Teacher Variables and Student Mathematics Learning Related to Manipulative Use

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ABSTRACT

It has been suggested that teachers' instructional practices mediate the relationship between teacher background variables and student learning (Fennema & Franke, 1992; Mewborn & Cross, 2007). That is, an indirect relationship between these variables exists in which teacher variables are related to teacher instructional practices; and teacher instructional practice is then related to student learning. One instructional practice that has been supported as a way to help build students' mathematical understanding is the use of manipulatives (e.g., Hiebert et al., 1997). In this dissertation I investigate the role of manipualtive use as a mediator of the relationship between teacher variables and students' mathematics learning.

To examine the potential mediating role of manipulative use, I conducted two separate quantitative studies. In the first study, I examined the relationship between teacher variables and the frequency with which teachers use manipulatives in their classroom activities. Using data from 503 in-service elementary teachers, I investigated the relationship between manipulative use and teachers' beliefs about manipulatives, the grade level they teach, their age and experience, as well as the interrelationship among these teacher variables. Teachers' beliefs and grade level were found to be important predictors of manipulative use. In the second study, I examined the relationship between manipulative use and mathematics learning of elementary-aged students (K-5). Data for this study were drawn from the Early Childhood Longitudinal Study (ECLS), and analyzed using a two-level hierarchical linear model and graphical techniques. I also investigated the moderating effects of student home language on this relationship. A positive relationship between the frequency of manipulative use and student mathematics learning was found. Home language was not found to moderate this relationship.

Results from this manuscript dissertation seem to support the claim of Fennema and Franke (1992) and Mewborn and Cross (2007) about the mediating role of teacher instructional practice on the relationship between teacher variables and student learning. In particular, combining the results from the two studies, teachers holding positive beliefs about manipulatives tend to use these devices more often in their classroom activities, and teachers in lower grades tend to use manipulatives more often than teachers in the upper grades; and when students use manipulatives more often in their mathematics lessons, their mathematical learning tends to increase. Thus, together these findings suggest a tenable indirect relationship between teacher variables and student learning. That is, teacher variables are related to manipulative use which in turn is related to students' mathematics learning. Together these two studies provide a more complete picture of teacher factors and student outcomes as they relate to manipulative use in elementary mathematics classrooms. Implications and future directions for research on manipulative use in the teaching and learning of mathematics are discussed.

DEDICATION

To

My parents, María Hersilia and Víctor Julio,

My husband, Jesús Hernán,

My sisters and brother, Martha Patricia, Miriam Janet, Hilda Rosario, Lucía Victoria, and Víctor Mauricio,

My nephews, Julián Enrique, Jorge Andrés, Diego Alexander, Carlos Fernando, Andrés Felipe, José Luis, and Oscar Mauricio,

My nieces, Erika Paola, Nancy Janet, María Patricia and Ana María

Your love and support during all these years have always given me the strength to continue and not give up my dreams. I hope my experience will also inspire you to pursue your dreams and to dream high. Because of all your support and love this is not me reaching a degree, this is a degree for the whole family. Los amo mucho!!

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Chapter 1. Introduction

It has been suggested that teachers' instructional practices may serve as a mediator of the relationship between teacher variables and student learning (Fennema & Franke, 1992; Mewborn & Cross, 2007). That is, instead of a direct relationship between teacher variables and student learning, teacher variables influence classroom instruction which ultimately influences student learning. In Figure 1, a mediation model is presented to represent this relationship. The direct relationship is represented by the dotted arrow. The meadiating role of instructional practice is represented by the indirect path from teacher variables through instructional practice (gray arrow), to student learning (black arrow). The goal of this dissertation is to evaluate the mediating role of an instructional practice in mathematics classrooms at elementary school level. To determine the extent of the mediating role of instructional practice it is necessary to examine the two individual relationships (paths) between (1) teacher variables and teacher instructional practice, and (2) teacher instructional practice and student learning. Finding a relationship for each of these paths provides evidence of the mediating role of instructional practice.

One teaching strategy that has received continued support for mathematics instruction is the use of manipulatives (Gravemeijer, 2002; Hiebert et al, 1997). Teaching strategies involving manipulatives may also serve as a mediating factor between teacher variables and student learning. In my dissertation, I used the framework in Figure 1 to ground two studies to investigate manipulative use in mathematics classrooms as a mediator between teacher variables and student learning. In the first study, I investigated the relationship between teacher variables, such as beliefs and other background characteristics, and teachers' manipulative use in the classroom (see gray arrow in Figure 1). In the second study, I investigated the relationship between manipulative use in mathematics classrooms and student learning (see black arrow in Figure 1). By investigating the indirect path in Figure 1, I aim to evaluate manipulative use as a mediator between teacher variables and student learning.

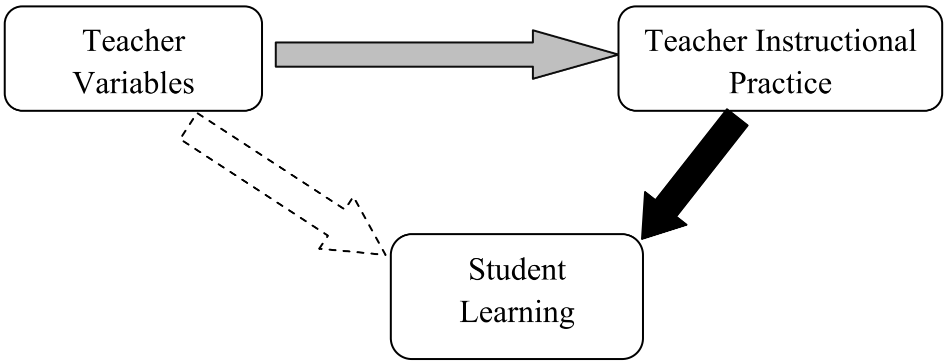

Figure 1.
Manipulative use as a mediator between teacher variables and student learning.

In general, teaching is a complex process (Barkatsas & Malone, 2005; Beswick, 2005; Ernest, 1989; Raymond, 1997; Thompson, 1984), and there are several factors that affect teacher instructional practices. Teacher beliefs have been found to influence teacher instructional practices (Nespor, 1987; Peterson, Fennema, Carpenter, & Loef, 1989; Wilkins, 2008). In addition to beliefs, research has also found teacher background characteristics as possible factors influencing teaching practices (Gilbert & Bush, 1988; Opdenakker & van Damme, 2006; Wilkins, 2008).

Considering manipulative use in particular, teacher variables, such as beliefs and background characteristics, have been found to be related to manipulative use in mathematics classrooms. For example, if a teacher believes that manipulatives are for "fun" times, then the tools will likely not be used to promote students' mathematical understanding (Moyer, 2001).

Moreover, if an upper grade level teacher does not use manipulatives because of a belief that manipulatives are not necessary for “older” students, then students may not have opportunities to support their learning process through the use of manipulatives. In fact, it has been found that upper grade level teachers tend to use manipulatives less often than their peers in lower grade levels (Malzahn, 2002; Raphael & Wahlstrom, 1989; Weiss, 1994). Studies analyzing the relationship between other teacher variables, such as experience, and manipulative use have found inconsistent results. Some indicate that experienced teachers use manipulatives more often than their colleagues (Gilbert & Bush, 1988), but other studies such as Howard et al. (1997) found no differences. Recent studies attempt to understand how teachers use manipulatives (Moyer & Jones, 2004; Moyer, Salkind, & Bolyard, 2007); however, possible factors affecting teachers’ instructional decisions related to manipulative use have not been deeply analyzed. For example, possible interrelations between different factors predicting manipulative use in mathematics classrooms have not been studied.

The relationship between teacher manipulative use and student learning is not clear because results from the research are inconsistent. For example, some studies have found positive results related to students’ achievement and the use of manipulatives to learn a concept (Bolyard, 2005; Suh, 2005; Suh & Moyer, 2007; Suydam & Higgins, 1977; Trespalacios, 2008). Other studies have not found positive results relating students’ performance to manipulative use (Drickey, 2006; McClung, 1998; Posadas, 2004). Moreover, results from meta-analyses are consistent with these varied results about the relationship between manipulative use and students’ achievement (see Fennema, 1972; Sowell, 1989). In addition, most studies measuring the relationship between manipulative use and students’ performance have investigated the use of these tools for only short periods of time (e.g., Bolyard, 2005; Reimer & Moyer, 2005; Suh, 2005; Suh & Moyer, 2007; Trespalacios, 2008), but according to Sowell (1989), manipulatives seem to be most effective if they are used for periods of time longer than a year. In addition, Willet (1988) indicates that change and growth over time should be measured when investigating student learning. Therefore, studies analyzing the impact of manipulative use on students' learning over long periods of time are needed.

Furthermore, the National Council of Teachers of Mathematics (NCTM) Equity Principle calls for support for all students' mathematics learning process. In this principle, NCTM (2000) states that "all students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study – and support to learn – mathematics" (p. 12). Therefore, students should not be impaired because of any difference, such as language (Khisty, 1995). Teachers should use strategies that support all students' mathematics learning. English Language Learners (ELL) experience many situations in their classrooms in which language can limit their possibilities to succeed. Cummins (1984) indicates that language acquisition is related to two continuous factors: context and cognitive demand of the situations. Strategies to help these students to reduce cognitive demand due to lack in language skills should be used in classrooms in order to help these students learn other subjects when language proficiency is limited (Herrell, 2000). Herrell (2000) proposes manipulative use as a strategy to be used in mathematics classrooms to help ELLs' mathematics learning process. However, few studies have investigated the relationship between manipulative use and ELLs' mathematics learning.

In summary, teacher instructional strategies have been proposed to mediate the relationship between teacher variables and student learning (Fennema & Franke, 1992; Mewborn & Cross, 2007). In this dissertation, I am evaluating *manipulative use* as a potential mediator of the relationship between these variables by individually investigating the relationship between teacher variables and manipulative use, and the relationship between manipulative use and student learning. The relationship between teacher variables and manipulative use has not been deeply studied, and studies have found mixed results. Moreover, studies investigating the relationship between manipulative use and student learning consider either learning measuring only two points in time or student achievement, and manipulatives are usually used for short periods of time.

Research Methods and Questions

To examine the potential mediating role of manipulative use, I conducted two quantitative studies to investigate the two paths in the mediator model (Figure 1). In the first study, titled “Elementary School Teachers’ Manipulative Use” and presented in Chapter 3, I examined the relationship between teacher variables and manipulative use. For this study, I used survey data from 503 in-service elementary school teachers from two school districts in the southeastern part of the United States. Teachers completed a survey at the beginning of a professional development program in 2000. In this study, the teacher variables included were teachers’ age, experience, grade level, and beliefs about manipulatives, and I investigated the relationship between them and manipulative use. Different regression models are used, which are useful to investigate the interrelationship between teacher variables.

In the second study, titled “Manipulative Use and Elementary School Students’ Mathematics Learning” and presented in Chapter 4, I evaluated the relationship between manipulative use and student learning with the Early Childhood Longitudinal Study (ECLS) database.I used hierarchical linear modeling techniques to examine the relationship between manipulative use and student learning. With this study design, I also investigated whether or not this relationship was moderated by home language. In order to control for differences in home language that exist within the larger sample, a second similar analysis is conducted with only the sub-sample of Hispanic students.

The overall aim of these studies was to answer the following general research questions:

(a) Is there a relationship between elementary school teachers' use of manipulatives and teachers' variables?

(b) Is there a relationship between elementary-aged students' manipulative use and their individual mathematics learning?

Outline of the Dissertation

In Chapter 2, I discuss literature relevant to both studies. This review focuses on student mathematics learning related to manipulative use. For example, I discuss the role of manipulative use for ELLs' mathematics learning processes. This review also focuss on the relationship between teacher variables and manipulative use. For example, I discuss teacher beliefs, grade level and experience as it relates to manipulative use. In Chapter 3, I present the first manuscript on the relationship between teacher beliefs and manipulative use. In Chapter 4, I present the second manuscript on the relationship between manipulative use and student mathematics learning. In Chapter 5, I synthesize findings from Chapters 3 and 4, and discuss conclusions and implications from both studies.

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Chapter 2. Literature Review

Manipulative use in the mathematics classroom is supported and informed by several bodies of literature. In this chapter, I present two of them which are related to the mediating role of manipulative use between teacher variables and student learning. First of all, I present literature related to the relationship between student learning and manipulative use. Learning models based on psychologists and learning theorists, such as Bruner and Piaget, support manipulative use and inform how these tools facilitate students' mathematics learning process in general. In addition, based on language acquisition theories, manipulative use may also help the mathematics learning processes of English language learners. In the first section of this chapter, I discuss literature related to the learning process and manipulative use.

The second body of literature presented in this chapter is literature related to the relationship between teacher variables and teaching practices in mathematics classrooms, particularly manipulative use. Research on the relationship between teacher variables and teacher instructional practices has analyzed the complex process of teacher's decision making during instruction. Some of this research focuses on general teacher variables encompassing teacher beliefs and background characteristics. Moreover, some research has focused on teachers' characteristics and relates them specifically to the teachers' instructional practice of manipulative use.

Manipulative Use and the Learning Process

Mathematics teachers should use teaching strategies that promote students' mathematical understanding (NCTM, 2000). One strategy intended for that purpose is the use of manipulative materials (Hiebert et al., 1997). Psychologists and learning theorists have supported the use of these tools during mathematics instruction for many years. Learning models based on ideas of some of these theorists, such as Bruner and Piaget, help to understand how these tools facilitate the learning process in mathematics classrooms.

In this section, I focus my discussion on how manipulative use can help students' mathematical learning processes. First, I present a brief history of manipulative use and those who support manipulative use. Second, I discuss some definitions of the term *manipulative* found in the literature, followed by a definition that encompasses the most important ideas. Third, I present a way that manipulative use and learning processes can be connected by discussing two models: (1) Information processing model, and (2) Experiential model. Finally, I present results from studies that relate manipulative use in mathematics classrooms to students' performance.

History of Manipulatives

The idea of using objects to enhance the learning process is not a new one. This ideology is founded upon John Locke's theory of learning from experience (Zuckerman, 2006). Locke (1841) states that learning, or getting ideas, comes from external objects and reflection. All of his theories about learning are based upon learner experiences where the learner is actively involved in his/her learning process. Several educational philosophers such as Pestalozzi, Froebel, Montessori, and Dewey (Zucherman, 2006) as well as Jean Piaget (Kohler, 2008) developed learning theories connected to Locke's theory. Pestalozzi implemented the idea of using hands-on materials for teaching and learning processes in schools (Szendrei, 1997). Pestalozzi, according to Resnick, Berg, and Silverman (1998), affirmed that by using their senses and participating in physical activities students learn and claimed "things before words, concrete before abstract" (Pestalozzi, 1803, cited in Resnick, Berg, & Silverman, 1998, p. 281). According to Brosterman (1997), in Pestalozzi's pedagogy "objects were used in the teaching of all classes and the primacy of books was greatly reduced. In arithmetic, tools of perception (apples, stones and so forth) were used to develop" (p. 21) mathematical operations and relations.

Others followed Pestalozzi's ideas on teaching. In 1837, Froebel, Pestalozzi's pupil, created the first kindergarten (Brosterman, 1997). This kindergarten was full of different physical objects called "gifts" that the young students could play with and learn from. Montessori and Piaget also continued working with Pestalozzi's idea (Resnick et al., 1998), but they used his ideas in different ways. For instance, in developing Montessori's method of sensory education (Gutek, 2003), she created objects called "didactic materials" that represent abstract concepts and help students learn from their senses (Zuckerman, 2006). These materials were for learners with the purpose of having students learning from their experiences according to their developmental stage. She paid special attention to the environment and how it provides learning opportunities for children. In this kind of environment, students are free to learn and teachers' main work is to observe students and address them in their thinking processes (Montessori, 1912).

Piaget's work was more in the psychological arena. His work focused on learners' minds, in particular, the developmental stages of learners. For example, he considered the developmental stages of youth ages 7-11 to be the concrete operational stage where children need and are able to have concrete experiences. Children over the age of 11 years old were in the formal operations stage where children are able to have more abstract thinking and need less concrete experiences. Piaget identified two different ways of having concrete experiences using objects: physical experience and logico-mathematical experience (Kohler, 2008). Physical experiences relate to looking for physical properties of objects that students manipulate, whereas logico-mathematical experiences refer to abstractions reached because of the interaction with the objects (Kohler, 2008). However, people have critiqued Piaget's stages (O'Hagan & Smith, 2002; Post, 1980; Resnick & Ford, 1981). For example, the need for concrete experiences is not limited to the 7-11-year-old learners; those experiences are beneficial for young students (O'Hagan & Smith, 2002) as well as for older students (Post, 1980). In addition, many factors may impact development, one being the experience a child/person has on a specific topic (Resnick & Ford, 1981). Therefore, the need for diverse experiences for learning at different ages is supported. Moreover, Montessori and Piaget share the idea that the teachers' role in the classroom is as a facilitator of students' experiences in constructing their own knowledge, and they both support the idea of using tools to help learning processes (Post, 1980; Zucherman, 2006).

U.S. education has been influenced by advocators of learning through experience such as Dewey, Bruner, and Dienes. Dewey supported the idea that education comes from experiences, but not all experiences are equally beneficial for the learning process (Dewey, 1938). Dewey (1938) states that teachers should provide experiences that engage students and make them want to have more experiences. For him, education should be "within, by and for experience" (p 17). Dewey, according to Ratner (1992), "argued that mathematics as a school subject needs to be viewed as a development from the experiences of student emotion, thought and action" (p. 106-107). Moreover, Bruner (1966) states that there are three representations for experiential learning: "*enactive representation"* or representation by action, "*iconic representation"* or using pictorial representations, and "*symbolic representation"* or using symbols as a representation of what is being learned. He also emphasizes that going from one stage to the next is a complex process (Bruner, 1966), as well as the need for the learner to participate actively in the process of building their knowledge (Post, 1980). Bruner's pupil Zoltan Dienes also had an influence in the field of experiential learning, specifically in mathematics. He proposed six stages in the process of learning mathematics. He based his theory on using the senses and concrete materials, stating that learning processes start when students are getting familiar with the environment and that the goal is for students to reach abstraction of mathematical structures (Dienes, 1973).

Definitions of Manipulatives

Teachers using the word *manipulatives* to describe tools used in their classroom may not be referring to the same devices. Sherman and Richardson (1995), studying 25 elementary school teachers, state that teachers have different interpretations of the term *manipulatives*. For some teachers, manipulatives are any concrete material students can touch (boxes, calculators, chalkboard), while for other teachers, manipulatives are tools such as geoboards, strips of papers, or rulers. The authors affirm that it is difficult to determine what teachers mean when they attest to using concrete materials in their instructional practices.

Teachers, however, are not alone in their varied definition of the term *manipulatives*. Researchers in mathematics education also differ on their definition. When they refer to manipulatives their definitions may describe concrete manipulatives, computer base/virtual manipulatives, or any kind of manipulatives. Definitions of manipulatives, specifically concrete manipulatives, found in the literature are not equivalent and are sometimes even contradictory. For example, Yeatts (1997) expresses that "manipulative materials are objects or things that appeal to several of the senses" (p. 7). According to this definition, a video is a manipulative because it appeals to visual and audio senses. For other researchers or educators, manipulatives are objects that can be touched, moved around, rearranged, or stacked (Clement, 2004; Hynes, 1986; Kennedy, 1986). In this case, books in a shelf are considered manipulatives because they can be organized though they may just be used to read or solve problem sets.

For other researchers and educators, manipulatives represent abstract mathematical ideas (Durmus & Karakirik, 2006; Heddens, 2005; Hynes, 1986; Moyer, 2001; Suh, 2005). Moreover, Moyer (2001) states that manipulatives are "objects designed to represent explicitly and concretely mathematical ideas that are abstract. They have both visual and tactile appeal and can be manipulated by learners through hands-on experiences" (p. 176). In this case, manipulatives are just commercial objects, such as algeblocks, pattern blocks, or computer base/virtual manipulatives. Strings or rubber bands used in activities to help students understand a mathematical concept are not manipulatives because they are not designed for that purpose. In this definition Moyer (2001) also refers specifically to two senses: tactile and visual instead of *several* senses as referred to in Yeatts (1997). Some definitions propose learners as users of these tools, for instance, Durmus, and Karakirik (2006) state that manipulatives are models of mathematical concepts that can be manipulated by students and appeal to their senses. However, Heddens' (2005) definition states students have to have the opportunity to use the objects and they should not be objects just for teacher demonstration.

The purpose for using these tools is also found in parts of some definitions given in the literature. In Uttal, Scudder, and Deloache (1987) and McNeil and Jarvin (2007), manipulatives are objects that help students learn or understand mathematics. Uttal et al. (1987) state that these objects are designed specifically to help students learn mathematics, but McNeil and Jarvin (2007) define them as any object that helps students understand mathematics, even if these objects were not designed for that purpose. In these definitions, it is not clear how manipulatives help students' learning processes. Textbooks can be manipulatives because they help students learn, so too can pencils, whiteboards, computers, calculators and notebooks can be considered manipulatives according to these definitions- even if students just use these objects to read, write symbols, draw pictures, or make calculations.

When referring to virtual manipulatives, the definitions are pretty similar and mostly based on Moyer, Bolyard, and Spikell's (2002) definition. They define a virtual manipulative as "an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge" (p. 373). In this definition, the interaction with the dynamic object refers to "us[ing] a computer mouse to actually slide, flip and turn the dynamic visual representation" (p. 373). Other people define virtual manipulatives as a computer-based representation of concrete manipulatives (e.g., Dorward, 2002), although a definition of concrete manipulatives is often not specified, assuming that concrete manipulatives are well-defined.

As a conclusion, the term *manipulatives* is generally used in different ways among teachers as well as among mathematics educators. For some, manipulatives are any concrete object including objects such as chalkboard, rulers, calculators, papers, or geoboards. Definitions of manipulatives involve different aspects such as their need to appeal to senses (Yeatts, 1997; Moyer, 2001), represent mathematical concepts (Durmus & Karakirik, 2006; Hynes, 1986; Moyer, 2001), how students use them (Heddens, 2005), or reasons for using these tools (McNeil & Jarvin, 2007; Uttal et al., 1987).

All these definitions show vast differences in what people, in general, mean when they describe these tools; however, from the previous discussion, a definition can be comprised that encompasses the main characteristics identified for these devices. Objects can be called *manipulatives when* they are used (stacked, moved around, arranged, etc) by students and/or teachers to represent mathematical ideas/concepts/relations. It is by the senses, mainly visual and tactile, that manipulative users can perceive the mathematical representations. To be called manipulatives, the objects should be *manipulated* and not just be displaying computations or graphics. Therefore, any object *can* be used as a manipulative, although an object *is not necessarily always* used as a manipulative.

Manipulative Use and Students' Learning

It is important for teacher to provide opportunities for students to learn mathematics with understanding as well as be active participants in the learning process (NCTM, 2000). Further, if teachers provide opportunities in which students can connect their previous knowledge to classroom activities, students may be better able to build their knowledge with understanding (Baroody & Coslick, 1998; Brooks & Brooks, 1999). Teachers can use various tools to help students make connections and achieve mathematical understanding. Manipulatives have been proposed as a way to make mathematics more concrete and manageable for students (Gravemeijer, 2002; Hiebert et al., 1997). Several models have been discussed to understand students' processes while learning. In this section I relate two of those models to students' learning processes, specifically when they are using manipulatives in mathematics classrooms. These models are the Information Processing Model and the Experiential Learning Model.

Information Processing Model. The use of tools, specifically manipulatives, is supported in theory by cognitive information processes. According to cognitive information processes (Bourne, Dominowski, Loftus, & Healy, 1986; Ormrod, 2004), we receive information through our senses and some of that information is stored in working memory. This process occurs because attention is given to that information. Once information is in working memory, it can be encoded and transferred to long-term memory (Ormrod, 2004).

However, there is a two-way interaction between working memory and long-term memory. Information is encoded to be transferred to long-term memory, and information is being retrieved from long-term memory to working memory (Ormrod, 2004). I discuss this model more in depth and relate it to manipulative use below.

