

Design of Mechanisms for Demand Response Programs

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Abstract—We prove the inefficiency (in the sense of Pareto) of the electricity system, as well as its resemblance with the tragedy of the commons. Also, we present a mechanism intended to achieve efficiency in the electricity consumption by means of economic incentives. The proposed incentives might be seen as an indirect revelation mechanism, in which users do not have to reveal private information about their preferences. Instead, a particular incentive is calculated for each user, based solely on its relative consumption. We conclude that the success of the proposed mechanism requires subsidies from external institutions, at least during the transition between an inefficient outcome and the efficient equilibrium.

Index Terms—Electricity market, dynamic pricing, game theory, mechanism design.

I. INTRODUCTION

Demand response (DR) approaches arise in the context of smart grids as mechanisms intended to encourage particular behaviors in users. The main goal of DR strategies is to achieve efficiency in the electricity system from the demand side, by promoting smart consumption in users [1]. Efficiency in the electricity system might benefit to both electricity utilities and users. Accordingly, the electricity utilities might be interested in supporting DR programs, due to some potential economic benefits (see [2]). Likewise, the expected electricity price reduction might motivate users to cooperate with DR programs. These premises have justified the use of economic incentives as a mean to induce behavioral changes in users. In particular, pricing mechanisms have been used extensively in the literature (see [3]–[7] and the references therein). The pricing mechanisms attempt to induce changes in the consumption habits by means of time varying electricity prices, which are aligned with the electricity demand. Specifically, higher prices applied in peak consumption hours might incentive users to redistribute their consumption. The objective of the incentives is to induce an optimal consumption, that maximizes an optimality criteria, such as the social welfare. An example of these mechanisms is the real time pricing scheme, which uses a marginal price signal as a mechanism to incentive users.

The design of DR approaches based on price incentives has been made using economical models, such as ideal markets,

where agents are price takers [8]. In ideal markets, the marginal price scheme might lead to an efficient equilibrium in the sense of Pareto, a.k.a., competitive equilibrium. However, the Pareto optimal might not be reachable in situations where agents are selfish and price anticipating. In that case, the Nash equilibrium is a preferred solution concept, because it allows to model stable outcomes in situations that induce strategic interactions among agents. In general, the Nash equilibrium is not equal to the competitive equilibrium. Additionally, it has been shown that the efficiency of an affine marginal price mechanism is reduced when the agents are price anticipating [9].

From this perspective, the design of mechanisms offers some theoretical tools intended to design rules (or incentives) to achieve of the desired outcomes in a strategical environment. Specifically, mechanisms can be designed to achieve the social optimal, regardless of the characteristics of users and their environment [7]. Particularly, the Vickrey-Clarke-Groves mechanism needs that users explicitly reveal their private information in order to solve the social welfare optimization problem. Furthermore, there are some mechanisms that consider the DR problem as a resource allocation problem, and assume that the utility is able to commit a desired quantity of energy to each user [10].

However, real situations exhibit some practical limitations, by which the DR objectives might be unreachable. On the one hand, the incentives design can be seen as a distributed optimization problem. Along this line, the information required to solve the problem (i.e., the users preferences) is distributed among the users. However, 1) there is no guarantee that the users would be willing to report true information; and 2) solving the problem might require an outstanding amount of computational resources. On the other hand, capturing all the required information from users might demand a nonviable communication infrastructure. These characteristics are discussed by Hurwicz in the seminal paper [11], where the resource allocation problem is analyzed.

In this work, we design a mechanism intended to achieve the Pareto efficient outcome in the DR problem. This mechanism is based on economical incentives that modify the price charged to each user. Specifically, we model the electricity market as a one-shot game which implements *ex-post* prices. Hence, the mechanism do not considers a negotiation procedure prior to the resource allocation. The proposed mechanism can be classified as an *indirect-revelation* mechanism, since users do not have to reveal their private information. Instead, our mechanism only requires a demand signal to compute

This work has been supported in part by Proyecto CIFI 2012, Facultad de Ingeniería, Universidad de los Andes, also by project Silice III Codensa-Colciencias under Grant P12.245422.006, and also by the program Jóvenes Investigadores 2012, by Colciencias

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the incentives. In particular, the proposed mechanism assumes an information exchange process that is carried out in a one-dimensional message space, rather than multidimensional information spaces as some mechanisms in the literature.

The main contributions of this paper are as follows: 1) we formulate the demand response problem as a tragedy of the commons dilemma, highlighting the efficiency loss in the electricity system when price signals are not controlled; 2) we propose a novel scheme of economic incentives for achieving optimal demand profiles in a population of strategic agents. In particular, we prove that under certain conditions, there is not a mechanism that satisfies the budget balance property in the electricity system.

