An Analytical Investigation on Deflections of Pratt Pattern Bridge Truss Posttensioned with External Tendons

Ravindra P M, Nagaraja P S

Abstract- Majority of the existing steel truss bridges all over the world are very old and more than 80 % of them inventoried in the United States are structurally deficient and/or functional obsolete. There is a need to strengthen these bridges in order to fulfill the present and future loading and traffic requirements. Posttensioning is one of the potential techniques to enhance the performance of these old steel bridges, as it creates redundancy in the structure and also, it is a simple, easy and economical method. In the present analytical study, determinate Pratt pattern of truss is posttensioned with external tendon layouts located below the bottom chord and their effectiveness in reducing deflection is studied.

Stiffness matrix for truss member and two-drape tendon are formulated. Posttensioned truss analysis is carried out in three stages: in first stage, for dead loads, in the second stage for dead loads and posttensioning loads and further in the last stage for other loads. The final deflections are obtained by superimposing the results of second and the third stage. External posttensioning reduced deflection and the reduction is more with the increase in distance between the bottom chord and the tendon. When compared to internal posttensioning along the bottom chord, external posttensioning is more effective in reducing deflections.

Index Terms: Bridges, Chord, Deflection, Posttensioning, Redundancy, Tendon

I. INTRODUCTION

The need for rehabilitation of existing steel truss bridges is a growing concern in many countries and has been emphasized in various research reports and publications. Many of these bridges were designed for relatively light loading and have narrow lane widths that are inadequate for present traffic; thus they face serious problems such as load and lane limitations and expensive premature bridge replacement. Rehabilitation is also required in situations where the preservation of bridge is necessary as a part of historic and cultural heritage. Although rehabilitation of a bridge includes all of its components such as substructure, superstructure, and approaches, the present study will treat only truss portion of the superstructure by adopting external posttensioning.

1. LITERATURE REVIEW

Use of the prestressing is one of the trends of the technical advancements in the field of construction, which ensures a higher level of engineering standards. The principle concept underlying prestressing is to artificially provide internal stresses of the sign opposite to those generated by the external loads.

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It has been reported in the literature that tendons used for prestressing usually take one of the following forms: wires, strands and bars (Troitsky, 1990; Belenya, 1977), and their configuration may be rectilinear, polygonal or curvilinear. Tendons may be internal or external. An internal tendon layout is one in which the tendons are placed within the truss system, where as in case of external tendon layout tendons are placed outside the truss system (Ayyub et. al, 1990).

Anchorages are the mechanical devices used to transmit the tendon force to the steel structure. They include the means of gripping and securing the tendon installation to the steel member. For tensioning and anchoring the tendons to the structure there are a number of different systems, some of which are patented (Belenya, 1977).

Diaphragms assure stability of the members during prestressing and the tendon should have close contact with diaphragms. Throughout their length, the members are connected to the tendon by diaphragms spaced at intervals of 40-50 times minimum radius of gyration of the member chord cross section during prestressing as reported by Troitsky (1990) and Belenya (1977). The member acts through the diaphragms on the tendon and remains straight under compressive loads. At its end edges the member has anchors that connect the tendon to the member. Prestressing concepts have been in use from times immemorial. Back in ancient times, the builders of the Roman triumphal arches provided an additional compressive load on pylon to cancel out the tensile stresses due to the arch thrust. In 1861, academician A. V. Gadolin suggested winding artillery barrels with hot high-strength wire which, after cooling, would compress the barrels and so reduce the tensile stresses in it.

All over the world, for strengthening of many of the existing steel truss bridges, technique of posttensioning has been used. To name a few, use of this technique in England, Rumania, Switzerland, USA, Indonesia is reported in the literature (for example, in England, strengthening of two steel bridges: Livery Street Bridge in Birmingham by Berridge and Lee, 1956; Monmouth Railway Truss Bridge by England Berridge, 1957).

