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Connecting IB to the Core

IB and the Common Core State Standards
Relationship studies: Resources to inform curriculum alignment

Mathematics standards

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IB PROGRAMMES
GLOBAL CONTEXTS
TEACHING AND LEARNING IN THE IB
THE IB LEARNER PROFILE®
Inquiry • Action • Reflection
Multilingualism and intercultural understanding · Global engagement
Broad and balanced, conceptual and connected curriculum
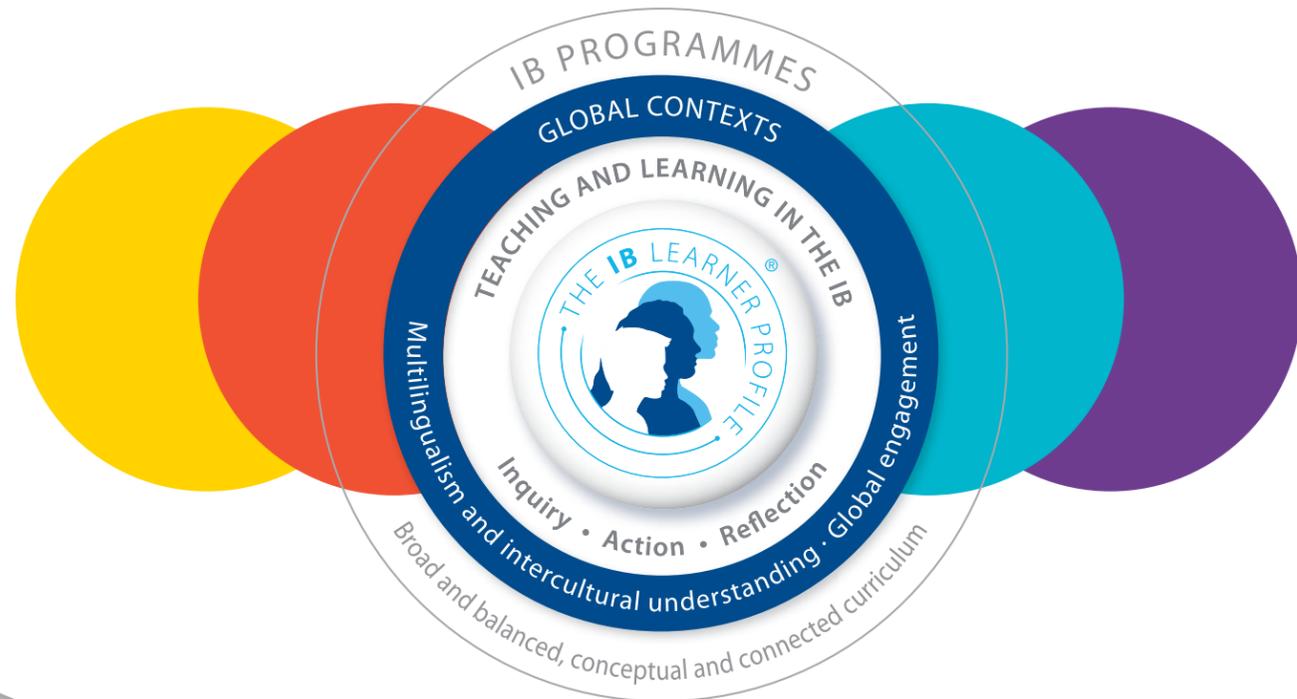
Connecting IB to the Core

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Mathematics standards

What is an IB education?

Connecting IB to the Core
IB and the Common Core State Standards

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Since 1968 the small group of international school educators who founded the International Baccalaureate (IB) have been analysing best practices in the field of education from around the world. They drew upon these best practices in order to build what would become the IB's Diploma Programme (DP). Today, over one million IB students from over 140 countries are held to the same rigorous academic expectations that formed the building blocks of the DP 45 years ago. The IB's experience working flexibly within the diversity of national education systems and curriculums across a global platform make it a senior statesman in the field of broad, standards-based academic reform movements such as the Common Core State Standards initiative (CCSS).

The IB and the CCSS share many goals, the foremost of which is their mutual emphasis on career and college readiness. An IB education not only holds students to the highest academic standards but also incorporates an understanding and appreciation of other cultures and points of view, and world language competency—precisely the soft skills in demand by the global economy. IB students demonstrate a strong competency in the context of global readiness and are, not surprisingly, sought after by colleges and universities for their soft skills as well as their hard-earned academic achievements.

IB World Schools have an advantage when adopting the CCSS. IB standards were selected as one of five international benchmarks against which to compare the CCSS in an influential study conducted by the Educational Policy Improvement Center (EPIC). The CCSS represents a shift in teaching from covering a wide breadth of content to a greater focus on depth of understanding and interdisciplinary approaches to teaching and learning. These very characteristics define what makes an IB education so effective. The shift in thinking and practice that many IB schools made on their journey to become IB World Schools are often the same shifts schools will need to make in transitioning to the CCSS.

Our goal in undertaking these relationship studies is to support IB educators in their efforts to align their curriculum to the CCSS. We are confident that these studies will provide a starting point to begin the important work of curriculum alignment in your own schools and serve as a touchstone to reassure you that transitioning to the CCSS will be that much easier due to the hard work that you and your colleagues have already done to meet the rigorous standards required to offer an IB education.

Although the CCSS initiative is unique to the United States, it will impact standards-based reform movements everywhere. The IB contributes a long-respected voice in the field of international education, adding a global dimension to the discussion around the CCSS. The IB will continue to draw upon school reform initiatives around the world, such as the CCSS, to ensure that it remains a leader in providing a pedagogically current international education based on research and best practices.

As always, we welcome your ideas and want to hear your reflections and feedback. All materials related to the IB and the CCSS will be posted and can be accessed through a dedicated webpage on the IB public website: http://www.ibo.org/iba/commoncore. All feedback related to the CCSS can be sent to a mailbox especially created to respond quickly to inquiries (commoncore@ibo.org).

Warm regards
Drew Deutsch
Director, IB Americas
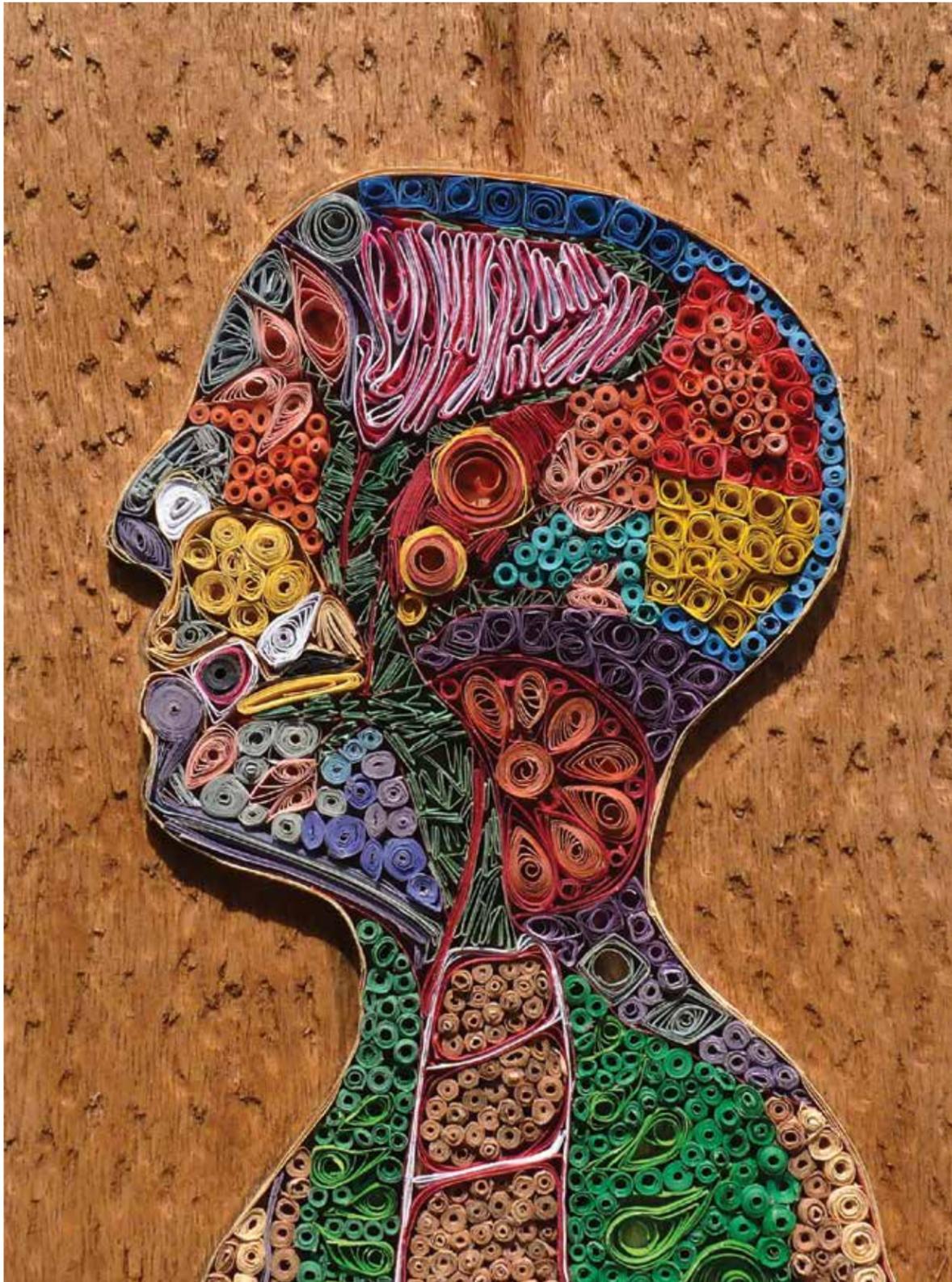
Artwork by: Eleazar Mendoza
Dwight D. Eisenhower High School
11th Grade

About the IB

The IB continuum of international education for 3 to 19-year-olds is unique because of its academic and personal rigour. Teaching and learning in all IB programmes— the Primary Years Programme (PYP), the Middle Years Programme (MYP), the Diploma Programme (DP) and the IB Career-related Certificate (IBCC)—grows from an understanding of education that celebrates the many ways people work together to construct meaning and make sense of the world.

An IB education is for the whole person, providing a well-rounded experience anchored by values and outcomes described in the IB learner profile. IB learners strive to become inquirers, knowledgeable, thinkers, communicators, principled, open-minded, caring, risk-takers, balanced and reflective. These attributes of internationally minded people represent a broad range of human capacities and responsibilities that go beyond intellectual development and academic success.

The IB's student-centred philosophy, with its focus on the interplay between inquiry, action and reflection, empowers students for a lifetime of learning, both independently and in collaboration with others. An IB education centres on learners, develops effective approaches to teaching and learning, and explores significant content within global contexts.

IB World Schools undertake rigorous authorization and evaluation processes to offer one or more IB programmes. The IB *Programme standards and practices* is a document that provides a set of criteria against which both the IB World School and the IB can evaluate success in the implementation of the four programmes. This foundational document for schools and the IB ensures quality and fidelity in the implementation of IB programmes.

About these resources

The IB recognizes that the implementation of the Common Core State Standards (CCSS) will have a significant impact on public schools in the US and in IB World Schools around the globe that follow a US curriculum. In order to support IB World Schools as they prepare for the CCSS, the IB commissioned studies to identify the broad relationships that exist between the overall expectations in the PYP mathematics scope and sequence, the MYP and DP aims and objectives for mathematics and the K–12 CCSS for mathematics.

The IB developed these studies in collaboration with IB educators, a hallmark of its relationship with its community. Educators with specialized knowledge of IB curriculums and the CCSS for mathematics worked closely with IB academic staff to produce these resources for the PYP, MYP and DP.

