THE ANALYSIS OF INDUCTION MOTORS WITH VOLTAGE CONTROL BY SYMMETRICALLY TRIGGERED THYRISTORS

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ABSTRACT

The application of thyristor switching to induction motor speed control has resulted in a number of unconventional supply systems. One such technique, which has been successfully employed in a number of applications, is variable voltage control by means of symmetrically triggered thyristors in the stator phases of a wye-connected machine. In this paper, an analytic method for predicting the steady-state performance of such a system is presented. The solution is obtained in closed form without the necessity for iteration to obtain the proper boundary conditions. In addition, it is shown that the symmetry of the solution permits an additional reduction in the computation time. Since matrix techniques are utilized throughout the analysis, the equations can be easily implemented into a digital computer program. Hence, the method is well suited to the evaluation of proposed motor designs when used in conjunction with thyristor voltage control. Measured torque-speed characteristics of a typical drive system are included and the results compared to an analytical solution. It is demonstrated that a computed solution will favorably predict the performance of an actual system.

INTRODUCTION

With the development of the thyristor, a variety of schemes have been evolved incorporating these devices for the purpose of controlling the speed of induction motors. Perhaps the most straightforward application, from a conceptual point of view, is the use of thyristors in the supply lines of an induction machine to adjust the effective voltage applied to the stator terminals. Since the output torque varies as the square of the air-gap emf, variation of the terminal voltage is basically an inexpensive and reliable means of adjusting the motor speed. When a relatively high rotor resistance is incorporated in the design of the induction machine, speed ranges of 5 to 1 can be readily obtained.[1]

In many cases, stator voltage speed control is not practical owing to the inherently poor efficiency of the scheme. At low speeds, as much as 18-20% of the full load power is dissipated as heat within the machine, and considerable care must be exercised to properly match the machine design to the application. Efficiency, however, must be weighed with a number of other important factors including initial cost, system power factor, reliability, and ease of maintenance. Hence, the method is being increasingly employed in low and medium power applications wherein the load torque varies as the square of the motor speed. Such applications are typically fan or pump loads in the 5 – 150 hp range.

Thyristors can be connected to induction machines in a variety of ways for the purpose of adjusting the terminal voltage. However, it has been demonstrated that a circuit containing a pair of thyristors connected back-to-back in each line of a wye-connected motor is superior to all other conventional connections if average torque per rms stator ampere is used as the performance criterion.[1] Although thyristor voltage control of induction motors has been in use for many years[2] and component parts, as well as completely designed systems, are commercially available, a detailed analysis and solution of such a scheme has not, as yet, appeared in the literature.

Despite the simplicity of the scheme, an analysis of even its steady-state performance is extremely complex owing to the difficulty of establishing a suitable set of boundary or initial conditions needed to generate a solution. An approximate analysis of a system with a delta-connected primary has been reported.[3] In this paper, an expression for average torque using the first harmonic component of stator current was derived. In this case, the harmonic components were measured directly from an actual system. Only relatively poor correlation of the computed and measured torque-speed curves was obtained for various firing angles. An exact analysis of the same delta-connected configuration has been attempted.[4] However, a solution of the resulting equations was not carried out, apparently owing to their complexity. In a recent book,[5] Takeuchi has analyzed a version of the wye-connected machine. The average torque, voltages, and currents are approximated as an infinite series of symmetrical components and computation of the resulting equations is tedious. Since the voltage across an open-circuited phase is neglected, results are variable and become inaccurate for large delay angles when the rotor is near maximum speed.

Implicit solutions by analog or digital simulation techniques have also been obtained.[6,7,8] However, such an approach does not easily lend itself to system design aspects which generally require a repeated number of steady state solutions.

These analyses seem to demonstrate that a precise solution of the stator and rotor currents is required for accurate prediction of torque-speed curves, power factor, efficiency and other important performance characteristics. In this paper, the steady-state solution of a wye-connected induction machine with back-to-back thyristors connected in the lines is developed in detail. State variable techniques[9] are utilized to generate, without iteration, a set of initial conditions which will yield the steady-state solution. The analysis differs considerably from related work[10] in that the equations defining the appropriate initial conditions are obtained by simple matrix algebra. Also, the open circuit conditions which occur are defined without reducing the rank of the resulting matrix differential equation. Hence, it is not necessary to define different sets of equations for each open-circuit condition. Since the solution is carried out entirely with matrices, the method is well suited for digital computer programming. It is shown that when attention is given to the symmetry of the system variables, it is possible to significantly reduce the computational effort required to obtain the solution.

BASIC ASSUMPTIONS

A simplified diagram of the system considered in this paper is given in Fig. 1. The system consists of a three-phase power source, three pairs of identical thyristors (silicon controlled-rectifiers), connected back-to-back in series with the phases of a three-wire wye-connected induction machine. The thyristor gate control signals used to trigger the thyristors are derived from zero crossings of the three-phase power source. Stator voltage control is achieved by alternately open-circuiting the three stator phases at instants of zero current.

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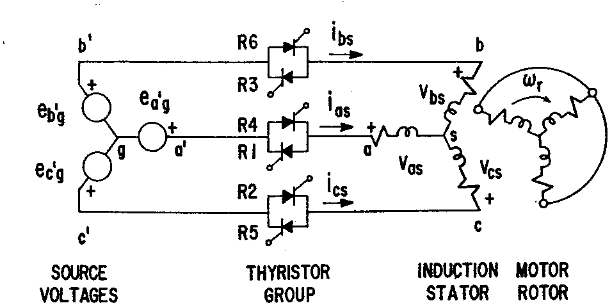


Fig. 1. System studied.

A frequently-used variation of this system incorporates three pairs of delta-connected thyristors in the neutral of the machine.[5] Performance of this system is identical to that in Fig. 1 and the analysis contained herein is applicable to this configuration with proper interpretation of the results.

In this paper it is assumed that:

(a) The power source may be considered as a set of balanced, sinusoidal three-phase voltages having zero source impedance.

(b) The six thyristors have identical characteristics, are symmetrically triggered and can be considered as a device which presents an infinite impedance in the blocking mode when the forward or anode-to-cathode voltage changes from positive to negative. The impedance changes to zero whenever a trigger pulse is applied, provided the forward voltage is positive.

(c) The induction machine is an idealized machine in which the stator and rotor windings are distributed so as to produce a single sinusoidal MMF wave in space when balanced sets of currents flow in the stator and rotor circuits.

(d) All parameters of the machine are assumed to be constant and saturation of the magnetic circuit is neglected.

(e) The system is in the steady-state. In particular, the rotor speed of the induction machine is assumed to be constant.

CONSIDERATIONS OF SYMMETRY

Because of the symmetrical nature of the voltages applied to the induction machine and the symmetry of the resulting line currents, a considerable reduction in the effort required to obtain a steady-state solution is possible. A typical steady-state solution of the voltage-controlled induction motor scheme of Fig. 1 is given in Fig. 2, which serves to set forth the notation used and to illustrate the symmetry of the solution.

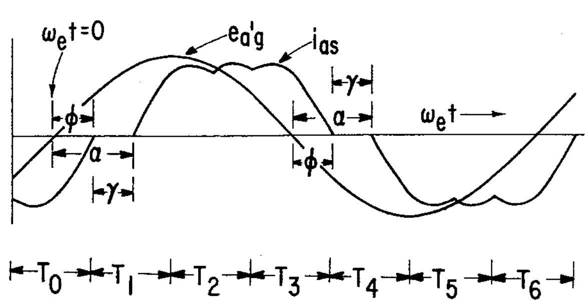


Fig. 2. Typical steady-state solution, $\alpha=60°$, $\gamma = 30°$, $\phi = 30°$.

