Evaluating Teachers Ranking Using Fuzzy AHP Technique

Hota H.S., Sirigiri Pavani, P.V.S.S. Gangadhar

ABSTRACT- Teachers are the backbone of any educational institution and responsible for quality education, a good teacher can produce good student but Indian institutions are very poor in terms of quality teachers, in spite of having well qualified faculty members in their institutions. There is always a question mark about quality teaching. A teacher with good academic records may not necessarily be a good teacher hence there should be a reliable technique to evaluate teachers quality for financial and administrative decision making .An institute management can take proper decision about teachers after choosing best teacher in their institution and also assign new responsibilities based on their quality.

Fuzzy AHP is a multi criteria decision making technique which is frequently used to find out ranking and can be applied to find out teachers ranking ,the quality of teacher is fuzzy in nature hence fuzzy AHP approach can better deal with this situation and finally decide ranking of the teachers based on the multiple conflicting criteria of the teachers. A teacher may have many qualities like communication ability, knowledge level ,interaction with students etc. but all these qualities are qualitative not quantitative which is little bit difficult to deal with traditional theory .Fuzzy logic can be used to deal this type of problem . In this research work fuzzy logic based MCDM method: fuzzy AHP is used to decide the ranking of teacher for further decision making. Data of small sample size of teachers are collected from educational institution.

Keywords: Fuzzy analytical hierarchy process (FAHP), Multi criteria decision making (MCDM).

1. INTRODUCTION

In the last couple of decades many graduates are being produced by educational institution every year but there is always question mark about quality of these graduates and also quality of educational institution in context of quality teacher. There must be continuous performing evaluations of a teacher for financial and administrative decision making. Which can be done either by any other teacher/expert or by an administrator.

A technique is required to evaluate teachers ranking to deal with fuzzy conflicting criteria like skill , knowledge level, Interaction etc different teachers may have different quality of all these.

II. MULTICRITERIADECISION MAKING (MCDM) METHOD

Multi criteria decision making (MCDM) is one of the technique which can be applied in better way to evaluate teachers performance and finally decide ranking of the teachers based on the multiple conflicting criteria of them, since almost all the qualities mentioned above are qualitative hence we need help of fuzzy theory where these can be explained in form of fuzzy linguistic variables.

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Say for example knowledge level of a teacher can be expressed as good, better and best linguistic variables.

A sample data collected from an institution is very small in size to check the effectiveness of the Fuzzy AHP approach in this domain in future the research work can be extended for more number of teachers and with multiple experts to evaluate them.

2.1 Fuzzy Analytic Hierarchy Process (FAHP) Method

One of the most popular analytical techniques for complex decision-making problem is the analytic hierarchy process (AHP).Analytic Hierarchy Process (AHP) is proposed by Satty (1980,2000), is an approach for decision making that involves structuring multiple choice criteria into a hierarchy, assessing the relative importance of these criteria, comparing alternatives for each criterion, and determining an overall ranking of the alternatives.

The output of the AHP is prioritized ranking indicating the overall preference for each of the decision alternatives eventually help the decision maker to select the best approach.

The FAHP method is an advanced analytical method which is developed from the AHP. In spite of the popularity of AHP, this method is often criticized for its inability to adequately handle the inherent uncertainty and imprecision associated with the mapping of the decision-makers perception to exact numbers. In FAHP method, the fuzzy comparison ratios are used to be able to tolerate vagueness. Decision maker wants to use the uncertainty while performing the comparisons of the alternatives. For taking uncertainties into consider ration fuzzy numbers are used instead of crisp numbers.