The CCSS define what students in mathematics should understand and be able to do by the end of each grade. The PYP and MYP provide curriculum frameworks that are designed to meet the developmental needs of students. These curriculum frameworks and the DP mathematics courses offer schools the flexibility to accommodate the demands of national or local requirements for mathematics.

Studies were commissioned to educators with specialized knowledge of IB curriculums and CCSS for mathematics. The purpose of these studies is to provide schools with a flexible resource to inform their own curriculum alignment. The studies employ the structure of the eight common core standards for mathematical practice to highlight the broad relationship with teaching and learning about mathematics in the PYP, MYP and DP. They provide a snapshot of the relationships and are not intended to be comprehensive in nature.

The IB anticipates that these resources will evolve with further implementation of the CCSS. IB educators will no doubt discover other aspects of the relationship between teaching and learning in IB programmes and the CCSS as they reflect upon their teaching and students' learning.
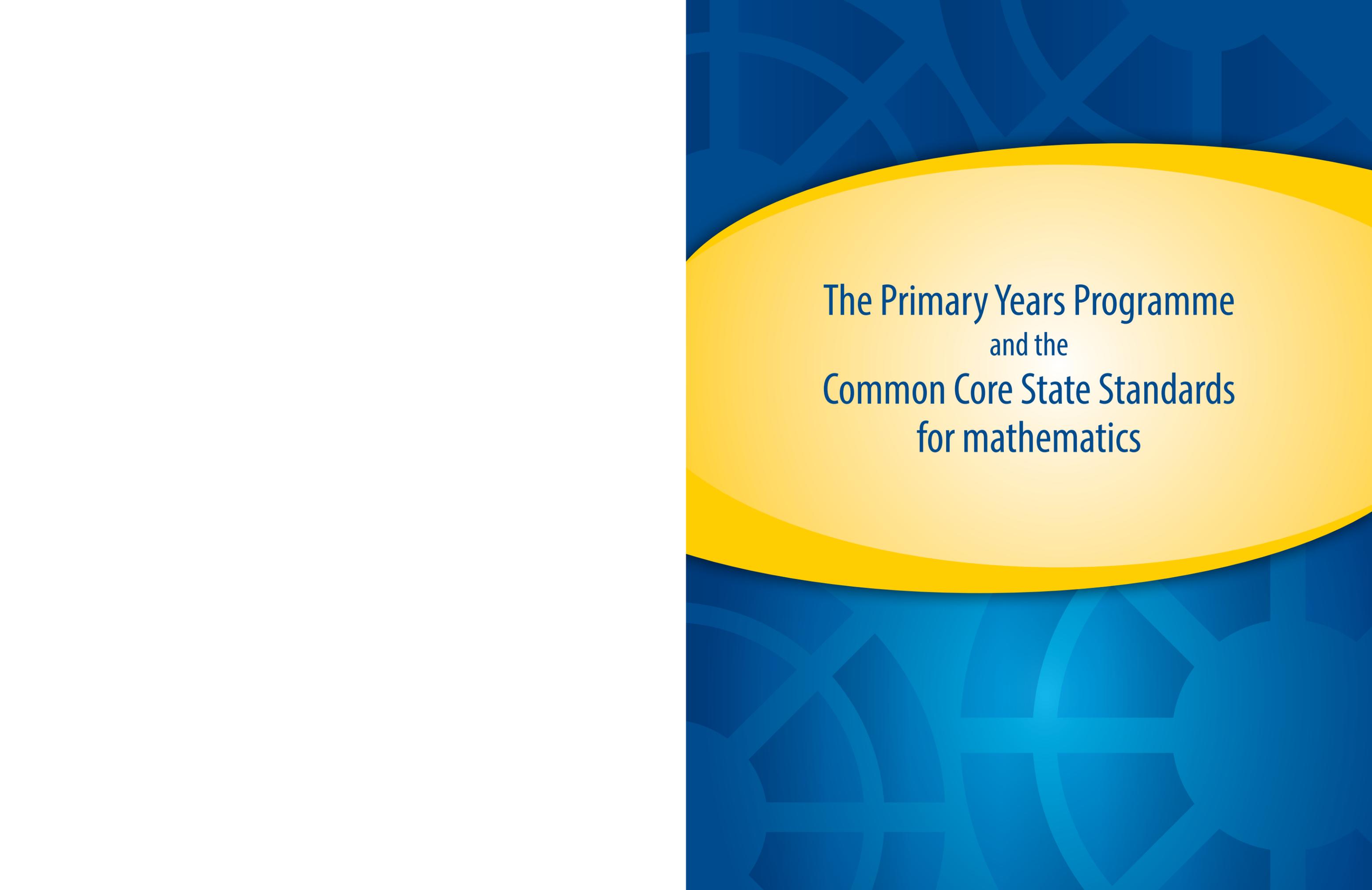
The Primary Years Programme
and the
Common Core State Standards
for mathematics
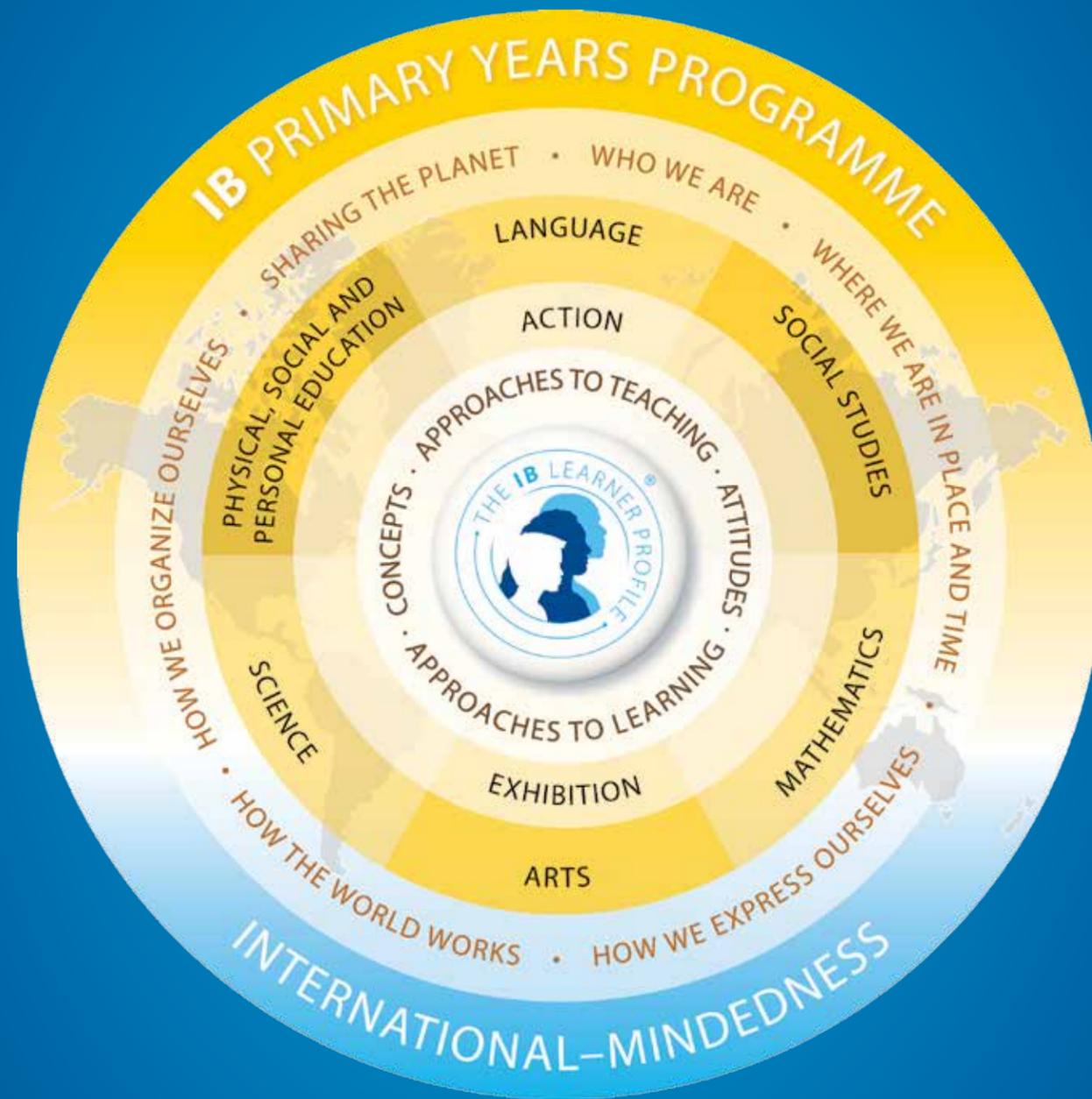
The PYP is designed for students aged 3 to 12. It focuses on the development of the whole child as an inquirer, both in the classroom and in the world outside.

It is a framework guided by six transdisciplinary themes of global significance, explored using knowledge and skills derived from six subject areas, as well as approaches to learning with a powerful emphasis on inquiry.

The design of the PYP is sufficiently flexible to accommodate the demands of national or local curriculums as schools develop their own programme of inquiry (POI). It follows therefore, that as a flexible and rigorous curriculum framework, the PYP offers teachers the opportunity to develop learning experiences for students that meet the demands set out by the CCSS.

The CCSS for mathematics "do not dictate curriculum or teaching methods" (NGA Center 2010: 5). The PYP, with its inquiry-based pedagogy, provides an effective framework for teaching and learning the CCSS.

The CCSS define what students should understand and be able to do by the end of each grade and the CCSS for mathematics place an emphasis on applying mathematics to the real world. Mathematical instruction in the CCSS includes both the proficiency in, and processes of, mathematics.

This following study employs the structure of CCSS to relate the eight standards for mathematical practice for kindergarten through grade five to a specific component of the PYP: the overall expectations and learning outcomes of the PYP *Mathematics scope and sequence*. The purpose is to provide schools with a resource to support their own curriculum alignment. The study demonstrates that the curriculum framework of the PYP supports the implementation of the CCSS in mathematics for kindergarten through grade five.

Introductory observations

The CCSS are a shift in the direction of mathematics education. They move beyond traditional "standards" to a focus on applying mathematics to real-life situations. Students are no longer learning content as isolated facts, but rather as tools to solve a wide range of problems. This shift resonates with the PYP *Changes in mathematics practices* (Appendix 2), which notes an increased emphasis on "real-life problem solving using mathematics".

Both the PYP and the CCSS use strands of mathematics to structure learning progression. In the PYP *Mathematics scope and sequence* the strands are: **data handling, measurement, shape and space, pattern and function** and **number** (Appendix 1).

The strands are divided into four phases. Each phase further identifies the following stages students typically follow when learning mathematics: *constructing meaning,* **transferring meaning into symbols,** and **applying with understanding.** The four phases form a developmental learning continuum detailing how students might move through the phases as they become more proficient in mathematics. It is important to note that these phases are not to be identified as grade equivalents and should allow for developmental differences. This will enable teachers to more accurately identify current levels of each student's development and plan learning experiences accordingly. The PYP *Mathematics scope and sequence* document states in the section "The structure of the PYP *Mathematics scope and sequence*" "… that the evidence of mathematical understandings are described in the behaviours or learning outcomes associated with each phase and these learning outcomes relate specifically to mathematical concepts, knowledge and skills. The learning outcomes have been written to reflect the stages a learer goes through when developing conceptual understanding in mathematics—constructing meaning, transferring meaning into symbols and applying with understanding".