The remainder of the paper is organized as follows. The problem statement as well as the introduction of two solution concepts is made in Section II. The design of the incentives mechanisms is presented in Section III, while the concluding remarks are presented in Section IV.

II. THE ELECTRICITY MARKET MODEL: ECONOMIC ENVIRONMENT

In this section, we introduce the electricity system model as well as two solution concepts, namely the competitive equilibrium and the Nash equilibrium. The main objective of this section is to highlight the characteristics of the solution concepts in terms of both the profit achieved by the population and the amount of resources spent.

A. General Framework

The electricity market model considers three main components, namely the producers, the consumers, and an independent system operator (ISO). Particularly, producers and consumers are constrained to sell and buy electricity, respectively. On the other hand, the ISO is a nonprofit entity in charge of regulating the electricity market, e.g., set the market clearing price [6].

In this case, we consider a population composed by N users that are subscribed to a unique producer. The population is defined as the set $\mathcal{V} = \{1, \dots, N\}$. We assume that the i^{th} user has a daily consumption $Q_i \geq 0$, where $i \in \mathcal{V}$. However, the consumption along a day is not uniform, because the electricity necessity of each user varies according to the time of the day, e.g., higher consumptions would be made in the night. Since the valuation given by each user to the electricity is a time varying function, it is convenient to divide a day in time periods that has roughly the same demand. Accordingly, let us divide a period of 24 hours in a set of T time intervals denoted $\tau = \{\tau_1, \dots, \tau_T\}$. Formally, we define the set τ as a partition of $[0, 24]$, where $\cup_{t \in \{1, \dots, T\}} \tau_t = \tau$ and $\cap_{t \in \{1, \dots, T\}} \tau_t = \emptyset$. Now, let q_i^t be the electricity consumption of the i^{th} user in the t^{th} time interval, such that $\sum_{t=1}^T q_i^t = Q_i$, where $q_i^t \geq 0$ and $t \in \{1, \dots, T\}$. Along these lines, the daily electricity consumption of the i^{th} user is represented by the vector $\mathbf{q}_i = [q_i^1, \dots, q_i^T]^\top \in \mathbb{R}_{\geq 0}^T$. On the other hand, the joint electricity consumption of the whole population is denoted by $\mathbf{q} = [q_1^\top, \dots, q_N^\top]^\top$. Without loss of generality, we assume

that the electricity consumption of the i^{th} user satisfies $q_i^t \geq 0$, in each time instant t .

The electricity necessity of the i^{th} user in a given time instant t might be represented by means of a *valuation function* $v_i^t : \mathbb{R} \rightarrow \mathbb{R}$, where $v_i^t(q_i^t)$ is the *valuation* that the i^{th} user gives to an electricity consumption of q_i^t units in the t^{th} time interval. Moreover, a daily valuation is equivalent to the sum of the electricity valuation perceived in each time interval τ_t , i.e., the daily valuation of the i^{th} user is the function $v_i : \mathbb{R}^T \rightarrow \mathbb{R}$, defined as $v_i(\mathbf{q}_i) = \sum_{t=1}^T v_i^t(q_i^t)$, where $t \in \{1, \dots, T\}$.

B. Solution concepts

1) *Competitive Equilibrium*: A competitive equilibrium considers that users are price taker. This implies that each user adjusts its consumption in order maximize its profit, assuming that prices are given. Let the vector $\boldsymbol{\lambda}$ be the daily electricity price defined as $\boldsymbol{\lambda} = [\lambda^1, \dots, \lambda^T]^\top$, where λ^t is the price in the t^{th} time interval. Thereby, the profit of the i^{th} user can be expressed as $U_i(\mathbf{q}_i) = v_i(\mathbf{q}_i) - \mathbf{q}_i^\top \boldsymbol{\lambda}$. On the other hand, profit of the producer is defined as $G = \mathbf{g}^\top \boldsymbol{\lambda} - \sum_{t=1}^T C(g^t)$, where the vector $\mathbf{g} = [g^1, \dots, g^T]^\top$ defines the daily power generation, g^t is the power generated in the t^{th} time period, and $C(g^t)$ is the generation cost associated with a production of g^t electricity units. We assume that the generation cost function $C(\cdot)$ is the same for all t .

In an ideal market, interactions among producer and users lead to an equilibrium in which generation equals demand, i.e., in a given time instant t , we have $g^t = \sum_{i=1}^N q_i^t$. Moreover, the generation costs equal generation profits, such that $G = 0$. According to the previous definitions, the consumers welfare function is maximized by solving [9]

$$\begin{aligned} & \underset{\mathbf{q}}{\text{maximize}} && \sum_{i=1}^N U_i(\mathbf{q}_i) = \sum_{i=1}^N \left(v_i(\mathbf{q}_i) - \sum_{t=1}^T C\left(\sum_{j=1}^N q_j^t\right) \right) \\ & \text{subject to} && q_i^t \geq 0, i = \{1, \dots, N\}, t = \{1, \dots, T\}. \end{aligned} \quad (1)$$

This optimization problem has a unique solution if the following conditions are satisfied:

Assumption 1.

