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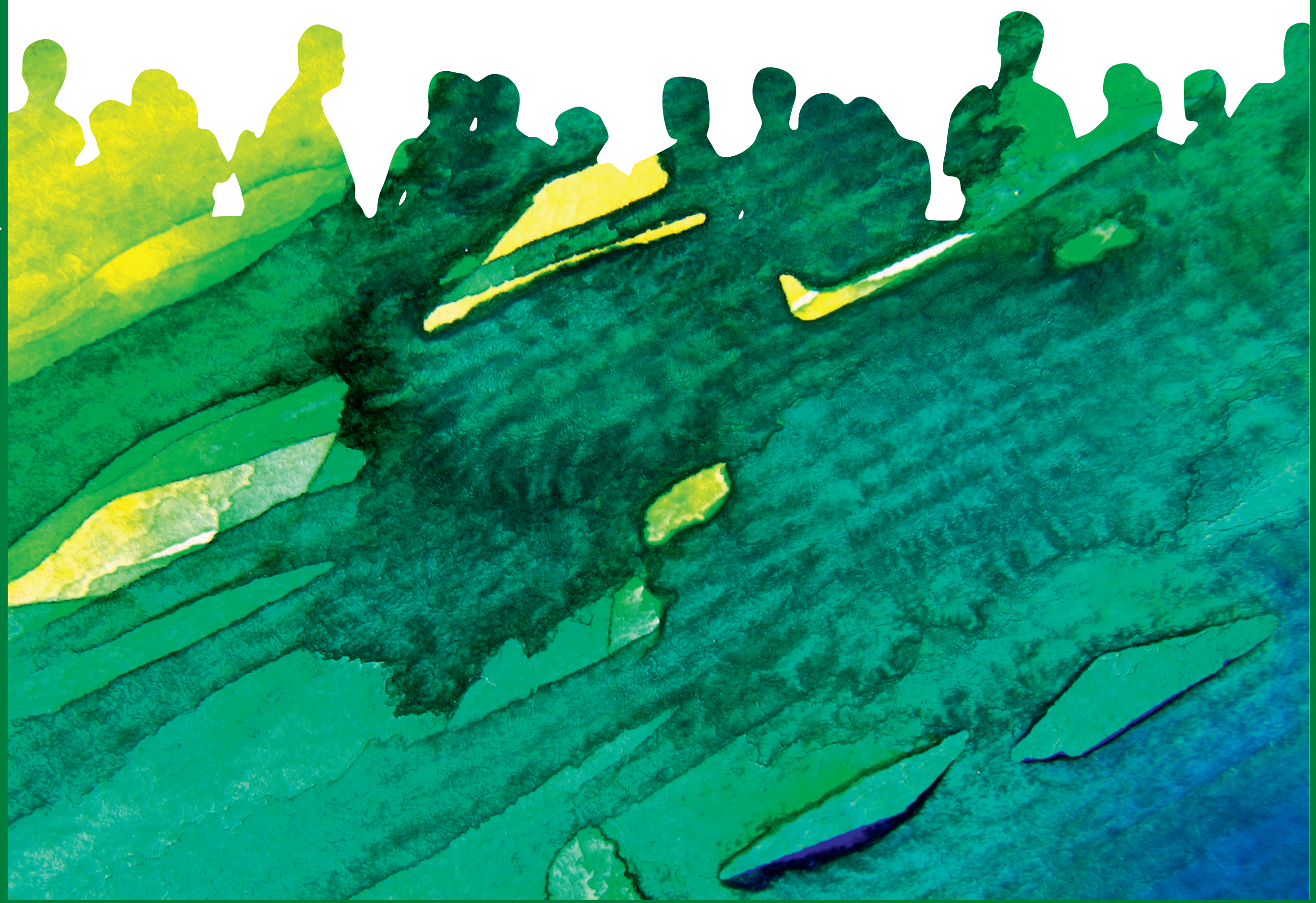
Brazilian Mortality and Survivorship

Life Tables

INSURANCE MARKET EXPERIENCE – 2010



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Contents

Presentation

This book presents the survival and mortality biometric tables as the product of a commissioned research by Fenaprevi – Federação Nacional de Previdência Privada e Vida [National Federation of Private Pension and Life] to the Laboratory of Applied Mathematics at the Mathematical Institute of the Federal University of Rio de Janeiro.

The research represents a milestone since Brazilian statistics are being applied for the first time, which means that pension funds and insurance institutions now have at their disposal a more reliable representation of the Brazilian population's profile. More than 300 million records from 23 insurance companies were used with detailed information on insurance contracts, broken down by gender, age and type of plan. These data were also crossed-referenced with the statistics from the register of the Death Control System at the National Registry of Information. No less important, were the team of researchers: some exceptional individuals who have international recognition, which provides credibility to the new biometric tables.

With these new tables, named BR-EMS, Brazilian institutions – insurance companies, pension companies, regulatory and supervisory agencies, universities, research centres and consulting firms – will have access to a fundamental tool for greater operational efficiency and solvency of the system.

The Brazilian College of Insurance, as an educational and research institution, is proud to publish and disseminate this research, thereby reinforcing its partnership agreement with the LabMA/UFRJ – Laboratory of Applied Mathematics at the Mathematical Institute of the Federal University of Rio de Janeiro.

Professor Claudio Contador, Ph.D.
Brazilian College of Insurance

1 | Introduction

During the first decade of the current century, the Brazilian life insurance market expanded at an accelerated rate. Insurance companies operating in Brazil were using foreign life tables, since there were no local life tables available.

Consequently, FenaPrevi – the National Federation of Open Pension Funds and Life Insurance Companies – commissioned LabMA/UFRJ – the Applied Mathematics Lab of the Mathematics Institute of the Federal University of Rio de Janeiro – to construct life tables for the Brazilian insurance market.

This project was followed by SUSEP – the Brazilian Supervising Insurance Authority.

The present work describes the methodology used and presents the survival and mortality tables constructed thereof, based on the experience of the Brazilian insurance life products market, for the years 2004, 2005 and 2006. Data was provided by a pool of Insurance Companies, representing an 82% share of the market.

These life tables were named Experiência do Mercado Segurador Brasileiro, BR-EMS (Experience of the Brazilian Insurance Market) and consist of four variants for mortality (male and female) and survivorship (male and female). The variants were given the following names: BR-EMSsb-v.2010-m, BR-EMSsb-v.2010-f, BR-EMSmt-v.2010-m e BR-EMSmt-v.2010-f, where “sb” denotes survivorship, “mt” mortality, “m” male and “f” female.

SUSEP established tables BR-EMS as Standard Life Tables for the Brazilian Insurance Market. SUSEP also demanded that they be reviewed in 2015.

We would like to thank all members of the Actuarial Committee of FenaPrevi who contributed with suggestions and discussions in all phases of the project. We also thank our students of LabMA/UFRJ, especially Paulo Vitor who was responsible for redrafting all the graphs in this book. This research was partially supported by FAPERJ.

2 | Brief history of Mortality Tables

Life tables have existed for a long time in human history. There is evidence that in ancient Rome, in the 3rd century B.C., the State collected and elaborated statistics for life and death, probably using life table schemes, calculating life expectancy at birth and other ages (Duchene & Wunsch, 1988). However, the first scientific references to life tables occur in the work of John Graunt, “Natural and political observations made upon the bills of mortality”, published in 1662 (apud David, 1998), and notably the seminal paper by astronomer Edmond Halley, in 1693 (apud Duchene & Wunsch, 1988) where the basis of actuarial science were laid.

Since then, numerous tables have been compiled for different countries and regions, as well as tables for specific purposes to support the actuarial work of insurance companies and pension funds.

Life tables are an important tool to help with the establishment of public policies, such as health, education, economic planning, workforce allocation, social security, insurance in general, and investment planning.

There are two issues when one considers constructing a life table for a specific population group:

i) The first concerns the data itself, mostly characteristics of the population at risk, e.g. age and sex, and statistics of deaths. For example, in Brazil, IBGE (National Central Statistical Agency) constructs life tables for the population as a whole, based on census data and deaths from the Civil Registry. It is well-known that there is under-registration of deaths, since not all deaths are reported appropriately.

