DRAFT - Situating Standards-Based Mathematics in Culturally and Linguistically Familiar Contexts: Mathematical Thinking in Spanish-Speaking First Graders

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Abstract

An exploration of young Spanish-speaking Mexican immigrant students' mathematical problem solving in bilingual classrooms shows students' ability to successfully engage in rich problem-solving tasks as required by the Equity Principle in the National Council of Teachers of Mathematics standards (NCTM, 2000). This study was conducted during students' first grade year and guided by sociocultural theory that believes children have the psychological and linguistic tools they need even at a young age to make sense of mathematics. Researchers used problems based in Cognitively Guided Instruction (Carpenter, et al., 1999) to qualitatively examine students' problem solving and communication. Findings demonstrate students' shifts toward learning mathematics with understanding and their increasingly sophisticated mathematical discourse in Spanish.

Introduction

Problem Statement

As young children move from spontaneous home environments into classrooms, their informal numeric activities are replaced by structured mathematical tasks. When the tasks are built around word problems located in familiar contexts, students have the opportunity to make connections between what they know informally about numbers and the formal mathematical concepts they are learning. The aim of this research is to probe deeper into student thinking when they are engaged in problem-solving tasks. This research examines how first grade Spanish-speaking students communicate their mathematical thinking in a Spanish language learning environment.

Significance of the Study

Many of today's mathematics curricula are linked closely with a reform movement based on the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 2000). These standards *begin* with the Equity Principle that requires *all* students to have opportunities to engage in quality classroom learning experiences to help them develop a deeper understanding of mathematics (Boaler, 2002b; NCTM, 2000). Unfortunately, equal access to quality mathematics experiences is not a reality for many students who live in poverty, come from immigrant communities, and/or speak a dialect or native language other than standard English (Jordan, Kaplan, Oláh, & Locuniak, 2006; Kamii, Rummelsburg, & Kari, 2005; National Assessment of Educational Progress [NAEP], 2005; Pappas, Ginsburg, & Jiang. 2002; Moschkovich, 2002; Lubienski, 2000; Ortiz-Franco, 1999; Khisty, 1997). When these students under-perform, they are placed in classrooms that stress drill and practice at the expense of more conceptually challenging tasks, further removing them from meaningful mathematics experiences (Kamii et al., 2005). Because research has found that many of these same children enter kindergarten and first grade with limited counting skills, limited experience in explaining their thinking, and reduced standard English vocabulary (Jordan et al., 2006; Lubienski, 2000; Pappas et al., 2003), the notion that they need to build both counting and language skills first has persisted.

While some research has shown that indeed Spanish-speaking Latino students and students from poor communities can fully engage in complex mathematical processes and that their general counting and discursive skills can improve through problem solving activities (Kamii et al., 2005; Villaseñor & Kepner, 1993; Turner, Celedón-Pattichis, Marshall & Tennison, in press), very little research has shown exactly how Spanish-speaking immigrant students successfully engage in these tasks in their native language. This study comes at a time when a perceived achievement gap is being blamed on Spanish language instruction, specifically bilingual education (Escamilla, Chavez, & Vigil, 2005; Collier & Thomas, 2004). For this reason, it is important to show that Spanish-speaking immigrant children have enormous potential for mathematical achievement when they are given access to high quality teaching and learning activities in their native language.

Theoretical Framework and Review of the Literature

The research presented in this paper focuses on the exploration of young Spanish-speaking students' mathematical thinking through what they do and what they say during problem solving in bilingual classrooms. The emphasis on problem solving and communication comes from reform mathematics and is influenced by sociocultural theory. Particular emphasis is given to sociocultural theory because of its argument for a strong relationship between language and cognitive development (John-Steiner & Mahn, 1996). This research is also informed by social constructivism in mathematics education and the research done by Cobb and Yackel (1996) and McClain and Cobb (2001) that explored the development of mathematical discourse and the use of representations as tools for learning in elementary classrooms. It is beyond the scope of this paper to argue for bilingual education. However, the importance of bilingual education, and especially early native language mathematics instruction, is an assumption that underlies this research and is supported by sociocultural theory.

Reform Mathematics

The reform movement in mathematics education has resulted in a shift from the teaching and learning of isolated skills and procedures to a focus on conceptual development and learning with understanding (NCTM, 2000; Hiebert & Carpenter, 1992). Learning with understanding engages students more deeply in the processes of mathematics while solving complex problems. Reform-minded researchers and educators, working together, were instrumental in publishing the influential first edition of the mathematics standards in 1989, followed by the revised *Principles and Standards for School Mathematics* (NCTM) in 2000 where the important areas of content knowledge and process skills for K-12 mathematics education are laid out.

Sociocultural Theory and Constructivism

Reform mathematics has roots in the both the cognitive and social sciences. Constructivism, which grew out of cognitive science, argues that children learn by actively and independently constructing knowledge in their interactions with the surrounding physical environment (Kamii et al., 2005; Kato, Kamii, Ozak & Nagahiro, 2002). From this point of view, children's psychological orientations and cognitive development move from an internal focus to an external focus as they become increasingly more social. Constructivism had a strong influence on the development of the NCTM standards and their emphasis on students as active learners. However, as will be explained in the next few paragraphs, it is scoiocultural theory that makes the important connections among cognitive development, language, and students' cultural and historical experiences.

Sociocultural theory is based on the work of Vygotsky (Kozulin, 1990) and argues that children's psychological and cognitive development move in the opposite direction from constructivism, from the social to the individual, and learning occurs when external social experiences are internalized (Nasir & Hand, 2006; John-Steiner & Mahn, 1996). Sociocultural theory supports the notion that humans begin life as intrinsically social beings. All our early thoughts and experiences are incorporated into our innate cognitive structures by the communication we have with our caregivers. As children grow, these early dialogues are internalized and become the basis for understanding new information. In this way, cognitive development is mediated by the linguistic interactions we have with our caregivers and community, and new information is continually interacting with the old to expand knowledge and refine its structure. This interaction can be applied to formal learning. When children enter school they are introduced to what Vygotsky called scientific or formal concepts. These formal concepts, broken down into the disciplines that form the basis for modern schooling, help children order the spontaneous or informal concepts they bring with them into the classroom.

In formal mathematics education from a sociocultural perspective, it is critical that students be allowed to connect the spontaneous concepts rooted in their earliest social and cultural interactions with the formal concepts presented in the classroom and use the spontaneous concepts as tools to understand what is being taught in school. These early psychological tools are structured linguistically and were built through social interactions. It is critically important to recognize that familiar language patterns have mediated the growth of these psychological structures. For a cognitively demanding area such as mathematics, there is no doubt that new information is best learned in the language of the home if the formal concepts are to give structure to the spontaneous concepts developed at home (Van der Veer & Valsiner, 1991; Nelson, 1991).

There is an unresolved tension between constructivists and sociocultural theorists in mathematics education research, where on the one hand the individual activity of the child is emphasized and on the other, the social dynamics of the learning environment. Cobb and Yackel (1996) attempt to refocus this conversation by emphasizing social constructivism, the activity of the individual within social contexts. In the description of their "emergent" theory they say, "Learning is a constructive process that occurs while participating in and contributing to the practices of the local community" (p. 220). Socioculuralists counter that a more nuanced understanding of sociocultural theory accounts of the variations of the individual within the environment (Van der Veer & Valsiner, 1994) through individual transformation of knowledge.

Research on NCTM Process Standards in School Mathematics

Constructivists, sociocultural theorists, and emergent theorists all agree that *active* engagement is essential for learning. When students actively engage in mathematics, they work on complex tasks using concrete tools, their own existing knowledge of the world, and use each other as resources to make sense of the mathematics (Cobb, Boufi, McClain, & Whitenack, 1997; Carpenter, Fennema, & Franke, 1996). Active engagement lies at the heart of the five process standards outlined in the NCTM *Standards* publication (2000). These processes are problem solving, communication, representations, reasoning, and connections. The first three have particular significance for this study and I will discuss them specifically. However, all five processes are interconnected, and even though I do not discuss reasoning and connections explicitly, reasoning permeates all problem solving, and the very nature of solving relevant problems makes connections between informal and formal knowledge possible.