- i. The valuation function $v_i^t(\cdot)$ is differentiable, concave, and non-decreasing.
- ii. The generation cost function $C(\cdot)$ is differentiable, convex, and non-decreasing.
- iii. The generation cost function can be expressed as $C(q) = qp(q)$, where $p(\cdot)$ is differentiable, convex, and non-decreasing.

In particular, the solution to Eq. (1) is feasible if the following assumption is satisfied.

Assumption 2.

$$\frac{\partial}{\partial q_i^t} U_i(0, \mathbf{q}_{-i}) = \frac{\partial}{\partial q_i^t} v_i^t(0) - p\left(\sum_{j \neq i} q_j^t\right) > 0,$$

for all $i \in \mathcal{V}$.

Note that if the population \mathcal{V} is finite, then $p(q) = \frac{C(q)}{q}$ corresponds to an average price scheme. In the remainder of

the document, we assume that the generation cost function has the form $C(q) = \beta q^2 + bq$.

Now, let $\boldsymbol{\mu} = [\boldsymbol{\mu}_1, \dots, \boldsymbol{\mu}_N]^\top$ be the optimal joint electricity consumption in the sense of Pareto, where the vector $\boldsymbol{\mu}_i \in \mathbb{R}^T$ corresponds to the optimal daily demand of the i^{th} user. Consequently, the following first order condition (FOC) is satisfied for every user $i \in \mathcal{V}$:

$$\left. \frac{\partial}{\partial q_i^t} \sum_{i=1}^N U_i(\mathbf{q}) \right|_{\mathbf{q}=\boldsymbol{\mu}} = \frac{\partial}{\partial q_i^t} v_i^t(q_i^t) - p\left(\sum_{j=1}^N q_j^t\right) - \left(\sum_{h=1}^N q_h^t\right) \frac{\partial}{\partial q_i^t} p\left(\sum_{j=1}^N q_j^t\right) \Big|_{\mathbf{q}=\boldsymbol{\mu}} = 0. \quad (2)$$

The equilibrium $\boldsymbol{\mu}$ is efficient in the sense of Pareto, since it maximizes the social welfare.

2) *Nash Equilibrium*: The Nash equilibrium is a solution concept used in situations in which agents take part of a strategic game. Particularly, in the electricity market, the consumption made by each agent has an impact on the overall energy cost, and therefore, influences the profit of other users. Since users are price anticipating, they individually maximize their profit taking into account the effect of its actions in the electricity price. In this case, the profit function of the i^{th} agent is defined as

$$W_i(\mathbf{q}_i, \mathbf{q}_{-i}) = v_i(\mathbf{q}_i) - \sum_{t=1}^T q_i^t p\left(\sum_{j=1}^N q_j^t\right), \quad (3)$$

where $p(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is the function of average prices and \mathbf{q}_{-i} is a vector composed by the daily consumption of all the individuals, except the i^{th} , i.e., $\mathbf{q}_{-i} = [q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_N]$. Thereby, the optimization problem of a price anticipating user is represented by:

$$\begin{aligned} & \underset{\mathbf{q}_i}{\text{maximize}} && W_i(\mathbf{q}_i, \mathbf{q}_{-i}) = v_i(\mathbf{q}_i) - \sum_{t=1}^T q_i^t p\left(\sum_{j=1}^N q_j^t\right) \\ & \text{subject to} && q_i^t \geq 0, i = \{1, \dots, N\}, t = \{1, \dots, T\}. \end{aligned} \quad (4)$$

This problem has a unique solution as long as the Assumption 1 is satisfied. Furthermore, the unique solution is feasible if Assumption 2 is satisfied.

Now, let the vector $\boldsymbol{\xi} = [\boldsymbol{\xi}_1, \dots, \boldsymbol{\xi}_N]$ be the solution to the maximization problem in Eq. (4). Hence, $\boldsymbol{\xi}$ satisfies the following FOC

$$\left. \frac{\partial}{\partial q_i^t} W_i(\mathbf{q}_i, \mathbf{q}_{-i}) \right|_{\mathbf{q}=\boldsymbol{\xi}} = \frac{\partial}{\partial q_i^t} v_i^t(q_i^t) - p\left(\sum_{j=1}^N q_j^t\right) - q_i^t \frac{\partial}{\partial q_i^t} p\left(\sum_{j=1}^N q_j^t\right) \Big|_{\mathbf{q}=\boldsymbol{\xi}} = 0, \quad (5)$$

where the vector $\boldsymbol{\xi}_i$ in \mathbb{R}^T is the demand profile of the i^{th} agent, for all $i \in \mathcal{V}$. If Eq. (5) is satisfied for all agent $i \in \mathcal{V}$, we say that $\boldsymbol{\xi}$ is the Nash equilibrium of the game described by Eq. (3).

