

A Deterministic Inventory Model for Deteriorating Items with Selling Price Dependent Demand and Parabolic Time Varying Holding Cost under Trade Credit

Vipin Kumar, Gopal Pathak, C.B.Gupta

Abstract- *In the present study, we have developed a deterministic inventory model for deteriorating environment under the price dependent demand with parabolic time varying holding cost and trade credit. Supplier offers a credit limit to the customer and during that span of time no interest is charged, but after the expiry of the specified time limit, the supplier will charge some interest. The customer has the reserve capital to make the payments at the beginning, but however, he decides to take the advantage of the credit limit facility provided by the supplier. This study has main focus to establish the mathematical model for an inventory system under the above conditions. At the end of the paper, numerical examples are provided to illustrate the problem and sensitivity analyses have been carried out for showing the effect of variation in the parameters. Mathematical subject classification: 90B05*

Keywords: *Deterioration, Price dependent demand, Trade credit, parabolic varying holding cost*

I. INTRODUCTION

In the Economic Order Quantity (EOQ) model, we assume that the supplier must be paid for the items as soon as the items are received by the customer. But however, in practice, it may not be true. In the modern business transactions, it is frequently seen that a supplier will allow a certain fixed span of time for the settlement of the amount owed to him for the items supplied. Normally, there is no charge if the outstanding amount is settled within the allowed fixed settlement period. After the expiry of this period, interest is charged. Recently Haley and Higgins (1973), Kingsman (1983), Chapman et al. (1985), Bregman (1993) examined the influence of the trade credit on the optimal inventory policy. Furthermore, Goyal (1985) explored a single item economic order quantity model under the conditions of permissible delay in payments. Chung (1998) studied the same model as Goyal (1985) and developed an alternative approach to find a theorem to determine the EOQ under conditions of permissible delay in payments. Aggarwal and Jaggi (1995) extended Goyal's model to the case of deterioration, Jamal et al. (1997) generalized Aggarwal and Jaggi (1997) to the case of allowable shortage, Kumar, M et al. (2008) developed an EOQ model for the time dependent demand rate under the trade credits.

Kumar, M et al. (2009) presented an inventory model for power demand rate incremental holding cost under permissible delay in payments and Kumar et al. developed an inventory model for quadratic demand rate, inflation with permissible delay in payments. Chen and Kang (2010) proposed an integrated inventory models considering permissible delay in payment and variant pricing strategy, M. Liang et.al. (2011) developed an optimal ordering quantity under the advance sales and permissible delays in payments, C.K. Jaggi (2011) developed a pricing and replenishment policies for imperfect quality deteriorating items under inflation and permissible delay in payments. Ramji Porwal and C.S.Prasad,(2012) studied the parabolic varying holding cost. In practice, the deterioration is applicable to many inventories like blood banks, fashion goods, agricultural products, medicines, highly volatile liquids like gasoline; alcohol and turpentine undergo physical depletion over time through the process of evaporation, electronic goods, radioactive substances, photographic films, grains and many more deteriorate through a gradual loss of potentials or utilities with the passage of time. So decay or deterioration of physical goods in stock is a very realistic feature and inventory researchers felt the need of this factor into consideration. Shah and Jaiswal (1977) presented an inventory model for items deteriorating at a constant rate, Covert and Philip (1973), Deb and Chaudhuri (1986), Kumar, M et al. (2009) developed an inventory model with time dependent deterioration rate. Some of the recent work in this field has been done by Chung and Ting (1993), Hariga (1996), Giri and Chadhuri (1997), Jalan and Chadhuri (1999). In the classical inventory models, the demand rate is assumed to be constant. In realistic world, the demand for physical goods may be time dependent, stock dependent and price dependent. Selling price plays a very important role in the field of inventory system. Burwell (1997) developed an economic lot size model for price dependent demand under quantity and freight discounts, Mondal et al. (2003) presented an inventory system of ameliorating items for price dependent demand rate, You (2005) developed an inventory model with price and time dependent demand, Teng et al. (2005) developed an inventory model with price dependent demand rate.

