore linked a fairly complex pattern of choice shifts to the Allais paradox using the “intermediate” device of a strictly concave transformation function. Several steps are missing, of course. We need to show that a strictly concave f is not just a convenient link, it is a *necessary* link. In addition, our predicted pattern of choice shifts is valid for *all* risky lotteries, not just those that take on two outcomes.

The reader interested in the complete argument is therefore invited to study the formal proof. The proof actually indicates that the Allais paradox can be stated in even weaker form without losing the equivalence of the theorem: one need only impose the paradox for “small” shifts of probability from the low to the middle outcome. However, the original version of the Allais paradox refers to another special case: one in which *all* the weight is removed from the low outcome for one of the lotteries (recall the example in Section 2.2). This case is implied by the strict concavity of the transformation function f , but the reverse implication does not generally hold. One can easily amend the proof here to show that if the *coexistence of strictly concave and strictly convex segments in f* can be ruled out by

some other condition, our predicted pattern of the choice shifts is equivalent to the original, restricted form of the Allais paradox.¹⁵

3 Discussion

By Theorem 1, any individual who exhibits the Allais paradox exhibits choice shifts, but does so in a *particular, testable pattern*. As an instance of this pattern, consider the case of unanimity, and suppose that in the absence of unanimous agreement the fallback option is the risky outcome. It is very easy to see that $b = 0$ for this case, so by Theorem 1, a risky shift will occur.

On the other hand, suppose that a unanimity vote is required to replace a sure status-quo with a risky alternative. Then $b = 1$. Theorem 1 states that a cautious shift will occur.¹⁶

The above prediction can be applied in a variety of situations. For instance, empirical evidence shows that 75% of U.S. households do not hold stocks despite the high equity premium (see Michael Haliassos and Carol Bertaut (1995)): another manifestation of the well known equity premium puzzle. To understand this bias towards relative safety, it may help to view a household as a two-person group that requires unanimity to make risky investments. In the absence of unanimous agreement, say that some safe strategy is adopted — a savings account or a CD. Then b is effectively 1, and so our model predicts that whenever the spouse is almost indifferent between risky and safe investing, [s]he would tend to decide against the risky option. Thus households would exhibit a bias towards relatively safe investment opportunities, accepting risky investments only when these offer a relatively high premium.¹⁷

Indeed, the reasoning behind our explanation yields a corollary: controlling for wealth and other factors, households with two or more members will tend to make more cautious investments compared to the investments made by unitary households. We know of no data that confirm or deny this assertion, but it is certainly testable.

More generally, our argument brings together two strands of literature on household behavior: one that models households as collective decision units (see Fran cois Bourguignon and Pierre-André Chiappori (1992) and Maurizio Mazzocco (2004)), and another that attempts to explain the equity-premium puzzle with rank-dependent preferences (but remains within the representative agent paradigm).¹⁸ Our particular emphasis of the effect of the Allais paradox on group decision-making opens up a new and possibly significant connection. If indeed, as the experimental literature (see, e.g., Colin Camerer

(1995)) has shown, most people are prone to Allais-type behavior, then we should not be “puzzled” by large equity premia. In short, under our approach, the equity premium is a puzzle only so far as Allais behavior is considered a paradox.

What determines the likely outcome (such as safety in the example above) in case a particular individual isn’t pivotal? In many situations, social convention might drive such an outcome. For example, in a football team, huddled in the final seconds of a close game, players would expect a risky play ($b \simeq 0$), and such an expectation might itself cause a risky shift for each player.¹⁹ Or consider a team of physicians that debates the use of a new experimental drug (successful in some past cases but leading to complications in others) versus a conventional option (which, say, alleviates the pain but does not cure the illness). The belief of doctors about their peers opinions (and hence the parameter b) may depend on the situation of the patient or the usual practices in their hospital.