The method is proposed by Chen and Hwang(1992)this method involves following steps:

i) Converting linguistic terms to fuzzy numbers: This step systematically converts linguistic term into their corresponding fuzzy numbers. It contains eight conversion scales. The conversion scales were proposed by synthesizing and modifying the works of Wenstop(1976), Bass and Kwakernaak(1977), Efstathiou and Rajkovic (1979), Kerre (1982) and Chen(1988).

ii) Converting Fuzzy Numbers to Crisp Scores:- This step uses a fuzzy scoring approach that is a modification of the fuzzy ranking approaches proposed by Jain(1976) and Chen(1985).The crisp score of fuzzy number 'M' is obtained as follows:

$$\mu_{max}(x) = \begin{cases} x, 0 \le x \le 1 \\ 0, otherwise \end{cases}$$

$$\mu_{min}(x) = \begin{cases} 1-x, 0 \le x \le 1 \\ 0, otherwise \end{cases}$$

The fuzzy max and fuzzy min of fuzzy numbers and are defined in a manner such that absolute location of fuzzy numbers can be automatically incorporated in the comparison cases. The right score of each fuzzy number M_i is defined as:-$\mu_R(M_1) = Sup[\mu_{max}(x) \wedge \mu_{M1}(x)]$

And the left score is:-$\mu_L(M_1) = Sup[\mu_{min}(x) \wedge \mu_{M1}(x)]$

The total score of a fuzzy number M_i is defined as:-

$$\mu_T(M_1) = [\mu_R(M_1) + 1 - \mu_L(M_1)]/2$$

iii) Demonstration of the method:- Now, the 5-point scale is considered to demonstrate the conversion of fuzzy number into crisp scores. To demonstrate the method, a 5-point scale having the linguistic terms like low, below average, average, above average and high as shown in figure 2 are considered.

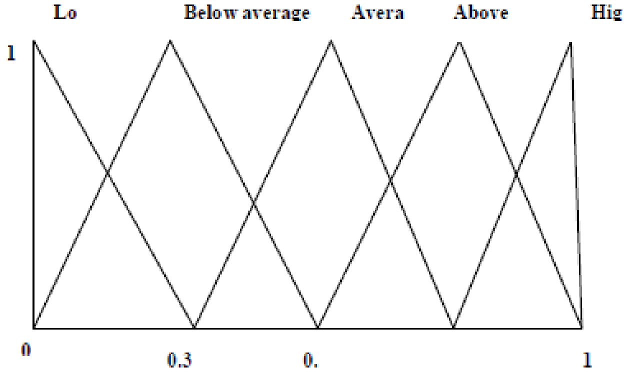

Fig 1:-Linguistic terms to fuzzy numbers conversion (5-point scale)

Table 1:5-pont scale of linguistic terms to fuzzy number conversion

Linguistic Terms	Fuzzy Number
Low	M_1
Below average	M_2
Average	M_3
Above average	M_4
High	M_5

From figure1,membership functions of M_1,M_2,M_3,M_4 and M_5 are written as:

$$\mu_{M1}(x) = \begin{cases} 1, x = 0 \\ \dfrac{(0.3 - x)}{(0.3)}, 0 \leq x \leq 0.3 \end{cases}$$

$$\mu_{M2}(x) = \begin{cases} \dfrac{(x - 0)}{(0.25)}, 0 \leq x \leq 0.3 \\ \dfrac{(0.5 - x)}{(0.25)}, 0.25 \leq x \leq 0.5 \end{cases}, \mu_{M3}(x)$$

$$= \begin{cases} \dfrac{(x - 0.3)}{(0.2)}, 0.3 \leq x \leq 0.5 \\ \dfrac{(0.7 - x)}{0.2}, 0.5 \leq x \leq 0.7 \end{cases}$$

$$\mu_{M4}(x) = \begin{cases} \frac{(x-0.5)}{(0.25)}, 0.5 \leq x \leq 0.75 \\ \frac{(1.0-x)}{0.25}, 0.75 \leq x \leq 1.0 \end{cases}$$

$$\mu_{M5}(x) = \begin{cases} \dfrac{(x - 0.7)}{(0.3)}, 0.7 \leq x \leq 1.0 \\ 1, x = 1 \end{cases}$$

The right, left and total scores are computed as follows for M_1

$$\mu_R(M_1) = Sup[\mu_{max}(x)^\wedge \mu_{M1}(x)] = 0.23$$

$$\mu_L(M_1) = Sup[\mu_{min}(x)^\wedge \mu_{M1}(x)] = 1.0$$

$$\mu_T(M_1) = [\mu_R(M_1) + 1 - \mu_L(M_1)]/2 = 0.115$$

Similarly, the right, left and total scores are computed for M2,M3, M4 and M5 and are tabulated in table 2 (a) .