> *The CCSS are a shift in the direction of mathematics education. They move beyond traditional "standards" to a focus on applying mathematics to real-life situations.*
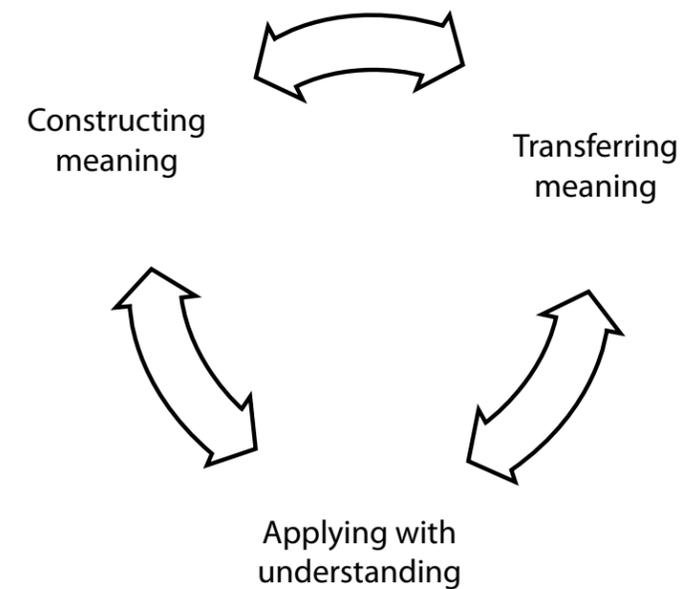
Figure 1
How children learn mathematics

The CCSS for mathematics "set grade-specific standards" (NGA Center 2010: 4) of what students should understand and be able to do. The CCSS for mathematical content are divided into domains "which are larger groups of related standards" (NGA Center 2010: 5) identified as **operations and algebraic thinking, number and operations in base 10, measurement and data** and **geometry.** Kindergarten also includes the domain of **counting and cardinality.**
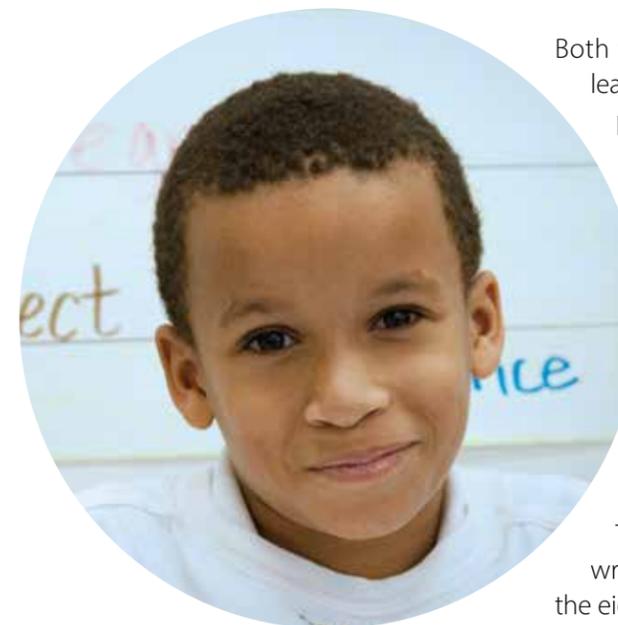
Both the CCSS and PYP identify the importance of conceptual learning in mathematics. The conceptual framework of the PYP promotes a deep understanding of mathematical principles and the use of transdisciplinary themes ensures that connections with the real world are made.

In the PYP, mathematics is viewed as a tool to support inquiry and provides a global language through which students make sense of the world around them. Mathematics helps explain the why and how, and is a process of thinking. The overall expectations of PYP mathematics include processes and proficiency, as does the CCSS.

The comprehensive philosophy and approach of the PYP's written, taught and assessed curriculum is highly visible within the eight "Standards for mathematical practice" as evidenced in the following section.

Standards for mathematical practice

1. Make sense of problems and persevere in solving them.

"Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution" (NGA Center 2010: 6).

Both the PYP *Mathematics scope and sequence* and the "Standards for mathematical practice" recognize the many ways in which students construct mathematical understandings.

Students learn mathematics by constructing meaning through ever-increasing levels of abstraction, starting with exploring their own personal experiences, understandings and knowledge. The PYP *Mathematics scope and sequence* identifies constructing meaning as the first stage in all four phases in every strand. The PYP recognizes the need for students to learn using concrete problems before they can work at an abstract level. The nature of learning through inquiry in the PYP naturally promotes making sense of problems and persevering in solving them.

> *The PYP recognizes the need for students to learn using concrete problems before they can work at an abstract level. The nature of learning through inquiry in the PYP naturally promotes making sense of problems and persevering in solving them.*

2. Reason abstractly and quantitatively.

"Mathematically proficient students make sense of quantities and their relationships in problem situations" (NGA Center 2010: 6). When students truly understand mathematics conceptually, they can apply their mathematical understanding to a new problem and use mathematical symbols and language to explain their thinking. While the CCSS for mathematics explicitly refer to the ability to "decontextualize" and "contextualize", development of this ability is implicit in all PYP mathematical strands and phases. Both the PYP *Mathematics scope and sequence* and the "Standards for mathematical practice" have similar expectations of students to reason abstractly and quantitatively.

In order to be able to use mathematics as a tool, students need to be able to go from the specifics of the situation to the more abstract mathematics underlying the problem. The PYP *Mathematics scope and sequence* identifies the second stage in all four phases of every strand as "transferring meaning into symbols". The third stage is to "apply with understanding". These two stages in every strand move students' understanding from the concrete to the abstract.

3. Construct viable arguments and critique the reasoning of others.

"Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments" (CCSS 2010: 6).

Both the PYP *Mathematics scope and sequence* and the CCSS for mathematical practice encourage not only that students are able to comprehend mathematical concepts and content, but also exhibit and demonstrate the processes and new understandings developed. Students explain their thinking using numbers, models, graphs, words and relationships. In addition, students should then be able to apply new knowledge and understandings in a variety of situations that allow for further understanding as well as the development of multiple perspectives.

The PYP *Mathematics scope and sequence* encourages students and teachers to use certain processes of mathematical reasoning as they progress through the three stages of learning mathematics. These processes include:

- using patterns and relationships to analyse the problem situations upon which they are working

- making and evaluating their own and each other's ideas

- using models, facts, properties and relationships to explain their thinking

- justifying answers and the processes by which they arrive at solutions.

In this way, students validate the meaning they construct from their experiences with mathematical situations. By explaining their ideas, theories and results, both orally and in writing, they invite constructive and critical feedback, as well as lay out alternative models of thinking for the class (see the section "What the PYP believes about learning mathematics").

The IB learner profile attributes of communicator and thinker further reinforce the construction of viable arguments and the ability to critique the reasoning of others. The ability to conjecture and justify conclusions is essential if students are to be able to fully experience mathematics.

4. Model with mathematics.

"Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (NGA Center 2010: 7).

Both the PYP *Mathematics scope and sequence* and the CCSS for mathematical practice require students to explain their answers both by the use of estimation and by precise computation.

The PYP *Mathematics scope and sequence* reinforces that modelling using manipulatives provides a valuable scaffold for constructing meaning about mathematical concepts. It identifies modelling as one of the processes students and teachers use throughout the three stages of learning mathematics. It is specifically mentioned as a way to develop conceptual understanding in the following strands: **data-handling, shape and space** and **number.**

In the **pattern and function** strand, students progress in phase one to understand patterns and sequences in everyday situations. In phase two, they represent patterns using numbers and other symbols. In phase three, they sense real-life situations using mathematical representations. In phase four, symbolic rules are used to analyse and represent patterns.

In the shape and space strand, students to use two-dimensional and three-dimensional models to explain properties of various shapes. In the **number** strand, students use fractions in real-life situations and use estimation to check the reasonableness of their answers.

PYP educators provide regular opportunities for students to use a range of manipulatives as well as discuss and negotiate their developing understanding with others.

> *PYP educators provide regular opportunities for students to use a range of manipulatives as well as discuss and negotiate their developing understanding with others.*

5. Use appropriate tools strategically.

"Mathematically proficient students consider the available tools when solving a mathematical problem" (NGA Center 2010: 7). Mathematics has always relied on tools of some form, for example, paper and pencil, slide rule or computer software. The goal is to help students know when and how to use tools appropriately. Both the PYP *Mathematics scope and sequence* and the CCSS standards for mathematical practice encourage students to employ appropriate tools. This includes using technology to solve real-life problems in a strategic and meaningful manner that will assist with the development of mathematical concepts within a relevant context. Learners are asked to integrate new mathematical knowledge in a variety of situations that require in-depth understanding and application of new learning.

Use of mathematical tools is evident throughout the PYP, and provides authentic opportunities for students to interact in realistic contexts with mathematical tools playing a key role. The use of tools is encouraged and evident in the PYP *Mathematics scope and sequence* strands and is particularly emphasized in the **data handling** and **measurement** strands. For example, in the **measurement** strand in phase three, students "select and use appropriate tools and units of measurement" and in phase four learners "develop and describe formulas to find area, perimeter and volume" (see the section "Learning continuum for measurement"). In the **data handling** strand, students in all phases learn how to sort, categorize and interpret data with a variety of charts and graphs. By phase three and four, students are able to choose which type of graph best depicts the data for representation (see the section "Learning continuum for data handling").

The PYP *Mathematics scope and sequence* notes that while tools commonly used should be utilized in authentic ways to solve problems, care should be taken to ensure that students have a strong understanding of the concepts embedded in the problem to ensure meaningful engagement with the tools and develop a fuller understanding of the solution posed. Doing so ensures that students use appropriate tools strategically.

> *The goal is to help students know when and how to use tools appropriately. Both the PYP Mathematics scope and sequence and the CCSS standards for mathematical practice encourage students to employ appropriate tools. This includes using technology to solve real-life problems in a strategic and meaningful manner that will assist with the development of mathematical concepts within a relevant context.*

6. Attend to precision.

"Mathematically proficient students try to communicate precisely to others" (NGA Center 2010: 7).

Both PYP mathematics and the CCSS for mathematics recognize the importance of communicating mathematical concepts and understandings in an effective, knowledgeable manner. The two mathematics courses also require students to communicate precisely using mathematical terminology, including labels and notation. Attention to detail when recording mathematical data and solutions is essential to both, and students are provided with multiple opportunities to ensure attention to the recording and reporting of data and information.

The PYP Mathematics scope and sequence recognizes that in order to be useful, number facts (for addition, subtraction, multiplication and division) must be recalled automatically with precision. The PYP notes that the degree of precision needed when calculating depends on how the result will be used. In the PYP **data handling** strand, as early as phase one, students use tally marks or pictographs to accurately depict data. Phase four of the **measurement** strand addresses precision in the degree of accuracy for their measurements as well as their mathematical notation. In the **number** strand, students have automatic recall of facts and use strategies to solve problems and check for accuracy. Phase four of the **shape and space** strand requires students to use scale accurately to enlarge and reduce shapes as well as encouraging them to create and manipulate shapes. By doing this, students align their natural vocabulary with more formal mathematical vocabulary and begin to appreciate the need for this precision.

Artwork by: Emma Downey
Caldwell Heights Elementary
2nd Grade

7. Look for and make use of structure.

"Mathematically proficient students look closely to discern a pattern or structure" (NGA Center 2010). Both the PYP *Mathematics scope and sequence* and the CCSS for mathematical practice require students to utilize patterns to explain mathematical thinking in geometry and with numbers. The **pattern and function** strand in the PYP *Mathematics scope and sequence* sets expectations in each phase that students will look for, and make use of, structure. The guide also notes that "By analysing patterns and identifying rules for patterns it is possible to make predications" (see the section "Learning continuum for pattern and function").
In the **number** strand, students use operations and the order of operations to solve problems. In the **data handling** strand, students gather and display various types of data, seek ways in which best to represent that data and learn to identify patterns in order to analyse data.

In the PYP *Mathematics scope and sequence* **shape and space** strand, students learn the properties of regular and irregular polyhedra and can use models to visualize real world situations by phase four. Students are able to use ratios and scale to create models to explain larger or smaller shapes.

8. Look for and express regularity in repeated reasoning.

"Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts" (NGA Center 2010: 8).

