

Analysis of Relationship between Resistances in Delta Circuit to Determine Total Resistance

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Abstract Electric circuit has a very important role in electronics field progressing. From simple to complex form of circuits, actively support the establishment of new innovations in engineering technology. To build a good concept of circuit, it is necessary to have proper understanding on basic concept and circuit analysis. Starting from Ohm and Kirchhoff's laws, another circuit laws and formulas are derived from analysis and theorems, to give better calculation and solution in problems. This paper will do an analysis on relationship between resistances in delta circuit, and apply basic electric circuit theory to calculate the total resistance. This analysis is conducted by doing comparison and calculation to derive the expected formula.

Keywords: Δ – Y circuit, resistances, total resistance

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1. Introduction

To analyze an electric circuit problem especially a complex circuit, all we have to learn is the basic formula called Ohm's law. This is the very important thing that we need to know first in order to help us solving the problems for electric circuits. From this law we derive another important formulas to support the circuit analysis; Kirchhoff Voltage Law (KVL), Kirchhoff Current Law (KCL), Voltage divider, and Current divider. With these various derivative laws, the circuit solution becomes easier by the developing circuit methods and theorems.

Two basic electric circuits known in circuit analysis are Resistance and Δ – Y circuits. These two kind of circuits will mainly discussed and applied in this paper [1,4,5,6,7].

2. Discussion

2.1. Basic Theory

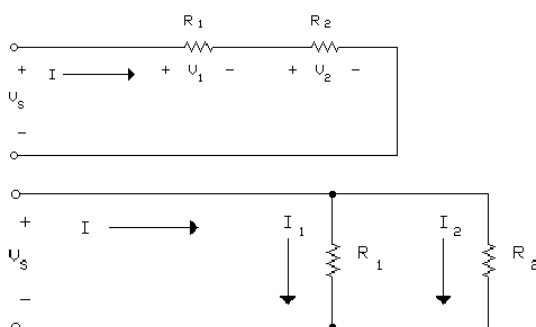


Figure 1. Resistance circuits: (a) series, and (b) parallel

There are two common resistance circuits: series and parallel, as shown in figure 1. Using the right formula for series and parallel, primarily we can calculate the total resistance for either single or combination form of electric circuit [1,4,5,6,7].

Series resistance has the same value of current I , and different value of voltage V , while parallel resistance has different value of I and one value of V [1,2,6,7,8].

$$V = I.R_1 + I.R_2 + \dots + I.R_n$$

$$V = I.(R_1 + R_2 + \dots + R_n) \quad (1)$$

$$V = I.R_{total}$$

$$R_{total} = R_1 + R_2 + \dots + R_n$$

$$V = I_1.R_1 = I_2.R_2 = \dots = I_n.R_n$$

$$R_n I = I_1 + I_2 + \dots + I_n$$

$$\frac{V}{R_{total}} = \frac{V}{R_1} + \frac{V}{R_2} + \dots + \frac{V}{R_n} \quad (2)$$

$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

It usually happens that the connection of each resistance is not in series or parallel, but in a form of delta (Δ) or wye (Y). In this case, we should use specific formula to get the right result [1,2,3].

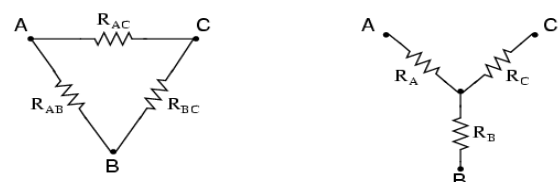


Figure 2. Δ and Y connections

To calculate a total resistance from an electric circuit having delta connection, mainly the circuit need to be converted from delta (Δ) to wye (Y) [2,3,6,8].

$$\begin{aligned} R_A &= \frac{R_{AB} \cdot R_{AC}}{R_{AB} + R_{AC} + R_{BC}} \\ R_B &= \frac{R_{AB} \cdot R_{BC}}{R_{AB} + R_{AC} + R_{BC}} \\ R_C &= \frac{R_{AC} \cdot R_{BC}}{R_{AB} + R_{AC} + R_{BC}} \end{aligned} \quad (3)$$

3. Aims of Study

This paper will conduct a simple analysis of applying the theory of basic electric circuit, to figure out the resistances relationship in delta circuit, to solve any problems related to total resistance. By using the derived formula, it gives another quick and best solution for circuit analysis in relation to resistance and $\Delta - Y$ concept. Hopefully this nice description of theory will build our critical thinking and bring spirit to always finding good solutions in electronics world.