   Therefore, one usually uses a correction factor to account for this error. The literature reports several techniques to estimate under-registration (UN, Manual X, 1983). For instance, it could be a uniform correction for all ages, or alternatively, specific corrections for certain age groups, such as children and the older population. In Brazil, there is evidence of stronger under-registration for the extreme age-groups, so that one should usually perform corrections differentiated for the extremely young and extremely old age-groups.

In the present study, although data was sourced from administrative records within insurance companies, LabMA/UFRJ decided to countercheck the death information with government data available in systems of the *Ministério da Previdência Social* (Social Security Administration).

ii) The second issue involves choosing an adequate model to describe the mortality pattern. For a given age-group (x), deaths can be considered as random variables with Binomial distributions, B(N_x,q_x), with a given size parameter, N_x, and an unknown probability of death, q_x, to be estimated. If the age-group x is large, one may consider a Poisson approximation to the Binomial. In this area of knowledge, it is quite common to use non-parametric techniques for estimating the vector probability of death in all ages. In general these methods involve some kind of smoothing. The UN published Manual X: *Indirect techniques for demographic estimation* (1983), where a family of life tables was established according to different regions of the world, based on the experience of 158 life tables. These families were indexed by a single parameter, making their use somehow limited.

On the other hand, there have always been many parametric models for life tables. The first models were simple mathematical curves, such as the linear model proposed by DeMoivre. Gompertz (1825) based on the hypothesis of a life force diminishing with age. He defined the life force as the inverse of the instant mortality rate: μ_x^{-1}, where $\mu_x = -l'_x / l_x$, hence solving the differential equation for μ_x^{-1},

$$\frac{d\mu_x^{-1}}{dx} = -k\mu_x^{-1},$$

where k is a positive constant. Solving for μ_x, one finds $\mu_x = Bc^x$. From the definition of μ_x in terms of l_x the solution of the differential equation is $l_x = l_0 \cdot g^{c^x}$, where g and c are positive constants.

Later in the 19th century, Makeham generalized Gompertz's model with the introduction of a constant factor to the instant mortality rate, independent of age, to represent deaths by accidents. Models based on Gompertz/Makeham proposals are still very much in vogue and have proven to produce very good results. It so happens that, in many instances, these models are particularly good for describing mortality for certain age groups, but not for all. For instance, the model proposed by Heligman and Pollard has a very important component for middle and old ages which is basically Gompertz/Makeham.

Other authors followed different paths to establish mortality laws. For instance, the Weibull distribution, which describes the failure pattern of interacting complex and multiple dynamical systems, may also be used for human beings.

Yet another approach would be to model different age brackets using specific mortality models. For instance, Heligman and Pollard used three components, each one dominant in a certain age bracket: children, young adults and adults.

For all methods used for estimating life tables, one usually starts with a process of smoothing the crude rates, initially calculated from the raw data, $(\hat{q}_x)$ $x = 0, 1, 2...$ In many cases, crude rates tend to oscillate as a function of age, which is not plausible as a theoretical model for mortality. Since the ageing process is continuous in time, one should expect a rather continuous mortality function. Therefore, the true non-observable parameters of a life table, $(\tilde{q}_x)$, should be continuous as a function of age, and the crude rates could be considered as sample estimates of these non-observable parameters. The greater the population, the greater is the precision of estimate.

Graduation can be defined as the set of principles and methods used to smooth the crude mortality rates to generate a mortality function, presenting desirable features such as avoiding spikes and depression, as well as being monotone above a certain age.

The analysis of different models for life-tables constitutes an important part of the present work. The next chapter mentions some of these techniques and analyzes their pros and cons, converging on the chosen model.

3 | Mortality graduation

3.1 Introduction

This chapter covers some graduation methods for mortality tables. The present brief review is based on the literature, which is listed in the bibliography. Usually the idea is to obtain a smooth curve of mortality rates, monotonically increasing after a certain age, typically around 30 years.

There are several possible classifications of graduation methods. The first possibility is between parametric and non-parametric methods.

Parametric methods are rather efficient, once one is able to describe the mortality phenomenon by a set of equations with a given (not too large) number of parameters. With the right parametric family one can rationalize the mortality behavior of a population, sometimes by associating the parameters with certain mortality characteristics.

Furthermore, non-parametric methods are smoothing methods that either directly transform crude rates through running averages or medians into a smooth sequence, or else this kind of sequence is obtained by an optimization process. Parametric and non-parametric methods can be combined in a complementary fashion, by proceeding initially with a non-parametric method, which in turn feeds a parametric procedure.

There are several other possible classifications of graduation methods, according to whether they incorporate, or not, the time dimension. In what follows, are listed some of the alternative classifications found in the literature, without the time dimension.

Copas and Haberman (1983) classify the graduation methods which do not involve the time dimension in three groups:

1. Graphical methods;
2. Parametric methods; and
3. Summation and adjusted-average methods

Benjamin and Pollard (1980) consider 5 large groups:

1. Graphical methods;
2. Summation and adjusted-average methods;
3. Graduation methods using mathematical formulae;
4. Graduation by reference to a standard life table; and
5. Osculatory interpolation and splines (abridged and model life tables).

Abid, Kamhawey & Alsalloum (2005) tally actuarial graduation methods into nine groups, fractioning some of groups mentioned by Copas/Haberman and Benjamin/Pollard:

1. Graphical method;
2. Summation and adjusted-average methods;
3. Kernel's method;
4. The method of osculatory interpolation;
5. The spline method;
6. The curve fitting or parametric method;
7. Graduation by reference to a standard table;
8. Difference equation method; and
9. Linear programming method.

3.2 Distributions

The most commonly used distributions for modeling deaths and survivors in life table models for a homogenous population group (same sex and age) are the binomial and the Poisson distributions. The death of a person can be modeled by a Bernouilli (0,1) distribution, and the sum of independent Bernouilli distributions is a Binomial distribution B(N_x, q_x), where N_x is the number of living individuals of age x and q_x is the probability of death at age x. The maximum likelihood estimator for qx is given by the observed mortality rate

$$\hat{q}_x = \frac{O_x}{N_x}.$$

The use of MLE for q_x does not guarantee the smoothness of the curve of $\hat{q}_x$ as function of x. Therefore, one rarely uses these MLEs directly but, rather, employs a smoothing method for the graduation of life tables. This smoothing process is equivalent to the introduction of constraints in an optimization process. The probability of death usually concerns a given time interval (most commonly one year). In a given population, not all individuals are exposed to the risk for the full year. Nevertheless, an individual exposed during a whole year is equivalent to n individuals exposed to periods that sum up to one year. The total age group exposure usually turns out to be not an integer number unless one uses exposure units measured in shorter units such as months or days.

When time is considered as a continuous variable and, therefore, the function to be adjusted is the force of mortality, the usual distribution model is Poisson. The Poisson and the Binomial distributions are equivalent when E_x is large and q_x is close to zero. For finite populations and particularly for older age groups, this is not usually the case. The usual MLE for both distributions are the same, but the one corresponding to the Poisson distribution presents a larger variance. Therefore, when jointly estimating mortality for many ages under smoothing constraints, one may assign a smaller weight to higher ages with the Poisson model.