In the steady-state, an induction motor operates as an inductive load. Hence, a phase current typically lags its respective line-to-neutral voltage by a phase angle ϕ. The delay from a point of zero phase voltage to the conduction of the succeeding thyristor in that phase is generally termed the delay angle α. The delay from the instant the phase current reaches zero to the firing of the succeeding thyristor in that phase is defined in this paper as the current delay angle or hold-off angle γ. These angles are illustrated in Fig. 2. Since the thyristors are fired symmetrically, γ is the same for all phases.

At any instant of time, the differential equations defining the state of the system are linear since the rotor speed of the induction motor has been assumed constant. In addition, the voltages fed to the six controlled-rectifiers form a balanced polyphase set. All controlled-rectifiers are triggered in a symmetrical fashion with respect to a current zero and the induction motor is itself symmetrical. Hence, the stator line-to-neutral voltages and phase currents form balanced sets. That is to say, the voltage v_{bs} across phase bs of the induction machine and the current i_{bs} in this phase are identical in form to the corresponding voltage and current in phase as and phase delayed by $2\pi/3$ radians. Similarly, the voltage v_{cs} and i_{cs} are identical in form to v_{bs} and i_{bs} and phase delayed by a further $2\pi/3$ radians. Furthermore, since the hold-off angle for each of the two thyristors in a given phase is identical, each voltage and current is half-wave symmetric.

Because of this symmetry, not only the hold-off angle γ, but also the phase angle ϕ and the delay angle α associated with the turn-off of each thyristor are identical. In order to facilitate the analysis, it is convenient to divide a full period into six equal intervals, T_1, T_2, ..., T_6, each having a duration of $\pi/3$ radians. These intervals are also noted in Fig. 2. Intervals 1 to 6 are initiated when the thyristors R1 to R6, as indicated in Fig. 1, enter the blocking mode. An extra interval T_0, occurring one interval prior to T_1, has been included for purposes of analysis.

On the basis of three-phase symmetry and half-wave symmetry, the following conclusions can be obtained.

(a) Only five system states can exist for a wye-connected three-wire machine as follows:

(1) All three phases connected to the source voltage; i_{as}, i_{bs}, and i_{cs} are non-zero.

(2) Phase as disconnected from the source voltages; $i_{as} = 0$.

(3) Phase bs disconnected; $i_{bs} = 0$.

(4) Phase cs disconnected; $i_{cs} = 0$.

(5) All three phases disconnected from the source voltages; $i_{as} = i_{bs} = i_{cs} = 0$.

(b) A solution for any $\pi/3$ interval is sufficient to uniquely define all variables over an entire period.

(c) By virtue of (b), only three system states need be investigated in detail. If interval T_1 is chosen as the interval over which a solution is desired, these states are (1), (2), and (5) as defined in (a) above.

(d) Only two distinct modes of operation exist for a wye-connected, three-wire machine and these modes are defined by the hold-off angle γ, as follows:

(1) First mode: $0 < \gamma < \pi/3$. When γ ranges between 0 and $\pi/3$, either three or two windings of the induction motor are connected to the power source. The sequence of system states during intervals T_1 to T_6 are 2-1-4-1-3-1 as noted in (a).

(2) Second mode: $\pi/3 < \gamma < 2\pi/3$. During this mode, either two or one winding of the induction motor is connected to the power source. Since the machine is wye-connected, all three stator currents are zero for the latter connection. The sequence of system states during a full period beginning with interval T_1 is then 5-2-5-4-5-3.

A trivial case exists for $2\pi/3 < \gamma < \pi$, when either one winding or zero windings are connected to the source. For either connection, the three stator currents are zero; and, hence, this mode is not of interest. However, if the induction motor has a neutral return path to the power source, it is clear that three modes of operation will exist.

At this juncture, it is possible to proceed towards a solution utilizing either the delay angle or hold-off angle as the basic parameter. However, the two operating modes are related directly to the hold-off angle. It will be shown that a closed form of solution for a set of initial conditions can be found when γ and ϕ are selected as the basic parameters, whereas the use of α and ϕ results in a need for an iterative solution. If necessary, results can be readily converted from one form to another once a solution has been obtained.

STATE VARIABLE ANALYSIS

In the analysis of problems involving induction machinery, it has proven useful to transform the equations which describe the behavior of the machine to d-q axes fixed either on the stator or the rotor, or rotating in synchronism with the applied voltages. When actual phase voltage and currents are used as variables, time-varying coefficients appear in the resulting differential equations owing to the sinusoidal variation of mutual inductance with displacement angle. However, a transformation of voltage and current variables to d-q axes results in a set of constant coefficients in the resulting differential equations when the rotor speed is constant.

In the case of the induction motor with stator voltage control, switching of phase currents takes place in the stator of the machine. Hence, it is desirable to fix the d-q axes in the stator. These axes were originally termed the α-β axes by Stanley.[11] However, since the subscript α is also used to denote the delay angle, the notation of Krause and Thomas[12] will be employed. These equations, expressed in per unit, wherein the d-q axes of the arbitrary reference frame are fixed in the stator (d^s-q^s) axes are given in matrix form by

$$\begin{bmatrix} v_{qs}^s \\ v_{ds}^s \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + \frac{p}{\omega_b} x_s & 0 & \frac{p}{\omega_b} x_m & 0 \\ 0 & r_s + \frac{p}{\omega_b} x_s & 0 & \frac{p}{\omega_b} x_m \\ \frac{p}{\omega_b} x_m & \frac{\omega_e - \omega_r}{\omega_b} x_m & r_r' + \frac{p}{\omega_b} x_r' & \frac{\omega_e - \omega_r}{\omega_b} x_r' \\ -\frac{\omega_e - \omega_r}{\omega_b} x_m & \frac{p}{\omega_b} x_m & -\frac{\omega_e - \omega_r}{\omega_b} x_r' & r_r' + \frac{p}{\omega_b} x_r' \end{bmatrix} \times \begin{bmatrix} i_{qs}^s \\ i_{ds}^s \\ i_{qr}'^s \\ i_{dr}'^s \end{bmatrix} \tag{1}$$

or in the corresponding vector-matrix form as

$$\bar{v} = \bar{X} \frac{p}{\omega_b} \bar{i} + \bar{R}\bar{i} \tag{2}$$

In these equations, the superscript s is employed to denote that the d-q axes have been fixed in the stator. A p denotes the operator d/dt. Although six equations are generally required to completely define the machine response, the two zero-sequence equations have been immediately omitted since the sum of stator as well as rotor currents are zero. Also, in Eq. 1, ω_r is the electrical angular velocity of the rotor and is assumed constant, ω_e is the electrical angular velocity of the source voltages and ω_b is the base electrical angular velocity used to obtain the per unit machine parameters. The voltages v_{ds}^s and v_{qs}^s are an equivalent set of voltages, often written v_α and v_β,[11] related to the phase voltages by the equations

$$v_{qs}^s = v_{as} \tag{3}$$

$$v_{ds}^s = \frac{1}{\sqrt{3}} (v_{cs} - v_{bs}) \tag{4}$$

where in order to obtain Eq. 3 it has been noted that the sum of the stator line-to-neutral voltages of a three wire-wye-connected machine is zero.[12]

The $d^s - q^s$ stator currents are related to the stator phase currents by

$$i_{qs}^s = i_{as} \tag{5}$$

$$i_{ds}^s = \frac{1}{\sqrt{3}} (i_{cs} - i_{bs}) \tag{6}$$

The stator referred d-q rotor currents are related to the rotor phase currents by

$$i_{qr}'^s = \frac{N_r}{N_s} [i_{ar} \cos\theta_r + \frac{1}{\sqrt{3}} (i_{cr} - i_{br}) \sin\theta_r] \tag{7}$$

$$i_{dr}'^s = \frac{N_r}{N_s} [-i_{ar} \sin\theta_r + \frac{1}{\sqrt{3}} (i_{cr} - i_{br}) \cos\theta_r] \tag{8}$$

Again, the sum of the stator currents and the sum of the rotor currents have been assumed to be zero. In eqs. 7 and 8, N_r/N_s is the effective rotor to stator turns ratio, θ_r denotes the relative displacement in electrical radians of the ar rotor axis with respect to the as axis.