Instead of assigning arbitrary values for various attributes, this fuzzy method reflects the exact linguistic descriptions in terms of crisp scores. Hence, it gives better approximations that are widely used.

Table 2 (a) Membership function of M_1,M_2,M_3,M_4,M_5 (b) Linguistic terms with their corresponding crisp scores.

(a)

i	$\mu_R(M_i)$	$\mu_L(M_i)$	$\mu_T(M_i)$
1	0.23	1.0	0.115
2	0.39	0.8	0.295
3	0.58	0.59	0.495
4	0.79	0.4	0.695
5	1.0	0.23	0.895

(b)

Linguistic Term	Fuzzy Number	Crisp Score
Low	M_1	0.115
Below average	M_2	0.295
Average	M_3	0.495
Above average	M_4	0.695
High	M_5	0.895

III. FORMULATION OF FUZZY AHP

The hierarchy to evaluate teacher ranking based on various conflicting criteria is shown in figure 2, at the top of the hierarchy the goal is placed to decide ranking with three different criteria at the next level ,these are: communication, knowledge and interaction and at last the various alternatives: teacher1, teacher2 and teacher3 are placed .After formation of hierarchy for the problem we can apply fuzzy AHP method to evaluate teachers ranking which consists various steps these are:

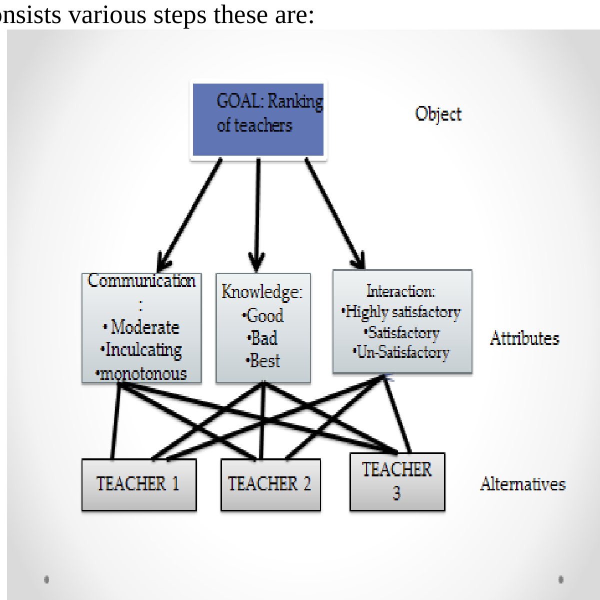

Fig 2: Hierarchy of evaluation of teachers ranking.

STEP 1:- Various criteria and its linguistic terms along with alternatives are tabulated in table 3.For the experimental purpose we have considered only three alternatives and three criteria ,problem can be extended for more number of alternatives and criteria in the same manner and can be used in real sense for managerial decision making process.

Table 3: Various criteria and alternatives for Fuzzy AHP

Teachers	Communication	Knowledge	Interaction
T1	Moderate	Better	Non-Satisfactory
T2	Inculcating	Good	Satisfactory
T3	Monotonous	Best	Highly - Satisfactory

Instead of 5-point scale as explained above we have considered here 3-point scale for conversion of fuzzy linguistic term into crisp and corresponding crisp value calculated using equation 1 is depicted in table 4 (a).Various linguistic terms of three different criteria are also shown in table 3.