Numerous experimental investigations on trusses prestressed by tendons have been carried out in USSR as reported by Belenya (1977). Most of the investigations were performed at the S M Kirov Ural Polytechnical Institute, Byelorussian Polytechnical Institute, Moscow Construction Engineering Institute and others. Tests were carried out on models of trusses and on whole trusses under laboratory and field conditions, with single and multi-stage prestressing. Trusses were loaded by jacks with the aid of tensioning devices. The aim of the tests was to compare the actual behaviour of trusses with the theoretical assumptions, to check various designs and to refine the prestressing techniques. All methods for creating a prestress in experimental trusses, inclusive of multi-stage prestressing, were proved to be practically feasible. They noticed deviation between the calculated and actual (through tests) values of stresses and deflections. Deviations in deflections are better than that of stresses and the actual test deflections were less than calculate values.

Apart from USSR, experimental investigations on posttensioning of steel trusses have also been carried out and reported by several researchers and to mention a few, by Yadlosky et. al (1982) and Karkare et. al (1997).

The technique of posttensioning is not only limited to bridge structures made up of steel; but also have been applied to strengthening of wooden roof trusses (Langlois et. al, 2006). There is a report by Karkare et. al (1997) that, the technique has been applied to Glass Fiber Reinforced Polymer (GFRP) tubular truss to improve its efficiency.

Instead of using high strength steel tendons for posttensioning, use of Carbon Fibre Reinforced Polymer (FRP) tendons also has been reported (Phares et. al, 2003).

II. OBJECTIES OF THE PRESENT STUDY

The objective of this analytical investigation is to formulate a method for the posttensioned truss analysis and to study the effect of external posttensioning in reducing the deflections. It is also aimed at comparing the effectiveness of the external posttensioning with internal posttensioning in reducing the deflections.

III. METHODOLOGY OF POSTTENSIONED TRUSS ANALYSIS AND RESULTS

The details of the truss considered for the present analytical study are shown in Fig.1. The Pratt pattern of truss is considered for the present analytical investigation which is one of the most commonly used type in truss bridges. The truss is statically determinate which is having eight panels each of span 6.0 m, height 7.5 m, and a load of 600 kN is acting at each joint along the bottom chord. The Modulus of Elasticity of truss members is taken as 200 GPa and for tendon is 160 GPa. The area of cross section of all the truss members is presented in Table 1 and the cross sectional area of tendon is 600 mm^2. The truss is posttensioned with tendon with an initial stress of 1120 N/mm^2 and the corresponding force is 672 kN.

The truss is posttensioned with three external tendon layouts which are placed below the bottom chord at distances of 2 m, 4 m and 6 m as shown in by dotted lines in Fig. 3(a), Fig. 3(b) and Fig. 3(c) respectively and the truss before posttensioning is shown in Fig. 2. Additional joints B_1 and B_7 are created just vertically below L_1 and L_7 by adding the extra members L_1B_1, L_2B_1, L_6B_7 and L_7B_7. Ends of the external tendons are connected to joints L_0 and L_8 of the truss and passed through the pulleys attached to additional joints B_1 and B_7, so that the path of the tendon is along $L_0B_1B_7L_8$.

Direct stiffness matrix approach is adopted for the posttensioned truss analysis: for the truss members the stiffness matrix is of size 4×4, since it is having system degree of freedom as 4; whereas for tendon its size is 8×8, as its system degree of freedom is 8. Stiffness matrix for the truss members is considered from the literature (Weaver and Gere, 1986) and for the tendon, stifffness matrix is formulated from the fundamentals. Posttensioned truss analysis is performed in three stages. In the first stage, only dead load is considered. In the second stage, posttensioned load applied at the truss joints is accounted apart from dead load. In the third stage truss is analysed for live, impact and any other loads. The final deflections are obtained by superposing the results of second and third stage. Computer Programs for the analysis of posttensioned truss are generated in MATLAB and deflections before and after posttensioning are obtained which are presented in Table 2.
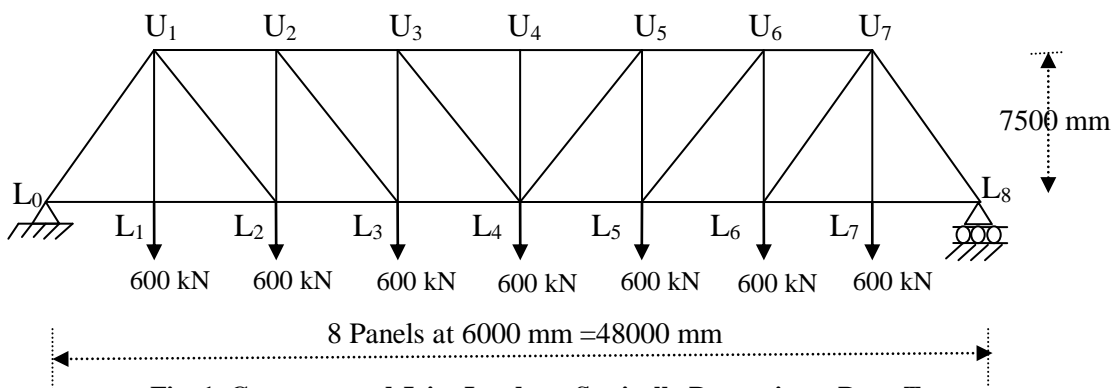