Problem Solving

Problems that draw on students' knowledge of the world and provide mathematical contexts for real life situations help students build connections among mathematical ideas. These problems provide the link between students' informal or spontaneous knowledge and students' formal structuring of this knowledge into the discipline of mathematics (Cobb et al., 1997; Carpenter, Fennema & Franke, 1996; Kozulin, 1990). Constructivist research emphasizes the importance of problem solving to facilitate cognitive development. In their study with first graders from low socioeconomic backgrounds, Kamii et al. (2005) promoted the use of problem solving games to help develop students' abilities to classify objects, create series, and uncover spatial and temporal relationships.

Carpenter et al. (1996) have done extensive research with young children and problem solving and have developed a framework for understanding how children approach problems. Drawing on both constructivist and sociocultural theories, they argue that even young children have a wide range of experiences and knowledge to help them construct their own solutions to word problems in a supportive social learning environment. As a framework to guide teachers, Cognitively Guided Instruction (Carpenter, Fennema, Franke, Levi & Empson, 1999) lays out basic problem types and students' strategies. CGI problem types were used as a basis for student activities in this research.

Oral Communication

A second process critical to teaching and learning for understanding is oral communication. When students learn to describe their thinking and justify their answers, the result is a greater understanding of their own thinking and reasoning (NCTM 2000; Siegler, 2000; Cobb et al., 1997). Additionally, when students discuss their solutions out loud, other students have access to their thinking and can borrow these strategies for their own problem solving. From a sociocultural perspective, the source of learning lies in social contexts where students' internal understanding and structuring of ideas is based on the external structures they encounter through social interactions (Kieran, 2001; Lerman, 2001; O'Conner, 1998; Sfard, 2001; Van Oers, 2001). The continuous interplay among each student's personal characteristics, past experiences, and the social and cultural contexts in which learning is unfolding, creates a dynamic of learning that results in the internalization and transformation of new ideas from the social to the individual (Van der Veer & Valsiner, 1994).

As students struggle to explain their reasoning, they also struggle to organize and clarify their thinking, thus supporting the underlying premise of sociocultural theory of the relationship between thought and language (John-Steiner & Mahn, 1996). When students have a more organized and clearer idea of their own thinking they can then make connections among ideas and increased their metacognitive awareness (Hiebert & Carpenter, 1992; NCTM, 2000). Researchers in Finland and the U. S. have found a positive correlation between metacognition in primary age children and the rate of their mathematical development, particularly students' ability to solve word problems (Kamii et al., 2005; Aunola, Leskinene, Lerkkanan & Nurmi, 2004). In addition, research in psycholinguistics asserts that language acquisition in humans links two distinct domains found in intelligent animals and preverbal humans, the exact recognition of small numbers and the distinction between small and large sets, leading to humans' ability to solve exact, large-number arithmetic problems (Spelke & Tsivkin, 2001).

Representations

According to the NCTM *Standards*, "Representing ideas and connecting the representations to mathematics lies at the heart of understanding mathematics" (NCTM, 2000, p. 136); therefore, students should have multiple opportunities to create a variety of representations about mathematical situations. Students gain fresh insight into their own thinking and have greater flexibility of expression when they are encouraged to create drawings of word problems and their solutions (Kamii et al., 2005; Kendrick & McKay; 2004). Greeno and Hall (1997) found that young children often have partial understanding of problems, and through their drawings develop a greater awareness of the contexts and dynamics of situations. Drawing also provides an external structure to help students internalize a mathematical situation, leading to greater understanding of a problem (Hiebert and Carpenter, 1992). In addition, pictorial representations mark a transitional step between concrete and abstract mathematical thinking (Kammi et al., 2005; Outhred & Sardelich, 2005; Kato et al., 2002; Fennell & Rowan, 2001).

Research Questions

In my research I ask, how do first grade Spanish-speaking students communicate their mathematical thinking in their native language during problem solving? What strategies do they use to attack problems? How do they represent the problem situations in drawings? How do they talk about the mathematics of the problems? Can they make connections among mathematical ideas in their solutions?

This research is based on the assumption that using familiar contexts to engage students gives them the opportunity to explore mathematics from their own linguistic and psychological perspectives. The research also assumes that students' mathematical thinking will be more transparent when the confounding effects of language and culture are reduced.

Description of the Case

Setting

La Joya Elementary School[2] is located in a large urban area in the Southwestern United States and serves children from predominantly Mexican immigrant and low-income families. La Joya responds to the educational needs of its 86% Spanish-speaking student population with its kindergarten through fifth grade bilingual program. Data for this study was collected in two first grade bilingual classrooms at La Joya where mathematics instruction was conducted exclusively in Spanish.

La Joya promotes a maintenance Spanish-English bilingual program with the goal of long-term Spanish language development for native speakers. Both first grade classroom teachers in this study are themselves native Spanish speakers and conduct their math lessons in Spanish. Naomi is from Peru and Helena is from Puerto Rico. Their classrooms have many visual aids for student mathematics learning including a number line around the top of the walls, a 100 chart, geometric shapes, and math counters and cubes. Naomi has several homemade materials for teaching mathematics such as magnetized base ten blocks and coins attached to cookies sheets. Naomi's room reflects the emphasis she places on multiple tools to facilitate mathematics learning. Helena's

[2] La Joya is a pseudonym.

room has motivational posters on the walls, posters of words to say "hello" in many languages, and a pennant picture of her whole class. This room reflects a strong multicultural perspective. In both classrooms students sit together in groups of desks or at tables and they work in groups.

Participants

The eight first graders in this study were all participants in research during their kindergarten year where CGI was used as a framework to teach problem solving (see Turner et al. in press for a complete description of this study). Six of the students are female and two are male. The eight students were chosen for this study because of their experiences the previous year and because their teachers, Naomi and Helena, expressed interest in developing students' problem solving abilities. All eight students speak Spanish as a first language. One student, Omar, is dominant in English, and two, Ana and Gerardo, are bilingual in Spanish and English. The other five students, Dolores, Gina, Briza, Yolanda and Jenna, are English Language Learners. These students represent a range of mathematical ability from low to high. All of the eight students have been enthusiastic research participants and are experienced with our methods of data collection, including videotaping while solving problems and explaining their thinking. Unfortunately, Dolores, the student who is at the most emergent stage in her mathematics development, chose not to participate in the May 2007 interviews and data on her is not included in this paper.

The Research Team

As a white, middle class English speaker and Spanish learner I am an outsider to this bilingual elementary school environment. I first became acquainted with the student participants in this study when they were in kindergarten. The CEMELA research team conducted a CGI investigation in this classroom during the 2005-2006 school year (Turner, et al., in press). The time I spent collecting data and observing mathematical teaching and learning in the Spanish-language kindergarten classroom left me with some clear assumptions about the power of problem solving to support students' mathematical development. I saw students make sense of the mathematics and gain confidence in their solution strategies by explaining and justifying their answers. I wanted this momentum to continue in the first grade and therefore designed the study described in this paper. Other members of the research team include the post-doctoral fellow in the CEMELA program and a faculty advisor. Both of these individuals are Latinas and are native Spanish speakers.