When peak rated line-to-neutral voltage and peak rated line current are chosen as base quantities, the electromagnetic torque expressed in terms of the d^s-q^s variables is[12]

$$T_e = x_m (i_{qs}^s i_{dr}'^s - i_{ds}^s i_{qr}'^s) \tag{9}$$

In addition to the equations defining operation of the induction machine, equations expressing the ac source voltages may be written in differential equation form. Since the machine is supplied by a balanced, polyphase voltage source, the per unit source voltages may be expressed

$$e_{ag}' = V_m \sin \omega_e t \tag{10}$$

$$e_{bg}' = V_m \sin (\omega_e t - 2\pi/3) \tag{11}$$

$$e_{cg}' = V_m \sin (\omega_e t + 2\pi/3) \tag{12}$$

where V_m denotes the peak value of the source voltages expressed in per unit. The electrical angular velocity of the source voltages,

ω_e, has been assumed equal to the base angular velocity throughout this study.

The source voltages e'_{ag}, e'_{bg} and e'_{cg} may, in turn, be obtained from the pair of first order differential equations

$$\frac{p}{\omega_e} e_1 = e_2 \tag{13}$$

$$\frac{p}{\omega_e} e_2 = -e_1 \tag{14}$$

where $e_1(0) = 0$, $e_2(0) = V_m$. In matrix form, Eqs. 13 and 14 become

$$\frac{p}{\omega_e} \bar{e} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \bar{e} \tag{15}$$

where

$$\bar{e} = [e_1, e_2]^T \tag{16}$$

and T denotes the transpose. The source voltages are related to the e_1, e_2 set of voltages by

$$e'_{ag} = e_1 \tag{17}$$

$$e'_{bg} = -\frac{1}{2} e_1 - \frac{\sqrt{3}}{2} e_2 \tag{18}$$

$$e'_{cg} = -\frac{1}{2} e_1 + \frac{\sqrt{3}}{2} e_2 \tag{19}$$

Eqs. 1 and 9 are sufficient to properly express the behavior of the induction machine provided that the d^s-q^s axis voltages are known. The identity of these voltages depend upon the system state. Although five system states exist, it has been noted that only system states 1, 2 and 5 need be defined to completely specify the solution. Hence, it is necessary to relate the d^s-q^s axes voltages only to these system conditions.

System state 1; i_{as}, i_{bs} and i_{cs} nonzero

In state 1, the machine is simply supplied by a conventional polyphase source. The line-to-neutral voltages across the terminals of the machine are

$$v_{as} = e'_{ag} - v_{sg} \tag{20}$$

$$v_{bs} = e'_{bg} - v_{sg} \tag{21}$$

$$v_{cs} = e'_{cg} - v_{sg} \tag{22}$$

However, since the sum of the stator line-to-neutral voltages and the sum of the source voltages are zero, it is readily noted that

$$v_{sg} = 0 \tag{23}$$

Utilizing Eqs. 3 and 4, the direct quadrature axis voltages are given by

$$v_{qs}^s = e_1 \tag{24}$$

$$v_{ds}^s = e_2 \tag{25}$$

The voltage forcing function vector $\bar{v}$ in Eq. 2 can now be related to the source voltage vector $\bar{e}$ by the matrix equation

$$\bar{v} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}^T \bar{e} \tag{26}$$

which, for convenience, can be expressed in the form

$$\bar{v} = \bar{C}_1 \bar{e} \tag{27}$$

Hence, the differential equations of the system for state 1 may be expressed in matrix notation as

$$\frac{p}{\omega_b} \bar{i} = -\bar{X}_1^{-1}\bar{R}\bar{i} + \bar{X}_1^{-1}\bar{C}_1\bar{e} \tag{28}$$

$$\frac{p}{\omega_e} \bar{e} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \bar{e} \tag{29}$$

It will be shown that the reactance matrix $\bar{X}$ as defined by Eq. 2 is valid only for system state 1. Thus, the subscript 1 has been appended to the reactance matrix in Eq. 28 although the corresponding matrices in these two equations are identical.

By partitioning, these equations may be written as a single matrix equation

$$\frac{p}{\omega_e} \begin{bmatrix} \bar{i} \\ \bar{e} \end{bmatrix} = \begin{bmatrix} -\bar{X}_1^{-1}\bar{R} & \bar{X}_1^{-1}C_1 \\ \bar{0} & \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \end{bmatrix} \times \begin{bmatrix} \bar{i} \\ \bar{e} \end{bmatrix} \tag{30}$$

where 0 is a 2 x 4 matrix of zeros and it has been noted that in this application $\omega_e = \omega_b$. Eq. 30 is expressed in the standard state-variable format familiar to control engineers.[10] It is normally described by the single compact equation

$$\frac{p}{\omega_e} \bar{x} = \bar{A}\,\bar{x} \tag{31}$$

In this application, the column vector $\bar{x} = [\bar{i}^T, \bar{e}^T]^T$ and contains the system state variables. $\bar{A}$ is the 2 x 2 partitioned system matrix of Eq. 30.

System state 2; $i_{as} = 0$

When the stator current i_{as} reaches zero, thyristor R_1 enters the blocking state and the voltage which appears across phase as is an induced emf owing to mutual coupling with the other stator and rotor phases. By virtue of Eq. 5 when $i_{as} = 0$, then $i_{qs} = 0$, and the open circuit voltage may be expressed in the d^s-q^s axes as[12]

$$v_{qs}^s = \frac{p}{\omega_e} x_m i'^s_{qr} \tag{32}$$

Since the bs and cs phases remain connected, the d^s axis voltage is again given by

$$v_{ds}^s = \frac{1}{\sqrt{3}} (e'_{cg} - e'_{bg}) = e_2 \tag{33}$$

When Eq. 32 is substituted into the general induction machine expression, Eq. 1, and is cancelled with the similar term on the right-hand side of the equation, the system differential equations expressed in state-variable form for system state 2 may be written

$$\frac{p}{\omega_e}\begin{bmatrix}\bar{i}\\ \\ \bar{e}\end{bmatrix} = \begin{bmatrix}-\bar{X}_2^{-1}\bar{R} & \bar{X}_2^{-1}\bar{C}_2\\ 0 & \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\end{bmatrix} = \begin{bmatrix}\bar{i}\\ \cdot\\ \bar{e}\end{bmatrix} \tag{34}$$

where $\bar{X}_2$ is the reactance matrix

$$\bar{X}_2 = \begin{bmatrix} x_s & 0 & 0 & 0\\ 0 & x_s & 0 & x_m\\ x_m & 0 & x_r' & 0\\ 0 & x_m & 0 & x_r'\end{bmatrix} \tag{35}$$

and $\bar{C}_2$ is the voltage connection matrix

$$\bar{C}_2 = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix}^T \tag{36}$$

Eq. 34 may now be represented compactly by the state variable equation

$$\frac{p}{\omega_e}\bar{x} = \bar{B}\bar{x} \tag{37}$$

where $\bar{B}$ is the 2 x 2 partitioned matrix of Eq. 34.

