Solar Energy Vol. 47, No. 4, pp. 287-297, 1991
Printed in the U.S.A.

0038-092X/91 $3.00 + .00

MONTHLY-AVERAGED DAILY SHADING FACTOR FOR A COLLECTOR FIELD

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Abstract—An analytical expression is derived for the monthly-averaged daily shading factor $\bar{f}$ for a field of flat plate collectors. The elements of the collector field are oriented towards the equator and tilted with a constant value of tilt angle. Uniform spacing is maintained between various rows of collectors in the field. The expression of $\bar{f}$ is found to depend on the site latitude angle, month, length and width of each collector, spacing between any successive rows of collectors, number of collectors in a row, and the value of the tilt angle for collectors. Graphical presentation for the value of $\bar{f}$ at wide range of its independent variables is given. The graphs and the analytical expression of $\bar{f}$ assist in improving the accuracy of predicting the monthly-averaged daily solar radiation received by a collector field.

1. INTRODUCTION

Long-term performance of a single flat plate collector depends on the monthly-averaged daily radiation $\bar{H}_t$ received by the collector surface. The value of $\bar{H}_t$ is usually estimated using the following commonly accepted isotropic model.

$$\frac{\bar{H}_t}{\bar{H}} = \left(1 - \frac{\bar{H}_d}{\bar{H}}\right)\bar{R}_b + \frac{1}{2}\frac{\bar{H}_d}{\bar{H}} \times (1 + \cos\beta) + \tfrac{1}{2}\rho_{gr}(1 - \cos\beta) \quad (1)$$

where the meanings of the various symbols are given in the nomenclature list. In a previous work[1] we showed that the above equation for $\bar{H}_t$ must be corrected before being applied to a collector field. The correction is necessary to account for shading of beam irradiance by other collectors in the preceding rows and also to account for shielding of the collector from ground and sky by collectors in the preceding rows. It was shown[1] that each term in the right hand side of eqn (1) is multiplied by one or more correction factors. To account for shading of beam irradiance the first term in eqn (1) must be multiplied by $(1 - \bar{f})$, i.e., to read

$$\left(1 - \frac{\bar{H}_d}{\bar{H}}\right)\bar{R}_b(1 - \bar{f})$$

where $\bar{f}$ is the monthly-averaged daily shading factor for the N elements row. The expression of $\bar{f}$ is given as follows[1]

$$\bar{f} = \frac{1}{\bar{H}_b\bar{R}_b}\frac{1}{M}\sum^{M}\int_{t_{sr}}^{t_{ss}} f\,\frac{\cos\theta}{\cos\theta_z}\,G_t\,\mathrm{d}t. \quad (2)$$

In the above expression the instantaneous shading factor f is defined as follows

$$f = \frac{\text{shaded area of all elements in the row}}{\text{total area of all elements in the row}}. \quad (3)$$

Calculation of the daily beam radiation received by a tilted, partially or fully shaded surface have been studied previously by several investigators. Applebaum and Bany[2] studied the geometry of shading of vertical and inclined poles. They then derived an analytical expression for the instantaneous shading factor f (see eqn 3) for south facing flat plate collector fields. Utzinger and Klein[3] used the same expression given by eqn (2) to determine $\bar{f}$ for vertical flat plate receivers shaded by overhangs. After deriving the expression for f, they used it in eqn (2) to determine $\bar{f}$ by carrying out a numerical integration. Finally, they established charts to provide $\bar{f}$ at various parametric conditions. Jones and Bukhart[4] considered a large flat plate collector field with rows of infinite width. With this simplification, they derived an analytical expression for the daily beam and daily total radiation on collector surfaces. Sharp[5] also studied a simple overhang on a surface at any tilt and azimuth and derived an expression for the daily- or monthly-averaged daily beam radiation received by the surface. Since his results are only for a special geometry they could not be applied therefore for collector fields. Recently, Elsayed and Al-Turki[6] studied in detail the calculation of the shading factor f (eqn 3) for equator facing collector fields of finite or infinite row width. A simple analytical expression is derived for f using the separation of variable technique. The results could be used to determine the instantaneous shading factor f for any collector in a row or for a whole row as caused by one or more collectors in a preceding row.

The objective of the present paper is to derive a mathematical expression for $\bar{f}$ for collector fields with elements oriented towards the equator and tilted with any arbitrary tilt angle.

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2. INSTANTANEOUS SHADING FACTOR

Consider a collector field of several rows of N elements per row and with collectors aligned behind each other and oriented toward the equator (to south in the northern hemisphere), as shown in Fig. 1. The spacing between rows is maintained at Δy and each element has width W and length L. In this geometry we have showed in a previous work[6] that the shading factor for any row (other than the first) is given as follows:

$$\left.\begin{aligned} f &= (1-\tan|G_1|)\left(1-\tan G_2\cdot\frac{NW}{L}\right), \\ &\quad \text{at } -1 \leqslant \tan G_1 \leqslant 1 \\ &\quad \text{and } 0 \leqslant \tan G_2 \leqslant \frac{L}{NW} \\ f &= 0; \quad \text{at} \quad |\tan G_1| \geqslant 1 \\ &\quad \text{and/or} \quad \tan G_2 \geqslant \frac{L}{NW} \end{aligned}\right\} \quad (4)$$

where the angles G_1 and G_2 are defined as follows[6]

$$\tan G_1 = \frac{\Delta y/(NW)\cdot\sin\beta\sin\theta_z\sin\psi}{\cos\theta_z\cos\beta+\sin\theta_z\cos\psi\sin\beta}, \quad (5a)$$

$$\tan G_2 = \frac{\Delta y/(NW)\cdot\cos\theta_z}{\cos\theta_z\cos\beta+\sin\theta_z\cos\psi\sin\beta}. \quad (5b)$$

By simple algebraic manipulation the angles G_1 and G_2 could be expressed in the following forms

$$\tan G_1 = \left(\frac{\Delta y}{NW}\right)\sin\beta\frac{\sin\omega}{\lambda+\cos(\phi-\beta)\cos\omega}, \quad (6a)$$

$$\tan G_2 = \left(\frac{\Delta y}{NW}\right)\cos\phi\;\frac{\cos\omega-\cos\omega_s}{\lambda+\cos(\phi-\beta)\cos\omega} \quad (6b)$$

where ω_s and λ are expressed as follows

$$\omega_s = \cos^{-1}(-\tan\delta\tan\phi), \quad (7a)$$

$$\lambda = \tan\delta\left[\tan\phi\cos(\phi-\beta)-\frac{\sin\beta}{\cos\phi}\right]. \quad (7b)$$