Design of the Study

Methodological Framework

This is an exploratory study of students' mathematical thinking based in grounded theory where the purpose of the research is to develop a theory "closely related to the context of the phenomenon being studied" (Creswell, 1998, p. 56). In this case, the phenomenon studied is students' mathematical learning through problem solving in the first grade. To promote the phenomenon, I suggested weekly math lessons based on CGI problems. A typical lesson in each classroom began with an introduction to the problem type in a whole group setting by the teacher. After the introduction to the lesson, teachers and researchers continued the work facilitating small groups. Facilitators asked students to draw their problem solutions and explain their thinking. Data collected through classroom observation fieldnotes, video taped student interviews, and student work was analyzed to develop themes that characterize the way the participants communicated their thinking about mathematics.

Definitions and Limitations

The mathematical word problems given to students were all based on problem types defined by Carpenter et al. (1999) in their teacher handbook on Cognitively Guided Instruction. These problems were carefully developed to reflect contexts that would be familiar to the focal students. Problem types included join, separate, compare, part-part-whole, multiplication, partitive division, measurement division, and multi step problems. Student communication was broken down into three main types: oral, written and pictorial.

Limitations of this study come from my level of Spanish speaking ability, student health and family issues, and the dynamics of school life beyond my control. Although I tried to repeat what students said during interviews and asked them for clarification, it is possible I may have missed some of the subtleties of their oral language. Because I am not a fluent Spanish speaker, all translations of Spanish used by students were checked by the other members of the research team. In addition, student health and family issues may have impacted on their learning while in school or caused them to be absent, thereby affecting their data. Special school and classroom activities always took precedence over my research and at times interfered or prevented data collection.

Validity

I am only *assuming* students are communicating their thinking in what they say, draw, and write. Some students may believe that in order to be successful they should solve problems the teacher's way. Students' interpretations of what the teacher wants may influence their thinking. To verify the conclusions developed from this data, all themes were discussed with other members of the research team.

I also *assumed* that the problem-solving experiences we provided reflected students' cultural and linguistic experiences. Changing the items and names in the problems did not change the basic structure of the problems that came out of mainstream mathematics education. For cultural and linguistic relevance, I relied on the teachers and their knowledge of students' lives to modify the problems I suggested.

Methods of Data Collection

Overview and outline of time spent in the field

From October 2007 until May 2007, at least two of the three members of the research team visited each classroom weekly for approximately one hour. These visits encompassed the entire math lesson and were followed by debriefings with the teacher. After these visits I created a computer file of field notes about the overall lesson and the activity of each of the focal students. Individual student interviews based on CGI problem types were conducted in November 2006 and the end of April 2007. In addition to regular classroom visits, I conducted video taped interviews in Spanish with the eight focal students in pairs on February 6, 2007, using one of the more advanced problem types that we had not covered in class. On March 6, 7 and 21, 2007, I conducted unrecorded individual interviews in Spanish with each of the eight focal students using a variety of comparison problems. I took field notes during all the interviews and typed them up in detail afterwards.

Methods of Analysis

The first round of analysis on this data is considered preliminary and further analysis will be conducted that includes a more detailed examination of students' use of language in relation to problems solving. From this preliminary analysis, trends present in classroom field notes, the transcription and interview files, students' drawings and their writing on these drawings were developed by "selective open coding" (Emerson et al. 1995, p. 155), that isolated the actions and products of the eight focal students. During open coding, I created analytic memos to record the ideas I had at the time. After open coding, I moved to "axial coding" (Creswell, 1998, p. 57), to consolidate codes, create categories and develop themes. As suggested by Emerson et al., (1995, p. 147), I continued to concentrate on what the students were doing. When I next moved to "selective coding" (Creswell, 1998, p. 57), the development of subsequent themes was influenced by the theoretical connections outlined in my literature review (Emerson et al., p. 164).

Organization of data

I created an extensive classroom behaviors and artifacts table for every student. These tables contain specific information about what focal students did during each day's lesson, the problems they explored, their drawings, writings and equations. In a separate file, I created a listing of codes for each focal student based on their pre-assessment interviews in, the pairs' interviews, and their individual interviews. By comparing the table for each student with the codes from her or his individual work, I was able to isolate patterns for individuals and among students.

Coding scheme

Codes were based on behaviors and products. These were categorized by what focal students said and did during whole group problem solving, small group problem solving, and pairs and individual interviews. What they drew and wrote about the problem were included in these categories. Open codes were organized into categories and these categories in turn were grouped into seven axial codes.

Themes

The following five themes grew out of a closer examination of axial and selective codes.

1) Students are learning mathematics with understanding as reflected in their more flexible thinking and their use of multiple concepts and strategies to solve and represent problems (Hiebert & Carpenter, 1992). Students' increasing sense of number is demonstrated by facility with base ten thinking (NCTM, 2000).

2) Students are developing mathematical discourse in their native language, Spanish, and their second language, English. An exploration of students' language gives a window into their thinking (Lerman, 2001; Sfard, 2001). Student explanations of their solution strategies help them organize and consolidate their thinking, an important goal in standards-based mathematics teaching and learning (NCTM, 2000).

3) Students are developing an impressive amount of conceptual knowledge, but the depth of this knowledge is relatively superficial and the links they are making between concepts are still weak. Some of the more advanced concepts appear to be partial (Greeno & Hall, 1997) in that students cannot apply them consistently to all modes of expression. The data show that students are still in the process of developing the strongly connected network of mathematical understanding promoted by Hiebert and Carpenter (1992).

4) Students' successful use of strategies with smaller numbers does not translate to larger numbers, and therefore, working with large numbers presents a different type of challenge to young students. Even though these students can count large numbers and write them correctly, they have difficulty applying what they know about small-number calculations to large numbers. This is consistent with research from psycholinguistics that posits small number accuracy and large number estimation lie in two different cognitive domains (Spelke & Tsivkin, 2001).

5) Students' pictorial representations give insight into how they are thinking about problem situations, a finding that became apparent when students were introduced to certain types of compare problems. (Kendrick & McKay, 2004)

Interpretation of Data

While students' quantitative success in problem solving is not the focus of this study, it gives valuable background information on students' overall mathematical development and the effectiveness of problem solving activities. By the end of the school year seven of the eight focal students were able to successfully solve multi-step, multiplication, and partitive and measurement division problems. Most were able to solve start and change unknown problems (Carpenter et al., 1999) even though they had very limited experience with these problem types in the classroom. They were able to represent problems in various ways, extend concepts to new situations, and make connections among ideas. Classroom observations confirmed that compare problems were the most challenging type for this age group.

The following table shows students' advanced strategy use and percent of correct answers from the November 2006 interviews and the May 2007 interviews. When the focal students began CGI problem solving in kindergarten, they used direct modeling of the problem situation as the most common strategy to find their solutions. This is consistent with the CGI literature (Carpenter et al., 1993), which says students being with the most basic direct modeling and then advance toward counting, recalled facts, and derived facts. When students directly model a problem, they represent all numbers with, for example, manipulatives, fingers, or in drawings. Students' use of the other more advanced strategies indicates an increasing sense of number (Carpenter et al., 1999).

As can be see in the table below by the low percentage of advanced strategies, many students continued to use direct modeling in November of first grade. However, by the end of first grade, all students had progressed to using at least some advanced strategies. The combination of advanced strategies and correct answers gives a fuller picture of their mathematical development.

Table 1 – First Grade Data
Comparison of Advanced Strategies to Correct Answers

Student	**Advanced Strategies Nov. 2006**	**Correct Answers Nov. 2006**	**Advanced Strategies May 2007**	**Correct Answers May 2007**
Brisa	0%	92%	33%	89%
Ana	0%	86%	22%	96%
Yolanda	71%	71%	75%	92%
Omar	64%	86%	81%	86%
Jenna	0%	63%	36%	86%
Gina	36%	93%	43%	100%
Gerardo	31%	81%	44%	90%
Averages	**29%**	**82%**	**48%**	**91%**

Overall, students were very successful problem solvers by the end of first grade, based on the CGI problem types used in the individual interviews. Using this understanding of their problem solving ability, the following themes were developed.