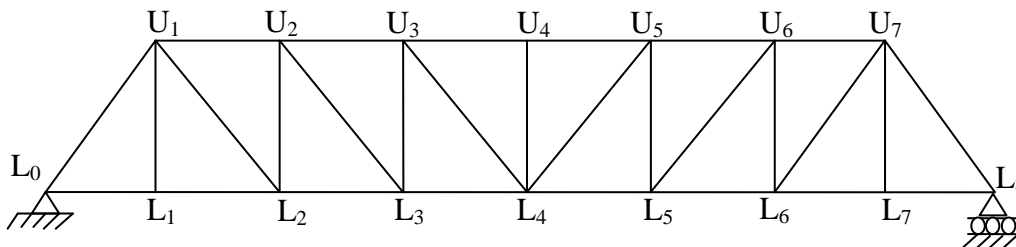**Fig. 1: Geometry and Joint Loads on Statically Determinate Pratt Truss**

Fig. 2: Pratt Truss without Posttensioning

Fig. 3(a): External Tendon with *h*=2 m

Fig. 3(b): External Tendon with *h*=4 m
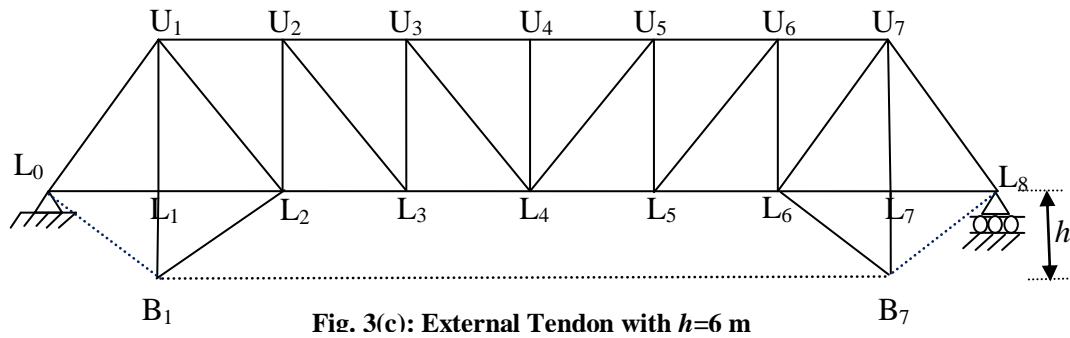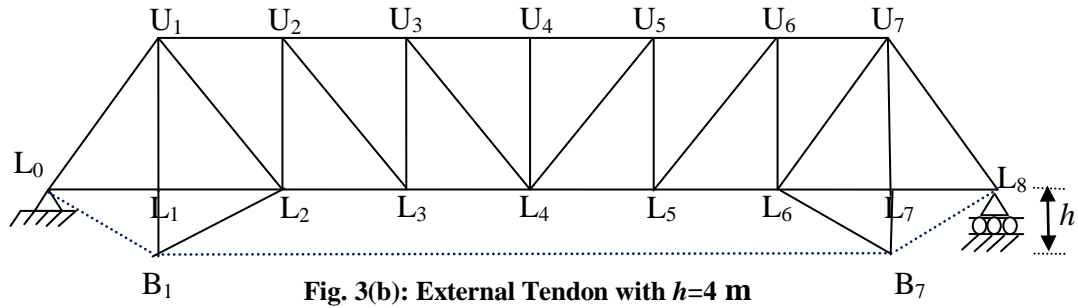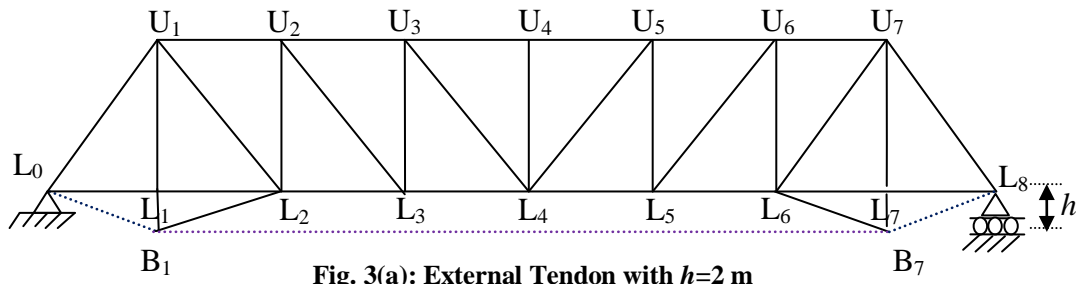

