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# Evaluating Some Types of Definite Integrals

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**Abstract** This paper studies three types of definite integrals. The infinite series forms of these definite integrals can be obtained using Parseval's theorem. In addition, we provide some examples to do calculation practically, and Maple is used to calculate the approximations of some definite integrals and their solutions for verifying our answers.

***Keywords:*** *definite integrals, infinite series forms, Parseval's theorem, Maple*

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## 1. Introduction

In calculus and engineering mathematics, there are many methods to solve the integral problems including change of variables method, integration by parts method, partial fractions method, trigonometric substitution method, etc. In this paper, we study the following three types of definite integrals which are not easy to obtain their answers using the methods mentioned above.

$$\int_0^{2\pi} \exp(2r\cos x)\cos^2(r\sin x)dx \tag{1}$$

$$\int_0^{2\pi} \exp(2r\cos x)\sin^2(r\sin x)dx \tag{2}$$

$$\int_0^{2\pi} \exp(2r\cos x)dx \tag{3}$$

where $r$ is any real number. We can obtain the infinite series forms of these definite integrals using Parseval's theorem; this is the major result of this paper (i.e., Theorem A). Adams et al. [1], Nyblom [2], and Oster [3] provided some techniques to solve the integral problems. Yu [4-26], Yu and B. -H. Chen [27], and T. -J. Chen and Yu [28,29,30] used complex power series method, integration term by term theorem, differentiation with respect to a parameter, and generalized Cauchy integral formula to solve some types of integrals. In this article, some examples are used to demonstrate the proposed calculations, and the manual calculations are verified using Maple.

## 2. Main Results

A notation, a definition and some formulas used in this paper are introduced below.

### 2.1. Notation

Let $z = a + ib$ be a complex number, where $i = \sqrt{-1}$, and $a, b$ are real numbers. $a$, the real part of $z$, denoted as Re(z), and b, the imaginary part of z, denoted as Im(z).

### 2.2. Definition

Suppose that $f(x)$ is a continuous function defined on $[0, 2\pi]$, the Fourier series expansion of $f(x)$ is $\frac{1}{2}a_0 + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx)$, where $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)dx$, $a_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos kxdx$, and $b_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin kxdx$ for all positive integers k.

### 2.3. Formulas

#### 2.3.1. Euler's Formula

$e^{ix} = \cos x + i\sin x$, where $x$ is any real number.

#### 2.3.2. DeMoivre's Formula

$(\cos x + i\sin x)^n = \cos nx + i\sin nx$, where $n$ is any integer, and $x$ is any real number.

#### 2.3.3. Taylor Series Expansion of Complex Exponential Function

$e^z = \sum_{k=0}^{\infty}\frac{1}{k!}z^k$, where $z$ is any complex number.

The following is an important theorem used in this article, which can be found in [[31], p 428].

### 2.4. Parseval's Theorem

If f(x) is a continuous function defined on [0, 2π] and f(0)=f(2π). Suppose that the Fourier series expansion of $f(x)$ is $\frac{1}{2}a_0 + \sum_{k=1}^{\infty}(a_k \cos kx + b_k \sin kx)$, then $\frac{1}{\pi}\int_0^{2\pi} f^2(x)dx = \frac{{a_0}^2}{2} + \sum_{k=1}^{\infty}({a_k}^2 + {b_k}^2)$.

Before the major result can be derived, a lemma is needed.

### 2.5. Lemma

*Suppose that $r$ is any real number. The Fourier series expansions of the following trigonometric functions are:*

$$\exp(r\cos x)\cdot\cos(r\sin x) = 1+\sum_{k=1}^{\infty}\frac{r^k}{k!}\cos kx, \qquad (4)$$

$$\exp(r\cos x)\cdot\cos(r\sin x) = 1+\sum_{k=1}^{\infty}\frac{r^k}{k!}\sin kx, \qquad (5)$$

*Proof*

$$\begin{aligned}
&\exp(r\cos x)\cdot\cos(r\sin x)\\
&= \mathrm{Re}[\exp(r\cos x)\cdot\exp(ir\sin x)](by\ Enler's\ \ formula)\\
&= \mathrm{Re}[\exp(r\exp ix)]\\
&= \mathrm{Re}[\sum_{k=0}^{\infty}\frac{1}{k!}(r\exp ix)^k](u\sin g\ \ Formula\ 2.3.3.)\\
&= \mathrm{Re}[\sum_{k=0}^{\infty}\frac{r^k}{k!}\exp ix](by\ DeMoivre's\ \ formula)\\
&= \sum_{k=0}^{\infty}\frac{r^k}{k!}\cos kx(by\ Euler's\ \ formula)\\
&= 1+\sum_{k=0}^{\infty}\frac{r^k}{k!}\cos kx.
\end{aligned}$$