By definition, the angle G_1 is an equivalent solar azimuth angle and the angle G_2 is an equivalent solar altitude angle[6]. The angle G_2 is symmetric around solar noon while the angle G_1 is symmetric in magnitude only around solar noon. The change in the sign of G_1 is only due to the change in the sign of the solar hour angle ω before and after solar noon. The expression of f in eqn (4) could then be rewritten as follows

$$f = \left[1-\frac{\Delta y}{NW}\sin\beta\frac{|\sin\omega|}{\lambda+\cos(\phi-\beta)\cos\omega}\right] \times\left[1-\frac{\Delta y}{L}\frac{\cos\omega-\cos\omega_s}{\lambda+\cos(\phi-\beta)\cos\omega}\right],$$
$$0 \leqslant \omega_1 \leqslant |\omega| \leqslant \omega_2 \leqslant \omega_{st} \quad (8a)$$

$$f = 0, \quad |\omega| \leqslant \omega_1 \text{ or } |\omega| \geqslant \omega_2 \quad (8b)$$

where ω_1 and ω_2 are the boundaries of ω for the daily shading period after solar noon (shading period is symmetric around solar noon). Values of ω_1 and ω_2 are determined as given in Section 4. The sunset solar hour angle ω_{st} for the collector surface is determined from the following eqn[7]

$$\omega_{st} = \max\{\omega_s, \cos^{-1}[-\tan\delta\tan(\phi-\beta)]\}. \quad (9)$$

3. THE MONTHLY-AVERAGED DAILY SHADING FACTOR

In the present section the expression of f given by eqns (8a) and (8b) is used in eqn (2) and integration

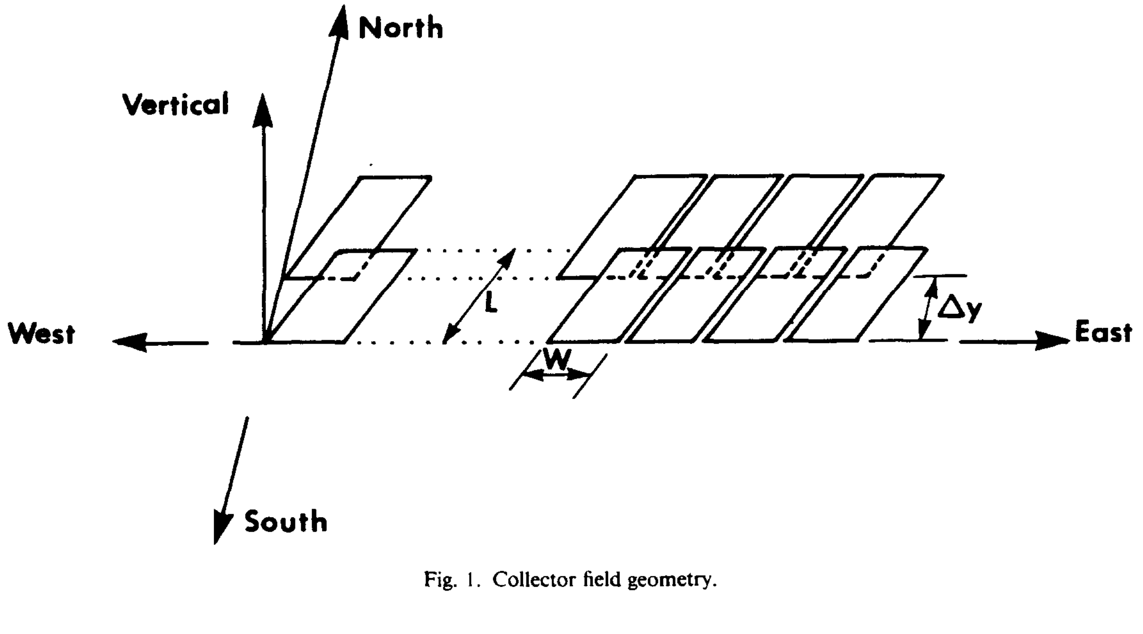

Fig. 1. Collector field geometry.

is carried out to find $\bar{f}$. Following the same procedure used by Liu and Jordan[8] and Klein[9] in the determination of $\bar{R}_b$, the extraterrestrial irradiance on a horizontal plane G_o is used as the weighing irradiance instead of actual value of G_t in eqn (2). The expression for G_o is given as follows[10].

$$G_o = G_{sc}\left[1 + 0.033\cos\left(\frac{360n}{365}\right)\right]\cos\theta_z \quad (10)$$

where G_{sc} is the solar constant and n is the Julian day. Equation (2) then takes the following form

$$\bar{f} = \frac{G_{sc}}{\bar{H}_o\bar{R}_b}\frac{1}{M} \times \sum^{M}\int_{t_{sr}}^{t_{ss}}\left[1 + 0.033\cos\left(\frac{360n}{365}\right)\cos\theta f\,\mathrm{d}t\right]. \quad (11a)$$

Using the average day $\bar{n}$ of the month as suggested by Klein[9], eqn (11a) becomes

$$\bar{f} = \frac{G_{sc}}{\bar{H}_o\bar{R}_b} \times \left[1 + 0.033\cos\left(\frac{360\bar{n}}{365}\right)\right]\int_{t_{sr}}^{t_{ss}}\cos\theta f\,\mathrm{d}t. \quad (11b)$$

Using the following expressions for $\bar{H}_o$ and $\bar{R}_b$[9],

$$\bar{H}_o = \frac{24 \times 3600}{\pi}G_{sc}\left[1 + 0.033\cos\left(\frac{360\bar{n}}{365}\right)\right] \times \left[\cos\phi\cos\delta\sin\omega_s + \frac{\pi\omega_s}{180}\sin\phi\sin\delta\right], \quad (12)$$

$$\bar{R}_b = \frac{\cos(\phi-\beta)\cos\delta\sin\omega_{st} + \dfrac{\pi}{180}\omega_{st}\sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\sin\omega_s + \dfrac{\pi\omega_s}{180}\sin\phi\sin\delta} \quad (13)$$

the expression for $\bar{f}$ then becomes

$$\bar{f} = \frac{J}{\cos(\phi-\beta)\cos\delta\sin\omega_{st} + \dfrac{\pi}{180}\omega_{st}\sin(\phi-\beta)\sin\delta} \quad (14)$$

where

$$J = \frac{1}{2}\int_{-\omega_s}^{\omega_s}\cos\theta f\,\mathrm{d}\omega = \int_0^{\omega_s}\cos\theta f\,\mathrm{d}\omega \quad (15)$$

since both f and θ are symmetric around solar noon. Substituting the expression for $\cos\theta$[10] and the expression of f using eqn (8) yields

$$J = \int_{\omega_1}^{\omega_2}\{\sin(\phi-\beta)\sin\delta + \cos(\phi-\beta)\cos\delta\cos\omega\} \times \left\{1 - \frac{\Delta y}{NW}\sin\beta\cdot\frac{\sin\omega}{\lambda + \cos(\phi-\beta)\cos\omega}\right\} \times \left\{1 - \frac{\Delta y}{L}\frac{\cos\omega - \cos\omega_s}{\lambda + \cos(\phi-\beta)\cos\omega}\right\}\mathrm{d}\omega. \quad (16)$$

Carrying out the above integration, the expression of $\bar{f}$ given by eqn (14) becomes

$$\bar{f} = \frac{1}{\cos(\phi-\beta)\cos\delta\sin\omega_{st} + \dfrac{\pi}{180}\omega_{st}\sin(\phi-\beta)\sin\delta} \times \left\{C_1\frac{\pi}{180}(\omega_2 - \omega_1) + C_2(\sin\omega_2 - \sin\omega_1) - C_3(\cos\omega_2 - \cos\omega_1) + C_4[I(\omega_2) - I(\omega_1)] - C_5\ln\left(\frac{\eta + \cos\omega_2}{\eta + \cos\omega_1}\right) + C_6\left[\frac{1}{\eta + \cos\omega_2} - \frac{1}{\eta + \cos\omega_1}\right]\right\} \quad (17)$$