In these examples and in many similar situations, one course of action is usually considered the social norm: in one situation the norm may call for risk-taking, while in another the norm may involve caution. Typically, a supermajority (perhaps full consensus) will be required to overturn the norm. Indeed, some of the pioneering studies on choice shifts focused on decision problems of this type (for a survey of these studies see Pruitt (1971a,b)). The basic finding in these studies is that for items on which the widely held values favor the risky alternative, unanimous group decisions are more risky than the average of the initial individual decisions. Group decisions tend to be more cautious on items for which widely held values favor the cautious alternative.

In group decisions in which no overt social or cultural values are at stake (e.g. a decision between investment opportunities, job candidates), it is often the case that an option serves as a focal point whenever it is believed to be supported by the majority. In such cases, our model predicts that shifts would occur in the direction of the “focal” option. This prediction is consistent with experimental studies that demonstrate choice shifts using lotteries with monetary prizes. In particular, Davis and Hintz (1982) provides a survey of experimental evidence demonstrating that in binary decision problems, the direction of choice shifts in groups is largely predicted by the preferences of the majority of individuals.

Shifts towards the majority option have been reported in a series of recent studies by Cass R. Sunstein and his collaborators on mock juries (see David Schkade, Sunstein and Daniel Kahneman (2001) and Sunstein, Reid Hastie, John W. Payne and Schkade (2002)). These studies have shown that “when a majority of individuals initially favored little punishment, the jury’s verdict showed a ‘le-

niency shift', meaning a verdict that was systematically lowered than the median rating of individual members before they started to talk with one another. But when the majority of individual jurors favored strong punishment, the group as a whole produced a 'severity shift', meaning a rating that was systematically higher than the median rating of individual members before they started to talk" (Sunstein (2004)).²⁰

Systematic biases in judicial decisions have also been found in comparing the decision of federal judges in isolation and in panels of three.²¹ Sunstein (2004) reports that the vote of a democratic/republican judge is influenced by the configuration of the other judges in the panel. On a given issue, judges deciding on their own would tend to vote according to their "political affiliation", but when facing a related decision in a panel containing a majority of judges from the opposing party, they tend to shift their decision in the other direction.

Investment clubs may also be thought of as an example in which group members exhibit a choice shift towards a focal option, one which is believed to be supported by the majority of members. An estimated 11% of the US population is involved in an investment club (Brooke Harrington (2001)). Approximately 60% of investment club members are women (Harrington (2001)), which contrasts sharply with the percentage of women who invest in the stock market on their own (Brad M. Barber and Terrance Odean (2001)). In addition, Barber and Odean (2001) present empirical evidence that suggests that women are far less confident than men are with regard to taking risk on the stock market.²² These findings suggests that women, while unwilling to invest in stocks on their own, accept the risk of these investments when deciding in a group with other women. This may be explained by noting that since the women meet as an "investment club", each expects *some* risky investment to be made, hence each would lessen the weight she puts on bad outcomes and be more willing to accept risk.²³

4 Concluding Remarks

In this paper we focused on choice shifts from the perspective of an individual decision-maker. A natural sequel would be to conduct an *equilibrium* analysis of choice shifts. One way to proceed in this direction is to introduce a distribution of preference types, such that all types are disappointment averse and at least one type is indifferent between the risky and safe lotteries. We could then derive the symmetric Bayesian Nash equilibria of games induced by different supermajority rules. Theorem

1 implies that under unanimity there would exist a unique equilibrium in undominated strategies in which the status-quo is implemented. This is because unanimity has the special feature that the outcome in a non-pivotal event is independent of the players' beliefs. However, this feature is not shared by other supermajority rules, which suggests that such rules may induce multiple equilibrium outcomes.

Our paper suggests a parsimonious explanation for why group members may base their decisions on non-pivotal events. An important conclusion that comes out of this is that systematic shifts that individuals exhibit when participating in group decisions may simply be another manifestation of the well-known Allais paradox. Hence, such shifts are to be viewed as anomalous only in as much as the Allais "paradox" is thought of as truly paradoxical.