Both the PYP *Mathematics scope and sequence* and the CCSS for mathematical practice require students to have a strong number sense so that they can determine patterns caused by repeated numbers and justify the relationship between the numbers.

In the PYP *Mathematics scope and sequence* **pattern and function** strand, repeated reasoning is clearly evident in all phases. Although not explicitly stated, the **number** strand incorporates repeated reasoning in that estimation, order of fractions, equivalent fractions to percentages and factors are all examples of repeated reasoning. When students are using algebra to solve problems, they utilize patterns and number sense.

> *Both the PYP* Mathematics scope and sequence *and the CCSS for mathematical practice require students to have a strong number sense.*

Summary

The areas of convergence between the requirements of the CCSS for mathematical practice and the PYP *Mathematics scope and sequence* are evident upon review. While the phases of learning that exist within the PYP are more developmental in nature, both the PYP and the CCSS require attention to the important processes of learning and the ability to communicate new understandings.

The study also highlights some areas of distinction in the PYP that are identified by commitment to the development of international-mindedness and student-led inquiry, which are essential to the PYP and provide opportunities for students and teachers to appreciate the global dimension of mathematics. The IB learner profile, together with the five essential elements of the PYP—knowledge, concepts, skills, attitudes and action—informs all planning, teaching and assessing in the PYP. The PYP approaches to learning coupled with the learner profile promote the qualities expected in 21st century learners and international-mindedness.

As students investigate mathematics and its application to the real world, the PYP provides an authentic framework for exploring the CCSS for mathematical practice. The PYP *Mathematics scope and sequence* purports, "The power of mathematics for describing and analysing the world around us is such that it has become a highly effective tool for solving problems. It is also recognized that students can appreciate the intrinsic fascination of mathematics and explore the world through its unique perceptions" (2009: 1).

With the implementation of the CCSS for mathematical practice, IB practitioners will discover other aspects of the relationship between the PYP *Mathematics scope and sequence* and the CCSS as they reflect upon their teaching and students' learning.

> *The power of mathematics for describing and analysing the world around us is such that it has become a highly effective tool for solving problems.*
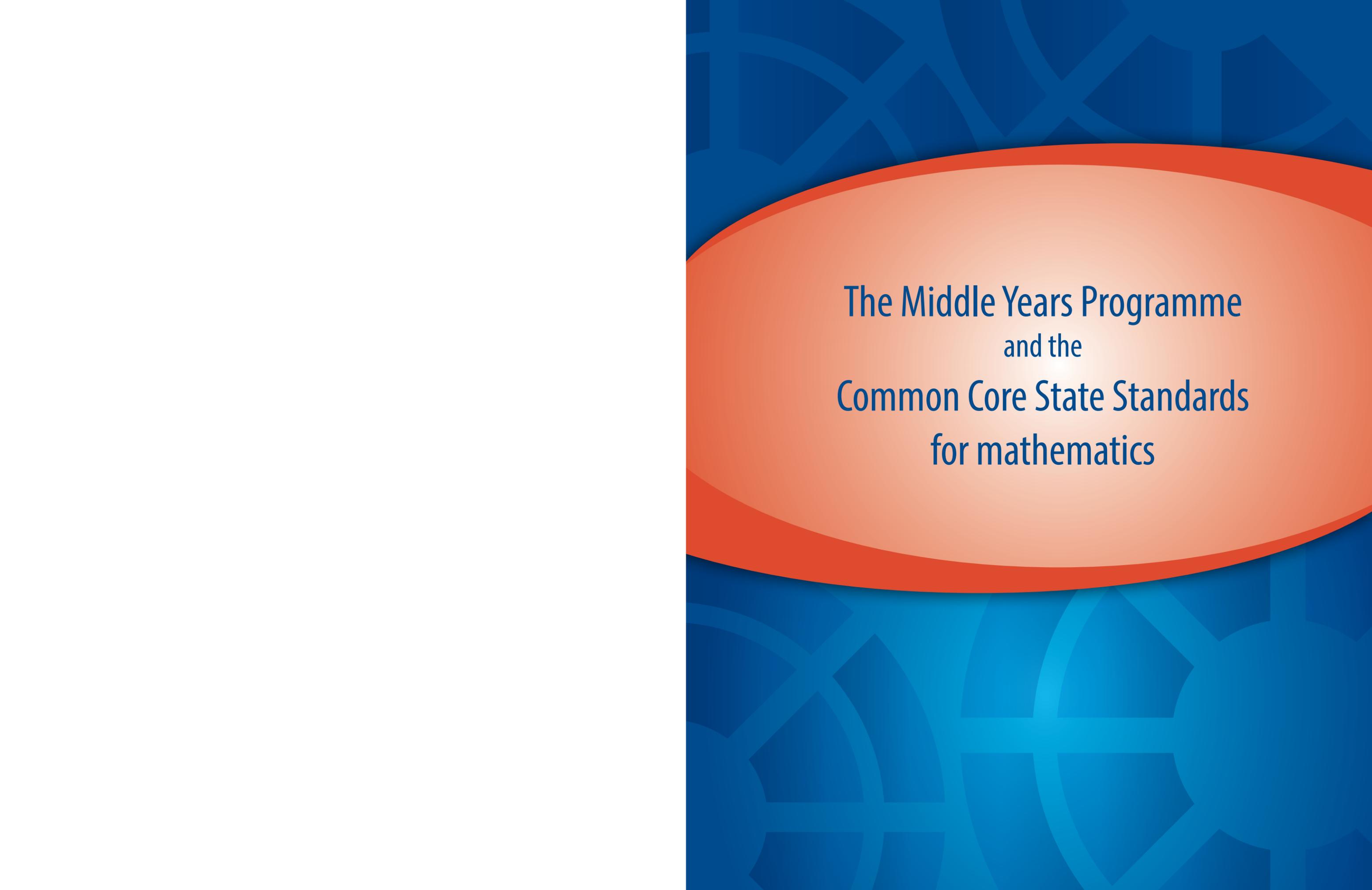
The Middle Years Programme
and the
Common Core State Standards
for mathematics

First teaching from September 2014

The MYP is designed for students aged 11 to 16. It provides a coherent and comprehensive curriculum framework of learning. The MYP encourages students to become creative, critical and reflective thinkers. The MYP emphasizes intellectual challenge, encouraging students to make connections between their studies in traditional subjects and to the real world. It fosters the development of skills for communication, intercultural understanding and global engagement, qualities that are essential for life in the 21st century.

The MYP is flexible enough to accommodate the demands of national and local curriculums. It builds on the knowledge, skills and attitudes developed in the PYP and prepares students to meet the challenges of the DP and the IBCC. The MYP's rigorous curriculum framework, within which schools develop their own units of study, ensures that students can meet the learning objectives demanded in the CCSS.

The CCSS for mathematics "do not dictate curriculum or teaching methods" (NGA Center 2010: 5). The MYP, therefore, is an effective framework for teaching and learning for the CCSS. MYP schools develop their own units of study incorporating the content of the CCSS using the MYP philosophy of teaching and learning.

The CCSS define what students should understand and be able to do by the end of each grade and the CCSS for mathematics place an emphasis on applying mathematics to the real world. These standards set out to develop a clear definition of what students need to know to succeed in university and in their future careers.

Studies demonstrating the broad relationship between the overall expectations in the PYP *Mathematics scope and sequence*, the aims and objectives of mathematics in the MYP and DP with the CCSS standards of mathematical practice were commissioned to educators with specialized knowledge of IB curriculums and CCSS. The purpose of these studies is to provide schools with resources to inform their own curriculum alignment.

The following relationship study employs the structure of the CCSS to relate the eight standards for mathematical practice in grades 6 to 10 to specific components of the MYP mathematics as identified in the *Mathematics guide* (2013).

Introductory observations

This study demonstrates that the aims and objectives of MYP mathematics for years 1 to 5 clearly support the implementation of the CCSS in mathematics grades 6 to 10.

The emphasis on both application and inquiry are tenets of the MYP. The conceptual framework of the MYP promotes that deep understanding of mathematical principles and the global contexts infuse the realworld in mathematics education. From the aims of MYP mathematics to its objectives and criteria, together with its stance on the use of technology, students in MYP classrooms are experiencing the type of learning described by the CCSS.

The CCSS are a shift in the direction of mathematics education. They move beyond traditional "standards" to a focus on applying mathematics to real-life situations. Students are no longer learning content as isolated facts, but rather as tools to solve a wide range of problems (both mathematical and real-world). At the same time, the CCSS focus more on students understanding of mathematical principles and involvement in the exploration of mathematical concepts to grasp better the mathematics they are learning. The IB learner profile's ten attributes are the IB mission statement translated into a set

of learning outcomes for the 21st century, promoting life-long learning and international-mindedness. The IB learner profile is integral to teaching and learning in the MYP because it represents the qualities of effective learners and internationally minded students. The MYP approaches to learning (ATL) are organized into five skill areas: **thinking, communication, social, self-management** and **research.** The ATL, coupled with the learner profile, promote the qualities expected in 21st century learners and promote international-mindedness. As students explore mathematics and its application to the real world as promoted by the CCSS, these fundamental components of all IB programmes provide an authentic framework for exploring the CCSS standards of mathematical practice.

Standards for mathematical practice

1. Make sense of problems and persevere in solving them.

"Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution" (NGA Center 2010: 6).

A main goal of mathematics education is to help students use it as a tool to solve problems. In order to do so, they must be willing to persist from the initial introduction to the problem to its eventual solution. This requires students to have at their disposal a wide range of strategies and a willingness to constantly evaluate their work in case they need to change their approach.

Two of the stated aims of MYP mathematics are "to develop logical, critical and creative thinking" and "to develop perseverance in problem solving". Both the CCSS and the MYP encourage the use of appropriate technology in the solution of problems. Students in MYP Common Core State Standards for mathematics classes are assessed on their ability to select and apply problem-solving techniques in a wide variety of contexts including unfamiliar situations (objective A). Students are also required to identify relevant elements of real-life problems, select and apply appropriate mathematics correctly and reflect on whether their answer makes sense in the context of the problem (objective D). The use of investigations (objective B) and the use of inquiry as a teaching method help students to develop that perseverance since they are accustomed to looking for and finding relationships in mathematics without direct dissemination from the teacher. All of these are supported and further developed by the ATL skill of **thinking** as well as the learner profile characteristics of being inquirers, thinkers, reflective and knowledgeable.

"Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution"

2. Reason abstractly and quantitatively.

"Mathematically proficient students make sense of quantities and their relationships in problem situations." (NGA Center 2010: 6)

In order to be able to use mathematics as a tool, students need to be able to go from the specifics of the situation to the more abstract mathematics underlying the problem. They then must be able to manipulate symbols in order to come to a solution.

The aims of MYP mathematics include helping students "develop powers of generalization and abstraction" as well as "develop an understanding of the principles and nature of mathematics". While the MYP does not explicitly separate abstract and quantitative reasoning, the use of investigations (objective B) allows students to study and manipulate quantities in order to establish the relationships between them. The focus on applying mathematics to the real world (objective D) affords students the opportunity to move between decontextualizing and contextualizing a situation. They do much the same as they develop the ATL skill of **thinking.**

> *Mathematically proficient students make sense of quantities and their relationships in problem situations.*

3. Construct viable arguments and critique the reasoning of others.

"Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments" (NGA Center 2010: 6).

The ability to conjecture and justify conclusions is essential if students are to be able to fully experience mathematics. They need to be able to communicate and logically defend their own conclusions as well as evaluate those of others.