In this paper, we have developed an economic order quantity inventory model for deteriorating items in which the deterioration rate and holding cost are linear but shortages are allowed and are fully backlogged. Demand rate is a function of selling price with permissible delay in payments

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II. ASSUMPTION AND NOTATION

- The deterioration rate is time varying. $\theta(t) = \theta \cdot t$ is inventory deterioration rate
- Shortages are allowed and are fully backlogged
- p is the selling price per unit time
- a is parameter used in demand function which hold the condition $a > p$
- The demand rate is the function of selling price.
 $f(p) = (a - p) > 0$
- The holding cost is parabolic with time dependent
 $h(t) = (h + \alpha t^2)$ where $h > 0, \alpha > 0$
- Replenishment is instantaneous
- Lead time is zero.
- Delay in payment is allowed
- C_1 is the inventory shortage cost per unit time
- C_2 is the unit cost of an item.
- A is the ordering cost of an order
- T is the length of the cycle
- t_1 is the length of the period with positive stock of the item
- q is the order quantity per unit cycle
- I_e is the interest earned per Rs./unit time
- I_p is the interest paid per Rs./unit time, $I_p > I_e$
- M is the permissible delay in payments in the settlement of the account

III. MODEL FORMULATION

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -(a - p), 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dI(t)}{dt} = -(a - p), t_1 \leq t \leq T \quad (2)$$

With boundary condition $I(t) = 0$ at $t = t_1$

The solutions of above differential equation are

$$I(t) = (a - p) \left[(t_1 - t) + \theta \left(\frac{t_1^3}{6} - \frac{t_1 t^2}{2} + \frac{t^3}{3} \right) + \theta 2t1540 - t13t212 + t1t48 - t515 \right] \quad (3)$$

$$I(t) = (a - p)(t_1 - t) \quad 0 \leq t \leq t_1 \quad (4)$$

The total profit of the system consists of the following element

- Annual ordering cost = A
- Net stock loss cost due to deterioration = I_D
- Unit cost of the item order quantity per cycle = q
- Net annual holding cost = H
- Net annual shortage cost = SC
- Interest charged = IP
- Interest earned = IE

Since the inventory model considers delay in payment therefore there arise two different cases:

CASE I: $M \leq T_1$ (payment at or before total depletion of Inventory)

CASE II: $M \geq T_1$ (payment at or before total depletion of Inventory)

CASE I: $M \leq T_1$

(payment at or before total depletion of Inventory)

In this case

(i) The total number amount of deteriorating units during the period $[0, t_1]$ is given by

$$I_D = \int_0^{t_1} \theta(t) \cdot I(t) dt = (a - p) \left[\theta \frac{t_1^3}{6} + \theta^2 \frac{t_1^5}{40} \right] \quad (5)$$

(ii) The total amount of ordering cost during the period $[0, T]$ is given by

$$q = C_2 \left[I_D + \int_0^T f(p) dt \right]$$

$$= C_2(a - p) \left[T + \theta \frac{t_1^3}{6} + \theta^2 \frac{t_1^5}{40} \right] \quad (6)$$

(iii) The total amount of holding cost during the period $[0, t_1]$ is given by

$$HC = \int_0^{t_1} h(t)I(t) dt = \int_0^{t_1} (h + \alpha t^2)(a - p) \left[(t_1 - t) + \theta \left(\frac{t_1^3}{6} - \frac{t_1 t^2}{2} + \frac{t^3}{3} \right) \right] dt$$

$$= (a - p)h \left[\frac{t_1^2}{2} + \frac{\theta t_1^4}{12} + \frac{\theta^2 t_1^6}{90} \right] + (a - p)\alpha \left[\frac{t_1^4}{12} + \frac{\theta t_1^6}{90} + \frac{\theta^2 t_1^8}{840} \right] \quad (7)$$

