GRADE 6 MATH: RATIO REASONING

UNIT OVERVIEW
This three week unit introduces and develops concepts around ratios. Students learn to use ratios in their reasoning, and they use the language of ratios and rates to describe relationships and solve a wide variety of problems.

Task Details
Task Name: *Mixed Paint*, *Jolly Ranchers and Lemonheads*, *Photo Copies*, and *Boxed Oranges*
Grade: 6
Subject: Mathematics
Depth of Knowledge: 2

Task Description: The sequence of tasks asks students to use their understandings of ratios, rates, and ratio reasoning to solve mathematical problems.

Standards:
6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
6.RP.2 Understand the concept of unit rate a/b associated with a ratio a:b with b≠0, and use rate language in the context of a ratio relationship.
6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

Standards for Mathematical Practice:
MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.4 Model with mathematics.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.
MP.8 Look for and express regularity in repeated reasoning.

Materials Needed:
The Best Deal Lesson Outline and Task
Piano Task
IMPACT MATHEMATICS, COURSE 1, ©GLENCOE McGRAW-HILL

COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through this year's Common Core pilots that beginning with rigorous assessments drives significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

Acknowledgements: Mya James, Grade 6 Teacher, JHS 383 Philippa Schuyler, Toni Cargill, Grade 6 Teacher, JHS 383 Philippa Schuyler, Mary Lawton, Assistant Principal, JHS 383 Philippa Schuyler, Barbara Sanders, Principal, JHS 383 Philippa Schuyler, Margaret Bailey Tang, Achievement Coach, Children First Network 112, Kathy Rehfield-Pelles, Network Leader, Children First Network 112, Joshua Dragoon, Senior Instructional Coach, Office of Achievement Resources, Amy Schless, NYCDOE Math Common Core Fellow, Sean Blanks, NYCDOE Math Common Core Fellow, Shelly Kryger, NYCDOE Math Common Core Fellow

Input was given from the Academic Evaluation Team in the DOE Central Office and the Common Core Fellows.

Department of Education
Dennis M. Walcott, Chancellor

COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

GRADE 6 MATH: RATIO REASONING

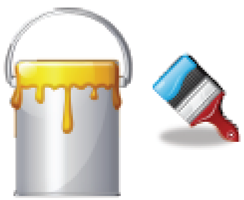
PERFORMANCE TASK

NYC Department of Education
Dennis M. Walcott, Chancellor

institute for learning

1. Mixed Paint

Mark was mixing blue paint and yellow paint in the ratio of 2:3 to make green paint. He wants to make 45 liters of green paint. He began to make a table to help him think about the problem, but is unsure of what to do next.



Liters of Blue Paint	Liters of Yellow Paint	Liters of Green Paint
2	3	5
4	6	10

a. Explain in words how to continue to place values into the table.

b. Write an explanation in words to Mark about how he can use his table to find how many liters of blue paint and how many liters of yellow paint will he need to make 45 liters of green paint.

2. Jolly Ranchers and Lemonheads

The candy jar below contains Jolly Ranchers (the rectangles) and Lemonheads (the circles). Answer each of the following questions:

a. What is the ratio of Jolly Ranchers (the rectangles) to Lemonheads (the circles)?

b. Suppose you have a larger jar of Jolly Ranchers and Lemonheads in which the ratio of the two candies is equivalent to the ratio in this jar. How many Jolly Ranchers and Lemonheads might be in the jar? Use mathematical reasoning to justify your response.

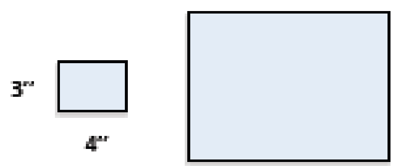
c. Suppose you have a new candy jar with the same ratio of Jolly Ranchers to Lemonheads as shown above, but it contains 100 Jolly Ranchers. How many Lemonheads are in the jar? Use mathematical reasoning to justify your response.

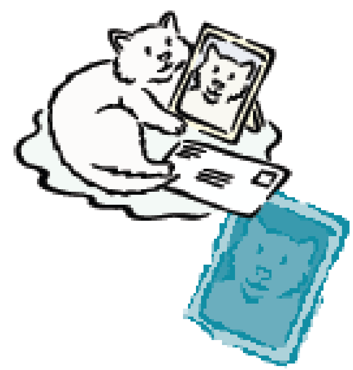
(Adapted from Smith, Silver, Stein, Boston Henningsen & Hillen (2005, p. 26))

3. Photo Copies

Katie and Jacob are enlarging pictures for the school yearbook on the photocopier. The ratio of the width to the length of the enlarged photo will be the same as the ratio of the width to the length of the original photo.

One of the photographs that they want to enlarge is a 3" x 4" photo. Katie says that she can enlarge the photo to 9" x 12", but Jacob disagrees. He says it will be 11" x 12". Who is correct? Explain your reasoning in words.





4. Boxed Oranges

A local store sells oranges at a price of 6 for $9.00.

a. Abbey wants to send 24 oranges to her grandmother. How much will Abbey spend if she buys the oranges at the local store? Explain your reasoning in words.

b. Abbey decides she does not have enough money to buy 24 oranges. She can buy only 15 oranges. How much do 15 oranges cost? Explain your reasoning in words.

GRADE 6 MATH: RATIO REASONING

RUBRIC

Scoring Guide

1. Mixed Paint

Mark was mixing blue paint and yellow paint in the ratio of 2:3 to make green paint. He wants to make 45 liters of green paint. He began to make a table to help him think about the problem, but is unsure of what to do next.

Liters of Blue Paint	Liters of Yellow paint	Liters of Green paint
2	3	5
4	6	10

a. Explain in words how to continue placing values into the table.

b. Write an explanation in words to Mark about how he can use his table to find how many liters of blue paint and how many liters of yellow paint he will need to make 45 liters of green paint.

Sixth Grade CCSS for Mathematical Content

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

The CCSS for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Three-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of all parts of the task. Minor omissions may exist, but do not detract from the correctness of the response. Minor arithmetic errors may be present, but no errors of reasoning appear.

<table>
<tr><td colspan="2">Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution.</td></tr>
<tr><th>Mathematical Practices</th><th>Evidence-Based Traits: Mathematical Content</th></tr>
<tr><td>Construct viable arguments and critique the reasoning of others:
Provides a logical argument that makes clear that the table must be extended in a multiplicative way, possibly indicating ‘doubling’ or ‘tripling’ or ‘going up by the 2, 3 and 5 tables.’</td><td>Response indicates an understanding of the multiplicative relationship among part, part and whole.
a. Correctly explains that values can be added to the table by multiplying each entry by the same number (i.e., part, part and whole).</td></tr>
<tr><td>Reason abstractly and quantitatively: Correctly abstracts the 2:3 ratio as part-part from the context, and recognizes the 5 as representing the “whole” or amount of green paint. Works accurately within the table to reach 45 green liters, then accurately states the need for 18 blue liters and 27 yellow liters of paint.</td><td rowspan="5">Response indicates an understanding of the need to attend to the multiplicative relationship among part, part and whole.
Correctly explains to Mark that values can be added to the table in any of the following ways:
b. Extending the table, adding values to it in a 2-3-5 pattern until the ‘whole’ (green) column reaches 45, signifying 18 blue liters and 27 yellow liters yield 45 liters of green paint. OR
c. Extending the table by multiplying entries in a trial-and error pattern until the ‘green’ column reaches 45, signifying 45 liters of green paint. OR
d. Extending the table by noting that, since 2-3-5 is a member of the table and 45 is 9 times 5, then 2 and 3 must also be multiplied by 9, suggesting 18 blue liters and 27 yellow liters yield 45 liters of green paint. (May do a similar process when the green paint reaches 15, i.e., multiply each entry by 3.)
e. Extending the table by noting, e.g., that the 2-3-5 pattern can be multiplied by 8, yielding 16-24-40 and then adding 2-3-5 to obtain 18-27-45 or the 2-3-5 pattern can be multiplied by 10, yielding 20-30-50 and then subtracting 2-3-5 to obtain 18-27-45 (or any accurate combination of multiplication and addition/subtraction).</td></tr>
<tr><td>Construct viable arguments and critique the reasoning of others:
Provides a logical argument that the entries in the table must be extended in some multiplicative fashion until the number of liters of green paint reaches 45.</td></tr>
<tr><td>Model with mathematics: Possibly writes number sentences describing the reasoning used, e.g., 2 x 9 = 18, etc.</td></tr>
<tr><td>Attend to precision: Correctly extends the table and labels the final result as 18 blue liters and 27 yellow liters; possibly also notes 45 liters of green paint.</td></tr>
<tr><td>Look for and make use of structure: Possibly uses the multiplicative structure of ratios to scale up to 45 (see d and e to right).</td></tr>
</table>

Look for and express regularity in repeated reasoning: Possibly uses the multiplicative structure of ratios as a shortcut to scale up to 45 quickly (see d and e to right).	

Liters of Blue Paint	Liters of Yellow paint	Liters of Green paint
2	3	5
4	6	10
10	15	25
20	30	50
18	27	45

a. Multiply 2, 3 and 5 by 2 then 3 and 4 or any number

b. Mark can multiply 2 X 5 so multiply 3 and 5 by 5
10, 15, 25 doubled is 20, 30, 50 too much
so back down by a group of 2, 3, 5 to 45

Two-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of all parts of the task. Some of the necessary connections between the context and the table may be missing.

<table>
<tr><td colspan="2">Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution.</td></tr>
<tr><th>Mathematical Practices</th><th>Evidence-Based Traits: Mathematical Content</th></tr>
<tr><td>Construct viable arguments and critique the reasoning of others:
Provides an argument that attempts to say that the table must be extended in a multiplicative way, but fails to use explicit language like ‘doubling’ or ‘tripling’ or ‘going up by the 2, 3 and 5 tables.’</td><td>Response indicates an understanding of the multiplicative relationship among part, part and whole.
a. Attempts to explain that values can be added to the table by multiplying each entry by the same number (i.e., part, part and whole).</td></tr>
<tr><td>Reason abstractly and quantitatively: Correctly abstracts the 2:3 ratio from the context. Works multiplicatively within the table, but fails to recognize how to use the 45 in the table.

Construct viable arguments and critique the reasoning of others:
Argues that the entries in the table must be extended in some multiplicative fashion.

Model with mathematics: Possibly writes number sentences describing the reasoning used, e.g., 2 x 9 = 18, etc.

Attend to precision: Extends the table and labels the final result as blue liters, yellow liters; possibly also notes number of liters of green paint.

Look for and make use of structure: Possibly uses the multiplicative structure of ratios to scale up to some number (see d and e to right).