Theme 1: Students are learning mathematics with understanding.

Mathematics learned with understanding is reflected in students' abilities to tackle challenging problems by applying what they know in new and different ways (Hiebert & Carpenter, 1992). Students with mathematical understanding show a flexibility of thinking in how they approach problems, how they recognize and use patterns and relationships, and how they express their answers. Below Gina, Omar, Yolanda and Ana show their increasing understanding and flexibility.

Gina Applies Base Ten Thinking

Fieldnotes, 3/6/07 (individual student interviews in Spanish):

First problem: Gina had 67 pieces of gum and her friend Reinata had 76. Who had more? How many more?

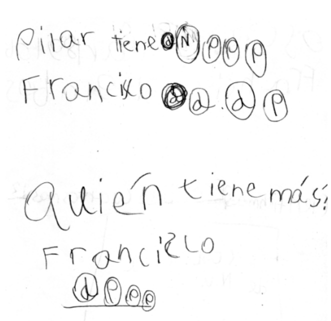
Gina wrote the numbers on her paper with initials for the names of the two children in the problem. She thought for a few minutes and then said "nueve [9]". I asked her why 9 was the answer and she said because if it was 10 then the second number would be 77 so it had to be 9. I repeated her answer (in Spanish) to make sure I heard it correctly and she agreed. When I asked her about who had fewer pieces of gum and how many fewer, she knew right away that if her friend Reinata had 9 more she would have 9 fewer.

In the above example, Gina dealt with large numbers by applying what she knows about the patterns and relationships in our number system. After writing 67 and 76 on her paper, she was able to recognize a pattern and realized that if she added 10 to 67 she would have 77. Hence, her answer had to be nine.

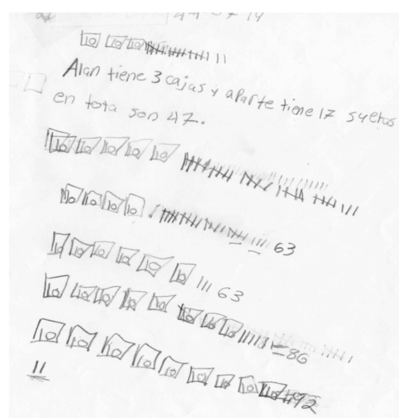
Omar, Yolanda, and Ana Use Flexible Notation

In Omar's drawing below he uses a coin notation to solve a comparison problem between 31 balloons and 18 balloons. He is using "d" for dime (10), "N" for nickel (5), and "P" for penny (1). He writes 18 as "dNPPP" and 31 as "dddP". He even expresses his answer to how many more in this way, writing 13 as "dPPP."

In Yolanda and Ana's pictures that follow we see this same flexibility in expression. The girls were solving multiplication problems to reinforce base ten thinking. For example, the first question asked how many crayons are there in three boxes of crayons with ten in each box, and 17 single crayons? Yolanda solved the problems with boxes of ten and tally marks. When she counted the groups for her answer, she first counted by tens, then by fives, and finally by ones. She wrote, "Alan tiene 3 cajas y aparte tiene 17 suetos en tota son 47." [Alan has 3 boxes and in addition he has 17 singles in total there are 47.][3] When the problem asked how many crayons in four boxes of ten and 23 single crayons, Yolanda first drew four ten-boxes and 23 singles then redrew the problem with six ten-boxes and 3 singles. Ana drew ten boxes as well to express the answers for these problems, and after the first problem she was able to just listen to the numbers while the problem was being given and draw the all ten boxes initially. Similar to Omar's method, she also expressed the first answer, 47, using coin notation.

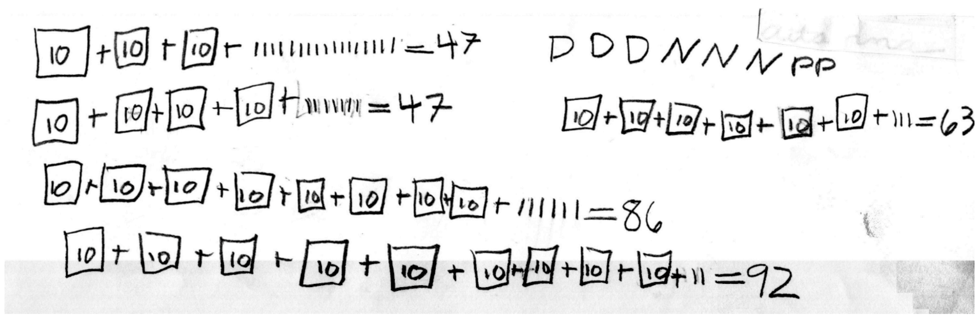


Omar's comparison problem solution



Yolanda groups by 10s and 5s

3 All translations from Spanish to English were done by the author.



Ana uses coin notation and groups of tens

Although tally marks, coin notation, and counting by tens has been encouraged in their Everyday Mathematics curriculum, Omar, Yolanda, and Ana are applying these concepts to new situations. These examples of their work show they were able to use ideas learned in their regular classroom and apply them to the novel problem situations we introduced during our visits.

Theme 2: Students are developing mathematical discourse in their first language, Spanish, and second language, English.

Below are examples of student explanations from the end of the year interviews conducted in May 2007. Because Brisa, Gina, and Yolanda are Spanish dominant, their interviews were conducted in Spanish. Gerardo is a balanced bilingual and chose English as the language for the interview. These interviews have not been fully analyzed using discourse analysis; however, the preliminary findings demonstrated in the examples below show that students are able to explain their thinking and problem solving strategies in relation to the models and diagrams they use for problem solving. When their calculations relate to the details of the story, this demonstrates their mathematical reasoning and the sense they are making of the problems (Cortina, 2006). The CGI types below include a multi-step multiplication problem, an addition/join problem with one of the addends (the change) unknown, a simple subtraction problem, and a subtraction problem with the subtrahend (the change) unknown.

Brisa solves 7 x 10 + 6: Seven bags of marbles with ten marbles in each bag and six single marbles.

In this problem Brisa uses base ten thinking skills to recognize that she can simply count by tens for the bags and then count by ones to add in the single marbles. In her drawing she included all the marbles in the bags, not as an aid to problem solving, but possibly as an aid to visualization. Her reasoning is clear in her explanation as she relates her problem solving strategy to the structure of the story. Because there were ten marbles in each bag, she counted the bags by ten and then counted in the six single marbles.

> Interviewer: Este problema es de canicas y de bolsitas. Pero en este caso, otra vez, hay 7 bolsitas, pero en cada bolsita hay 10 canicas. Y, también, hay 6 canicas sueltas. Siete bolsitas, pero hay 10 en cada bolsita y hay 6 canicas sueltas. ¿Cuántos hay en total? ((Brisa draws bags again and puts 10 dots in each.)) Hay muchas.
> *This problem is about marbles and bags. But in this case, once again, there are 7 bags, but in each bag there are 10 marbles. And also, there are 6 single marbles. Seven bags, but there are 10 in each bag and there are 6 single marbles. How many in all? There are a lot.*
>
> Brisa: …y 6 sueltas. ((She draws 6 circles.)) ¿Cuántas en total?
> *…and 6 singles. How many in all?*
>
> Interviewer: Sí, en total. ((Brisa counts each bag as though counting by ones. Then she counts the circles.))
> *Yes, in all.*
>
> Brisa: Setenta y seis
> *Seventy six*
>
> Interviewer: Muy bien. Son muchas. ¿Cómo supiste?
> *Very good. There are a lot. How did you know?*
>
> Brisa: Porque puse diez…había…Ashley tenía 7 bolsitas de canicas, en cada bolsita tenía 10. Y conté de 10 en 10 hasta 70 y habían 6 sueltas y le [sic] conté 70 hasta 76.

Because I put 10…there were…Ashley had 7 bags of marbles, in each bag she had 10. And I counted by 10s up to 70 and there were 6 singles and I counted from 70 up to 76.