(iv) The total amount of shortages cost during the period $[0, t_1]$ is given by

$$SC = -C_1 \int_{t_1}^T I(t) dt = \frac{1}{2} C_1(a - p)(T - t_1)^2 \quad (8)$$

(v) Interest payable during the period $[M, t_1]$

$$IP_1 = C_2 I_p \int_M^{t_1} I(t) dt = C_2(a - p) \cdot I_p \cdot \left[\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{12} + \frac{\theta^2 t_1^6}{90} \right) - M t_1 - M^2 - \theta M t_1^3 + M^4 - M^3 t_1 - \theta M^2 t_1^2 - M^6 - M^5 t_1 \right] \quad (9)$$

(vi) Interest earned during the period $[0, t_1]$

$$IE_1 = C_2 I_e \int_0^{t_1} (a - p) t dt = C_2 I_e(a - p) \frac{t_1^2}{2} \quad (10)$$

Total profit per unit time $P(T, T_1, p)$

$$= p(a - p) - \frac{1}{T} [A + SC + HC + q + IP_1 - IE_1]$$

$$= p(a - p) - \frac{1}{T} \quad (11)$$

Let $t_1 = \beta T, 0 < \beta < 1$

Hence, the profit function becomes $P(T, p)$

$$= p(a - p) - \frac{1}{T} \left[A + C_1 \frac{1}{2} (a - p) T^2 (1 - \beta)^2 + a - p h \beta^2 T^2 + \theta \beta^4 T^4 + \theta^2 \beta^6 T^6 + (a - p) \alpha \beta^4 T^4 + \theta \beta^6 T^6 + \theta^2 \beta^8 T^8 + C_2 a - p T + \theta \beta^3 T^3 + \theta^2 \beta^5 T^5 + C_2 a - p \cdot I_p \cdot \beta^2 T^2 + \theta \beta^4 T^4 + \theta^2 \beta^6 T^6 - M \beta T - M^2 - \theta M \beta^3 T^3 + M^4 - M^3 \beta T - \theta M^2 \beta^5 T^5 - M^6 - M^5 \beta T - C_2 I_e(a - p) \beta^2 T^2 \right] \quad (12)$$

Our object is to maximize the profit function (T, p) . The necessary conditions for maximizing the profit are $\frac{\partial P(T, p)}{\partial T} = 0$ and $\frac{\partial P(T, p)}{\partial p} = 0$ i.e.

$$\left[-\frac{A}{T^2} + C_1 \frac{1}{2} (a - p) (1 - \beta)^2 + (a - p) h \left[\frac{\beta^2}{2} + \frac{\theta \beta^4 T^2}{4} + \theta^2 \beta^6 T^4 + (a - p) \alpha \beta^4 T^4 + \theta \beta^6 T^6 + \theta^2 \beta^8 T^8 + C_2 a - p + \theta \beta^3 T^3 + \theta^2 \beta^5 T^5 + C_2 a - p \cdot I_p \cdot \beta^2 + \theta \beta^4 T^4 + \theta^2 \beta^6 T^6 - M \beta T - M^2 - \theta M \beta^3 T^3 + M^4 - M^3 \beta T - \theta M^2 \beta^5 T^5 - M^6 - M^5 \beta T - C_2 I_e(a - p) \beta^2 \right] \right] = 0 \quad (13)$$

And

$$(a - 2p) - \frac{1}{T} \left[-C_1 \frac{1}{2} T^2 (1 - \beta)^2 - h \left[\frac{\beta^2 T^2}{2} + \frac{\theta \beta^4 T^4}{12} + \theta^2 \beta^6 T^6 - \alpha \beta^4 T^4 + \theta \beta^6 T^6 + \theta^2 \beta^8 T^8 - C_2 T + \theta \beta^3 T^3 + \theta^2 \beta^5 T^5 - C_2 \cdot I_p \cdot \beta^2 + \theta \beta^4 T^4 + \theta^2 \beta^6 T^6 - M \beta T - M^2 - \theta M \beta^3 T^3 + M^4 - M^3 \beta T - \theta M^2 \beta^5 T^5 - M^6 - M^5 \beta T + C_2 I_e \beta^2 \right] \right] = 0 \quad (14)$$