5 Appendix

Proof of Theorem 1. Let \mathbf{p} and \mathbf{p}' be two lotteries. Denote the union of their supports by $\{1, \dots, n\}$. In what follows we assume without loss of generality that outcome $k + 1$ is strictly preferred to outcome k . Let $\{w_k\}$ and $\{w'_k\}$ be the associated collection of worse-than weights. Then, if Δ denotes the utility difference between the two lotteries,

$$\begin{aligned}\Delta &= \sum_{k=1}^n \{[f(w_{k+1}) - f(w_k)] - [f(w'_{k+1}) - f(w'_k)]\} u(k) \\ &= \sum_{k=1}^n [T_{k+1} - T_k] u(k),\end{aligned}$$

where $T_k \equiv f(w_k) - f(w'_k)$ for each k . Notice that $T_1 = T_{n+1} = 0$. Using this information, we see that

$$\Delta = \sum_{k=2}^n T_k c(k), \tag{4}$$

where $c(k) \equiv u(k - 1) - u(k) < 0$ for all $2 \leq k \leq n$.

We proceed in two steps.

STEP 1. The Allais paradox holds if and only if f is strictly concave.

Suppose that f is strictly concave. Let $\mathbf{p} = (1 - \alpha - \gamma, \alpha + \gamma, 0)$ and $\mathbf{p}' = (1 - \beta - \gamma, \gamma, \beta)$. (4) implies that

$$\Delta = c(2)[f(1 - \alpha - \gamma) - f(1 - \beta - \gamma)] + c(3)[1 - f(1 - \beta)]. \tag{5}$$

When $\gamma = 0$, it is assumed that $\mathbf{p} \sim \mathbf{p}'$, so that

$$c(2)[f(1 - \alpha) - f(1 - \beta)] + c(3)[1 - f(1 - \beta)] = 0 \quad (6)$$

Using (6) in (5), we see that

$$\begin{aligned} \frac{\Delta}{-c(2)} &= [f(1 - \beta - \gamma) - f(1 - \alpha - \gamma)] + [f(1 - \alpha) - f(1 - \beta)] \\ &= [f(1 - \alpha) - f(1 - \alpha - \gamma)] - [f(1 - \beta) - f(1 - \beta - \gamma)], \end{aligned} \quad (7)$$

and the right-hand side of this expression is positive by strict concavity of f . Consequently, the Allais paradox holds.

Conversely, assume the Allais paradox. To prove that f is strictly concave, it suffices to show that there exists $\epsilon > 0$ such that for every x and y in $(0, 1)$ with $0 < y - x < \epsilon$,

$$f(x) - f(x - \gamma) > f(y) - f(y - \gamma) \quad (8)$$

for all $\gamma \in (0, x]$. To this end, pick any three outcomes that are strictly ranked (nondegeneracy of u — see footnote 11 — permits this). Clearly, there exists $\epsilon > 0$ such that if x and y are in $(0, 1)$ and $0 < y - x < \epsilon$

$$(y, 0, 1 - y) \succ (x, 1 - x, 0).$$

Using the connectedness of the outcome space, however, we can shift the best of the three outcomes close enough to the intermediate outcome such that

$$(y, 0, 1 - y) \sim (x, 1 - x, 0).$$

Now (with $\alpha = 1 - x$ and $\beta = 1 - y$) all the conditions of the Allais paradox are satisfied, so that we can follow the same argument leading to (7) to assert that for all $\gamma \in (0, x]$, (8) holds.

STEP 2. Part 2 of the theorem holds if and only if f is strictly concave.

Assume f is strictly concave. Fix a pair (\mathbf{r}, \mathbf{s}) and a probability of being pivotal a . The decision problem our individual faces in a group decision $\mathbf{g} = (a, b)$ is a choice between the two compound lotteries \mathbf{r}^* and \mathbf{s}^* , which are defined formally in (2) and (3).