Senses can receive an unlimited amount of information that stay in sensory memory in the same way it was received (visual, tactile, etc) (Bourne et al., 1986; Ormrod, 2004). Sensory information stays there for a very short time, although there are disagreements on the amount of time. For example, according to Ormrod (2004), information remains in the sensory register for no more than four seconds. She states that auditory information may last between two and four seconds. However, Bourne et al. (1986) state that information may stay there for no more than one second. When using manipulatives in mathematics classrooms, students receive information though several senses, such as visual and tactile senses, but this information stays in the sensory memory for a very short period of time. The question remains, how is information transferred to working memory without losing it?

To maintain information and move it to working memory, it is necessary to pay attention to it. We have a limited capacity of attention so we select some of the sensory information to pay attention to and process (Bourne, et al., 1986; Ormrod, 2004). Attention is influenced by different stimuli such as size, unusualness, intensity, and personal significance (Ormrod, 2004). Using manipulatives in mathematics classrooms may help students pay attention to the information that can be abstracted from the objects if, for example, these tools hold a significance to them and their learning process. For instance, if these tools were used in experiences that students enjoyed before or are familiar objects to the students, they have some significance.

After the information is gathered in sensory memory, it is then moved to working memory, which is also called short-term memory (Bourne et al., 1986). Information in working memory is what we are aware of at any given time (Bourne et al., 1986). Working memory also has a limited capacity (7±2 units of information) and short duration (5-20 sec) (Miller, 1956; Ormrod, 2004). This stage in information processing is called short-term memory because of the short time that information can be kept there; others call it working memory because they see it as a place where processes are taking place (Ormrod, 2004). Two strategies are suggested to hold information in working memory, repeating aloud to help keep auditory information, which activates the phonological loop, and using visuals to maintain information, which is the same as activating the visuospatial sketchpad (Ormrod, 2004). Using manipulatives, students may activate the visuospatial sketchpad through the visual representation of these tools.

Information in working memory needs to be moved to long-term memory for learning to occur. We encode information from working memory to transfer it, encoding the information in different ways such as making it meaningful, organizing it, visualizing it, elaborating something from it, or rehearsing it (Ormord, 2004). In the case of using manipulatives, we can encode the information using two of these ways: making it meaningful and using visual representations. Processing new information in a meaningful way is usually related to connecting it to previous knowledge (Ormord, 2004). However, students, working with manipulatives could also make mathematical content meaningful by connecting it with their experiences of using manipulatives. Students also code information in a visual way. Students using manipulatives can see a “visual” representation of the abstract content helping students make connections between content and such representation. These connections allow students to manipulate the mathematical content in a more concrete and manageable way.

When using manipulatives, encoding information using visualization includes meaningful relationships between actions and meanings. These relationships could include the meaning of objects within the activity as well as relations to the mathematical idea. Visual representation without meaningful processes may not help process information to long term memory. Students using manipulatives may be able to reach mathematical understanding (Gravemeijer, 2002) because they could be processing information in meaningful and visual ways.

In summary, the information processing model describes how information is transferred from sensory register to long term memory. Initially, attention is needed to transfer information to working memory; then it needs to be encoded to transfer it to long term memory so it can be used later. Encoding can be done in different ways. Manipulatives could help to receive information through the senses and move it to working memory. Manipulatives could help to encode information to long term memory from working memory by making it meaningful and representing it visually.

Experiential learning model. In the Experiential Learning Model (ELM), Kolb (1984) integrates Lewin's, Dewey's and Piaget's models of learning. Some similarities between these three models identified by Kolb are the idea of learning as a process, experience as its resource or input, and knowledge as being created through the learning process or output of the process.

Based on these and other principles, Kolb proposes ELM as a four-stage cycle composed by concrete experience (CE), reflective observation (RO), abstract conceptualization (AC), and active experimentation (AE) (Figure 2). In his model, there are two distinct ways of grasping experience or *prehension*; one is abstract, called *comprehension*, and the other one is concrete or tangible, called *apprehension*. Kolb, Boyatzis, and Mainemelis (2001), when talking about experiential learning, state that in each experience people either receive information concretely by “experiencing the concrete, tangible, felt qualities of the world relying on our senses and immersing ourselves in concrete reality” (p. 228), or receive it in symbolic or abstract ways, not through our senses. The EL model also presents two different ways of *transforming* the experience; one is called *intention* which is transforming information by reflection, and the other is called *extension* which is transforming information by action (Kolb, 1984).

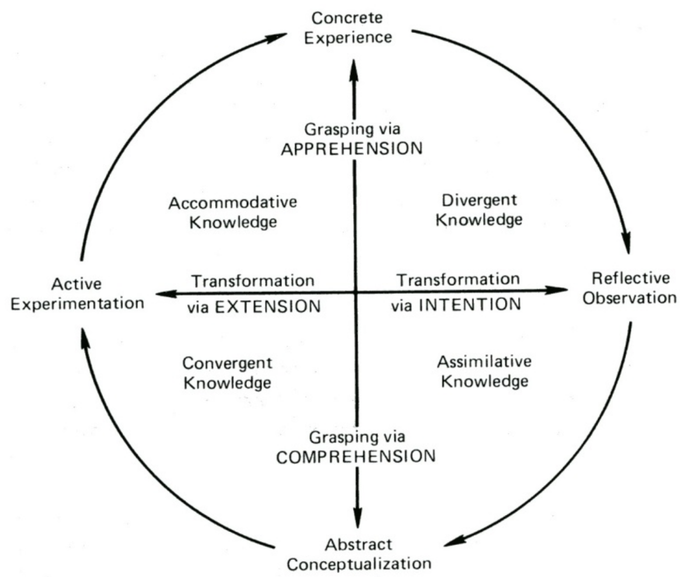

Figure 2. Experimental Learning Cycle.
Kolb, David A., EXPERIENCTIAL LEARNING: Experience As a Source of Leraning & development, ©1984, p. 24. Reprinted by persmission of Pearson Education, Inc., Upper Saddle River, NJ.

Neither prehension nor transformation by themselves produces knowledge, but both are needed for learning to occur (Kolb, 1984). Kolb (1984) states that “the simple perception of experience is not sufficient for learning; something must be done with it. Similarly, transformation cannot represent learning, for there must be something to be transformed, some state or experience that is being acted upon” (Kolb, 1984, p 42). However, processing (transforming) some type of information (prehension) is needed to generate knowledge. Kolb (1984) states that “learning is the process whereby knowledge is created through the transformation of experience. Knowledge results from the combination of grasping and transforming experience" (p. 41). The model suggests that the four combinations of prehension and transformation create different types of knowledge (see Figure 2): CE and RO form divergent knowledge, AC and RO build assimilative knowledge, AC and AE form convergent knowledge and CE and AE create accommodative knowledge (Kolb, 1984). It may, however, be incomplete to simply have a combination of one prehension and one transformation in the creation of knowledge, a combination of two prehension and a transformation or a prehension and two transformations produce a better learning (Kolb, 1984). Moreover, "the combination of all four of the elementary learning forms produces the highest level of learning, emphasizing and developing all four modes of the learning process. (Kolb, 1984, p. 66)"

Applying Kolb's (1984) and Kolb et al.'s (2001) ideas of experiential learning to manipulative use for learning mathematics, suggests the information received from experiences with these tools requires transforming to learn mathematics. Information received through experiences using manipulatives is intended to be mostly concrete; to learn students must then transform that information either by intention or extension. For example, in an activity involving Cuisenaire rods, intentional transformation of apprehension may be gained through reflection on one's own actions. In the case of transforming apprehension by extension, the information is transformed just by observing the objects and working with them; there is no reflection on actions but on the objects.

The new knowledge is integrated into the previous knowledge and students may then be able to abstract new ideas and reach comprehension from the transformation of information by doing activities with the tools. Then students may try to test the new ideas using the Cuisenaire rods or evaluate the ideas using previous knowledge so they are having a new concrete experience. In this way, the EL cycle is completed by using manipulatives and a higher level of learning is achieved by the experiences with manipulatives.

Manipulative Use and Mathematics Learning Process of English Language Learners (ELL).

The NCTM Equity Principle, presented in *Principles and Standards for School of Mathematics* (NCTM, 2000), calls for support of every student's mathematics learning process. NCTM (2002) states that "all students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study – and support to learn – mathematics" (p. 12). Therefore, students should not be impaired because of language (Khisty, 1995), and teachers should use strategies that support all students' learning. English Language Learners (ELL) often experience learning situations in school in which language may limit their possibilities to succeed.

Theories on language acquisition discuss processes that occur while learning a language. Some people in language acquisition also discuss strategies to help ELL with subjects such as mathematics. In this section, I discuss ideas about language acquisition and how manipulative use is proposed as a teaching strategy to help ELL mathematics learning.

There are varied but similar theories about the process of language learning/acquisition. For example, Krashen (1981) discusses the difference between language acquisition and "conscious language learning" (p. 2). These two processes differ in the awareness of the individual on gaining the knowledge of a new language. Krashen relates acquisition to a natural and meaningful interaction with the new language, one in which the individual is more focused on communicating and understanding the message than on the structure and sounds of the language. According to Krashen, in the acquisition process, corrections are result of the process, experience in the language, and the feeling of correctness by the individual. However, conscious language learning, for him, is more related to the formal learning of explicit rules, and to the correction of errors. A difference between acquisition and learning for a second language student involves the awareness of the results of the process.

Chomsky (2006) points out the difference between *competence* and *performance* in the process of learning a language. The first term, competence refers to the system of rules associated with a language and can be related to what Krashen refers to as conscious learning because it focuses on the awareness and knowledge of the rules of the specific language. The term performance, used by Chomsky, refers to the observed language use by the individual. It is similar to Krashen's acquisition process because language use can be observed through the interactions with others and factors that impact that use. Performance is not focused on grammatical rules, but rather on the context such as the situation and speaker-audience interaction.

These two theories of language acquisition/learning refer to the process of gaining proficiency in a language and mixing experiencing language (acquisition or performance) and knowing rules (learning or competence) to help individuals to master the language. Mastering a language is an on-going and continuous process; it therefore goes from being cognitively demanding to cognitively undemanding while becoming proficient in the new language (Cummins, 1984). The process of mastering a language, according to Cummins (1984) and Cummins and Swain (1986), is related to two factors: cognitive demand and context. The relationship between these two factors is represented in Figure 3.

According to Cummins (1984) and Cummins and Swain (1986), the vertical axis represents the level of cognitive demand needed for communicating, which relates directly to proficiency in language. Lower proficiency indicates that a communicative activity demands more cognitive processes. For the horizontal axis, the activities placed on the left side are those more embedded in a context, in which the participants can negotiate word meanings based on situations or context. For those activities, communication is not mainly based on linguistic rules of the language but is supported by different tools available in the situation. Thus, these situations involve active participation of individuals engaged in the situation. On the other hand, context reduced situations do not involve cues from the environment. For those situations, communication is more dependent on the accurate use of linguistic rules to avoid misinterpretations in the communication.

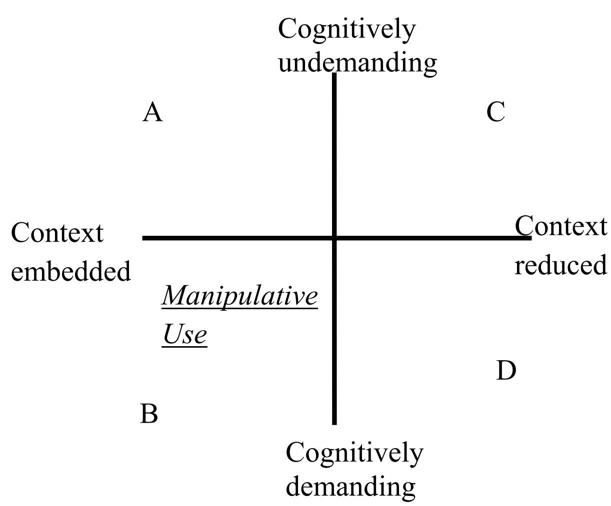

Figure 3. Manipulative use in Cummins' quadrants.

Adapted from *Cummins, Jim, and Swain, Merrill, BILINGUALISM IN EDUCATION: Aspects of theory, research and practice., ©1986, p. 135. Persmission granted from Pearson Education, Inc., Logman Group Limited. London, UK*

The interaction between these two factors is described by the four quadrants, called Cummin's quadrants: (A) cognitively undemanding and context embedded, (B) cognitively demanding and context embedded, (C) cognitively undemanding and context reduced, and (D) cognitively demanding and context reduced. Within these four quadrants, Cummins (1984) and Cummins and Swain (1986) point out that both context and cognitive demand are aspects to consider in the relationship between students' language proficiency and academic achievement. This theoretical framework has been used to describe appropriate strategies for English language learners (ELL).

Following Cummins'(1984) and Cummins' and Swain's (1986) quadrants descriptions, we can describe situations that occur for ELLs in school. If teachers have second language students in their classrooms, activities may need to be more embedded in context to help students decrease the cognitive demand required for language and communication. Hands-on activities have been found to be appropriate instructional methods for this particular type of students requiring high cognitive load due to lack of language proficiency (Cummins, 1998). Herrell (2000), using the Cummins' quadrants, presents 50 teaching strategies that fit a situation or quadrant. She includes manipulative use for mathematics learning in quadrant (B), cognitively demanding and context embedded situations, as a possible strategy to help ELLs. Herrell describes manipulatives as devices that students can move around and that help to "support [students'] thinking and learning" (p. 126). In addition, activities involving manipulatives have been suggested to help ELLs (see Lee, Silverman, & Montoya, 2002; Lee & Jung, 2004).

In summary, experiences in a second language environment should involve context embedded situations to reduce cognitive demand due to lack of proficiency in the language. This could be beneficial in ELL environments in which high cognitive load is required for learning content as well as for communicating or using language. In mathematics instruction for English language learners, the cognitive load required for communication may be reduced through the use of manipulatives which may provide context embedded situations during instruction.

Manipulative Use and Student's Performance

In this section, I discuss results from different studies that attempt to evaluate the relationship between manipulative use and student performance. The effectiveness of instructional techniques or tools on students' overall learning has most often been measured through achievement scores on tests and tasks. Measuring the effectiveness of student manipulative use has been done in a similar manner. The effectiveness of manipulative use has been studied using several different research designs, such as pretest-posttest with control group design (e.g., Drickey, 2006; McClung, 1998; Posadas, 2004), and pretest-posttest without control group design (e.g., Bolyard, 2005; Reimer & Moyer, 2005; Suh & Moyer, 2007; Suh, 2005; Trespalacios, 2008). While some researchers have found a positive relationship between manipulative use and student achievement (e.g., Bolyard, 2005; Suh, 2005; Trespalacios, 2008), others have found partial positive results (Reimer & Moyer, 2005), no differences between groups of students using and not using manipulatives (Drickey, 2006; Posadas, 2004), or even lower results in manipulative user's groups (McClung, 1998).

Suydam and Higgins (1977) analyzed results from 23 studies that compared groups of students using manipulatives and groups not using these tools. In 11 studies, students using manipulatives scored significantly higher than the non-manipulative group of students; in 10 studies there was no significant difference between the groups; and in two studies the students not using manipulatives scored higher. After analyzing confounding variables present among studies such as differences between teachers and the goals of studies, the authors concluded that groups of students using manipulatives should have higher average achievement than groups of students not using manipulatives.

However, other studies using the same design have found lower performance by the treatment group (see Fennema, 1972; Sowell, 1989; McClung, 1998). McClung (1998) compared two classes of high school students enrolled in an Algebra I course. The treatment group used Algeblocks while the control group used worksheets for practice. In this study, both groups worked for nine weeks in cooperative learning groups and the teaching method was similar. For the control group, the teaching strategy was "lecture, homework and in class work sheets" (p. 27); for the treatment group worksheets were replaced with manipulatives. Student achievement was assessed before and after treatment using a test constructed by the teacher. McClung found that the control group scored significantly higher than the treatment group. According to the author, two possible factors affecting the lower results of the manipulative group of students were the teachers' lack of knowledge about using these tools in their classroom as well as students' lack of familiarization with the devices. It may also be the case that the manipulatives were not used to help build concepts, but only for practicing what was taught by traditional lecture instruction.

The majority of the studies found in the literature examine the relationship between students' mathematics performance and manipulative use using pretest-posttest designs. In one study, Posadas (2004) included multiple measures of achievement during the treatment period, in other words, she used a repeated measures control design. In this study, Posadas (2004), aimed to examine if manipulative use and visual cues help ELLs' mathematics learning. Sixty-four Hispanic students who failed four of the mathematics objectives of the Texas Assessment of Academic Skills (TAAS) participated in this study. The objectives assessed were: "use of the operator of Multiplication to solve problems", "use of the operation of Division to solve problems", "estimate solution to a problem situation", and "determine solution strategies and analyze or solve problems" (p. 49). One day per week for 5 weeks, students worked on a specific objective. During instruction, students in the treatment groups used either manipulatives or visual cues. Participants' performance was measured 6 times during the 5 weeks of treatment, once before treatment and once every week of treatment. Posadas found no significant difference between the treatment groups and the control group performances. One possible reason for these results, as stated by Posadas, was the short period of time that students were given to use the tools.

Other studies have used pretest-posttest without control group designs to measure the success of manipulative use. Among these studies, some found significant improvement in students' scores when they used manipulatives during learning activities (e.g., Bolyard, 2005; Suh & Moyer, 2007; Suh, 2005; Trespalacios, 2008; Whitmire, 2006). Trespalacios (2008) compared two generative instructional strategies, answering questions and generating examples using virtual manipulatives to teach fractions to third grade students during one day. In this repeated measures study, students' knowledge of fractions was tested once before the treatment and twice after. He found that students' scores, in both groups, increased significantly after treatment and were maintained two weeks after the instruction.

In another study, Suh and Moyer (2007) studied differences in the impact of two types of manipulatives, physical and virtual, on students' achievement in algebra. According to the authors in both groups, teachers first introduced the manipulatives to their students. Then, students in the physical manipulative group completed algebraic equations on a worksheet prepared by the teacher. Students in the virtual manipulative group completed algebraic equations created by computer software. They found that both groups of students achieved higher scores on their posttest after one week of using the manipulatives. However, without a control group it is difficult to attribute learning solely to manipulative use.

Yet not all the studies using pretest-posttest without control group designs have found positive relationships between manipulative use and student performance (e.g., Remier & Moyer, 2005). In a study by Remier and Moyer (2005), the teacher gave students the opportunity to manipulate virtual base ten blocks applets before the treatment. This was done to provide students with experience manipulating similar applets to those that were going to be used in the study. During treatment, the teacher introduced students to an applet for working with fractions. On a worksheet, the teacher provided instructions for the applet and the exercises to be solved using the virtual manipulatives. All materials were designed by the teacher. Reimer and Moyer found students' conceptual knowledge scores increased significantly after the treatment, but students' procedural knowledge scores did not.

In summary, studies measuring the impact of manipulative use have employed different designs and often found different results. Some studies use control groups to measure the impact of manipulative use, while other studies did not. Studies also varied in the amount of time students used manipulatives, e.g., one day (Trespalacios, 2008), one week (Suh & Moyer, 2007), five weeks (Posadas, 2002), or as long as nine weeks (McClung, 1998). Moreover, in all the studies discussed, the analyses involved either achievement at a single point in time or student learning measured using only two time points. Measuring learning based on just two achievement measures does not necessarily provide an accurate measure of learning since the two points determine a fixed line while a regression fitted to more than two points may provide a more acurate representation of achievement growth over time. No studies have evaluated individual students' variations in growth or change, i.e., individual student's learning, for an extended period of time, when studying manipulative use.

Learning versus Achievement

In education, the terms achievement and learning are often used synonymously, however, although they are related, they are different. Student achievement refers to a student's knowledge at a certain point in time; for example, a student's score on a midterm exam represents the achievement of that student at that specific time. Achievement represents a snap-shot of what students know. On the other hand, learning represents a change in performance (Driscoll, 2005). Driscoll explains that learning is "a persisting change in human performance or performance potential" (p. 9). For Snelbecker (1985), the term learning refers to change and process. How an individual changes her/his performance in a certain situation is a way to infer how much learning has occurred (Snelbecker, 1985). Again, achievement at a certain time point represents the performance of a student at that specific time. However, a student's change in achievement indicates the student's learning process.

Student learning is the purpose for instructional activities (Willett, 1988). Student learning could be used as a measure to evaluate manipulative use as an instructional strategy in mathematics classrooms. According to Willett (1988), learning "implies *growth* and *change*" (p. 346). In order to evaluate the impact of teaching practices, it is necessary to use measures of change and growth (Willett, 1988). Several studies have examined the impact of varied factors on students' learning processes (e.g., Chang, Singh, & Mo, 2007; Ma & Wilkins, 2007; Wilkins & Ma, 2002, 2003). In these studies, student learning is modeled using multiple measures of student achievement over time; that is, these are longitudinal studies.

Regarding manipulative use, there are no long-term longitudinal studies evaluating the impact of manipulative use on student learning to date. Moreover, Sowell (1989), in a meta-analysis of 60 studies, found that when students use manipulatives for a period of time of at least a year that students achievement increases. However, no recent studies involving manipulative use for long periods of time exist. In order to evaluate the effects of manipulative use on student learning, it is necessary to examine the relationship between manipulative use and student learning over a long period of time, such as throughout elementary school years.

Manipulative Use and Teacher Characteristics

In general, teachers' instructional practices have been found to be influenced by different variables such as their beliefs, knowledge, context, attitudes, and previous experiences (Barkatsas & Malone, 2005; Beswick, 2005; Ernest, 1989; Thompson, 1984; Wilkins, 2008). In particular, teachers' use of manipulatives has also been found to be related to teacher variables. In this section, I first discuss some of the general literature related to teacher beliefs and their impact on teaching strategies. Then, I discuss literature on teacher beliefs and manipulative use in mathematics classrooms. Finally, I discuss literature related to teaching strategies and manipulative use.

Teachers' Beliefs and Teaching Strategies

One factor that has been shown to influence teaching practices in mathematics classrooms is teacher's beliefs (Ernest, 1989; Nespor, 1987; Peterson et al., 1989; Philipp, 2007; Raymond, 1997; Speer, 2008; Thompson, 1984, 1992; Wilkins, 2008). In her qualitative study, Thompson (1984) wanted to relate the conceptions, also referred to as beliefs, about mathematics and mathematics teaching of three junior high school teachers to their teaching practices. In her study, Thompson (1984) collected data about teaching practices from direct observation and about teacher beliefs through individual interviews. Even though not all of the participants showed a direct relationship between their beliefs about teaching mathematics and their teaching practices, the author found a strong relationship between teachers' beliefs about mathematics and their teaching practices. In addition, she found that the relationship between beliefs and teaching practices not to be a simple one. She identified differences between the three teachers' beliefs about mathematics. In two of the three teachers, she found relationships between teacher beliefs about teaching mathematics and their teaching practices.

For example, Jeanne, one of the teachers in the study, believed mathematics is an exact and static subject wherein established rules and concepts are connected. Kay, another of the participants, believed that mathematics (1) is established by an axiomatic method, (2) may change, and (3) assists other areas such as science. The other teacher, Lynn, had similar beliefs to Jeanne's; mathematics as an established axiomatic subject. However, she believed mathematics is unchangeable subject, and she visualized mathematics as a set of unrelated rules that help to find an answer (Thompson, 1984). These teachers differed in their teaching behavior. According to the author, Jeanne's classes were based on concepts, connections between them and following existing rules. Kay's classes were more related to problem-solving activities connected to other areas, and Lynn's practices were mostly presenting a procedure and having students follow her presentation. Moreover, Lynn's practices reflected her beliefs about mathematics, but not mathematics teaching. Even though she believed, for example, that activities requiring logical reasoning and that students' participation is important in mathematics classes, she did not propose these types of activities to her students or give opportunities for active student involvement during mathematics instruction (Thompson, 1984). For Jeanne and Kay, beliefs about mathematics teaching and teaching practices were found to be related.

In Thompson's (1984) study, the teaching practices of all three teachers were found to be related to their beliefs, including Lynn's practices. However, as presented in Thompson's (1984) study, a teacher's beliefs about mathematics and mathematics teaching can be opposite. Therefore, teachers' actions may reflect some of their beliefs and oppose others. In general in Thompson's study, teachers' actions are related to some of their beliefs about mathematics and teaching mathematics.