These are ideals that are clearly reflected in the MYP mathematics aims of helping students to "develop the ability to reflect critically upon their own work and the work of others" and "communicate clearly and confidently in a variety of contexts". Objective C (communicating) helps students develop their abilities to communicate complete, coherent and concise lines of reasoning as well moving between different representations and it also promotes presenting work using a logical structure. Once again, objective B (investigating) is assessed and holds students accountable for finding patterns, representing them and justifying their conclusions. With **logic** as one of three key concepts, and a focus on inquiry-based learning, MYP mathematics students are exposed to a wide range of opportunities and content where they will develop their powers of argumentation and justification.

4. Model with mathematics.

"Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (NGA Center 2010: 7).

In order for students' mathematical knowledge and understanding to be useful (as well as to motivate them to want to learn more), they need to be able to apply mathematics in authentic ways to problems in the real world. It is no longer enough to simply be skilled in procedures.

As stated in the MYP mathematics aims, students need to be able to "apply and transfer skills to a wide range of situations, including real life, other areas of knowledge and future developments". Through the MYP global contexts, the fundamental concept of **holistic learning** and in developing the ATL thinking skill of **transfer,** students explore how content relates to other courses and the world around them. Students in MYP mathematics seldom ask the question "When am I ever going to use this?" because they know the answer will be evident throughout the unit. Objective D (applying) clearly outlines the expectation that all students should be able to model authentic real-life problems using mathematics and that they should be assessed on their ability to do so. It also requires students to analyse whether or not their answer makes sense in real-life as well as justify their degree of accuracy. By pairing this objective with objective C (communicating) students are then pushed to be able to use the multiple representations described in the CCSS and move effectively between them.

> *Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace*

5. Use appropriate tools strategically.

"Mathematically proficient students consider the available tools when solving a mathematical problem" (NGA Center 2010: 7).

Mathematics has always relied on tools of some form, for example, paper and pencil, a slide rule or computer software. The goal is to help students know when and how to use tools appropriately.

The CCSS and the MYP promote the use of technology in the application of mathematics and the MYP goes one step further to also promote its use in the communication of mathematics. Information and communication technology (ICT) is useful not just when considering complex calculations, but also to "investigate data and mathematical concepts; obtaining rapid feedback when testing out solutions; [and] observing patterns and making

generalizations" (*Mathematics guide* 2013: 9). These are not just tools for "doing" mathematics, but also for teaching and learning it.

All of the objectives (A, B and D) that require students to "select and apply" problem-solving strategies recognize that students may also select various tools in order to implement their plan. The ATL skill of **research** is further evidence of the MYP's commitment to helping students learn to use responsibly all of the strategies available to them.

6. Attend to precision.

"Mathematically proficient students try to communicate precisely to others" (NGA Center 2010: 7).

Both communicating and calculating in mathematics require students to take care in choosing their words and symbols and in the execution of operations. To not do so would hinder both the student's success and their ability to transmit what they know to someone else.

With a fundamental concept of **communication,** and a learner profile characteristic and ATL skill focusing on communicating, MYP students learn the necessity for communicating precisely to others. The aims of MYP mathematics include the ability to "communicate clearly", something also evident in the MYP mathematics objective C (communicating) where students improve their ability to be clear, concise and coherent. Objective D (applying) also requires students to attend to accuracy by asking them to justify their level of precision.

> *Mathematically proficient students try to communicate precisely to others*

7. Look for and make use of structure.

"Mathematically proficient students look closely to discern a pattern or structure" (NGA Center 2010: 8).

Whereas traditional state standards would have required students to simply "know" how to factor a trinomial, for example, the CCSS now want students to be involved in discovering its patterns, in understanding what makes something "factorable". Students, then, need practice at arriving at these conclusions, something the MYP's focus on inquiry-based learning promotes. The MYP conceptual framework also contributes to this type of learning as it forces teachers and students to go beyond the mere acquisition of skills and reach for an understanding of the mathematics underlying those skills. With one of three key concepts being **form,** students will

be attending to the structure of an entity to discern its properties and any inherent patterns. Furthermore, objective B (investigating) also requires students to take part in investigations where they are then assessed on their ability to discern patterns and describe them as general rules. This puts the student in the position of creating knowledge rather than simply being a recipient of someone else's knowledge.

8. Look for and express regularity in repeated reasoning.

"Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts" (NGA Center 2010: 8).

Students are asked to look for patterns themselves in order to establish rules. Rather than simply giving students "the formula", they will understand it better when they are involved in its formulation or discovery. The MYP focus on inquiry and objective B (investigating) promotes this important ability. Students are regularly looking for patterns in data and then communicating their findings to an audience. In an MYP mathematics classroom, students are challenged to discover concepts on their own or in groups and then extend their knowledge to other situations.

> *Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts*

Summary

MYP mathematics is not a set of mandatory tasks but rather a philosophy of teaching that exemplifies what the CCSS are attempting to accomplish. With a focus on concepts and applications, students learn not only where the mathematics comes from, but where it can be used in the real world. By promoting a student-centred approach based on inquiry, action and reflection in the MYP, students can experience mathematics as described by the CCSS.

The "Standards for mathematical content" give specific objectives for student learning, while the MYP has a suggested mathematical framework that describes the types of topics that students might learn in a MYP mathematics classroom (without prescribing them). The framework includes the four branches of number, algebra, geometry and trigonometry, and statistics and probability, very similar to the number system, expressions and equations, geometry, and statistics and probability clusters described for CCSS grades 6 through 8. Some grades also include the clusters of ratio and proportional relationships (grades 6 and 7) that then develop into functions in grade 8. These topics are specifically suggested in the number and algebra branches in the MYP. Once in high school, students develop the 8th grade clusters further, though now algebra and modelling have been added. These two clusters are also evident in the mathematics framework provided by the MYP.

Reading through the "Standards for mathematical content", the MYP philosophy is very apparent. All of the elements of the MYP, including aims, objectives, assessment criteria, conceptual framework, focus on inquiry and approaches to learning, support the implementation of the one element that is not explicitly given in MYP mathematics: content. That content and its accompanying practices are clearly laid out in the CCSS.

This study also highlights some areas of distinction in the MYP that are identified by a commitment to the development of international-mindedness and the IB learner profile attributes that provide opportunities for students and teachers to appreciate the global dimension of mathematics. MYP mathematics also promotes that students will "enjoy mathematics, develop curiosity and begin to appreciate its elegance and power" (MYP *Mathematics guide* 2013).

Teaching MYP mathematics clearly supports and extends the teaching of the CCSS for mathematics.

> *" Students will enjoy mathematics, develop curiosity and begin to appreciate its elegance and power. "*
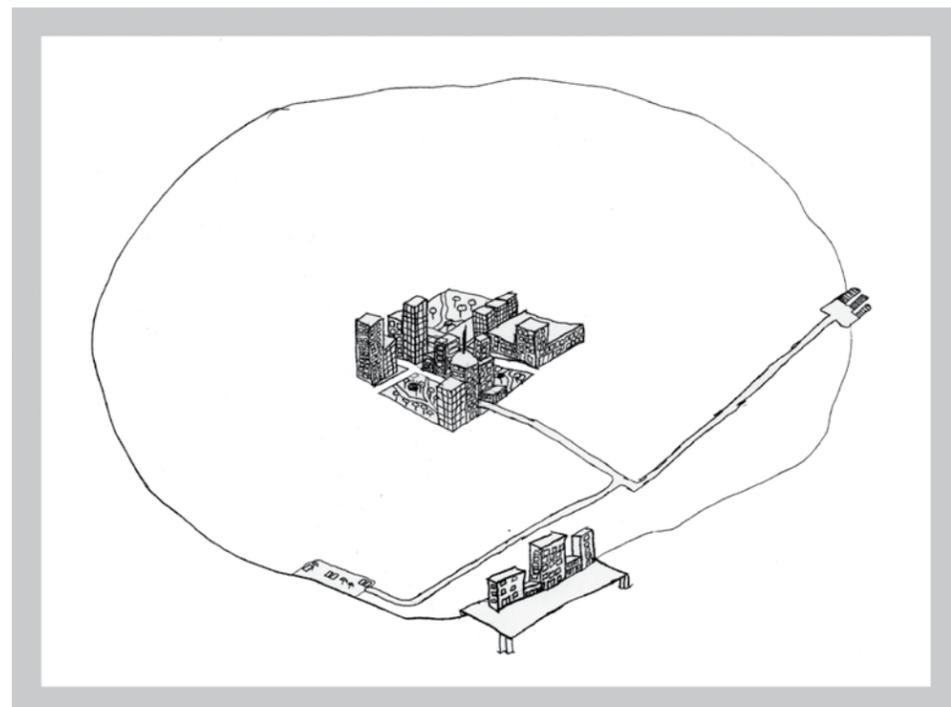
Artwork by: Aiden Malcolm Rose
Park IB School
2nd Grade

The Diploma Programme
and the
Common Core State Standards
for mathematics

IB DIPLOMA PROGRAMME

STUDIES IN LANGUAGE AND LITERATURE

LANGUAGE ACQUISITION

THEORY OF KNOWLEDGE

INDIVIDUALS AND SOCIETIES

APPROACHES TO TEACHING

EXTENDED ESSAY

THE IB LEARNER PROFILE

APPROACHES TO LEARNING

MATHEMATICS

SCIENCES

CREATIVITY, ACTION, SERVICE

THE ARTS

INTERNATIONAL–MINDEDNESS
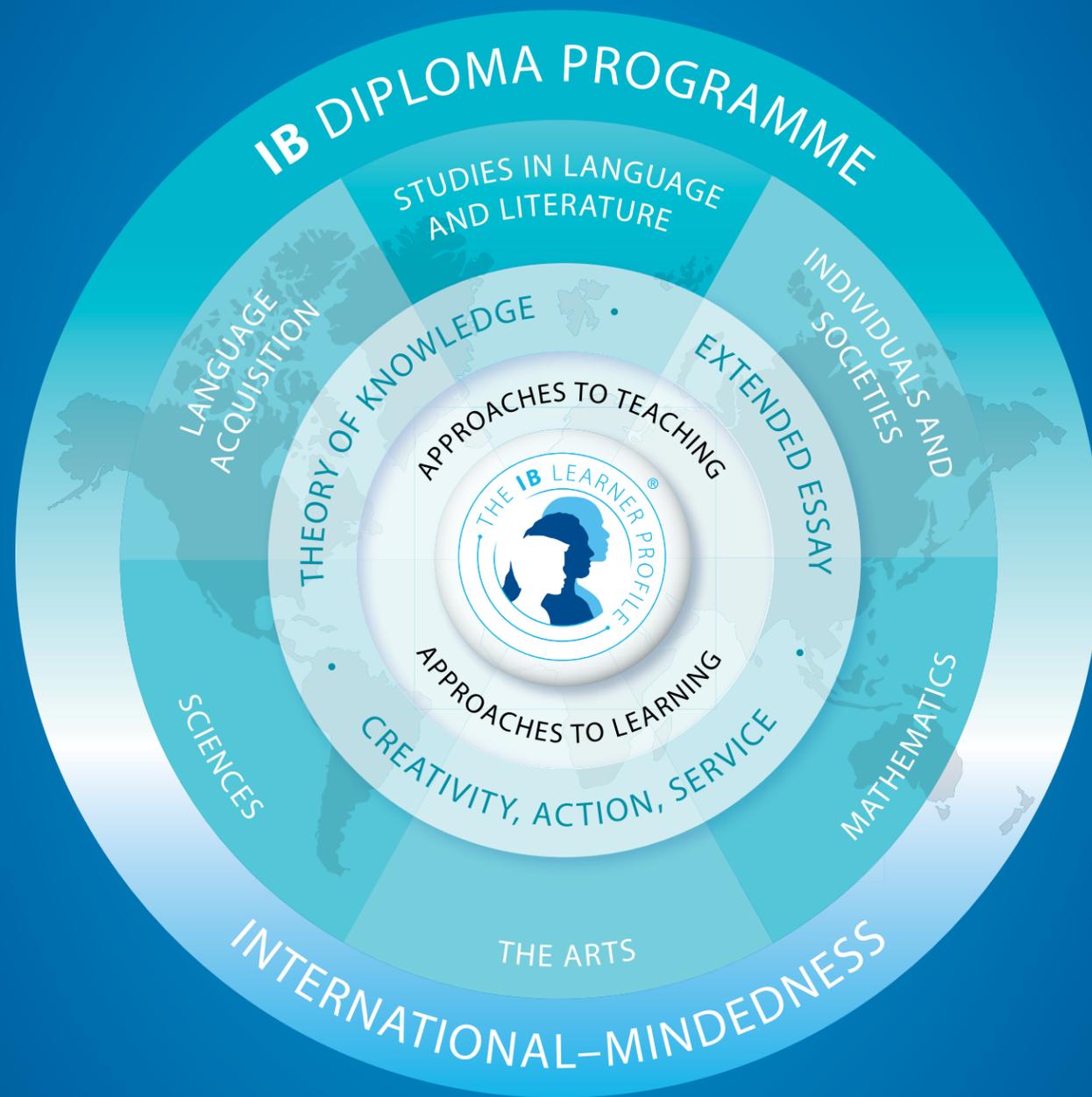
The DP is an academically challenging and balanced programme of education with internal assessment and final examinations that prepares students, aged 16 to 19, for success at university and life beyond. It has been designed to address the intellectual, social, emotional and physical well-being of students. The programme has gained recognition and respect from the world's leading universities.