The likelihood function for a Binomial is:

$$L(\underline{q}) = \prod_{x=0}^{w} \binom{E_x}{O_x} (q_x)^{O_x} (1-q_x)^{E_x - O_x}$$

$$L(\underline{q}) = \prod_{x=0}^{w} \binom{E_x}{O_x} \left(\frac{q_x}{1-q_x}\right)^{O_x} (1-q_x)^{E_x}$$

$$L(\underline{q}) = \exp\left[\sum_{x=0}^{w} \ln\binom{E_x}{O_x} + O_x \ln\left(\frac{q_x}{1-q_x}\right) + E_x \ln(1-q_x)\right]$$

The likelihood function for a Poisson is:

$$L(\underline{\mu}) = \prod_{x=0}^{w} \frac{(\mu_x)^{O_x} e^{-\mu_x}}{O_x!}$$

$$L(\underline{\mu}) = \prod_{x=0}^{w} \frac{(E_x q_x)^{O_x} e^{-E_x q_x}}{O_x!}$$

$$L(\underline{\mu}) = \prod_{x=0}^{w} \exp[O_x \ln E_x + O_x \ln q_x - E_x q_x - \ln O_x!]$$

3.3 Criteria for model choice

Due to the intrinsic characteristics of the project that instigated this book, the selection of methodology to be adopted was guided by the following *desiderata:*

i) parsimony criterion (Ockham's razor) – given a choice, follow the simplest theory which solves the problem;

ii) intelligibility criterion – easy to understand and communicate methodology;

iii) replicability criterion – results should be replicable by other researchers;

iv) stability criterion – universally accepted and tested methodology;

v) transparency criterion – methodology should be fully documentable;

vi) self-sufficiency criterion – methodology should not depend on a single or experimental software.

In the case of a family of time dependent life tables, one would also need:

vii) criterion of compatibility between static and dynamic tables – methodology should allow for a time evolution of the static life tables.

By considering all the above listed criteria, the chosen model was Heligman & Pollard.

According to Beltrão and Sugahara (2004) model life tables should present the following three properties: a) they should be simple and easy to use as, for example, the Coale-Demeny family, the United Nations models, Brass *logit* model and the system of Lederman; b) they should be able to describe any specific age mortality pattern found in a real population; and c) they should present the best possible adjustment when comparing real and predicted mortality rates.

4 | Database construction

4.1 The insured population

The population dataset used to pursue the life tables study was provided individually by 13 economic conglomerates, comprising 23 insurance companies. These companies are: Mapfre Nossa Caixa Vida e Previdência S.A., Brasilprev Seguros e Previdência S.A., HSBC Vida e Previdência S.A., Unibanco AIG Vida e Previdência S.A., Sul América Seguros, Icatu Hartford Seguros S.A., Itaú Vida e Previdência S.A., Itaú Seguros S.A., Bradesco Seguros S.A., Finasa Seguradora S.A., Caixa Seguradora S.A., Mapfre Vera Cruz Vida e Previdência S.A., Generali do Brasil, Unibanco AIG Seguros S.A., HSBC Seguros S.A., Sul América Seguros de Vida e Previdência S.A., Aliança do Brasil, MARES-Mapfre Riscos Especiais Seguradora S.A., Bradesco Vida e Previdência S.A., Caixa Vida e Previdência S.A., AIG Brasil Companhia de Seguros, CAPEMI e GBOEX. The insured population considered covers over 82% of the Brazilian Life Insurance market.

Every insurance company is required to annually send its life insurance data to SUSEP – Brazilian Insurance Authority, individually listing each insured person, following a given protocol. The protocol may vary over time (SUSEP regulations n. 197 for 2004 data, n. 312 and 322 for 2005 data and n. 335 for 2006 data), but requires basically the same information. The information is organized in two major groups of forms: one for the insured individual and the other for the beneficiaries of the insurance benefits. The forms for the insured individuals are divided into death and survival coverages. Each individual is identified by his/hers CPF – tax registration ID, sex and birth date. For each calendar year, the insurance company informs the exposure period, and the eventual death for each insured person, in every insurance product, in separate files.

According to Brazilian Law, life insurance products are classified as:

1. Defined Benefit Pension Plan (PPT – Previdência Privada Tradicional);
2. Variable Contribution Pension Plan (PBL – Plano Gerador de Benefício Livre);
3. Accumulation and Annuities (FGB – Fundo Gerador de Benefício);
4. Variable Contribution Pension Plan (VGL – Vida Gerador de Benefício Livre);
5. Group life insurance – Corporations (VGA – Vida em Grupo – empregado/empregador)
6. Group life insurance – Associations (VGB – Vida em Grupo – associações);
7. Group life insurance – Insurance Clubs (VGC – Vida em Grupo – clubes de seguro);
8. Personnal Accidents (AP – Acidentes Pessoais);
9. Private Pensions (PP – Previdência Privada); and
10. Individual life insurance (VI – Vida Individual).

Products 1 to 4 are considered savings products, while products 5 to 7 are life insurances. In the present work, an insured person with any of the savings products is classified in the *survivorship group*, whereas a person solely with a life insurance product is classified in the *mortality group*. The last three products, which were present up to 2004, relate to a previous classification.

LabMA/UFRJ received data covering the years 2004, 2005 and 2006. There were 300.337.582 records from these corporations, which corresponded to a total of 21.7 million men and 17.8 million women. Table 1 presents this population by year.

TABLE 1. POPULATION DISTRIBUTED BY YEAR

Year	Male	Female	Total
2004	11.401.537	7.965.070	19.366.607
2005	13.985.764	11.514.878	25.500.642
2006	16.851.702	13.821.050	30.672.752

4.2 The "Tábuas" database

Given the Project's aim and dimension, one can clearly see the need for a Database Management System, mainly due to:

i. Management of large datasets elected to construct the life tables;

ii. Make queries, consolidations, filtering and data analysis, simultaneously over the whole dataset;

iii. Storing with security, assuring confidentiality and isolation of companies data subsets;

The database developed within the project was named "Tábuas". In what follows, a series of actions for database construction are commented:

i. Data gathering and transformation of data subsets from the several participating insurance companies;

ii. Storage and security procedures, as well as routines to assure confidentiality;

iii. Data model of the "Tábuas" database; and

iv. Integration routines of several data sources.

Data gathering and transformation

Each participating insurance company sent its data sets to LabMA/UFRJ, recorded according to SUSEP`s protocol. Each data set was individually incorporated into the "Tábuas" database. LabMA/UFRJ adopted a different database schema for each insurance company and calendar year due to confidentiality and data isolation. The C language was used to program some routines to import each company data set into the corresponding database schema. The whole process, as conceived, is robust and replicable.

Storage, security and confidentiality procedures

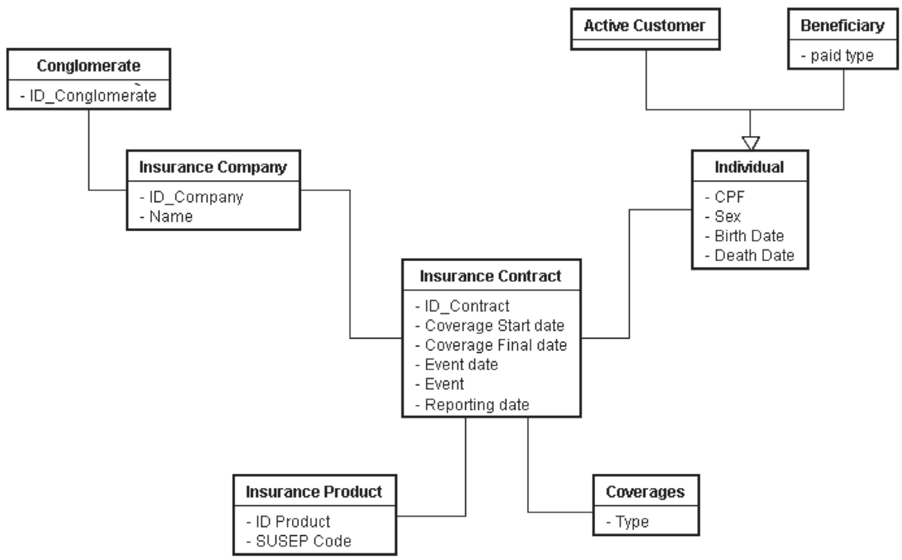
The life table project had a major requisite of complete confidentiality of the information received from the insurance companies. To ensure that this requisite is fully satisfied the "Tábuas" database environment was set to partition all data access. A firewall was set to protect the database environment, as well as the network of computers used in project development which deal directly or indirectly with the data. Every external data access was forbidden, while all internal accesses were logged due to security reasons and to allow error tracking. Backup procedures were implemented, particularly, with respect to the integration and data analysis processes.