Look for and express regularity in repeated reasoning: Possibly uses the multiplicative structure of ratios as a shortcut to scale up to some number quickly (see d and e to right).</td><td>Response indicates an understanding of the need to attend to the multiplicative relationship among part, part and whole: any of the 3-point strategies for building the table may be used, but response does not "abstract and reason quantitatively" with the 45, i.e., does not use it appropriately in the problem.

Attempts to explain to Mark that values can be added to the table in any of the following ways:
b. Extending the table, adding values to it in a 2-3-5 pattern, but failing to end when the ‘whole’ column reaches 45; stopping at a ‘good’ place to stop, possibly where space runs out or yellow paint reaches 45 liters. OR
c. Extending the table, multiplying entries in a trial-and error pattern, but failing to end when the ‘whole’ column reaches 45, OR
d. Extending the table by noting that, since 2-3-5 is a member of the table, then any multiple of these numbers will work, e.g., 20-30-50, but failing to end when the ‘whole’ column reaches 45.
e. Extending the table by noting that any accurate combination of multiplication and addition will work, but failing to end when the ‘whole’ column reaches 45.</td></tr>
</table>

Liters of Blue Paint	Liters of Yellow paint	Liters of Green paint
2	3	5
4	6	10
6	9	15
8	12	20
10	15	25
12	18	30
14	21	35
16	24	40
18	27	45
20	30	
22	33	
24	36	
26	39	
28	42	
30	45	
32		
34		
36		
38		
40		
42		
44		

a. You can do the 2 3 5 tables.

b. Go to 45 like I did.

One-Point Score:

The response accomplishes some of the prompted purposes and communicates the student's mathematical understanding. The student's strategy and execution meet only a few of the content demands of the task. The necessary connections between the context and the table are missing.

<table>
<tr><td colspan="2">Make sense of problems and persevere in solving them.
Work is missing representations of the problem or understanding of the constraints of the problem given in the text.</td></tr>
<tr><th>Mathematical Practices</th><th>Evidence-Based Traits: Mathematical Content</th></tr>
<tr><td>Construct viable arguments and critique the reasoning of others:
Provides an argument that the table must be extended additively.</td><td>Response lacks an understanding of the multiplicative relationship among part, part and whole.
a. Attempts to explain that values can be added to the table by repeatedly adding 2, 3 and 5.</td></tr>
<tr><td>Reason abstractly and quantitatively: Correctly abstracts the 2:3 ratio from the context. Works additively within the table, and/or fails to recognize how to use the 45 in the table.</td><td rowspan="6">Response indicates an understanding of the need to extend the table, but fails to recognize the multiplicative relationship, and/or fails to cope with the role of the 45 in the problem.
Attempts to explain to Mark that values can be added to the table in any of the following ways:
b. Extending the table additively in a 2-3-5 pattern, possibly failing to end when the 'whole' column reaches 45, stopping at a 'good' place to stop, possibly where space runs out or yellow paint reaches 45 liters. OR
c. Extending the table additively in a 2-3-5 pattern, but extending two or all three columns to 45 or more liters of paint.
d. Using language suggesting multiplication or proportions, but failing to indicate how this can be accomplished (accompanied by an additive explanation for part a).</td></tr>
<tr><td>Construct viable arguments and critique the reasoning of others:
Argues that the entries in the table must be extended in some additive fashion.</td></tr>
<tr><td>Model with mathematics: Possibly writes number sentences describing the reasoning used, e.g., 2 + 2 + 2 = 6, etc.</td></tr>
<tr><td>Attend to precision: Extends the table accurately; may or may not label the final result as blue liters, yellow liters, green liters.</td></tr>
<tr><td>Look for and make use of structure: Fails to use the multiplicative structure of ratios to scale up to some number.</td></tr>
<tr><td>Look for and express regularity in repeated reasoning: Fails to use the multiplicative structure of ratios as a shortcut to scale up to some number quickly.</td></tr>
</table>

B	Y	G
2	3	5
4	6	10
6	9	15
8	12	20
10	15	25
12	18	30
14	21	35
16	24	40
18	27	45
~~20~~	~~30~~	

a. For every 2 liters of Blue and 3 liters of yellow paint there is 5 liters of green paint. Every time you go around you add 2 to the blue, 3 the yellow, and then you add together to get green.

b. If Mark knows the ratio is 2:3, he should use porportions to count up to the amount of the green paint: 45 liters.

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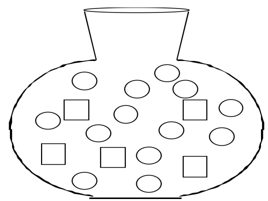
Scoring Guide

2. Jolly Ranchers and Lemonheads

The candy jar contains Jolly Ranchers (the rectangles) and Lemonheads (the circles). Answer each of the following questions:



a. What is the ratio of Jolly Ranchers (the rectangle) to Lemonheads (the circles)?

b. Suppose you have a larger jar of Jolly Ranchers and Lemonheads in which the ratio of the two candies is equivalent to the ratio in this jar. How many Jolly Ranchers and Lemonheads might be in the jar? Use mathematical reasoning to justify your response.

c. Suppose you have a new candy jar with the same ratio of Jolly Ranchers to Lemonheads as shown above, but it contains 100 Jolly Ranchers. How many Lemonheads are in the jar? Use mathematical reasoning to justify your response.

(Adapted from Smith, Silver, Stein, Boston Henningsen & Hillen (2005, p. 26)

Sixth Grade CCSS for Mathematical Content

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.3 Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

The CCSS for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Three-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of all parts of the task. Minor omissions may exist, but do not detract from the correctness of the response. Minor arithmetic errors may be present, but no errors of reasoning appear.

Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution.

Mathematical Practices	Evidence-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Correctly abstracts the 5:13 ratio from the context. **Look for and express regularity in repeated reasoning:** Possibly maintains oversight of the problem and attends to detail by noting that 5 + 13 = 18 and there are 18 candies in the jar.	Task Part a Response indicates an understanding of the concept of ratio. a. Correctly notes the ratio of 5 JR:13 L.
Reason abstractly and quantitatively: Correctly abstracts the 5:13 ratio from the context. Quantitatively scales up appropriately to some other number in part b, and to 100 Jolly Ranchers in part c. Correctly interprets the results as the number of Jolly Ranchers and/or Lemonheads. **Construct viable arguments and critique the reasoning of others:** Provides a logical argument that makes clear that the ratio must be handled in a multiplicative fashion, possibly indicating 'doubling' or 'tripling' or multiplying by 20 for part b.	Task Parts b and c Response indicates ability to use ratio and rate reasoning to solve real-world and mathematical problems; response demonstrates an understanding of the multiplicative structure in a ratio. Correctly scales the 5:13 ratio in any of the following ways (e.g., scaling by a multiple of 10 and indicating there are 50 Jolly Ranchers and 130 Lemonheads in the jar. a. Scaling up the 5:13 ratio in tabular form. b. Multiplying 5 and 13 by the same number. c. Scaling up the 5:13 ratio in fraction form d. Using a proportion or proportional reasoning (e.g., $\frac{5\ JR}{13\ L} = \frac{x}{130}$ or $\frac{5\ JR}{13\ L} = \frac{50}{x}$)
Model with mathematics: Possibly writes number sentences describing the reasoning used, e.g., 5 x 6 = 30, and 13 x 6 = 78, etc. or writes equivalent fractions or a proportion. **Attend to precision:** Correctly multiplies, scales up, creates equivalent fractions, etc. and appropriately labels the final results as Jolly Ranchers and/or Lemonheads.	Response indicates ability to use ratio and rate reasoning to solve real-world and mathematical problems; response demonstrates an understanding of the multiplicative structure in a ratio. Correctly scales the 5:13 ratio to 100 Jolly Ranchers in any of the following ways e. Multiplying 5 and 13 by 20, since 5 x 20 = 100. f. Scaling up the 5:13 ratio in fraction form, e.g., $\frac{5\ JR}{13\ L} = \frac{10\ JR}{26\ L}$ until the number of Jolly Ranchers

Look for and make use of structure: Uses the multiplicative structure of ratios, equivalent fractions, and/or proportions, tables or multiplication statements to scale up.	equals 100. g. Using a proportion or proportional reasoning (e.g., or $\frac{5\ JR}{13\ L} = \frac{100}{x}$) h. Scaling up the 5:13 ratio in tabular form until the number of Jolly Ranchers equals 100. i. Possibly drawing a picture of several sets of 5 rectangles and 10 circles, until switching to one of the multiplicative strategies above or until the number of rectangles equals 100.
Look for and express regularity in repeated reasoning: Uses the multiplicative structure of ratios, equivalent fractions, and/or proportions, tables or multiplication statements as a shortcut to scale up directly to the target value, e.g., 10: 26 = (10 x 2): (26 x 2)	

a. 5:13
Jolly Ranchers Lemonheads

b. smaller jar 5:13
larger jar 10:26
JR. LH

The larger jar is the double size of the smaller jar.

c. ~~we~~ ~~100~~
~~we know~~ ~~10x10=100~~
We know 5x20=100 so you would have to do 20x13 to equal 260. The ratio would be 100:260 260 Lemonheads

Two-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of all parts of the task. Some of the necessary connections between the context and the ratio may be missing.

<table>
<tr><td colspan="2">Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution, but indicates a struggle to interpret parts of the context correctly.</td></tr>
<tr><th>Mathematical Practices</th><th>Evidence-Based Traits: Mathematical Content</th></tr>
<tr><td>Reason abstractly and quantitatively: Correctly abstracts the 5:13 ratio from the context.</td><td rowspan="2"><u>Task Part a</u>
Response indicates an understanding of the concept of ratio.
a. Correctly notes the ratio of 5:13.</td></tr>
<tr><td>Look for and express regularity in repeated reasoning: Possibly maintains oversight of the problem and attending to detail by noting that 5 + 13 = 18 and there are 18 candies in the jar.</td></tr>
<tr><td>Reason abstractly and quantitatively: Correctly abstracts the 5:13 ratio from the context. Quantitatively scales up appropriately to some other number in part b, but fails to recognize the significance of the results in the context of the problem and/or quantitatively scales up in part c, but fails to appropriately interpret the meaning of the 100 in the context.</td><td rowspan="2"><u>Task Parts b and c</u>
Response indicates an understanding of the multiplicative structure in a ratio: any of the 3-point strategies for scaling the 5:13 ratio may be used, but response does not recognize the significance of the results in the context of the problem, e.g., switches Jolly Ranchers with Lemonheads or does not return to the context to explain the solution.
Correctly scales the 5:13 ratio in any of the following ways, but fails to recognize the significance of the results in the context of the problem
a. Scaling up the 5:13 ratio in tabular form.
b. Multiplying 5 and 13 by the same number.
c. Scaling up the 5:13 ratio in fraction form
d. Using a proportion or proportional reasoning.</td></tr>
<tr><td>Construct viable arguments and critique the reasoning of others: Provides a logical argument that makes clear that the ratio must be handled in a multiplicative fashion, possibly indicating ‘doubling’ or ‘tripling’ or multiplying the number of both Jolly Ranchers with Lemonheads by the same number.</td></tr>
<tr><td>Model with mathematics: Possibly writes number sentences describing the reasoning used, e.g., 5 x 6 = 30, and 13 x 6 = 78, etc. or writes equivalent fractions or a proportion.</td><td>Response indicates an understanding of the multiplicative structure in a ratio: any of the 3-point strategies for scaling the 5:13 ratio may be used, but response does not "abstract and reason quantitatively" with the 100, i.e., does not use it appropriately in the problem.</td></tr>
</table>