The solution of (13) and (14) will gives T^* and P^* . The value of T^* and p^* , so obtained is the optimal value of

$P^*(T, p)$ of the average net profit, is determined by (12) provided the sufficient conditions satisfy for maximum $P(T, p)$ are
 $\frac{\partial^2 P(T, p)}{\partial T^2} < 0, \frac{\partial^2 P(T, p)}{\partial p^2} < 0$ and $\frac{\partial^2 P(T, p)}{\partial T^2} \frac{\partial^2 P(T, p)}{\partial p^2} - \frac{\partial^2 P(T, p)}{\partial T \partial p} > 0$

CASE II $M \geq T_1$

(payment at or before total depletion of Inventory)

In this case, the costs i.e. ordering cost, holding cost, shortages cost remain unaltered and given by (6), (7), and (8) respectively. The interest payable per cycle is zero i.e. $IP_2 = 0$ when $t_1 < M < T$ because the supplier can be paid in full at time M the permissible in delay.

$$\begin{aligned} IE_2 &= C_2 I_e \int_0^{t_1} f(p) \cdot t \cdot dt + C_2 I_e (M - t_1) \int_0^{t_1} f(p) \cdot dt \\ &= C_2 I_e (a - p) t_1 (M - \frac{t_1}{2}) \end{aligned}$$

Total profit per unit time

$$\begin{aligned} P(T, T_1, p) &= p(a - p) - \frac{1}{T} [A + SC + HC + q + IP_2 - IE_2] \\ &= p(a - p) - \frac{1}{T} \left[A + C_1 \frac{1}{2} (a - p) (T - t_1)^2 + (a - p) h T^2 + \theta t_1 122 + \theta t_1 1412 + \theta t_1 1690 + (a - p) a t_1 1412 + \theta t_1 1690 + \theta t_1 18840 + C_2 a - p T + \theta t_1 136 + \theta t_1 1540 - C_2 I_e (a - p) t_1 (M - t_1 2) \right] \end{aligned} \quad \dots$$

(15)

Let $t_1 = \beta T, 0 < \beta < 1$

Hence, the profit function becomes

$$\begin{aligned} P(T, p) &= p(a - p) - \frac{1}{T} \left[A + C_1 \frac{1}{2} (a - p) T^2 (1 - \beta)^2 + a - p h \beta^2 T^2 + \theta \beta^4 T^4 + \theta \beta^6 T^6 + (a - p) a \beta^4 T^4 + \theta \beta^6 T^6 + \theta \beta^8 T^8 + C_2 a - p T + \theta \beta^3 T^3 + \theta \beta^5 T^5 - C_2 I_e (a - p) (M \beta T - \beta^2 T^2) \right] \end{aligned} \quad \dots$$

(16)

Our object is to maximize the profit function (T, p) . The necessary conditions for maximizing the profit are $\frac{\partial P(T, p)}{\partial T} = 0$ and $\frac{\partial P(T, p)}{\partial p} = 0$ i.e.

$$\begin{aligned} \left[-\frac{A}{T^2} + C_1 \frac{1}{2} (a - p) (1 - \beta)^2 + (a - p) h \left[\frac{\beta^2 T^2}{2} + \frac{\theta \beta^4 T^4}{4} + \theta \beta^6 T^6 + a - p a \beta^4 T^4 + \theta \beta^6 T^6 + \theta \beta^8 T^8 - C_2 T + \theta \beta^3 T^3 + \theta \beta^5 T^5 + C_2 I_e (a - p) \beta^2 T^2 \right] \right] &= 0 \end{aligned} \quad \dots$$

$$+ C_2 a - p T + \theta \beta^3 T^3 + \theta \beta^5 T^5 + C_2 I_e (a - p) \beta^2 T^2 = 0 \quad (17)$$