The “worse-than” weights of the two compound lotteries are, of course, functions of the group decision parameter \mathbf{g} , so write them as $w_k(\mathbf{r}^*, \mathbf{g})$ and $w_k(\mathbf{s}^*, \mathbf{g})$. To economize on notation, however, let $R_k(\mathbf{g}) \equiv w_k(\mathbf{r}^*, \mathbf{g})$ and $S_k(\mathbf{g}) \equiv w_k(\mathbf{s}^*, \mathbf{g})$ for every k , with the understanding that we will simply

write R_k and S_k when the context is clear. Similarly, let $w_k \equiv w_k(\mathbf{r})$ for every outcome k . Without loss of generality \mathbf{r} places positive weight on outcomes 1 and n , so that

$$0 < w_k < 1 \text{ for every } 2 \leq k \leq n. \quad (9)$$

It is easy to see that

$$\begin{aligned} R_k = R_k(\mathbf{g}) &= [a + (1-a)(1-b)]w_k \text{ for } k \leq s \\ &= [a + (1-a)(1-b)]w_k + b(1-a) \text{ for } k > s, \end{aligned} \quad (10)$$

while

$$\begin{aligned} S_k = S_k(\mathbf{g}) &= (1-a)(1-b)w_k \text{ for } k \leq s \\ &= (1-a)(1-b)w_k + [a + b(1-a)] \text{ for } k > s. \end{aligned} \quad (11)$$

Let $V(\mathbf{r}^*, \mathbf{g})$ and $V(\mathbf{s}^*, \mathbf{g})$ be the expected payoffs from voting \mathbf{r} and from voting \mathbf{s} in the group problem \mathbf{g} . Define $\Delta(\mathbf{g}) \equiv V(\mathbf{r}^*, \mathbf{g}) - V(\mathbf{s}^*, \mathbf{g})$. Then, invoking (4),

$$\Delta(\mathbf{g}) = \sum_{k=2}^n T_k c(k), \quad (12)$$

where $T_k \equiv f(R_k) - f(S_k)$ for each k . In fact, if we define $\lambda \equiv (1-a)(1-b)$,

$$T_k = f([a + \lambda]w_k) - f(\lambda w_k) \text{ for } k \leq s, \quad (13)$$

and

$$T_k = f([a + \lambda]w_k + [1-a-\lambda]) - f(\lambda w_k + [1-\lambda]) \text{ for } k > s. \quad (14)$$

Substitution of (13) and (14) into (12) yields

$$\Delta(\mathbf{g}) = \sum_{k=2}^s [f([a + \lambda]w_k) - f(\lambda w_k)]c(k) - \sum_{k=s+1}^n [f(\lambda w_k + [1-\lambda]) - f([a + \lambda]w_k + [1-a-\lambda])]c(k). \quad (15)$$

Recalling (9), the observation that $f(x+y) - f(x)$ is declining in x when f is strictly concave and $y > 0$, and the fact that $c(k) < 0$, simple inspection of (15) reveals that $\Delta(\mathbf{g})$ is strictly increasing in λ for given a .²⁴ Because λ and b are negatively related, this proves that $\Delta(\mathbf{g})$ is strictly decreasing in b , which establishes the first statement in part 2.

To establish the remainder of part 2, notice that $a = 1$ (and $\lambda = 0$) is equivalent to the individual decision problem. Using this parameter in (15), we obtain

$$\sum_{k=2}^s f(w_k)c(k) - \sum_{k=s+1}^n [1 - f(w_k)]c(k) = 0. \quad (16)$$

describes the indifference condition between \mathbf{r} and \mathbf{s} . In this case, we simply verify the end-point conditions

$$\lim_{b \rightarrow 0} \Delta(\mathbf{g}) > 0 \text{ and } \lim_{b \rightarrow 1} \Delta(\mathbf{g}) < 0. \quad (17)$$