Fig. 3(c): External Tendon with *h*=6 m

Fig. 3: Determinate Pratt Truss Posttensioned with Different External Tendon Layouts

Table 1: Member Data of Statically Determinate Pratt Truss

Member	Area (mm²)	Length (mm)
L_0L_1	25428	6000.00
L_1L_2	25428	6000.00
L_2L_3	25428	6000.00
L_3L_4	25428	6000.00
U_1U_2	23408	5100.00
U_2U_3	23408	5100.00
U_3U_4	23408	5100.00
L_0U_1	23408	9604.69
L_1U_1	3796	7500.00
L_2U_1	12660	9604.69
L_2U_2	9992	7500.00
L_3U_2	7468	9604.69
L_3U_3	5604	7500.00
L_4U_3	3796	9604.69
L_4U_4	2680	7500.00
L_1B_1	6600	2000.00 for h=2 m 4000.00 for h=4 m 6000.00 for h=6 m
L_2B_1	6600	6324.56 for h=2 m 7211.10 for h=4 m 8485.28 for h=6 m

Table 2: Deflections of Statically Determinate Pratt Truss before and after Posttensioning (in mm)

| Joint		Before Posttensioning	After Posttensioning with External Tendons		
			h=2 m	h=4 m	h=6 m
L_0	x	0.00	0.00	0.00	0.00
	y	0.00	0.00	0.00	0.00
L_1	x	1.98	0.71	0.65	0.63
	y	-32.86	-24.36	-21.29	-19.76
L_2	x	3.96	1.42	1.29	1.25
	y	-52.96	-43.97	-39.75	-35.54
L_3	x	7.36	3.46	2.98	2.63
	y	-76.86	-66.27	-61.25	-56.25
L_4	x	11.61	6.34	5.52	4.85
	y	-89.03	-78.19	-72.90	-67.64
L_5	x	15.86	9.22	8.06	7.08
	y	-76.86	-66.27	-61.25	-56.25
L_6	x	19.25	11.25	9.75	8.45
	y	-52.96	-43.97	-39.75	-35.54
L_7	x	21.24	11.96	10.40	9.08
	y	-32.86	-24.36	-21.29	-19.76
L_8	x	23.22	12.67	11.05	9.71
	y	0.00	0.00	0.00	0.00
U_1	x	24.84	18.62	16.81	15.15
	y	-26.93	-21.01	-18.86	-17.04
U_2	x	21.14	15.25	13.77	12.44
	y	-56.34	-47.35	-43.13	-38.91
U_3	x	16.53	10.94	9.80	8.80
	y	-78.87	-68.28	-63.26	-58.26
U_4	x	11.61	6.34	5.52	4.85
	y	-89.03	-78.19	-72.90	-67.64
U_5	x	6.69	1.73	1.25	0.91
	y	-78.87	-68.28	-63.26	-58.26
U_6	x	2.07	-2.58	-2.72	-2.73
	y	-56.34	-47.35	-43.13	-38.91
U_7	x	-1.62	-5.95	-5.77	-5.45
	y	-26.93	-21.01	-18.86	-17.04

