Introduction to Detection Theory*

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Abstract

This is an introduction to Detection Theory. This module gives a brief overview of the problems associated with signal transfer–specifically, the effects that noise produces in a signal during transmission.

1 Introduction

The intent of **detection theory** is to provide rational (instead of arbitrary) techniques for determining which of several conceptions–models–of data generation and measurement is most "consistent" with a given set of data. In digital communication, the received signal must be processed to determine whether it represents a binary "0" or "1"; in radar or sonar, the presence or absence of a target must be determined from measurements of propagating fields; in seismic problems, the presence of oil deposits must be inferred from measurements of sound propagation in the earth. Using detection theory, we will derive signal processing algorithms which will give good answers to questions such as these when the information-bearing signals are corrupted by superfluous signals (noise).

The detection theory's foundation rests on statistical hypothesis testing (*Cramér, 1946, Chapter 35*[1]; *Lehman, 1986*[2]; *Poor, 1988, Chapter 2*[3]; *van Trees, 1968, pp 19-52*[4]). Given a probabilistic model (an event space Ω and the associated probabilistic structures), a random vector r expressing the observed data, and a listing of the probabilistic models–the **hypotheses**–which may have generated r, we want a systematic, optimal method of determining which model corresponds to the data. In the simple case where only two models– $\mathcal{M}_0$ and $\mathcal{M}_1$–are possible, we ask, for each set of observations, what is the "best" method of deciding whether $\mathcal{M}_0$ or $\mathcal{M}_1$ was true? We have many ways of mathematically stating what "best" means: we shall initially choose the average cost of each decision as our criterion for correctness. This seemingly arbitrary choice of criterion will be shown later **not** to impose rigid constraints on the algorithms that solve the hypothesis testing problem. Over a variety of reasonable criteria, one central solution to evaluating which model describes observations–the likelihood ratio test–will persistently emerge; this result will form the basis of **all** detection algorithms.

Detection problems become more elaborate and complicated when models become vague. Models are characterized by probability distributions, and these distributions suffice in the likelihood ratio test. Vagueness does not refer to this stochastic framework; rather, it refers to uncertainties in the probability distribution itself. The distribution may depend on unknown parameters, like noise power level. The distribution most certainly depends on signal structure; suppose that is partially or completely unknown? The most difficult (and interesting) problems emerge when uncertainties arise in the probability distributions themselves. For example, suppose the only model information we have is through data; how would an optimal detector be derived then?

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Along the way we will discover that a general geometric picture of detection emerges: Ease of a detection problem depends on how "far apart" the models are from each other. This geometric framework turns out to be elaborate, but underlies modern detection theory and forms links to information theory.

References

[1] H. Cram[U+FFFD] *Mathematical Methods of Statistics*. Princeton University Press, Princeton, NJ, 1946.

[2] E. L. Lehmann. *Testing Statistical Hypotheses*. John Wiley & Sons, New York, second edition. edition, 1986.

[3] H. V. Poor. *An Introduction to Signal Detection and Estimation*. Springer-Verlag, New York, 1988.

[4] H. L. van Trees. *Detection, Estimation, and Modulation Theory, Part I*. John Wiley & Sons, New York, 1968.