To do this, note that $\Delta(\mathbf{g})$ is continuous so simply put $b = 0$ ($\lambda = 1 - a$) and then $b = 1$ ($\lambda = 0$) in (15). Pursuing the former exercise first, we see that when $b = 0$,

$$\Delta(\mathbf{g}) = \sum_{k=2}^s [f(w_k) - f([1-a]w_k)]c(k) - \sum_{k=s+1}^n [f([1-a]w_k + a) - f(w_k)]c(k). \quad (18)$$

Now by strict concavity, $f([1-a]w_k + a) > (1-a)f(w_k) + af(1) = (1-a)f(w_k) + a$, and $f([1-a]w_k) > (1-a)f(w_k)$, so

$$\begin{aligned} \Delta(\mathbf{g}) &> \sum_{k=2}^s [f(w_k) - (1-a)f(w_k)]c(k) - \sum_{k=s+1}^n [(1-a)f(w_k) + a - f(w_k)]c(k) \\ &= a \left\{ \sum_{k=2}^s f(w_k)c(k) - \sum_{k=s+1}^n [1 - f(w_k)]c(k) \right\} = 0, \end{aligned}$$

using (16). Alternatively, if $b = 1$, then $\lambda = 0$ and

$$\Delta(\mathbf{g}) = \sum_{k=2}^s f(aw_k)c(k) - \sum_{k=s+1}^n [1 - f([aw_k + [1-a]])]c(k). \quad (19)$$

Once again, strict concavity tells us that $f(aw_k + [1-a]) > af(w_k) + (1-a)$ and $f(aw_k) > af(w_k)$, so that

$$\begin{aligned} \Delta(\mathbf{g}) &< \sum_{k=2}^s af(w_k)c(k) - \sum_{k=s+1}^n [1 - af(w_k) - (1-a)]c(k) \\ &= a \left\{ \sum_{k=2}^s f(w_k)c(k) - \sum_{k=s+1}^n [1 - f(w_k)]c(k) \right\} = 0, \end{aligned}$$

using (16) once again. This completes the proof of part 2, given part 1.

It remains to show that part 2 of Theorem 1 implies that f is strictly concave.

Denote by x a typical low outcome, by s a safe outcome and by y a typical high outcome, with $u(x) < u(s) < u(y)$. Let \mathbf{r} denote a typical risky lottery that places probability p on x and $1 - p$ on y . Then for group parameters $\mathbf{g} = (a, b)$,

$$\Delta(\mathbf{g}) = [f((a + \lambda)p) - f(\lambda p)]c(s) + [f((a + \lambda)p + (1 - a - \lambda)) - f(\lambda p + (1 - \lambda))]c(y),$$

where $c(s) \equiv u(x) - u(s)$, and $c(y) \equiv u(s) - u(y)$.

Now suppose that there is an open interval I such that $f'(z)$ is nondecreasing on I . Find parameters $\hat{g} = (\hat{a}, \hat{b})$ and a risky weight \hat{p} such that

$$[\hat{\lambda}\hat{p}, \hat{\lambda}\hat{p} + (1 - \hat{\lambda})] \subseteq I. \quad (20)$$

where $\hat{\lambda}$, just as before, is $(1 - \hat{a})(1 - \hat{b})$. This is very easy to do. As a consequence, all the four numbers

$$\hat{\lambda}\hat{p}, \quad (\hat{a} + \hat{\lambda})\hat{p}, \quad (\hat{a} + \hat{\lambda})\hat{p} + (1 - \hat{a} - \hat{\lambda}) \text{ and } \hat{\lambda}\hat{p} + (1 - \hat{\lambda})$$

lie in I as well.