System state 5; $i_{as} = i_{bs} = i_{cs} = 0$

In cases where the hold-off angle γ is large, two of the stator phase currents, will be zero whenever two of the controlled-rectifiers are simultaneously in the forward-biased blocking mode. Since the machine is wye-connected, no currents flow in the stator winding during this interval. However, since currents continue to flow through the short-circuited rotor windings, voltages may appear across the three stator phases owing to mutual coupling. By virtue of Eqs. 5 and 6, when the three stator phase currents are zero, then both i_{qs}^s and i_{ds}^s are zero. The voltages which appear across the stator phase may be represented in the d^s-q^s axes as

$$v_{qs}^s = \frac{p}{\omega_e} x_m i_{qr}'^s \tag{38}$$

$$v_{ds}^s = \frac{p}{\omega_e} x_m i_{dr}'^s \tag{39}$$

It is readily established that the system differential equation for system state 5 is

$$\frac{p}{\omega_e}\begin{bmatrix}\bar{i}\\ \\ \bar{e}\end{bmatrix} = \begin{bmatrix}-\bar{X}_5^{-1}\bar{R} & \bar{0}^T\\ \bar{0} & \begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\end{bmatrix} x \begin{bmatrix}\bar{i}\\ \\ \bar{e}\end{bmatrix} \tag{40}$$

where

$$\bar{X}_5 = \begin{bmatrix} x_s & 0 & 0 & 0\\ 0 & x_s & 0 & 0\\ x_m & 0 & x_r' & 0\\ 0 & x_m & 0 & x_r'\end{bmatrix} \tag{41}$$

Hence, the state variable equation for the condition $i_{as} = i_{bs} = i_{cs} = 0$ is

$$\frac{p}{\omega_e}\bar{x} = \bar{C}\bar{x}$$

where $\bar{C}$ is the 2 x 2 partitioned system matrix of Eq. 40.

Solution for mode 1; $0 < \gamma < \pi/3$

When the hold-off angle ranges between 0 and $\pi/3$, either three windings of the induction machine are connected to the source voltages or two windings are connected while the third is disconnected. If t_o denotes a time reference in interval T_o defined such that $\phi + \gamma - \pi/3 \leq \omega_e t_o \leq \phi$, by referral to Fig. 2, the condition of the system can be noted as state 1. The solution to Eq. 31 for any time t where $\omega_e t_o \leq \omega_e t \leq \phi$, expressed in terms of the system initial conditions at $t = t_o$, is

$$\bar{x}(\omega_e t) = \epsilon^{\omega_e(t-t_o)\bar{A}}\,\bar{x}(\omega_e t_o) \tag{43}$$

The matrix exponential function $\exp[\omega_e(t-t_o)\bar{A}]$ appearing in the solution is generally termed the state transition matrix by control engineers. Computation of this quantity will be discussed in a subsequent section.

When $\omega_e t = \phi$, thyristor R_1 enters the blocking mode and the system is in state 2. The solution to Eq. 37, expressed in terms of the system condition at $\omega_e t = \phi$, for $\phi \leq \omega_e t \leq \phi + \gamma$ is

$$\bar{x}(\omega_e t) = \epsilon^{(\omega_e t-\phi)\bar{B}}\,\bar{x}(\phi) \tag{44}$$

During the period $\phi + \gamma \leq \omega_e t \leq \omega_e t_o + \pi/3$, the system is again in state 1 and the solution in terms of the system conditions at $\omega_e t = \phi + \gamma$ is

$$\bar{x}(\omega_e t) = \epsilon^{(\omega_e t-\phi-\gamma)\bar{A}}\,\bar{x}(\phi+\gamma) \tag{45}$$

Eqs. 43, 44, and 45 serve to completely define the operation of the system over the interval $\omega_e t_o \leq \omega_e t \leq \omega_e t_o + \pi/3$. However, the solution is not complete until the initial condition vector $x(\omega_e t_o)$ is known. This task may be carried out by utilizing the symmetry of the solution.

Since Eq. 44 is valid for the condition $\omega_e t = \phi$, and Eq. 45 is valid for $\omega_e t = \phi + \gamma$

$$\bar{x}(\phi) = \epsilon^{(\phi-\omega_e t_o)\bar{A}}\,\bar{x}(\omega_e t_o) \tag{46}$$

and

$$\bar{x}(\phi+\gamma) = \epsilon^{\gamma\bar{B}}\,\bar{x}(\phi) \tag{47}$$

Hence, Eq. 45 can be written

$$\bar{x}(\omega_e t) = \epsilon^{(\omega_e t-\phi-\gamma)\bar{A}}\,\epsilon^{\gamma\bar{B}}\,\epsilon^{(\phi-\omega_e t_o)\bar{A}}\,\bar{x}(\omega_e t_o) \tag{48}$$

where, $\phi + \gamma \leq \omega_e t \leq \omega_e t_o + \pi/3$. In particular, when $\omega_e t = \omega_e t_o + \pi/3$, the solution of Eq. 48 is

$$\bar{x}(\omega_e t_o+\pi/3) = \epsilon^{(\omega_e t_o-\phi-\gamma+\pi/3)\bar{A}}\,\epsilon^{\gamma\bar{B}}\,\epsilon^{(\phi-\omega_e t_o)\bar{A}}\,\bar{x}(\omega_e t_o) \tag{49}$$

It is shown in the Appendix that regardless of the form of the solution,

$$\bar{x}(\omega_e t_o+\pi/3) = \bar{T}\,\bar{x}(\omega_e t_o) \tag{50}$$

where $\bar{T}$ can be regarded as a time translation matrix which relates the solution at any two instants which are phase displaced by $\pi/3$ radians. The expression for the translation matrix $\bar{T}$ is given by Eq. 78. Utilizing Eq. 50, Eq. 49 can be expressed in the form

$$\left[\bar{T} - \epsilon^{(\omega_e t_o-\phi-\gamma+\pi/3)\bar{A}}\,\epsilon^{\gamma\bar{B}}\,\epsilon^{(\phi-\omega_e t_o)\bar{A}}\right]\bar{x}(\omega_e t_o) = \bar{0} \tag{51}$$

where $\bar{0}$ in this case is a 6 x 1 column vector of zeros. Because of the required symmetry of the solution, Eq. 51 is a necessary condition for steady-state operation. In addition, this equation may be utilized to solve for the unknown initial condition $\bar{x}(\omega_e t_o)$.

Thus far, Eq. 51 is valid for any time t_o where $\omega_e t_o$ is in the interval T_o and $\phi + \gamma - \pi/3 \leqslant \omega_e t_o \leqslant \phi$. If t_o is selected to be the particular initial time $\omega_e t_o = \phi$ or the initiation of interval T_1, the first element of the state vector $\bar{x}$ must be zero since at this instant i_{as}, and thus i_{qs}^s, is identically equal to zero. With this choice of initial time, Eq. 51 reduces to

$$\left[\bar{T} - \epsilon^{(\pi/3-\gamma)\bar{A}}\,\epsilon^{\gamma\bar{B}}\right]\bar{x}(\phi) = \bar{0} \tag{52}$$

or for simplicity

$$\bar{W}\,\bar{x}(\phi) = \bar{0} \tag{53}$$

Eq. 53 can be written in partitioned form as

$$\begin{bmatrix} \bar{W}_1 & \bar{W}_2 \\ \bar{W}_3 & \bar{W}_4 \end{bmatrix} \times \begin{bmatrix} \bar{i}(\phi) \\ \bar{e}(\phi) \end{bmatrix} = \bar{0} \tag{54}$$

It has been noted by Fath[7] that matrices $\bar{W}_3$ and $\bar{W}_4$ are identically zero and hence cannot be utilized further. When the vector $\bar{i}(\phi)$ is solved in terms of the source voltage vector $\bar{e}(\phi)$ using the upper two matrices

$$\bar{i}(\phi) = \bar{W}_1^{-1}\,\bar{W}_2\,\bar{e}(\phi) \tag{55}$$

or simply

$$\bar{i}(\phi) = \bar{Y}\,\bar{e}(\phi) \tag{56}$$

Since $i_{qs}^s = 0$ when $\omega_e t_o = \phi$, the first row of Eq. 56 must equal zero. If y_{11} denotes the (1,1) element and y_{12} the (1,2) element in the 4 x 2 matrix $\bar{Y}$ defined by Eq. 55, then

$$y_{11}e_1 + y_{12}e_2 = 0 \tag{57}$$

However, by virtue of Eqs. 13 and 14, $e_1(\phi) = V_m \sin\phi$ and $e_2(\phi) = V_m \cos\phi$. Substituting these two expressions in Eq. 57 yields the final result

$$\phi = \tan^{-1}(-y_{12}/y_{11}) \tag{58}$$

Having computed ϕ, the remaining unknown currents in the vector $\bar{i}(\phi)$ are readily obtained by means of Eq. 56.