Other studies have analyzed the relationship between specific teachers' beliefs and/or specific teaching practices (Beswick, 2005; Speer, 2008; Wilkins, 2008). For example, in a quantitative study with 481 participants, Wilkins (2008) studied the relationship between teachers' beliefs about inquiry-based teaching methods and the use of those methods in the mathematics classroom. In this study, he evaluated relationships between teachers' attitude toward mathematics and mathematic teaching, beliefs about the effectiveness of inquiry-based instruction and the use of these instructional practices. Using a path analysis on Ernest's (1989) theoretical model, he found a positive relationship between teachers' beliefs about the effectiveness of inquiry-based instruction and the use of this type of instruction. This relationship was the strongest one found in the model when compared to the other variables (such as content knowledge and attitude). Moreover, Wilkins (2008) found that teachers' attitude toward mathematics and mathematics teaching also had a positive relationship to the use of inquiry-based practices. The relationship between mathematics content knowledge and inquiry-based teaching practices, however, was found to be negative.

In a quantitative study with 25 teachers, Beswick (2005) studied the relationship between teachers' beliefs about problems-solving and constructivist learning practices. In this study, data were collected from teachers' reported beliefs and students' reports of events in these teachers' classroom. Teachers were asked to complete a beliefs survey about mathematics teaching and mathematics as subject matter. Then teachers were asked to administer the constructivist learning environment survey (CLES with small changes) to their own students in two classes. The results of the teacher belief survey were scored indicating their level of Problem-solving view; so higher scores indicate a higher problem-solving orientation view (Beswick, 2005). In this study, higher responses from the CLES rather than lower results in this survey indicated that the respondent understands that the classroom environment is more constructivist. The results of this study indicate that the classrooms of the group of teachers that had a tendency toward problem-solving had students who saw their classroom as more of a constructivist environment. These teachers viewed "mathematics as more than computation and were less inclined to believe that it was their role to provide answers or even clear solution methods" (p. 54) and they expressed high beliefs about problem-solving situations and student-centered activities. These teachers were graded higher by their students as having created a constructivist environment than how students of the other teachers did. These results, according to Beswick (2005), indicate a relationship between teachers' reported beliefs about constructivist environments and their practices as determined by their students (Beswick, 2005).

Other studies have found inconsistencies regarding the relationship between teacher beliefs and teaching practices (e.g., Raymond, 1997), possibly due to the complexity of beliefs systems. As Leatham (2006) explains, beliefs are "sensible systems" and researchers need to understand and recognize this when studying beliefs. Beliefs are held in different ways and they may change over time (Green, 1971). Green (1971) explains an organization of beliefs that may help to explain actions, describing this organization as a system in different dimensions.

Beliefs can be organized into a three-dimensional system: by logical structure, by psychological strength, and/or by clusters (Green, 1971). By logical structure, according to Green (1971), people organize beliefs as primary or derivative; this type of organization is if a belief is being held by itself or if it is derived from another. An example of this organization of beliefs would be a teacher believing that mathematics teachers should allow students to use manipulatives in their classes (derivative belief) because the use of these tools in mathematics classes helps students to learn mathematics (primary belief).

Through psychological strength, according to Green (1971), people organize beliefs as central or peripheral. This classification is related to the degree of importance that these beliefs have to the individual and therefore how open to change the beliefs are. The more open beliefs are, the more peripheral they are. An example that shows this categorization is that a teacher may believe finishing the curriculum is needed for students to succeed; the same teacher may value manipulative use in learning mathematics and also believe that activities involving manipulative use require more time than traditional lecture. Whether a belief is central or peripheral would be indicated by the actions of the teacher; if he chooses not to use manipulatives because of time, the first belief will be stronger than the belief that manipulatives help students learn mathematics.

The last structure in which Green (1971) classifies beliefs is by clusters. In this dimension he states that we hold our beliefs in clusters and those clusters are separate; there is no relation between clusters. An example of this dimension has been shown previously, a cluster represents to time management in the classroom and another cluster represents how students learn. In this case, a person who knows what value the teacher gives manipulative use in the classroom may find a contradiction in the teacher's actions when his/her teaching strategies do not involve manipulative use. If the teacher belief about time management is not known, the teacher seems to be teaching against his/her teaching beliefs. Different clusters may have different psychological strength when in turn determines people's actions. How people, particularly teachers, hold their beliefs may change over time (Green, 1971). Knowing that the beliefs system is changeable and multidimensional may help trainers when they try to help teachers reassess their beliefs.

The communication of beliefs between researchers and teachers, what teachers want to say and what they are actually saying, are factors impacting the results of studies about teachers' beliefs (Leatham, 2006). Leatham (2006) says that teachers may have problems reporting their beliefs and researchers may struggle trying to interpret teachers' beliefs from their actions, interpretations and/or words. Leatham (2006) emphasizes viewing beliefs as a "sensible system" in which some beliefs are stronger than others—an idea similar to Green's (1971). Therefore, understanding teachers' beliefs systems is important to recognize difficulties in the communication of beliefs between teachers and researchers (Leatham, 2006). Knowing their beliefs and making teachers reflect on their different beliefs and actions may foster professional development and assist teachers in changing their beliefs in order to improve their teaching practices to help students learn.

Teachers' Beliefs and Manipulative Use

Knowing that teachers' beliefs and teachers' teaching practices are related according to the literature discussed above, it is interesting to examine the results in regards to beliefs about manipulatives. Information regarding teachers' beliefs about manipulative use relates to teachers' reasons for using these tools in the classroom (see Bolyard, 2005; Crawford & Brown, 2003; Suh & Moyer, 2007).

Teachers may use manipulatives in their classroom but the reasons they believe they should use these tools are not highly studied in the literature. Suydam (1984), reporting results from questionnaire data, states that “most teachers indicate that they believe that manipulative materials should be used for mathematics instruction” (p. 27). She does not, however, explain or present reasons that teachers have for using these tools in their classroom, but other researchers have presented some reasons.

Manipulatives have been proposed as tools in mathematics classrooms because these tools may help students learn mathematics with understanding (Hiebert et al., 1997), but teachers and mathematics educators have different reasons for using these tools in classrooms. For example, some mathematics educators assume that these tools motivate students (Bolyard, 2005; Cain-Castro, 1996; Crawford & Brown, 2003; Ferrer-Weiss, 2005; Herbert, 1985; Reimer & Moyer, 2005; Suh & Moyer, 2007), and some teachers believe these tools serve as a reward for students (Moyer, 2001). In this section of the document, I identify reasons that teachers and mathematics educators report for using manipulatives in their mathematics classrooms. I first present teachers’ reasons for manipulative use as identified in the literature. Then, I present some reasons found by researchers in mathematics education. Reasons from teachers and researchers seem to be similar, but teachers’ information is not detailed enough to get a clear understanding of their reasons.

Howard, Perry and Tracey (1997), studying survey answers from 939 teachers of different levels (603 primary and 336 secondary), compared manipulative use between primary and secondary teachers. They found that a majority of the teachers (94% primary teachers and 84% secondary teachers) report that these tools are beneficial for students’ learning process. However, the benefits of using these devices are not explained in the document. Teachers may thus believe that manipulatives help without being clear as how. Another reason teachers report using manipulatives is that students enjoy them (64% primary teachers and 53% secondary teachers). Teachers believe these tools motivate students because they enjoy using them but explanations of how or why students enjoy manipulating these devices are not presented and may not be clear for teachers. Additional reasons indicated by teachers include school mathematics policy (15% primary teachers and 8% secondary teachers) and syllabus mandate (10% primary teachers and 4% secondary teachers) but these reasons seem to have less influence on teachers' decision to use manipulatives in their classrooms. It is not clear if the teachers in this study chose many of these options or if they wrote reasons on their own (open-ended question). If teachers have chose from a set of given options, their expression of reasons may be limited. Teachers may have other more important reasons for why and how teachers use these tools in their classrooms. If authors coded teachers' responses, their codification process is not described in the document; readers cannot have a clear idea of teachers' thoughts and authors' coding process of teachers' reasons for using manipulatives in their classrooms.

Sherman and Richardson (1995) found that 76% (19 of 25 teachers) of their participants report using manipulatives and they summarize teachers' reasons for using these tools. The most common reason (31% or 6 of the 19 teachers using manipulatives) was that students using manipulatives have opportunities to learn "via senses of touch and sight provid[ing] students a strong foundation for understanding a concept" (p. 30). The second most common reason (26% or 5 of the 19 teachers) was the concrete manipulative property; teachers identify that mathematical concepts can be perceived in a concrete way when using manipulatives. Improvement in students' conceptual understanding was reported as a reason for 2 of the 19 teachers (11%). However, as Sherman and Richardson (1995) express, how manipulatives actually help to improve understanding was not specified by the teachers.

The reasons given by teachers in this study are similar to what Howard, Perry and Tracey (1997) found as the most common reason: benefit to students' learning process. Sherman and Richardson (1995) provide more information about how teachers see manipulative use helping students' processes. The reasons for manipulative use reported in Sherman and Richardson (1995) were: (1) by using several senses (2) to help give concreteness to abstract concepts and (3) conceptual understanding. Even though these three reasons could be together when teachers explain their reasons of using manipulatives in their classrooms, separating them gives us more information about teachers understanding of manipulatives. For example, a teacher reported that through using these tools, students "can see and feel concepts" (Sherman & Richardson, 1995, p. 30) and that manipulatives allow students to see and feel concepts in a concrete way. These tools help introduce what is abstract in mathematics as well as increase students' understanding. However, these teachers may value the use of senses in learning more than the fact that manipulatives give a concrete representation of mathematical concepts or increase students' understanding. In this open-ended question, researchers interpreted teachers' reported reasons for using these tools and seem to include one reason for each teacher who reports using manipulatives in her/his classroom.

More recent researchers have been presenting more ideas to support the use of manipulatives in mathematics classrooms (e.g., McNeil & Jarvin, 2007). McNeil and Jarvin (2007) present three of them: manipulatives provide additional resources, manipulatives help to connect mathematics with real-world experiences, and manipulatives help memory and understanding because of the physical movements required while using these tools. In this theoretical document, McNeil and Jarvin (2007) support their ideas on some empirical studies to inform teachers about manipulatives research. Knowing these ideas, one possible reason teachers may be willing to use manipulatives in their classrooms when they are looking for ways to help students' mathematical understanding (McNeil & Jarvin, 2007).

Many other reasons motivate manipulative use in mathematics classrooms. One is the trend toward students' involvement in mathematics instruction because students are "active" when they work with these tools (Hatfield, 1994; Suydam, 1984). Motivation is another reason to support manipulative use in mathematics classes because lessons including manipulatives are fun and engage students in class activities that involve work with these tools (Moyer, 2001; Herbert, 1985). For example, Moyer (2001) studied how 10 teachers used manipulatives in their classroom and found that some of these teachers used manipulatives with the purpose of giving students enjoyment and fun. After analyzing data from teachers' interviews, Moyer (2001) found that teachers refer to manipulative use as play time or reinforcement time which they differentiate from "real math" (p. 185) where "they taught rules, procedures and algorithms using textbooks, notebooks, worksheets, and paper and pencil tasks" (p.185). She states that because students seem to like activities involving manipulative use, teachers use those activities as rewards for students' behavior (Moyer, 2001) instead of as active tools in helping them to learn mathematical concepts.

Teachers may want to use manipulatives if they are looking for ways to involve students in mathematics classes (Hatfield, 1994; Suydam, 1984; Suydam & Higgings, 1977), make mathematics lessons more enjoyable (Herbert, 1985; Moyer, 2001), and/or help students learn mathematics (McNeil & Jarvin, 2007; Suydam, 1984; Suydam & Higgings, 1977). However, the way to use these tools in order to accomplish those objectives is not clearly established in the literature. Teachers may assume that just because they use these tools, their lessons are enhanced, successful and entertaining. The use of manipulatives, however, does not guarantee learning (Ball, 1992; Baroody, 1989; Fennema, 1972; Kamii, Lewis, & Kirkland, 2001).

Teaching strategies and manipulative use.

Research relating instructional practices and teacher variables has found teacher's beliefs to be highly related to teacher instructional practices (Wilkins, 2008). However, there are other teacher variables that are also related to instructional practices (Ernest, 1989). Therefore, in addition to identifying teacher beliefs about manipulative use, it is important to look for other aspects related to manipulative use in the literature. Also found in the literature is: information related to ways of using manipulatives (see Howard, Perry, & Tracey, 1997), classroom settings while manipulatives are used in mathematics classrooms (Sherman & Richardson, 1995), and factors impeding manipulative use in mathematics classrooms (Gilbert & Bush, 1988; Hatfield, 1994; Sherman & Richardson, 1995). Moreover, research has found that teacher characteristics are related to manipulative use (Gilbert & Bush, 1988; Howard, et al., 1997; Opdenakker & Van Damme, 2006; Weiss, 1994).

Ways of using manipulatives. Studies report that teachers use manipulatives in different ways. For example, Howard, Perry and Tracey (1997), reporting separate results for primary school teachers and secondary school teachers, state that the most common way to use manipulatives for both grade levels is by demonstration from the teacher (83% primary and 67% secondary teachers). In that study, for elementary school teachers, the second most common way of using manipulatives is that students choose ways to use these tools (71% teachers), whereas agreement between teacher and students is the second most common way (59% teachers) for the secondary school teachers. Other reasons reported are for remedial support and checking work by students. This classification was focused mostly on which students used the manipulatives and how much control teachers had over their use.

Classroom settings and manipulative use in mathematics classrooms. Sherman and Richardson (1995) studied whether manipulatives are used for a whole class or a small group and whether they develop a concept or practice skills. They found that a majority of teachers (89%) use manipulatives to develop a concept for the whole class, while just 37% of the teachers reported that they used manipulatives to develop concepts for small groups. Forty seven percent of teachers reported using manipulatives for practice in whole class or in small groups. Individual practice was the least frequent way that teachers reported using manipulatives (11%).

Factors impeding manipulative use. There are some studies that report factors that teachers identify as impeding their use of manipulatives in their classrooms. Some of the factors that affect teachers are: teachers' knowledge about the tools, classroom management, availability of materials, time, management of materials, and cost of them (Gilbert & Bush, 1988; Hatfield, 1994; Sherman & Richardson, 1995). These factors are ranked differently in the literature.

Teacher competence or knowledge in using manipulatives is reported, by teachers, as a factor in all three studies. However, Gilbert and Bush (1988) report that this is not as important as lack of time needed to use these materials. Lack of time is considered to be an important factor because some teachers understand that activities involving manipulatives require more time (Sherman & Richardson, 1995) or, as reported by a teacher in Gilbert and Bush's (1988) study, "time could be better utilized with other instructional approaches" (p. 466). Classroom management is another factor teachers report; it seems manipulative work is viewed as a strategy where students could get out of control because they may get "overly enthusiastic working with manipulatives" (Sherman & Richardson, 1995, p. 33). When students are using manipulatives in their classroom they "would talk much more in mathematics classes-with their peers in small groups as well as in whole-class discussions" (Cohen & Ball, 1990, p. 235). In addition, some teachers avoid using manipulatives because parents and others at the school believe that manipulatives are games and are counterproductive to teaching (Sherman & Richardson, 1995).

Teacher characteristics and manipulative use. Teacher characteristics have been found to be related to manipulative use. For example, teacher grade level has been found to be related to manipulative use (Howard et al., 1997; Kloosterman & Harty, 1987; Weiss, 1994). The frequency with which teachers' use manipulatives in their classrooms differs between the elementary, middle and high school level. Weiss (1994) reported that, in general, the use of manipulatives in mathematics classes increased from the mid 1980's to 1993; however, the frequency with which teachers used manipulatives was found to differ by grade level. Elementary school teachers were found to use manipulatives more often than middle school teachers. High school teachers were found to use manipulatives the least. In addition, Howard, Perry and Tracey (1997) comparing elementary and secondary mathematics school teachers' views of manipulatives in mathematics instruction, found that just 4% of the secondary teachers reported using manipulatives in every lesson while 55% of their colleagues at the elementary level reported manipulative use in every lesson.

Other studies considering the use of manipulatives at the elementary school level have found further differences between elementary school teachers (Gilbert & Bush, 1988; Howard et al., 1997; Kloosterman & Harty, 1987; Malzahn, 2002; Suydam, 1984). Howard, et al. (1997) found that manipulative use varied by grade at the elementary school level, with K-4 grade teachers using manipulatives more often than their colleagues in grades 5 and 6. In addition, Malzahn (2002), analyzing national data, reported that the use of manipulatives was different for grade K-2 students and grade 3-5 students. For the K-2 group, it was found that more than 50% of the students used manipulatives in all or almost all of their mathematics classes, and just 15% of the other group used these tools that often. This finding is consistent with results from Kloosterman and Harty (1987) who investigate data from 301 elementary school principals. They found manipulatives used more often in K-2 grade classrooms than in 3-5 grade classrooms. In addition, Suydam (1984) found teachers of second grade or higher reported less use of manipulatives than teachers in grades K-1, and use decreased as the grade level increased. Moreover, for primary grade level teachers, Gilbert and Bush (1988), working with teachers from first to third grade, found that the use of manipulatives decreased among teachers in these three grade levels, but they did not report how the three groups of teachers statistically compared to each other across grades.

Based on past research, manipulative use is related to the grade level of the teacher. However, because of inconsistencies in these results, it is possible that there are other teacher characteristics, not controlled in prior research, that interact with or explain these differences. For example, perhaps teachers who believe that the use of manipulatives is important for children's learning tend to teach in the lower grades and vice versa. In this case, teachers' beliefs influence their choices and not just the elementary curriculum. Instead of just knowing that manipulative use is related to grade, and by considering the interrelationship of several teacher variables, in addition to grade level, we may better understand why teachers' choose to use manipulatives.

Other teachers' background characteristics have been found to be related to teaching practices (Gilbert & Bush, 1988; Howard, Perry & Tracey, 1997; Opdenakker & van Damme, 2006). Opdenakker and van Damme (2006) found teachers' characteristics, such as classroom management skills and job satisfaction, to be related to their teaching style. For example, they found that teachers with high job satisfaction tend to give more support to their students, and those with good classroom management skills have a good relationship with their students. Specifically considering the use of manipulatives as a teaching practice, several studies have investigated its relationship with elementary school teachers' characteristics (Gilbert & Bush, 1988; Howard, Perry, & Tracey, 1997; Raphael & Wahlstrom, 1989). The results of these studies are mixed. For instance, Gilbert & Bush (1988) studied 220 teachers of first, second and third grade from 12 states in the United States and found that less experienced teachers used manipulatives more often. On the other hand, Raphael and Wahlstrom (1989), using data from 103 8 grade teachers in Ontario from the Second International Mathematics Study, found that experienced teachers used manipulatives more often than less experienced teachers. In addition, Howard et al. (1997) studied 900 elementary and secondary mathematics teachers from the southwestern suburbs of Sydney and the North Coast of New South Wales, and found no relationship between manipulative use and elementary mathematics teaching experience. Moreover, no relationship was found between manipulative use and other teacher characteristics such as gender, position in school, school system, and teacher preparation (Howard et al.).

Connecting the Literature Review and the Two Studies

This literature review encompasses two bodies of literature related to the indirect relationship between teacher variables and student learning mediated by manipulative use in mathematics classrooms. Each body of literature is associated to one of the two relationships created in the indirect path. The first one relates teacher variables to manipulative use, and the second relates manipulative use to student learning.

Several studies have investigated the relationship between teacher variables and manipulative use. Studies involving manipulative use have explored teacher beliefs (Howard, Perry & Tracey, 1997; Moyer, 2001; Sherman & Richardson, 1995), grade level (Gilbert & Bush, 1988; Kloosterman & Harty, 1987; Malzahn, 2002; Weiss, 1994), and other background characteristics (Gilbert & Bush, 1988; Howard, Perry & Tracey, 1997; Opdemakker & Van Damme, 2006). However, no studies have investigated interaction between all those factors and related then to manipulative use in the mathematics classroom. In Chapter 3, I present a manuscript of a study of teacher variables (i.e., grade, teacher's age, experience, and beliefs) that may be related to manipulative use at elementary school level. In this study, in addition to studying the relationship between those factors and manipulative use, I also investigate their interrelations as predictors. By investigating their interrelations, interaction between variables is controlled and outcomes better show the variables, among those included, that are highly related to manipulative use.

In reference to student learning, the literature shows that manipulative use seems to be related to student learning. However, findings are mixed and inconclusive. Learning models such as the Information processing model and the Experiential learning model are two of the models where manipulative use can be placed as tools to help student mathematics learning. However, studies measuring manipulative use's effectiveness have found inconclusive results. Different research designs have been used to investigate this relationship, as well as different treatment time. However, no studies were found to use growth and change to measure student learning while using manipulatives for a period longer than a year. In Chapter 4, I present a study that relates student mathematics learning, operationalized as achievement growth over time, to manipulative use at elementary school level. In this study, in addition to studying the relationship between manipulative use and student mathematics achievement growth, students' home language is used as a potential moderator.

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Chapter 3. Elementary School Teachers' Manipulative Use

Mathematical learning that focuses on understanding is an important goal of school mathematics (NCTM, 2000). As suggested by Hiebert et al. (1997) there are many tools that teachers can use to help students develop mathematical understanding. One such set of tools is physical materials such as base-ten blocks or fraction tiles. These are examples of mathematical manipulatives. In the literature, there is no unique definition for mathematical manipulatives. For example some definitions describe manipulatives as objects that can be touched, moved about and rearranged (Kennedy, 1986); other definitions suggest that manipulatives can also be stacked (Clement, 2004); while others, such as Moyer (2001), emphasizes that these objects should represent "explicitly and concretely mathematical ideas that are abstract" (p. 176). Mathematical manipulatives offer students a way of understanding abstract mathematical concepts by enabling them to connect the concepts to more informal concrete ideas.

Even though manipulatives have been proposed as useful tools to help students gain mathematical understanding, teachers differ in how they use manipulatives in their mathematics classrooms (Hatfield, 1994; Howard, Perry, & Tracey, 1997). In education, differences in practices have been related to several factors, such as teachers' beliefs, content knowledge and attitude (Ernest, 1989; Wilkins, 2008). In relation to teachers' use of manipulatives as part of their instructional practice, studies have focused on teachers' characteristics such as experience (Gilbert & Bush, 1988), grade level (Weiss, 1994), and beliefs (Moyer, 2001), however, these studies considered these factors separately. Therefore, it is not clear if and how these different factors interrelate to influence teachers' use of manipulatives. In this study, we will investigate how three factors, grade level, teachers' background characteristics and teachers' beliefs about manipulatives affect how often elementary school teachers use manipulatives during their mathematics instruction. In addition, this study will examine the interrelationship of these variables as they relate to manipulative use. In particular, this study will consider the following research questions:

1) Is there a relationship between teachers' use of manipulatives and the grade level that they teach?
2) Is there a relationship between teachers' use of manipulatives and teachers' background characteristics (e.g., age, years experience)?
3) Is there a relationship between teachers' use of manipulatives and their beliefs about manipulatives?
4) How do teachers' grade level, beliefs, and other background characteristics interrelate as predictors of teachers' manipulative use?

Manipulatives in Mathematics Instruction

Even though many teachers and educators support the use of manipulatives (NCTM, 2000; Hiebert et al., 1997; Kennedy, 1986; Suydam & Higgins, 1977), results from studies about the effectiveness of manipulative use are inconclusive. While some studies report that students benefit from using manipulatives in their mathematics lessons (Bolyard, 2005; Clements, 2007, 1999; Moyer, 2001; Suh, 2005; Suydam, 1984; Suydam & Higgins, 1977; Trespalacios, 2008), others have found an opposite result or no result at all (Drickey, 2006; McClung, 1998; Posadas, 2004; also see Fennema, 1972; Sowell, 1989).

Bain, Lintz, and Word (1989) found that all "effective teachers" in their study, those whose students' achievement was in the top 15%, reported using manipulatives to teach mathematics, and 98% of them specifically state they use concrete objects in their classrooms.

More recently, Clements (2007) used the *Building Blocks Assessment of Early Mathematics, PreK-K* to evaluate students performance after using "building blocks" software and found that students who used manipulatives outperformed students in the control group.

Moreover, the idea that the use of manipulatives in mathematics classrooms helps students to develop mathematical understanding is supported in the literature (Kennedy, 1986; Reimer & Moyer, 2005). For instance, Kennedy (1986) concludes that "[l]earning theories and evidence from research and classroom practice support the use of manipulative materials to help children learn and understand mathematics" (p. 7). In a research study with third grade students using virtual manipulatives to learn fractions, Reimer and Moyer (2005) state that there was a significant improvement in students' conceptual knowledge of fractions after working with the virtual manipulatives.