<table>
<tr>
<td>Attend to precision: Correctly multiplies, scales up, creates equivalent fractions, etc. but fails to appropriately label the final results as Jolly Ranchers and/or Lemonheads.</td>
<td rowspan="3">Scales the 5:13 ratio in any of the following ways, but fails to recognize that the 5 must be scaled up to 100 or fails to scale to 100 appropriately.
e. Multiplying 5 and 13 by the same number.
f. Scaling up the 5:13 ratio in fraction form or by using a proportion.
g. Scaling up the 5:13 ratio in tabular form; stopping at a ‘good’ place to stop, possibly where space runs out
h. Possibly drawing a picture of several sets of 5 rectangles and 10 circles, until switching to one of the multiplicative strategies above.</td>
</tr>
<tr>
<td>Look for and make use of structure: Uses the multiplicative structure of ratios, equivalent fractions, and/or proportions, tables or multiplication statements to scale up.</td>
</tr>
<tr>
<td>Look for and express regularity in repeated reasoning: Uses the multiplicative structure of ratios, equivalent fractions, and/or proportions, tables or multiplication statements as a shortcut to scale up directly to the target value, e.g., 10: 26 = (10 x 2): (26 x 2)</td>
</tr>
</table>

a. 5:13

b. Small Jar = 5:13
Large Jar = 10:26

c. 5x20=100 so you would have to do 13x20 too. and you get 260 as your answer For 13x20 = 260
So the answer is 100:260.

One-Point Score:

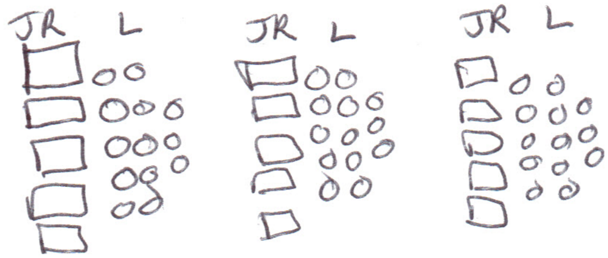
The response accomplishes some of the prompted purposes and communicates the student's mathematical understanding. The student's strategy and execution meet only a few of the content demands of the task. The necessary connections between the context and the ratio are missing.

Make sense of problems and persevere in solving them.
Work is missing representations of the problem or understanding of the constraints of the problem given in the context.

Mathematical Practices	Evidence-Based Traits: Mathematical Content
Reason abstractly and quantitatively: May fail to abstract the 5:13 ratio from the context.	<u>Task Part a</u> Response indicates *some understanding of ratio, but not the part-to-part nature of this relationship*. a. Notes a ratio of 5:18, or 13:5 or 18:5, etc.
Look for and express regularity in repeated reasoning: Possibly maintains oversight of the problem and attending to detail by noting that 5 + 13 = 18 and there are 18 candies in the jar.	
Reason abstractly and quantitatively: Abstracts the 5:13 ratio from the context, but works additively and *fails to recognize the significance of the results in the context of the problem* and/or *fails to appropriately interpret the meaning of the 100 in the context.*	<u>*Task Parts b and c*</u> *Response indicates an attempt to work with ratio, but fails to recognize the multiplicative relationship, typically by reverting to addition, and/or fails to cope with the constraints of the problem.* Attempts to reason in any of the following ways: b. *'Scaling up' the 5:13 ratio in tabular form using addition, not multiplication.* c. *Adding the same number to 5 and 13.* d. *'Scaling up' the 5:13 ratio in fraction form using addition, not multiplication.*
Construct viable arguments and critique the reasoning of others: *Argues that ratio must be tended to in some additive fashion.*	
Model with mathematics: Possibly writes number sentences describing the reasoning used, e.g., 5 + 95 = 100, and 13 + 95 = 108, etc. or attempts to write equivalent fractions, but uses addition rather than multiplication.	*Response indicates an attempt to work with ratio, but fails to recognize the multiplicative relationship, typically by reverting to addition, and/or fails to cope with the constraints of the problem.* Attempts to reason in any of the following ways: e. *Adding 95 to 5 and 13, since 5 + 95 = 100.* f. *'Scaling up' the 5:13 ratio to 100 Jolly Ranchers in tabular form using addition, not multiplication.* g. *'Scaling up' the 5:13 ratio in tabular form using addition; stopping at a 'good' place to stop, possibly where space runs out.* h. *Possibly drawing a picture of several sets of 5 rectangles*
Attend to precision: *Incorrectly adds, creates non-equivalent fractions with addition, etc. and fails to appropriately label the final results as Jolly Ranchers and/or Lemonheads.*	

Look for and make use of structure: Fails to use the multiplicative structure of ratios to scale up to some number	*and 13 circles, until switching to one of the additive strategies above.*
Look for and express regularity in repeated reasoning: Fails to use the multiplicative structure of ratios as a shortcut to scale up to some number quickly.	

a.



5+5+5
13+13+13

b.

5+5+5+5+5 = 25
13+13+13+13+13 = 65 25JR with 65 L

c.

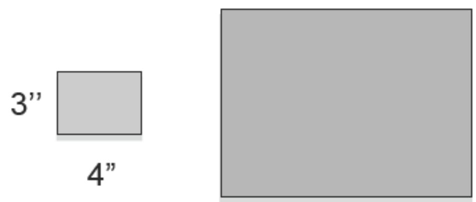
25+75 = 100 JR
65+75 = 140 L

Scoring Guide

3. Photo Copies

Katie and Jacob are enlarging pictures for the school yearbook on the photocopier. The ratio of the width to the length of the enlarged photo will be the same as the ratio of the width to the length of the original photo.

One of the photographs that they want to enlarge is a 3” x 4” photo. Katie says that she can enlarge the photo to 9” x 12”, but Jacob disagrees. He says it will be 11” x 12”. Who is correct? Explain your reasoning in words.



Sixth Grade CCSS for Mathematical Content

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

6.RP.3a Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

The CCSS for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
6. Attend to precision
7. Look for and make use of structure

Three-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of all parts of the task. Minor omissions may exist, but do not detract from the correctness of the response. Minor arithmetic errors may be present, but no errors of reasoning appear.

Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution.

Mathematical Practices	Evidence-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Correctly abstracts the 3:4 ratio from the context. Works accurately with a table, a ratio, multiplication or proportion statements, then accurately notes that the 9" x 12"photo is the enlargement. **Construct viable arguments and critique the reasoning of others:** Provides the logical argument that only Katie's method is the enlargement; backs up the argument by clearly explaining the multiplicative strategy used to test and respond, e.g., scaling, multiplication, proportion, etc. **Model with mathematics:** Possibly models with a table or writes number sentences describing the reasoning used, e.g., 9 = 3 x 3 or (3 x 3):(4 x 3) = 3:4, etc. **Attend to precision:** Correctly extends the table, multiplies, uses ratios or proportions; labels the final result as 9 inches and 12 inches **Look for and make use of structure:** Uses the multiplicative structure of ratios, equivalent fractions, and/or proportions to scale up.	Response indicates an understanding of the concept of a ratio and the ability to use ratio and rate reasoning to solve real-world and mathematical problems. Argues that Katie has used a multiplicative method to determine the enlargement of the photo is 9" x 12", and/or Jacob has used an additive method to determine the enlargement of the photo is 11" x 12." accompanied by one of the following pieces of evidence: a. Scaling up the 3:4 ratio in tabular form until the 9:12 ratio is reached. b. Multiplying 3 x 3 = 9 and 4 x 3 = 12, or (3 x 3):(4 x 3) = 3:4; therefore, the 9" x 12"photo is the enlargement, c. Scaling up the 3:4 ratio in fraction form, e.g., $\frac{3"}{4"} = \frac{6"}{8"}$ until the $\frac{9"}{12"}$ ratio is reached. d. Recognizing that both Katie and Jacob claim the length is 12"; then using a proportion or proportional reasoning. $\frac{3"}{4"} = \frac{x}{12"}$ or $\frac{3"}{x} = \frac{4"}{12"}$ e. Possibly reasoning that Jacob's method is additive by indicating 3 + 8 = 11 and 4 + 8 = 12 while Katie's method is multiplicative by indicating, e.g., 3 x 3 = 9 and 3 x 4 = 12. f. Possibly reasoning that Jacob's method is additive by indicating 3 + 1 = 4 and 11 + 1 = 12, while Katie's method is multiplicative

Katie is correct. Jacob thinks since the 3 and 4 in the smaller photo is counting up by 1, the larger photo should be 11" x 12".

Katie knows that 3" x 4" is 3 inches as the width and 4 inches as the length. Katie probably used porportional reasoning and did 3x3 to get 9 and 3x4 to equal 12.

Two-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of all parts of the task. Some of the necessary connections between the context and the table may be missing.

<table>
<tr><td colspan="2">Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution.</td></tr>
<tr><th>Mathematical Practices</th><th>Evidence-Based Traits: Mathematical Content</th></tr>
<tr><td>Reason abstractly and quantitatively: Correctly abstracts a ratio from the context. Works multiplicatively, but fails to use the quantities appropriately.

Construct viable arguments and critique the reasoning of others:
Provides a logical argument that both length and width in an enlargement must be tended to multiplicatively. Fails to note that only Katie's method is multiplicative.

Model with mathematics: Possibly writes number sentences describing the reasoning used, e.g., $\frac{4"}{3"} = \frac{16"}{12"}$, etc.

Attend to precision: Correctly extends a table, multiplies, or uses ratios or proportions; labels the final result as inches.

Look for and make use of structure: Possibly uses the multiplicative structure of ratios to scale up to some number (see c - e to right).