Equation 56, together with 58, generate the required set of initial conditions needed to obtain the steady-state solution. It is of interest to note the added significance of the equations which have been derived. Although numerous switchings occur over a complete cycle, the system equations for any period are always linear. A necessary requirement for a linear system is that a linear relationship be maintained between the forcing function (source voltages) and the response (resulting currents). That is to say, if the terminal voltage is doubled, the machine currents should double as well. It is immediately apparent from Eq. 56 that this condition is satisfied for the instant $\omega_e t = \phi$. It follows that the result is also true for all successive time instants. Also, it is recalled that for a linear system, the phase angle between voltage and current is independent of the magnitude of the forcing function and depends only upon system parameters. Eq. 58 verifies this requirement.

Having obtained one point on the steady-state solution curve, the solution for any successive instant is readily evaluated by means of Eqs. 44 and 45. If n_2 is the number of solution points desired during the hold-off period γ of T_1, then for $\phi \leqslant \omega_e t \leqslant \phi + \gamma$

$$\bar{x}(\phi+n\gamma/n_2) = \epsilon^{\bar{B}\gamma/n_2}\,\bar{x}[\phi+(n-1)\gamma/n_2] \tag{59}$$

where $n = 1, 2, \ldots, n_2$;

During the remainder of time interval T_1, $\phi + \gamma \leqslant \omega\, t \leqslant \phi + \pi/3$ and all three machine terminals are connected to the source. If n_1 is the number of solution points desired during system state 1

$$\begin{aligned}\bar{x}[\phi+\gamma+(\pi/3-\gamma)n/n_1] &= \\ \epsilon^{\bar{A}(\pi/3-\gamma)n_1}\,&\bar{x}[\phi+\gamma+(\pi/3-\gamma)(n-1)/n_1]\end{aligned} \tag{60}$$

where $n = 1, 2, \ldots, n_1$.

Having computed the solution for the $\pi/3$ interval T_1, the solution for the remaining five intervals is immediately obtained by means of the recursion relationship, Eq. 50. These solution points require no actual computation and only involve systematic rotation of the values obtained for the first interval.

Although a d^s-q^s axes solution is often sufficient, it is generally desirable to obtain the actual phase voltages and currents. The required inverse relationships needed to transform system voltage from the d^s-q^s variables to the phase variables are

$$v_{as} = v_{qs}^s \tag{61}$$

$$v_{bs} = -\frac{1}{2}v_{qs}^s - \frac{\sqrt{3}}{2}v_{ds}^s \tag{62}$$

$$v_{cs} = -\frac{1}{2}v_{qs}^s + \frac{\sqrt{3}}{2}v_{ds}^s \tag{63}$$

In system state 1, the d^s-q^s axes voltages are related to the source voltage vector $\bar{e}$ by Eqs. 24 and 25. In system state 2, the as phase is open circuited and

$$v_{qs}^s = x_m \frac{p}{\omega_e} i_{qr}'^s \tag{64}$$

The quantity $(p/\omega_e)\, i_{qr}'^s$ is readily evaluated by computing the third row of the state variable equation defining the open circuited condition, Eq. 37. A set of transformation equations, analogous to Eqs. 61-63, is used to obtain the stator phase currents. The rotor currents can be obtained, if desired, by using the inverse expression for Eqs. 7 and 8.

Solution for mode 2; $\pi/3 < \gamma < 2\pi/3$

When the hold-off angle γ becomes greater than $\pi/3$, the three terminals of the machine are no longer simultaneously connected to the source voltages. Either one or two terminals are connected and Eqs. 37 and 42 describe the operation of the machine during the first time interval T_1. If an initial time t_o is again chosen such that $\omega_e t_o = \phi$, it can be readily shown that a necessary condition for steady-state operation is

$$\left[\bar{T} - \epsilon^{(2\pi/3-\gamma)\bar{B}} \epsilon^{(\gamma-\pi/3)\bar{C}}\right] \bar{x}(\phi) = \bar{0} \tag{65}$$

where $\bar{T}$ is again the time translation matrix defined by Eq. 78. As was the case for mode 1, $i_{qs}^s = 0$ when $\omega_e t_o = \phi$ so that Eqs. 53-58 can again be employed to solve for a suitable initial condition vector. In this case, $\bar{W}$ is the bracketed 6 x 6 matrix of Eq. 65.

COMPUTER SOLUTION OF THE SYSTEM EQUATIONS

In implementing a computer solution of the equations which have been derived, four matrix exponential functions must be evaluated in order to complete the solution. When γ is less than $\pi/3$, these expressions are: exp $[(\pi/3-\gamma)\bar{A}]$ exp $(\gamma\bar{B})$, exp $(\gamma\bar{B}/n_2)$ and exp $[(\pi/3-\gamma)\bar{A}/n_1]$ from Eqs. 52, 59 and 60, respectively. Although the computation of these functions is straightforward, a familiarity with numerical analysis is required. Briefly, the matrix exponential is defined by a power series expansion of the matrix argument similar in form to the familiar power series expansion of the scalar exponential ϵ^a. Accuracy can be obtained to an arbitrary number of significant digits by calculating enough terms in the power series. However, the number of terms required for a specified accuracy depends greatly on the matrix argument and if convergence is slow, round-off errors may introduce erroneous results. Fortunately, when the induction machine parameters are expressed in per unit, as in this paper, the system matrix is conditioned so that convergence is very rapid. When six digit accuracy is desired, the number of terms required in the power series expansion for the matrix exponentials used in Eq. 52 is typically ten. For the same accuracy, five terms are typically required for the matrix exponentials used in Eqs. 59 and 60 when n_1 or n_2 is approximately ten.

An excellent introduction to the computation of the matrix exponential is given by Liou.[13] A more efficient method used to obtain the computer results described herein is given by Fath.[14]

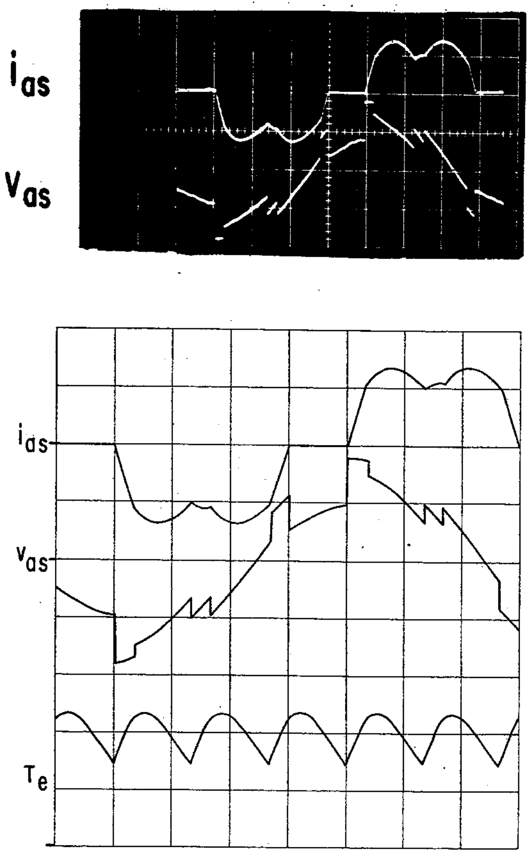
COMPARISON OF COMPUTED AND TESTED RESULTS

In order to verify the equations which have been developed, the solution obtained from a computer analysis was compared to the tested results of an actual system. The induction machine used for purposes of comparison was a 1/3 hp, 220 volt, 50 Hz, 4 pole, wound-rotor machine. Since both rotor and stator of the machine was random-wound, this choice represents a severe test of the accuracy of the analytical method. The measured parameters of the machine expressed in per unit using rated voltage as base volts and 375 watts as base power are: $r_s = 0.0566$, $r_r' = 0.069$, $x_s = 1.0318$, $x_r' = 1.0318$, $x_m = 0.969$. In order to more closely approximate an induction machine which might be utilized in a practical application, an external resistance $r_{ext}' = 0.0562$ was connected to the machine via slip rings. Hence, the total effective rotor resistance was increased to 0.1252 pu. Rated voltage at rated frequency was applied to the machine so that $V_m = 1.0$ and $\omega_e = 314$ rad/s.