3) *Nash Equilibrium Inefficiency*: Now we are interested in analyzing the properties of both the competitive equilibrium $\boldsymbol{\mu}$ and the Nash equilibrium $\boldsymbol{\xi}$ that arise in the context of an electricity system. Particularly, we find that the competitive equilibrium achieves the maximum social profit by using a lower amount of resources, compared with the Nash equilibrium. However, the Nash equilibrium is a stable outcome regardless of its inefficiency. This result is stated in the following lemma.

Lemma 1. *Suppose that Assumptions 1 and 2 are satisfied. Also, consider that $\boldsymbol{\mu}$ is the Pareto efficient equilibrium of the system and $\|\cdot\|_1$ is the L^1 -norm. Then, a game of the form stated in Eq. (3) has a unique Nash equilibrium, namely $\boldsymbol{\xi} = [\boldsymbol{\xi}_1^\top, \dots, \boldsymbol{\xi}_N^\top]^\top$, that satisfies the following conditions.*

- i. $\|\boldsymbol{\xi}\|_1 > \|\boldsymbol{\mu}\|_1$ for all $i \in \mathcal{V}$,
- ii. $\boldsymbol{\xi}$ is not efficient in the sense of Pareto.

Proof sketch: Since Assumptions 1 and 2 are satisfied, the optimization problems in Eq. (1) and (4) have a unique solution, which we refer as $\boldsymbol{\xi}$ and $\boldsymbol{\mu}$, respectively. Now, the proof of numeral (i) is made by contradiction. First, let us assume that the total energy consumed by a price-taker individual is greater or equal than the total energy consumed by a strategic agent, i.e., $\|\boldsymbol{\mu}_i\|_1 \geq \|\boldsymbol{\xi}_i\|_1$. Since the valuation function $v_i^t(\cdot)$ is non-decreasing and concave at any time t , we have that $\frac{\partial}{\partial q_i^t} v_i^t(q_i^t) > \frac{\partial}{\partial q_i^t} v_i^t(q_i^t + c)$, for some constant $c > 0$. Consequently, the inequality

$$\sum_{i=1}^N \frac{\partial}{\partial q_i^t} v_i(\boldsymbol{\xi}_i) \geq \sum_{i=1}^N \frac{\partial}{\partial q_i^t} v_i(\boldsymbol{\mu}_i) \quad (6)$$

is satisfied for any time t . Since the price function $p(\cdot)$ is convex and increasing, then the following inequalities are satisfied: $\sum_{i=1}^N p\left(\sum_{j=1}^N \xi_j^t\right) \leq \sum_{i=1}^N p\left(\sum_{j=1}^N \mu_j^t\right)$ and $\frac{\partial}{\partial q_i^t} p\left(\sum_{j=1}^N \xi_j^t\right) \leq \frac{\partial}{\partial q_i^t} p\left(\sum_{j=1}^N \mu_j^t\right)$. We can use the previous expressions, along with Eq. (2), (5), and (6) to verify that

$$\sum_{i=1}^N \frac{\partial}{\partial q_i^t} v_i(\boldsymbol{\xi}_i) < \sum_{i=1}^N \frac{\partial}{\partial q_i^t} v_i(\boldsymbol{\mu}_i) \quad (7)$$

Seeing that Eq. (6) and (7) are in contradiction, we conclude that $\|\boldsymbol{\mu}_i\|_1 < \|\boldsymbol{\xi}_i\|_1$, for all $i \in \mathcal{V}$. Particularly, from the previous result we can conclude that the energy consumed in the equilibrium of the game is greater than the optimal energy consumption, i.e., $\|\boldsymbol{\mu}\|_1 < \|\boldsymbol{\xi}\|_1$.

Now, the proof of numeral (ii) is made by direct proof. From Assumption 1 we know that the competitive equilibrium is unique, and corresponds to the best possible outcome for the population. Hence, the competitive equilibrium is efficient in the sense of Pareto. On the other hand, from numeral (i) we conclude that $\|\boldsymbol{\mu}\|_1 < \|\boldsymbol{\xi}\|_1$. Therefore, $\boldsymbol{\xi} \neq \boldsymbol{\mu}$, which implies that the total consumption at the Nash equilibrium $\boldsymbol{\xi}$ is not efficient in the sense of Pareto. ■

Lemma 1 reveals that the electricity system presented in Section II conforms a social dilemma similar to the ‘tragedy of the commons’ [12]. This situation arises when a shared resource is overused by the people, even if they know the negative consequences of overusing it. In the case of the

electricity system, the Pareto efficient outcome μ maximizes the population welfare, using less electricity resources than the equilibrium of the game ξ . However, at least one agent has incentives to deviate from the Pareto efficient outcome. Therefore, the social welfare maximizer is not a stable point. This fact illustrates the requirements for implementing incentives.