At the same time, choose the supports x, y and s so that $c(s)$ and $c(y)$ satisfy the equation

$$\Delta(g) = [f((\hat{a} + \hat{\lambda})\hat{p}) - f(\hat{\lambda}\hat{p})]c(s) + [f((\hat{a} + \hat{\lambda})\hat{p} + (1 - \hat{a} - \hat{\lambda})) - f(\hat{\lambda}\hat{p} + (1 - \hat{\lambda}))]c(y) = 0. \quad (21)$$

Now differentiate $\Delta(g)$ with respect to b in a small interval around \hat{b} (keeping all other parameters constant). Letting $\lambda = (1 - \hat{a})(1 - b)$, we see that that

$$-\frac{1}{1 - \hat{a}} \frac{\partial \Delta(g)}{\partial b} = \hat{p}[f'((\hat{a} + \lambda)\hat{p}) - f'(\lambda\hat{p})]c(s) - (1 - \hat{p})[f'((\hat{a} + \lambda)\hat{p} + (1 - \hat{a} - \lambda)) - f'(\lambda\hat{p} + (1 - \lambda))]c(y).$$

But for all such b (in a small enough interval containing \hat{b}) (20) holds. Therefore $f'((\hat{a} + \lambda)\hat{p}) \geq f'(\lambda\hat{p})$ and $f'((\hat{a} + \lambda)\hat{p} + (1 - \hat{a} - \lambda)) \leq f'(\lambda\hat{p} + (1 - \lambda))$. Using this in the expression above along with the fact that $c(s)$ and $c(y)$ are negative, we see that

$$\frac{\partial \Delta(g)}{\partial b} \geq 0$$

for all b in some interval containing \hat{b} . Together with (21), this contradicts part 2 of the statement of the theorem. ■

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Footnotes

1. A notable exception is a recent experimental literature that compares the quality of decision making across groups and individuals (e.g., David J. Cooper and John H. Kagel (2004)).
2. Stoner's study was based on a questionnaire with 12 hypothetical "life situations", which were originally designed by Michael A. Wallach and Nathan Kogan (1959, 1961) to investigate individual risk-taking propensities.
3. Shifts have also been found in studies using choices between lotteries with monetary prizes — see Davis and Hintz (1982) and the references therein.
4. More generally, the literature on non-expected utility has focussed on two sorts of preferences: the Yaari-Quiggin class and the class of "betweenness-satisfying" preferences. The latter has preferences in which a mixture between two lotteries is ranked "in between" those lotteries. To motivate our focus on RDP we note that even the weakest version of betweenness, mixture

symmetry (proposed by Soo-Hong Chew, Larry G. Epstein and Uzi Segal (1991)), cannot accommodate choice shifts. For under mixture symmetry, an individual who is indifferent between some pair of lotteries, is also indifferent between each of these lotteries and any mixture of both.

5. Observe that this description is compatible with a large class of aggregation rules within the group. Also note that in an extended framework, a and b are endogenously determined by the “equilibrium” behavior of all group members.
6. An exercise in a similar spirit may be found in Daisuke Nakajima (2004), which connects the Allais paradox to experimental findings that the Dutch auction raises more revenue than a first-price sealed-bid auction.
7. Indeed, in most of the experiments, and in many of the examples we discuss in Section 3, group members arrived at a decision *after* the relevant information was either presented by an outside party or revealed during deliberation.
8. For a formal model of the latter type of interaction, see Enriqueta Aragones, Itzhak Gilboa, Andrew Postlewaite and David Schmeidler (2005).
9. When preparing the final draft of this paper, Doug Bernheim drew our attention to Joel Sobel (2005), which takes up this line of reasoning.
10. There have been many axiomatic derivations of rank-dependent utility. Restrictions of space prevent us from doing justice to all the contributors to this line of research. The most recent and most general axiomatization is offered by Mohammed Abdellaoui (2002). References to previous studies can be found in this paper.
11. Nondegeneracy simply means that utility values are distinct over at least two outcomes, and therefore — by the continuity of u and connectedness of the outcome space — over a continuum of them.
12. Note that individuals don’t necessarily vote within the group problem. Depending on the context, one may be modelling votes, advice, command or suggestion.
13. The equivalence between the Allais paradox and strict concavity of f was proved by Segal (1987). While we work with a weaker version of the Allais paradox, we do not emphasize this