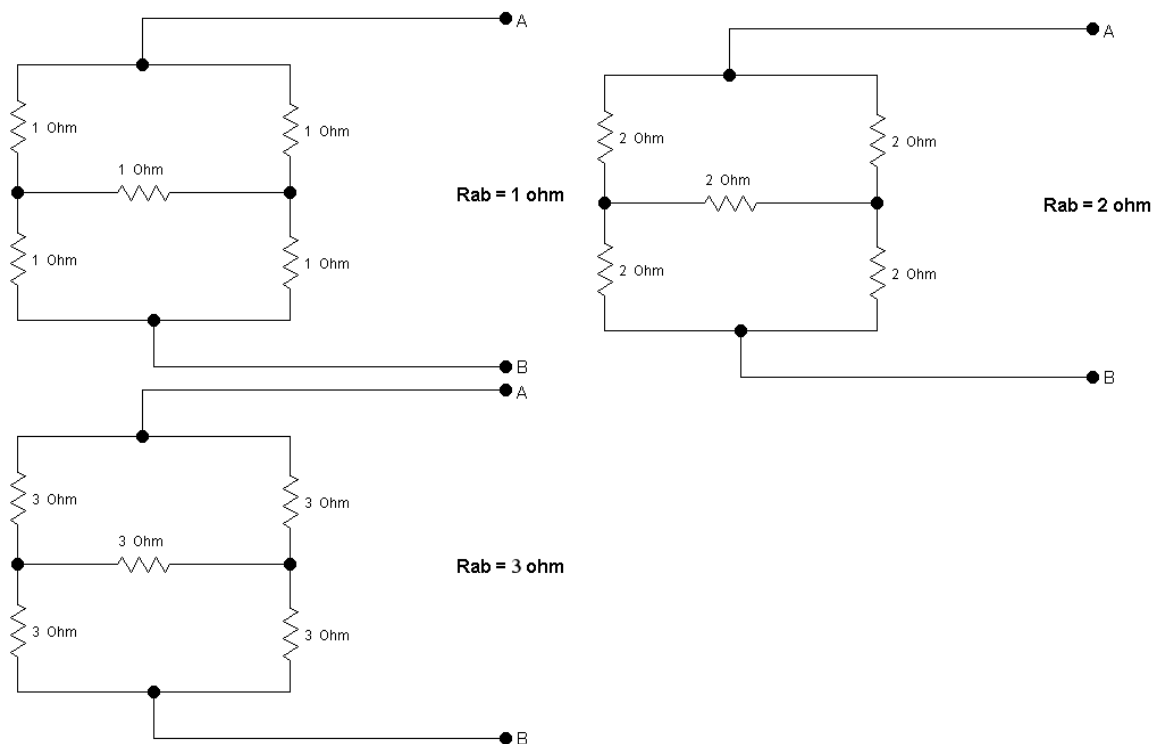


Figure 3. Resistance circuits having Δ model and same value of resistances

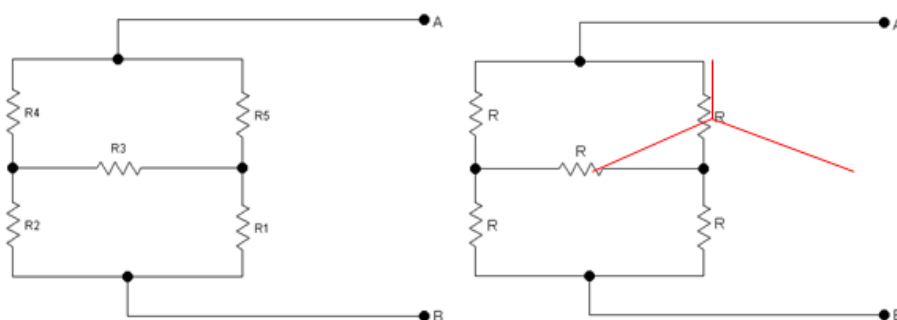


Figure 4. the Δ form on the upper side is changed into Y form and all resistances named with R