From the Chen and Hwang(1992) method membership function of M_1,M_2 and M_3 are written as:-

$$\mu_{M1}(x) = \begin{cases} 1, x = 0 \\ \frac{(0.3-x)}{(0.3)}, 0 \le x \le 0.3 \end{cases} \mu_{M2}(x)$$

$$= \begin{cases} \frac{(x-0.3)}{(0.2)}, 0.3 \le x \le 0.5 \\ \frac{(0.7-x)}{0.2}, 0.5 \le x \le 0.7 \end{cases}$$

$$\mu_{M3}(x) = \begin{cases} \frac{(x-0.7)}{(0.3)}, 0.7 \le x \le 1.0 \\ 1, x = 1 \end{cases} \qquad \ldots(1)$$

Table 4 (b) is obtained with the help of table 3 and 4 (a) which show the crisp data for corresponding fuzzy linguistic terms and matrix is known as decision making matrix (DMM).

Table 4: (a) Crisp value of fuzzy numbers in 3-point scale
(b) Decision making matrix for teachers ranking

Teachers	Communication	Know	Inter
T1	0.75	0.2323	1
T2	0.66	1	0.553
T3	01	0.1284	1

(b)

Fuzzy Number	Crisp Score
M1	0.115
M3	0.495
M5	0.895

STEP 2: Consistency checking :Consisting ratio (CR) checking is required to check whether the weights assign based on expert reasoning is correct or not, usually its value is less then 0.1 which shows that the weights are consistent.

A relative importance matrix to assign weights for comparing criteria with criteria is shown below ,in this matrix diagonal elements are always zero because a criteria compared with same will always be 1.Also $a_{ij}= a_{ji}$where a is an element of matrix

$$\begin{matrix} communication \\ knowledge \\ interaction \end{matrix} \begin{bmatrix} commu & knowl & inter \\ 1 & 5 & 3 \\ 1/5 & 1 & 1/2 \\ 1/3 & 2 & 1 \end{bmatrix}$$

With the help of above matrix we can calculate geometric mean (GM) as follows:

$GM_1=(1*5*3)^{1/3}$ = 2.4659 , $GM_2=(1/5*1*1/2)^{1/3}$ = 0.4641 and $GM_3=(1/3*2*1)^{1/3}$ = 0.873

Hence total Geometric mean (GM) = GM1 + GM_2+ GM_3= 3.79

Calculating normalized weights

W_1 = 2.46/3.79 = 0.649,W_2 = 0.46/3.79 = 0.121 and W_3= 0.87/3.79 = 0.229

Consistency can now be checked using following formulae :

$A_3=A_1*A_2$ …(1)

Where A1 is relative importance matrix and A_2 is weigh matrix obtained from equation 2

$$A_3 = \begin{bmatrix} 1 & 5 & 3 \\ 1/5 & 1 & 1/2 \\ 1/3 & 2 & 1 \end{bmatrix} * \begin{bmatrix} 0.649 \\ 0.121 \\ 0.229 \end{bmatrix} = \begin{bmatrix} 1.914 \\ 0.36 \\ 0.678 \end{bmatrix}$$

Also $A_4 = A_3/A_2$ …(2)

$$A_4 = \begin{bmatrix} 1.914 \\ 0.36 \\ 0.678 \end{bmatrix} / \begin{bmatrix} 0.649 \\ 0.121 \\ 0.229 \end{bmatrix} \quad A_4 = \begin{bmatrix} 2.949 \\ 2.975 \\ 3.0818 \end{bmatrix}$$

Calculating average of A_4i.eλ_{max}

$$\lambda_{max} = \frac{2.949+2.975+3.0818}{3} = 3.001$$

Then $CI = \frac{(\lambda_{max} - n)}{n-1}$, $CI = \frac{3.001-3}{2}$ = 0.0005 where n is size of matrix

And $CR = \frac{CI}{RI} = \frac{0.0005}{0.52}$= 0.00096<0.1

Where RI is Random index already given for specified number of criteria, for three criteria value is 0.52. Since value of CR is less then 0.1 hence the weights are consistent.