Gina solves a Join, Change Unknown Problem: $9 + ? = $18:

Gina used interlocking cubes in this problem to find out how many more dollars Marian needs to buy a doll that costs $18 dollars when she only has $9. As Gina is manipulating the cubes to make 18 with ten and 8, she sees that she can remove one cube from the 10 rod, add it to the 8 rod and then she has nine and nine. From this model it is obvious to Gina that if Marian has nine dollars, she needs nine more to make the 18 she needs to buy the doll. Gina's explanation is not clear without the gestures associated with manipulating the cubes, nevertheless, she is developing the ability to explain the reasoning she employs to solve the problem.

Interviewer: OK Let's see, Marian. Marian quiere comprar un juguete. ¿Qué tipo de juguete? ¿Una peluche? ((shakes head)) No. Muñeca? ((nods)) OK, OK. Marian quiere comprar una muñeca que cuesta 18 dólares. Ella tiene 9 dólares. ¿Cuántos dólares mas necesita Marian para poder comprar la muñeca? ((Gina builds a rod of 8 cubes)). ¿Qué estas pensando?
OK, Let's see, Marian. Marian wants to buy a toy. What kind of toy? A stuffed animal? No. A doll? OK, OK. Marian wants to buy a doll that costs 18 dollars. She has 9 dollars. How many more dollars does Marian need to be able to buy the doll? What are you thinking?

Gina: …ocho… ((She puts another cube on the rod)) nueve, nueve.
…eight…nine, nine

Interviewer: ¿Por qué, por qué la respuesta es nueve?
Why, why is the answer nine?

Gina: Porque ella tiene 9… como si le ponen con los demás es 10, y le van a quedar ocho, y como ella tienen 9 le quité uno y se lo puse aquí, y ahora le quedan 9. ((She has 18. She took one from the 10 to make a rod of 8 then saw that she had 9 on the other rod and know that 9 and 9 was 18.))
Because she has 9…because if they put it with the rest it is 10, and they are left with eight, and because she had 9 and I removed one and put it here, and now she is left with 9.

Yolanda solves a Separate Result Unknown problem: 35 – 15 = ?.

In this example, Yolanda is more focused on manipulating the numbers and employing her knowledge of grouping and counting by fives than reasoning about the problem. Yolanda has demonstrated a sophisticated number sense in past interviews and it appears that she has a clear numerical image in her mind about the number relations involved in this problem. This may be why she does not need to use cubes or paper and pencil to find a solution. She uses her fingers only as an aid in keeping track of her counting. Her explanation focuses on the nature of the number relationships instead of the structure of the problem.

> Interviewer: Aquí es un problema similar, pero voy a cambiar los números un poco. ¿Quieres usar los bloques? ((indicates base ten blocks)) o cualquiera. Tu papá tenía 35 galletas, un gran pila, un gran montón de galletas, y tenía mucho hambre y se comió 15.
> *Here is a similar problem, but I am going to change the numbers a little. You want to use blocks? Or whatever. You father had 35 cookies, a big pile, a big mountain of cookies, and he was very hungry and he ate 15.*
>
> Yolanda: Treinta y cinco?
> *Thirty five?*
>
> Interviewer: Treinta y cinco y se comió 15. ¿Quieres usar los bloques aquí ((base ten blocks)) o tu mente?
> *Thirty five and he ate 15. You want to use the blocks here? Or your mind?*
>
> Yolanda: ((Shakes her head no. She puts her fingers in front of her, looks up, and is silently using her fingers to calculate in her head)).
>
> Interviewer: ¿Cuántos le quedaron?
> *How many does he have left?*
>
> Yolanda: ((Thinks while holding up her fingers)) veinte?
> *Twenty?*
>
> Interviewer: ¿Cómo supiste tan rapido veinte? Sin…
> *How did you know twenty so quickly, without…*

Yolanda: Usé los dedos y puse 35 menos 10 es treinta…oh ((shakes her head)) menos 5 es treinta y menos otra cinco es veinte cinco y menos otro cinco es veinte.
I used my fingers and put 35 minus 10 is 30…oh minus 5 is 30 and minus another 5 is 25 and minus another 5 is 20.

Gerardo solves a Separate Change Unknown problem, 12 – ? = 5.

Finally, Gerardo makes a clear connection between his diagram of the structure of the problem, which he uses for solving the problem, and the way he explains his solution strategy. Using the diagram as an aid to explanation, he is able to recreate his thinking process and justify his answer by connecting his solution strategy into the actions of the problem.

Interviewer: This is my problem here. ((G smiles)) We will let you tell a problem later. OK, Monkey Boy had 12 marbles and then he gave you some of his marbles and he had 5 left.

Gerardo: How much?

Interviewer: How many did he give you? He had 12 marbles to start and then he gave you some. ((Gerardo starts drawing lines across top of paper.)) He had 5 left. After he gave you some… ((Gerardo counts in whispers from 1 to about 12)) How many did he give you? ((Gerardo counts the lines on his paper from left to right, makes a dividing line, then counts the lines to the right.))

Gerardo: Seven

Interviewer: Tell me about your thinking and tell me if you're sure it's the right answer, you think it's the right answer? Tell me about it.

Gerardo: Um, I think it's the right answer.

Interviewer: Why?

Gerardo: Because when I put twelve, um, I started counting them and where I got to 5, right here where I started, ((indicating drawing), I put a line and then I keeped going until over here so I could know much did he give me and then I started counting the ones that he give me.

Theme 3: Students are developing impressive conceptual knowledge, but the depth of their knowledge is superficial and/or partial, and the links they are making between concepts are still weak.

The first two observations are of Yolanda, the most abstract thinker and conceptually mature of the focal students. In this first excerpt, she shows the rest of the class how to solve a comparison story where the difference between two sets is unknown. In the second excerpt, she solves a comparison problem where the smaller set is unknown and the difference is known.

Yolanda Solves With Ones Counting, But Explains With Tens

Fieldnotes, 1/31/07 (Naomi's class, whole group activity):

Yolanda goes up to the front of the group and retells the comparison story using the problem numbers 20 and 35 as the two known sets. She says that if you count up by 1s from 20 to 35 you get 15. Naomi (the teacher) asks her to show her solution to the rest of the class with magnetized base ten blocks on the white board. Yolanda starts with three base ten rods on the white board and 5 cubes to make 35. Then she removes two rods, removing 20 from the 35. She counts the rod and cubes that are left to get 15, the answer.

Fieldnotes 3/7/07 (Naomi's class, small group activity, author facilitating):

When I changed the first comparison number to 24 and 10 less, Yolanda counted backwards quickly on her fingers by 1s to get 14. When I asked her to explain her answer, she went up to the large 100s chart that hangs on an easel near the front of the class and showed me how starting at 24 and moving directly backwards (in this case directly up) by 10, is 14.

In the above examples, we see that Yolanda is building a network of conceptual understanding (Hiebert & Carpenter, 1992), but there is inconsistency between her solution strategies and explanations. In the first example, Yolanda counted by ones to get the correct answer to the first problem, but then explained her solution with base ten blocks. Even though she was able to use the blocks, it is not clear that she was thinking in groups of ten to solve the problem. She was still relying on counting forward and backward by ones as her primary solution strategy. In the second example, Yolanda once again relied on counting backwards by ones to find the answer, but chose another tool, the 100s chart, to show how she solved the problem.

Gina Represents Two Comparison Problems

The next two fieldnote excerpts are of Gina, a mature and thoughtful mathematical problem solver. Here she is solving comparison problems.