DP students must choose one subject from each of five groups, ensuring breadth of knowledge and understanding in their best language (language A), additional languages (language B), the social sciences, the experimental sciences and mathematics. Students may choose either an arts subject from group 6, or a second subject from groups 1–5. DP subjects can be taken at higher level (HL) or standard level (SL). In addition to disciplinary and interdisciplinary study, the DP features three core elements that broaden students' educational experience and challenge them to apply their knowledge and skills.

Students take written examinations at the end of the programme, which are marked by external IB examiners. Students also complete assessment tasks in the school, which are either initially marked by teachers and then moderated by external moderators or sent directly to external examiners. Assessment is criterion-related, which means student performance is measured against pre-specified assessment criteria based on the aims and objectives of each subject curriculum, rather than the performance of other students taking the same examinations.

The IB learner profile's ten attributes are the IB mission statement translated into a set of learning outcomes for the 21st century. The IB learner profile is integral to teaching and learning in IB programmes as it represents the qualities of effective learners and internationally minded students.

The CCSS for mathematics "define what students should understand and be able to do in their study of mathematics" (CCSS 2010: 4) and place an emphasis on applying mathematics to the real world. These standards set out to develop a clear definition of what students need to know to succeed in university and in their future careers.

The following relationship study employs the structure of the CCSS to relate the CCSS for mathematics for grades 11 to 12 to specific components of the DP mathematics aims and assessment objectives identified in the DP mathematics subject guides.

> *The CCSS define what students should understand and be able to do in their study of mathematics.*

Introductory observations

Both the DP and the CCSS expect a high degree of skills development and content knowledge in mathematics. The DP mathematics courses provide opportunities for students to develop mathematical concepts in a coherent manner while applying their mathematical knowledge to solve realistic problems in context.

The CCSS for mathematics "do provide clear signposts along the way to the goal of college and career-readiness for all students". They are designed to be relevant to the real world, representing the knowledge and skills students need to be prepared for college and a career. The CCSS for mathematics were developed to help mathematics education become more focused and coherent, allowing for more clarity and specificity. They include consistent rigorous content and application of knowledge through higher-order skills, so all students are prepared to succeed in our global economy and society.

The three main mathematics courses in the DP identified below support the CCSS goals noted above.

Mathematical studies SL: A standard level course designed for students with a wide range of abilities. This course concentrates on the applications of mathematics, notably statistics.

Mathematics SL: A standard level course designed for students who have achieved reasonable proficiency in mathematical techniques and who may require the use of some mathematics in further study.

Mathematics HL: A higher level course designed for students with good proficiency in mathematics.

These three main DP courses represent a progression of understanding and application of mathematical concepts and skills.

The aims for DP mathematics (Appendix 4) are broader than the CCSS and include references to ethics and international-mindedness, but in general terms have similar aspirations. The DP assessment objectives for mathematics (Appendix 4) meanwhile provide explicit expectations for students and teachers related to **knowledge and understanding, problem-solving, communication and interpretation, technology, reasoning, inquiry** and **investigative approaches.** Furthermore, the internal assessment component of each DP course provides an opportunity for students to use higher-order reasoning skills while communicating in a clear and coherent manner.

The CCSS define what students "should understand and be able to do by the end of each grade" and include detailed expectations for students. Appendix A: Designing high school mathematics courses based on the Common Core State Standards identifies four model course pathways in mathematics based on the CCSS. These "can be a useful foundation for discussing how best to organize the high school standards into courses".

As students explore mathematics and its application to the real world as promoted by the CCSS, these fundamental components of all IB programmes provide an authentic framework for exploring the CCSS for mathematics.

Standards for mathematical practice

1. Make sense of problems and persevere in solving them.

"Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution" (NGA Center 2010: 6).

This mathematical practice provides opportunities for students to struggle with problems, search for strategies and solutions on their own and learn to evaluate their own results. DP mathematics courses aim to "develop logical, critical and creative thinking, and patience and persistence in problem-solving" (aim 4) which closely relates to this first mathematical practice.
In the DP, students seek ways to justify and explain their solutions to problems. These explanations allow opportunities to observe students' mathematical thinking.

The assessment objectives for DP mathematics courses clearly show that problem-solving is central to learning mathematics and students need to persevere when learning concepts and skills through non-routine and open-ended problems. This is evident in assessment objective 2, which states students need to "recall, select and use their knowledge of mathematical skills, results and models in both real and abstract contexts to solve problems".

The internal assessment provides an opportunity for students to understand a problem, develop and carry out a plan to solve the problem then end with an opportunity to reflect back on what worked and did not work while making use of different approaches including the use of technology. This mathematical practice is further developed through the IB learner profile attributes of being inquirers, thinkers, reflective and knowledgeable.

> " *Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution* "

2. Reason abstractly and quantitatively.

"Mathematically proficient students make sense of quantities and their relationships in problem situations" (CCSS 2010: 6). In order to be able to use mathematics as a tool, students need to be able to go from the specifics of the situation to the more abstract mathematics underlying the problem.

The intention of this standard for mathematical practice is for students to reason with models or pictorial representations to solve problems, convert situations into symbols to appropriately solve problems as well as convert symbols into meaningful situations. This mathematical practice is clearly addressed in the aims and assessment objectives of DP mathematics courses.

One of the stated aims of all DP mathematics courses is that students "employ and refine their powers of abstraction and generalization" (aim 5). At the same time, one of the assessment objectives requires students to use reasoning skills to manipulate mathematical expressions and modelling in real and abstract contexts. Another clearly states that students are able to "construct mathematical arguments through use of precise statements, logical deduction and inference, and by the manipulation of mathematical expressions" (assessment objective 5).

3. Construct viable arguments and critique the reasoning of others.

"Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments" (NGA Center 2010: 6).

The ability to conjecture and justify conclusions is essential if students are to be able to fully experience mathematics. They need to be able to communicate and logically defend their own conclusions as well as evaluate those of others.

Students learn to create arguments that rely on logical thinking and reasoning skills. This is central to all of the DP mathematics courses, as illustrated in the aim to "communicate clearly and confidently in a variety of contexts" (aim 3). Several assessment objectives also relate to this mathematical standard, as in assessment objective 3 where students should be able to "transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation".

Another dimension to this mathematical standard is its relationship to the IB learner profile. The attribute of thinking promotes critical and creative **thinking** to approach complex problems, and the attribute of **communicators** promotes the expression of ideas confidently in a variety of modes of communication. Developing a classroom culture that supports and nurtures mathematical discourse provides an environment for constructing arguments and critiquing the reasoning of others.

> " *Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments* "

4. Model with mathematics.

"Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (NGA Center 2010: 7).

Throughout the aims and objectives of the DP mathematics courses, students are expected to demonstrate an inquiry and modelling approach by using their knowledge of mathematics to model both real and abstract contexts to solve problems. "Apply and transfer skills to alternative situations, to other areas of knowledge and to future developments" (aim 3) is particularly relevant to this mathematical standard.

As specifically stated in DP mathematics HL and SL, students should be able to "investigate unfamiliar situations, both abstract and real world, involving organising and analyzing information, making conjectures, drawing conclusions and testing their validity" (assessment objective 6). In mathematical studies students should be "organizing and analysing information or measurements, drawing conclusions, testing their validity, and considering their scope and limitations" (assessment objective 6). Students have opportunities to explore mathematical relationships in real world contexts through regular classroom assignments and mini projects or explorations.

Artwork by: Rosana Pozo
Escuela Bella Vista
18 Years Old

5. Use appropriate tools strategically.

"Mathematically proficient students consider the available tools when solving a mathematical problem" (CCSS 2010: 7).

Mathematics has always relied on tools of some form, for example, paper and pencil, slide rule or computer software. The goal is to help students know when and how to use tools appropriately.

The aim of this mathematical practice is for a student to select and use a combination of tools, including technology, to explore and solve a problem as well as justify their tool selection and problem solution. Students in the DP mathematics courses are expected to use technology accurately, appropriately and efficiently both to explore new ideas and to solve problems as stated in the DP aim "to appreciate how developments in technology and mathematics have influenced each other" (aim 7).

Students are encouraged to demonstrate the proper uses of technology through the internal assessment component of all DP mathematics courses in order to explore and deepen their understanding of mathematical concepts. Students have opportunities to choose and use a variety of tools to solve problems through investigative tasks. There are extensive references in all the DP mathematics guides on the use of technology for teaching and learning.

6. Attend to precision.

"Mathematically proficient students try to communicate precisely to others" (NGA Center 2010: 7). The intent of this mathematical practice is for a mathematically proficient student to be able to communicate mathematics precisely in a clear and coherent manner. This includes the proper use of mathematical terminology and symbols. Within the DP mathematics courses, students are expected to communicate and interpret mathematics. Specifically stated in the assessment objectives, students will demonstrate the ability to: transform common realistic contexts into mathematics, comment on the context, sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology and record methods, solutions and conclusions using standardized notation. Additionally, one of the DP mathematics aims is to enable students "to communicate clearly and confidently in a variety of contexts" (aim 5). Students are provided with multiple opportunities to practice precision through mini explorations or projects in preparation for the internal assessment.

7. Look for and make use of structure.

"Mathematically proficient students look closely to discern a pattern or structure" (NGA Center 2010: 8). Students have to be inquirers and use their knowledge of mathematical facts, concepts and techniques to discover how some complicated concepts are a composition of several other concepts. Students discover patterns, functions and general formulas through investigative tasks. Students are expected to discuss the reasonableness of their results as part of these investigative tasks.

Mathematically proficient students are able to compose and decompose number situations and relationships through observed patterns in order to simplify solutions.

8. Look for and express regularity in repeated reasoning.

"Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts" (NGA Center 2010: 8).

Students not only need to be able to look for obvious patterns but also use reasoning strategies for obvious patterns. They need to be able to discover deep, underlying relationships, for example, uncover a model or equation that unifies the various aspects of a problem such as a discovery of an underlying function. DP students are required to "apply and transfer skills to alternative situations, to other areas of knowledge and to future developments" (aim 6).

Students in the DP mathematics courses develop an appreciation of the "elegance and power of mathematics" (aim 1) while employing and refining their powers of abstraction and generalization.

> *Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts*

Summary

The DP mathematics courses and the CCSS mathematical standards focus on preparing students for college and career readiness by developing thinkers, inquirers and communicators through mathematics. The DP assessment objectives for mathematics: knowledge and understanding, problem-solving, communication and interpretation, technology, reasoning and inquiry or investigative approaches are clearly reflected in the CCSS eight mathematical practices.