And

$$\begin{aligned} (a - 2p) - \frac{1}{T} \left[-C_1 \frac{1}{2} T^2 (1 - \beta)^2 - h \left[\frac{\beta^2 T^2}{2} + \frac{\theta \beta^4 T^4}{12} + \theta \beta^6 T^6 - a \beta^4 T^4 + \theta \beta^6 T^6 + \theta \beta^8 T^8 - C_2 T + \theta \beta^3 T^3 + \theta \beta^5 T^5 + C_2 I_e M \beta T - \beta^2 T^2 \right] \right] &= 0 \end{aligned} \quad \dots$$

(18)

The solution of (17) and (18) will give T^* and P^* . The value of T^* and p^* , so obtained is the optimal value of $P^*(T, p)$ of the average net profit, is determined by (16) provided the sufficient conditions satisfy the maximum $P(T, p)$ are

$$\frac{\partial^2 P(T, p)}{\partial T^2} < 0, \frac{\partial^2 P(T, p)}{\partial p^2} < 0 \text{ and } \frac{\partial^2 P(T, p)}{\partial T^2} \frac{\partial^2 P(T, p)}{\partial p^2} - \frac{\partial^2 P(T, p)}{\partial T \partial p} > 0$$

Numerical Example:

Example 1:- The parameters of the product are $A = 200, a = 100, M = 0.055, C_1 = 1.2, C_2 = 20, h = 0.4, \alpha = 0.1, \beta = 0.95, \theta = 0.01, IE_1 = 0.15, IE_2 = 0.12$

Solution:- Based on these input data, the computer outputs are as follows

Profit (P)=1468.5174, Sealing price(p^*) = 60.670231, Time(T^*) = 2.553845
Ordering Quantity (q)=101.51740

Example 2:- The parameters of the product are $A = 200, a = 100, M = 0.035, C_1 = 1.2, C_2 = 20, h = 0.4, \alpha = 0.1, \beta = 0.95, \theta = 0.01, IE_1 = 0.15, IE_2 = 0.12$

Solution: Based on these input data, the computer outputs are as follows

Profit (P)=1398.4059, Sealing price(p^*) = 60.425375, Time(T^*) = 1.868655

Ordering Quantity(q)=74.322079

Sensitivity analysis:

Table-1

Parameters	% Change	Profit(P)	Sealing Price(p^*)	Time (T^*)	Ordering Quantity(q)
θ	-50	1473.241	60.66047	2.689730	106.36234
	-20	1470.347	60.66634	2.604534	103.24616
	20	1466.757	60.67409	2.507215	99.672178
	50	1464.233	60.67982	2.443705	97.331766
C_1	-50	1468.593	60.66962	2.554752	101.42475
	-20	1468.548	60.66998	2.554207	101.40179
	20	1468.487	60.67047	2.553482	101.37121
	50	1468.442	60.67083	2.552938	101.34828
C_2	-50	1904.280	55.54248	2.770461	124.53276
	-20	1636.177	58.61922	2.624967	109.70268
	20	1309.601	62.72219	2.498931	93.992971
	50	1087.474	65.80373	2.441682	84.213467
h	-50	1477.893	60.59511	2.671955	106.37262
	-20	1472.214	60.64081	2.599707	103.31958
	20	1464.889	60.69886	2.509740	99.531016
	50	1459.570	60.74045	2.446721	96.885526
A	-50	1512.742	60.45115	1.946686	77.408256
	-20	1484.840	60.59087	2.344711	93.134334
	20	1453.396	60.74243	2.734719	108.51736
	50	1432.367	60.84105	2.968669	117.73139
a	-50	136.6175	36.19430	3.705103	52.173579
	-20	783.0326	50.79048	2.850498	84.239057
	20	2355.963	70.58987	2.341999	116.63221
	50	4064.589	85.50727	2.111437	137.04505
β	-50	1505.188	60.54905	3.907106	154.56148
	-20	1490.568	60.56824	3.110647	123.53389
	20	1441.528	60.81227	2.124290	84.181835
	50	1396.202	61.06617	1.671668	65.969051
α	-50	1470.878	60.67490	2.646033	105.10644
	-20	1469.432	60.67187	2.588332	102.77759
	20	1467.638	60.66883	2.521991	100.10215
	50	1466.377	60.66712	2.478373	98.344274