particular connection as a novel feature of the paper. On the other hand, the equivalence with choice shifts is at the heart of our exercise. In passing, it is worth noting that the strict concavity of f has been linked to other facets of individual behavior. Abdellaoui (2002) and Yaari (1987) connect concave f to a notion of risk-aversion, appropriately defined for the RDP class. Peter P. Wakker (2001, 2005) links the concavity of f to a failure of the sure-thing principle, which he calls “pessimism”.

14. The idea of a “revealed probability trade-off” was introduced in Abdellaoui (2002).
15. For instance, a further axiomatization of RDP is provided by Simon Grant and Atsushi Kajii (1998), which results in a transformation function that has constant elasticity. Constant elasticity precludes the coexistence of strictly convex and strictly concave segments.
16. Of course, unanimity represents a special case in which the nonpivotal event is uniquely pinned down by some default. In other group settings, an individual’s expectation regarding the non-pivotal event may depend on her beliefs about the other members in the group (and so captured best by “interior” values of b).
17. To be sure, a fully developed application of our model would require us to explain why an individual household member faces a parameter such as a : some probability between 0 and 1 that she may be pivotal. One interpretation — the one that fits most easily into our model — is that there some uncertainty about what the partner(s) of that member will decide to do. This seems reasonable: while there is presumably a process of deliberation in which all the pros and cons are discussed, each member may still be uncertain as to the effect of her opinion on the final investment decision. It is also possible to view the situation as one in which a spouse becomes reluctant to pressure his/her partner into a risky investment, if there is uncertainty regarding what that partner would do if called on to decide unilaterally. By supporting the safe status quo, the spouse avoids the possibility of feeling responsible for unfavorable results.
18. See Narayana R. Kocherlakota (1996) for a survey.
19. Wallach, Kogan and Daryl J. Bem (1962) use this decision problem as an example of a group decision in which the risky option is considered to be the social norm.
20. In our opinion, the systematic shifts uncovered by Sunstein and others cannot be explained by the information-driven biases of the various jury theorems in the political economy literature.

Jurors approach the trial with no prior *private* information, they all observe the same evidence and arguments during the trial, and any asymmetry in the interpretation of this information will most likely wash out in the deliberation process.

21. These studies have focused on cases in which judges were confronted with the decision to “uphold an administrative agency’s interpretation of law, so long as those interpretations do not violate clear congressional instructions and so long as those interpretations are ‘reasonable’ ” (Sunstein (2004)). One may view the decision to uphold the agency’s interpretation as the safe alternative, while offering a different interpretation may be reasonably viewed as a riskier alternative.
22. Noticing that women are reluctant to invest in the stock-market on their own, some commercial banks in Israel have begun organizing investment clubs where women meet to discuss potential investments; see Noa Greenberg (2001).
23. Another interpretation of the what happens in investment clubs is that they allow individuals to aggregate information and share the burden of conducting research. However, this may not be a compelling answer. Most investment clubs tend to be composed of individuals who do not have prior expertise in investing. They also tend to be composed of homogeneous populations. Moreover, there other standard alternatives that are available to individual investors who do not have the time or expertise to invest in financial research, such as actively managed mutual funds. Indeed, recent studies have shown that investment clubs underperform relative to individual investors drawn from the same population (Barber and Odean (2000)). This seems to run counter to the intuition that investment clubs aggregate information, or at least that they do so effectively.
24. For $2 \leq k \leq s$, identify x with λw_k and y with aw_k . For $s + 1 \leq k \leq n$, identify x with $(a + \lambda)w_k + (1 - a - \lambda)$ and y with $a(1 - w_k)$, but note in this latter case that x decreases as λ increases.