Remark 1. *The discrimination of time periods is made in order to model different preferences of users along the day. However, the consumption made in a given time period t is considered independent from the consumption made in a different time period. That is, the optimization problems described in Eq. (1) and (4) can be separated in T independent optimization problems. In particular, we can do this generalization because the electricity is not considered as a limited resource that has to be allocated. Therefore, without loss of generality, we can analyze the case of $T = 1$ and the results obtained can be extended to cases with $T > 1$. Accordingly, the notation used hereafter does not have into account any particular time period.*

III. ECONOMIC INCENTIVES

In the previous section, we have observed that a society with strategic agents is not able to achieve the social optimal, i.e., when the users are price anticipating, the stable outcome is inefficient in terms of profits and resources usage. Accordingly, the design of economic incentives is required in order to guarantee that the efficient equilibrium is a stable outcome. In this section, we design and analyze an *indirect-revelation mechanism* to tackle the DR problem.

The design of incentives is strongly related to the mechanism design. Mechanism design arises in the context of strategic situations that establish a game among some agents. However, the main objective in mechanism design is find some rules that guarantee the realization of a particular objective function, rather than predict the outcome of a game, as in classical game theory [13], [14]. Mechanism design is applied mainly in scenarios with hidden information, e.g., situations such as auctions and resource allocation problems. In these situations, the players of the game have private information, which is required by a central entity to find the optimal outcome of the game.

One of the most commonly used mechanisms is the *direct-revelation mechanism*. A direct-revelation mechanism proposes a game in which players are asked explicitly to reveal the information required to calculate the ideal outcome, i.e., the strategy of each agent consists on revealing (truthfully or untruthfully) its private information (see Vickrey second price auction) [14]. In general, direct-revelation mechanisms are characterized by being inefficient with respect to both communication and computation resources [15]. Such characteristics would be unsuitable for the DR problem, since its implementation in a large population might require an nonviable amount of resources.

On the other hand, an indirect-revelation mechanism proposes a game in which the outcome is calculated through information about the agents, which implicitly contains the

preferences of each agent. In particular, the demand profile of each user contains information about its preferences. Accordingly, although the preferences of each user remain unrevealed, the consumption information can be considered as truthful information for assigning incentives. Since prices are set *ex-post* (i.e., prices are set once the consumption has been made), intentionally false consumptions would lead to losses rather than gains. In this case, we assume that the utility has information about the consumption profile of each user. This is reasonable, because that information is obtained through smart meters to calculate the electricity bill.

An initial intuition about the mechanism, is that incentives should reflect the contribution that an agent makes to the population surplus. The main idea is to redistribute the social gains among agents, according to the contribution of each one to the population welfare. Thus, an agent that cooperates is provided with higher incentives than another agent that do not cooperates. From Lemma 1, we know that the welfare of the population increases as the agents approach to the Pareto optimal outcome. Hence, the profit loss that the i^{th} user experiences when it unilaterally deviates from the inefficient equilibrium ξ_i can be compensated through the benefits induced in the rest of the population. In this way, an agent would be incentivized to cooperate for the sake of the population. In order to align the users welfare function with the population objective function, we consider incentives of the form

$$I_i(\mathbf{q}) = \left(\sum_{h \neq i} q_h \right) \left(h_i(\mathbf{q}_{-i}) - p \left(\sum_{j=1}^N q_j \right) \right), \quad (8)$$

where $h_i(\mathbf{q}_{-i})$ is a design term that has some relevance with respect to the properties of the mechanism (as stated below). The incentives modify the price charged to each user. In particular, we consider that the profit of the i^{th} agent in a game with incentives is defined as

$$\hat{W}_i(q_i, \mathbf{q}_{-i}) = v_i(q_i) - q_i p \left(\sum_{j=1}^N q_j \right) + I_i(\mathbf{q}). \quad (9)$$

These incentives are calculated having into account the relative consumption of each user with respect to the population consumption profile. The form of this incentive is related to the price used in the Vickrey-Clarke-Groves mechanism [14] and some utility functions used in potential games [16]. Let ω be the equilibrium of this game, such that satisfies the FOC:

$$\begin{aligned} \frac{\partial}{\partial q_i^t} \hat{W}_i(\mathbf{q}) \Big|_{\mathbf{q}=\omega} &= \frac{\partial}{\partial q_i^t} v_i(q_i) \\ - p \left(\sum_{j=1}^N q_j^t \right) - \left(\sum_{h=1}^N q_h \right) \frac{\partial}{\partial q_i^t} p \left(\sum_{j=1}^N q_j^t \right) \Big|_{\mathbf{q}=\omega} &= 0. \end{aligned} \quad (10)$$