4. Research Method

For this kind of analysis, I try to compare several forms of resistance circuits having delta model, see the relationships, and applying the exact method and formula to derive another formula equations for calculating total resistance.

5. Analysis and Results

Using the standard of resistance and $\Delta - Y$ formula, we can derive different equations and solve the total resistance in electric circuit problems modeled in specific nodes and branches [2,3,6,8].

Pay attention to the electric circuit problem in figure 3. It is a specific form of circuit that usually we met when dealing with resistance circuit problems. It has a Δ model both on the upper side and bottom side of the circuit. To solve the circuit, we have to change the Δ form into Y form as shown in figure 4, and then do the parallel resistance calculation. Because all the resistance values are the same, we can states that all resistance equal to R, equation 4.

Using the formula of $\Delta - Y$, we get the result of each resistance in Y model as follows:

$$R_{\Delta Y} = \frac{R}{3} \quad (4)$$

The total resistance now can be calculated using the combination of series and parallel formula, and we get the result in equation 5 as follows:

$$R_{ab} = \frac{R}{3} + \frac{\left(R + \frac{R}{3}\right) \cdot \left(R + \frac{R}{3}\right)}{2 \cdot \left(R + \frac{R}{3}\right)}$$

$$R_{ab} = \frac{R}{3} + \frac{\left(R^2 + \frac{R^2}{3} + \frac{R^2}{3} + \frac{R^2}{9}\right)}{2R + \frac{2R}{3}}$$

$$R_{ab} = \frac{R}{3} + \frac{\left(\frac{9R^2}{9} + \frac{3R^2}{9} + \frac{3R^2}{9} + \frac{R^2}{9}\right)}{\frac{6R}{3} + \frac{2R}{3}}$$

$$R_{ab} = \frac{R}{3} + \frac{\left(\frac{16R^2}{9}\right)}{\frac{8R}{3}} \quad R_{ab} = \frac{8R}{24} + \frac{16R}{24} = \frac{24R}{24} = R$$

Suppose we change the value of two resistances into $(1+R)$, again we will get the result of each resistance in Y model and calculate the total resistance to derive the result in equation 6 as follows:

$$R_{\Delta Y} = \frac{R}{3} \quad R_{ab} = \frac{R}{3} + \frac{\left(1 + R + \frac{R}{3}\right) \cdot \left(1 + R + \frac{R}{3}\right)}{2 \cdot \left(1 + R + \frac{R}{3}\right)}$$

$$R_{ab} = \frac{R}{3} + \frac{\left(1 + R + \frac{R}{3} + R + R^2 + \frac{R^2}{3} + \frac{R}{3} + \frac{R^2}{3} + \frac{R^2}{9}\right)}{2 + 2R + \frac{2R}{3}}$$

$$R_{ab} = \frac{R}{3} + \frac{\left(1 + 2R + R^2 + \frac{2R}{3} + \frac{7R^2}{9}\right)}{\frac{6 + 6R + 2R}{3}}$$

$$R_{ab} = \frac{R}{3} + \frac{\left(\frac{9 + 18R + 9R^2 + 6R + 7R^2}{9}\right)}{\frac{6 + 8R}{3}}$$

$$R_{ab} = \frac{R}{3} + \frac{(9 + 24R + 26R^2)}{18 + 24R}$$

$$R_{ab} = \frac{72R^2 + 90R + 27}{72R + 54}$$

$$R_{ab} = \frac{(72R + 54) \cdot (R + 0,5)}{72R + 54} = R + 0,5 = \frac{R + (1 + R)}{2}$$