"Tábuas" Database Schema

The main objectives of the database model conceived for the life table project were to aid in the statistical processing and to construct a temporal profile for each individual (a time line in the Lexis Diagram).

The "Tábuas" database was logically and physically divided in two levels. The first level concerns the process of gathering, importing, transforming and integrating data. The second level is the basis for conducting data analysis required for the construction of life tables.

The "Tábuas" database schema consists of eight entities as shown in the next figure. The central entity of the model is the "Insurance Contract", which associates each "Individual" with "Insurance Product" and "Coverages", related to "Insurance Company", being part of a "Conglomerate". Hence, the entity "Insurance Contract" relates each product bought by an individual, if he is an "Active", or else, if he is a "Beneficiary", to the benefit paid. Each record of the entity "Insurance Contract" concerns an insurance product, and up to three "Coverages".



Database integration

The next step is to integrate the various data sets, associated with the numerous insurance companies into a single database. The integration process dealt with heterogeneities among the schemas for the different calendar years. During the process, the most evident problem was related to the fact that the "Insurance Product" list for 2004 was different from that of 2005 and 2006.

The data conversion and integration processes go through successive steps of filtering and clustering using database views. To implement the integration of the database as a whole, 178 database views were programmed. The highest database view hierarchical level integrates all the data collected from the insurance companies. All data analysis routines run on this database view.

Data quality check

In this step a data analysis was performed in order to check the quality of the data gathered. To do that, three different procedures were implemented:

i. Consistency check of stocks and flows;

ii. CPF – tax registration ID validation check; and

iii. Population and death age and sex distribution check.

This last check was performed for each combination of calendar year, "Insurance Company", "Insurance Product" and "Coverage".

Consistency check of stocks and flows

In the first check performed, stocks and flows were verified for consistency: stock at the beginning of the year plus new entries should match stock at the end of the year less withdrawals.

This Consistency check was performed for the aggregated database by sex and, later on, tallying by the combination of:

i. Insurance company (23 companies);

ii. Calendar year (2004, 2005 and 2006);

iii. Insurance product (defined by SUSEP in 7 categories);

iv. Coverage (2 types);

v. Activity status (active or beneficiary); and

vi. Age.

The second consistency check verified information in adjacent years: stock at the end of a given calendar year should match the stock at the beginning of the following year. This check was performed for the link between 2004 and 2005, and between 2005 and 2006. This consistency check was also performed for the aggregated database by sex and, later on, tallying by the same combination described above. These analyses showed some inconsistencies, which were not particularly important for the project.

Tax registration ID validation check

The validation check of the CPF (tax ID) information was first performed using the internal consistency check of the tax ID. The CPF consists of a set of 9 digits followed by 2 control digits. The 2 control digits can be obtained as a function of the previous 9 digits (IRS defines the function). In the validation check, CPFs that did not conform to the control digit verification were considered *invalid* and correspond records were analyzed separately. Afterwards, some combinations that did conform to the control digit verification but were easily recognizable fakes, such as 000.000.001-71, 111.111.111-11, 222.222.222-22, were also considered invalid. In the "Tábuas" database, 13.7% of the registers contained invalid tax ID numbers, which corresponded to less than 0,1% of the individuals of the whole database.

Identification of individuals

As already mentioned, all records were aggregated using individual identification key: CPF – tax registration ID, sex and birth date. From these records, approximately 39.5 million distinct individuals were identified with a valid CPF. Table 2 presents the total of individuals by coverage and calendar year.

TABLE 2. INDIVIDUALS BY COVERAGE AND CALENDAR YEAR

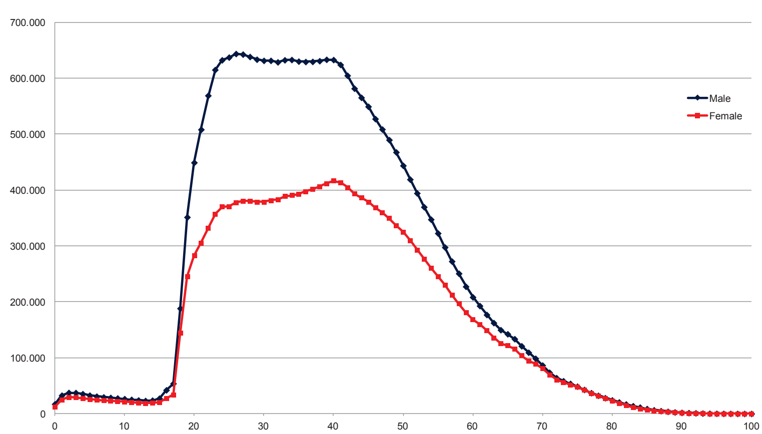
Year	Mortality group		Survivorship group		Total
	Male	Female	Male	Female	
2004	8.438.854	5.942.603	2.962.683	2.022.467	9.366.607
2005	11.234.464	9.440.083	2.751.300	2.074.795	25.500.642
2006	13.930.229	11.579.173	2.921.473	2.241.877	30.672.752

5 | Selection of subpopulations

5.1 Population distribution and evolution in time

Graph 1 presents the age-sex distribution of the studied insured population exposure during the 2004/2006 period. Most of the population, both male and female, lies between the ages of 20 and 80. Below 18 there is very little exposure, while above 80 the continuous decline is to be expected. The mode occurs around forty-something, being slightly higher for females. The male population is almost twice as large as the female.

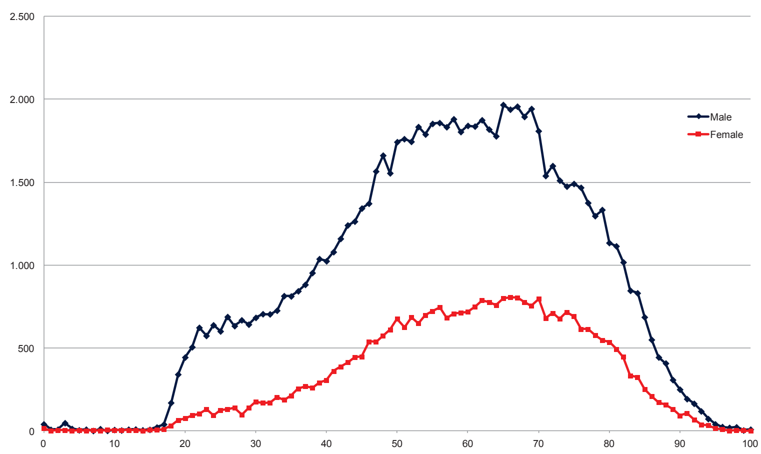
GRAPH 1. DISTRIBUTION OF THE EXPOSURE TO RISK BY AGE AND SEX – INSURED BRAZILIAN POPULATION – 2004 TO 2006



Since past experience with Brazilian data tends to show under registration of deaths in similar databases, the insurance companies' data was challenged with available government data in the *Cadastro Nacional de Informações Sociais* (CNIS – National Registry of Social Data) and *Sistema de Controle de Óbitos* (SISOBI – National Death Registry System) of the *Ministério da Previdência Social* (Social Security Administration), both operated by Dataprev – *Empresa de Tecnologia e Informações da Previdência Social.*

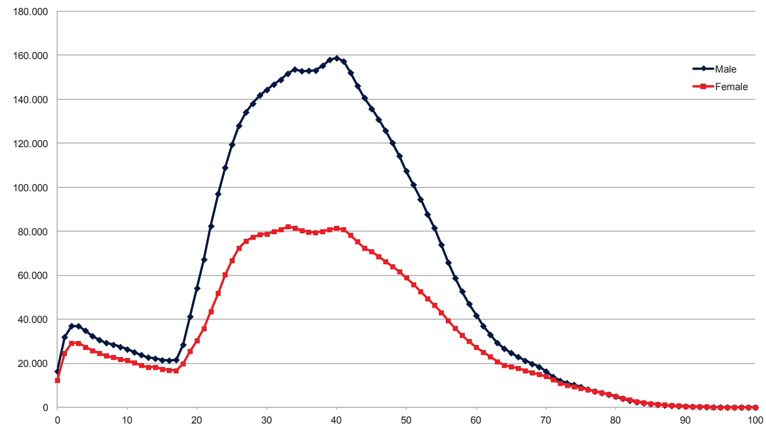
During this period 575.838 deaths were registered (after cross-checking with the Government death registration System CNIS/SISOBI). Graph 2 presents the corresponding age/sex distribution.