Look for and express regularity in repeated reasoning: Possibly uses the multiplicative structure of ratios as a shortcut to scale up to some number quickly, e.g., in a table or number sentence.</td><td>Response indicates an understanding of the concept of a ratio and the ability to use ratio and rate reasoning to solve real-world and mathematical problems; any of the 3-point strategies for solving the problem may be used, but response does not carefully "abstract and reason quantitatively", i.e., does not use the quantities appropriately.

a. Scaling up the 3:4 ratio in tabular form but failing to stop at the 9:12 ratio.

b. Reversing the numbers and working with a 4: 3 ratio in tabular form, stopping when the 3 scales to 12. Concluding neither person has determined the correct enlargement

c. Reversing the 3: 4 ratio; using a 4: 3 ratio and attempting to scale up 4 to 9, 11, or 12, e.g., 4 x 2.5 = 9 and is 3 x 2.5 =7.5 while 4 x 2.75 =11 and 3 x 2.75 =8.25; therefore, neither person has determined the correct enlargement.

d. Scaling up the 4:3 ratio in fraction form or by using proportions and deciding neither person has determined the correct enlargement, e.g., $\frac{4"}{3"} = \frac{16"}{12"}$.

e. Possibly reasoning that Jacob's method is additive by indicating 3 + 8 = 11 and 4 + 8 = 12 but failing to check and note that Katie's method is multiplicative in any way.

f. Possibly reasoning that Jacob's method is additive by indicating 3 + 1 = 4 and 11 + 1 = 12, but failing to check and note that Katie's method is multiplicative in any way.</td></tr>
</table>

~~Jake~~
Jacob ~~is~~ thinks that since 3 and 4 are 1 number apart that the answer should be too.

But since you are multipling the larger photo should be 4 apart since you multiplied 4 times.

We know that 3x4 is 3 inches as the with and 4 inches as the lenght

If we use porpotional resoning to figure out 3:4 then just x 3 by 4 to 12 and 4 by 4 to 16

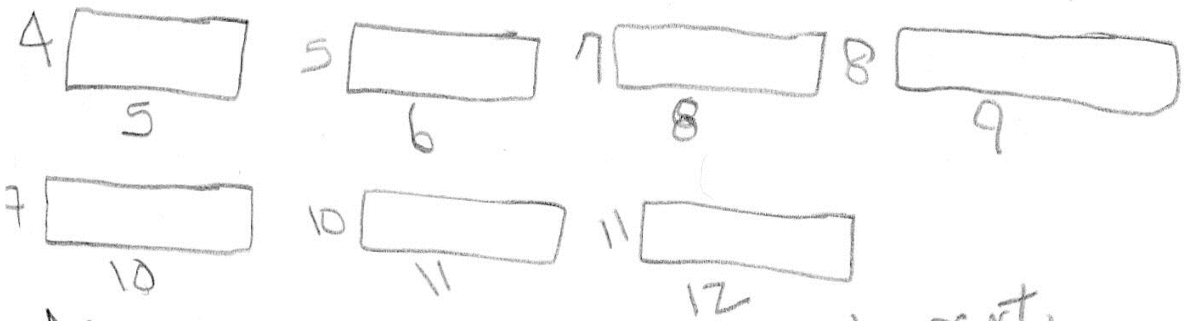
One-Point Score:

The response accomplishes some of the prompted purposes and communicates the student's mathematical understanding. The student's strategy and execution meet only a few of the content demands of the task. The necessary connections between the context and a multiplicative structure are missing.

Make sense of problems and persevere in solving them.
Work is missing representations of the problem or understanding of the constraints of the problem given in the text.

Mathematical Practices	Evidence-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Correctly abstracts a ratio from the context. Works multiplicatively, *but fails to use the quantities appropriately.*	Response indicates *a lack of ability to use ratio and rate reasoning to solve real-world and mathematical problems, possibly because of a misunderstanding of the multiplicative relationship between lengths and widths when enlarging a picture or a failure to use the quantities appropriately.* a. Creating and extending a table, but failing to recognize the multiplicative relationship, and/or failing to cope with the role of the dimensions of the enlargement in the problem. b. Extending the table *additively* in a 3: 4 or 4: 3 pattern, *but stopping at a 'good' place to stop, possibly where space runs out.* c. Possibly reasoning that Jacob's method is correct by indicating 3 + 8 = 11 and 4 + 8 = 12 *but Katie's method is not, since 3 + 8 ≠ 9.* d. Possibly reasoning that Jacob's method is correct by indicating that, since width + 1 = length (3 + 1 = 4) in the smaller photo, the same must be true in the larger photo. Jacob's method works, since 11 + 1 = 12 but Katie's method does not, since 9+ 1 ≠ 11
Construct viable arguments and critique the reasoning of others: *Argues improperly that Jacob's additive method produces the enlargement while Katie's does not.*	
Model with mathematics: Possibly writes number sentences describing the reasoning used, e.g., 3 + 8 = 11 and 4 + 8 = 12, etc.	
Attend to precision: Extends the table accurately, *but incorrectly uses a 4: 3 ratio and may or may not label the quantities as inches.*	
Look for and make use of structure: Fails to use the multiplicative structure of ratios to scale up to some number.	
Look for and express regularity in repeated reasoning: Fails to use the multiplicative structure of ratios as a shortcut to scale up to some number quickly.	

The l and w has to stay 1 apart so you go like this:



At the end they are still 1 apart.
Katie's doesn't work becuz they are 3 apart.

Scoring Guide

4. Boxed Oranges

A local store sells oranges at a price of 6 for $9.00.

a. Abbey wants to send 24 oranges to her grandmother. How much will Abbey spend if she buys the oranges at the local store? Explain your reasoning in words.

b. Abbey decides she does not have enough money to buy 24 oranges. She can only buy15 oranges. How much do 15 oranges cost? Explain your reasoning in words.

Sixth Grade CCSS for Mathematical Content

Understand ratio concepts and use ratio reasoning to solve problems.

6.RP.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.
6.RP.2	Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.3a	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
6.RP.3b	Solve unit rate problems including those involving unit pricing and constant speed.

The CCSS for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Three-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of all parts of the task. Minor omissions may exist, but do not detract from the correctness of the response. Minor arithmetic errors may be present, but no errors of reasoning appear. The student clearly grasps the connection between ratio and the context given in the task.

<table>
<tr><td colspan="2">Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution.</td></tr>
<tr><th>Mathematical Practices</th><th>Evidence-Based Traits: Mathematical Content</th></tr>
<tr><td>Reason abstractly and quantitatively: Correctly abstracts the 6 oranges : $9 ratio from the context; creates a representation for parts a and b, solves the problem and correctly notes the meaning of the results in the context of the problem.</td><td rowspan="3">Response indicates an understanding of the concept of ratio by correctly noting the ratio of 6 oranges to $9.00 using pictures, tables or ratio notation.

Response indicates an understanding of the multiplicative structure in a ratio.

Correctly scales the 6 oranges to $9.00 ratio in any of the following ways:
a. Possibly drawing a diagram of several sets of 6 oranges at $9 for each set, until four sets of oranges are drawn (24 oranges) and four sets of $9 are formed. Possibly pairing each orange with one dollar, then splitting the $3 into 6 groups of $0.50 (perhaps drawing coins) to determine each orange costs $1.50 (unit rate); then multiplying $1.50 X 24.
b. Dividing $9.00 by 6 to arrive at the unit rate of $1.50 per 1 orange; then multiplying $1.50 X 24.
c. Scaling up the 6:9 ratio in tabular form until a total of 24 oranges is reached.
d. Scaling up the 6:9 ratio in fraction form, e.g., $\frac{6\ OR}{\$9} = \frac{12\ OR}{x18}$ until the number of oranges equals 24.
e. Multiplying 6 and 9 by 4, since 6 x 4 = 24.
f. Using a proportion or proportional reasoning (e.g., $\frac{6\ OR}{\$9} = \frac{24}{x}$ or $\frac{6\ OR}{24\ OR} = \frac{\$9}{x}$)</td></tr>
<tr><td>Construct viable arguments and critique the reasoning of others: Provides an explanation noting that, for each group of 6 oranges, $9 must be spent; clearly explaining the strategy used (diagram, unit rate, table, ratio and possibly proportion) for each part of the problem.</td></tr>
<tr><td>Model with mathematics: Possibly writes number sentences describing the reasoning used, e.g., 6 x 2.5 = 15 and 9 x 2.5 = 22.50, etc. or writes equivalent fractions or a proportion.</td></tr>
<tr><td>Attend to precision: Correctly multiplies, scales up, creates equivalent fractions, etc. and appropriately labels the final results as oranges and/or dollars.</td><td>Response indicates an understanding of the multiplicative structure in a ratio.

Correctly scales the 6 oranges to $9.00 ratio in any of the following ways:</td></tr>
</table>

Look for and make use of structure: Uses the multiplicative structure of ratios, equivalent fractions, and/or proportions to scale up (see b – f and h – l to right).	g. Possibly drawing a diagram of several sets of 6 oranges at $9 for each set, noting that three sets contain more than 15 oranges, then halving the third set of oranges and the $9.00. Possibly pairing each orange with one dollar, then splitting the $3 into 6 groups of $0.50 (perhaps drawing coins) to determine each orange costs $1.50 (unit rate); then multiplying $1.50 X 15.
Look for and express regularity in repeated reasoning: Uses the same structure of ratios, equivalent fractions, and/or proportions in both parts of the problem; possibly uses information gained in part a to shortcut decision-making in part b (e.g., see g to the right).	h. Dividing $9.00 by 6 to arrive at the unit rate of $1.50 per 1 orange; then multiplying $1.50 X 15. i. Scaling up the 6:9 ratio in tabular form until reaching 18, then noting the need to halve 6 oranges and $9.00, either adding 3 to 12 or subtracting 3 from 18, until a total of 15 oranges is reached; possibly using the results from the diagram in part a and adding or subtracting appropriately. j. Scaling down and up the 6:9 ratio in fraction form, e.g., $\frac{6\ OR}{\$9} = \frac{2\ OR}{\$3}$ until the number of oranges equals 15, possibly using similar strategies as noted above. k. Multiplying 9 by 2.5, since 6 x 2.5 = 15. l. Using a proportion or proportional reasoning (e.g., $\frac{6\ OR}{\$9} = \frac{15}{x}$ or $\frac{6\ OR}{15\ OR} = \frac{\$9}{x}$)

a. It would cost $36.00 for her to send 24 oranges to her grandmother. We know that 6 x 4 = 24 and 4 x 9 = $36 So abbey has to pay $36.00 For 24 oranges.

b. It costs $22.50 to buy 15 oranges. We since 2 packs (12 oranges) costs $18.00 and you have 3 oranges left you can cut the price in half and got $4.50. $18.00 + $4.50 = $22.50

Two-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of all parts of the task. Some of the necessary connections between the context and the ratio may be missing.

Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution, *but indicates a struggle to interpret parts of the context correctly.*

Mathematical Practices	Evidence-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Correctly abstracts the 6 oranges : $9 ratio from the context; creates a representation for parts a and b. Quantitatively scales up appropriately to some other number, *but fails to recognize the significance of the results in the context of the problem* and/or *fails to appropriately scale to 24 or to find a mechanism to scale to 15.* **Construct viable arguments and critique the reasoning of others:** Provides a logical argument that makes clear that the ratio must be handled in a multiplicative fashion, possibly indicating 'doubling' or 'tripling' or *multiplying both the number of oranges and cost by the same number.* **Model with mathematics:** Possibly writes number sentences describing the reasoning used, e.g., 6 x 2.5 = 15 and 9 x 2.5 = 22.50, etc. or writes equivalent fractions or a proportion. **Attend to precision:** Correctly multiplies, scales up, creates equivalent fractions, etc. *but fails to appropriately label the final results as oranges and/or dollars* **Look for and make use of structure:** Uses the multiplicative structure of ratios equivalent fractions, and/or proportions to scale up (see c, d, g and h to right).	Response indicates an understanding of the multiplicative structure in a ratio: any of the 3-point strategies for scaling the 6: 9 ratio may be used, *but response does not "abstract and reason quantitatively" with the 24 OR the 15, i.e., does not use one or the other appropriately in the problem.* Correctly scales the 6: 9 ratio in any of the following ways, *but fails to recognize that the 6 must be scaled up to 24:* a. Possibly drawing a diagram of several sets of 6 oranges at $9 each set, *but failing to stop at 24 oranges.* b. Dividing $9.00 by 6 to arrive at the unit rate of $1.50 per 1 orange, *but failing to multiply the price per orange by 24.* c. Scaling up the 6:9 ratio in tabular form, in fraction form, or by using a proportion, but failing to stop at 24 oranges. d. Multiplying 6 and 9 by the same number, *but not by 4.* Correctly scales the 6: 9 ratio in any of the following ways, *but fails to recognize that the 6 must be scaled up to 15 or fails to find a mechanism for scaling to 15.* e. Possibly drawing a diagram of several sets of 6 oranges at $9 each set; *possibly approximating a cost between $18 and $ 27.* f. Scaling up the 6:9 ratio in tabular form, in fraction form, or by using a proportion, but failing to recognize the need to consider half of a set and half of the cost; *possibly approximating a cost between $18 and $ 27.* g. Multiplying 6 and 9 by the same number, *but not by 2.5.*

Look for and express regularity in repeated reasoning: Uses the same structure of ratios, equivalent fractions, and/or proportions in both parts of the problem; possibly attempts to use information gained in part a to shortcut decision-making in part b (e.g., notes 12 oranges cost $18 and 18 oranges cost $27, then *possibly approximating a cost between $18 and $ 27*.	

a. 6x4=24 and 9x4=36

So ~~Abby~~ Abby needs to pay $36.00 for the 24 oranges.

b. 6x2=12 and 9x2=18 and 9x3=27

we need 3 more oranges so the price is between $18.00 and $27.00 for the 15 oranges.

One-Point Score:

The response accomplishes some of the prompted purposes and communicates the student's mathematical understanding. The student's strategy and execution meet only a few of the content demands of the task. The necessary connections between the context and the ratio are missing.

<table>
<tr><td colspan="2">Make sense of problems and persevere in solving them.
Work is missing representations of the problem or understanding of the constraints of the problem given in the context.</td></tr>
<tr><th>Mathematical Practices</th><th>Evidence-Based Traits: Mathematical Content</th></tr>
<tr><td>Reason abstractly and quantitatively: May fail to abstract the 6: 9 ratio from the context or abstracts the 6: 9 ratio from the context, but works additively and fails to recognize the significance of the results in the context of the problem and/or fails to appropriately interpret the meaning of the 24 and 15 in the context.</td><td>Response indicates some understanding of ratio, but not the rate nature of this relationship, e.g., by noting a ratio of 6:15.

Response indicates an attempt to work with ratio, but fails to recognize the multiplicative relationship, typically by reverting to addition, and/or fails to cope with the constraints of the problem.</td></tr>
<tr><td>Construct viable arguments and critique the reasoning of others: Argues that the situation must be tended to in some additive fashion.</td><td rowspan="5">Attempts to reason in any of the following ways:
a. Adding 18 to 6 and 9, since 6 + 18 = 24 or adding 9 to 6 and 9, since 6 + 9 = 15.
b. 'Scaling up' the 6: 9 ratio to $24 and $15 in tabular form using addition, not multiplication.
c. 'Scaling up' the 6: 9 ratio in tabular form using addition; stopping at a 'good' place to stop, possibly where space runs out
d. Possibly drawing a diagram of several sets of 6 oranges at $9 each set, until switching to one of the additive strategies above, particularly in part b.</td></tr>
<tr><td>Model with mathematics: Possibly writes number sentences describing the reasoning used, e.g., 6 + 18 = 24, and 9 + 18 = 27, etc. or attempts to write equivalent fractions, but uses addition rather than multiplication.</td></tr>
<tr><td>Attend to precision: Incorrectly adds, creates un-equivalent fractions with addition, etc. and fails to appropriately label the final results as oranges and/or dollars.</td></tr>
<tr><td>Look for and make use of structure: Fails to use the multiplicative structure of ratios to scale up to some number</td></tr>
<tr><td>Look for and express regularity in repeated reasoning: Fails to use the multiplicative structure of ratios; uses different strategies in parts a and b or fails to use information gained in part a to shortcut decision-making in part b.</td></tr>
</table>

oranges	$
6	$ 9
12	$ 15
18	$ 21
24	$ 27
15	$ 18

a. 24 oranges cost 27 dollars.

b. 15 cost 18 dollars.

GRADE 6 MATH: RATIO REASONING

ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points. The student work shows examples of student understandings and misunderstandings of the task.

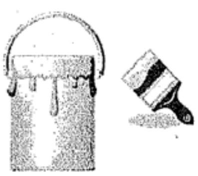
Score Point 2
The student extends the table and clearly explains the patterns that can be followed to add more values to the table. S/he identifies that there is a ratio between the liters of blue and yellow paint. The student provides a limited explanation in part *b* but does clearly indicate the information in the table. The student recognizes the structure of the relationship between the blue and yellow paint and is able to extend the relationship/pattern.

Assessment 1
Grade 6

institute for learning

1. Mixed Paint

Mark was mixing blue paint and yellow paint in the ratio of 2:3 to make green paint. He wants to make 45 liters of green paint. He began to make a table to help him think about the problem, but is unsure of what to do next.



Liters of Blue Paint	Liters of Yellow Paint	Liters of Green Paint
2	3	5
4	6	10
6	9	15
8	12	20
10	15	25
12	18	30
14	21	35
16	24	40

Blue paint	Yellow paint	Green pai
18	27	45

Student extends the table of equivalent ratios. (6.RP.3a)

a. Explain in words how to continue to place values into the table.

Well, you can add the first two numbers and will get you 5. The Blue paint is going by two's. The yellow paint is going by threes. The Gree paint is going by fives.

Student shows understanding of the concept of a ratio and is able to show this in the table, as well as in the explanation for part *a*. (6.RP.1, 6.RP.3a)

b. Write an explanation in words to Mark about how he can use his table to find how many liters of blue paint and how many liters of yellow paint will he need to make 45 liters of green paint.

You can use this table to help you find out the ratio and how many liters of blue and yellow paint to make the green you want.

Student explanation in part *b* does not fulfill the requirement of constructing a viable argument. (MP.3)

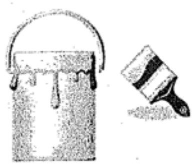
Score Point 2
The student correctly extends the table of values until there are 45 liters of green paint and explains that extending the pattern requires adding down. The student doesn't attend to precise language in the explanation, indicating that "you have to add the numbers going down," in the explanation of how to continue the values in the table. The student identifies the number of liters of yellow and blue paint required to make 45 liters of green paint.

Assessment 1
Grade 6

institute for learning

1. Mixed Paint

Mark was mixing blue paint and yellow paint in the ratio of 2:3 to make green paint. He wants to make 45 liters of green paint. He began to make a table to help him think about the problem, but is unsure of what to do next.



Liters of Blue Paint	Liters of Yellow Paint	Liters of Green Paint
2	3	5
4	6	10
6	9	15
6	12	20
15	15	25
12	18	30
14	21	35
16	24	40
18	27	45

Student correctly extends table of equivalent ratios. (6.RP.3a)

a. Explain in words how to continue to place values into the table.

You have to add the numders going down.

Student does not attend to precision or use ratio language in explanation for part *a*. (6.RP.1, MP.6)

b. Write an explanation in words to Mark about how he can use his table to find how many liters of blue paint and how many liters of yellow paint will he need to make 45 liters of green paint.

You will have to add dy 5 until You get to 45. After that You can add 3 all the way to 27 and add 2 all the way to 18.

Explanation in part *b* uses ratio and rate reasoning through tables of equivalent ratios. (6.RP.3)

Score Point 1
The student correctly identifies a pattern to continue the values in the column for blue paint. The student writes an incomplete explanation of how to find the number of liters of blue and yellow paint required to make 45 liters of green paint. While the student does provide a method for solving the problem, namely extending the table using the pattern until you get to 45 liters of green paint, s/he does not actually solve the problem.

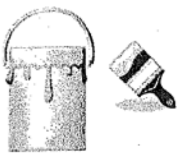
Assessment 1
Grade 6

institute for learning

1. Mixed Paint

Mark was mixing blue paint and yellow paint in the ratio of 2:3 to make green paint. He wants to make 45 liters of green paint. He began to make a table to help him think about the problem, but is unsure of what to do next.

Liters of Blue Paint	Liters of Yellow Paint	Liters of Green Paint
2	3	5
4	6	10



a. Explain in words how to continue to place values into the table. The blue paint must go up by 2

Student does not demonstrate complete understanding of the concept of ratio, only notes the increase in blue paint. Student also does not sufficiently use ratio language in explanations (6.RP.1).

b. Write an explanation in words to Mark about how he can use his table to find how many liters of blue paint and how many liters of yellow paint will he need to make 45 liters of green paint.

you have to add 2 liters of blue paint to 3 liters of yellow paint to get 5 liters of green paint so add 2 blue plus 3 yellow tell you get 45 liters

While the student makes some sense of the problem, s/he does not persevere in solving it. (MP.1)

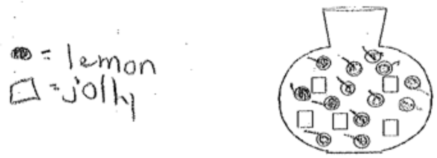
Score Point 3
The student identifies the correct ratio of Jolly Ranchers to Lemonheads in part *a*. The student provides a correct answer (including notation for doubling the number of each candy) for part *b*. The student correctly scales the number of Lemonheads when there are 100 Jolly Ranchers using multiplication (multiplying each part of the original ratio in part a by twenty). The work demonstrates a correct process for finding the scaling factor (dividing 100 by 5) and using the resulting quotient as a multiplicative scaling factor for the number of Lemonheads.

Assessment 1
Grade 6

institute for learning

2. Jolly Ranchers and Lemonheads

The candy jar below contains Jolly Ranchers (the rectangles) and Lemonheads (the circles). Answer each of the following questions:



a. What is the ratio of Jolly Ranchers (the rectangles) to Lemonheads (the circles)?