Fieldnotes 2/13/07 (Helena's class, small group activity, author facilitating):

Gina began with confidence on the problem where Fernando had 7 candies and she (Gina) had 4 more than Fernando. She drew a number line with jumps to indicate moving from 7 to 11 and the letters F and G to label which child had which number. Below the number line she drew two row of 7 circles, one on top of the other, a vertical line after the circles, labeled one row "G," and after this row 4 more circles to make 11. She wrote "**yo tengo 11 dulses más que Fernando**." [**I have 11 candies more than Fernando**.] She should have said that she had 11 candies.

(Fieldnotes continued) On the next problem with 15 candies for Fernando and 6 more for Gina, she used the same strategy, starting at 15 and jumping on the number line to 21. She did the same kind of drawing with this one as well, using circles to represent the candies and a vertical line to indicate where she started adding, but the 6 more candies are not clear. Her additional circles do not match her answer. She wrote, **"yo tengo 21 dulses y Frnando tiene 15". [I have 21 candies and Fernando has 15.]** This time she got the wording for the answer correct.

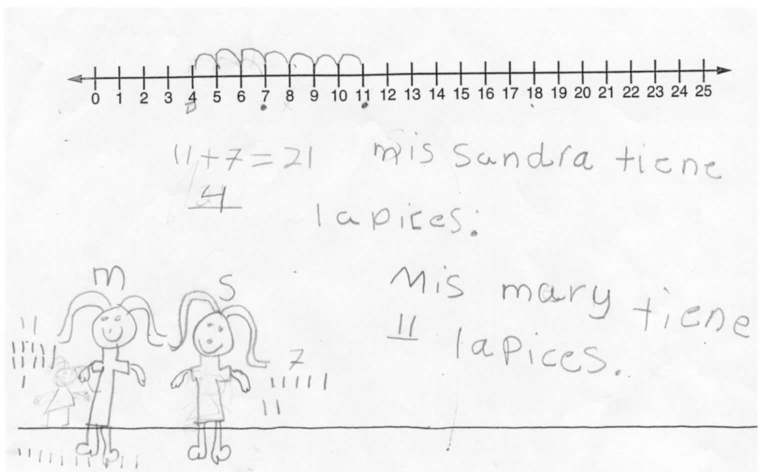
In Gina's first example she drew the correct solution, made a correct model for the problem, but wrote an incorrect "más" [more] into her solution sentence. There is a significant conceptual difference in meaning between saying "I have 11 candies more than Fernando" and saying, "I have 11 candies." However, in her next example she wrote the answer correctly, but made a mistake in her model. Her numeric answer to both problems was correct.

Jenna Works With a Comparison Problem

Jenna is a mature and academically motivated student with medium high mathematics ability. She is still a concrete thinker during problem solving.

Fieldnotes 2/28/07 (Naomi's class, whole group):

The problem is: The author (Mary) has 11 pencils and Sandra has 7 pencils less than Mary. The question is how many pencils does Sandra have? In Jenna's drawing she has put dots on the number line under the 11 and under the 7, but she has jumped back on the number line to the 4, which is the correct answer. Since this was a whole group activity, I don't know if she had help from another student or adult. We were all circulating among the students and they were talking to each other. She has written on her paper: "11 + 7 = 21 mis Sandra tiene <u>4</u> lapices. Mis mary tiene <u>11</u> lapices." [11 + 7 = 21 Miss Sandra has 4 pencils. Miss Mary has 11 pencils.] She has draw two stick figures, M with 11 pencils and S with 7 pencils, where it should have been M with 11 pencils and S with **4 pencils.** I think she was trying to show how many pencils M and S have together with her equation.



Jenna's drawing of the pencil comparison problem.

Jenna's drawing, similar to the example from Gina, shows that the sense students are making of the problems appears to be only *partial*. Jenna has written the correct answer, but this is not represented in her drawing. She has shown the answer on the number line, but her equation is incorrect to answer either the question of how many pencils Sandra has or how many pencils M and S have together.

Theme 4: The link between concepts, strategies, and successful work with small numbers does not translate to larger numbers.

Yolanda Struggles to Find a Strategy for Large Numbers

Fieldnotes, 3/7/07 (individual interview with Yolanda):

Problem: Yolanda has 94 pencils and Ana has 10 less. How many pencils does Ana have? Yolanda began by trying to count backwards with her fingers but she gave up on this rather quickly. Could it be an issue of the big numbers again? Then she wrote 94 – 10 as a vertical subtraction problem, covered the ones with her fingers, but still was not able to go further. Then she tried to count backwards on the number line 10 times. She knew it was 10 backwards but came up with 83 one time and 86 the next. She clearly did not see the relationships of tens.

Large numbers present a new challenge to students. Just because they can count with these numbers and write them, does not mean that they can work with them in the same way as with smaller numbers. We know from classroom observations that Yolanda has great facility with base ten thinking and using multiple strategies for smaller numbers. However, here we see that she was not able to think about large numbers in the same way as small ones.

Gerardo Works with Base Ten Concepts

The next example focuses on Gerardo. He is an outgoing Spanish-English bilingual student with a high degree of classroom confidence and verbal facility. In this example he attempts to solve multiplication problems with groups of ten.

Fieldnotes 3/21/07 (Helena's class):

Problem: Fernando tiene 2 cajas de crayolas. Hay 10 crayolas en cada caja. Tambien el tiene seis crayolas mas. ¿Cuántas crayolas tiene Fernando en total? [Fernando has 2 boxes of crayolas. There are 10 crayolas in each box. There are also 6 more crayolas. How many crayolas does Fernando have in all?]

Helena (the teacher) begins by drawing a light bulb on the board and asking the students to turn on their minds. She has Anthony come up to draw the first part of the problem, after they all read it together. Anthony draws:

10 10
□ □

Helena asks what the significance of the "10s" is and Gerardo says "dies adentro." [ten inside] Then Gerardo goes up and adds "+" between the boxes, and "+ 111111 = 26". He draws a box around the answer. It is now:

10 10
□ + □ + |||||| = 26

Helena asks who can answer in words. Gina says, "Fernando tiene 26 crayolas en total." [Fernando has 26 crayolas in all.]

(three problems later in the lesson)

The problem is to find out how many crayons are in 12 boxes, with 10 in each and 6 single crayons. *I have suggested to Helena that we go beyond 100 to see what the students can do.* Gerardo volunteers. Helena tells him to use the ten boxes already there from the previous problem. She has erased the lines and the previous answer so that all is left are the 10 boxes with "10" above them and "+" in between, as below.

10 10 10 10 10 10 10 10 10 10
□ + □ + □ + □ + □ + □ + □ + □ + □ + □

Gerardo adds after the boxes (above): **"+ □ + 10 = 201006"**

We see in this extended excerpt from fieldnotes that Gerardo was confident with the smaller numbers and able to apply base ten thinking to find the answer, 26. When the numbers became large, however, he was confused both by adding to what was already on the board and also trying to manage the writing of a number greater than 100. All he needed to do was add his previous solution, 26, to the row of ten boxes to get 126. However, he did not make this connection.

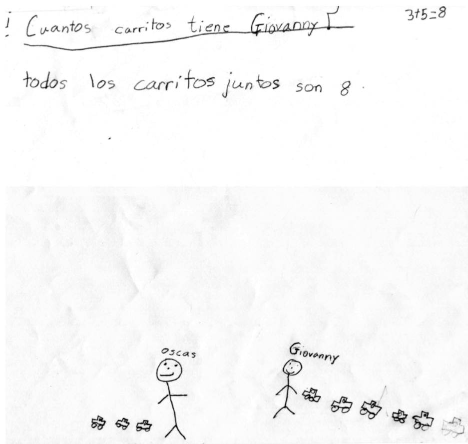
Theme 5: Pictorial representations give valuable insight into how students are thinking about the problems.