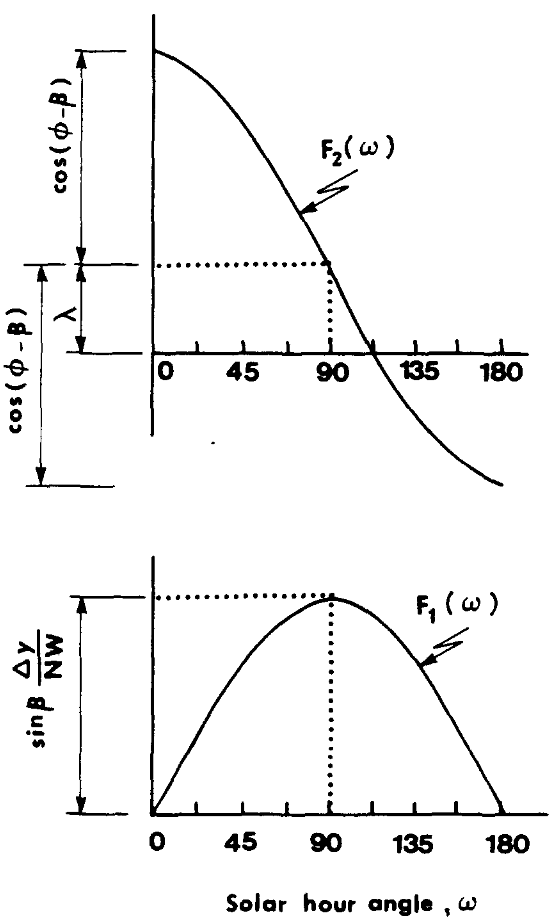

Fig. 2. Presentation of the functions $F_1(\omega)$ and $F_2(\omega)$.

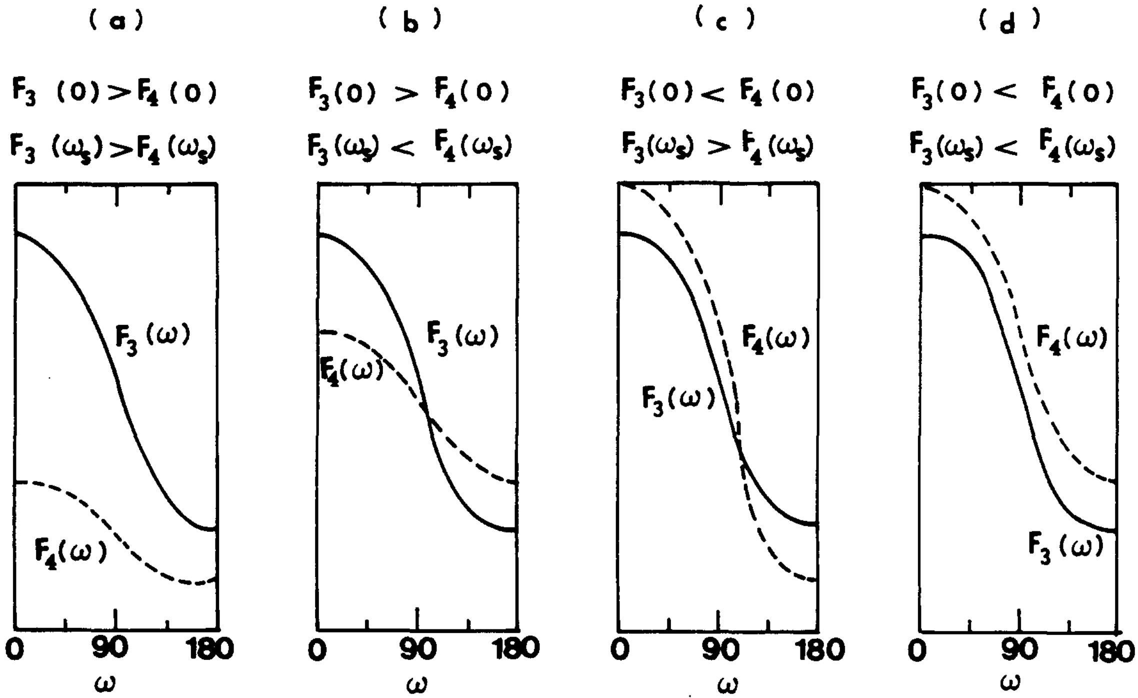

Fig. 3. Presentation of the functions $F_3(\omega)$ and $F_4(\omega)$.

where

$$\eta = \frac{\tan\delta}{\cos\phi}\left(\sin\phi - \frac{\sin\beta}{\cos(\phi-\beta)}\right) \tag{18a}$$

$$C_1 = \frac{\sin\delta}{\cos(\phi-\beta)}\left\{\sin(\phi-\beta)\left[\cos(\phi-\beta) - \cos\phi\,\frac{\Delta y}{L}\right] - \sin\beta\,\frac{\Delta y}{L}\right\} \tag{18b}$$

$$C_2 = \cos\delta\left[\cos(\phi-\beta) - \cos\phi\,\frac{\Delta y}{L}\right] \tag{18c}$$

$$C_3 = \sin\beta\cos\delta\,\frac{\Delta y}{NW}\left[\frac{\Delta y}{L}\cdot\frac{\cos\phi}{\cos(\phi-\beta)} - 1\right] \tag{18d}$$

$$C_4 = \cos\phi\sin\delta\,\frac{\Delta y}{L}\left[\tan(\phi-\beta) - \tan\phi + \frac{\sin\beta}{\cos\phi\cos(\phi-\beta)}\right] \tag{18e}$$

$$C_5 = \frac{\Delta y}{NW}\,\frac{\sin\beta\sin\delta}{\cos(\phi-\beta)}\left\{\left[\tan(\phi-\beta)\cos\phi + \frac{2\sin\beta}{\cos(\phi-\beta)} - \sin\phi\right]\frac{\Delta y}{L} - \sin(\phi-\beta) - \left[\frac{\sin\beta}{\cos\phi} - \tan\phi\cos(\phi-\beta)\right]\right\} \tag{18f}$$

$$C_6 = \frac{\Delta y}{NW}\,\frac{\Delta y}{L}\,\frac{\sin^2\beta\sin^2\delta}{\cos\delta\cos^2(\phi-\beta)}\left[\tan\phi - \tan(\phi-\beta) - \frac{\sin\beta}{\cos\phi\cos(\phi-\beta)}\right]. \tag{18g}$$

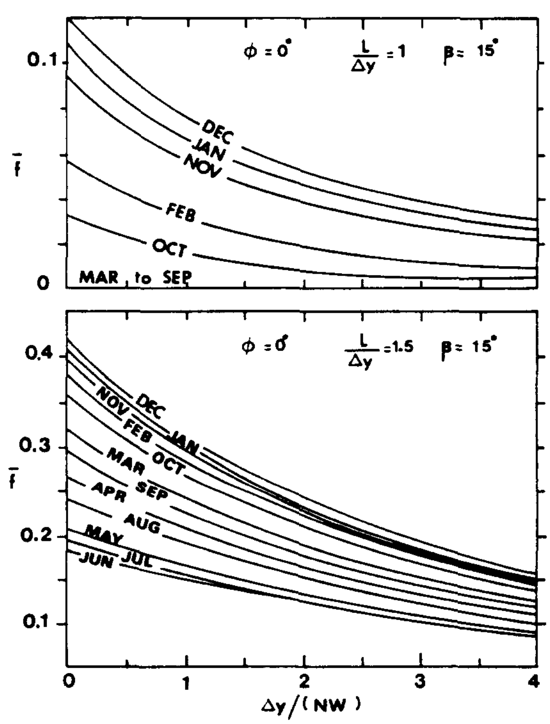

Fig. 4. Variation of $\bar{f}$ with $\Delta y/(NW)$ at $\phi = 0°$, $\beta = 15°$, and $L/\Delta y = 1$ and 1.5.