STEP 4: Pair –Wise Comparison : Pair wise comparison of alternative to alternative is performed for each criteria as below :

i) Pair wise comparison matrix for criteria Communication

$$\begin{matrix} Teacher1 \\ Teacher2 \\ Teacher3 \end{matrix} \begin{bmatrix} Teacher1 & Teacher2 & Teacher3 \\ 1 & 0.495 & 0.895 \\ 1/0.495 & 1 & 0.895 \\ 1/0.895 & 1/0.895 & 1 \end{bmatrix}$$

Now calculating Geometric mean(GM)for i^{th} row:-

$GM_1=(1*0.495*0.895)^{1/3}$=0.7623

$GM_2=(1/0.495*1*0.895)^{1/3}$=1.2182

$GM_3=(1/0.895*1/0.895*1)^{1/3}$=1.0767

Total Geometric mean=3.05

Hence the normalized weights are:w_1=0.7623/3.05=0.249,w_2=1.2182/3.05=0.398 and w_3=1.0767/3.05=0.352

Now consistency checking by using equation (1)and(2) as below:-

So the

$$A_3 = \begin{bmatrix} 1 & 0.495 & 0.895 \\ 1/0.495 & 1 & 0.895 \\ 1/0.895 & 1/0.895 & 1 \end{bmatrix} * \begin{bmatrix} 0.249 \\ 0.398 \\ 0.352 \end{bmatrix} = \begin{bmatrix} 0.7614 \\ 1.2167 \\ 1.0752 \end{bmatrix}$$

$$\text{And } A_4 = \begin{bmatrix} 0.7614 \\ 1.2167 \\ 1.0752 \end{bmatrix} \div \begin{bmatrix} 0.249 \\ 0.398 \\ 0.352 \end{bmatrix} = \begin{bmatrix} 3.074 \\ 3.005 \\ 3.053 \end{bmatrix}$$

And maximum value λ_{max} that is the average of matrix A4:-

$$\lambda_{max} = \frac{3.074+3.005+3.053}{3} = 3.044$$

Then CI = = = 0.022

And CR = = = 0.04<0.1

Hence the weights are consistent.

ii)Pair wise comparison matrix for criteria Knowledge

$$\begin{matrix} Teacher1 \\ Teacher2 \\ Teacher3 \end{matrix} \begin{bmatrix} Teacher1 & Teacher2 & Teacher3 \\ 1 & 0.895 & 0.115 \\ 1/0.895 & 1 & 0.115 \\ 1/0.115 & 1/0.115 & 1 \end{bmatrix}$$

Now calculating Geometric mean(GM)for i^{th} row:-

$GM_1=(1*0.895*0.115)^{1/3}$=0.4686

$GM_2=(1/0.895*1*0.115)^{1/3}$=0.50464

$GM_3=(1/0.115*1/0.115*1)^{1/3}$=4.2280

Total Geometric mean=5.2012

Hence the normalized weights are: w_1=0.4686/5.2012=0.090,w_2=0.50464/5.2012=0.0970 and w_3=4.2280/5.2012=0.81288

Now consistency checking by using equation (1)and(2) as below:-

So the

$$A_3 = \begin{bmatrix} 1 & 0.895 & 0.115 \\ 1/0.895 & 1 & 0.115 \\ 1/0.115 & 1/0.115 & 1 \end{bmatrix} * \begin{bmatrix} 0.090 \\ 0.0970 \\ 0.812 \end{bmatrix} = \begin{bmatrix} 0.2701 \\ 0.2908 \\ 2.438 \end{bmatrix}$$

And $A_4 = \begin{bmatrix} 0.2701 \\ 0.2908 \\ 2.438 \end{bmatrix} \div \begin{bmatrix} 0.090 \\ 0.0970 \\ 0.812 \end{bmatrix} = \begin{bmatrix} 3.001 \\ 2.997 \\ 3.002 \end{bmatrix}$

And maximum value λ_{max} that is the average of matrix A4:-

$$\lambda_{max} = \frac{3.001+2.997+3.002}{3} = 3$$

Then CI = = = 0

And CR = = = 0/0.52=0<0.1

Hence the weights are consistent.