However, positive results have not always been found. Sowell (1989) and Fennema (1972), using meta-analysis, found several studies that report negative results. In a more recent study, McClung (1998), comparing a group of students using Algeblocks to a group of students not using them, found that the students not using the manipulatives scored significantly higher on the posttest than those using the Algeblocks. The teaching method for the non-manipulative group was "lecture, homework and in-class work sheets" (p. 27) and while the manipulative group was treated similarly, instead of worksheets the students used manipulatives. According to the author, some possible factors affecting the lower results of the manipulative group were the teacher's lack of knowledge about using these tools in their classroom as well as students' lack of familiarization with the devices.

Manipulatives by themselves cannot bring about understanding (Baroody, 1989; Ball, 1992; Kamii, Lewis, & Kirkland, 2001; Viadero, 2007). The importance of using manipulatives is the quality of student's thinking promoted by using them (Kamii, Lewis, & Kirkland, 2001), as well as the abstractions that can be reached from the interaction with these objects, as promoted by Piaget (Kohler, 2008). How teachers design their classroom activities involving manipulatives will ultimately affect the success of their use on student understanding.

Teachers' Use of Manipulatives

Results from studies considering manipulative use are varied. One of the reasons for these results could be how teachers use them in their classrooms. Not all teachers use manipulatives, and those who use these tools do not necessarily use them the same way. For example, some teachers use manipulatives more frequently than others. Making decisions related to instructional practice is complex and may be related to several factors such as teachers' content knowledge, beliefs, background characteristics, or attitude (Ernest, 1989; Wilkins, 2008). Factors identified in the literature as affecting how often teachers use manipulatives in their classroom are grade level (Howard, et al., 1997; Weiss, 1994), teachers' background characteristics (Gilbert & Bush, 1988; Howard, et al., 1997; Opdenakker & Van Damme, 2006), and beliefs (Gilbert & Bush, 1988; Howard, et al., 1997; Raphael & Wahlstrom, 1989).

Grade Level

The frequency with which teachers' use manipulatives in their classrooms differs between the elementary, middle and high school level. Weiss (1994) reported that, in general, the use of manipulatives in mathematics classes increased from the mid 1980's to 1993, however, the frequency with which teachers used manipulatives was found to differ by grade level. Elementary school teachers were found to use manipulatives more often than middle school teachers. High school teachers were found to use manipulatives the least. In addition, Howard, Perry, and Tracey (1997) comparing elementary and secondary mathematics school teachers' views of manipulatives in mathematics instruction, found that just 4% of the secondary teachers reported using manipulatives in every lesson while 55% of their colleagues at the elementary level reported manipulative use in every lesson.

Other studies considering the use of manipulatives at the elementary school level have found further differences between elementary school teachers (Gilbert & Bush, 1988; Howard, et al., 1997; Kloosterman & Harty, 1987; Malzahn, 2002; Suydam, 1984). Howard, et al. (1997) found that manipulative use varied by grade at the elementary school level, with K-4 grade teachers using manipulatives more often than their colleagues in grades 5 and 6. In addition, Malzahn (2002), analyzing national data, reported that the use of manipulatives was different for grade K-2 students and grade 3-5 students. For the K-2 group, it was found that more than 50% of the students used manipulatives in all or almost all of their mathematics classes, and just 15% of the other group used these tools that often. This finding is consistent with results from Kloosterman and Harty (1987) in an investigation on reports from 301 elementary school principals. They found that manipulatives were used more often in grade K-2 classrooms than in grades 3-5 classrooms. In addition, Suydam (1984) found that teachers of second grade or higher reported less use of manipulatives than teachers in grades K-1, and use decreased as the grade level increased. Moreover, for primary grade level teachers, Gilbert and Bush (1988), working with teachers from first to third grade, found that the use of manipulatives decreased between teachers in these three grade levels, but they did not report how the three groups of teachers statistically compared to each other across grades.

Based on past research, manipulative use is related to the grade level of the teacher. However, because of inconsistencies in these results, it is possible that there are other teacher characteristics, not controlled for in prior research, that interact with or explain these differences.

For example, perhaps teachers who believe that the use of manipulatives is important for children's learning tend to teach in the lower grades and vice versa. In this case, teachers' beliefs influence their choices and not just the elementary curriculum. Instead of just knowing that manipulative use is related by grade, by considering the interrelationship of several teacher variables, in addition to grade level, we may better understand why teachers' choose to use manipulatives.

Teacher Background

Teachers' background characteristics have been found to be related to teaching practices (Gilbert & Bush, 1988; Howard, Perry & Tracey, 1997; Opdenakker & Van Damme, 2006). Opdenakker and Van Damme (2006) found that teachers' characteristics such as classroom management skill and job satisfaction are related to their teaching style. For example, they found that teachers with high job satisfaction tend to give more support to their students, and those with good classroom management skills have good relationships with their students. Specifically considering the use of manipulatives as a teaching practice, several studies have investigated its relationship with elementary school teachers' characteristics (Gilbert & Bush, 1988; Howard, Perry & Tracey, 1997; Raphael & Wahlstrom, 1989). The results of these studies are mixed. For instance, Gilbert & Bush (1988) studied 220 teachers of first, second and third grade from 12 states in the United States and found that less experienced teachers used manipulatives more often. On the other hand, Raphael and Wahlstrom (1989), using data from 103 8th grade teachers in Ontario from the Second International Mathematics Study, found that experienced teachers used manipulatives more often than less experienced teachers. In addition, Howard et al. (1997) studied 900 elementary and secondary mathematics teachers from the southwestern suburbs of Sydney and the North Coast of New South Wales, and found no relationship between manipulative use and elementary mathematics teaching experience. Moreover, no relationship was found between manipulative use and other teacher characteristics such as gender, position in school, school system, and teacher preparation (Howard et al.).

Teachers Beliefs

Teachers' beliefs about mathematics and mathematics teaching have been identified as an important factor associated with classroom practice (Ernest, 1989; Nespor, 1987; Thompson, 1984, 1992; Wilkins, 2008). For example, Nespor (1987) presents an example of a professor who believed that his students would be more interested in the study of math if its applications were addressed in class; therefore, he designed his instruction to incorporate mathematical applications. Thompson (1984) found teachers' beliefs to be an important factor related to teachers' instructional behavior. In her study, she collected data related to teachers' instructional practices through direct observation and data related to teachers' beliefs from individual interviews. Even though not all of the participants showed a direct relationship between mathematics teaching beliefs and teaching practices, the author reported a strong relationship between beliefs about mathematics and teaching practices. In addition, she found that the relationship between beliefs and teaching practices is not a simple one to understand.

Understanding this relationship has been the focus of many studies in mathematics education. For example, Ernest (1989) developed a theoretical model of mathematics teacher's knowledge, attitudes, and beliefs that suggests the complexity of teachers' decision making process associated with their teaching practices. Wilkins (2008), based on this model, used path analysis in a study of 481 elementary teachers and found teachers' beliefs to be the strongest predictor of teaching practices, among teachers' content knowledge, attitudes, and other background characteristics.

Research specifically considering manipulative use and its relationship to teachers' beliefs has provided inconclusive evidence (Perkkilä, 2003; Suydam, 1988; Moyer, 2001; Howard et al., 1997). For example, in a research report, Suydam (1984) indicates that even though teachers believe manipulatives to be tools that could help students develop meaning in mathematics, second grade or higher level teachers do not use manipulatives very often. In addition, Perkkilä (2003), who collected data from six first and second grade teachers, found teachers' beliefs about manipulatives and their practices to be inconsistent. Although teachers believed that manipulatives are helpful, there were other factors that led teachers to not use these tools (Perkkilä, 2003).

Moreover, teachers hold certain general beliefs about manipulatives and their use. For example, Perkkilä (2003) states that for teachers in their study, "the use of manipulatives in teaching/learning situations was often regarded as useful for promoting the view that 'math is fun'" (p. 3). In a qualitative study of manipulative use, Moyer (2001) found that teachers associated manipulatives with 'fun math' and not with 'real math,' and as a result, in general, they did not use manipulatives for teaching and learning concepts. These findings also suggest that teachers' beliefs and actions, in particular manipulative use, are related.

The Present Study

Mathematical understanding is essential for elementary school students, and manipulatives are tools that teachers can use to help students build understanding. Research suggests several factors that may affect elementary school teachers' use of these devices, in particular, grade level that they are teaching, teachers' background characteristics and teachers' beliefs. No studies to date have analyzed the interrelationship of these three variables and manipulative use. In the current study, we investigated the relationship between teachers' use of manipulatives in their mathematics instruction and their beliefs about manipulatives, grade they teach, and teachers' age and teaching experience, and further, investigated the interrelationship among these variables. In this investigation we wanted to answer the following research questions:

1. Is there a relationship between teachers' use of manipulatives and the grade level that they teach?
2. Is there a relationship between teachers' use of manipulatives and teachers' age and years of experience?
3. Is there a relationship between teachers' use of manipulatives and their beliefs about manipulatives?
4. How do teachers' grade level, beliefs and other background characteristics interrelate as predictors of teachers' manipulative use?

Methods

Data

This study analyzed data collected from 530 in-service elementary school teachers from two school districts in the southeastern part of the United States. These teachers were participants in a professional development experience.

Measures

Teachers were asked to complete a survey at the beginning of the professional development experience. The survey contained items related to their beliefs, attitudes, and instructional practices associated with mathematics and mathematics teaching. Three questions on the survey were related to teachers' use of manipulatives in the classroom using the following prompt for each: "In your mathematics lessons, how often do you usually ask students to do each of the following? (1) Use manipulatives to solve exercises or problems; (2) Use manipulatives to explore a concept; and (3) Engage in hands-on mathematical activities." These items were rated on a 5-point Likert scale (1=*Never*, 2=*Rarely* [a few times a year], 3=*Sometimes* [once or twice a month], 4=*Often* [once or twice a week], and 5=*All or almost all mathematics lessons*). These three items were found to have internal consistency as a single measure of manipulative use (Cronbach's $\alpha = .90$). These items were used to create a MANIPULATIVE variable which was used as an indicator of the frequency with which teachers used manipulatives in their mathematics lessons.

Teacher background characteristics included in this study are age of teachers (AGE) (1=*under 25,* 2=*25-29,* 3=*30-39*, 4=*40-49,* 5=*50-59,* 6=*60 or more*), and teaching experience (EXPERIENCE) (1= *0-2 years*, 2= *3-5 years*, 3=*6-10 years*, 4=*11-15 years*, 5=*16-20 years*, 6=*21-25 years*, 7=*26 or more years*). To facilitate interpretation of these two variables (AGE and EXPERIENCE) their values were recoded. Values for the variable AGE were recoded by taking the value in the middle of the range for each of the options. For the lower endpoint we used a value of 23 and for the upper endpoint we used a value of 62. Therefore, the coded values are *23 minimum*, *27*, *35*, *45 55*, and *62 maximum.* The values of EXPERIENCE were coded by taking the value in the middle for the first six options and adding two more years to the minimum value of option 7. Then, the new values for EXPERIENCE are *1*, *4*, *8*, *13, 18*, *23*, and *28.* Teachers' grade level (GRADE) (0=Kindergarten to 5=Grade 5) was also included as a variable.

Three questions on the survey were related to teachers' beliefs about manipulatives. Two were based on the following prompt: "In your opinion, how important are each of the following for effective mathematics instruction in the grades you teach? (1) Provide concrete experiences before abstract concepts (CONCRETE); (2) Have students participate in appropriate hands-on activities (HANDSON). These items were rated on a 4-point Likert scale (1=*not important*, 2= *somewhat important*, 3=*fairly important*, 4=*very important*). Teachers were also asked to indicate their level of agreement with the following item using a 6-point Likert scale (1=*Strongly disagree*… 6=*Strongly agree*): (3) Since older students can reason abstractly, the use of manipulatives becomes less necessary (OLDER).

Results and Discussion

From the total of 530 teachers, 27 of them did not complete all the items from the questionnaire. Teachers with missing items were removed from the sample resulting in a working sample of 503 teachers. Descriptive statistics for the variables for the teachers are shown in Tables 1 and 2.

To investigate possible differences in teachers' manipulative use by grade level, a one-way analysis of variance (ANOVA) was conducted. The results show that there are significant differences ($F = 48.22$, $p < 0.05$) in teachers' use of manipulatives by teacher grade. Using a Dunnett's T3 post hoc analysis, we found three grade groups with statistically significant mean differences (see Table 2). The first group is the Kindergarten teachers who were found to use manipulatives most often. The second group consisted of Grade 1-2 teachers, and the third group included Grade 3-5 teachers. Grade 3-5 teachers were found to use manipulatives least often.

In order to consider the relationship between manipulative use and teacher grade, background, and beliefs, a series of regression models were estimated, culminating in a final predictive model including all the variables. Model 1 considered only grade level allowing us to investigate previous findings related to grade level (e.g., Malzahn, 2002, Howard et al., 1997; Gilbert & Bush, 1988; Suydam, 1984). Based on results from the ANOVA three dummy coded variables were created KINDER for kindergarten teachers, FIRSTSECOND for first and second grade teachers, and THIRDFIFTH for third through fifth grade teachers. The variable FIRSTSECOND was used as the contrast group (i.e., FIRSTSECOND not included in model). The variable MANIPULATIVE was regressed on KINDER and THIRDFIFTH. This first model basically replicates the ANOVA, but allows us to control for the effects of grade in subsequent models.

Model 2 was the background model and MANIPULATIVE was regressed on AGE and EXPERIENCE, and it allowed us to investigate the relationship between those background characteristics and manipulative use and compare it to previous findings (e.g., Howard et al., 1997; Raphael & Wahlstrom, 1989; Gilbert & Bush, 1988). For model 3, the beliefs model, MANIPULATIVE was regressed on the three teacher beliefs about manipulatives (CONCRETE, OLDER and HANDSON); this model allowed us to investigate the relationship between teachers' beliefs about manipulatives and manipulative use and compare them to previous findings (Perkkilä, 2003; Suydam, 1988; Moyer, 2001; Howard et al., 1997). The final model included all seven variables, so MANIPULATIVE was regressed on all the variables KINDER, THIRDFIFTH, AGE, EXPERIENCE, CONCRETE, OLDER and HANDSON. This model extends previous research by investigating the interrelationship among all the variables and MANIPULATIVE, and how each variable is related with MANIPULATIVE while the other variables are controlled in the model. The results of each model are described below.

From Model 1 (see Table 3), as expected from the ANOVA, grade variables explain a significant proportion of variance (F=112.54, $p < 0.01$) with KINDER ($\beta = 0.19$, $p<0.01$) and THIRDFIFTH ($\beta = -0.445$, $p <0.01$) statistically different from first-second grade teachers. In addition, as expected, the contrast group, first and second grade teachers' frequency of manipulative use, was in the middle of the other two groups.

From Model 2, AGE (β = -1.61, $p < 0.05$) was found to be a statistically significant predictor of manipulative use. According to the results, older teachers, on average, tend to use manipulatives less often than younger teachers. There is also a statistically significant relationship between EXPERIENCE (β = 0.14, $p < 0.05$) and manipulative use. Experienced teachers tend to use manipulatives more often than novice teachers. However, the amount of variance explained by AGE and EXPERIENCE is relatively small, indicating that these background characteristics have very little predictive power in teacher manipulative use.

From Model 3, there is a significant proportion of variance explained by teacher beliefs (F = 36.67, $p < 0.01$), but just two of the three variables, HANDSON (β = 0.40, $p < 0.01$) and OLDER (β = -0.08, $p < 0.05$), were found to be statistically significant predictors of manipulative use. Based on these results, teachers who believe that it is important for students to participate in appropriate hands-on activities use manipulatives more often than teachers who do not agree with the statement. Also, teachers who do not agree with the statement: "Since older students can reason abstractly, the use of the manipulatives becomes less necessary" tend to use manipulatives more often than teachers who agree with it, which suggest that teachers who think that older students do not need manipulatives tend to use manipulatives less often than their colleagues who agree with the statement.

The Final Model including all seven variables, beliefs about manipulatives (CONCRETE, OLDER and HANDSON), grade variables (KINDER and THIRDFIFTH), and teacher characteristics (AGE and EXPERIENCE) explains a significant proportion of variance in the MANIPULATIVE variable (F = 49.72, $p < 0.01$). In this model, four out of the seven variables were found to be statistically significant predictors of manipulative use. Those variables were OLDER (β = -0.10, $p < 0.05$), HANDSON (β = 0.29 $p < 0.01$), KINDER (β = 0.17 $p < 0.01$) and

THIRDFIFTH ($\beta = -0.39$, $p < 0.01$). Interaction effects associated with the variables were not found to be statistically significant, and were excluded from the model.

Considering the interrelationship among the variables, while AGE and EXPERIENCE were statistically significant predictors of manipulative use in model 1, after controlling for grade and beliefs, these variables were no longer statistically significant. Further, while the overall direction of the relationship between KINDER, THIRDFIFTH, OLDER, HANDSON and MANIPULATIVE remained the same, the magnitude of the coefficients was reduced indicating possible shared variance among these variables (see Table 3). An examination of the correlations between HANDSON and THIRDFIFTH (see Table 1) finds a statistically significant, although small, negative correlation between these variables indicating that teachers in grades 3-5 tend not to believe in the importance of having children participate in appropriate hands-on activities. Consistent with this finding there is a small positive correlation between HANDSON and KINDER indicating that teachers in kindergarten tend to believe that it is important for children to participate in hands-on activities. Together these findings suggest that while grade does predict manipulative use, that some of the differences may be further due to the beliefs of teachers who choose to teach in these grades.

Conclusion

The aim of this study was to investigate the relationship between teacher grade, background characteristics, and beliefs, and the interrelationship among these variables, and how often teachers use manipulatives in their mathematics instruction. Results from this study suggest that teachers' beliefs and the grade level that they teach are important predictors of how often elementary school teachers use manipulatives in their mathematics instruction. However, teacher background characteristics were not found to be consistent predictors of manipulative use.

Kindergarten teachers were found to use manipulatives more often than the other grade-level teachers, and teachers in grades 3-5 were found to use manipulatives least often. Further, this relationship does not change when teachers' background characteristics and beliefs were controlled. This result is consistent with previous research showing less use of manipulatives in the higher grades at the elementary school level (Gilbert & Bush, 1988; Howard, et al., 1997; Kloosterman & Harty, 1987; Malzahn, 2002). However, results of this study indicate further differences at the primary level as Kindergarten teachers were found to use manipulatives more often than teachers in grades 1-2 which is different from previous research (Howard, et al., 1997; Kloosterman & Harty, 1987; Malzahn, 2002) and provides evidence of a tendency for a decrease in manipulative use between teachers of first, second and third grade (cf. Gilbert & Bush, 1988).

Although teachers' years of classroom experience and age were found to be related to manipulative use when considered alone, after controlling for teacher grade and beliefs these variables were no longer statistically significant predicators of manipulative use. This finding helps to clarify the mixed findings of previous research (cf. Gilbert & Bush, 1988; Howard, et al., 1997, Raphael & Wahlstrom, 1989) and suggests that teacher background characteristics alone do not help explain the important instructional mechanisms that determine teachers' choice to use manipulatives in their classroom. Consistent with results indicating a relationship between teachers' beliefs and their instructional practices (Moyer, 2001; Nespor, 1987; Thompson, 1984, 1992; Wilkins, 2008), teachers' beliefs about manipulative use were found to be related to their use of manipulatives in their mathematics instruction. Teachers who tend to believe that it is important to have students participate in appropriate hands-on activities for effective mathematics instruction tend to use manipulatives more frequently in their mathematics lessons. Further, teachers who tend to believe that the use of manipulatives with older students is less necessary were found to use manipulatives less often. These relationships remained salient even after teachers' background characteristics and grade were controlled.

By considering the interrelationship among previously studied variables related to manipulative use, we were better able to determine those variables that are more likely to add to our understanding of teachers' manipulative use with further study; that is, based on this study, teacher beliefs and teacher grade level. Further study should focus on why grade level makes a difference. For example, there may be grade level curricular differences that influence the use of manipulatives. However, findings from this study also hint at a relationship between teachers' beliefs and the grade level that they teach. While this study was not able to indentify this relationship as a strong one, the evidence suggests a possible avenue for further study to better understand why teachers choose to use manipulatives in their classroom.

Limitations of Study

It is important to point out several limitations of the present study. First, this study was conducted as a secondary analysis on data that were collected for a purpose other than the study of manipulative use. Thus, there were only a few questions related to manipulatives available for the current study. Second, the word *manipulative* was not defined on the survey, thus teachers may have had different conceptions of manipulatives and their uses. Third, the data collected was self-reported and teachers' reported practices may be different than what might be observed in their classroom. Finally, it was not possible to identify the curriculum that teachers were using at the time the survey was competed, which could be another factor affecting the frequency of manipulative use during mathematics instruction. Future research would benefit from additional questions related to manipulative use and teachers' beliefs. In addition, the use of a common definition for teachers to think about when answering questions about manipulatives might help to alleviate differences in conceptions. In addition to the self-reported data it would be helpful to observe some teachers to see how they use manipulatives in their classroom or ask more specific questions that would allow teachers to describe how they are used in their classroom. Finally, asking teachers about the curriculum that they use might help to control for a curriculum effect associated with teachers' use of manipulatives.

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Table 1. Descriptive statistics

Variable	1	2	3	4	5	6	7	8	*M*	*SD*
1. MANIPULATIVE	-								4.12	0.80
2. AGE	-.06	-							40.10	10.73
3. EXPERIENCE	.02	.75**	-						12.45	9.10
4. CONCRETE	.11*	.02	.06	-					3.80	0.49
5. HANDSON	.42**	-.04	-.02	.23**	-				3.85	0.39
6. OLDER	-.13**	.03	-.01	-.00	-.12**	-			4.64	1.22
7. KINDER	.39**	-.04	-.01	.06	.15**	.01	-		0.20	0.40
8. FIRSTSECOND	.22**	-.08	-.05	.02	.10*	.02	-.38**	-	0.36	0.48
9. THIRDFIFTH	-.53**	.10*	.06	-.07	-.23**	-.02	-.45**	-.66**	0.44	0.50

Note: $N = 503$; $^{*}p < .05$, $^{*}p < .01$

Table 2. Means for manipulative use by grade.

	Grade					
	K	1	2	3	4	5
MANIPULATIVE	4.73[a]	4.44[b]	4.25[b]	3.82[c]	3.53[c]	3.53[c]
N	102	96	84	76	76	69

Note: Means with same letter are not statistically significant from each other.

Table 3. Standardized coefficients for each regression model

Variable	Model 1 β	Model 2 β	Model 3 β	Final model β
KINDER	$.19^{***}$			$.17^{***}$
THIRDFIFTH	$-.45^{***}$			$-.39^{***}$
AGE		$-.16^{*}$		-.07
EXPERIENCE		$.14^{*}$		$.10^{a}$
CONCRETE			.02	.00
OLDER			$-.08^{*}$	$-.10^{**}$
HANDSON			$.40^{***}$	$.29^{***}$
R^2	.31	.01	.18	.41

Note: $^{*}p<0.05$, $^{**}p<0.01$, $^{***}p<0.001$, $^{a}p=0.057$

Chapter 4. Manipulative use and elementary school students' mathematics learning

If teachers provide opportunities in which students can connect their previous knowledge to the instructional activities, students may be better able to build their knowledge with understanding (Baroody & Coslick, 1998; Brooks & Brooks, 1999). It is important for students to learn mathematics with understanding as well as be active participants in their learning process, and teachers should provide opportunities in their classrooms to support this (NCTM, 2000). The experiential learning model by Kolb (1984) can be utilized to explain how active involvement in the process may support student mathematical learning in mathematics classrooms. Participating in experiences allows students to build their mathematical knowledge because "knowledge is created by transforming the experience" (Kolb 1984, p. 42). Kolb's model is a four-stage cycle composed by concrete experience, reflective observation, abstract conceptualization, and active experimentation. Knowledge is created by grasping experience, either by concrete experience or abstract conceptualization, and transforming it, either by reflective observation or active experimentation. Each of the four combinations is an elementary learning form, and "the combination of all four of the elementary learning forms produces the highest level of learning, emphasizing and developing all four modes of the learning process" (Kolb, 1984, p. 66).