GRAPH 2. DEATHS BY AGE AND SEX – INSURED BRAZILIAN POPULATION – 2004 TO 2006

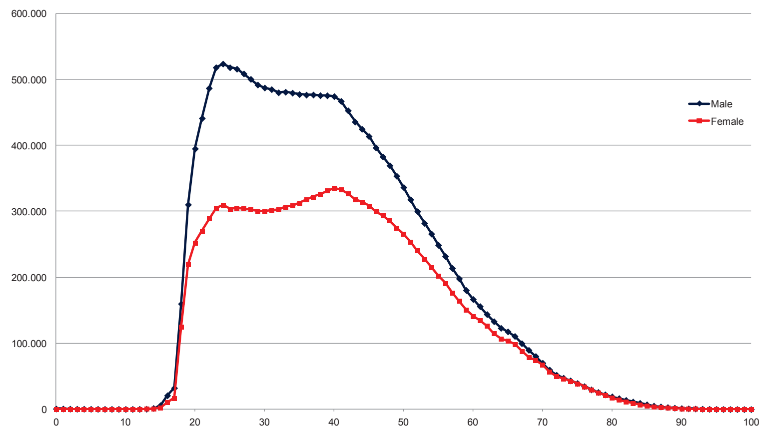


Graph 3 to graph 6 correspond to disaggregating male and female population according to coverage. There are 17.4 million males and 14.5 million females in the death coverage group, while 4.3 million males and 3.3 million females are in the survivorship group. Although the second group is much smaller than the first one, it is still large enough for estimation purposes. The magnitude of this database is more than enough to estimate life tables for the Brazilian insurance market.

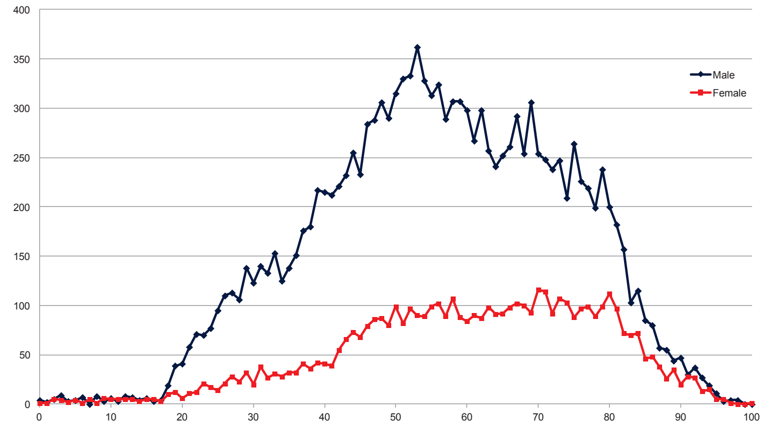
GRAPH 3. DISTRIBUITION OF THE EXPOSURE TO RISK BY AGE AND SEX – SURVIVORSHIP COVERAGE POPULATION – 2004 TO 2006



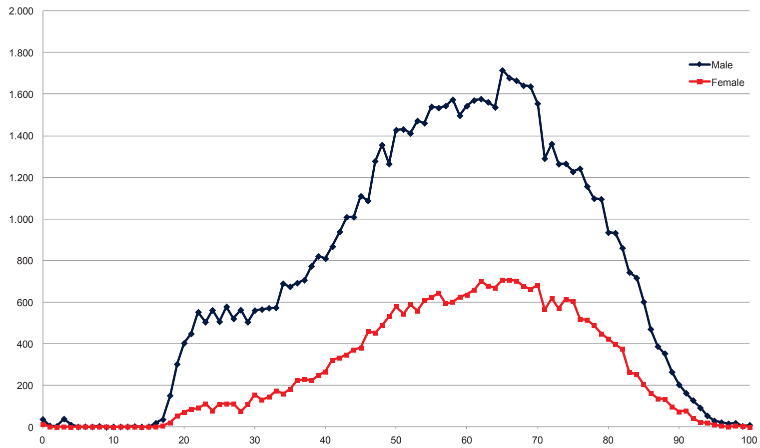
GRAPH 4. DISTRIBUTION OF THE EXPOSURE TO RISK BY AGE AND SEX – DEATH COVERAGE POPULATION – 2004 TO 2006



GRAPH 5. DEATHS BY AGE AND SEX – SURVIVORSHIP COVERAGE POPULATION – 2004 TO 2006



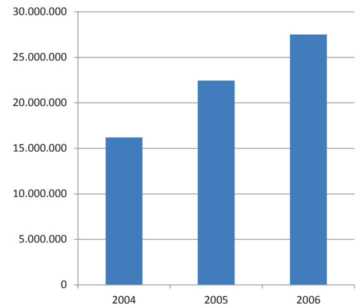
GRAPH 6. DEATHS BY AGE AND SEX – DEATH COVERAGE POPULATION – 2004 TO 2006



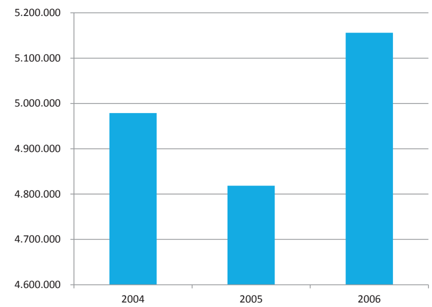
Graph 7 presents the number of individuals disaggregated by coverage and calendar year.

GRAPH 7.

INDIVIDUALS IN THE DATABASE BY CALENDAR YEAR – DEATH COVERAGE



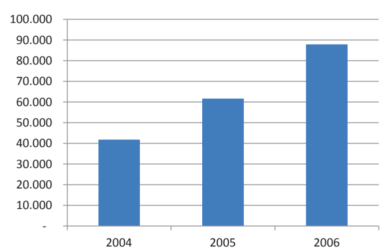
INDIVIDUALS IN THE DATABASE BY CALENDAR YEAR – SURVIVORSHIP COVERAGE



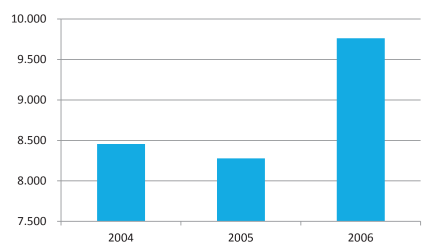
Graph 8 below show the distribution of death for the three years classified by coverage.

GRAPH 8.

DEATHS IN THE DATABASE BY CALENDAR YEAR – DEATH COVERAGE



DEATHS IN THE DATABASE BY CALENDAR YEAR – SURVIVORSHIP COVERAGE



5.2 IBNR estimates

Table 3 presents the distribution of delays in reporting death events. One finds that 83% of deaths are reported within 100 days of occurrence. Therefore, one may assume that all deaths for 2004 and 2005 were already reported. For 2006, the same can be assumed since the data were tabulated in July 2007. Moreover, since there is also a cross-check with the Government death registration System (CNIS/SISOBI), there is no need for IBNR considerations.

TABLE 3. DISTRIBUTION OF DELAYS IN REPORTING DEATH EVENTS

Year	Delay	Deaths	% of Deaths
2004	0 to 100 days	45.925	86,16%
2004	100 days to 1 year	6.208	11,65%
2004	1 to 2 years	822	1,54%
2004	2 to 3 years	133	0,25%
2004	Over 3 years	215	0,40%
2005	0 to 100 days	57.550	86,33%
2005	100 days to 1 year	6.473	9,71%
2005	1 to 2 years	2.359	3,54%
2005	2 to 3 years	106	0,16%
2005	Over 3 years	176	0,26%
2006	0 to 100 days	59.060	76,51%
2006	100 days to 1 year	15.484	20,06%
2006	1 to 2 years	2.201	2,85%
2006	2 to 3 years	266	0,34%
2006	Over 3 years	178	0,23%

5.3 Consideration on extreme age groups

Although the insured population as a whole is very large, with the extreme age groups, the population is rather sparse and demands special attention. Total figures are still significant, though (see table 4).