In Fig. 3, a comparison of the actual machine phase voltage and current with the results obtained from a digital computer solution is shown. In order to facilitate a comparison, the calculated results have been converted from the per unit form to actual units. The hold-off angle γ was set at 45° ($\pi/4$) so that the induction machine was operating in mode 1. Also, for this case, the load torque $T_L = 1.0$ N-m, $\omega_r = 1325$ r/min. Although not available for measurement in the physical system, the instantaneous electro-magnetic torque obtained from the computer solution is also shown. A close comparison of computed and tested results is clearly evident.

In Fig. 4, the hold-off angle was increased to 70°. For this case, the machine was unable to overcome even its internal losses so that $T_L \cong 0$, and $\omega_r/\omega_e = 0$. Again, a close comparison is apparent.

Calculated torque-speed curves are given in Fig. 5 for constant values of γ. Again, actual rather than per unit values have been plotted. Measured values of average torque are denoted by an x or Δ. Due to the characteristic of the loading device, it was not possible to reliably measure torque beyond the effective breakdown values. Also, the current rating of the thyristors used prevented checking high load conditions for small γ. However, satisfactory correlation is evident over the range of operation measured. It can be noted that negligible torque is generated when γ is greater than 60° and mode 2 operation of the induction machine is clearly of little practical importance.

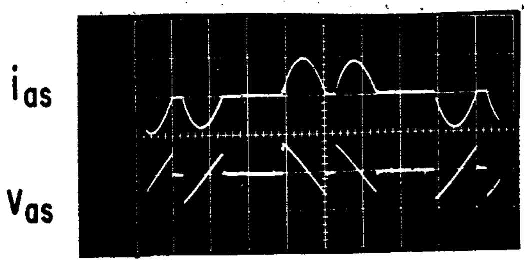


(b) Computed results

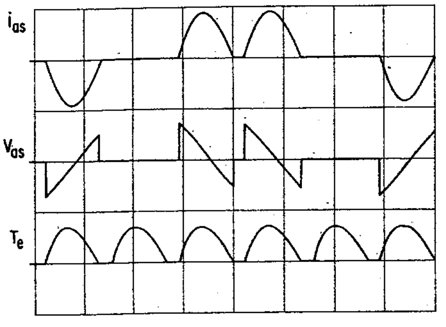
Fig. 3. Comparison of computed and measured results, $\gamma = 45°$, $\omega_r = 1325$ r/min. Scale: i_{as} - 1.0 A/division, v_{as} - 100 V/div., T_e - 0.5 N-m/div.

CONCLUSIONS

This paper has developed a direct, straightforward method of calculating the steady-state behavior of an induction motor operating with thyristors in the stator phases. The method is free from



(a) Actual system



(b) Computed results

Fig. 4. Comparison of computed and measured results, $\gamma = 70°$, $\omega_r=0$. Scale: i_{as} - 1.0 A/division, v_{as} - 100 V/div., T_e - 0.1 N·m/div.

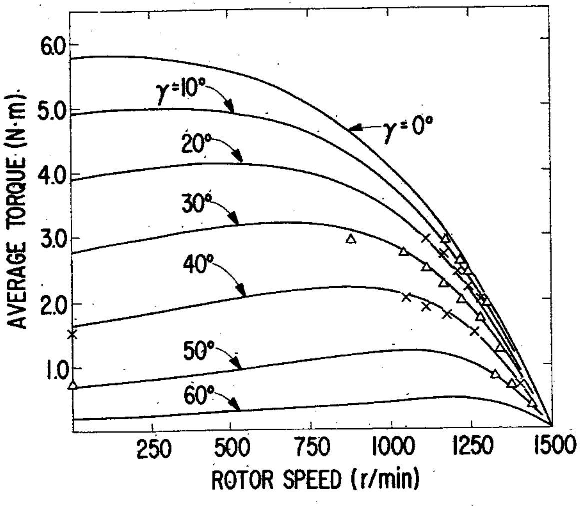


Fig. 5. Torque-speed curves for changes in γ.

the maze of complex components which has characterized the solution of this and similar problems in the past.

Two distinct modes of operation are shown to exist for a wye-connected, three-wire machine. These modes are independent of the machine parameters and depend only on the hold-off angle γ. Complete solutions have been obtained for both modes of operation. However, on the basis of this study, the second mode of operation does not appear to be of practical importance. The results of a digital computer solution are plotted and compared with the results obtained from an actual system. Good correlation is shown to exist over a wide range of delay angles.

Modern state variable techniques have been utilized throughout this analysis, and it is shown that this technique permits a closed form solution for a suitable set of initial conditions. Since matrix methods are exclusively employed, the analysis is well suited to a digital computer solution. This problem and solution vividly demonstrates that state variable methods provide the analyst with a new and powerful tool for the study of modern static ac and dc drives. It is expected that this technique will find increasing use in the analysis of such systems wherein the complex switching of the power converter defines numerous modes of operation.

ACKNOWLEDGEMENTS

The author wishes to thank the University of Manchester Institute of Science and Technology for facilities provided. Acknowledgement is made to the U. K. Science Research Council for the award of a grant in support of research in the area of variable speed drives. The author is also greatly indebted to Mr. J. Hindmarsh for his assistance in obtaining the experimental results.

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APPENDIX

When the stator currents i_{as}, i_{bs} and i_{cs} are half-wave symmetric

$$i_{as}(\omega_e t + \pi) = - i_{as}(\omega_e t) \quad (66)$$

$$i_{bs}(\omega_e t + \pi) = - i_{bs}(\omega_e t) \quad (67)$$

$$i_{cs}(\omega_e t + \pi) = - i_{cs}(\omega_e t) \quad (68)$$

The assumption of three-phase symmetry implies

$$i_{bs}(\omega_e t + 2\pi/3) = i_{as}(\omega_e t) \quad (69)$$

$$i_{cs}(\omega_e t + 2\pi/3) = i_{bs}(\omega_e t) \quad (70)$$

$$i_{as}(\omega_e t + 2\pi/3) = i_{cs}(\omega_e t) \quad (71)$$

Increasing the argument of Eqs. 69-71, and substituting the result in 66-68, yields

$$i_{as}(\omega_e t + \pi/3) = - i_{bs}(\omega_e t) \quad (72)$$

$$i_{bs}(\omega_e t + \pi/3) = - i_{cs}(\omega_e t) \quad (73)$$

$$i_{cs}(\omega_e t + \pi/3) = - i_{as}(\omega_e t) \quad (74)$$

The equivalent d^s-q^s stator currents at time instants $\omega_e t$ and $\omega_e t+\pi/3$ are related to the phase currents by Eqs. 6 and 7.