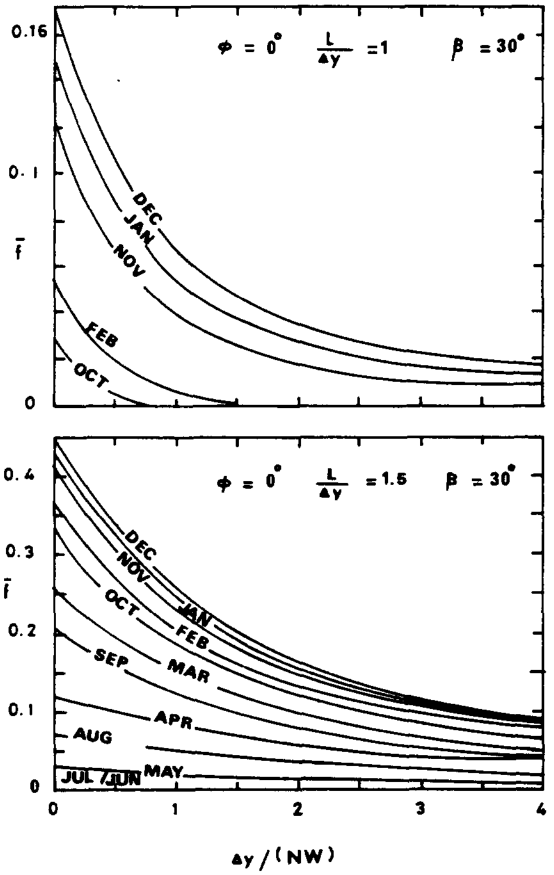

Fig. 5. Variation of $\bar{f}$ with $\Delta y/(NW)$ at $\phi = 0°$, $\beta = 30°$, and $L/\Delta y = 1$ and 1.5.

The function $I(\omega)$ depends on the value of the coefficient η as follows

$$|\eta| > 1: \quad I = \frac{2}{\sqrt{\eta^2 - 1}} \tan^{-1}\left[\frac{\eta - 1}{\sqrt{\eta^2 - 1}} \tan\left(\frac{\omega}{2}\right)\right];$$

$$\eta = 1: \quad I = \tan\left(\frac{\omega}{2}\right);$$

$0 < |\eta| < 1$:

$$I = \frac{1}{2\sqrt{1 - \eta^2}} \ln\left[\frac{(1 - \eta)\tan\left(\frac{\omega}{2}\right) + \sqrt{1 - \eta^2}}{(1 - \eta)\tan\left(\frac{\omega}{2}\right) - \sqrt{1 - \eta^2}}\right]^2;$$

$$|\eta| = 0: \quad I = \ln\left[\tan\left(\frac{\pi}{4} + \frac{\omega}{2}\right)\right];$$

$$\eta = -1: \quad I = \cot\left(\frac{\omega}{2}\right). \qquad (19)$$

4. DAILY SHADING PERIOD

In the previous section the expression for $\bar{f}$ is found to depend on the boundaries ω_1 and ω_2 of the afternoon part of the shading period on the average day of the month. We shall now determine the expressions for ω_1 and ω_2.

4.1 *Condition for G_1*

The angle G_1 is defined as an equivalent solar azimuth angle measured from south to west[6]. Thus, G_1 is symmetric in magnitude around solar noon but has negative sign before solar noon and positive sign in the afternoon. For shading to exist at any instance during a day, the angle G_1 must satisfy the following inequality indicated by eqn (4)

$$-1 \leq \tan G_1 \leq 1.$$

This condition is reduced to the following inequality, when only considering the afternoon period of the day

$$0 \leq \sin\beta \frac{\Delta y}{NW} \sin\omega \leq \lambda + \cos(\phi - \beta)\cos\omega$$

or

$$0 \leq F_1(\omega) \leq F_2(\omega) \qquad (20)$$

where eqn (6A) is used to replace $\tan G_1$ in the in-

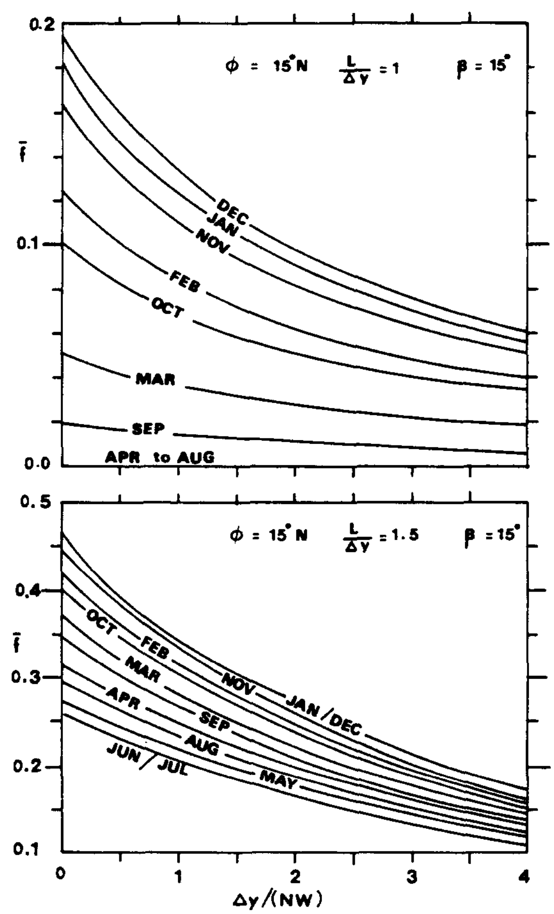

Fig. 6. Variation of $\bar{f}$ with $\Delta y/(NW)$ at $\phi = 15°$N, $\beta = 15°$ and $L/\Delta y = 1$ and 1.5.