5 jolly ranchers / 13 lemon heads

Student demonstrates understanding of the concept of ratio. (6.RP.1)

b. Suppose you have a larger jar of Jolly Ranchers and Lemonheads in which the ratio of the two candies is equivalent to the ratio in this jar. How many Jolly Ranchers and Lemonheads might be in the jar? Use mathematical reasoning to justify your response.

13 × 2 = 26

5 jolly × 2 / 13 lemon × 2 = 10 jolly ranchers / 26 lemon heads

c. Suppose you have a new candy jar with the same ratio of Jolly Ranchers to Lemonheads as shown above, but it contains 100 Jolly Ranchers. How many Lemonheads are in the jar? Use mathematical reasoning to justify your response.

5J × 20 / 13L × 20 = 100 Jolly ranchers / 260 Lemon heads

100 ÷ 5 = 20

5 × 20 = 100

13 × 20 = 260

Student adequately uses ratio and rate reasoning to solve problems. (6.RP.3)

(Adapted from Smith, Silver, Stein, Boston Henningsen & Hillen (2005, p. 26)

3

Score Point 2
The student identifies the correct ratio of Jolly Ranchers to Lemonheads in part *a*. The student provides a correct answer (double the number of each candy) for part *b* but does not provide a justification for the answer. The student correctly identifies the number of Lemonheads when there are 100 Jolly Ranchers using multiplication (multiplying each part of the ratio in part *b* by ten). The response scored a Level 2 because there was limited reasoning used to justify the student's responses.

Assessment 1
Grade 6

institute for learning

2. Jolly Ranchers and Lemonheads

The candy jar below contains Jolly Ranchers (the rectangles) and Lemonheads (the circles). Answer each of the following questions:

a. What is the ratio of Jolly Ranchers (the rectangles) to Lemonheads (the circles)?

5:13

Student shows understanding of the concept of ratio. (6.RP.1)

b. Suppose you have a larger jar of Jolly Ranchers and Lemonheads in which the ratio of the two candies is equivalent to the ratio in this jar. How many Jolly Ranchers and Lemonheads might be in the jar? Use mathematical reasoning to justify your response.

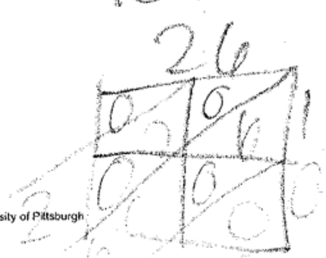
It might be 10:26

Student does not attend to precision by providing correct answer without justification. (MP.6)

c. Suppose you have a new candy jar with the same ratio of Jolly Ranchers to Lemonheads as shown above, but it contains 100 Jolly Ranchers. How many Lemonheads are in the jar? Use mathematical reasoning to justify your response.

10x10=100

100:260



(Adapted from Smith, Silver, Stein, Boston Henningsen & Hillen (2005, p. 26))

3

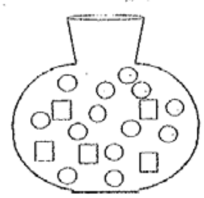
Score Point 1
The student incorrectly identifies the ratio of Jolly Ranchers to Lemonheads as 5 to 14. The student doubles the number of Jolly Ranchers and Lemonheads to get an equivalent ratio of 10 to 28, based on the incorrect answer in part *a*. S/he doesn't explain or use mathematical reasoning to justify any of the answers provided. The student incorrectly scales up the ratio in part *c*. The student's work shows a limited understanding of how to scale ratios.

Assessment 1
Grade 6

institute for learning

2. Jolly Ranchers and Lemonheads

The candy jar below contains Jolly Ranchers (the rectangles) and Lemonheads (the circles). Answer each of the following questions:



a. What is the ratio of Jolly Ranchers (the rectangles) to Lemonheads (the circles)?

5 Jolly ranchers
14 Lemonheads

Student lacked attention to precision. S/he made a computational error in part *a*, which led to incorrect answers in both parts *a* and *b*. (MP.6)

b. Suppose you have a larger jar of Jolly Ranchers and Lemonheads in which the ratio of the two candies is equivalent to the ratio in this jar. How many Jolly Ranchers and Lemonheads might be in the jar? Use mathematical reasoning to justify your response.

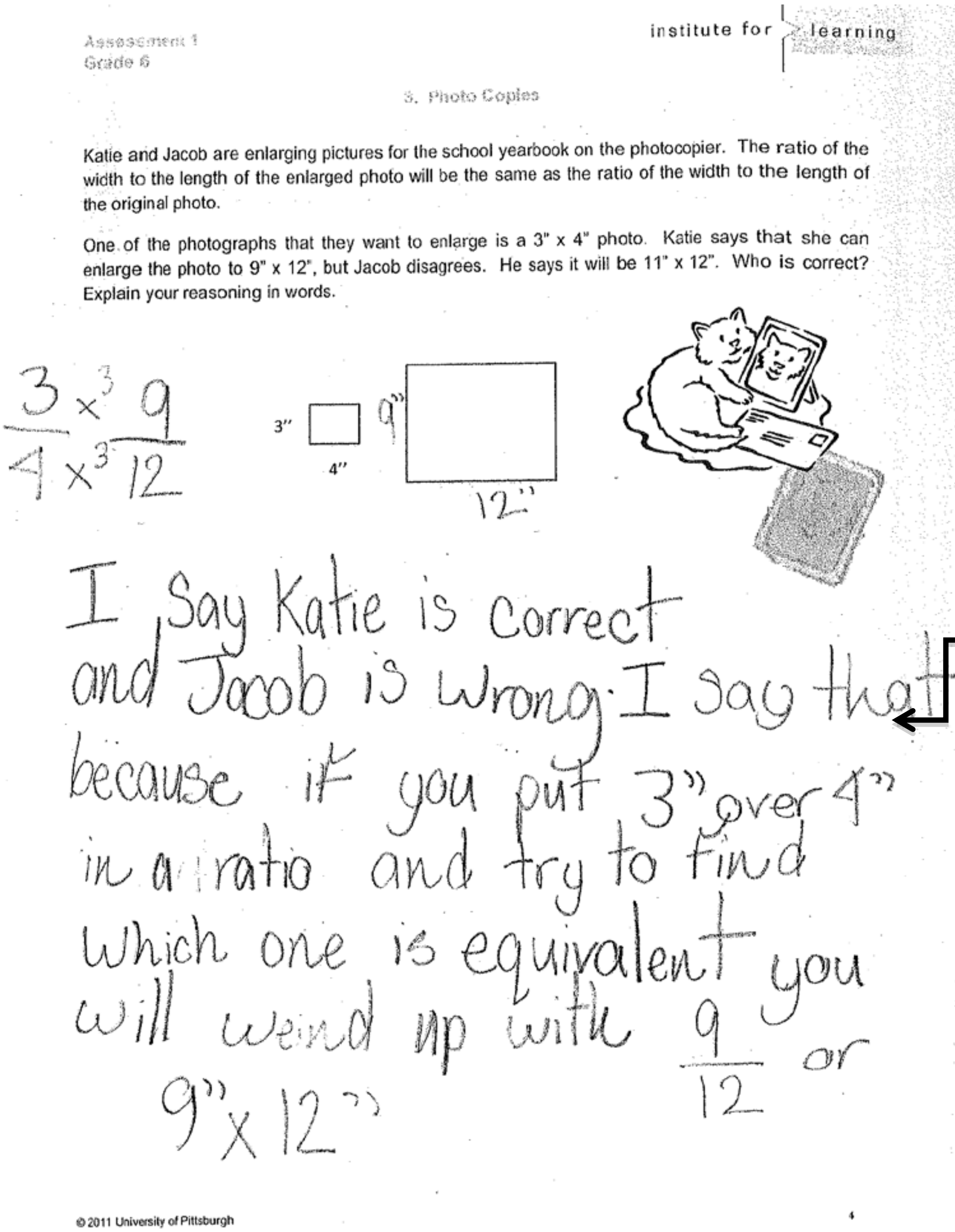
10 Jolly ranchers
28 lemonheads

c. Suppose you have a new candy jar with the same ratio of Jolly Ranchers to Lemonheads as shown above, but it contains 100 Jolly Ranchers. How many Lemonheads are in the jar? Use mathematical reasoning to justify your response.

100 Jolly ranchers
56 lemonheads

Student lacks understanding of the concept of equivalent ratios and proportional reasoning. (6.RP.1,3)

Score Point 3
The student correctly multiplies the dimensions of the original picture by 3 to obtain the correct dimensions of the enlargement. The student identifies that Katie is correct and explains a process for obtaining an equivalent ratio (scaling a fractional representation) with a connection to the precise dimensions of the enlargement.



Assessment 1
Grade 6

institute for learning

3. Photo Copies

Katie and Jacob are enlarging pictures for the school yearbook on the photocopier. The ratio of the width to the length of the enlarged photo will be the same as the ratio of the width to the length of the original photo.

One of the photographs that they want to enlarge is a 3" x 4" photo. Katie says that she can enlarge the photo to 9" x 12", but Jacob disagrees. He says it will be 11" x 12". Who is correct? Explain your reasoning in words.

$\frac{3 \times 3}{4 \times 3} \; \frac{9}{12}$

3" 4" 9" 12"

I Say Katie is Correct and Jacob is Wrong. I say that because if you put 3" over 4" in a ratio and try to find which one is equivalent you will weind up with $\frac{9}{12}$ or 9" x 12"

4

Student uses ratio and rate reasoning to solve a real-world problem and constructs a viable argument. (MP.3)

Score Point 2
The student sets up two proportions, one for Katie's answer and one for Jacob's. The student then cross-multiplies to test for proportionality. The student correctly states that Katie is right but does not explain the process or his/her reasoning in words.

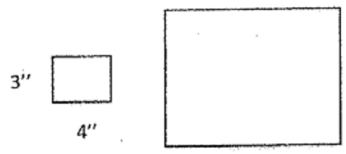
Assessment 1
Grade 6

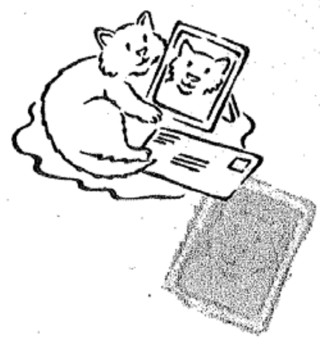
institute for learning

3. Photo Copies

Katie and Jacob are enlarging pictures for the school yearbook on the photocopier. The ratio of the width to the length of the enlarged photo will be the same as the ratio of the width to the length of the original photo.

One of the photographs that they want to enlarge is a 3" x 4" photo. Katie says that she can enlarge the photo to 9" x 12", but Jacob disagrees. He says it will be 11" x 12". Who is correct? Explain your reasoning in words.