Since Eq. (2) and (5) are equivalent, the Nash equilibrium ω of the game with incentives is equal to the competitive equilibrium μ . Therefore, the Nash equilibrium of the game with incentives ω is efficient in the sense of Pareto. Note that the equilibrium of the game with incentives do not depends on $h_i(\mathbf{q}_{-i})$, since $h_i(\mathbf{q}_{-i})$ is independent of the term q_i . However, the form of $h_i(\mathbf{q}_{-i})$ influences the

welfare of the i^{th} individual. For example, $h_i(\mathbf{q}_{-i}) = 0$ lead to a unrealistic pricing mechanism in which each user is charged with the total generation cost $C(\sum_{j=1}^N q_j)$, i.e., $W_i(\mathbf{q}) + I_i(\mathbf{q}) = v_i(q_i) - C(\sum_{j=1}^N q_j)$. A more reasonable mechanism is provided by the Clarke pivot rule [14], that calculates incentives with respect to the contribution made by an agent to the society. Inspired in the Clarke pivot rule, we propose

$$h_i(\mathbf{q}_{-i}) = p\left(\sum_{j \neq i} q_j + f(\mathbf{q}_{-i})\right), \quad (11)$$

where $f(\mathbf{q}_{-i})$ is a function that represents the alternative behavior of the i^{th} agent. Although the Clarke pivot rule defines $f(\mathbf{q}_{-i}) = 0$, we find that the case with $f(\mathbf{q}_{-i}) \neq 0$ provide some interesting properties. In particular, we consider that $f(\mathbf{q}_{-i})$ is a linear combination of the vector \mathbf{q}_{-i} of the form

$$f(\mathbf{q}_{-i}) = \sum_{j \neq i} \alpha_j q_j, \quad (12)$$

where $\alpha_i \in \mathbb{R}$ for all $i \in \mathcal{V}$. The next result shows that when we consider an affine marginal price function, there is no function $f(\cdot)$ of the form in Eq. (12), such that the mechanism of incentives is budget balanced. That is, the sum of the incentives on the population can not be equal to zero. Therefore, the mechanism requires subsidies from an external agent.

Theorem 1. *Suppose that Assumptions 1 and 2 are satisfied. Also consider that $p(z) = \beta z + b$, where $z \in \mathbb{R}$, $\beta > 0$, and $b \geq 0$, and a population of two or more agents. Then, there does not exists a function $f(\cdot)$ of the form in Eq. (12), such that the incentives mechanism described by Eq. (8) and (11) satisfies the budget balance property.*

Proof: This proof is made by contradiction. First, we assume that there exists a function $f(\cdot)$ such that the mechanism is budget balanced, i.e., $\sum_{i=1}^N I_i(\mathbf{q}) = 0$. Now, we express the incentives in matrix form. On that purpose, we first we define $[f(\mathbf{q}_{-1}), \dots, f(\mathbf{q}_{-N})]^\top = F\mathbf{q}$, as a vector with the i^{th} element equal to $f(\mathbf{q}_{-i})$. In particular, $F = (e\alpha^\top - \text{diag}(\alpha_1, \dots, \alpha_N))$, $\alpha = [\alpha_1, \dots, \alpha_N]^\top$, $\text{diag}(\alpha_1, \dots, \alpha_N)$ is a diagonal matrix, and e is a vector in \mathbb{R}^N with all its components equal to 1.

Since $p(\cdot)$ is an affine function, Eq. (8) can be expressed as $\sum_{i=1}^N I_i(\mathbf{q}) = \beta \sum_{i=1}^N \left(\sum_{j \neq i} q_j\right) (f(\mathbf{q}_{-i}) - q_i)$. This can be rewritten in matrix form as $\sum_{i=1}^N I_i(\mathbf{q}) = \beta \mathbf{q}^\top \Phi (F\mathbf{q} - \mathbf{q})$, where $\Phi = (ee^\top - I)$ and I is the identity matrix in $\mathbb{R}^{N \times N}$.