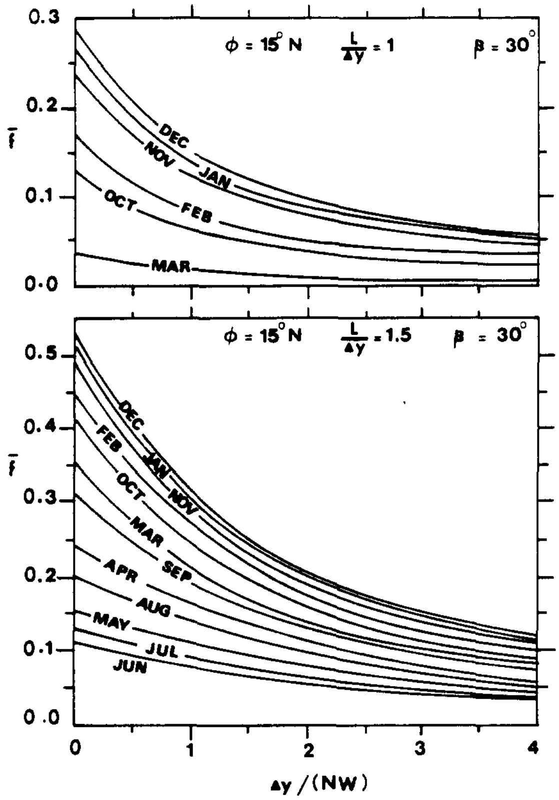

Fig. 7. Variation of $\bar{f}$ with $\Delta y/(NW)$ at $\phi = 15°N$, $\beta = 30°$, and $L/\Delta y = 1$ and 1.5.

equality. The functions $F_1(\omega)$ and $F_2(\omega)$ are defined as follows

$$F_1(\omega) = \sin\beta\,\frac{\Delta y}{NW}\sin\omega \tag{21}$$

$$F_2(\omega) = \lambda + \cos(\phi - \beta)\cos\omega. \tag{22}$$

The lower and upper limits ω'_1 and ω'_2 for the value of ω between solar noon to ω_s that satisfy the inequality in eqn (20) are now determined with the help of Fig. 2.

a) $\lambda + \cos(\phi - \beta) \leq 0$; no possible shading, let

$$\omega'_1 = \omega'_2 = 0 \tag{23a}$$

b) $\lambda + \cos(\phi - \beta) \geq 0$;

$$\omega'_1 = 0,\ \omega'_2 = \min(\omega'_{2+}, \omega'_{2-}) \tag{23b}$$

where ω'_{2+} and ω'_{2-} are the roots of solving the equation

$$\sin\beta\,\frac{\Delta y}{NW}\sin\omega = \lambda + \cos(\phi - \beta)\cos\omega. \tag{24}$$

Values of ω'_{2+} and ω'_{2-} are given as follows: $b \geq 4ac$:

$$\omega'_{2+} = \cos^{-1}\left\{\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right\} \tag{25a}$$

$$\omega'_{2-} = \cos^{-1}\left\{\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right\} \tag{25b}$$

$b < 4ac$: no roots, take

$$\omega_{2+} = \omega_{2-} = \omega_{st}.$$

The coefficients a, b, and c are given by the following expressions

$$a = \sin^2\beta\left(\frac{\Delta y}{NW}\right)^2 + \cos^2(\phi - \beta) \tag{26a}$$

$$b = 2\lambda\cos(\phi - \beta) \tag{26b}$$

$$c = \lambda^2 - \sin^2\beta\left(\frac{\Delta y}{NW}\right)^2. \tag{26c}$$

4.2 *Condition for G_2*

As given in [6], the angle G_2 is an equivalent solar altitude angle. The angle G_2 is symmetric around solar noon. Using eqns (4) and (6b), the condition for G_2 to cause shading at a given instance in a day is that

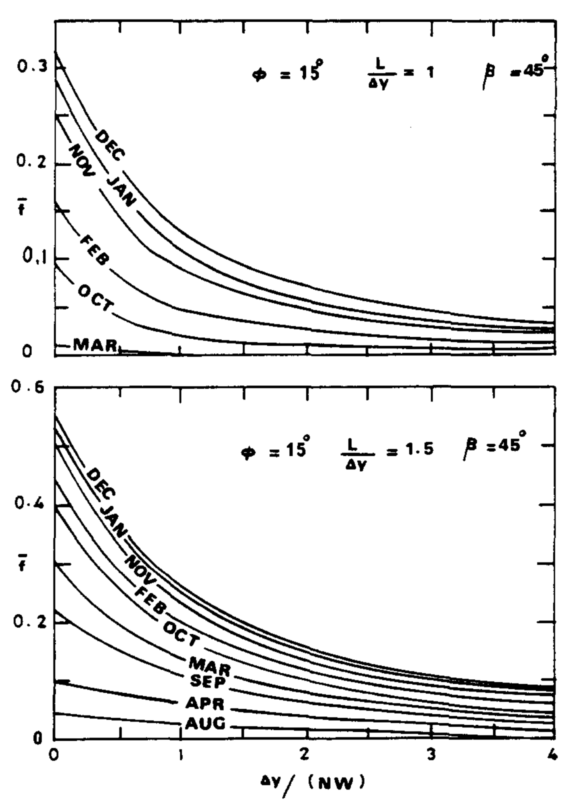

Fig. 8. Variation of $\bar{f}$ with $\Delta y/(NW)$ at $\phi = 15°N$, $\beta = 45°$, and $L/\Delta y = 1$ and 1.5.

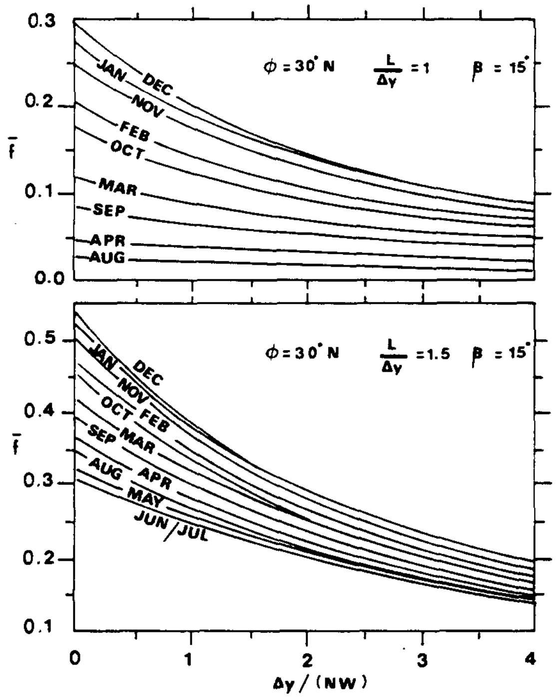

Fig. 9. Variation of $\bar{f}$ with $\Delta y/(NW)$ at ϕ = 30°N, β = 15°, and $L/\Delta y$ = 1 and 1.5.