For the experiences provided in the classroom, teachers can use various tools to help students achieve knowledge and understanding. One of the tools is concrete objects such as mathematical manipulatives (Hiebert et al., 1997). Manipulative use could be the "concrete experience" (Kolb, 1984) students can have for their mathematics learning process, and they could construct their knowledge from that experience. Algeblocks, geometric shapes, base-ten blocks, and pattern blocks are examples of manipulatives that may be found in mathematics classrooms. In general, manipulatives are defined as objects that can be touched, moved about, and rearranged or stacked (Clements, 2004; Hynes, 1986; Kennedy, 1986). In addition to physical manipulatives, there are also virtual manipulatives available to be used in mathematics classrooms, such as those provided by the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vLibrary.html) and the Manipula Math with Java (http://www.ies.co.jp/math/java/index.html). Virtual manipulatives are interactive computer representations of the concrete or physical manipulatives (Dorward, 2002) that students can manipulate to help them construct mathematics knowledge (Moyer, Bolyard, & Spikell, 2002). Both physical and virtual manipulatives give students an opportunity to work with representations of mathematical ideas and construct mathematical understanding (see Hiebert, et al., 1997; Moyer, Bolyard, & Spikell, 2002)..

Studies investigating the relationship between manipulative use and student mathematical performance have been mixed; some studies have found a positive relationship (Bolyard, 2005; Suh, 2005; Trespalacios, 2008), while others have found a negative (McClung, 1997), partial (Reimer & Moyer, 2005) or no relationship (Drickey, 2006; Posadas, 2004). These mixed results suggest that there may be other factors affecting the relationship between manipulative use and students' mathematics learning. One such factor is the amount of time that students use these tools during their mathematics instruction. For example Sowell (1989), in a meta-analysis of 60 studies, found that when manipulatives are used for a period longer than a year students' performance is affected positively; however, most studies to date do not involve such a long treatment period.

Willett (1988) indicates that when evaluating the impact of a teaching strategy, study designs should include several measures of achievement in order to analyze change over time or learning. To date, there are no longitudinal studies analyzing student's mathematics learning as described by Willett, in order to examine the role of manipulatives in such learning.

The NCTM Equity Principle, presented in *Principles and Standards for School of Mathematics* (NCTM, 2000), calls for support of every student's mathematics learning process. NCTM (2002) states that "all students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study – and support to learn- mathematics" (p. 12). Therefore, students should not be impaired because of language (Khisty, 1995), and teachers should use strategies that support all students' learning. English Language Learners (ELL) affront learning situations in their classrooms in which language may limit their possibilities to succeed. Cummins (1986) indicates that language acquisition is related to two continua factors: context and cognitive demand of the situations. Strategies to help these students reduce cognitive demand due to lack in language have been advocated to be used in classrooms (Cummins, 1998; Herrell, 2000). Herrell (2000) proposes manipulative use as a strategy to be used in mathematics classrooms to help ELLs' mathematics learning process.

The present study examines the relationship between student mathematics learning and manipulative use at the elementary school level. In addition, this study also explores socio-economic status (SES), race-ethnicity, and home language as possible moderators of this relationship. The research questions are:

1. Is there a relationship between elementary aged students' manipulative use and students' mathematics learning?
2. Does this relationship vary by SES, race-ethnicity, and home language?

3. Is there a relationship between Hispanic elementary aged students' manipulative use and these students' mathematics learning?
4. Do home language and SES moderate the relationship between Hispanic elementary aged students' manipulative use and these students' mathematics learning?

Background Literature

In this section, we first discuss how manipulative use and students' mathematics performance have been studied. Then, we discuss manipulative use as a potential strategy to help ELLs in their mathematics learning process. Finally, we discuss differences between mathematics learning and mathematics achievement, how they are measured, and the best way to evaluate the effectiveness of instructional strategies (i.e., manipulative use).

Manipulative Use and Student's Performance

The effectiveness of instructional techniques, strategies or tools on students' learning in general has most often been measured through the use of achievement scores on tests and tasks. Measuring the effectiveness of student manipulative use has been done in a similar manner. The effectiveness of manipulative use has been studied using several different research designs, such as pretest-posttest with control group design (e.g., Drickey, 2006; McClung, 1998; Posadas, 2004), and pretest-posttest without control group design (e.g., Bolyard, 2005; Reimer & Moyer, 2005; Suh & Moyer, 2007; Suh, 2005; Trespalacios, 2008). While some researchers have found a positive relationship between manipulative use and student achievement (e.g., Bolyard, 2005; Suh, 2005; Trespalacios, 2008), others have found mixed results (e.g., Reimer & Moyer, 2005), no results (e.g., Drickey, 2006; Posadas, 2004), or negative results (e.g., McClung, 1998).

Suydam and Higgins (1977) analyzed results from 23 studies that compared groups of students using manipulatives and groups not these tools. In 11 studies, students using manipulatives scored significantly higher than the non-manipulative group of students; in 10 studies there was no significant difference between the groups; and in two studies the students not using manipulatives scored higher. After analyzing confounding variables present between studies such as differences between teachers and the goals of studies, the authors concluded that groups of students using manipulatives should have higher average achievement than groups of students not using manipulatives.

However, other studies using the same design have found lower performance by the treatment group (e.g., McClung, 1998, also see Fennema, 1972; Sowell, 1989; McClung, 1998). McClung (1998) compared two classes of high school students enrolled in an Algebra I course. The treatment group used Algeblocks while the control group used worksheets for practice. In this study, both groups worked for nine weeks in cooperative learning groups and the teaching method was similar. For the control group, the teaching strategy was “lecture, homework and in class work sheets” (p. 27), for the treatment group worksheets were replaced with manipulatives. Student’s achievement was assessed before and after treatment using a test constructed by the teacher. McClung found that the control group scored significantly higher than the treatment group. According to the author, two possible factors affecting the lower results of the manipulative group of students were the teacher’s lack of knowledge about using these tools in their classroom as well as students’ lack of familiarization with the devices. However, it may also be the case that the manipulatives were not used to help build concepts, but only for practicing what was taught by traditional lecture instruction.

The majority of the studies found in the literature study students' mathematics performance and manipulative use using pretest-posttest designs. In one study, Posadas (2004) included multiple measures of achievement during the treatment period, in other words, used a repeated measures control design. In this study, Posada (2004), aimed to study if manipulative use and visual cues help ELLs' mathematics learning. Sixty-four Hispanic students who failed in four mathematics objectives of the Texas Assessment of Academic skills (TAAS) participated in this study. The objectives assessed were: "use of the operator of Multiplication to solve problems", "use of the operation of Division to solve problems", "estimate solution to a problem situation", and "determine solution strategies and analyze or solve problems" (p. 49). One day per week, for 5 weeks students worked on a specific objective, and during instruction either manipulatives or visual cues were used for either one of the treatment groups. Participants' performance was measured 6 times in the 5 weeks of treatment, one before treatment and one every week of treatment. Posadas found no significant difference between the treatment groups and the control group performances. One possible reason for these results, as stated by Posadas, was the short period of time that students were given to use the tools.

Other studies have used pretest-posttest without control group designs to measure the success of manipulative use. Among these studies, some found significant improvement in students' scores when they used manipulatives during learning activities (e.g., Bolyard, 2005; Suh & Moyer, 2007; Suh, 2005; Trespalacios, 2008; Whitmire, 2006). Trespalacios (2008) compared two generative instructional strategies, answering questions and generating examples, using virtual manipulatives to teach fractions to third grade students during one day. In this repeated measures study, students' knowledge of fractions was tested once before the treatment and twice after it. Trespalacios found that students' scores, in both groups, increased significantly after the treatment and were maintained two weeks after the instruction.

In another study, Suh and Moyer (2007) studied differences in the impact of two types of manipulatives, physical and virtual, on students' achievement in algebra. According to the authors, in both groups, teachers first introduced the manipulatives to their students. Then, students in the physical manipulative group completed some algebraic equations on a work sheet prepared by the teacher; and students in the virtual manipulative group completed algebraic equations created by the computer software used. They found that both groups of students achieved higher scores on their posttest, after a week of using the manipulatives. However, without a control group it is difficult to attribute learning to the manipulative use.

Yet, not all the studies using pretest-posttest without control group designs have found positive relationships between manipulative use and students' performance (e.g., Remier & Moyer, 2005). In a study by Remier and Moyer (2005) a teacher gave students the opportunity to manipulate virtual base ten blocks applets before the treatment. This was done to provide students experience with manipulating similar applets to those that were going to be used in the study. During treatment, the teacher introduced students to an applet for working with fractions. On a worksheet, the teachers provided instructions for the applet and the exercises to be solved using the virtual manipulatives. All materials were designed by the teacher. Reimer and Moyer found students' conceptual knowledge scores increased significantly after the treatment, but students' procedural knowledge scores did not.

In summary, studies measuring the impact of manipulative use have employed different designs and often found different results. Some studies use control groups to measure the impact of manipulative use, while other studies did not. Studies also varied in the amount of time using manipulatives, e.g., one day (Trespalacios, 2008), one week (Suh & Moyer, 2007), five weeks (Posadas, 2002), or as long as nine weeks (McClung, 1998). Moreover, in all the studies discussed, the analyses involved only groups mean comparisons of achievement (i.e., group learning growth and change). No studies conducted have examined individual students' variations in growth or change, that is, student's learning.

Learning versus Achievement

In education, the terms achievement and learning are often used synonymously, however, although they are related, they are different. Achievement refers to a student's knowledge at a certain point in time; for example, a student's score on a midterm exam represents the achievement of that student at that specific time. Achievement represents a snap-shot of what students know. On the other hand, learning represents a change in performance (Driscoll, 2005). Driscoll explains that learning is "a persisting change in human performance or performance potential" (p. 9). For Snelbecker (1985) the term learning refers to change and process. How an individual changes her/his performance in a certain situation is a way to infer how much learning has occurred (Snelbecker, 1985). Again, achievement at a certain time point represents the performance of a student at that specific time. However, a student's change in achievement indicates the student's learning process.

Student learning is the reason of instructional activities (Willett, 1988). Student learning could be used as a measure to evaluate manipulative use as an instructional strategy in mathematics classrooms. According to Willett (1988), learning "implies *growth* and *change*" (p. 346). In order to evaluate the impact of teaching practices, it is necessary to use measures of change and growth (Willett, 1988). Several studies have examined the impact of varied factors on students' learning process (e.g., Chang, Singh, & Mo, 2007; Ma, & Wilkins, 2007; Wilkins,

& Ma, 2002, 2003). In these studies student learning is modeled using multiple measures of student achievement over time; that is they are longitudinal studies.

Regarding manipulative use, there are, to date, no long term longitudinal studies evaluating the impact of manipulative use on student learning during. Moreover, Sowell (1989), in a meta-analysis of 60 studies, found that when students use manipulatives for a period of time of at least a year that their achievement is increased. However, no recent studies involving manipulative use for long periods of time have been found in the literature. In order to evaluate the effects of manipulative use on student learning, it is necessary to examine the relationship over a long period of time, for example elementary school years.

Manipulative use and English language learners (ELL)

In mathematics classrooms, students whose home language is not English may need to deal with situations in which language obstructs their ability to communicate with peers and the teacher (Khisty, 1995). Theories on language acquisition provide a framework for understanding the process that students go through while learning a language. By understanding the process we can better understand how particular instructional strategies (e.g., manipulative use) can help ELLs while learning subjects such as mathematics. In this section, I discuss two theories about language acquisition and how manipulative use fits into these theories as a teaching strategy for aiding ELLs' mathematics learning.

There are varied but similar theories about the process of language learning/ acquisition. For example, Krashen (1981) discusses a difference between language *acquisition* and *learning* or "conscious language learning" (p. 2). These two processes differ in the awareness of the individual on gaining the knowledge of a new language. Krashen relates acquisition to a natural and meaningful interaction with the new language, in which the individual is more focused on communicating and understanding the message than on the structure and sounds of the language. According to Krashen, in the acquisition process, corrections are result of the process, experience in the language, and the feeling of correctness by the individual. However, conscious language learning is more related to formal learning of explicit rules, as well as to the correction of errors (Krashen, 1981). A difference between acquisition and learning for a second language student involves the awareness and focus on the results of the process.

Chomsky (2006) points out the difference between *competence* and *performance* in the process of learning a language. The first term, competence, which refers to the system of rules associated with a language, can be related to what Krashen refers to as conscious learning because it focuses on the awareness and knowledge of the rules of the specific language. The term performance, used by Chomsky, refers to the observed language use by the individual. It is similar to Krashen's acquisition process because the use can be observed by the interactions with others and factors that impact that use. The performance is not focused on grammatical rules, but on the context such as the situation and speaker-audience interaction.

These two theories of language acquisition/learning refer to the process of gaining proficiency in a language, and mixing both experiencing language (acquisition or performance) and knowing rules (learning or competence) helps individuals to master the language. Mastering a language is an on-going and continuous process, moving from being a cognitively high demanding to a cognitively undemanding process while becoming proficient in the new language (Cummins, 1984). The process of mastering a language, according to Cummins (1984) and Cummins and Swain (1986), is related to two factors: level of cognitive demand and context. Cognitive demand refers to how many processes are needed for communication because of lack of proficiency, and context refers to the environment and tools, besides words, that can be used for the ELLs to communicate.

The interaction between these two factors is described by the the Cummin's quadrants: (A) cognitively undemanding and context embedded, (B) cognitively demanding and context embedded, (C) cognitively undemanding and context reduced, and (D) cognitively demanding and context reduced. Within these four quadrants, Cummins (1984) and Cummins and Swain (1986) point out that both context and cognitive demand are aspects to consider in the relationship between student's language proficiency and academic achievement. This theoretical framework has been used to describe appropriate strategies for English language learners (ELL).

Following Cummins'(1984) and Cummins' and Swain's (1986) quadrant descriptions, we can describe situations that occur for ELLs in school. If teachers have ELLs in their classrooms, activities may need to be more context-embedded to help students decrease the cognitive demand required for language and communication. Hands-on activities have been found to be appropriate instructional methods for this particular type of students in cognitively high demanding situations (Cummins, 1998). Herrell (2000) presents 50 teaching strategies that fit on different situations identified in Cummins' quadrants. She includes manipulative use for mathematics learning in quadrant B, cognitively demanding and context-embedded situation, as a possible strategy to help ELLs; and she describes manipulatives as devices that students can move around and help to "support [students'] thinking and learning" (p. 126).

The idea of including manipulatives in Cummins' quadrant B could be supported by information processing theory because manipulatives could help to encode information in a meaningful and/or visual way to store it in long term memory. Processing new information in a meaningful way is related to connecting it to previous knowledge (Ormord, 2004). However, students, working with manipulatives, could also make the mathematical content meaningful by connecting it with their experiences of using manipulatives. Students also code the information in a visual way. Students using manipulatives can have a “visual” representation of the abstract content and that helps students to make connections with content to such representation. These connections allow students to manipulate the mathematical content in a more concrete and manageable way. Therefore, ELLs in a classroom where manipulatives are used in classroom activities do not depend just on the language. Activities incorporating manipulatives provide a context that may help these students’ mathematics learning process.

In addition to the information processing theory, the experiential learning (EL) model by Kolb (1984) may also be related to the inclusion of manipulatives in quadrant B. There are four stages in the EL model: concrete experience, reflective observation, abstract conceptualization and active experimentation. In his model, there are two distinct ways of grasping experience or *prehension*; one is an abstract method, called *comprehension*, and the other one is a concrete or tangible way, called *apprehension.* The model also describes two different ways of *transforming* the experience, one is called *intention* which is transforming information by reflection, and the other is called *extension* which is transforming information by action (Kolb, 1984). To build knowledge, prehension and transformation should occur (Kolb, 1984).

Therefore, in cognitively demanding situations, if EL the model is used, ELLs could support their learning in the context provided by the concrete experience and/or active experimentation. Context could by embedded in the situation by using manipulatives and reflect on their actions, this is having a concrete experience and transforming by intention. As well as by using manipulatives to verify what has been comprehended, this is having an abstract experience and transforming it by extension.

Studies investigating the impact of manipulative use on the mathematics learning for ELLs are rare. Posadas (2004) investigated this relationship with 64 Hispanic students. In this five week longitudinal study, Hispanic students' achievement was measured 6 times, starting with a pre-assessment and then 5 more times, once a week, every week during the treatment. The study involved a control group, and two treatments groups (a) visual cues and (b) manipulative use. Comparing group mean differences, Posada found no significant difference between the two treatments and the control group. The author proposes the short treatment time as one of the possible reasons for the non-significant results. This is consistent with previous studies indicating the need to use manipulatives for long periods of time in order to impact mathematics learning (see Sowell, 1989).

In summary, experiences in a second language environment can be enhanced through context embedded situations to reduce cognitive demand due to a lack of proficiency in a language. This could be beneficial in ELL environments in which cognitive demand is high for learning content as well as for communicating or using the language. In mathematics instruction for English language learners, the cognitive demand required for communication may be reduced through the use of manipulatives providing context-embedded instruction. With the use of tools such as manipulatives that has been suggested to help ELLs (Lee, Dilverman, & Montoya, 2002; Lee, & Jung, 2004), the benefits would extend to all students.

Methods

Data

This study used the Early Childhood Longitudinal Study 1998/2004 (ECLS Kindergarten class of 1998-1999) database from the National Center for Education Statistics (NCES, 2006). The ECLS, a nationwide longitudinal dataset, was designed to provide data from students in

Kindergarten during the academic year 1998-99 and followed those students until eighth grade. Data include information from different sources including teachers, parents, administrators, and students. This information includes different aspects of students' development, as well as their classroom, school and home environment. In total, the data set contains seven waves of collected data for the participants: two times during the Kindergarten year, two times during first grade, and one time during each of the third, fifth and eighth grade years.

We used data from four waves to examine the longitudinal growth rates in mathematics achievement of students during their elementary school years. The four waves represent (a) baseline measure, Kindergarten Spring semester 1999; (b) First follow-up, first grade Spring semester 2000; (c) Second follow-up, third grade Spring semester 2002; (d) Third follow-up, fifth grade Spring semester 2004.

From the initial sample at the Kindergarten year (1998-1999), 83% of that sample was contacted during the spring semester of fifth grade in 2003-2004 which represents 17,527 students. The data was intended to be used to generalize to the whole population of U.S. students that were in Kindergarten during 1998-1999 school year. In order to accomplish this, NCES created sampling weights based on student characteristics and their relative relationship to the U.S. population of Kindergarteners. Weights were used in the present study to allow us to generalize to the U.S. Kindergarten population from the 1998-1999 academic year. For these four waves, students who (a) have assessment data in at least one wave or (b) who were excluded for all assessments because of a disability were assigned a weight different from zero (NCES, 2006). In total, this includes 10,673 students. In this study, all statistics refer to the weighted sample, unless otherwise indicated. From these students, 84.5% are students whose home language is English. The sample consisted of 57.6% Caucasian students, 16.0% African

American students, 18.8% Hispanic students, 2.8% Asian students, 4.6% from other race-ethnicity groups, and 3% no identified race.

Measures

Mathematics achievement. The dependent variable was mathematics achievement scores. The ECLS mathematics assessment included items from different sources such as national and state standards, commercial assessments, other NCES studies, and some created specifically for the study. The mathematics assessment included items related to topics such as number sense, properties and operations, measurement, geometry, spatial sense, data analysis, statistics and probability, patterns, algebra and functions; and the items involved conceptual and procedural knowledge and problem solving (NCES, 2006).

Students' English proficiency was tested at Kindergarten and First grade by using the Oral Language Development Scale before the mathematics assessment. Students who did not have adequate English proficiency and whose home language was Spanish were assessed using a Spanish version of the instrument (NCES, 2006). No translated version of the instruments were used for Third or Fifth grade data collection as it was felt that most students show English proficiency after First grade (NCES, 2006).

After collecting Fifth grade data, students' scores from Kindergarten through Fifth grade were adjusted to be used in longitudinal studies. Using item response theory (IRT) techniques, the ECLS staff adjusted test scores to make comparisons possible between the waves. In this technique, an *item characteristic function* is used to relate item performance to examinee characteristics (Hambleton, 1990). In the ECLS data set, those IRT mathematics scores range between 1 and 153 and represent the mathematical knowledge each student had at a certain time during her/his elementary school.

In this study, the IRT scores were used to measure growth in mathematics knowledge, or mathematics learning. We created a linear growth trajectory for each student using the four achievement scores of each student. Each student's linear trajectory was created by regressing student's achievement on time. The slopes of these lines then represent rate of change over time or learning. The operationalization of mathematics learning is described in more detail in the statistical analysis section.

Independent Variables. The primary independent variable was manipulative use. In each fall semester data was collected on the frequency that students used manipulatives in their mathematics classroom. At the Kindergarten and First grade level, two questions measuring this frequency were used. One item asked about student's use of geometric manipulatives, and the other asked about student's use of counting manipulatives. The exact teacher prompts were: *How often do children in this classroom do each of the following math activities: (a) work with geometric manipulatives,* and *(b) work with counting manipulatives.* These items were ranked on a 6- point Likert scale (1 = *Never*, 2 = *Once a month or less*, 3 = *two or three times a month*, 4 = *once or twice a week*, 5 = *three or four times a week*, 6 = *daily*). The maximum value of the two items per student as her/his measure of frequency of manipulative use was used. For example if a teacher indicated that a student used geometric manipulatives *once or twice a week* and indicated that the student *never* used the counting manipulatives, in total the student was using manipulatives in mathematics instruction at least once or twice a week.

For the third and fifth grade level, frequency using manipulatives was measured using a 4-point Likert scale (1 = *Almost every day*, 2 = *Once or twice a week*, 3 = *Once or twice a month*, 4 = *Never or hardly ever*). The teacher prompts were: *How often do children in your class engage in the following: work with manipulatives e.g., geometric shapes.* Because the teachers were asked about their students' manipulative use in a different way at different grades, values for each grade were recoded to create comparable measures across all four grades. The new values for manipulative use variables were coded as low, medium and high frequency use. In Table 4 the initial values and new values are presented. The new *low* value merges the values 1 and 2 (*Never* and *once a month or less*) for the measures in the Kindergarten and First grade, and the value 4 (*Never or hardly* ever) for the measures in the Third and Fifth grade as "*Never/hardly ever*". The new *medium* value merges the values 3 and 4 (*two or three times a month* and *once or twice a week*) for the measures in the Kindergarten and First grade, and the values of 2 and 3 (o*nce or twice a week* and *once or twice a month*) for the measures in the Third and Fifth grade, as "*between two and eight times per month*". The new *high* value merges the values of 5 and 6 (*three or four times a week* and *daily*) for the measures in the Kindergarten and First grade, and the value of 1 (a*lmost every day*) for the measure in the Third and Fifth grade, as "*almost every day*". Finally, from the new manipulative variables, 3 dummy variables were created. For LOW manipulative variable, a student obtains 1 value, for any grades in which s/he used manipulatives "never or hardly ever", and 0 otherwise. The same was done for the MEDIUM and HIGH manipulative use variables.

Student background variables came from the student questionnaire, including students' socio-economic status (SES), race-ethnicity, and home language (English or non-English). SES and race-ethnicity were composite variables created by NCES and included in the database; SES is a continuous variable ranging between -4.75 and 2.75. It was created using information about parents' household possessions, education, income and occupation during 1998-1999 school year. The race-ethnicity variable was composed from two items, one asking about belonging to any of the five race categories (White, African American, American Indian or Alaskan Native, Asian, Native Hawaiian or other Pacific Islander), and the other one asking if they were Hispanic or not (NCES, 2006). From the race-ethnicity variable in the data set, five dummy variables were created (Caucasian, African-American, Hispanic, Asian, Other). During Kindergarten Fall semester, student's home language was identified by asking parents if languages other than English were spoken at home. In the original data set, the values for the home language variable were *non-English*= 1 and *English*= 2. For this study, these values were recoded as *non-English*= 0 and *English*= 1.

In this study, we wanted to examine growth in students' mathematics learning through their elementary school years. Therefore, because we considered the first wave as the starting point for that growth, we coded the grade levels as 0 = *Kindergarten Spring semester*, 1 = *First Grade Spring semester*, 3 = *Third Grade Spring semester*, 5 = *Fifth Grade Spring semester*. This allows us to interpret the intercept of the regressed lines average achievement at Kindergarten level, and help us to evaluate the growth and change students have through the years. Because no data were collected in second and fourth grade, it was necessary to code grades to reflect the appropriate time between data points.