Katie: 36 36 $\frac{3}{4} = \frac{9}{12}$ ✓

Jacob: 36 44 $\frac{3}{4} \times \frac{11}{12}$ ✗

ANSWER: KATIE IS CORRECT

Student clearly demonstrates understanding of ratios, but fails to provide appropriate explanation (6.RP.1, MP.3).

Score Point 2
This student shares a correct method for scaling up the measurements of the dimensions of the photographs. The student correctly identifies that multiplying both dimensions by three will result in final picture size that Katie argues will result. The student does not provide a clear connection between the reason for the operation (namely, the constant ratio that is stated in the problem) and scaling a fractional representation.

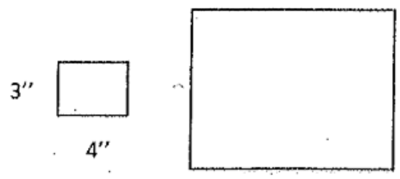
Assessment 1
Grade 6

institute for learning

3. Photo Copies

Katie and Jacob are enlarging pictures for the school yearbook on the photocopier. The ratio of the width to the length of the enlarged photo will be the same as the ratio of the width to the length of the original photo.

One of the photographs that they want to enlarge is a 3" x 4" photo. Katie says that she can enlarge the photo to 9" x 12", but Jacob disagrees. He says it will be 11" x 12". Who is correct? Explain your reasoning in words.



Katie is right because if you times 3x3=9 and 4x3=12 its 9x12.

Student does not attend to precision by not specifying units of measure. (MP.6)

Student demonstrates understanding of scaling up, but does not show understanding of the use of ratio to solve a real-world problem. (6.RP.1, 3)

Score Point 3
The student correctly connects the multiplicative scaling factor between the number of oranges (noting that, "6 times 4 is 24"), noting that the cost should be multiplied by four. To determine the cost of 15 oranges in part *b*, the student halves the cost of 6 oranges (to find that 3 oranges costs $4.50) and then multiplies that number by 5 to get $22.50. The student correctly identifies these processes in the work shown above the information presented in the problem. The written explanation could be improved by attending to the mathematical language of the unit (e.g., ratio, multiplication, and scaling).

Assessment 1
Grade 6

institute for learning

15 22.50
3 4.50
Oranges

A local store sells oranges at a price of 6 for $9.00.

a. Abbey wants to send 24 oranges to her grandmother. How much will Abbey spend if she buys the oranges at the local store? Explain your reasoning in words.

Abbey would spend 36$ because she is buying 4 times the amount theyre sold in 6 times 4 is 24 so it ought to cost 4 times as much and 4 times 9 is 36.

Student demonstrates complete understanding of using equivalent ratios and reasoning to solve real-world problems. (6.RP.3)

b. Abbey decides she does not have enough money to buy 24 oranges. She can buy only 15 oranges. How much do 15 oranges cost? Explain your reasoning in words.

15 oranges cost 22.50$ because 15 oranges costs and is 5 times what half of the amount they are sold in costs and is half what they are sold in is 3 for 4.50 4.50 times 5 is 22.50.

Student demonstrates ability to decontextualize quantities in the problem to arrive at correct solution and articulate meaning. (MP.2)

Score Point 1
The student provides incorrect answers in both parts of the problem and does not explain reasoning or show work. The written statements do not provide insight into the process the student used to solve the problem; the student merely restates the information from the problem.

Assessment 1
Grade 6

institute for learning

4. Boxed Oranges

A local store sells oranges at a price of 6 for $9.00.

a. Abbey wants to send 24 oranges to her grandmother. How much will Abbey spend if she buys the oranges at the local store? Explain your reasoning in words.

If he wants to buy 24 oranges for his grandmother then she will need to spend $39.

b. Abbey decides she does not have enough money to buy 24 oranges. She can buy only 15 oranges. How much do 15 oranges cost? Explain your reasoning in words.

If she wants to buy 15 cause she not have enough to buy them then she will have to spend like $18

Student demonstrates lack of understanding of ratio and rate reasoning (6.RP.1). Does not adequately make sense of the problem (MP.1).

GRADE 6 MATH: RATIOS REASONING

INSTRUCTIONAL SUPPORTS

The Instructional Supports on the following pages include a unit with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

Unit Outline: Grade 6 Math

Grade 6 Math: Introduction to Ratios & Proportional Relationships

Unit Overview

This unit should follow a longer unit that introduces and develops an understanding of ratio concepts and how to use ratio and rate language to describe relationships. Specifically, students should be able to recognize and describe ratios and rates by looking for structure and using precise language. In this unit, students will build on those skills as they extend their understanding to using ratio and rate reasoning to solve real-world mathematical problems.

This unit is supported by students' prior knowledge and study of multiplication and division (and expected fluency with multi-digit division), fractions (including using fractions to solve problems, ordering fractions and finding equivalent fractions, and multiplying fractions by whole numbers), and measurement (i.e., converting measurements within a system) completed in Grades 3-5 under the Common Core standards.

Additional opportunities to deepen and apply the knowledge, skills, and understandings developed in this unit should be provided. Specifically, this unit should be followed by further study of the application of ratio reasoning to unit conversions and understanding and using percents to solve problems to ensure full alignment of the curriculum to 6.RP.3, a standard that offers opportunity for in-depth study. Additionally, this year, students will also explore the connection between ratios and proportional relationships and expressions and equations, using variables to represent two quantities in a real-world problem that change in relationship to one another. Students will write an equation to express a dependent variable in terms of an independent variable using graphs and tables, and relate these to the equation (6.EE.9).

Unit Length

The sequence of lessons, tasks, and assessments included in this unit is intended to take two to three weeks of classroom instruction. This timeline is designed to ensure sufficient time for deep investigation and opportunities for application and assumes that ratios and proportional relationships were introduced early in the year.

Common Core Content Standards

- **6.RP.1** Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.
- **6.RP.3** Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- **6.RP.3a** Make tables of equivalent ratios relating quantities with whole-number

measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. ➢ **6.RP.3b** Solve unit rate problems including those involving unit pricing and constant speed. **STANDARDS FOR MATHEMATICAL PRACTICE** ➢ **MP.1** Make sense of problems and persevere in solving them. ➢ **MP.2** Reason abstractly and quantitatively. ➢ **MP.3** Construct viable arguments and critique the reasoning of others. ➢ **MP.4** Model with mathematics. ➢ **MP.6** Attend to precision. ➢ **MP.7** Look for and make use of structure. ➢ **MP.8** Look for and express regularity in repeated reasoning.	
BIG IDEAS/ENDURING UNDERSTANDINGS ➢ Reasoning with ratios involves attending to and coordinating two quantities. ➢ Ratios have a multiplicative relationship. ➢ In a multiplicative relationship, if one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied by the same factor to maintain proportional reasoning. The same multiplicative relationship can be applied to other situations and will result in quantities that maintain the same proportional reasoning.	**ESSENTIAL QUESTIONS** ➢ What is a rate? How are rates used to make comparisons? ➢ What is a unit rate? When is it appropriate to use unit rates to solve mathematical or real-world problems? ➢ What is the difference between a ratio and a rate? ➢ How can I use tables of equivalent ratios, tape diagrams, double number lines, the division method, or equations to find and compare rates of change?
CONTENT ➢ Relationship between fractions and ratios ➢ Scaling up and scaling down of ratios ➢ Rates, including unit rates ➢ Ratio and rate problems	**SKILLS** ➢ Apply multiplicative reasoning to explain the concept of ratio. ➢ Use ratio language to describe a ratio relationship between two quantities. ➢ Distinguish between rate and

➢ Proportional reasoning ➢ Representations of ratios, including ratio tables, tape diagrams, double number line diagrams, and equations ➢ Speed ➢ Equivalent ratios	ratio. ➢ Apply concepts of rate and ratio. ➢ Make tables of equivalent ratios. ➢ Solve unit rate problems, including those involving unit pricing and constant speed. ➢ Express ratios in different forms. ➢ Solve proportions using equivalent fractions. ➢ Use tables to compare and scale ratios.

Vocabulary/Key Terms

Ratio, rate, unit rate, equivalent ratios, per, "for every," "for each," "for each 1," scaling, unit, unit price, multiplicative relationship, ratio table, table, compare, multiplicative relationship

Formative Assessments and Instructional Tasks

Piano Task

This task should be used as a pre-assessment to determine students' understanding of ratios and their ability to use ratio language and a variety of tools to represent ratios, including making tables, finding equivalent ratios, using tape diagrams, double number lines, or equations. In this task, students are given a picture of a portion of a keyboard and are asked to identify the ratio of black keys to white keys (part-part ratio). Using this ratio reasoning, students are then asked to determine how many black keys appear on a keyboard with 35 white keys. Lastly, students are asked to determine how many black keys appear on a keyboard with a total of 240 keys (part-to-whole ratio).

The Best Deal

This formative assessment task asks students to compare three ratios to determine the cheapest price per can of soda. Students are expected to use the concepts of ratios and unit rates to compare the ratios and justify their answers with mathematical reasoning. (6.RP.3b)

Final Performance Task

Students were asked to complete four problems to assess the standards identified in this unit. The tasks, *Photo Copies*, *Mixed Paint*, *Jolly Ranchers and Lemonheads*, and *Boxed Oranges,* are provided. These tasks collectively assess students' understanding and ability to

- interpret what problems involving ratios, rates, and proportional relationships are asking them to find;
- use ratios and rates to solve these problems using a variety of methods, including making tables, finding equivalent ratios, using tape diagrams, double number lines, or equations;
- describe ratio relationships between two quantities.

Learning Plan & Activities

Unit Introduction

Students should be introduced to this unit by finding a variety of comparisons that can be "seen" easily in everyday life, including ratios of boys to girls in a classroom, vowels to consonants in words, sizes of pictures enlarged on a computer screen or photocopier, etc. Additional examples that can be introduced include speed (miles/hour) and concentration (juice concentrate/water). Students should be expected to use the language of ratios and rates and should be held accountable for the precise use of that language as they begin to understand these concepts.

Suggested Flow of the Unit

As you develop lessons for this unit, the following sequence of essential questions and activities should be considered. Also, we suggest connecting your lesson to the enduring understandings listed above to ensure that students are developing a deep understanding of these concepts.

Implement Pre-Assessment: Piano Task

Additional Supports:
The multiplicative relationship of ratios can be a source of challenge for students. Teachers should anticipate this challenge and look for evidence of misunderstanding in student work. We suggest modeling multiple strategies and providing opportunities for students to work with these strategies to help them understand the relationship.

Additional Supports
Model using tools such as tape diagrams, double line diagrams, and/or tables to help students see the multiplicative relationships between ratios. Provide time and opportunities for students to engage in guided practice and hands-on activities that allow students to explore this relationship.