IV. DISCUSSIONS OF RESULTS
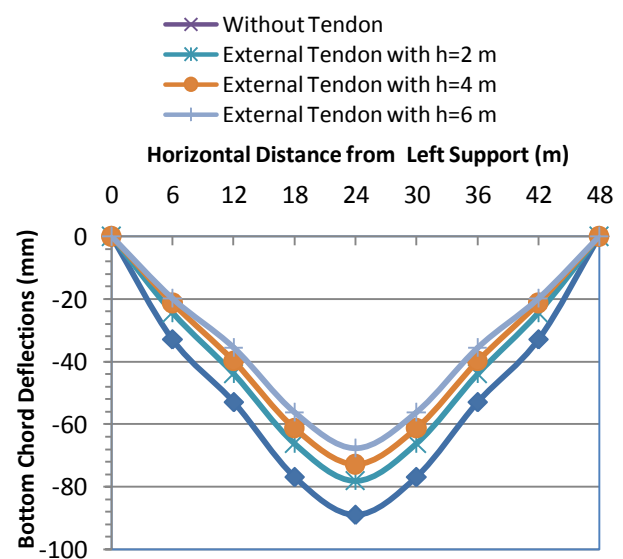
Fig. 4: Vertical Deflections along the Bottom Chord of Determinate Pratt Truss (mm)

Third column of Table 2, presents horizontal (x) and vertical (y) deflections for conventional (without posttensioning) truss shown in Fig. 2; whereas fourth, fifth and sixth columns gives

respective values after posttensioning for the truss shown in Fig. 3(a), Fig. 3(b) and Fig. 3(c) respectively.

i. Along Bottom Chord: Both vertical and horizontal deflections are reduced after posttensioning.

a. Vertical deflections:

Comparing y-values in fourth, fifth and sixth columns with their respective values in second column and also from Fig. 4, it is clearly noticed that along all the bottom chord joints, deflections are reduced after posttensioning. Before posttensioning downward deflections were 32.86 mm, 52.96 mm, 76.86 mm and 89.03 mm at joints L_1, L_2, L_3 and L_4 respectively; whereas after posttensioning the corresponding values brought down to:
i. 24.36 mm, 43.97 mm, 66.27 mm and 78.19 mm for h=2 m, which lead to a percentage reduction of 25.88, 17.00, 13.80 and 12.18 respectively.
ii. 21.29 mm, 39.75 mm, 61.25 mm and 72.90 mm for h=4 m, which lead to a percentage reduction of 35.21, 24.94, 20.31 and 18.12.
iii. 19.76 mm, 35.54 mm, 56.25 mm and 67.64 mm for h=6 m, which lead to a percentage reduction of 39.87, 32.89, 26.81 and 24.02.

b. Horizontal Deflections: It is observed from Table 2 that, the horizontal deflections along the bottom chord joints before and after posttensioning are zero at joint L_0 (located at 0 m horizontal distance from the left support) and maximum at joint L_8 (located at 48 m horizontal distance from the left support) and at all the remaining joints, the horizontal deflections are increasing with the increase in horizontal distance from the left support. Horizontal deflections at joints L_1, L_2, L_3, L_4, L_5, L_6 and L_7 for conventional truss were 1.98 mm, 3.96 mm, 7.36 mm, 11.61 mm, 15.86 mm, 19.25 mm, 21.24 mm and 23.22 mm respectively. After posttensioning, the respective deflections have been reduced to:
i. 0.71 mm, 1.42 mm, 3.46 mm, 6.34 mm, 9.22 mm, 11.25 mm, 11.96 mm and 12.67 mm for h=2 m, which accounts to 64.14, 64.14, 52.99, 45.39, 41.87, 41.56, 43.69 and 45.43 percentage.
ii. 0.65 mm, 1.29 mm, 2.98 mm, 5.52 mm, 8.06 mm, 9.75 mm, 10.40 mm and 11.05 mm for h=4 m, which accounts to 67.17, 67.42, 59.51, 52.45, 49.18, 49.35, 51.04 and 52.41 percentage.
iii. 0.63 mm, 1.25 mm, 2.63 mm, 4.85 mm, 7.08 mm, 8.45 mm, 9.08 mm and 9.71 mm for h=6 m, which accounts to 68.18, 68.43, 64.27, 58.23, 55.36, 56.10, 57.25 and 58.18 percentage.