Analyses

Preliminary graphical analysis. Because our aim is to investigate the relationship between manipulative use and mathematics achievement growth, we started by analyzing the relationship at each grade. Students' manipulative use is likely to be different from grade to grade. For example, how often Kindergarten students used manipulatives in mathematics instruction will probably be different than how often they used manipulatives during their fifth grade year. Therefore, the impact of manipulative use on mathematics learning may also be different at different grade levels. Since data were collected at the end of each academic year, the analysis of the effect of manipulative use in a specific grade includes looking at the possible impact on mathematics achievement growth between that grade and the previous one, as well as at the achievement change between the grade manipulatives are used and the grade after. For example, high manipulative use during first grade may impact the growth in mathematics achievement from Kindergarten to first grade, as well as between first grade and the third; but the impact of high manipulative use during the fifth grade would impact just the growth in mathematics achievement from third to fifth grade.

To understand the immediate effects of manipulative use, information was separated by identifying groups of students using manipulatives with a specific frequency at each grade level (see Ma & Wilkins, 2007). This graphical analysis was the first step in our analysis of the longitudinal data. The impact of students' manipulative use on their mathematics achievement was analyzed by looking at (a) scores at the grade level where manipulative use was measured, (b) growth between that time and the previous time measured, and (c) growth between that time and the next time manipulative use was measured. This analysis involves data at specific points in time. Further analysis using data at all times followed and is described below.

Statistical Analyses. After analyzing data separately per grade, we modeled student's mathematics learning during their elementary school year. While the graphical analysis evaluated the immediate impact of the manipulative use in a specific year, the statistical analyses involve longitudinal models to evaluate the impact of manipulative use in the mathematics achievement growth during the elementary school years. The statistical analysis involved different phases. In the first phase, the analysis focused on the relationship between manipulative use and the mathematical learning of all students through their elementary school preparation and how the relationship was potentially moderated by student's home language, race-ethnicity and socio economic status (SES). The second phase involved the analysis of only Hispanic students. For the model related to this subsample, we again analyzed the relationship between student's manipulative use and mathematics achievement growth and how the relationship was moderated by student's SES and home language. We chose to analyze the sub-sample of Hispanic students because the home language for all these students is likely to be the same, i.e., Spanish. In addition, NCES provides Kindergarten and First grade students with the opportunity to present the assessment tests using a Spanish version in case of lack of English proficiency. Therefore, for this group, students' scores were less likely to be affected by the language on the assessment instrument and may reflect more about students' mathematical understanding.

A two-level hierarchical linear model (HLM) (see Hedeker, 2004; Raudenbush & Bryk, 2002) was employed to examine the relationship between manipulative use and growth in mathematics achievement at the elementary school level. Having four waves of data, we used a linear individual growth model (Raudenbush & Bryk, 2002). Level 1 of our model was a set of separate linear regressions, one per student. These equations regressed students' four mathematics achievement test scores on time measured according to the grade that data were collected (Kindergarten = 0, First grade = 1, Third grade = 3, and Fifth grade = 5), frequency of manipulative use (LOW, MEDIUM, and HIGH), and interactions between time and frequency of manipulative use.

In this study, three dummy variables for manipulative use were used. In order to maintain non collinearity it is necessary to include only two of them to the model and compare to the third. In this case, any of three possible combinations could be used to compare manipulative use. In order to determine the best model we used measure of model deviance (Hox, 2002; Raudenbush & Bryk, 2002), number of participants with sufficient data to model random effects, and theoretical judgment. Below we discuss the selected model based on these criteria; results of the analysis are discussed later.

Level 1 (growth level) The first level involved the growth in mathematics achievement scores within students, which was specified by the variables of time (*grade*), two frequency variables and the interaction variables; LOW was used as the comparison group and thus not included in the model. With this model specification, we examine the effects of manipulative use and its interaction with *grade*.

The linear level-1 or within subjects level was specified as:

$$Y_{ij} = \pi_{0i} + \pi_{1i}\,\text{grade}_{ij} + \pi_{2i}\,\text{MEDIUM}_{ij} + \pi_{3i}\,\text{HIGH}_{ij} +$$

$$\pi_{4i}\,(\text{MEDIUM}_{ij})(\text{grade}_{ij}) + \pi_{5i}\,(\text{HIGH}_{ij})(\text{grade}_{ij}) + e_{ij}$$

Where Y_{ij} is the mathematics achievement score for student *i* at time *j*, time variable indicates the *grade* that the measure was collected (*grade* values are 0, 1, 3, and 5), MEDIUM_{ij} indicates whether the frequency of manipulative use for student *i* at time *j* was MEDIUM or not with values of 0 or 1, respectively; HIGH_{ij} indicates whether the frequency of manipulative use for student *i* at time *j* was HIGH or not with values of 0 or 1, respectively. The parameter π_{0i} indicates the initial mathematics achievement for student *i* when time is zero and the use of manipulative was LOW.

The parameter π_{1i} is the coefficient for the time variable and represents the rate of growth (i.e., mathematics learning) for student *i* throughout the elementary school grades. As suggested by Ma and Wilkins (2007), it specifically represents the “natural growth” reflecting cognitive maturity of students during the elementary school level. The parameters π_{2i} and π_{3i} are the coefficients for the medium and high manipulative use frequency variables and represents the overall main effect of those variables on mathematics achievement. The effect of manipulative use is weighted in the same way for students at Kindergarten than for any other grade, then the values of those parameters are affected by grade level. Consequently, the values of these parameters cannot be meaningfully interpreted. Further, these values do not represent the growth added due to manipulative use. Added growth is represented by the interaction between growth and manipulative use, and that is because student's manipulative use may vary by time and mathematics achievement due to those changes varies as well (Hedeker, 2004). Therefore, interaction variables were included in the model and are explained below.

The parameters π_{4i} and π_{5i} are coefficients of the interactions between time and medium and high manipulative use. Different from natural growth, they represent the added growth due to medium and high manipulative use, respectively, compared to low manipulative use. To better understand these parameters, assume that student *i* at time *j* has used manipulatives almost daily, that is HIGH=1 and MEDIUM=0. In this case, the model at level 1 simplifies to:

$$Y_{ij}=(\pi_{0i} + \pi_{3i}) +(\pi_{1i} + \pi_{5i})\ grade_{ij} + e_{ij}$$

In this way, and similar to the models presented by Ma and Wilkins (2007), we can identify the parameters related to growth in mathematics achievement, π_{1i} and π_{5i}. This model, in addition to allowing us to study the growth of mathematics achievement through elementary school years, permits us to investigate the effects of manipulative use on that growth. In this case, π_{1i} is the parameter indicating the natural growth or growth due to maturity and π_{5i} indicates the growth due to using manipulatives more often. If π_{5i} is statistically significant, it means that using manipulatives more often adds significantly more to growth in mathematics achievement in addition to the natural growth during the elementary school years than students using manipulatives less. Similarly, the parameter π_{4i} is related to the growth added by medium manipulative use compared to low manipulative use. The term e_{ij} is the error term. In HLM models such as the one in this study, the e_{ij}'s are assumed to be independent and normally distributed with mean zero and variance σ^2 (Hedeker, 2004).

Level 2 (between student level). At the second level we intend to model the between-student variability in the growth terms. We include home language (HOMELANGUAGE) as a potential moderator of mathematics growth. In addition, SES and race-ethnicity variables were included as statistical controls. In this model, CAUCASIAN was used as a control variable, and we compare results of students of any other race-ethnicity variables to the Caucasian students. Thus, the second-level model, or between student model, was specified as:

$$\pi_{0i} = \beta_{00} + \beta_{01}\,SES_i + \beta_{02}\,AFRICANAMERICAN_i + \beta_{03}\,HISPANIC_i$$
$$\beta_{04}\,ASIAN_i + \beta_{05}\,OTHER_i + \beta_{06}\,HOMELNGUAGE_i + r_{0i}$$
$$\pi_{4i} = \beta_{40} + \beta_{41}\,SES_i + \beta_{42}\,AFRICANAMERICAN_i + \beta_{43}\,HISPANIC_i$$
$$\beta_{44}\,ASIAN_i + \beta_{45}\,OTHER_i + \beta_{46}\,HOMELNGUAGE_i + r_{4i}$$
$$\pi_{5i} = \beta_{50} + \beta_{51}\,SES_i + \beta_{52}\,AFRICANAMERICAN_i + \beta_{53}\,HISPANIC_i$$
$$\beta_{54}\,ASIAN_i + \beta_{55}\,OTHER_i + \beta_{56}\,HOMELNGUAGE_i + r_{5i}$$

Where β_{00}, β_{40}, and β_{50} are slope parameters for growth, and interaction between *grade* and the two manipulative use variables (MEDIUM and HIGH) respectively. The parameters β_{01}, β_{41}, and β_{51} represent the effect of the variable SES on the coefficients π_{0i}, π_{4i} and π_{5i}, the interaction between manipulative use and growth variables respectively; the set of parameters from β_{02} to β_{05}, β_{42} to β_{45} and β_{52} to β_{55}, represent the effect of the race-ethnicity variables on the coefficients π_{0i}, π_{4i} and π_{5i} respectively; the parameters β_{06}, β_{46}, and β_{56} represent the effect of the variable HOMELANGUAGE on the coefficients π_{0i}, π_{4i} and π_{5i} respectively; and the values

r_{0i}, r_{4i}, and r_{5i} are random individual-specific errors (Hedeker, 2004). Statistically significant values of any of the race-ethnicity parameters indicate the race-ethnicity group shows a different effect on growth than the Caucasian group. Statistically significant values for the HOMELANGUAGE variable indicate that home language is a moderator of growth, that is student achievement growth is different due to student home language (English and non-English).

The statistical analysis, in this study, has several steps. First, with all the students included in the longitudinal data set, a base model was estimated in which no variables were included at the second level. The goal of this first step was to model the relationship between manipulative use and mathematics learning. Second, with the same sample, the variables SES, HOMELANGUAGE and four race-ethnicity dummy variables (AFRICANAMERICAN, HISPANIC, ASIAN, and OTHER) were included in the second level model. With this, we wanted to attempt to explain the variability in students' mathematics achievement growth and the interaction between manipulative use and growth.

After completing the analysis for the general population, we did the same analysis for the subsample of Hispanic students. In this case at the second level, we only included SES and HOMELANGUAGE. The goal was to model the relationship between manipulative use and mathematics learning, as well as to explain variability in student's mathematics achievement growth and the interaction between manipulative use and growth for the Hispanic group.

Results

The results from the data analysis are presented in three parts. First, we present descriptive analyses, including a graphical analysis of the relationship between the mathematics achievement and manipulative use and a correlational analysis between manipulative use and mathematics achievement at each grade level. Finally, the results from the longitudinal analyses are presented and discussed.

Descriptive Analysis

The data were separated to analyze the impact of manipulative use according to the use by grade level. In Table 5, we present the weighted descriptive statistics of mathematics achievement and manipulative use by grades at the elementary school level (Kindergarten, First grade, Third grade, and Fifth grade). As expected mathematics achievement increases while the grade level increases. The mean of mathematics achievement increased from 32.68 points in Kindergarten to 111.73 points in Fifth grade. In Kindergarten, 69.1% of the students used manipulatives almost every day, while in Fifth grade just 9.4% of the students used manipulatives as often. The group of students who never or hardly ever used manipulatives in mathematics classes increased from 0.7% in the students at Kindergarten to 11.6% in Fifth grade. The percentage of students who used manipulatives between two and eight times per month also changed over time, increasing from 30.2% in Kindergarten to 79.0% in Fifth grade. These values are also represented visually in Figure 4. HIGH manipulative use is represented in the bottom of the cylinders, and LOW manipulative use is found in the top, shades of gray are used to differentiate the manipulative use frequencies.

Graphical analysis

To study the impact of each of the three manipulative use frequency modes on mathematics achievement by the grade level students, we followed the analysis of Ma and Wilkins (2007). In our study, this analysis involved comparing mathematics achievement slopes between manipulative use groups at different grade level, as well as achievement at the specific point in time. For example, for first grade level, we observe the slopes on mathematics achievement for each of the three manipulative use group, and we compare these slopes for the time between Kindergarten and First grade as well as the time between first grade and third grade; moreover the achievement of the three groups during the first grade is also discussed. The discussions of the four graphics are presented below, starting with Kindergarten comparisons (Figure 5 and Table 6).

Manipulative use at Kindergarten. The analysis at this grade involves the achievement at the end of Kindergarten and the change, if any, or slopes between Kindergarten and first grade. On average, students using manipulatives in different frequency seem to have no differences in their achievement at the Kindergarten level (See Figure 5 and Table 6). Mathematics achievement, at the end of Kindergarten, seems to not be related to manipulative use. Similar results can be observed on achievement growth or change between Kindergarten and First grade. Slopes for the three groups appear to be similar.

Manipulative use at First grade. On average, First grade students using manipulatives in different frequency seem to have slight differences in their achievement in that grade (See Figure 6 and Table 7). Mathematics achievement, at the first grade level, appears to be slightly related to manipulative use. In general, students who use manipulatives almost every day (HIGH) seem to have lower achievement than students in the other two groups. First grade students using manipulatives less often have higher achievement than their peers.

Achievement growth results of these groups vary between Kindergarten-First grade and First-Third grade years. For Kindergarten to First grade, achievement growth of these groups seems to be similar, but with some slightly differences. The LOW group seems to have a little higher growth than the other groups, and the HIGH group seems to obtain the lowest growth. Although the slopes are relatively the same, a possible reason for this relationship is that some first grade teachers may use manipulatives for remediation purposes as found in Howard, Perry and Tracey (1997). Students' achievement growth between first grade and third grade showed growth for all groups with some differences. These differences are stronger than between Kindergarten and First grade. Students who used manipulatives less often (LOW), during their first grade year, presented the lowest increment in mathematics achievement growth. However, students who used manipulative between two and eight times per month presented the highest growth in mathematics achievement.

Manipulative use at third grade. Third grade results are slightly different than at the first grade. On average, third grade students using manipulatives in different frequency groups seem to have some differences in their achievement (See Figure 7 and Table 8). The group of students who use manipulatives almost every day (HIGH) appears to have similar achievement on the assessment as students using manipulatives less often (LOW). Third grade students using manipulatives between two and eight times per month (MEDIUM) have the highest achievement at that grade level.

Achievement growth results are different between first-third grade and third-fifth grade years for these groups. Analyzing achievement change between first and third grade, it seems to be different between manipulative use groups. Students using manipulatives almost every day (HIGH) and between two and eight times per month (MEDIUM) show slightly higher growth than their peers in the LOW group. Evaluating the growth between third and fifth grade achievement scores, a different pattern is observed. Students' mathematic achievement growth seems to be similar between groups. Moreover, third grade students who use manipulative between two and eight times per month maintain the highest achievement at fifth grade. Achievement of the low and high manipulative users is similar.

Manipulative use at fifth grade. At this grade, manipulative use seems to be related to achievement. Higher use seems to be related to lower achievement (See Figure 8 and Table 9). The LOW group, students who use manipulatives *never or hardly ever*, appears to have higher mathematics achievement than their peers. At this grade level, the lowest average in mathematics achievement corresponds to the group of students who use manipulatives more often (HIGH). A reason for these results may be that students at this grade level were asked to use manipulatives mostly for remediation purposes (see Howard, Perry & Tracey, 1997). Comparing the graphs of the achievement during the elementary school years, Figure 8 is the only one that does not show any interaction, that is, students using manipulatives *never or hardly ever* (LOW) present the highest achievement throughout all years, while students using manipulatives *almost every day* (HIGH) always show the lowest achievement.

Correlational Analysis

From the descriptive and graphical analysis we can see that there are differences in how often manipulatives are used at different grades in elementary school and how the use of these tools may relate to mathematics achievement at different grades. Having found possible differences in the relationship between mathematics and manipulative use by grade level, we calculated the correlation between mathematics achievement and manipulative use by each grade. The Pearson correlation coefficients were all less than 0.10 (see Table 10) for each grade level. Therefore, no statistically significant correlations between manipulative use and mathematics achievement was found by grade level suggesting no relationship between manipulative use and mathematics achievement (at a single point in time).

Longitudinal Analysis

The longitudinal model used in this study allowed us to identify the added growth in mathematics achievement due to manipulative use (see Ma & Wilkins, 2007). Including two of the three frequency variables (LOW, MEDIUM and HIGH), we estimated different longitudinal models to select the one that best fit to the data. The criteria used for this selection were the deviance scores for the models, number of participants with sufficient data to model random effects, and theoretical judgment. In addition, we evaluated the parameters for statistically significant variability (i.e., significant random effects). Models in which the two added growth in mathematics achievement variables and the natural growth variable were allowed to vary randomly were not successfully estimated; therefore, models tested did not allow these three variables to vary randomly at the same time. In order to compare the added growth due to LOW, MEDIUM and HIGH manipulative use, we decided that our models should either allow two of the interaction variables (LOW*grade, MEDIUM*grade and HIGH*grade) to vary randomly or neither of them. As a result, we tested six models allowing either both of those parameters or just grade parameters to vary.

From the analysis of the six models, the model with the lowest deviance score (324,742.19) and highest number of participants (10271 students) was the one with the frequency variables MEDIUM and HIGH, allowing the parameters for the intercept and the *grade* variable to vary randomly. None of the six models result in significant random variability for both of the added growth variables, thus the effects for these variables where fixed. Table 11 shows the deviance scores, participants with complete data to calculate random effects, and variables with statistical significant variability for the six models estimated.

Analysis for all students. As discussed earlier, the parameters π_{1i}, π_{4i}, and π_{5i}, indicate the general mathematics learning growth, mathematics learning growth due to MEDIUM manipulative use, and mathematics learning growth due to HIGH manipulative use, respectively. For the mathematics learning growth parameter (see Base model on Table 12), results indicate that the growth increment was statistically significant, adding 13.72 points to mathematics achievement annually ($p<0.001$). In Table 12, results of the added growth due to MEDIUM and HIGH manipulative use, compared to mathematics learning growth due to LOW, are shown. These results indicate that the added growth due to MEDIUM and HIGH manipulative use were statistically significant and higher than the added growth due to LOW manipulative use. On average, MEDIUM manipulative use added 2.11 points ($p<0.001$) to mathematics achievement, and HIGH manipulative use added 5.11 points ($p<0.001$) to mathematics achievement, annually. Comparing growth added by MEDIUM manipulative use to growth added by HIGH manipulative use, it was found that HIGH manipulative use added significantly more growth than MEDIUM manipulative use ($p<0.05$). Using the largest standard error between the added growth variables ($SE = 0.24$), we calculated the 95% confidence interval to make the comparison between the added growth variables. The confidence interval for the added growth due to MEDIUM manipulative use was from 1.63 to 2.59, and the confident interval for the added growth due to HIGH manipulative use was from 4.63 to 5.59. The lack of overlap between these intervals indicates a difference between the groups.

Next we used student level variables to model the variability in mathematics learning (π_{1i}), added growth due to MEDIUM manipulative use (π_{4i}) and added growth due to HIGH manipulative use (π_{5i}). As explained before, interaction variables, MEDIUM*grade and HIGH*grade, were fixed, so they were not allowed to vary randomly. The mathematics learning variable was allowed to vary randomly in the model. After controlling for SES, race-ethnicity and home language, mathematics learning growth remain statistically significant (β=14.21, p<0.001) and increased from the base model (see Full model on Table 12). SES and race-ethnicity were found to explain the variability of this parameter. Statistically significant results for SES indicate that there are differences in mathematics learning growth due to SES. The increment in growth was 0.98 points per a standard unit increment in SES. Therefore, students with high SES tend to have a higher growth rate in mathematics achievement. Race-ethnicity variables were found to explain significant variability in mathematics learning growth. Results indicate that African Americans and Asian students have statistically significant different mathematics growth rates than their Caucasian peers at elementary school level. Overall, growth for African Americans was 1.4 points lower than the Caucasians, while overall, growth for Asians was 0.76 point higher than their Caucasian peers. Mathematics learning for Other race-ethnicity groups was not found to be significantly different from the Caucasian group. Home language was not found to explain variability in natural growth, indicating that students with different home language have similar growth rate in mathematics achievement.

Considering growth due to manipulative use, after controlling for SES, race-ethnicity, and home language, mathematics learning growth due to MEDIUM manipulative use (π_{4i}) remained statistically significant (β=2.22, p<0.001) and increased from the base model (see Full model on Table 12). SES and race-ethnicity were found to explain a significant amount of variability in this parameter. Statistically significant result for SES indicates differences in mathematics learning growth can be different between students due to their SES. The increment in growth is 0.26 points per standard unit increment in SES (p<0.05), indicating that students with higher SES tend to increase their mathematics achievement more when they use manipulatives between two and eight times per month. Race-ethnicity variables were found to explain variability in added growth due to MEDIUM manipulative use. Results indicate that African Americans and Other (American Indian or Alaskan Native, Native Hawaiian or other Pacific Islander) students add significantly less growth due to MEDIUM manipulative use than their Caucasian peers at the elementary school level. Overall, added growth for African Americans was statistically significant and 0.54 points lower than the Caucasians ($p<0.05$); and overall, added growth for the Other group was statistically significant and 0.86 points lower than their Caucasian peers ($p<0.05$). Other race-ethnic groups were found to not have a significant different in added growth due to MEDIUM manipulative use when compared to the Caucasian group. Home language was not found to explain variability in added growth due to MEDIUM manipulative use, indicating that students with different home language have similar growth rate due to MEDIUM manipulative use.

Similar results were found for mathematics growth due to HIGH manipulative use (π_{5i}). After controlling for SES, race-ethnicity and home language, mathematics learning growth due to HIGH manipulative use remained statistically significant ($\beta=5.66$, $p<0.001$) and increased (see Full model on Table 12). SES and race-ethnicity were found to explain a statistically significant amount of variability for this parameter. The increment in growth was of 0.87 points per standard unit increment in SES ($p<0.05$), indicating that students with higher SES tend to increase their mathematics achievement more when they use manipulatives almost every day. Race-ethnicity variables were found to explain variability in mathematics learning growth due to HIGH manipulative use. Results indicate that Asian students have significantly lower mathematics growth than their Caucasian peers due to HIGH manipulative use at the elementary school level. Overall, added growth for Asians was statistically significant and 2.0 points lower than the

Caucasians. Added growth of other race-ethnic groups was not found to be different from the Caucasian group. Home language was not found to explain variability in added growth due to HIGH manipulative use, indicating that students with different home language have similar growth due to HIGH manipulative use.

Analysis for Hispanic students. For this part of the analysis we only considered those students whose race-ethnicity was indicated as Hispanic. Being the race-ethnicity group which most likely uses the same language (Spanish) at home, this group seems to be more alike than others with a home language different than English. In addition, Spanish speaking students that showed low English proficiency at Kindergarten and First grade used assessment instruments in Spanish. Therefore, by using this group of students, we may reduce assessment language effect on students' scores.

For this subsample, as explained earlier, we utilized the same model as for the full sample. For the mathematics learning growth parameter, results (see Base model on Table 13) indicate that growth was statistically significant for the Hispanic students, adding 14.19 points to mathematics achievement annually (p<0.001). Added growth due to MEDIUM and HIGH manipulative use compared to mathematics learning growth due to LOW is also shown in Table 13. Added growth due to MEDIUM and HIGH manipulative use was higher and statistically significant than the added growth due to LOW manipulative use for the Hispanic students. On average, MEDIUM manipulative use added 1.12 points (p<0.001) to mathematics achievement, and HIGH manipulative use added 4.14 points (p<0.001) to mathematics achievement, annually.

Comparing growth added by MEDIUM manipulative use to growth added by HIGH manipulative use, we have found that HIGH manipulative use added significantly more growth than MEDIUM manipulative use (p<0.05). Similar to our procedure in the model for the full sample, to compare the added growth of MEDIUM and HIGH manipulative use, we calculated the 95% confidence intervals using the largest standard error for the two parameters estimates (SE = 0.49). Confidence interval for added growth due to MEDIUM manipulative use is from 0.14 to 2.1, and for added growth due to HIGH manipulative use is from 3.16 to 5.12. The lack of overlap between these intervals indicates a difference between the groups.