Table-2

Parameters	% Change	Profit(P)	Sealing Price(p^*)	Time (T^*)	Ordering Quantity (q)
θ	-50	1400.443	61.19067	1.90282	75.53429
	-20	1399.213	61.19174	1.881999	74.79597
	20	1397.609	61.19326	1.855714	73.86196
	50	1396.431	61.19448	1.837011	73.19604
C_1	-50	1398.461	61.1921	1.869088	74.3405
	-20	1398.428	61.19233	1.868828	74.32943

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	20	1398.384	61.19264	1.868483	74.31471
	50	1398.351	61.19287	1.868223	74.30367
C_2	-50	1859.289	55.85398	2.237782	100.5293
	-20	1574.626	59.06172	1.982267	82.90372
	20	1232.447	63.32139	1.784134	67.3162
	50	1002.086	66.51675	1.695717	58.796
h	-50	1405.112	61.14514	1.923715	76.66863
	-20	1401.064	61.17375	1.89014	75.23708
	20	1395.779	61.21097	1.847849	73.43673
	50	1391.895	61.23824	1.817839	72.16113
A	-50	1460.216	60.82587	1.349835	53.77437
	-20	1420.829	61.06048	1.684638	67.03986
	20	1377.988	61.31171	2.032102	80.78529
	50	1350.099	61.47305	2.249123	89.36004
a	-50	99.25641	37.01865	2.947642	42.28697
	-20	724.2234	51.38882	2.136423	63.40367
	20	2275.875	71.05944	1.683187	83.88268
	50	3971.221	85.92136	1.487293	96.52358
β	-50	1476.863	60.74542	3.127173	123.9823
	-20	1435.545	60.97431	2.296335	91.35495
	20	1357.709	61.43676	1.555588	61.78778
	50	1293.237	61.83023	1.231535	48.76456
α	-50	1399.295	61.19688	1.890622	75.21285
	-20	1398.758	61.19419	1.877234	74.66993
	20	1398.059	61.19085	1.860335	73.98472
	50	1397.547	61.18853	1.848304	73.49702

Results of the paper

The study of above tables reveals the following interesting facts with the increment in parameters $\theta, C_1, C_2, h, A, a, \beta, \alpha$ by -50% to 50 % (as -50%, -20%, 20%, 50%)

1. We notice that the **total profit P** decreases, when the parameters $\theta, C_1, C_2, h, A, \beta, \alpha$ increases whereas if a increases then **P** also increases .
2. The value of **selling price p^*** of the system is increasing with the increase of the parameters $\theta, C_1, C_2, h, A, a, \beta, \alpha$ whereas **p^*** decreases with the increse of α .
3. A decrement in the **initial inventory level q** is observed when the parameters $\theta, C_1, C_2, h, A, a, \beta, \alpha$ increases whereas **q** increases if **A** and **a** increases.

4. The **time (T^*)** also keeps decrement if the parameters $\theta, C_1, C_2, h, A, a, \beta, \alpha$ increases whereas (**T^***) increases if **A** increases.

Conclusion The prime objective of this study is the formulation of a deterministic inventory model for the deteriorating items under price dependent demand rate, time varying holding cost and when the supplier offers a trade credit for a specified period. The supplier offers credit period to the retailer who has the reserve money to make the payments at the time of receiving items from supplier, but however he decides to avail the privilege of credit limits. Shortages are allowed and are completely backlogged. Finally, we provide the numerical example and sensitivity analysis for the illustration and inferences of the theoretical results.

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