ii. **Along Top Chord:** Along top chord also change in deflections is noticed after posttensioning.

a. Vertical deflections: Deflections are reduced after posttensioning as seen from fourth, fifth and sixth columns when compare to their corresponding values in second column. Vertical deflections at joins U_1, U_2, U_3 and U_4 come down from 26.93 mm, 56.34 mm, 78.87 mm and 89.03 mm to:
i. 21.01 mm, 47.35 mm, 68.28 mm and 78.19 mm respectively for h= 2 m, which lead to a percentage reduction of 21.98, 15.96, 13.43 and 12.18.
ii. 18.86 mm, 43.13 mm, 63.26 mm and 72.90 mm respectively for h= 4 m, which lead to a percentage reduction of 29.97, 23.45, 19.79 and 18.12.

iii. 17.04 mm, 38.91 mm, 58.26 mm and 67.64 mm respectively for h= 6 m, which lead to a percentage reduction of 36.72, 30.94, 26.13 and 24.03.

b. Horizontal Deflections: From Table 2, it is noticed that, the horizontal deflections along the top chord joints are maximum at joint U_1 (located at 6 m horizontal distance from left support), and at all the remaining joints, the horizontal deflections are decreasing with the increase in horizontal distance from left support. Decrease in deflections is noticed and at some joints, nature of deflection also has been changed after posttensioning. At joints U_1, U_2, U_3, U_4, U_5, U_6 and U_7 of truss before posttensioning they are 24.84 mm, 21.14 mm, 16.53 mm, 11.61 mm, 6.69 mm, 2.07 mm and -1.62mm respectively. The effect of posttensioning is to alter these values respectively to:
i. 18.62 mm, 15.25 mm, 10.94 mm, 6.34 mm, 1.73 mm, -2.58 mm and -5.95 mm for h=2 m.
ii. 16.81 mm, 13.77 mm, 9.80 mm, 5.52 mm, 1.25 mm, -2.72 mm, and -5.77mm for h=4 m.
iii. 15.15 mm, 12.44 mm, 8.80 mm, 4.85 mm, 0.91 mm, -2.73 mm, and -5.45 mm for h=6 m.

Ravindra and Nagaraja (2013) considered the determinate Pratt truss with the same geometry and loading as we considered in the present analytical work and studied analytically the effect of internal posttensioning on deflections in which the tendon is placed along the bottom chord of the truss. Even after internal posttensioning also, along both the bottom and top chord the vertical deflections at all the joints are reduced. Along the bottom chord joints L_1, L_2, L_3 and L_4 the percentage reduction in vertical deflections is 10.07, 9.37, 7.55 and 6.51 respectively, and along the top chord joints U_1, U_2, U_3 and U_4 the percentage reduction in vertical deflections is 12.29, 8.82, 7.35 and 6.51 respectively. The percentage reduction in horizontal deflections along the bottom chord joints L_1, L_2, L_3, L_4, L_5, L_6 and L_7 is 52.02, 52.27, 42.12, 35.66, 32.66, 32.26, 34.13 and 35.66 respectively. Comparing the reduction in deflections after external posttensioning from the present study with the reductions of deflections after internal posttensioning available in the literature (Ravindra and Nagaraja, 2013), it is seen that the percentage reduction in deflections is more in external posttensioning than in internal posttensioning. Hence efficacy of external posttensioning is more than internal posttensioning in reducing deflections.

V. CONCLUSIONS

The following are the major conclusions drawn from the present analytical investigation.
- Vertical deflections along both the top and bottom chords were reduced after external posttensioning.
- As the distance between the tendon ends and the joint is increased the percentage reduction in vertical deflections after posttensioning was decreased.
- Both the bottom and top chord horizontal deflections were also reduced after external posttensioning.
- The vertical distance between the tendon and the bottom chord (h) is having influence on both the vertical and horizontal deflections: as h is increased the percentage reduction in deflections also increased.
- Percentage reduction in deflections after external posttensioning was more than their respective reductions after internal posttensioning. Hence efficacy of external posttensioning is more than internal posttensioning in reducing deflections.

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