Next we used student level variables to model the variability in mathematics learning (π_{1i}), added growth due to MEDIUM manipulative use (π_{4i}) and added growth due to HIGH manipulative use (π_{5i}). As explained before, the mathematics learning variable was allowed to vary randomly in the model, and the parameters related to growth due to manipulative use were fixed. After controlling for SES and home language, mathematics learning growth due to MEDIUM manipulative use remained statistically significant (β=14.59, p<0.001) and increased for Hispanic students (see Full model on Table 13). SES was found to explain a significant amount of variability in this parameter. Statistically significant result for the SES indicates differences in mathematics learning growth can be different between Hispanic students due to their SES. The increment in growth is 0.88 points per standard unit increment in SES. Therefore, Hispanic students with high SES tend to have a higher growth rate in mathematics achievement. Home language was not found to explain variability in Hispanic students' mathematics learning, indicating that Hispanic students with different home language have similar achievement growth.

Considering growth due to manipulative use, after controlling for SES, and home language, mathematics learning growth due to MEDIUM manipulative use remained statistically significant (β=1.05, p<0.01). Mathematics growth due to HIGH manipulative use also remained statistically significant (β=4.34, p<0.01) and increases from the base model (see Full model on Table 13). SES and home language were not found to explain a significant amount of variability in these parameters, added growth due to MEDIUM or HIGH manipulative use. Therefore, Hispanic students using manipulatives between two and eight times per month (MEDIUM) have similar growth due to manipulative use, regardless of SES or home language. Similar results were found for Hispanic students using manipulatives almost every day (HIGH).

Discussion and Conclusions

The purpose of this study was to examine the relationship between elementary school students' mathematics learning and manipulative use. Further we wanted to examine home language as a potential moderator of this relationship. Using the ECLS data, we examined this relationship in several ways. First, we analyzed differences in manipulative use by grade level by looking at descriptive statistics of the data; second, we analyzed the immediate impact of manipulative use on mathematics achievement and growth between two consecutive time points. Finally we conducted a longitudinal analysis to study the students' mathematics learning growth during the elementary school years, as well as the relationship between manipulative use and this learning. For the final analyses we investigated this relationship initially including all the students in the sample, and then only for Hispanic students.

Graphical analysis.

In this analysis, we examined the relationship between manipulative use and mathematics achievement in three ways: at the specific time the manipulative use was measured, at any two consecutive times comparing growth between the three groups. In general, we observed no relationship between manipulative use and manipulative achievement scores. At Kindergarten, the three groups have similar achievement scores which could indicate no relationship between the variables at this grade, but for the other grades results were different. For example, at first grade, the relationship seems to be negative; that is, more manipulative use is related to lower achievement. However, in Third grade students using manipulatives between two and eight times per month obtained the highest achievement. Interestingly, students using manipulatives almost every day did not seem to obtain the highest achievement in any of the grade levels.

Considering growth, we found no general relationship between manipulative use at a particular grade and student achievement growth between two points in time. For example, students using manipulatives almost every day at Kindergarten present similar growth between that grade and First grade than students using manipulatives at any other frequency. Students in the never or hardly ever manipulative use group at First grade seemed to grow faster than their peers between Kindergarten and First grade. However, that group has the lowest growth between First grade and Third grade.

These inconclusive results on the relationship between manipulative use and achievement or improvement between two consecutive times may be a representative of inconsistent results found in the literature. If the analysis is focused on achievement, in general, we would report no relationship between variables. If we focus on growth or change between two consecutive times, results may depend on which year students used manipulatives. Positive results may be found for students using manipulatives at Fifth grade, comparing growth during Third and Fifth grade. However, it is different for achievement growth between Kindergarten and First grade for students using manipulatives at first grade. In that case no relationship between manipulative use and achievement growth was found. Therefore, looking just to the Fifth grade data, studies may report positive relationship as presented in some studies (Bolyard, 2005; Suh, 2005; Trespalacios, 2008). Moreover, investigating First grade manipulative use, there may be no relationship as reported by Posadas (2004).

Because no substantial relationship between manipulative use and (1) achievement at a time point or (2) two consecutive time points were found, we decided to evaluate the relationship between manipulative use and students' achievement using a correlational analysis. From this analysis, we found no substantial relationship between mathematics achievement and manipulative use at any grade level examined.

Longitudinal Analysis

Graphical analysis considered average mathematics achievement at each time point separately. Growth and change in mathematics achievement scores were needed to study the relationship between manipulative use and student mathematics learning, since the goal was to evaluate the effectiveness of this instructional strategy (Willett, 1988). Here mathematics learning is represented as the growth rate of mathematics achievement over time.

From the longitudinal analyses, in general, we found that there is a positive relationship between manipulative use and student learning for elementary school students. This relationship was also found when only using the Hispanic sample. SES and race ethnicity were found to moderate this relationship for the full sample of students, indicating that higher SES is related to higher growth and some race-ethnicity groups differ in achievement growth from their Caucasian peers. Home language was not found to moderate the relationship between manipulative use and student's mathematics learning. When considering only the Hispanic sample, SES and home language were, again, not found to moderate the relationship between manipulative use and mathematics learning. From this result, we conclude that manipulative use helps all students independent of their home language.

Students whose home language is not English have similar gains in achievement compared to their English home language peers. Language proficiency can be a limitation for mathematics learning of ELLs (Kisthy, 1995), but manipulative use has been proposed to help ELLs in their learning in mathematics classrooms (see Cummins, 1998; Herrell, 2000; Lee, Silverman & Montoya, 2002, Lee & Jung, 2004). Since there are no differences in mathematics learning growth due to home language, results from this study seem to support this proposal.

This study also provides methodological evidence for the use of longitudinal studies when evaluating student learning. From the contrasting results found from the graphical, correlational, and longitudinal analyses, we verified that change and growth should be examined when evaluating the effectiveness of instructional strategies. In this study, we initially found no general relationship between manipulative use and mathematics achievement or growth between two consecutive points. However, for that analysis we used group mean achievement scores of each one of the three manipulative use groups. As indicated by Willett (1987), learning indicates change and growth on student's achievement.

From the previous discussion of the results, we can draw three main conclusions from this study. First, there is a relationship between manipulative use and mathematics learning. Student mathematics learning tends to be higher for students who use manipulatives more often during their elementary school years. This result is consistent with pervious results that found manipulatives as tools helping students in their learning mathematics process (e.g., Bolyard, 2005; Suh, 2005; Suh & Moyer, 2007; Trespalacios, 2008), but inconsistent with studies finding negative or no relationship between manipulative use and students' performance (e.g., McClung, 1998; Posadas, 2004). However, this is the only study, to date, investigating manipulative use over a long period of time, as suggested by Sowell (1989), and using multiple measures of students' performance; thus, modeling student learning.

Second, home language was not found to moderate this relationship which indicates that, the relationship between manipulative use and mathematics learning does not vary because of language spoken at home. It may be an indicator that manipulative use helps all students learning regardless of their home language, as well as language issues in mathematics classrooms may be reduced when manipulatives are used.

Third, when evaluating effectiveness of manipulative use, learning growth measures should be utilized. As recommended by Willett (1988), it is necessary to consider student learning to evaluate teaching strategies, and learning implies change and growth. Achievement at a specific point time should not be used to measure relationship between strategy and learning, since it does not indicate the impact of the strategy in the learning process. Moreover, using student change instead of group change allow us to value each student learning process, and this helps to study the impact of the teaching strategy on varied students.

Limitations and Further Research

This study represents secondary data analysis, using ECLS data that was collected by NCES. Therefore, the data was not collected for the purpose for this study. Information such as how manipulatives were used, topics for which manipulatives were used, or type of activities that involve manipulative use could not be analyzed. Moreover, the word manipulative was not defined in the question that asked about the frequency of use. Thus, there were likely differences in objects that teachers identified as manipulatives when they answered the questions. Finally, the data about students' manipulative use was reported by their teachers during the spring semester and it may be difficult for the teachers to calculate in an accurate way how often a specific student used the manipulatives during the whole year.

Future research would benefit from more specific questions about how students were using manipulatives in their classrooms and the types of activities in which students were involved while they used manipulatives. For example, as suggested by Kolb (1984) Experiential Learning model, if students are involved in a concrete experience, such as manipulative use, for learning to occur, they need to transform the information gathered through reflective observation. In case that students get information by abstract conceptualization, they may need to transform this information by active experimentation using the manipulatives to be able to construct knowledg. If transformation, either by reflective observation or active experimentation, does not occur, knowledge cannot be constructed (Kolb, 1984). Therefore, activities using manipulatives may need to also provide oportunities for students to transform the information received through the manipulation of the objects or test the abstract information by using manipulatives. Just because students use manipulatives in their mathematics lessons it is not necessarity the case that these tools are helping students learn mathematics (Ball, 1992; Baroody, 1989). Thus, investigating the relationship between the types of activities that involve manipulative use and student learning could help to identify those activities that best support learning.

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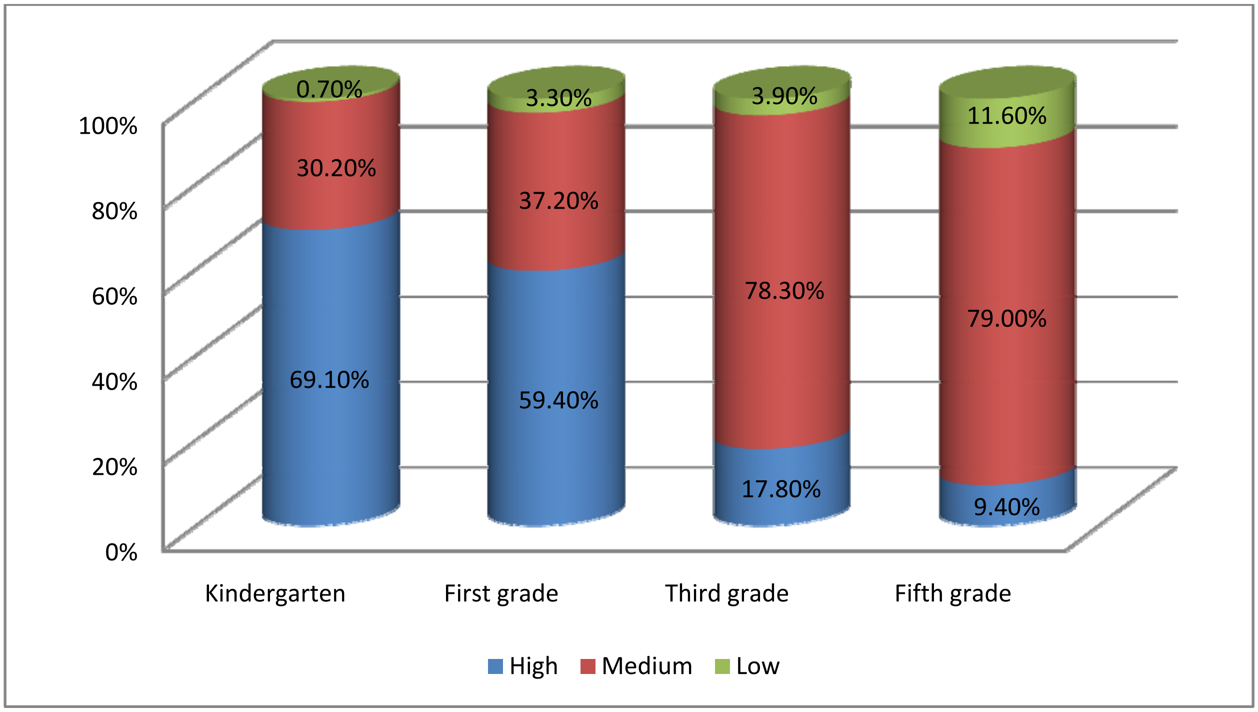

Figure 4. Manipulative Use by Grade Level

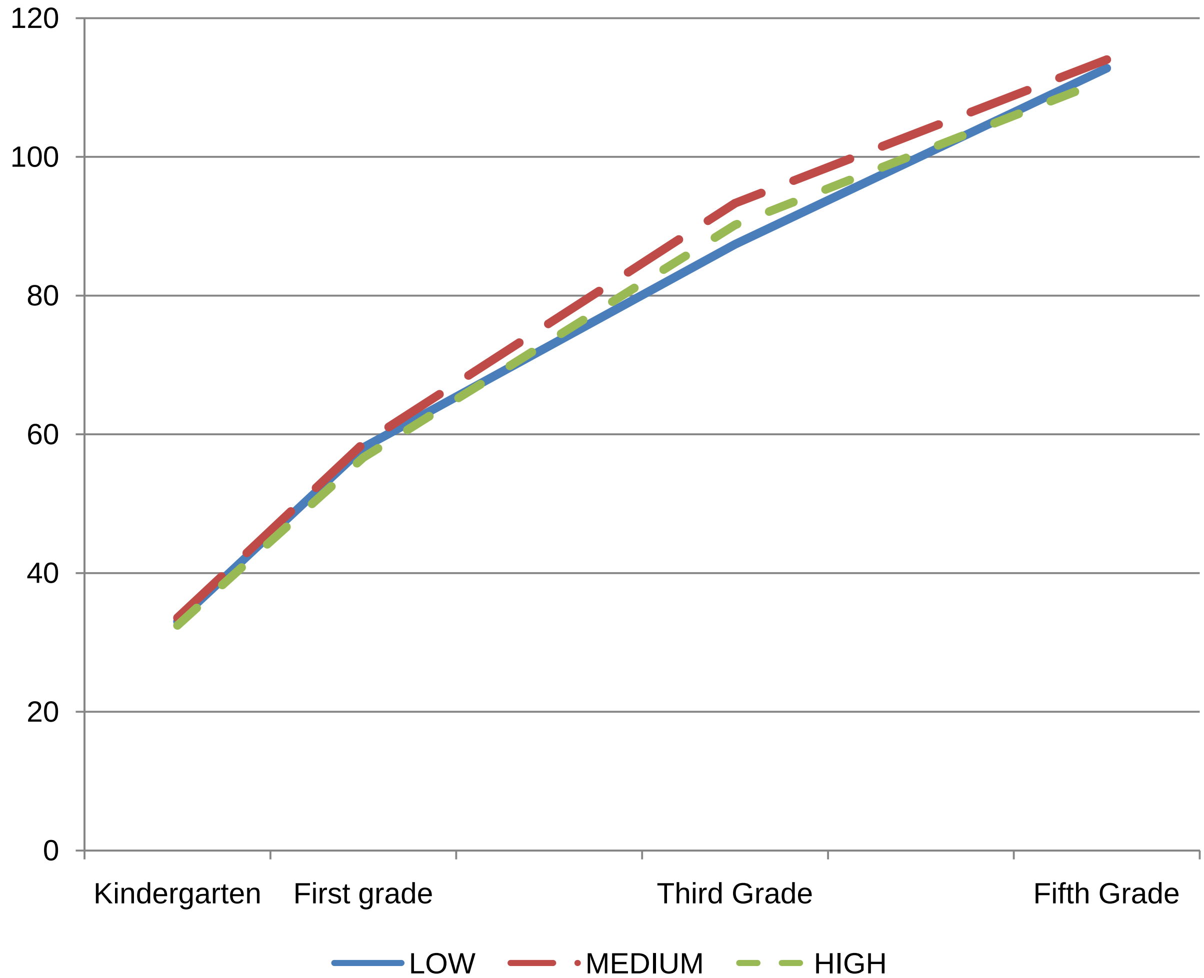

Figure 5. Growth in mathematics average achievement of group of students by manipulative use frequency at Kindergarten level

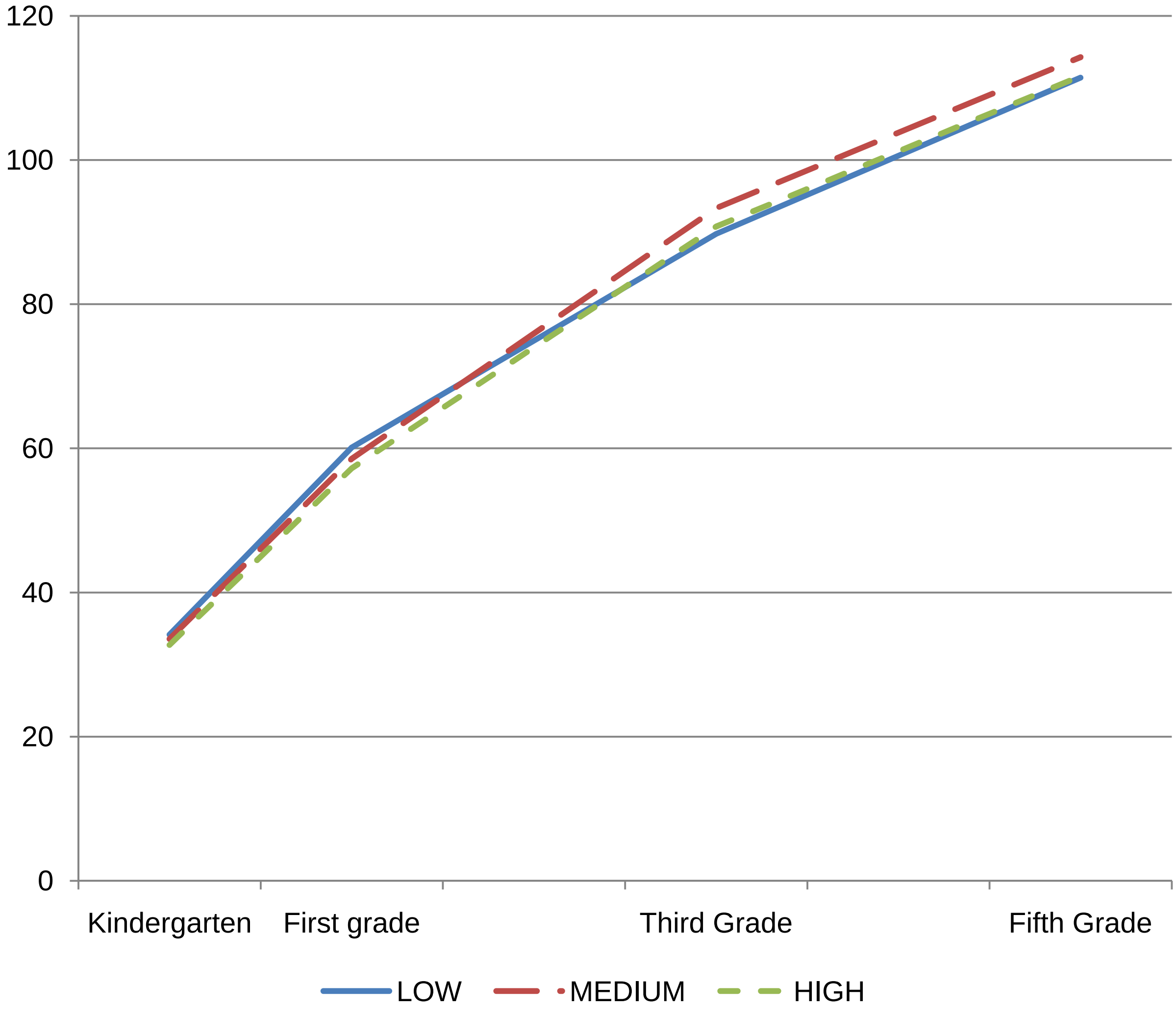

Figure 6. Growth in mathematics average achievement of group of students by manipulative use at First grade

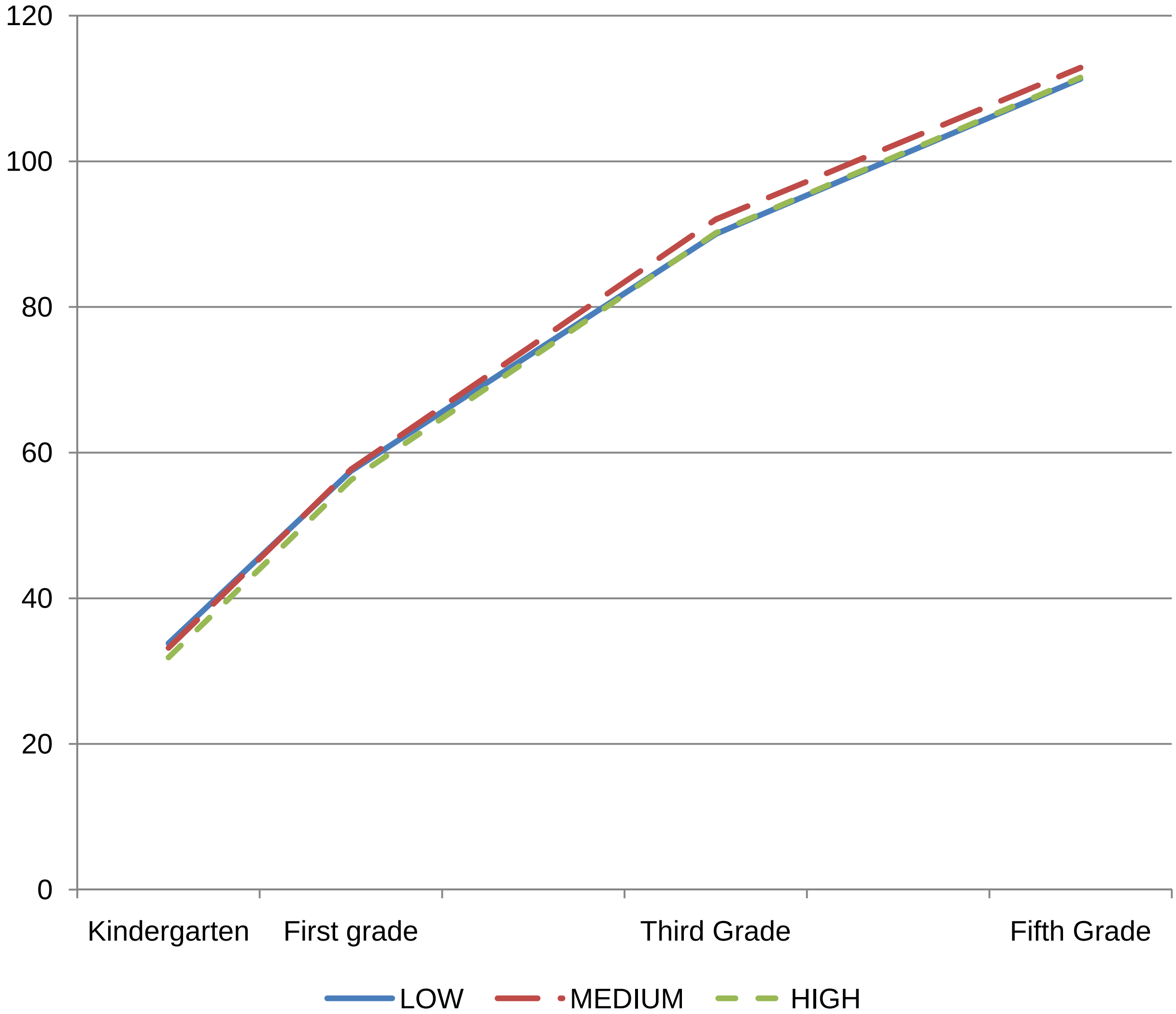

Figure 7. Growth in mathematics average achievement of group of students by manipulative use at Third grade

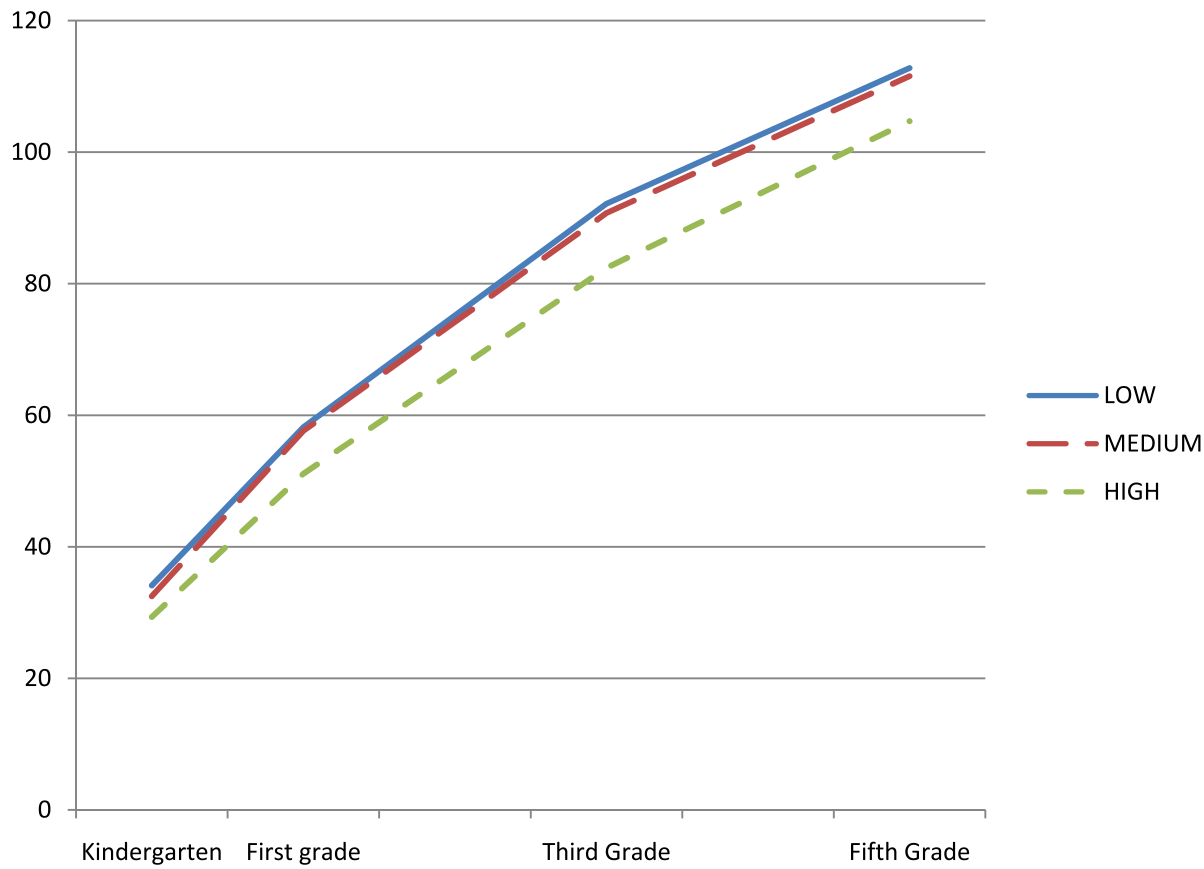

Figure 8. Growth in mathematics average achievement of group of students by manipulative use at Fifth grade

Table 4.