$$0 \leqslant \frac{\Delta y}{L} \cos \phi(\cos \omega - \cos \omega_s)$$

$$\leqslant (\lambda + \cos(\phi - \beta)\cos \omega) \quad (27)$$

or

$$0 \leqslant F_3(\omega) \leqslant F_4(\omega) \quad (28)$$

where:

$$F_3(\omega) = \frac{\Delta y}{L} \cos \phi(\cos \omega - \cos \omega_s) \quad (29)$$

$$F_4(\omega) = \lambda + \cos(\phi - \beta)\cos \omega. \quad (30)$$

By plotting the general trends for the functions $F_3(\omega)$ and $F_4(\omega)$ as shown in Fig. 3, the lower and upper limits ω_1'' and ω_2'' of the range of ω to satisfy the inequality in eqn (28) could be determined in accordance with the relative values of $F_3(0)$ and $F_4(0)$, and $F_3(\omega_s)$ and $F_4(\omega_s)$ as follows

a) $F_3(0) > F_4(0)$ and $F_3(\omega_s) > F_4(\omega_s)$; no possible shading, let

$$\omega_1'' = \omega_2'' = 0. \quad (31a)$$

b) $F_3(0) > F_4(0)$ and $F_3(\omega_s) < F_4(\omega_s)$:

$$\omega_1'' = \cos^{-1} P, \quad \omega_2'' = \omega_{st}. \quad (32b)$$

c) $F_3(0) < F_4(0)$ and $F_3(\omega_s) > F_4(\omega_s)$:

$$\omega_1'' = 0, \quad \omega_2'' = \cos^{-1} P. \quad (32c)$$

d) $F_3(0) < F_4(0)$ and $F_3(\omega_s) < F_4(\omega_s)$

$$\omega_1'' = 0, \quad \omega_2'' = \omega_{st}. \quad (32d)$$

In the above equations the parameter P is the solution of the equation $F_3 = F_4$, and is given as follows

$$P = \frac{(\Delta y/L)\cos \phi \cos \omega_s + \lambda}{(\Delta y/L)\cos \phi - \cos(\phi - \beta)}. \quad (33)$$

4.3 *Shading range for* ω

As given by eqn (4), shading takes place only when both the inequalities given by eqns (20) and (28) for G_1 and G_2, respectively are satisfied. In this case, the shading period is symmetric around solar noon. In the post solar noon period, shading extends from ω_1 to ω_2 where ω_1 and ω_2 are given as follows

$$\omega_1 = \max\{\omega_1', \omega_1''\} \quad (34)$$

$$\omega_2 = \max\{\omega_1, \min(\omega_2', \omega_2'')\}. \quad (35)$$

The determination of the lower and upper limits ω_1 and ω_2 of the shading period (after solar noon) during

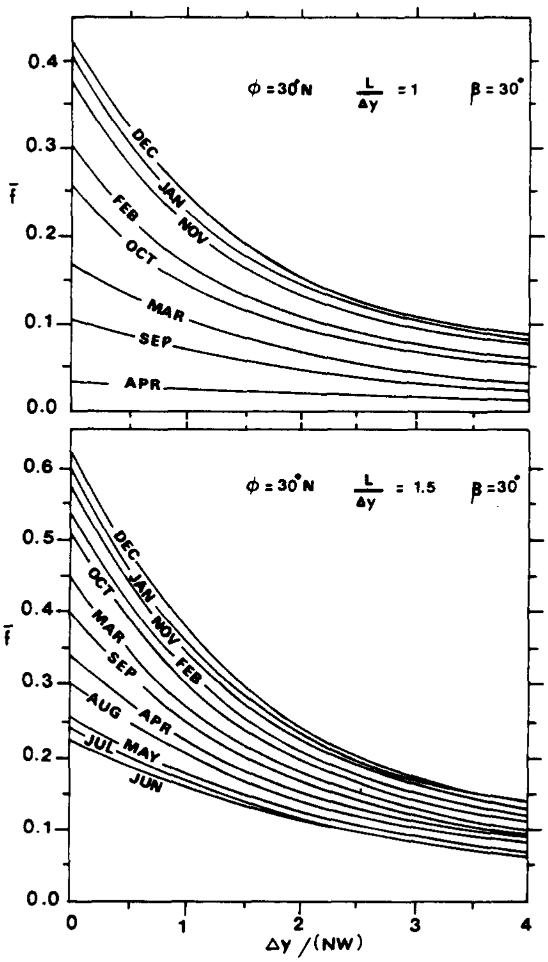

Fig. 10. Variation of $\bar{f}$ with $\Delta y/(NW)$ at ϕ = 30°N, β = 30, and $L/\Delta y$ = 1 and 1.5.

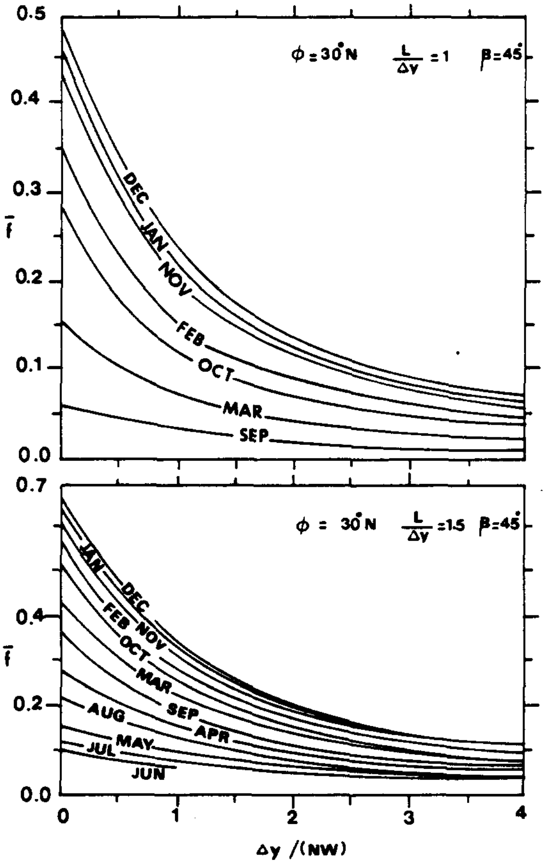

Fig. 11. Variation of $\bar{f}$ with $\Delta y/(NW)$ at ϕ = 30°N, β = 45° and $L/\Delta y$ = 1 and 1.5.

a given day n is completed. Values of ω_1 and ω_2 are used in eqn (17) to determine the monthly-averaged daily shading factor $\bar{f}$ for a collector row.

5. SIMPLIFIED EXPRESSION FOR THE CASE $\beta = \phi$

In many practical cases the collectors are tilted with tilt angle $\beta = \phi$. In this case, the expression for $\bar{f}$ is determined following the same procedure outlined previously for the general case of $\beta \Leftrightarrow \phi$. The expression of $\bar{f}$ in this case takes the following form

$$\bar{f} = \frac{1}{\sin \omega_{st}} \left[\left(1 - \frac{\Delta y}{L} \cos \phi \right) (\sin \omega_2 - \sin \omega_1) + \frac{\Delta y}{L} \cos \phi \cos \omega_s \frac{\pi}{180} (\omega_2 - \omega_1) + \frac{\Delta y}{NW} \sin \phi \left(1 - \frac{\Delta y}{L} \cos \phi \right) (\cos \omega_2 - \cos \omega_1) + \frac{\Delta y}{NW} \frac{\Delta y}{L} \sin \phi \cos \phi \cos \omega_s \ln \left(\frac{\cos \omega_2}{\cos \omega_1} \right) \right] \quad (36)$$

where ω_1 and ω_2 are then given as follows:

a) $\omega_s \leqslant 90$:

$$\frac{L}{\Delta y} > \cos \phi (1 - \cos \omega_s): \quad \omega_1 = 0, \quad \omega_2 = \min(\omega_{st}, \Omega_2) \quad (37)$$

$$\frac{L}{\Delta y} \leqslant \cos \phi (1 - \cos \omega_s):$$

$\Omega_1 > \Omega_2$: No shading, let

$$\omega_1 = \omega_2 = 0 \quad (38a)$$

$$\Omega_1 \leqslant \Omega_2: \quad \omega_1 = \Omega_1, \quad \omega_2 = \min(\omega_{st}, \Omega_2) \quad (38b)$$

b) $\omega_s > 90$:

$$\frac{L}{\Delta y} < \cos \phi (1 - \cos \omega_s):$$

No shading, let $\omega_1 = \omega_2 = 0$ (39a)

$$\frac{L}{\Delta y} \geqslant \cos \phi (1 - \cos \omega_s): \quad \omega_1 = 0, \quad \omega_2 = \min(\omega_{st}, \Omega_1, \Omega_2) \quad (39b)$$