The study also highlights some areas of distinction in the DP, as identified by commitment to the development of international-mindedness and attributes of the IB learner profile. This provides a strong foundation for students and teachers to appreciate the global dimension of mathematics.

Furthermore, DP mathematics aims to enable students to "enjoy mathematics, develop curiosity and begin to appreciate its elegance and power" (aim 1). This is a powerful testament to the role of mathematics in education and the impact it can have on the lives of students.

The three main DP mathematics courses clearly support and provide opportunities to extend the teaching of the CCSS of mathematical practice in grades 11 to 12. With the implementation of the CCSS mathematics, IB practitioners will discover other aspects of the relationship between the DP courses and the CCSS as they reflect upon their own teaching and their students' learning.

The IB Career-related Certificate and the Common Career Technical Core

The IB has developed a framework of international education that incorporates the vision and educational principles of the IB into local programmes, addressing the needs of students engaged in career-related studies. The IBCC is an academic educational framework designed to support schools and colleges that also offer career-related studies to their students.

A unique offering, the IBCC specifically addresses the needs of students who wish to engage in career-related education. The IBCC prepares students for flexibility and mobility in a range of employment opportunities as well as continuing lifelong learning through the integration of broad, general learning areas and specific career-related content in a framework of education.

The IBCC encourages these students to benefit from elements of an IB education through a selection of two or more DP courses in addition to a unique IBCC core, comprised of courses in: approaches to learning, community and service, a reflective project, and language development. The core framework is at the heart of the IBCC and enables students to enhance their personal and interpersonal development, with an emphasis on experiential learning.

This document briefly outlines the relationship between the IBCC and the career ready practices of the Common Career Technical Core (CCTC).

Relationship between the CCTC and the IBCC

The CCTC career ready practices are found predominantly within the IBCC core elements of approaches to learning, community and service, language development and reflective project. By examining both the stated aims and the defined content of each component, the 12 CCTC career ready practices are found to be explicitly embedded in the IBCC core.

The following table provides a brief overview of the overlap between the IBCC and the 12 CCTC career ready practices.

CCTC career ready practices	Corresponding IBCC core elements
1. Act as a responsible and contributing citizen and employee.	• Community and service
2. Apply appropriate academic and technical skills.	• Approaches to learning: personal development, thinking
3. Attend to personal health and financial well-being.	• Community and service • Approaches to learning: personal development
4. Communicate clearly, effectively and with reason.	• Approaches to learning: thinking, personal development, communication • Language development
5. Consider the environmental, social and economic impacts of decisions.	• Approaches to learning: thinking, personal development, communication, intercultural understanding • Reflective project • Community and service
6. Demonstrate creativity and innovation.	• Approaches to learning: thinking, communication • Reflective project
7. Employ valid and reliable research strategies.	• Reflective project • Approaches to learning: thinking
8. Utilize critical thinking to make sense of problems and persevere in solving them.	• Reflective project • Approaches to learning: thinking

CCTC career ready practices	Corresponding IBCC core elements
9. Model integrity, ethical leadership and effective management.	• Community and service • Approaches to learning
10. Plan education and career path aligned to personal goals.	• Approaches to learning: personal development, thinking
11. Use technology to enhance productivity.	• Approaches to learning: communication
12. Work productively in teams while using cultural global competence.	• Community and service • Approaches to learning: intercultural understanding, communication

Summary

The IBCC is a challenging and rewarding educational framework that demands the best from motivated students. It is evident that the IBCC clearly delivers the CCTC career ready practices through the broad and comprehensive IBCC core. Furthermore, the 10 attributes of the IB learner profile also support the application and development of the skills outlined in the 12 CCTC career ready practices.

" *The IBCC is an academic educational framework designed to support schools and colleges that also offer career-related studies to their students.* "

IB programmes
and the
Common Core State Standards
Application to Students with Disabilities

Both the IB and the CCSS provide statements on access to the curriculum for all students. The IB states that "difference and diversity are central in IB World Schools where all students enrolled in IB programmes should receive meaningful and equitable access to the curriculum ... and access to an appropriate education that affords students the opportunity to achieve personal potential" (Learning diversity in the International Baccalaureate programmes/Special educational needs within the International Baccalaureate programmes 2010: 2).

The CCSS document *Application to Students with Disabilities* states that the common core standards "provide an historic opportunity to improve access to rigorous academic content standards for students with disabilities." The CCSS notes that "students with disabilities ... must be challenged to excel within the general curriculum and be prepared for success in their post-school lives, including college and/or careers".

Artwork by: Helen Marcella Stritzel
William Palmer High School
11th Grade

The CCSS sets expectations for schools to incorporate support and accommodations to enable "students with disabilities to meet high academic standards and to fully demonstrate their conceptual and procedural knowledge and skills in mathematics, reading, writing, speaking and listening."

While the IB cannot be as explicit in its demands due to the legal and contextual issues of schools in such a wide range of countries, student access is supported through the following standards as detailed in the IB *Programme standards and practices* (2010).

Standard A.9: The school supports access for students to the IB programme(s) and philosophy.

Standard B2.8 The school provides support for its students with learning and/or special educational needs and supports their teachers.

Standard C1.6: Collaborative planning and reflection incorporates differentiation for students' learning needs and styles.

Standard C3.10: Teaching and learning differentiates instruction to meet students' learning needs and styles.

To further support schools in meeting these standards, the IB has identified four principles of good practice for promoting and supporting equal access: affirming identity and building self-esteem, valuing prior knowledge, scaffolding and extending learning. While supporting the *Programme standards and practices,* these principles also allow schools to use the learning approaches and strategies that are appropriate, or are legal requirements, within their own contexts.

The IB document *Candidates with assessment access requirements/special educational needs* outlines the principles and guidelines for applying access arrangements so that all DP examination candidates are allowed to demonstrate their ability under assessment conditions that are as fair as possible. Inclusive assessment arrangements allow all learners fair access to assessment without changing the demand and without devaluing the qualification. Arrangements may include additional time/rest, assistive technologies, scribes, readers, communicators, prompters, modifications (Braille, print sizes, coloured paper), audio recordings, transcriptions and assistance with practical work.

It is expected that reasonable adjustments and inclusive access arrangements as outlined above will be respected in the PYP, MYP and the IBCC, and these principles can be found in the programme guidelines.
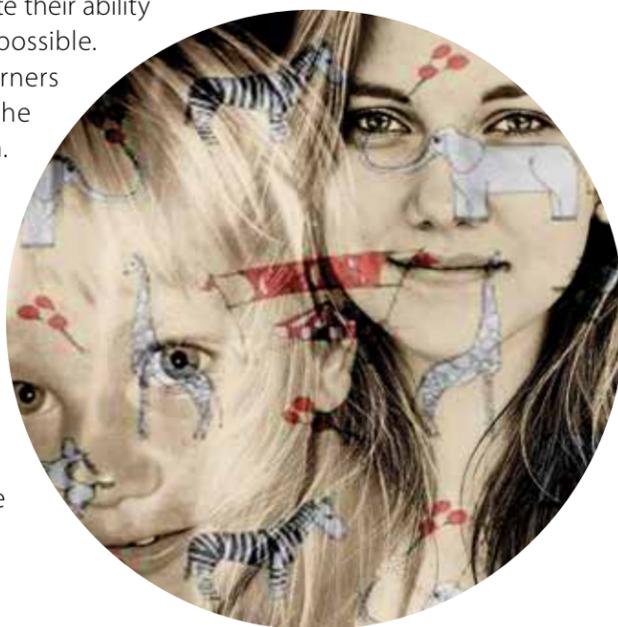
Artwork by: Camille Jackson
FDR American School of Lima
12th Grade

IB programmes and the CCSS both promote a culture of high expectations for all students. Additional supports and services are suggested by the IB and CCSS; for example: instructional supports such as those based on the principles of Universal Design for Learning (http://www.udlcenter.org/aboutudl) as well as instructional accommodations for which standards remain unchanged yet there are changes in materials or procedures. Through the supports, accommodations and inclusive arrangements identified above, students receive access to multiple means of learning and opportunities to demonstrate their knowledge, while the rigour and high expectations of the CCSS and IB programmes are maintained.

difference and diversity are central in IB World Schools where all students enrolled in IB programmes should receive meaningful and equitable access to the curriculum
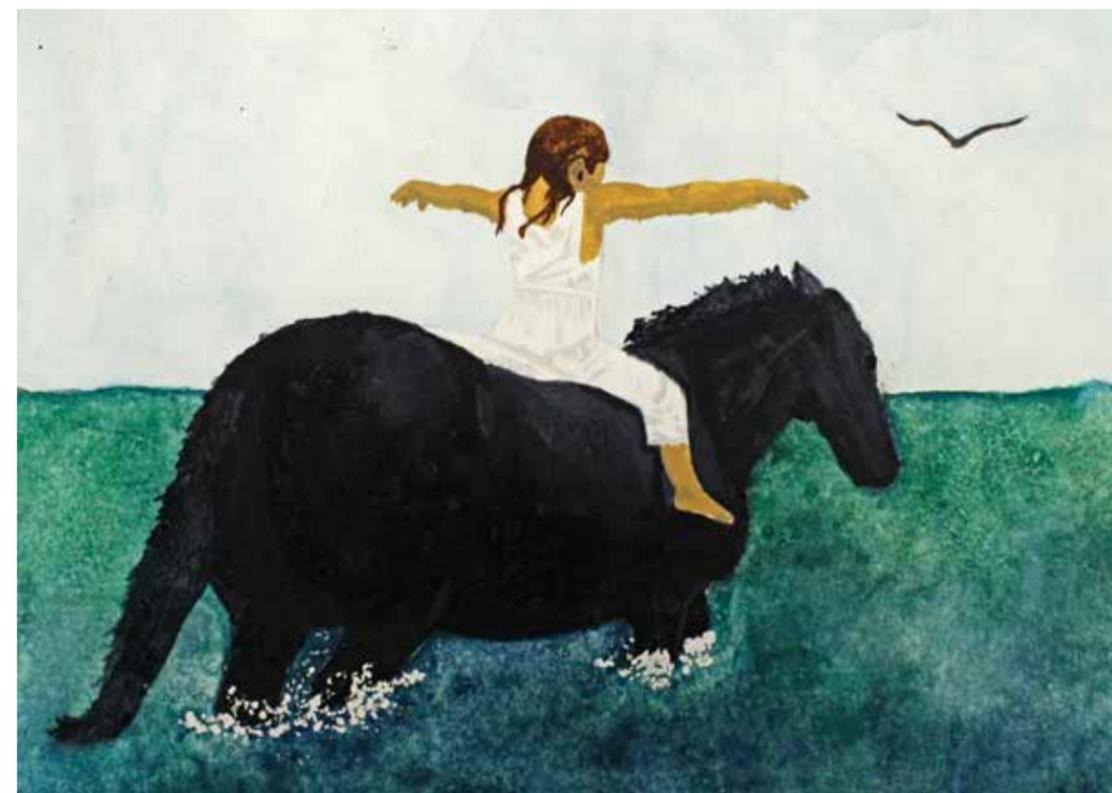
Artwork by: Madison Graber
Hutchinson High School
11th Grade

Appendices

Appendix 1: PYP mathematical strands
(*Making the PYP happen*: 86)

What do we want students to know?