New Frequency Measures Relating to Original Measures

New Labels	New values	K-1 grades original values	3-5 grades original values	New Description*
Low	1	1,2	4	Never/hardly ever
Medium	2	3,4	2,3	Between twice and eight times per month
High	3	5,6	1	Almost every day

Note. The new description is based on a combination of the original K-1 and 3-5 descriptions

Table 5.

Descriptive Statistics of Mathematics Achievement and Manipulative Use at Elementary School Level

	Kindergarten		First grade		Third grade		Fifth grade	
	M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Mathematics Achievement								
	32.68	11.59	57.10	16.70	90.87	21.96	111.73	22.21
N*	10,525		10,634		10,618		10,618	
Manipulative Use								
	%	*N**	*%*	*N**	*%*	*N**	*%*	*N**
LOW	0.7	74	3.3	359	3.9	438	11.6	685
MEDIUM	30.2	3,268	37.2	3,768	78.3	6,812	79.0	3,961
HIGH	69.1	6,807	59.4	5,333	17.8	1,401	9.4	383
Total		10,149		9,460		8,651		5,029

Note. * N values represent the actual number of participants in the sample. Other values (percentages, means and SD) correspond to the weighted sample sizes.

Table 6.

Mathematics achievement of students by manipulative use frequency at Kindergarten level

	Kindergarten			First Grade			Third Grade			Fifth Grade		
	M	*SD*	N	*M*	*SD*	N	*M*	*SD*	N	*M*	*SD*	N
LOW	33.04	16.16	73	58.09	16.75	73	87.39	17.71	73	112.78	16.63	73
MEDIUM	33.54	11.72	3,228	58.70	16.76	3,256	93.31	20.72	3,250	114.02	21.18	3,250
HIGH	32.46	11.45	6,713	56.67	16.51	6,782	90.18	22.27	6,774	111.19	22.32	6,779

Note. N values represent the actual number of participants in the sample. Other values (means and SD) correspond to the weighted sample sizes.

Table 7.

Mathematics achievement of students by manipulative use frequency at First grade level

	Kindergarten			First Grade			Third Grade			Fifth Grade		
	M	*SD*	N	*M*	*SD*	N	*M*	*SD*	N	*M*	*SD*	N
LOW	34.15	10.85	355	60.13	14.28	359	89.76	18.43	358	111.43	18.72	358
MEDIUM	33.55	10.89	3,728	58.57	15.45	3,754	93.32	20.57	3,753	114.27	20.15	3,752
HIGH	32.72	11.33	5,257	57.24	16.07	5,315	90.76	21.99	5,305	111.63	22.50	5,307

Note. * N values represent the actual number of participants in the sample. Other values (means and SD) correspond to the weighted sample sizes.

Table 8.

Mathematics achievement of students by manipulative use frequency at Third grade level

	Kindergarten			First Grade			Third Grade			Fifth Grade		
	M	*SD*	N	*M*	*SD*	N	*M*	*SD*	N	*M*	*SD*	N
LOW	33.83	12.82	434	57.46	15.84	436	90.01	20.59	436	111.30	21.28	436
MEDIUM	33.19	11.28	6,736	57.70	16.33	6,793	92.01	21.44	6,776	112.85	21.70	6,778
HIGH	31.88	12.31	1,380	56.24	17.72	1,395	90.15	23.09	1,395	111.49	22.58	1,393

Note. N values represent the actual number of participants in the sample. Other values (means and SD) correspond to the weighted sample sizes.

Table 9.

Mathematics achievement of students by manipulative use frequency at Fifth grade level

	Kindergarten			First Grade			Third Grade			Fifth Grade		
	M	*SD*	N	*M*	*SD*	N	*M*	*SD*	N	*M*	*SD*	N
LOW	34.12	11.14	671	58.17	16.26	678	92.15	21.86	683	112.79	21.64	683
MEDIUM	32.48	11.16	3,909	57.62	16.26	3,954	90.74	22.11	3,943	111.55	22.15	3,953
HIGH	29.33	10.38	379	51.09	13.88	380	82.40	22.01	377	104.70	22.29	374

Note. N values represent the actual number of participants in the sample. Other values (means and SD) correspond to the weighted sample sizes.

Table 10.

Correlation between mathematics achievement and manipulative use by grade level.

		Mathematics achievement			
		Kindergarten	First grade	Third grade	Fifth grade
Manipulative Use	Pearson's Correlation	-0.04	-0.05	-0.02	-0.08
	N	10,014	9,428	8,607	5,010

Note. N values represent the actual number of participants in the sample. Other values (means and SD) correspond to the weighted sample sizes.

Table 11.

Criteria for selecting the best model

Model Interaction Variable (*Parameter(s) Varying Randomly*)	Deviance Score	Participants	Significant Random Variability*
LOW-HIGH (*grade*)	325,428.54	10,271	*Grade*
LOW-HIGH (*LOW-HIGH*)	331,493.74	551	---
LOW-MEDIUM (*grade*)	326,841.48	10,271	*Grade*
LOW-MEDIUM (*LOW-MEDIUM*)	330,963.14	1,009	---
MEDIUM-HIGH (grade)	***324,742.19***	***10,271***	***Grade***
MEDIUM-HIGH (MEDIUM-HIGH)	328692.30	10,271	*MEDIUM*

Note. Bolded row indicates the best model. *Random variability column should be interpreted as the parameters that show a statistically significant random variability.

Table 12.

Natural and Added Growth to Mathematics Achievement by Manipulative Use Frequency

	Base Model		Final Model	
	β	*SE*	*β*	*SE*
Fixed effect				
Grade	13.72***	0.12	14.21***	0.27
SES			0.98***	0.10
HomeLanguage			0.28	0.24
African-Am			-1.40***	0.23
Hispanic			0.11	0.21
Asian			0.76**	0.27
Other			-0.35	0.31
MEDIUM * Grade	2.11***	0.16	2.22***	0.31
SES			0.26*	0.10
HomeLanguage			0.10	0.26
African-American			-0.54*	0.27
Hispanic			-0.50	0.26
Asian			-0.36	0.34
Other			-0.86*	0.39

Note. *** $p<0.001$. ** $p<0.01$, * $p<0.05$. Base Model in this table can be read as indicating the average added growth (β) of each manipulative use group to mathematics achievement. Final Model in this table can be read as indicating the added growth of each group after controlling for SES and home language. Moreover, SES and race-ethnicity values can be read as the impact of these variables on the growth for the specific group.

Table 12 (Continuation).

Natural and Added Growth to Mathematics Achievement by Manipulative Use Frequency (Cont.)

	Base Model		Final Model	
	β	*SE*	*β*	*SE*
Fixed effect				
HIGH * Grade	5.11***	0.24	5.66***	0.71
SES			0.87**	0.24
HomeLanguage			-0.20	0.67
African-American			-0.71	0.60
Hispanic			-0.62	0.57
Asian			-2.00*	0.80
Other			-1.00	0.54

Random Effect	Variance component	
	Base Model	Final Model
Intercept	121.78***	88.00***
Grade	6.46***	5.09***

Note. *** p<0.001. ** p<0.01, * p<0.05. Base Model in this table can be read as indicating the average added growth (β) of each manipulative use group to mathematics achievement. Final Model in this table can be read as indicating the added growth of each group after controlling for SES and home language. Moreover, SES and race-ethnicity values can be read as the impact of these variables on the growth for the specific group.

Table 13.

Natural Growth and Added Growth to Mathematics Achievement by Manipulative Use Frequency for Hispanic Students.

		Base Model		Final Model	
		β	*SE*	*β*	*SE*
Fixed Effects					
Grade		14.19***	0.21	14.59***	0.29
	SES			0.88***	0.21
	HomeLanguage			-0.38	0.33
MEDIUM * Grade		1.12***	0.27	1.05**	0.31
	SES			0.08	0.23
	HomeLanguage			0.36	0.33
HIGH * Grade		4.14***	0.49	4.34***	0.71
	SES			0.60	0.54
	HomeLanguage			0.35	0.90

Random Effect	Variance component	
	Base Model	Final Model
Intercept	73.66***	57.26***
Grade	7.24***	7.13***

Note. *** $p<0.001$, ** $p<0.01$. Base Model in this table can be read as indicating the average maturity growth (grade) and the added growth (MEDMan and HIGHMan) to mathematics achievement. Final Model in this table can be read as indicating the growth or added growth de to Medium or High manipulative use comparing to the growth due to LOW manipulative use after controlling for SES and home language. Moreover, SES and HomeLanguage values can be read as explaining variability values.

Chapter 5. Conclusions and Future Research

It has been suggested that teachers' instructional practices serve as a mediator between teacher variables and student learning (Fennema & Franke, 1992; Mewborn & Cross, 2007). In this case, instead of teacher variables being directly related to student learning (see Figure 9, dotted arrow), teacher variables are related to teachers' instructional practices (gray arrow) which in turn are related to student learning (black arrow). Therefore, an indirect path between teacher variables and student learning is created, with instructional strategies serving to mediate this relationship. In this dissertation, I investigated the mediating role of *manipulative use* on the relationship between teacher variables and student learning by conducting two studies, as represented in Figure 9.

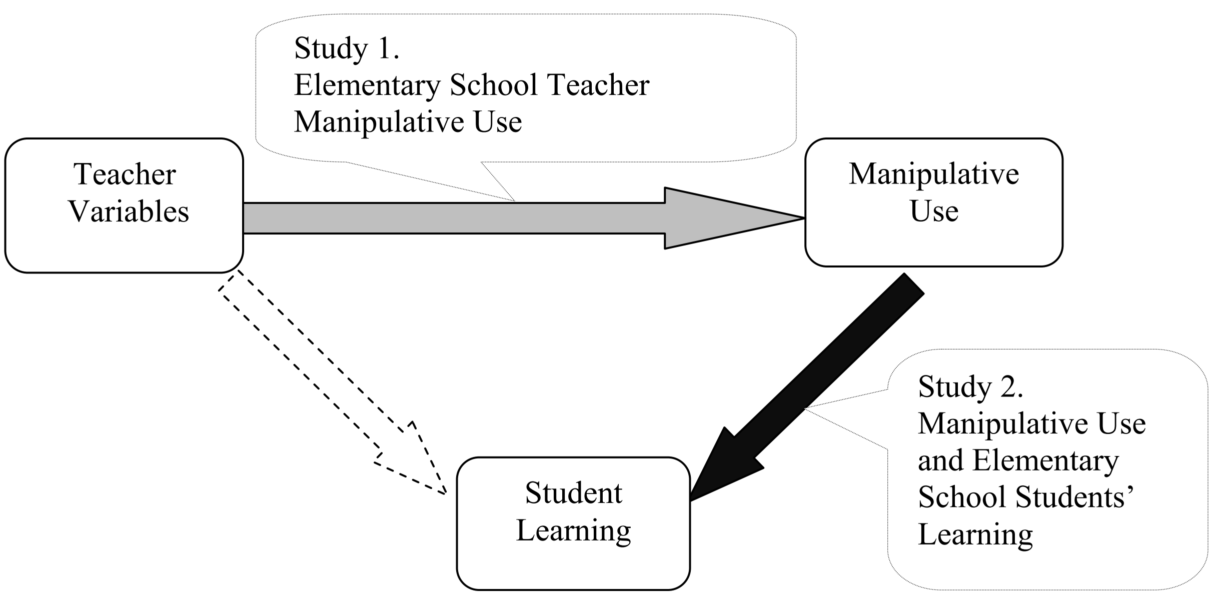

Figure 9.

Manipulative use as mediator of teacher variables and student learning.

Each study investigated a different relationship: Study 1, *Elementary School Teachers' Manipulative Use,* examined the relationship between teacher variables and the frequency of manipulative use; and Study 2, *Manipulative Use and Elementary School Students' Mathematics Learning,* examined the relationship between manipulative use and student learning. In this chapter, I first discuss findings and conclusions for each of the studies. Next, I discuss these findings according to the framework outlined in Figure 9. Finally, I discuss implications for future research.

Elementary School Teacher Manipulative Use.

To evaluate manipulative use as a mediator of the relationship between teacher variables and student learning, I first studied the relationship between teacher variables and manipulative use. This is the relationship and study identified by the gray arrow in Figure 9. In this study, presented in Chapter 3, six teacher variables were included, and those variables were: three teachers' beliefs; and teachers' experience, age, and the grade they teach. Interrelations between those variables were also analyzed to provide a better understanding of the relationship between those teacher variables and manipulative use.

Literature has identified teacher beliefs as a strong factor affecting teaching practices (Ernest, 1989; Thompson, 1984, 1992; Wilkins, 2008). However, this is not the only factor that may impact those practices (Ernest, 1989; Raymond, 1997; Wilkins, 2008). Literature relating teacher variables and manipulative use has explored various teacher characteristics such as experience (Gilbert & Bush, 1988; Raphael & Wahlstrom, 1989), or which grade level they teach (Howard, Perry, & Tracey, 1997; Malzahn, 2002; Weiss, 1994,), in addition to teachers' beliefs (Howard, et al, 1997; Sherman & Richardson, 1995). The results of the reseach literature examining the relationship between particular teacher variables (i.e., grade and experience) and manipulative use are inconclusive. Further, studies investigating more than one teacher variable did not necessarily examine the interrelationship among the variables.

As depicted in Figure 9, this study investigates one of the relationships needed to accomplish the goal of this dissertation, which is to analyze the role of manipulative use as mediator between teacher variables and student learning. The aim of this study was to investigate the relationship between grade level that teachers teach, teachers' age, teachers' experience, and their beliefs, as well as the interrelationship among these variables, and how often teachers use manipulatives in their mathematics instruction. Results from this study suggest that teachers' beliefs and the grade level that they teach are important predictors of how often elementary school teachers use manipulatives in their mathematics instruction. However, teacher age and years of experience were not found to be consistent predictors of manipulative use.

Kindergarten teachers were found to use manipulatives more often than the other grade-level teachers, and teachers in grades 3-5 were found to use manipulatives least often. Further, this relationship does not change when controlling for teachers' background characteristics and beliefs. This result is consistent with previous research showing less use of manipulatives in the higher grades at the elementary school level (Gilbert & Bush, 1988; Howard et al., 1997; Kloosterman & Harty, 1987; Malzahn, 2002). However, results of this study indicate further differences at the primary level. Kindergarten teachers were found to use manipulatives more often than teachers in grades 1-2 which is different from previous research (Howard, et al., 1997; Kloosterman & Harty, 1987; Malzahn, 2002) and provides evidence of a tendency for decrease in manipulative use between teachers of first, second and third grade (cf. Gilbert & Bush, 1988).

Although teachers' years of classroom experience and age were found to be related to manipulative use when considered alone, after controlling for teacher grade and beliefs these variables were no longer statistically significant predicators of manipulative use. This finding helps to clarify the mixed findings of previous research (cf. Gilbert & Bush, 1988; Howard, et al., 1997, Raphael & Wahlstrom, 1989) and suggests that teacher background characteristics alone do not help explain the important instructional mechanisms that determine whether or not teachers choose to use manipulatives in their classroom. Consistent with results indicating a relationship between teachers' beliefs and their instructional practices (Moyer, 2001; Nespor, 1987; Thompson, 1984, 1992; Wilkins, 2008), teachers' beliefs about manipulative use were found to be related to their use of manipulatives in their mathematics instruction. Teachers who tend to believe that it is important to have students participate in appropriate hands-on activities for effective mathematics instruction tend to use manipulatives more frequently in their mathematics lessons. Further, teachers who tend to believe that the use of manipulatives with older students is less necessary were found to use manipulatives less often. These relationships remained salient even after teachers' background characteristics and grade were controlled.

In summary, in general, manipulative use and teacher characteristics were found to be related. This finding provides evidence for the first path or relationship depicted in gray Figure 9.

Manipulative Use and Elementary School Student Learning

After evaluating the relationship between teacher variables and manipulative use, I examined the relationship between manipulative use and student learning. Verifying this relationship along with findings from Study 1 would provide evidence of a tenable mediating role of manipulative use on the relationship between teacher variables and student learning. In this second study, presented in Chapter 4, the relationship between manipulative use and student learning, operationalized as growth and change, was investigated.

Research on the relationship between manipulative use and student performance has been inconsistent. Some studies report a positive relationship or effect of manipulative use on achievement (Bolyard, 2005; Suh, 2005; Suh & Moyer, 2007; Trespalacios, 2008), but others have different results (McClung, 1997; Reimer & Moyer, 2005). However, no studies to date were found to involve manipulative use for a period of time of at least a year, as suggested by Sowell (1989). Moreover, studies found in the literature analyzed change in group means of achievement, while Willett (1988) suggest that to measure learning, change and growth of individual achievement should be used. Moreover when evaluating instructional strategies, individual learning, which implies change and growth, should be considered instead of just performance at a single point in time. Therefore, I investigated the relationship between manipulative use and student learning by modeling individual student mathematics learning throughout elementary school years.

We can draw three main conclusions from this study. First, there is a relationship between manipulative use and mathematics learning. Mathematics student learning tends to be higher for students who use manipulatives more often during their elementary school years. On one hand, this result is consistent with pervious results that found manipulatives to be tools that help students in their mathematics learning process (e.g., Bolyard, 2005; Suh, 2005; Suh & Moyer, 2007; Trespalacios, 2008). On the other hand, it is inconsistent with studies finding negative or no relationship between manipulative use and students' performance (e.g., McClung, 1998; Posadas, 2004). However, the most important difference in our study and previous studies is that, to date, it is the only study investigating manipulative use over a long period of time, as suggested by Sowell (1989), and using multiple measures of students' performance, thus, modeling student learning to evaluate the effectiveness of manipulative use.

Second, home language was not found to moderate the relationship between manipulative use and student learning which indicates that this relationship does not vary based on the language spoken at home. Manipulative use seems to help all students' mathematics learning regardless of their home language. Given this to be the case, and that manipulative use has been suggested as an instructional tool that could be beneficial in reducing the cognitive load for students with low language proficiency as well as increase the contextual environment of instructional activities (Cummins, 1998; Herrell, 2000), perhaps language issues in mathematics classrooms may be reduced when manipulatives are used.

Third, as recommended by Willett (1988), when evaluating the effectiveness of manipulative use, measures of achievement growth or learning should be utilized. Achievement at a specific point in time only provides a measure of status and thus does not provide an appropriate outcome measure to indicate whether a particular teaching strategy is effective since it does not show the impact of the strategy on the learning process. Moreover, it is beneficial to have more than two time points to observe the learning process in order to have a more reliable way to model individual student learning. In addition, using individual change and growth instead of group change and growth allows us to investigate individual change. Students likely differ in how their mathematical achievement change, and it may be influenced by their individual differences.

Based on this study, a relationship between manipulative use and student learning was found. Therefore, the second part of the indirect path between teacher variables and student learning represented in Figure 9 was verified.

Connecting the Studies

The two studies presented in this dissertation *seem* to support Fennema and Franke's (1992) and Mewborn and Cross' (2007) claim regarding the mediating role of teachers' instructional practices on the relationship between teacher variables and student learning. Study 1 found a positive relationship between teacher variables (i.e., beliefs and grade level) and manipulative use. Study 2 found that students' manipulative use was positively related to mathematics learning. Even though we cannot make a direct claim about manipulatives as a mediator because teachers and students who were the subjects in the studies presented in this dissertation were not related, by connecting these two studies we can conclude tentively that teacher characteristics are indirectly related to student learning as they are mediated by manipulative use in the mathematics classrooms.

In particular, joining the results from the studies, teachers holding positive beliefs about manipulatives tend to use these devices more often in the classroom; and when manipulatives are used more often in the mathematics classroom, students' mathematical learning tends to increase. That is, teachers' beliefs and grade level they teach are indirectly related to students learning as they are mediated by the use of manipulatives in the classroom.

Having the two studies together provides a bigger picture of manipulative use in mathematics lessons, specifically at the elementary school level. On one hand, if only the first study is analyzed, questions about relating students' learning and manipulative use in the classroom could arise. For example, asking for evidence about the relationship between manipulative use and students' mathematics learning. Moreover, because of the inconsistent findings from research about students' performance and its relationship to manipulative use in mathematics classrooms, people may not recognize the value of the first study. On the other hand, if just the second study is interpreted, to increase student learning, teachers may be asked to use manipulatives more often in their classrooms without considering other factors that could be impacting teachers' instructional practice. For example, teachers in different grades could be asked to increase the use of manipulatives in their mathematics instruction, even though, relative to the grade that they teach, they may believe that they already use manipulatives as often as their students need.

Future Research

Since my overall aim in this dissertation was to evaluate manipulative use as a mediator between teacher variables and student learning, the two studies conducted for this dissertation seem to support this role for manipulative use. However, I cannot make a strong claim from the results found in these studies. Therefore, I have identified different ways to improve and strengthen the findings associated with the relationship between teacher characteristics and student learning. The initial two suggestions are related to the main goal of this dissertation while the last one is related to just one of the two studies presented in this dissertation.

First, since participants in the presented studies are not related, a future study could investigate manipulatives as mediators between teacher characteristics and student learning by relating the teachers and the students. That is having as participants in the second study, students of the teachers participating in the first one.

Second, these studies have investigated the relationship between teacher characteristics and manipulative use, as well as manipulative use and student learning by focusing only on the frequency of using these tools. Other studies are needed to better understand the nature of the activities involving manipulatives. It is not just because manipulatives are being used that learning may occur (Ball, 1992; Baroody, 1989), but, more important, the students' thinking and reflection while using them that most affects learning and understanding (Kamii, Lewis, & Kirkland, 2001). To successfully construct knowledge,while using manipulatives need to provide learning experiences that engage students in acquiring information and transforming it (Kolb, 1984). According to Kolb (1984), the best learning cycle should involve gining information through concrete experience, reflect on that experience, reach abstract conceptualization, and then verify the ideas through active experimentation. For example, activities may involve manipulative use during the concrete experimentation and the active experimentation phases.

Third, considering the study relating teacher variables and manipulative use, investigating teacher definitions of these tools may help to better inform us about teacher beliefs about manipulatives and may also help us to understand how teachers use these tools in the classroom. Teachers probably have different definitions of manipulatives, as stated by Sherman and Richarson (1995), so teachers may report using manipulatives when they may actually be using different tools that would not be considered manipulatives by all teachers. Learning how teachers interpret the term may be related to what they use as manipulatives or how teachers use manipulatives in their classrooms.

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