i)Pair wise comparison matrix for criteria Interaction

$$\begin{matrix} Teacher1 \\ Teacher2 \\ Teacher3 \end{matrix} \begin{bmatrix} Teacher1 & Teacher2 & Teacher3 \\ 1 & 0.495 & 1 \\ 1/0.495 & 1 & 0.895 \\ 1 & 1/0.895 & 1 \end{bmatrix}$$

Now calculating Geometric mean(GM)for i^{th} row:-

$GM_1=(1*0.495*1)^{1/3}=0.7910$

$GM_2=(1/0.495*1*0.895)^{1/3}=1.2182$

$GM_3=(1*1/0.895*1)^{1/3}=1.0376$

Total Geometric mean=3.0468

Hence the normalized weights are:w_1=0.7910/3.0468=0.2596,w_2=1.2182/3.0468=0.3998 and w_3=1.0376/3.0468=0.3406

Now consistency checking by using equation (1)and(2) as below:-

So the

$$A_3 = \begin{bmatrix} 1 & 0.495 & 1 \\ 1/0.495 & 1 & 0.895 \\ 1 & 1/0.895 & 1 \end{bmatrix} * \begin{bmatrix} 0.2596 \\ 0.3998 \\ 0.3406 \end{bmatrix} = \begin{bmatrix} 0.7981 \\ 1.229 \\ 1.0469 \end{bmatrix}$$

And $A_4 = \begin{bmatrix} 0.7981 \\ 1.229 \\ 1.0469 \end{bmatrix} \div \begin{bmatrix} 0.2596 \\ 0.3998 \\ 0.3406 \end{bmatrix} = \begin{bmatrix} 3.0743 \\ 3.0740 \\ 3.0736 \end{bmatrix}$

And maximum value λ_{max} that is the average of matrix A4:-

$$\lambda_{max} = \frac{3.0743+3.0740+3.0736}{3} = 3.073$$

Then CI =0.036

And CR =0.070<0.1

Hence the weights are consistent.

STEP 5:A matrix is formed with the help of obtained weights in case of pair-wise comparison matrix for three different criteria as calculated in step 4 is:-

$$\begin{bmatrix} 0.2493 & 0.090 & 0.2596 \\ 0.3984 & 0.0970 & 0.3998 \\ 0.3521 & 0.8128 & 0.3406 \end{bmatrix}$$

So the final rank can be obtained as below:

$$\begin{bmatrix} 0.2493 & 0.090 & 0.2596 \\ 0.3984 & 0.0970 & 0.3998 \\ 0.3521 & 0.8128 & 0.3406 \end{bmatrix} \times \begin{bmatrix} 0.649 \\ 0.121 \\ 0.229 \end{bmatrix} = \begin{bmatrix} 0.2319 \\ 0.3617 \\ 0.4047 \end{bmatrix}$$

Deciding the rank according to the higher value of above matrix, hence ranking is T3,T2 and T1.

IV. FINDINGS AND CONCLUSION

Decision making is very common and necessary in day-to- day life. In every moment of life we have to take various decisions related to social, economical and others. In this situation multi criteria decision making (MCDM) can help us to decide ranking when criteria are conflicting in nature.

AHP is an effective problem solving multi criteria decision making method. Decision problem may contain various factors that need to be evaluated by linguistic variables. In classical AHP directly the numerical values of linguistic variables are used for evaluation. If the environment where the decision making process takes place is fuzzy, then fuzzy numbers are used for evaluation concerning some deviations of decision makers. Fuzzy AHP (FAHP) can deal this situation very well.

In this paper we have applied FAHP method to decide teachers ranking in educational institution .We have considered only 3 criteria and 3 alternatives for demonstration purpose. A pair wise comparison in between alternative to alternative is carried out for each criteria and finally weights obtained through this is used to decide ranking of teachers as T3 , T1 and T2 means the teacher T3 is the best ,this work can be extended in future for more numbers of criteria and teachers and comparison can be made with other fuzzy MCDM method like fuzzy TOPSIS method.

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