TABLE 4. EXTREME AGE GROUPS

Year	<18 years				>80 years			
	Male		Female		Male		Female	
2004	312.516	2,7%	261.699	3,3%	48.966	0,4%	32.757	0,4%
2005	397.698	2,8%	348.276	3,0%	67.312	0,5%	64.079	0,6%
2006	459.385	2,7%	398.777	2,9%	105.078	0,6%	106.779	0,8%

5.4 Selection criteria for exclusion of subpopulations

Subpopulations were defined as the combination of Insurance Company, Product, Type of Coverage, Sex and Calendar Year. Criteria adopted were:

i. Information given by the Insurance Company themselves;

ii. Comparison of Actual and Expected Number of Deaths under two extreme life tables (CSO 2001 and IBGE 2005);

iii. Visual examination of observed crude mortality curves for individual ages for all subpopulations.

Comparison of actual and expected number of deaths under two extreme life tables

For the purpose of comparison of observed mortality rates with extreme life tables, one assumes that CSO 2001 is an extreme table for low mortality rates, whereas the IBGE 2005, which describes the Brazilian population, is the other extreme for high mortality rates.

For each subpopulation, the ratio between the observed number of deaths and the theoretical number of deaths under each of the extreme tables calculated were denoted by o_e , and given by

$$o_e = \sum_x q_{x,e} \cdot E_{xspct}$$

where $q_{x,e}$ is the probability of death at age x of the extreme table e and E_{xspct} denotes the number of persons with age x, sex s, coverage c in insurance product p at time t. With ratios so defined, one assures that there is always a positive value for the denominator. For subpopulations with high mortality rates, close to the IBGE 2005 table, the ratio relative to IBGE should be close to unity and close to 10 (for men) and 5 (for women) when one considers the CSO 2001, since these are the average values of the ratios of probability of death between CSO and IBGE.

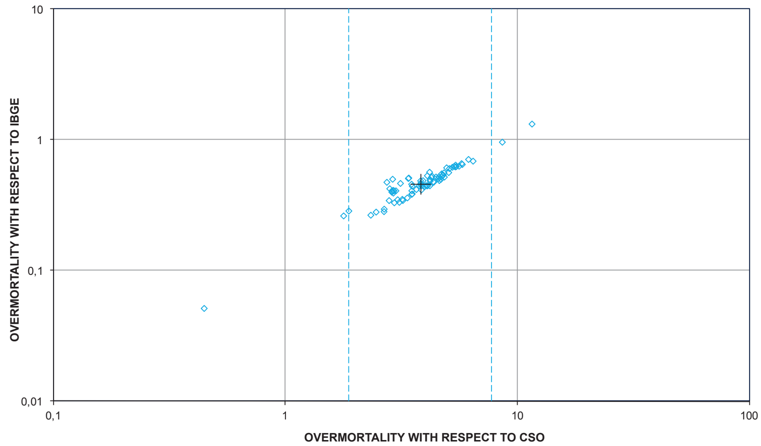
On the other hand, for subpopulations with low mortality rates, close to the CSO 2001 table, the ratio of actual to expected number of deaths should be close to the unity, whereas they are close to 1/10 (for men) and 1/5 (for women) when the IBGE table is considered.

These ratios were plotted in an x-y graph, with the ratio relative to IBGE in the x-axis and the ratio relative to CSO in the y-axis. One should expect, roughly, in the event all subpopulations were submitted to the same mortality pattern, that all points should be in a cloud around a central point, close to each other, probably close to a straight line. Points far away from the central point of the cloud should be discharged, since they would probably involve data errors and could bias the results.

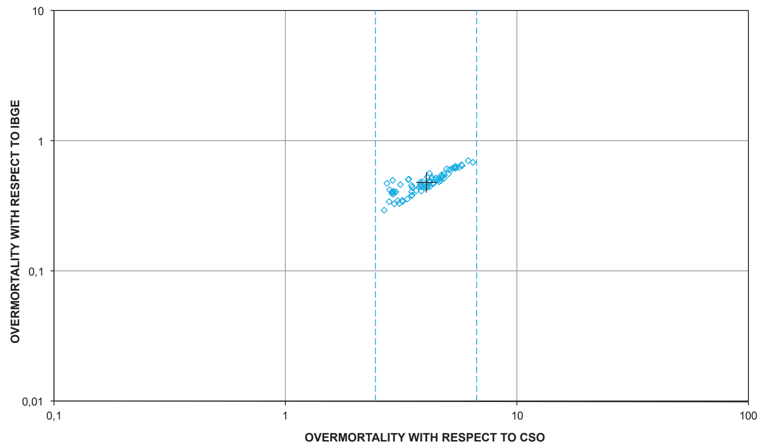
Since the numerator of each fraction is the actual number of deaths for each subpopulation, and since the number of deaths, considered as a random variable, is the sum of independent binomial random variables, one for each age, one may assume that the numerator of each fraction will be approximately normally distributed. Therefore, inspired by Tukey (1977), one can calculate bounds that will define subpopulation outliers. For the present case, all subpopulations outside these bounds were discharged. The process of establishing bounds and not considering subpopulation outliers was carried on interactively until no further outliers existed. Each bound was established by adding and subtracting 1.5 interquartile distance to the median, thus leaving roughly 95% of the points inside.

The graphs below present over mortality calculated with respect to the IBGE 2005 and CSO 2001 tables for subpopulations with death and survivorship coverage according to sex and calendar year. In these graphs, crosses indicate medians and the bounds are designated as dashed lines. These graphs do not show any temporal evolution in mortality.

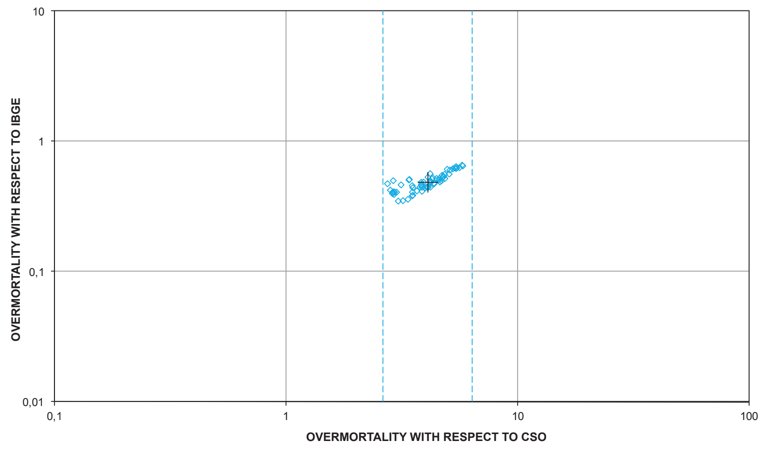
GRAPH 9. OVER MORTALITY WITH RESPECT TO IBGE 2005 AND CSO 2001 TABLES FOR MALES – SURVIVORSHIP COVERAGE – 1st ITERATION WITH FENCES – 2004 TO 2006