Yolanda and Ana's Drawings Show Subtle Gaps in Their Comprehension

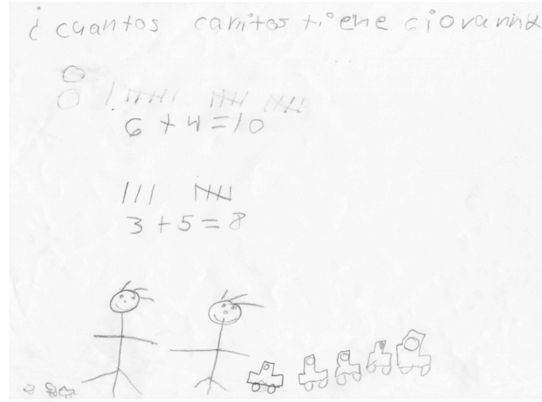
The following excerpt is from students' first experience with a type of compare problem where the difference is known and one of the reference sets is unknown. The two students are Yolanda and Ana. As mentioned previously, Yolanda is the most conceptually sophisticated of the focal students. Ana is the most academically mature. Both these girls are tripped up by this more conceptually difficult problem situation.

Fieldnotes 2/28/07 (Naomi's class, whole group):

The problem: Oscar has 3 toy cars and Giovanny has 5 more toy cars than Oscar. How many toy cars does Giovanny have? For this problem, Ana drew the same picture as Yolanda, showing Oscar with 3 cars and the Giovanny with 5 cars rather than the 8, which would have been the correct representation. Ana labeled the stick figures with names. She wrote the question from the board and then wrote for the answer, saying, "todos los carritos juntos son 8." [All of the cars together are 8.] (Indicating the 2nd student, Giovanny.



Ana's drawing



Yolanda's drawing

The importance of representations becomes apparent in this example. Yolanda and Ana were able to find the correct answer by manipulating the numbers, but their drawings send up a red flag about their comprehension. They have drawn three cars for Oscar and five cars for Giovanny, instead of the eight for Giovanny, which would have been the correct answer. These pictures give a clearer sense of actual student thinking than either their equations or what they wrote (Kendrick & McKay, 2004).

Conclusions

From the evidence presented in the five themes above, we see a remarkable amount of mathematical understanding among these seven first grade students. They are able to understand and solve complex comparison and multiplication type problems not usually introduced in first grade. Students' networks of mathematical understanding are growing and the links they are making among concepts are continually being strengthened. However, it is clear from what students do and say, what they write, and what they draw, that their networks are still in the formative stages at the peripheries where new information is being processed. We see evidence of the foundations necessary for breadth and depth of understanding, but not evidence of a well-constructed network solidly in place.

The above data also show that students have the psychological tools they need to tackle challenging problems and the linguistic sophistication to both comprehend problem situations and describe their thinking and solution strategies when problem solving takes places in their native language. They are building a foundation in reasoning and justification, process skills necessary for all advanced mathematics. Students' opportunity to build this knowledge within their native language clearly gives them an advantage they would not otherwise have if they had to make sense of both a second language and new mathematical ideas.

Finally, we see that assessment of student mathematical thinking and development is not straightforward. The data show that these first grade students can use manipulatives correctly, like base ten blocks, without evidence of a solid understanding of the concepts on which they are based. Students can produce a correct answer to a problem, but a closer examination of their drawings about the problem shows that they do not fully comprehend the situation. On the other hand, students may have a solid understanding of a problem, but not be able to produce a correct written statement about their answer since they are still developing writing in first grade.

I believe the evidence from the data collected in this study forces the educational community to question the validity of any single solution produced by first grade students. Students' complex networks of mathematical understanding cannot be assessed from only one perspective. Constructing mathematical knowledge is a complex, nonlinear process. To obtain a better understanding of the state of this knowledge, teachers need to approach assessment from multiple perspectives, including complex, contextually based word problems, to uncover the dynamic nature of this growing network and to give students the challenge they need to move their thinking forward.

Implications and Recommendations

The implications for these findings relate directly to bilingual education and equity in mathematics. We see that students learning in their native language levels the playing field for Spanish-speaking first grade students learning mathematics. These students from Mexican immigrant, Spanish-speaking backgrounds are impressive problem solvers and their young minds are a valuable resource that should not be wasted. Bilingual education and the opportunity to learn early mathematical concepts and number sense through problem solving in Spanish have provided these students with an equitable learning environment.

Further implications inform mathematical teaching and learning in primary classrooms. It is an incorrect assumption that if students can produce correct answers to a particular problem then they have incorporated the conceptual knowledge of that problem and can move on to more complex ideas. The findings from this study show that all students, even the most advanced, need multiple exposures to concepts and repeated opportunities to practice with a variety of challenging problem situations so that their growing knowledge base can become both flexible and secure. When the evaluation of students' knowledge is based on one-time assessments, the information obtained is highly questionable. One-time assessments, especially for young children who do not have well-developed academic speaking and writing skills, seriously undermine the measure of these students' abilities. In addition, they do not reveal important gaps students may have in their comprehension of mathematical ideas. On-going assessment coupled with repeated practice in a wide range of situations, as recommended by the NCTM Standards (NCTM, 2000), is the only valid way to assess students' developing mathematical knowledge.

Questions for Further Study

This study has only begun the exploration of students' mathematical thinking within bilingual contexts. Further analysis should probe more deeply into the language students are using to explain their thinking, how this mathematical discourse is developing over time, and how the discourse relates to the strategies students use to solve and represent problems. Further research is needed to obtain longitudinal data on the mathematical development of bilingual primary grade students. In addition, it is not enough to be satisfied with correct answers and the equations students write, and it is far too easy to skip the time-consuming step of asking students to justify their answers and create a representation, both important processes standards for mathematics (NCTM, 2000). As seen above, students' explanations and representations provide a unique window into their mathematical thinking and work with Cognitively Guided Instruction (Carpenter, et al., 1999) has given us a great deal of understanding on how young children model word problems. Additional studies should build on students' backgrounds in problem solving by employing larger numbers to push students beyond direct modeling and force them to use the advanced strategies that indicate a deeper sense of number.

References

Aunola, K., Leskinene, E., Lerkkanen, M-K., & Nurmi, J-E. (2004). Developmental dynamics of math performance from preschool to grade 2. *Journal of Educational Psychology*, 96(4), 699-713.

Baroody, A. (1999) Children's relational knowledge of addition and subtraction. *Cognition and Instruction,* 17(2), 137-175.

Boaler, J. (2002a). Exploring the nature of mathematical activity: Using theory, research and 'working hypotheses' to broaden conversations of mathematics knowing. *Educational Studies in Mathematics, 51*, 3-21.

Boaler, J. (2002b). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33*(4), 239-258.

Carpenter, T., Ansell, E., Franke, M., Fennema, E., & Weisbeck, L. (1993). Models of problem-solving: A study of kindergarten children's problem-solving processes. *Journal for Research in Mathematics Education,* 24(5), 427-440.

Carpenter, T., Fennema, E., & Franke, M. E. (1993). *CGI Cognitively Guided Instruction: Children's thinking about whole numbers.* Madison, WI: Wisconsin Center for Education Research.

Carpenter, T., Fennema, E., & Franke, M. E. (1996). Cognitively Guided Instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal*, 97(1), 3-20.

Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999). *Children's Mathematics: Cognitively Guided Instruction.* Portsmouth, NH: Heinemann.

Chao, S-J., Stigler, J., & Woodward J. A. (2000). The effects of physical materials on kindergarteners' learning of number concepts. *Cognition and Instruction,* 18(3), 285-316.

Clements, D., Swaminathan, S., Hannibal., M. A. Z. & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education,* 30(2), 192-212.

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective discourse and collective reflection. *Journal for Research in Mathematics Education,* 28(3), 258-277.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of development research. *Educational Psychologist,* 31, 175-190.

Collier, V. P. & Thomas, W. P. (2004). The astounding effectiveness of dual language education for all. *NABE Journal of Research and Practice, 2(2)*, 75-94.