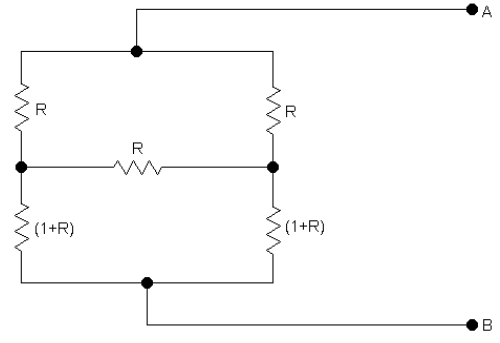


Figure 5. The two bottom side of resistances are given value $(1+R)$

Here are several simulations that have been conducted to prove the previous calculation results. They show the expected values from the current analysis, equation 6.

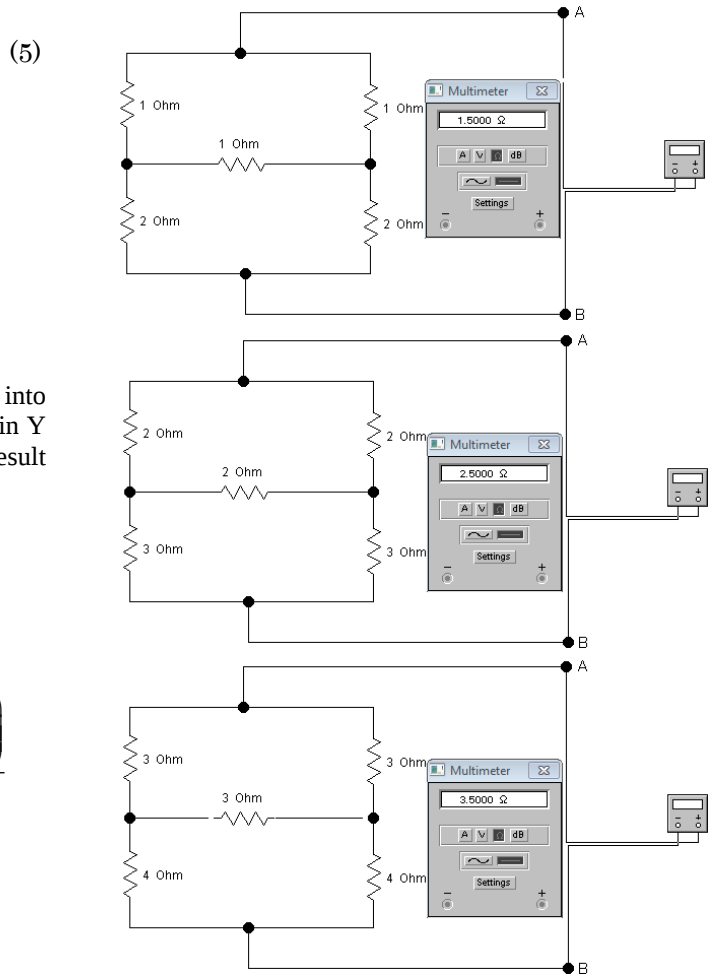


Figure 6. Circuit simulations using software electronics workbench 5.12

Continuing change the two resistances into $(2+R)$ and $(3+R)$, we will get the 'similar' pattern of result as described in the following equations:

$$R_{ab} \text{ for } (2+R) = \frac{R + (2+R)}{2} \quad (7)$$

$$R_{ab} \text{ for } (3+R) = \frac{R + (3+R)}{2} \text{ and etc} \quad (8)$$

It follows the same rules and pattern for the result of total resistance. These are unique relationships and can be applied for this special form of circuit problems.

6. Conclusion

From the specific relationship of resistances modeled in two delta connections, a standard and simple formula could be derived to solve the related problems for easier analysis and calculation. The value of resistances in this specific electric circuit are formed in special patterns and determined by the ratio between the two and three resistance values in the circuit. In the future, an advance analysis can be done to figure out more detail about this special form of circuit, and derive other formulas with different ratio of resistances.

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