Similarly,

$$\begin{aligned}
&\exp(r\cos x)\cdot\sin(r\sin x)\\
&= \mathrm{Im}[\exp(r\cos x)\cdot\exp(ir\sin x)]\\
&= \mathrm{Im}[\exp(r\exp ix)]\\
&= \sum_{k=0}^{\infty}\frac{r^k}{k!}\sin kx.
\end{aligned}$$

In the following, we determine the infinite series forms of the definite integrals (1), (2) and (3).

### 2.6. Theorem A

*Suppose that $r$ is any real number, the definite integrals:*

$$\int_0^{2\pi} exp(2r\cos x)\cos^2(r\sin x)dx = 2\pi+\pi\cdot\sum_{k=1}^{\infty}\frac{r^{2k}}{(k!)^2} \qquad (6)$$

$$\int_0^{2\pi} exp(2r\cos x)\sin^2(r\sin x)dx = \pi\cdot\sum_{k=1}^{\infty}\frac{r^{2k}}{(k!)^2} \qquad (7)$$

$$\int_0^{2\pi} exp(2r\cos x)dx = 2\pi+\pi\cdot\sum_{k=1}^{\infty}\frac{r^{2k}}{(k!)^2} \qquad (8)$$

*Proof* Using Parseval's theorem, (4) and (5), we obtain (6) and (7). Moreover, by the summation of (6) and (7), we obtain (8).

## 3. Examples

In the following, for the three types of definite integrals in this study, we provide some definite integrals and use Theorem A to determine their infinite series forms. On the other hand, we use Maple to calculate the approximations of these definite integrals and their solutions for verifying our answers.

If $r=5$ in (6), we obtain the definite integral:

$$\int_0^{2\pi}\exp(10\cos x)\cos^2(5\sin x)dx = 2\pi+\pi\cdot\sum_{k=1}^{\infty}\frac{5^{2k}}{(k!)^2} \qquad (9)$$

Next, we use Maple to verify the correctness of (9).
```
>evalf(int(exp(10*cos(x))*(cos(5*sin(x)))^2,x=0..2*Pi),18);
8848.97626723379613.
>evalf(2*Pi+Pi*sum(5^(2*k)/(k!)^2,k=1..infinity),18);
8848.97626723379613.
```

On the other hand, let $r=\sqrt{3}$ in (7), we have:

$$\int_0^{2\pi}\exp(2\sqrt{3}\cos x)\sin^2(\sqrt{3}\sin x)dx = \pi\cdot\sum_{k=1}^{\infty}\frac{(\sqrt{3})^{2k}}{(k!)^2} \qquad (10)$$

```
>evalf(int(exp(2*sqrt(3)*cos(x))*(sin(sqrt(3)*sin(x)))^2,x=0..2*Pi),18);
19.3490582735096348.
>evalf(Pi*sum(sqrt(3)^(2*k)/(k!)^2,k=1..infinity),18);
19.3490582735096347.
```

Finally, if $r=15/4$ in (8), then the definite integral:

$$\int_0^{2\pi}\exp\left(\frac{15}{2}\cos x\right)dx = 2\pi+2\pi\cdot\sum_{k=1}^{\infty}\frac{(15/4)^{2k}}{(k!)^2} \qquad (11)$$

```
>evalf(int(exp(15/2*cos(x)),x=0..2*Pi),18);
1684.90721246624587.
>evalf(2*Pi+2*Pi*sum((15/4)^(2*k)/(k!)^2,k=1..infinity),18);
1684.90721246624588.
```

## 4. Conclusion

This article uses Parseval's theorem to evaluate some definite integrals. In fact, the applications of this theorem are extensive, and can be used to easily solve many difficult problems; we endeavor to conduct further studies on related applications. In addition, Maple also plays a vital assistive role in problem-solving. In the future, we will extend the research topic to other calculus and engineering mathematics problems and solve these problems using Maple.

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