Eliminating the phase variables from the resulting six equations by means of the symmetry equations 72-74, the following equations for the variables are readily obtained.

$$i_{qs}^s(\omega_e t + \pi/3) = \frac{1}{2} i_{qs}^s(\omega_e t) + \frac{\sqrt{3}}{2} i_{ds}^s(\omega_e t) \quad (75)$$

$$i_{ds}^s(\omega_e t + \pi/3) = - \frac{\sqrt{3}}{2} i_{qs}^s(\omega_e t) + \frac{1}{2} i_{ds}^s(\omega_e t) \quad (76)$$

It is clear that a similar development applies for the stator phase voltages and referred rotor currents since transformation equations similar to 72-74 also apply for these variables. In addition to the machine variables, the source voltages e'_{ag}, e'_{bg}, and e'_{cg} are also half-wave and three-phase symmetric. Hence, a general symmetry relationship can be constructed which relates any of the machine currents or source voltages to these same variables phase displaced by $\pi/3$ radians. This relationship is expressed in matrix form as

$$\bar{x}(\omega_e t + \pi/3) = \bar{T}\, \bar{x}(\omega_e t) \quad (77)$$

where $\bar{x}$ is defined as $[i_{qs}^s, i_{ds}^s, i'^s_{qr}, i'^s_{dr}, e_1, e_2]^T$ and

$$\bar{T} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} \quad (78)$$

Discussion

Philip L. Alger (1758 Wendell Ave., Schenectady, N.Y. 12308): This paper presents for the first time a sound procedure for calculating the currents and voltages of a three-phase induction motor supplied through SCRs with delayed firing. This use of SCRs for voltage control is growing in importance and promises to be widely used in the future, particularly for induction motors driving fans and pumps, where feedback control of motor speed is required.

While I have not followed the analysis in the paper, I accept its conclusions as correct, based on the demonstrated agreement with test results and after talking with the author. A chief objection to this scheme of speed control with the usual high-resistance squirrel-cage motor is that the voltage must be brought down to less than 50% of normal, to bring the motor down to half speed; and in so doing the harmonics in the voltage and, therefore, in the motor current are quite large. It is my understanding that the high-frequency currents produced in the squirrel cage by these harmonics create high losses and increase the motor temperature, so that special provisions must be made for the ventilation of such motors. Also, in order to hold the current at half speed down to a low enough value, it is necessary that the full-load slip be of the order of 10%, making the motor efficiency much lower than normal.

As shown by several recent papers, particularly *Speed Control of Wound Rotor Motors with SCRs and Saturistors**, these difficulties can be overcome by the addition of Alnico bars in the slots of a squirrel-cage motor, giving the motor a flat speed-torque curve and a low starting current. Such a motor, when operated with SCR phase angle control, can be designed for a normal full-load slip of about 5%, and will come down to half speed when the effective voltage is reduced to perhaps 60% of rated voltage. Under these conditions, the harmonics in the motor voltage are much smaller for a given speed than in the case of the standard motor, and the induced high-frequency rotor currents are still smaller, because of the higher reactance introduced by the Alnico bars. Thus, as shown in the paper cited above, the effective currents and heating of the rotor are practically the same with SCR phase angle control as with sinusoidal voltage.

It is my opinion, therefore, that the use of squirrel-cage motors with Alnico bars in combination with SCR phase angle voltage control provides the best adjustable speed drive system for pumps and fans. It is true that the frequency converter with a standard motor provides better efficiency and much better performance when high torque at low speeds is required, but the lower cost and greater simplicity of the delayed firing SCRs make this system preferable for motors of moderate size.

Since the Saturistor impedance is non-linear with current and, when referred to primary, is independent of the speed, the analysis of its performance is quite different from that given in the author's paper. I hope that the author will extend his studies to this new case.

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*Philip L. Alger, William A. Coelho, and Mukund R. Patel, IEEE Trans. on Industry and General Applications, Vol. IGA-4, No. 5, Sept./Oct. 1968, pp. 477-485.

Ed. Gerecke (Freiestrasse, 212, 8032 Zurich, Switzerland): Dr. Lipo has reported an interesting analysis of the voltage controlled induction motor. However, in view of the considerable amount of work conducted here at the E.T.H. (Swiss Federal Institute of Technology) at Zurich/Switzerland, in the field of simulating induction motors on electronic computers, it seems that an information gap may exist between Schenectady and Zurich.

In 1956 we began to control induction motors by 6 thyratrons using the same configuration as in Fig. 1 of the paper of Dr. Lipo. In 1958, I asked Mr. Badr to simulate this system on an electronic computer. This was just at the end of the development of FORTRAN, and we had the impression that the time for programming and computation with a slow digital computer of that period would be too long. We chose, therefore, a small DONNER analog computer. Because simulation on this computer is executed by integration, we had to arrange the differential equations in such a way that on the left side of each equation appears the first derivatives of the unknown stator and rotor currents. This corresponds exactly to the today so-called state transition equations. Therefore, the differential equations on pages 41-51-72-79-80 of Mr. Badr's Doctor Thesis [15]

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for the three-phase motor and the two axis system (d-q) correspond with equation (1) of Dr. Lipo's Paper. We developed a simulation of a 5 kW motor with good accuracy and obtained the steady state solution for the stator currents, the stator and valve voltages, the torque $T_e(t)$ and the steady-state torque-speed curves for the induction motor and generator as a function of the firing angle α (Fig. 49). They are similar to the torque-speed curves on Fig. 5 of Dr. Lipo's paper. However, because he has chosen as parameter the hold-off angle γ, an exact comparison is not possible. Dr. Lipo should indicate that for every angle γ a corresponding angle α exists. Using a method similar to the OSSANA circle diagram for induction motors. Dr. Badr has also drawn the current loci in the complex plane for the first harmonic of the stator current. The cos ϕ factor goes down with firing control.

A very interesting feature is the transient behavior of the controlled induction motor, accelerating through or braking below synchronism, since the machine works during a short time as generator and has then a negative torque. A large oscillating torque is superposed over the steady state torque. The analog computer showed a great capability to display both these phenomena, as well as the influence of changes in the parameters and the electromagnetic torque $T_e(t)$, which cannot be measured directly.

Because Dr. Badr had connected the neutral points of the motor to the neutral point g of the generator (see Fig. 1), a small "ZERO-TORQUE" appeared. This fact was pointed out by Dr. Kovacs, who was a guest professor at our institute.

Fig. 5 of my paper [16] on the "Fifth International Analog Computation Meetings" [AICA] at Lausanne 1967, shows the block diagram of an induction machine in a closed control loop. Figs. 14 to 18 represent the steady state solution of the currents and the torque $T_e(t)$ for full load and no load; therefore, I can confirm the Figs. 3 and 4 of Dr. Lipo's paper. These were obtained by digital computation on the CDC-1604A computer of Control Data Corporation. The computing time for a period of 20 ms was 12 to 20 seconds. At slow motor speed, the computer did not arrive in a useful time at the steady state. Oscillations occur as a form of subharmonic.

The stimulation of iron saturation presents no special difficulty. Mr. Stürzinger [17] has simulated on the analog computer PACE-231R of EAI the behavior of the induction motor fed from a D.C. source by a self-commutating inverter.

Mr. Fatton [18] has simulated the transient state of a three-phase synchronous generator together with its water-turbine on a digital computer in the interval from no load to rated load as a nonlinear system.

As third IFAC president I quite agree with Dr. Lipo that the state variable methods of control engineers provide the analyst with a new and powerful tool. They give him a clear directive how to solve dynamic problems. We have applied it to the digital simulation of a high voltage D.C. power line together with the emitter and the receptor station. Every station has a three-phase thyristor bridge with 6 thyristors, which results in 64 + 64 = 128 different system states for the conducting and blocking valves. The power line of 1000 km length was simulated as a distributed system. Different kinds of transient phenomena could be studied such as switching on full load, no load and short-circuit, the behavior of automatic control and many kinds of defects of the valves.

When simulating such large power systems, we have always found 3 kinds of equations:

a) algebraic linear and nonlinear equations, i.e. from the 2 Kirchhoff laws,

b) linear and nonlinear first order differential equations in connection with the energy stores,

c) logical equations for the firing and the extinction of the valves.