Now, considering the budget balance condition, we have $\mathbf{q}^\top \Phi F\mathbf{q} = \mathbf{q}^\top \Phi \mathbf{q}$. This equation is satisfied if either $q_i = 0$ for all $i \in \mathcal{V}$, or if $F = I$. Note that F is a matrix with zeros in the diagonal, therefore, $F \neq I$. Accordingly, none of the aforementioned conditions are satisfied for all vector $\mathbf{q} \in \mathbb{R}_+^N$. Consequently, we conclude that the budget balance property can not be achieved by means of the incentives mechanism described by Eq. (8), (11), and (12). ■

Theorem 1 states the impossibility of finding a mechanism in the form of Eq. (8) and (12) that do not require external influence in form of subsidies or taxes. This result is not

unexpected, since the Myerson-Satterthwaite Theorem states the impossibility of mechanisms with ex post efficiency and without external subsidies, for a game between two parties [17]. Since there is not a mechanism such that the budget balance property is satisfied, then let us state some desirable conditions for the mechanism. First, we consider that all users are equivalent for the utility. Consequently, we make the following fairness assumption.

Assumption 3. *Incentives for the i^{th} and j^{th} agents are equivalent whether their consumption is the same, i.e., if $q_i = q_j$, then $I_i(\mathbf{q}) = I_j(\mathbf{q})$. Particularly, if $q_i = q_j$ for all $i, j \in \mathcal{V}$, then $I_i(\mathbf{q}) = I_j(\mathbf{q}) = 0$. On the other hand, a higher power consumption deserves a lower incentive, i.e., if $q_i < q_j$, then $I_i(\mathbf{q}) > I_j(\mathbf{q})$.*

We find that the a function $f(\mathbf{q}_{-i})$ that satisfies Assumption 3 corresponds to the average of \mathbf{q}_{-i} . This result is summarized in the following proposition.

Proposition 1. *Assume a population of $N \geq 2$ agents, incentives of the form in Eq. (8), (11), and (12), and that Assumption 3 is satisfied. Then the function $f(\cdot)$ has the form $f(\mathbf{q}_{-i}) = \frac{1}{N-1} \sum_{j \neq i} q_j$, for all $j, i \in \mathcal{V}$.*

Proof: Since the average price signal is defined as $p(z) = \beta z + b$, incentives from Eq. (8) might be rewritten as $I_i(\mathbf{q}) = (\sum_{j \neq i} q_j) \beta (f(\mathbf{q}_{-i}) - q_i)$. Now, let us consider an uniform population consumption profile \mathbf{q} in \mathbb{R}_+^N , such that $q_j = q_i = \sigma$, for all $i, j \in \mathcal{V}$. Now, we define two population profiles $\hat{\mathbf{q}} = \mathbf{q} + \delta e_i$ and $\tilde{\mathbf{q}} = \mathbf{q} + \delta e_j$, where e_i is an N-dimensional vector with the i^{th} entry equal to 1 and 0 otherwise, for all $j, i \in \mathcal{V}$. According to Assumption 3, the incentives given to the i^{th} and j^{th} individuals are the same, since $\hat{q}_i = \tilde{q}_j$ and $\hat{q}_{-i} = \tilde{q}_{-j}$. Therefore,

$$\begin{aligned} I_i(\hat{\mathbf{q}}) &= \beta \left((N-1)\sigma \right) \left(f(\hat{\mathbf{q}}_{-i}) - \sigma - \delta \right) \\ &= I_j(\tilde{\mathbf{q}}) = \beta \left((N-1)\sigma \right) \left(f(\tilde{\mathbf{q}}_{-j}) - \sigma - \delta \right) \end{aligned} \quad (13)$$

The previous equation is satisfied if $f(\hat{\mathbf{q}}_{-i}) = f(\tilde{\mathbf{q}}_{-j})$, that is equivalent to $\sigma \sum_{h \neq i} \alpha_h = \sigma \sum_{h \neq j} \alpha_h$. The last expression is satisfied only if $\alpha_i = \alpha_j = \kappa$, for all $j, i \in \mathcal{V}$.

Now, suppose that all agents have the same consumption profile, i.e., there is a uniform population consumption profile \mathbf{q} . According to the Assumption 3, $I_j(\mathbf{q}) = I_i(\mathbf{q}) = 0$. Therefore $I_i(\mathbf{q}) = (\sum_{j \neq i} q_j) \beta (f(\mathbf{q}_{-i}) - q_i) = 0$. Having into account that $q_j = q_i = \sigma$, we have that $I_i(\mathbf{q}) = \beta \left((N-1)\sigma \right) \left((N-1)\kappa\sigma - \sigma \right) = 0$. Therefore, $\kappa = \frac{1}{N-1}$, and consequently, $f(\mathbf{q}_{-i}) = \frac{1}{N-1} \sum_{j \neq i} q_j$. ■

According to Proposition 1, the unique mechanism that satisfies Assumption 3 is given by:

$$I_i(\mathbf{q}) = \beta \left(\sum_{j \neq i} q_j \right) \left(\frac{1}{N-1} \sum_{j \neq i} q_j - q_i \right) \quad (14)$$

for all $q_i \geq 0$, $i, j \in \mathcal{V}$. Hence, the population incentives can be expressed as

$$\sum_{i=1}^N I_i(\mathbf{q}) = \beta \mathbf{q}^\top A \mathbf{q}, \quad (15)$$

where $A = \left(\frac{-1}{N-1} ee^\top + \frac{N}{N-1} I \right)$.