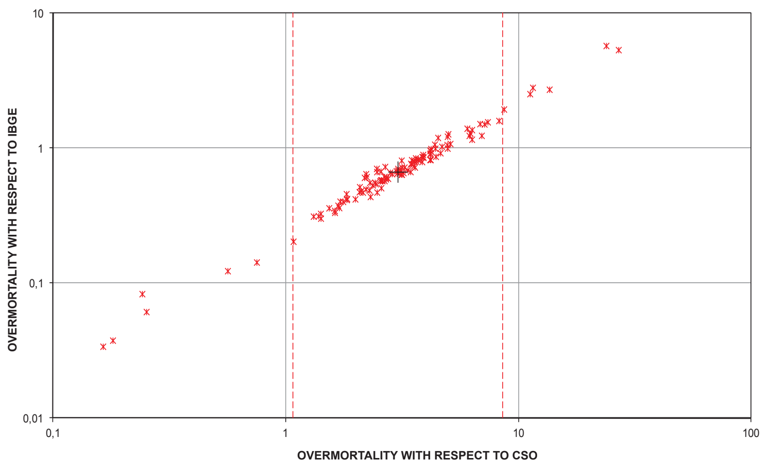
GRAPH 10. OVER MORTALITY WITH RESPECT TO IBGE 2005 AND CSO 2001 TABLES FOR MALES – SURVIVORSHIP COVERAGE – 2st ITERATION WITH FENCES – 2004 TO 2006



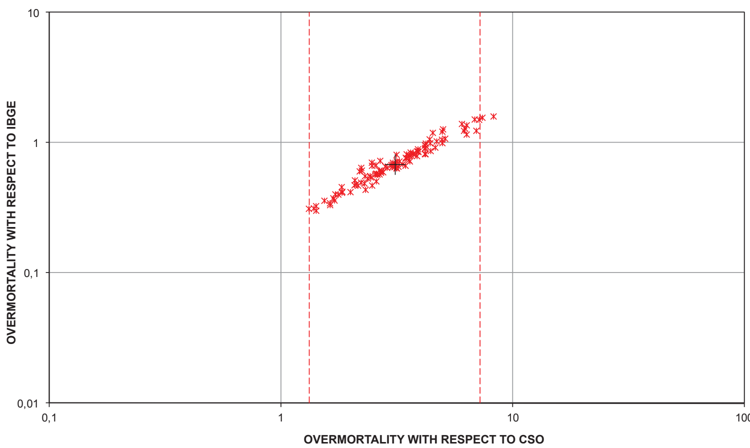
GRAPH 11. OVER MORTALITY WITH RESPECT TO IBGE 2005 AND CSO 2001 TABLES FOR MALES – SURVIVORSHIP COVERAGE – FINAL ITERATION WITH FENCES – 2004 TO 2006



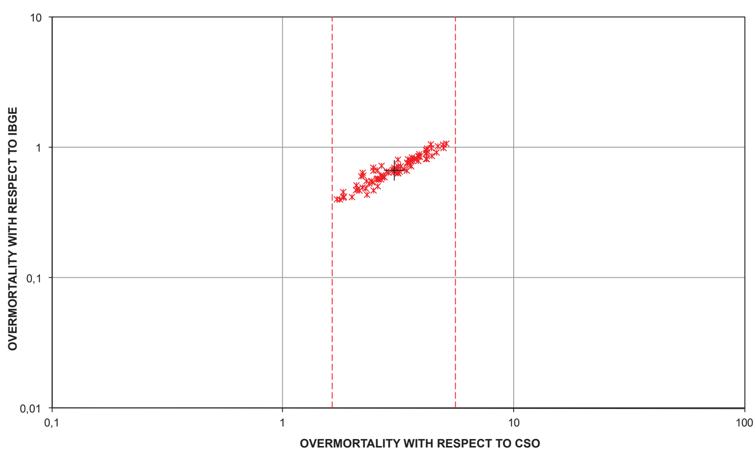
GRAPH 12. OVER MORTALITY WITH RESPECT TO IBGE 2005 AND CSO 2001 TABLES FOR FEMALES – DEATH COVERAGE – 1st ITERATION WITH FENCES – 2004 TO 2006



GRAPH 13. OVER MORTALITY WITH RESPECT TO IBGE 2005 AND CSO 2001 TABLES FOR FEMALES – DEATH COVERAGE – 2st ITERATION WITH FENCES – 2004 TO 2006



GRAPH 14. OVER MORTALITY WITH RESPECT TO IBGE 2005 AND CSO 2001 TABLES FOR FEMALES – DEATH COVERAGE – FINAL ITERATION WITH FENCES – 2004 TO 2006



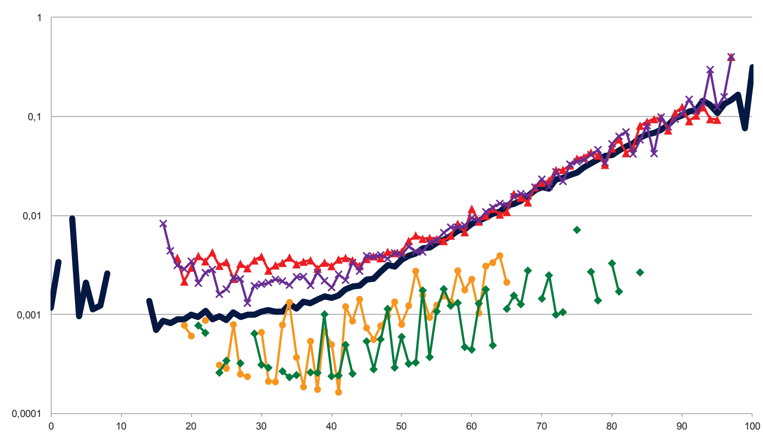
At the end of the elimination process, 5% of the male population were eliminated. For women, 10% were eliminated for the survivorship group and 22% for the mortality group.

Visual examination of observed crude mortality curves for individual ages for all subpopulations

After the elimination process by bounds, crude mortality rates for each combination of insurance company/product/calendar year/sex/coverage were calculated and graphed. Crude mortality rates were defined as the ratio of number of deaths to exposure at each age, classified according to sex and coverage. For a given calendar year, exposure is defined, for each age, as the total number of months lived in that age, expressed in years, for all individuals in the population.

For example, graph 15 presents crude mortality rates for the men's active populations (blue continuous line) as well as some of the combinations of insurance company/product/calendar year not yet eliminated. In this graph, two lines present extremely low values (see green and orange lines). These subpopulations were eliminated. Furthermore, two other lines present a plateau with mortality rates remaining constant for too many years, which is an impossibility (see red and violet lines). These subpopulations were eliminated also.

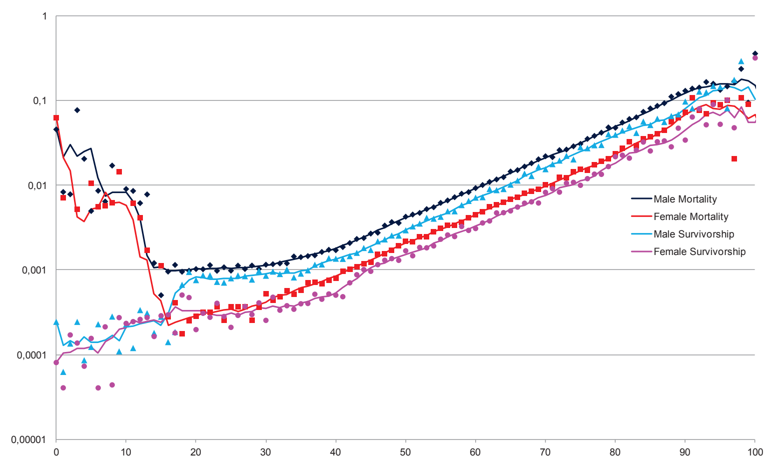
GRAPH 15. CRUDES RATES FOR ALL COMPANIES AND PRODUCTS – MALES – 2004 TO 2006



5.5 Differentiation of mortality curves according to sex and coverage

Graph 16 below shows a definite separation of curves describing mortality and survivorship for both men and women. These graphs also show that each curve has the shape one would expect from a regular mortality curve with a ***J*** shape and a structure similar to known mortality curves.