Data handling	Data handling allows us to make a summary of what we know about the world and to make inferences about what we do not know. • Data can be collected, organized, represented and summarized in a variety of ways to highlight similarities, differences and trends; the chosen format should illustrate the information without bias or distortion. • Probability can be expressed qualitatively by using terms such as "unlikely", "certain" or "impossible". It can be expressed quantitatively on a numerical scale.
Measurement	To measure is to attach a number to a quantity using a chosen unit. Since the attributes being measured are continuous, ways must be found to deal with quantities that fall between numbers. It is important to know how accurate a measurement needs to be or can ever be.
Shape and space	The regions, paths and boundaries of natural space can be described by shape. An understanding of the interrelationships of shape allows us to interpret, understand and appreciate our two-dimensional (2D) and three dimensional (3D) world.
Pattern and function	To identify pattern is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as generalized rules called "functions". This builds a foundation for the later study of algebra.
Number	Our number system is a language for describing quantities and the relationships between quantities. For example, the value attributed to a digit depends on its place within a base system. Numbers are used to interpret information, make decisions and solve problems. For example, the operations of addition, subtraction, multiplication and division are related to one another and are used to process information in order to solve problems. The degree of precision needed in calculating depends on how the result will be used.

Appendix 2: How mathematics practices are changing
(*Making the PYP happen*: 84)

How are mathematics practices changing?	
Increased emphasis on:	**Decreased emphasis on:**
connecting mathematical concepts and applications to learning	treating mathematics as isolated concepts and facts
manipulatives, to make mathematics understandable to students	rote learning, memorization and symbol manipulation
real-life problem solving using mathematics	word problems as problem solving
instruction built on what students know, what they want to know, and how they best might find out	instruction focused on what students do not know
a variety of strategies for possible multiple solutions—emphasis on process	one answer, one method, emphasis on answer
students being encouraged to speculate and pursue hunches	the teacher as the sole authority for right answers
a broad range of topics regardless of computational skills	computational mastery before moving on to other topics
mathematics as a means to an end	teaching mathematics disconnected from other learning
the use of calculators and computers for appropriate purposes	a primary emphasis on pencil and paper computations
programme of inquiry as the context for learning	the textbook as the context for learning
students investigating, questioning, discussing, justifying and journaling their mathematics	the use of worksheets
students and teachers engaged in mathematical discourse.	teacher telling about mathematics.

Appendix 3: MYP mathematics aims and objectives (2013)

The aims of the teaching and learning of MYP mathematics are to encourage and enable students to:

- enjoy mathematics, develop curiosity and begin to appreciate its elegance and power

- develop an understanding of the principles and nature of mathematics

- communicate clearly and confidently in a variety of contexts

- develop logical, critical and creative thinking

- develop perseverance in problem solving

- develop powers of generalization and abstraction

- apply and transfer skills to a wide range of situations, including real life, other areas of knowledge and future developments

- appreciate how developments in technology and mathematics have influenced each other

- appreciate the moral, social and ethical implications arising from the work of mathematicians and the applications of mathematics

- appreciate the international dimension in mathematics through an awareness of the universality of mathematics and its multicultural and historical perspectives

- appreciate the contribution of mathematics to other areas of knowledge

- develop the knowledge, skills and attitudes necessary to pursue further studies in mathematics

- develop the ability to reflect critically upon their own work and the work of others.

In MYP mathematics, the objectives reflect the fact that students should be able to know and use mathematics in a variety of contexts (including authentic real-life situations), perform investigations and communicate mathematics clearly.

A   Knowing and understanding

Knowledge and understanding are fundamental to studying mathematics and form the base from which to explore concepts and develop skills. This objective expects students to demonstrate knowledge and understanding of the concepts and skills of the four branches in the prescribed framework (number, algebra, geometry and trigonometry, and statistics and probability). It assesses the extent to which students can select and apply mathematics to solve problems in both familiar and unfamiliar situations in a variety of contexts.

In order to reach the aims of mathematics, students should be able to:

i.   demonstrate knowledge and understanding of the four branches of mathematics (number, algebra, geometry and trigonometry, statistics and probability)

ii.   select appropriate mathematics when solving problems

iii. apply the selected mathematics successfully when solving problems

iv.   solve problems correctly in both familiar and unfamiliar situations in a variety of contexts.

B   Investigating

Investigating patterns allows students to experience the excitement and satisfaction of mathematical discovery. Working through investigations encourages students to become risk-takers, inquirers and critical thinkers. The ability to inquire is invaluable in the MYP and contributes to lifelong learning.

In order to reach the aims of mathematics, students should be able to:

i.   select and apply mathematical problem-solving techniques to discover complex patterns

ii.   describe patterns as general rules consistent with findings

iii. prove, or test and justify, general rules.

C   Communicating

Mathematics provides a powerful and universal language. Students are expected to use appropriate mathematical language and different forms of representation when communicating mathematical ideas, reasoning and findings, both orally and in writing.

In order to reach the aims of mathematics, students should be able to:

i.   use appropriate mathematical language (notation, symbols and terminology) in both oral and written explanations

ii.   select appropriate forms of mathematical representation to present information

iii. move between different forms of mathematical representation

iv. communicate complete, coherent and concise mathematical lines of reasoning

v.   organize information using a logical structure.

D   Applying

MYP mathematics encourages students to see mathematics as a tool for solving problems in an authentic real-life context. Students are expected to transfer theoretical mathematical knowledge into real-world situations and apply appropriate problem-solving strategies, draw valid conclusions and reflect upon their results.

In order to reach the aims of mathematics, students should be able to:

i.   identify relevant elements of authentic real-life problems

ii.   select appropriate mathematics when solving authentic real-life problems

iii. apply the selected mathematics successfully to reach a solution

iv. justify the degree of accuracy of a solution

v.   justify whether a solution makes sense in real life.

Appendix 4: DP group 5 aims and assessment objectives

The aims of all DP mathematics courses are to enable students to:

- enjoy mathematics, and develop an appreciation of the elegance and power of mathematics

- develop an understanding of the principles and nature of mathematics

- communicate clearly and confidently in a variety of contexts

- develop logical, critical and creative thinking, and patience and persistence in problem-solving

- employ and refine their powers of abstraction and generalization

- apply and transfer skills to alternative situations, to other areas of knowledge and to future developments

- appreciate how developments in technology and mathematics have influenced each other

- appreciate the moral, social and ethical implications arising from the work of mathematicians and the applications of mathematics

- appreciate the international dimension in mathematics through an awareness of the universality of mathematics and its multicultural and historical perspectives

- appreciate the contribution of mathematics to other disciplines, and as a particular "area of knowledge" in the [theory of knowledge] (TOK) course.

The assessment objectives of all DP mathematics courses are:

- Knowledge and understanding: Recall, select and use their knowledge of mathematical facts, concepts and techniques in a variety of familiar and unfamiliar contexts

- Problem-solving: Recall, select and use their knowledge of mathematical skills, results and models in both real and abstract contexts to solve problems

- Communication and interpretation: Transform common realistic contexts into mathematics; comment on the context; sketch or draw mathematical diagrams, graphs or constructions both on paper and using technology; record methods, solutions and conclusions using standardized notation

- Technology: Use technology, accurately, appropriately and efficiently both to explore new ideas and to solve problems

- Reasoning: Construct mathematical arguments through use of precise statements, logical deduction and inference, and by the manipulation of mathematical expressions.

Mathematics HL and SL:

- Inquiry approaches: Investigate unfamiliar situations, both abstract and real-world, involving organizing and analysing information, making conjectures, drawing conclusions and testing their validity.

Mathematical studies SL:

- Investigative approaches: Investigate unfamiliar situations involving organizing and analysing information or measurements, drawing conclusions, testing their validity, and considering their scope and limitations.

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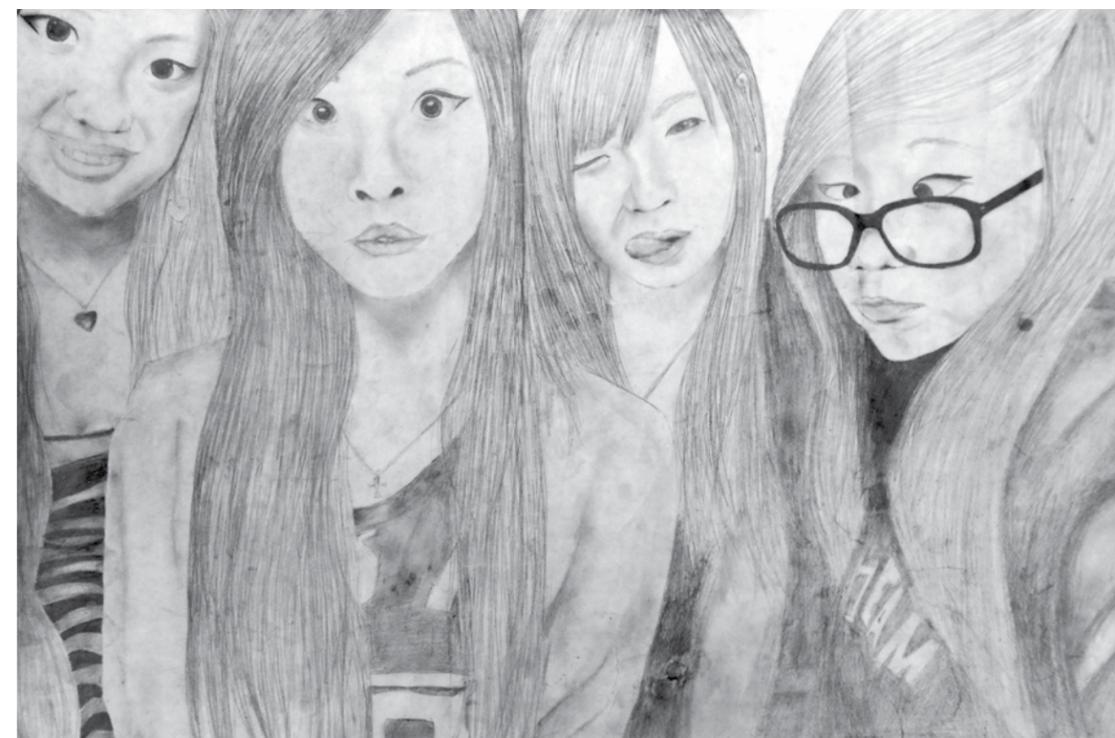
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IB learner profile

The aim of all IB programmes is to develop internationally minded people who, recognizing their common humanity and shared guardianship of the planet, help to create a better and more peaceful world.

As IB learners we strive to be:

INQUIRERS
We nurture our curiosity, developing skills for inquiry and research. We know how to learn independently and with others. We learn with enthusiasm and sustain our love of learning throughout life.

KNOWLEDGEABLE
We develop and use conceptual understanding, exploring knowledge across a range of disciplines. We engage with issues and ideas that have local and global significance.

THINKERS
We use critical and creative thinking skills to analyse and take responsible action on complex problems. We exercise initiative in making reasoned, ethical decisions.

COMMUNICATORS
We express ourselves confidently and creatively in more than one language and in many ways. We collaborate effectively, listening carefully to the perspectives of other individuals and groups.

PRINCIPLED
We act with integrity and honesty, with a strong sense of fairness and justice, and with respect for the dignity and rights of people everywhere. We take responsibility for our actions and their consequences.

OPEN-MINDED
We critically appreciate our own cultures and personal histories, as well as the values and traditions of others. We seek and evaluate a range of points of view, and we are willing to grow from the experience.

CARING
We show empathy, compassion and respect. We have a commitment to service, and we act to make a positive difference in the lives of others and in the world around us.

RISK-TAKERS
We approach uncertainty with forethought and determination; we work independently and cooperatively to explore new ideas and innovative strategies. We are resourceful and resilient in the face of challenges and change.

BALANCED
We understand the importance of balancing different aspects of our lives—intellectual, physical, and emotional—to achieve well-being for ourselves and others. We recognize our interdependence with other people and with the world in which we live.

REFLECTIVE
We thoughtfully consider the world and our ideas and experience. We work to understand our strengths and weaknesses in order to support our learning and personal development.

The IB learner profile represents 10 attributes valued by IB World Schools. We believe these attributes, and others like them, can help individuals and groups become responsible members of local, national and global communities.

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