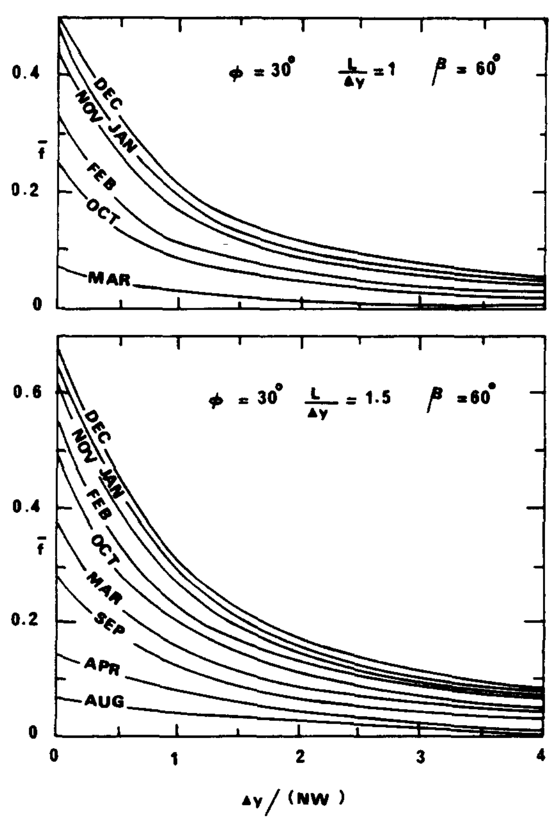

Fig. 12. Variation of $\bar{f}$ with $\Delta y/(NW)$ at ϕ = 30°N, β = 60°, and $L/\Delta y$ = 1 and 1.5.

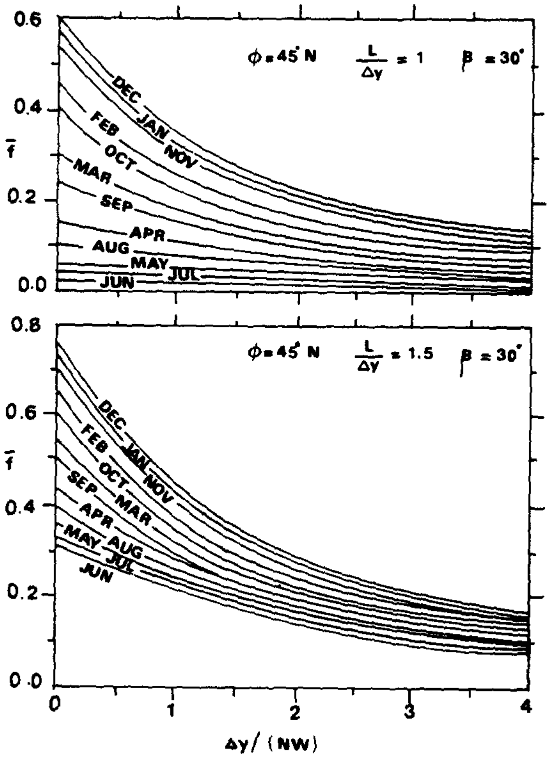

Fig. 13. Variation of $\bar{f}$ with $\Delta y/(NW)$ at $\phi = 45°$N, $\beta = 30°$, and $L/\Delta y = 1$ and 1.5.

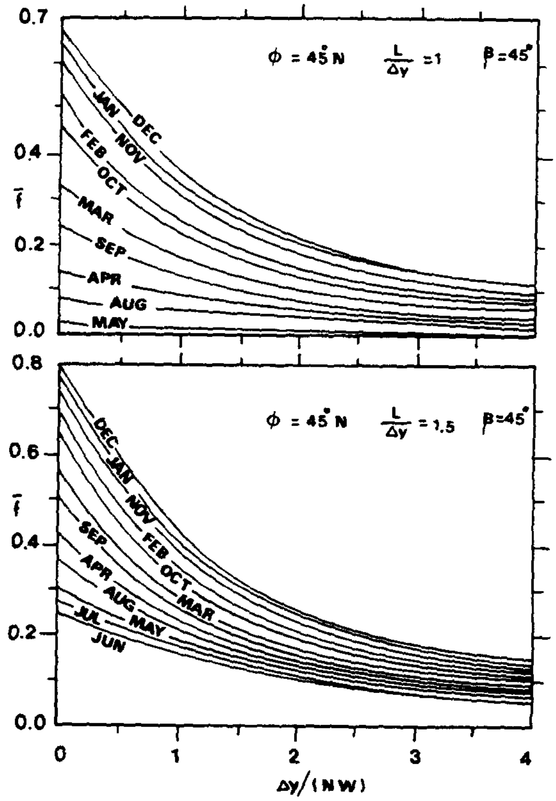

Fig. 14. Variation of $\bar{f}$ with $\Delta y/(NW)$ at $\phi = 45°$N, $\beta = 45°$, and $L/\Delta y = 1$ and 1.5.

The angles Ω_1 and Ω_2 are defined as follows

$$\Omega_1 = \cos^{-1}\left(\frac{\cos\phi\cos\omega_s}{\cos\phi - \Delta y/L}\right) \tag{40a}$$

$$\Omega_2 = \tan^{-1}\left(\frac{1}{\sin\phi\Delta y/NW}\right). \tag{40b}$$

The above expressions for $\bar{f}$, ω_1, and ω_2 are also verified by putting $\beta = \phi$ in the expressions for $\bar{f}$, ω_1, and ω_2 in Sections 3 and 4.

6. VERIFICATION OF THE RESULTS

A computer program is prepared to numerically evaluate the integral J given by eqn (15) and $\bar{f}$ from eqn (14). In this program the expression of f given by eqn (4) is used, and the lower limit ω_1 and the upper limit ω_2 to make $f = 0$ are determined numerically in accordance with the conditions for G_1 and G_2 in eqn (4). The results for ω_1 and ω_2 coincide with those obtained by eqns (34) and (35). The value of $\bar{f}$ obtained from the program is also the same as that obtained from the analytical expression by eqn (17).