Creswell, J. W. (1998). *Qualitative inquiry and research design: Choosing among five traditions.* Thousand Oaks CA: Sage Publications.

Eisenhart, M. & Borko, H. (1993). Contributions from cognitive psychology: Themes in cognitive psychological research on classrooms. In *Designing classroom Research: Themes, issues, and struggles* (23-40). Boston: Allyn & Bacon.

Emerson, R. M., Fretz, R. I., & Shaw, L. L. (1995). *Writing ethnographic fieldnotes.* Chicago: University of Chicago Press.

Escamilla, K., Chavez, L, & Vigil, P. (2005) Rethinking the "Gap:" High-stakes testing and Spanish-speaking students in Colorado. *Journal of Teacher Education*, 56(2), 132-144.

Fennema, E., Franke, M., Carpenter, T., & Carey, D. (1993). Using children's mathematical knowledge in instruction. *American Educational Research Journal,* 30(3), 555-583.

Fennell, F. & Rowan, T. (2001). Representation: An important process for teaching and learning mathematics. *Teaching Children Mathematics, 7*(4), 288-292.

Fuchs, L., Fuchs, D., Karns, K., Hamlett, C. & Katzaroff, M. (1999). Mathematics performance assessment in the classroom: Effects on teacher planning and student problem solving. *American Educational Research Journal, 36*(3), 609-646.

Greeno, J., & Hall, R. (1997). Practicing representation: Learning with and about representational forms. *Phi Delta Kappan, 78,* 361-367.

Hiebert, J. & Carpenter, T. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (65-97). Reston, VA: National Council of Teachers of Mathematics.

John-Steiner, V., & Mahn, H. (1996). Sociocultural approaches to learning and development: A Vygotskian framework. *Educational Psychologist, 31(4),* 191-206.

Jordan, N.C., Kaplan, D., Oláh, L. N., & Locuniak, M. N. (2006). Number sense growth in kindergarten: A longitudinal investigation of children at risk for mathematics difficulties. *Child Development, January/February 2006*, 77(1), 153-175.

Kamii, C., Rummelsburg, J., & Kari, A. (2005). Teaching arithmetic to low-performing, low-SES first graders. *Journal of Mathematical Behavior, 24*(1), 39-50.

Kato, Y., Kamii, C., Ozaki, K., & Nagahiro, M. (2002). Young children's representations of groups of objects: The relationship between abstraction and representation. *Journal for Research in Mathematics Education, 33*(1), 30-45.

Kendrick, M., & McKay, R. (2004). Drawing as an alternative way of understanding young children's constructions of literacy. *Journal of Early Childhood Education, 4,* 109-128.

Khisty, L. L. (1997). Making mathematics accessible to Latino students: Rethinking instructional practice. In J. Trentacosta and M. Kenney (Eds.), *Multicultural and gender equity in the mathematics classroom: The gift of diversity, 97th yearbook of the national council of teachers of mathematics.* Reston, VA: NCTM.

Kieran, C. (2001). The mathematical discourse of 13-year-old partnered problem solving and its relation to the mathematics that emerges. *Educational Studies in Mathematics, 46*(1&3), 187-228.

Kozulin, A. (1990). Thought and Language. In *Vygotsky's psychology: A biography of ideas* (151-193). Cambridge: Harvard University Press.

La Paro, K. & Pianta, R. (2000). Predicting children's competence in the early school years: A meta-analytic review. *Review of Educational Research, 70*(4), 443-484.

Lerman, S. (2000). The social turn in mathematics education research. In Jo Boaler (Ed.) *Multiple perspectives on mathematics teaching and learning.* Westport, CT: Ablex Publishing.

Lubienski, S. T. (2000). Problem solving as a means toward mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education, 31*(4), 454-482.

McClain, K. & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first-grade classroom. *Journal for Research in Mathematics Education, 32*(3), 236-266.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning, 4*(2&3), 289-212.

Nasir, N. i. S. & Hand, V. M. (2006). Exploring sociocultural perspectives on race, culture, and learning. *Review of Educational Research*, *76*(4), 449-475.

National Assessment of Educational Progress. (2005). *The Nation's Report Card: Mathematics 2005.* Retrieved on-line October 15, 2006 at: http://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2006453

National Center for Educational Statistics. (2006). *Teacher Qualifications, Instructional Practices, and Reading and Mathematics Gains of Kindergartners: Research and Development Report 2006-031*. Retrieved January 1, 2007 at: http://nces.ed.gov/pubs2006/2006031.pdf

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Nelson, K. (1991). The matter of time: Interdependencies between language and thought in development. In Gielman, S., Byrnes, J. (Eds.). *Perspectives on language and thought: Interrelations in development.* Cambridge: Cambridge Univ. Press, 278-318.

O'Conner, M. C. (1998). Language socialization in the mathematics classroom: Discourse practices and mathematical thinking. In M. Lampert & M. L. Blunk (Eds.), *Talking mathematics in school: Studies of teaching and learning*. New York: Cambridge University Press, 17-55.

Ortiz-Franco, L. (1999). Latinos, income, and mathematics achievement: Beating the odds. In Ortiz-Franco, L., Hernandez, N. & De La Cruz, Y. (Eds.) *Changing the faces of mathematics: Perspective on Latinos* (13-20). Reston, VA: NCTM.

Outhred, L. & Mitchelmore, C. (2000). Young children's intuitive understanding of rectangular area measurement. *Journal for Research in Mathematics Education, 31*(2), 144-167.

Outhred, L, & Sardelich, S. (2005). Problem-solving by kindergartners. *Teaching Children Mathematics*, *October 2005*, 146-154.

Pappas, S., Ginsburg, H. , & Jiang, M. (2002). SES differences in young children's metacognition in the context of mathematical problem solving. *Cognitive Development*, *18*(3), 431-450.

Saxe, G. B. (2002). Children's developing mathematics in collective practices: A framework for analysis. *Journal of the Learning Sciences, 11*(2&3), 275-300.

Sfard, A. (2001). There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning. *Educational Studies in Mathematics, 46*(1&3), 13-57.

Siegler, R. S. (2000). The rebirth of children's learning. *Child Development, 71*(1), 26-35.

Spelke, E. S., & Tsivkin, S. (2001). Initial knowledge and conceptual change: Space and number. In M. Bowerman & S. C. Levinson (Eds.). *Language Acquisition and*

Conceptual Development (70-97). Cambridge: Cambridge University Press.

Turner, E., Celedón-Pattichis, S., Marshall, M., & Tennison,A. (in press). "Fíjense amorcitos, les voy a contar una historia": The power of *story* to support solving and discussing mathematical problems among Latino/a kindergarten students. To appear in *Mathematics for ALL: Instructional Strategies for Diverse Classrooms: K-2*. Reston, VA: NCTM.

Turner, E., Celedón-Pattichis, S., & Marshall, M. (in press). Cultural and linguistic resources to promote problem solving and mathematical discourse among Latino/a kindergarten students. Paper to appear in NCTM TODOS Monograph. Reston, VA: NCTM.

Van der Veer, R. & Valsiner, J. (1991). The universe of words: Vygotsky's view on concept-formation. In *understanding Vygotsky: A quest for synthesis.* Cambridge: Blackwell, 256-283.

Van der Veer, R. & Valsiner, J. (1994). *The Vygotsky reader*. Cambridge, MA: Blackwell.

Van Oers, B. (2001). Educational forms of initiation in mathematical culture. *Educational Studies in Mathematics, 46*(1&3), 59-85.

Villaseñor, A. & Kepner, H. (1993). Arithmetic from a problem solving perspective: An urban implementation. *Journal for Research in Mathematics Education*, *24*, 62-70.

Warfield, J. (2001). Teaching kindergarten children to solve word problems. *Early Childhood Education Journal, 28*(3), 161-167.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, UK: Cambridge University Press.