In many cases, a hybrid computer consisting of an analog and a digital part, together with a memory CRO, has distinct advantages. In general, simulation of large power systems demand much skill and experience as well as a large high speed computer.

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P. C. Krause (Purdue University, Lafayette, Ind.): Dr. Lipo has solved a problem which has eluded other researchers. Moreover, he accomplishes this task directly with the simplicity of solution which cannot be equaled by any other method.

There are two features of this paper worth discussion. First, Dr. Lipo employs the d-q axis variables fixed in the stator. He does not complicate his analysis by using phase variables directly as is often suggested.[19,20] I attempted to point out the advantages of the d-q axis approach for this problem in my discussions of Refs. 7, 19 and 20. This paper by Dr. Lipo will place all other approaches to this problem in their proper perspective.

The second feature of this paper, which is actually linked to the first, is the concept of applying the open-circuit voltage to the d-q model for the purpose of maintaining a current zero. This concept enables one to develop the open-circuit voltage and to maintain a current zero without changing the parameters in the d-q model of the machine. Dr. Lipo and I have employed this idea in numerous analog computer simulations. The theory was first set forth in Ref. 9 and it was first employed in Ref. 21 to study the dynamic performance of an induction machine with series controlled rectifiers. Dr. Lipo has applied this idea to advantage in his analysis of steady-state performance.

Finally, if such information is readily available, it would be helpful to know how the speed-torque curves appear when α is used as a parameter rather than γ.

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W. Shepherd and H. Gaskell (University of Bradford, Bradford BD7 1DP, England): Dr. Lipo is to be congratulated on obtaining an analytic form of solution to a hitherto intractable problem. It is perhaps poetic justice that the thyristor extinction angle, which proved the stumbling block in an earlier attempt at solution (reference 4 of the paper), is used as a key parameter in the state-space approach.

The author's assumption of constant rotor speed with symmetrical phase-angle control is justified for machines of normal per-unit inertia and large conduction angle. For low inertia motors the double supply frequency torque harmonic which is usually obtained with phase-angle triggering causes speed ripple. Even for machines of normal per-unit inertia the use of large retardation of firing-angle has been found in some cases to cause second harmonic torque pulsations of such severity as to produce slight speed ripple. The performance of Fig. 3 which combines torque pulsations with constant, non-zero speed may therefore imply a motor of rather high inertia. It would be of interest in control applications to investigate at what value of the "figure of merit", torque/inertia ratio, the assumption of constant speed ceases to give accurate values of torque.

We would like to commend the attention of the author to the further class of problems whereby induction motor voltage control is obtained by integral-cycle triggering of thyristors. When the phase-current is interrupted for several supply cycles there is definite speed droop unless the motor has very large inertia. Such problems appear to be quite intractable of analytic solution but can be tackled digitally. The state-space approach, with appropriate transformations of variables, may provide a solution here also.

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T. A. Lipo: I wish to thank the discussors for their stimulating comments which have contributed both to the history of this problem and to trends in its future application.

Professor Alger has emphasized the use of Saturistors in the rotor circuit of an induction motor when operating from a variable voltage thyristor power supply. The use of nonlinear devices can, indeed, produce a significant reduction in high frequency losses for high slip conditions and an improved torque per rms ampere. However, work thus far has centered about the wound rotor motor in which the Saturistor is externally connected to the rotor current.* In order that the technique be practical, it would be necessary, as Professor Alger has noted, to incorporate the Saturistor in the rotor slots. Unfortunately, this introduces added machining and assembly operations to the construction of a squirrel cage rotor. It has not yet been established whether the improvement in performance warrants the added cost of material and manufacture.

Professor Gerecke has provided an impressive summary of his research on the same problem. I am aware of the simulation work carried out at E.T.H., Zürich. Since little information has passed between Zürich and Schenectady, I agree that an "information gap" exists. However, having pioneered in the simulation of ac machinery,[22,23,24] General Electric has developed the capability of simulating many of the problems described by Dr. Gerecke. Hence, an "information gap" should not be considered as a "technology gap."

Digital or analog simulation methods, indeed, provide the analyst with an alternative means to obtain a solution. In fact, the author completed a similar study of the same problem on an analog computer several years ago. The difference between simulation techniques and the state variable method used in this paper centers around how the solution is obtained.

Simulation is basically an implicit formulation of the system differential equations in which the equations are simply rearranged to accommodate the computer to be used. Initial conditions are guessed, and the steady-state solution is generated as a by-product of the transient solution obtained either by feedback or iteration. In this paper, an explicit analytic solution has been obtained whereby the initial conditions needed to generate the steady-state have been expressed directly in terms of system parameters (terminal voltage amplitude, slip, hold-off angle, and machine constants). The system currents are then solved as explicit functions of time.

Professor Gerecke has stated that computing time for simulating one cycle of the steady-state solution was 12 to 20 seconds. It should be noted that since the initial conditions needed to achieve steady state are not known a priori in a simulation, 10 to 20 cycles must typically be computed to pass through the initial transient. Hence, 120 seconds or more of computing time would be a more accurate figure for an entire computing run.

Using the explicit solution, machine currents are expressed as functions of time. Hence, the number of solution points computed per cycle for display purposes is arbitrary. Since Professor Gerecke does not mention the step size of the integration algorithm used for his simulation, a direct comparison is difficult. However, when as many as 200 solution points are computed per cycle, the time needed to obtain a full cycle of the steady-state solution is less than one second. The computer used during the study was a GE 600 series.

I concur with the statements of Professor Krause. It appears that phase coordinates are currently in favor, having been suggested for use in synchronous machine modeling[25] as well as for induction machines.[19,20] When space harmonics are neglected, I feel that d-q axis techniques continue to be the most straightforward and efficient method of analyzing ac machinery.

Professor Krause has suggested that speed-torque curves using the delay angle α as a parameter rather than γ would be useful. These curves are given in Fig. 6. The curves were obtained by extrapolating between solutions for constant γ. This technique is straightforward, and it should be emphasized that the procedure is not the same as iterating boundary conditions in order to arrive at a solution. A less rapid change in the characteristic with α than with γ can be noted. Rather than all curves terminating at zero torque and synchronous speed as was the case in Fig. 6, the curves intersect the curve for zero delay denoted by $\alpha = \phi$.

Dr. Shepherd and Dr. Gaskel have pointed out the necessity of constant rotor speed in the formulation of the solution. Such an assumption is, of course, required in order to arrive at a permissible set of linear differential equations. In this paper, it has been assumed that the hold-off angle γ associated with each of the six thyristors is identical. In this case, the lowest torque harmonic is six times supply

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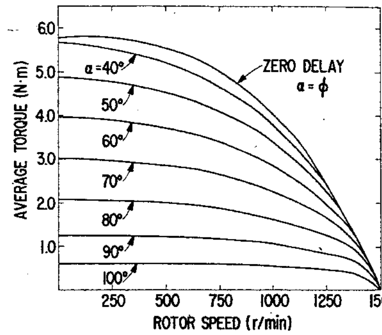

Fig. 6. Torque-speed curves for changes in α.

frequency, and the assumption of constant rotor speed is valid even for machines of relatively low inertia.

Imperfections in the thyristor gate circuitry or in the thyristors themselves can sometimes cause unsymmetrical firing. A second harmonic or even a fundamental component of electromagnetic torque can appear. The case of unsymmetrical firing is clearly not within the scope of this paper. However, the analysis techniques employed can be readily adapted to such a study. Since constant speed would again be assumed, it would then be quite important to establish a minimum torque/inertia ratio for each unbalanced condition. With an integral-cycle or burst firing type of speed control, speed deviation is of utmost importance, and a constant speed assumption is valid as over only a small portion of the speed range. At the present time, it appears that this problem is best studied using simulation techniques.

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