EQ: WHAT IS A UNIT RATE? WHEN IS IT APPROPRIATE TO USE UNIT RATES TO SOLVE MATHEMATICAL OR REAL-WORLD PROBLEMS?

EQ: HOW CAN I USE TABLES OF EQUIVALENT RATIOS, TAPE DIAGRAM, DOUBLE NUMBER LINES, THE DIVISION METHOD, OR EQUATIONS TO FIND AND COMPARE RATES OF CHANGE?

Rate: A rate is a comparison of two quantities using different types of measures.

Additional Supports
Students may find it difficult to understand that rates compare two types of measurement quantities. It may be helpful to have students discuss and understand the relationship between the two measurements. Also, students should be sure to label the different quantities as a strategy to make sense of the relationship and to reinforce their understanding of ratio relationships.

Additional Supports
Students may struggle with the concept of measurement if they have not done work around converting between measurements. Discuss different types of measurements that are familiar to students.

End of Rate Lessons Activity: Have students extend their understanding of ratios to rates by giving examples of rates they encounter in the real world. They should then express these rates using ratio and rate language and by using tools such as tape diagrams, double line diagrams, and/or tables.

Additional Supports
Design the end of lesson activities so that students are able to write, discuss, create, or draw to communicate their understanding of the concepts assessed in each activity.

EQ: WHAT IS A UNIT RATE? WHEN IS IT APPROPRIATE TO USE UNIT RATES TO SOLVE MATHEMATICAL OR REAL-WORLD PROBLEMS?

EQ: HOW CAN I USE TABLES OF EQUIVALENT RATIOS, TAPE DIAGRAM, DOUBLE NUMBER LINES, THE DIVISION METHOD, OR EQUATIONS TO FIND AND COMPARE RATES OF CHANGE?

Unit Rate: A unit rate compares a quantity of one measure to a single unit of another measure.

End of Unit Rates Lesson Activity: Have students revisit their examples of rates and then convert them into unit rates using ratio and rate language and each of the following tools: tape diagrams, double line diagrams, equivalent ratios, and the division method.

End of Unit Rates Lesson Activity 2: Have students describe a situation where it would be helpful/appropriate to know/determine the unit rate.

> **Additional Supports**
> Design the end of lesson activities so that students are able to connect their learning to real-world situations that are authentic and address common experiences and interests.

Implement Formative Assessment: The Best Deal Task

> **Additional Supports**
> Students may make their decisions about the best deal based on information in the problem other than the unit price (e.g., the wording of the problem, total cost, or the total number of items). It is important to have students justify their answers by making the connection to the cost per unit.

Implement Formative Assessment: Lawn Mowing Task (referenced below)

EQ: WHAT IS THE DIFFERENCE BETWEEN A RATIO AND A RATE?

Rates are special cases of ratios where the quantities being compared have two different measures. While all rates are ratios, all ratios are not rates.

End of Rate & Unit Rate Lessons Activity: Are all ratios also rates? Use mathematical reasoning to justify your answer and provide two real-world situations to support your claim.

Unit Outline: Grade 6 Math

Suggested Lessons

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LESSON	DESCRIPTION/UNIT CONTENT ADDRESSED
5.1: Ratios and Rates	The series of investigations in this lesson focuses on recognizing ratios, ordering them, and understanding equivalent ratios that should have been addressed in a previous unit. However, in Investigation 4, students also investigate the concept of unit rates. This lesson is recommended to take 4 days. **Investigation 1** provides students with a general background on ratios and provides students with practice identifying and expressing ratio relationships. *(Addresses Assumed Prior Knowledge)* **Investigation 2** delves into comparisons between ratios and provides opportunities for students to scale ratios. *(Addresses Assumed Prior Knowledge)* **Investigation 3** introduces students to ratio tables as a means to express equivalent ratios and as a tool for scaling up ratios. *(Addresses Assumed Prior Knowledge)* **Investigation 4** provides an opportunity for students to apply their understanding of scaling, rates, and ratios in mathematical problems involving cost, and more specifically, unit pricing.
Supplemental Task: *Lawn Mowing* Performance-Based Assessment, *Impact Mathematics, Course I Chapter 5 Resource Masters*, pp. 45-47	This assessment task was created by MARS (the Mathematics Assessment Resource Service), funded by a grant from the National Science Foundation. The task asks students to determine unit rates (speed) of two people mowing lawns and to use those rates to determine the amount of time it takes each of them to cut a certain area of lawn.

Suggestions for Instructional Strategies

- **Class discussions:** There should be strong emphasis on mathematical discourse in the classroom. Students should be held accountable for the use of precise vocabulary and meanings of numbers, terms, and variables and the quantities they represent. Time should be provided for students to investigate and discuss the numerous patterns and relationships among quantities that are related in ratios, rates, and proportions. This provides teachers with additional opportunities to assess student understanding.
    - In a problem such as *The Best Deal*, students should discuss the

various relationships present in the problem. One of the first opportunities for discussion would emerge around the equivalence between the ratios of price per can at Metromarket and ShopStop. Students should be encouraged to express themselves in mathematically precise language, describing *equivalence* or *equivalent ratios*. For students who are having difficulty writing explanations and/or justifications, consider providing them with opportunities for verbal expression first. Questioning and disagreement provide robust opportunities for developing "fluency" with student discourse in mathematics.

- Questions should be asked to assess and advance students' understanding of the meaning of the numbers in the ratios they identify (e.g., What does *x* represent in this ratio?), the value of and connection between ratios and unit rates (e.g., How is this ratio related to this unit rate? How are they similar and different? Why is it helpful to convert a ratio to a unit rate?) and proportionality (e.g., Why is this a proportional relationship? What is the relationship among quantities in a proportional relationship?).

STUDENTS SHOULD BE REQUIRED TO EXPLAIN AND/OR JUSTIFY THEIR THINKING IN CLASSROOM DISCUSSIONS. ENCOURAGE STUDENTS TO ELABORATE ON BRIEF ANSWERS (INCLUDING ONE WORD RESPONSES LIKE: "FIVE!" "YES," "NO," "SOMETIMES," "MULTIPLY," ETC.).

Unit Outline: Grade 6 Math

Resources

- Ratio table organizers (copies), two color chips, graph paper, rulers
- *Impact Mathematics,* Chapter 5
- Manipulatives and tools appropriate for students to use, if necessary (e.g., pattern blocks, blank ratio tables, two-color counters, etc.)
- *NCTM Illuminations*
  - Constant Cost per Minute interactive graph (6.2.1) http://www.nctm.org/standards/content.aspx?id=25092
- *Leonardo Da Vinci Activity*, from The Math Forum @ Drexel University (http://mathforum.org/alejandre/frisbie/math/leonardo.html)
- *Elementary and Middle School Mathematics: Teaching Developmentally (5th Edition)*, "Chapter 18: Developing Concepts of Ratio and Proportion," by John A. Van De Walle ©2004 Pearson Education.
- "Ratio Tables" from schools.nyc.gov/inquire provides a research summary about ratio tables and their use and sample activity using ratio tables to graph ratios.
- "Proportional Reasoning Lesson Study Toolkit" provides resources for conducting lesson study using lessons on proportional reasoning. It includes additional tasks and student work that can support the development of students' understanding of ratios, rates, and proportional relationships.
- (http://www.lessonresearch.net/NSF_TOOLKIT/pr_maintoolkit.pdf)

PIANO TASK

Pianos and pipe organs contain keyboards, a portion of which is shown below.

a) What is the ratio of black keys to white keys in the picture above?

b) If the pattern shown continues, how many black keys appear on a portable keyboard with 35 white keys?

c) If the pattern shown continues, how many black keys appear on a pipe organ with a total of 240 keys?

Lesson Outline: *The Best Deal*

Common Core Learning Standards Addressed:

6.RP.1 Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.

6.RP.2 Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b≠0*, and use rate language in the context of a ratio relationship.

6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

b. Solve unit rate problems including those involving unit pricing and constant speed.

Lesson Objectives

Students will be able to compare unit rates or ratios involving prices to solve a mathematical problem.

Context

While somewhat simpler than problems that emerge in everyday life, this problem asks students to engage in thinking similar to that required by shoppers searching for the "best deal." Consumers are often confronted with problems that involve comparing prices for the same product packaged in different ways (namely, different prices for difference size packages or different quantities of a product). This problem would be implemented as a means to introduce the concept of unit rate/unit price. Students already have informal ways of solving the problem, including division. The share component of the lesson should highlight the relationships between the aggregate price, the ratio of price to quantity, and the unit price.

Set-up of the Lesson

The lesson should be set up with a brief description of the problem (e.g., "I went to the market over the weekend, and I couldn't decide where to buy soda for my party"). In this way, students are engaged, and opportunities arise for students to ask questions that might support their engagement with the task. Be careful not to provide answers or guidance that reduce the cognitive demand of the task.

Sample Questions to Support/Deepen Engagement with the Task

To assess student thinking:

- What are you comparing?
- How did you determine the price of a single can?
- Can you explain your process in words?
- Did you notice anything when you first looked at the prices?
- Would a calculator help you solve this problem?
- What is the problem asking? What information do you have?

To help students engage more deeply with the mathematics of the problem:

- What is the relationship between the price per can and the number of cans you can buy per dollar?
- How else can you represent the price per can?
- How many cans of soda can you buy at each store with $10? How do you know?

WHICH STORE GIVES YOU THE BEST DEAL?

Metromarket	Wal-Town	ShopStop
18 cans of soda for $9	**10 cans of soda for $7.50**	**14 cans of soda for $7.00**

Show your work in the space provided, and justify your answer in words.

Show your work.

Justify your answer using mathematical reasoning.

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This problem asks students to compare three ratios of price to cans. Students can solve the problem by:

- comparing ratios of price to cans:

  ____ ____ ____ **and**

  ____ ____ ____ ____

- comparing among the ratios of cans to price
- comparing various representations of the ratio (e.g., ____, $9:18 can, $9 to 18 cans, etc.
- completing a ratio table that enables them to compare the price for a certain number of cans

Cans	Metromarket	Wal-Town	ShopStop
10	?	$7.50	?
14	?	?	$7.00
18	$9	?	?
…	…	…	…
140	?	$105.00	$70.00
180	$90	$135.00	?
…	…	…	…
630	$315	$472.50	$315.00

Students are expected to use mathematical reasoning to justify their answer. Responses should reflect the processes used and the reasons for the processes. Possible responses can include:

- an explanation of the unit rate (price per can) and a comparison of the unit rates of the three stores
- a comparison of the two stores' prices per can iteratively or as is done in the ratio table

As students are solving the problem, carefully select certain solution methods for presentation to the whole class. These methods should model the same underlying concepts and quantities in different ways and should provide opportunities for students to think and talk further about the task.

Conclude the activity by eliciting information about the *unit rate* from students to develop a shared definition with a link to a concrete example and shared experience.

FORMATIVE TASK: The Best Deal

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