7. GRAPHICAL PRESENTATION OF $\bar{f}$

Values of $\bar{f}$ calculated using eqn (17) with ω_1 and ω_2 are determined using eqns (34) and (35) for the

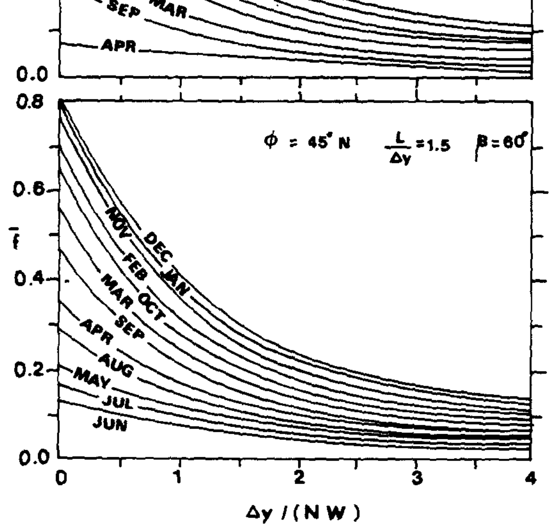

Fig. 15. Variation of $\bar{f}$ with $y/(NW)$ at $\phi = 45°$N, $\beta = 60°$, and $L/\Delta y = 1$ and 1.5.

Table 1. Summary of calculation results for a four month period

		$N = 3$			$N = 6$		
Month	$\bar{H}_{b,t}$ (MJ/m²)	$\bar{f}$	$\bar{H}^*_{b,t}$ (MJ/m²)	Error (%)	$\bar{f}$	$\bar{H}^*_{b,t}$ (MJ/m²)	Error (%)
March	10.62	0.124	9.3	14.1	0.183	8.68	22.4
June	7.13	0.000	7.13	0.0	0.000	7.13	0.0
September	10.39	0.085	9.51	9.3	0.122	9.12	13.9
December	7.67	0.323	5.19	47.8	0.482	3.97	93.0

average day of each month as recommended by Klein[9], with the following parametric conditions

ϕ	β	$\dfrac{L}{\Delta y}$	$\dfrac{\Delta y}{NW}$
0°	15°	1, 1.5	0.04 → 4
15°N	15°, 30°	1, 1.5	0.04 → 4
30°N	15°, 30°, 45°	1, 1.5	0.04 → 4
45°N	30°, 45°, 60°	1, 1.5	0.04 → 4

The results are plotted in Figs. 4–15. As indicated by the figures, the monthly-averaged daily shading factor increases by the increase of latitude angle, tilt angle, and/or the ratio $L/\Delta y$. Also, the value of $\bar{f}$ increases as the ratio $\Delta y/(NW)$ is decreased. Although values of $L/\Delta y = 1$ and 1.5 are only given in the charts, other values could be obtained by interpolation or extrapolation of the plotted results.

8. AN ILLUSTRATIVE EXAMPLE

To demonstrate the use of the correction factor $\bar{f}$ to improve the accuracy of predicting the monthly-averaged daily radiation received by collector surfaces, consider several rows of FPC located at Madison, WI. Each collector has a width $W = 0.5$ m, length $L = 1.5$ m, and spacing between rows $\Delta y = 1.5$ m. Using the same data and the results of example 2.16.1[10], the monthly-averaged daily beam radiation received per unit collector area is given as follows

$$\bar{H}_{b,t} = \left(1 - \frac{\bar{H}_d}{\bar{H}}\right)\bar{H}\bar{R}_b \qquad (41)$$

where the effect of shading is excluded. Including the effect of shading of a preceding row then gives

$$\bar{H}^*_{b,t} = \left(1 - \frac{\bar{H}_d}{\bar{H}}\right)\bar{H}\bar{R}_b(1 - \bar{f}). \qquad (42)$$

The percentage error in predicting $\bar{H}_{b,t}$ is

$$\Delta\bar{H}_{b,t} = 100(\bar{H}_{b,t} - \bar{H}^*_{b,t})/H^*_{b,t} \qquad (43)$$

Table 1 summarizes the results of calculations for rows with three and six elements for four months within the year where $\bar{f}$ is calculated using eqn (17) (it could be also determined by interpolation from Figs. 12 and 15). During the heating season (December) the error is about 47.8% for three elements rows and 93% for six elements rows. Increasing the spacing Δy between rows would decrease the effect of shading but with the penalty of increasing land area per unit collector area. An optimization procedure must then be followed by the designer searching for the optimum spacing between rows.

9. CONCLUSIONS

An analytical expression is derived to estimate the monthly-averaged daily shading factor for any row of collectors in a collector field. This expression is given by eqn (17) and could be applied for fields oriented toward the equator with collectors aligned behind each other and of N collectors in a row. To use the expression of $\bar{f}$ it is necessary to determine the solar hour angles at the start and end of the shading period, ω_1 and ω_2, respectively, on the afternoon part of the average day of each month. The expressions for ω_1 and ω_2 are given respectively by eqns (34) and (35). At the special case when the collectors are tilted with an angle equal to the latitude angle, i.e., $\beta = \phi$, the expressions for $\bar{f}$, ω_1, and ω_2 are simplified to those given in Section 5.

NOMENCLATURE

- $C_1 \rightarrow C_6$ coefficients defined by eqns (18B) to (18G)
- F_1 function defined by eqn (21)
- F_2 function defined by eqn (22)
- F_3 function defined by eqn (29)
- F_4 function defined by eqn (30)
- f instantaneous shading factor, see eqn (3)
- $\bar{f}$ monthly-averaged daily shading factor
- G_o extraterrestrial solar irradiance on a horizontal surface
- G_{sc} solar constant
- G_1 equivalent solar azimuth angle defined by eqn (5A)
- G_2 equivalent solar altitude angle defined by eqn (5B)
- $\bar{H}$ monthly-averaged daily solar total radiation on a horizontal surface
- $\bar{H}_b$ monthly-averaged daily beam radiation on a horizontal surface
- $\bar{H}_d$ monthly-averaged daily diffuse radiation on a horizontal surface
- $\bar{H}_o$ monthly-averaged daily extraterrestrial solar radiation on a horizontal surface
- $\bar{H}_t$ monthly-averaged daily solar total radiation on a tilted surface
- I function defined by eqn (19)
- J integral defined by eqn (15)
- L length of a single collector
- M number of days in a month
- N number of collectors in a row

n order of the day in the year
$\bar{n}$ order of the average day of the month in the year
P parameter defined by eqn (33)
$\bar{R}_b$ monthly-averaged daily ratio of beam radiation on a tilted surface to that on a horizontal surface
t solar time
t_{sr} sunrise solar time
t_{ss} sunset solar time
W width of a single collector
β tilt angle of collector with ground
δ solar declination angle
Δy spacing between collector rows, measured along S–N axis
η parameter defined by eqn (18A)
θ solar incidence angle on a surface
θ_z solar zenith angle
λ parameter defined by eqn (7B)
ρ_{gr} reflectance of ground
ϕ site latitude angle
ψ solar azimuth angle, measured from south to west
Ω_1 solar hour angle defined by eqn (40A)
Ω_2 solar hour angle defined by eqn (40B)
ω solar hour angle measured from solar noon, afternoon positive
ω_1 solar hour angle at the start of the daily afternoon shading period
ω_2 solar hour angle at the end of the daily afternoon shading period
ω_s sunset hour angle
ω_{st} sunset hour angle for a tilted surface

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