Based on the mechanism incentives defined in Eq. (14), we know that the sum of incentives of the population is greater than zero. Therefore, the mechanism requires subsidies. The following proposition describes this.

Proposition 2. *Suppose that Assumptions 1, 2, and 3 are satisfied. Given an incentives mechanism of the form in Eq. (14), then the incentives required by the population are positive, i.e., $\sum_{i=1}^N I_i(\mathbf{q}) \geq 0$, for all $\mathbf{q} \in \mathbb{R}_{\geq 0}^N$.*

Proof: First, consider $q_i^2 + q_j^2 - 2q_i q_j = (q_i - q_j)^2 \geq 0$ for all $q_i \in \mathbb{R}_{\geq 0}^T$. Hence, we have that $q_i^2 + q_j^2 \geq 2q_i q_j$. Now, summing in both sides of the previous equation we obtain $\sum_{i=1}^N \sum_{j \neq i} (q_i^2 + q_j^2) \geq \sum_{i=1}^N \sum_{j \neq i} 2q_i q_j$, which is equivalent to $(N-1) \sum_{i=1}^N q_i^2 \geq \sum_{i=1}^N \sum_{j \neq i} 2q_i q_j$. Reordering results

$$\sum_{i=1}^N q_i^2 \geq \frac{2}{N-1} \sum_{i=1}^N \sum_{j \neq i} q_i q_j. \quad (16)$$

Now, let $A_{j,i} = \frac{-1}{N-1}$ if $i \neq j$ and $A_{i,i} = 1$ for all $i, j \in \mathcal{V}$. Therefore, the incentives in Eq. (15) can be expressed as $\beta \mathbf{q}^\top A \mathbf{q} = \beta \sum_{i=1}^N q_i \left(\sum_{j=1}^N q_j A_{j,i} \right)$. This can be rewritten as

$$\beta \mathbf{q}^\top A \mathbf{q} = \beta \left(\sum_{i=1}^N q_i^2 + \frac{-1}{N-1} \sum_{i=1}^N q_i \left(\sum_{j \neq i} q_j \right) \right).$$

From Eq. (16), it can be seen that $\mathbf{q}^\top A \mathbf{q} \geq 0$, for all $\mathbf{q} \in \mathbb{R}_{\geq 0}^N$. ■

Remark 2. *Based on the previous result, we find that the social welfare reached by the population is greater when the incentives scheme is introduced. That is, since the Nash equilibrium of the game with incentives ω is efficient, we know that $\sum_{i=1}^N W_i(\xi) < \sum_{i=1}^N W_i(\omega) \leq \sum_{i=1}^N (W_i(\omega) + I_i(\omega))$. Consequently, $\sum_{i=1}^N W_i(\xi) < \sum_{i=1}^N \tilde{W}_i(\omega)$.*

Remark 3. *Let us consider a homogeneous population, composed by agents with equal preferences. In such population, the energy consumed at the equilibrium is the same for every agent. According to Assumption 3, at the equilibrium, the subsidies required from an external source are null. In particular, incentives would be required only to shift the system from an inefficient outcome toward the optimal equilibrium.*

Furthermore, as the population size increases, the total incentives of the population increase, i.e., the subsidies that the mechanism requires increase with the population size. Particularly, we have

$$\lim_{N \rightarrow \infty} \beta \mathbf{q}^\top A \mathbf{q} = \beta \mathbf{q}^\top \mathbf{q} > \beta \mathbf{q}^\top A \mathbf{q}.$$

IV. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper, we present the demand response problem as a social dilemma, known as the tragedy of the commons. We analyze the inefficiency of the Nash equilibrium that arises in the population, in terms of profit and resources consumed by the population in the equilibrium. Accordingly, we propose an indirect-revelation mechanism, based on the Clarke pivot

rule, which yields to an efficient demand profile. In the proposed mechanism, users are not asked to reveal their private preferences or utility functions. Instead, the mechanism uses consumption information to compute incentives for each agent. In particular, these signals are defined in a one-dimensional message space. One of the main contributions of this work, is to prove the impossibility of such mechanism to achieve budget balance, under certain restrictions on the mechanism form. This result suggests that the success of a demand response strategy is conditioned to external subsidies, at least during the transition toward the optimal equilibrium.

Future work will be focused on the analysis of the transition dynamics between inefficient and efficient outcomes. Also, it would be interesting to analyze the characteristics of the mechanism on different game settings.

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