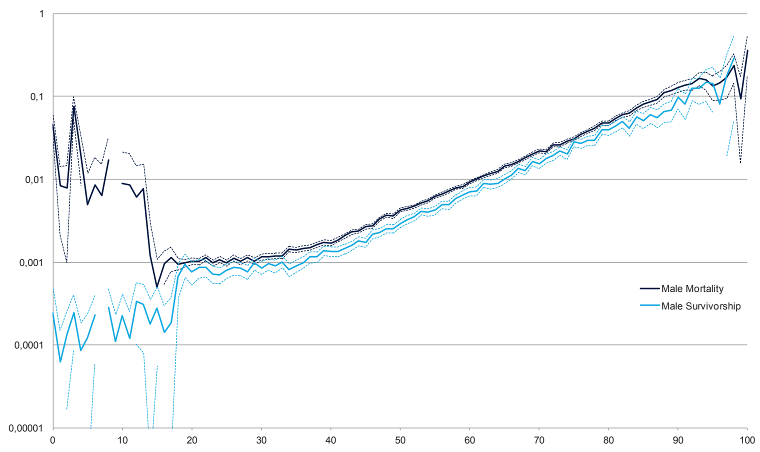
GRAPH 16. OBSERVED AND SMOOTHED DEATH RATES (MOVING AVERAGE OF 5) – 2004 TO 2006



Men

The separation between survivorship and death groups for the male population is very clear, not only considering observed mean value curves, but also when one considers the 95% confidence intervals (dashed lines in the graph) for the estimated values. This distance is particularly relevant between the ages of 15 and 90 (see graph 17).

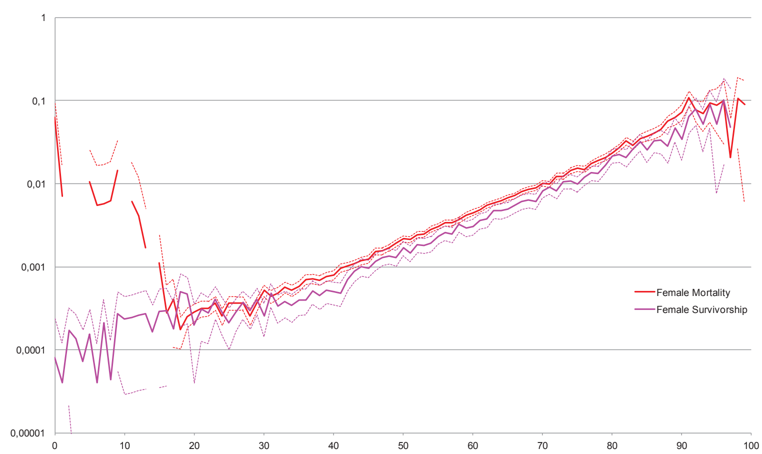
GRAPH 17. MALE CRUDE RATES BY AGE AND COVERAGE – 2004 TO 2006



Women

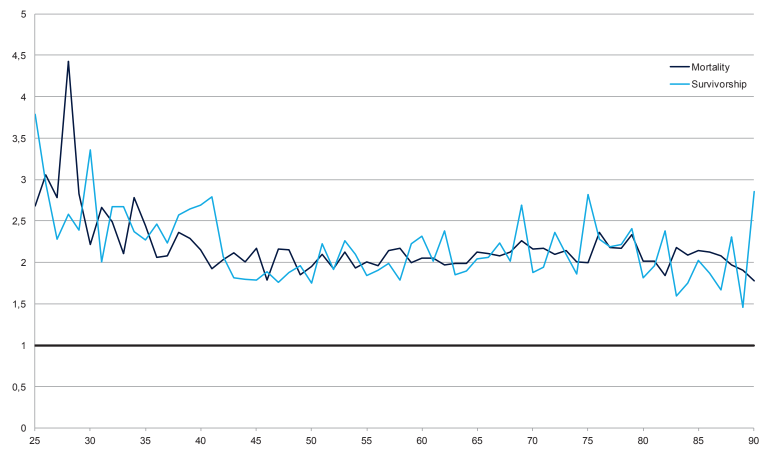
The separation between survivorship and death coverages for women is visually very clear, and also when one draws the 95% confidence intervals (see dashed lines) between the ages of 20 and 90 (see graph 18). Notice that the confidence intervals for women are larger than for men, since the number of women is smaller for almost all ages.

GRAPH 18. FEMALE CRUDE RATES BY AGE AND COVERAGE – 2004 TO 2006



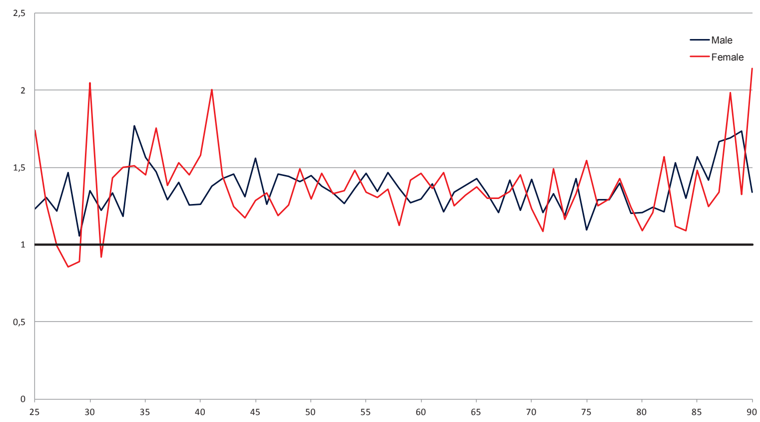
Graph 19 presents the male over mortality for each coverage group (survivorship and mortality). It is worth noticing that both coverage groups present the same pattern, which differs from the Brazilian population as a whole. Almost all values are above unity, which validates the separation between the sexes for each coverage group.

GRAPH 19. MALE OVER MORTALITY ACCORDING TO COVERAGE – 2004 TO 2006



Graph 20 presents the ratio between crude rates for death and survivorship coverage for each sex. The values are well above unity showing once again the coverage groups separation. Both curves, for males and females, are similar; the largest differences concern younger adults.

GRAPH 20. OVER MORTALITY OF POPULATION WITH DEATH COVERAGE TO POPULATION WITH SURVIVORSHIP COVERAGE BY SEX – 2004 TO 2006



Conclusions

The main conclusions reached are:

i. There is a very clear separation between male and female rates;

ii. There is a clear separation between the survivorship and death coverage groups, for both males and females;

iii. Crude rates follow a pattern similar to known life tables rates.

6 | Construction of Life and Survival Tables

6.1 Heligman & Pollard model

The model proposed by Heligman & Pollard (1980) has three components: child mortality, a "hump" for young adult mortality and a component for middle and old age mortality:

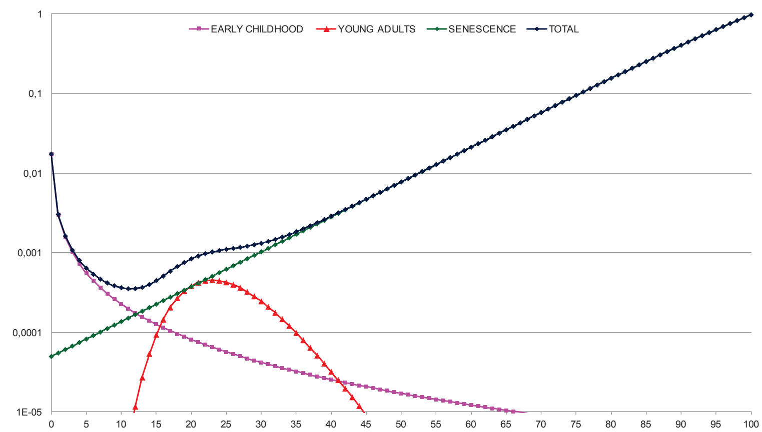
$$q(x) = A^{(x+B)^C} + De^{-E(\ln x - \ln F)^2} + \frac{GH^x}{(1 + KGH^x)}.$$

The Heligman & Pollard model can be viewed as a combination of different parametric models for describing human mortality. The second component is particularly useful for describing the mortality of young adults by external causes, a phenomenon which has been increasing since the middle of last century.

Since the model has nine parameters for the three components, it is very flexible and can approximate almost all known human mortality experiences. For these reasons, the Heligman & Pollard model was chosen for construction of mortality and survival tables.

Graph 21 shows the three components and the resultant for a typical case.

GRAPH 21. HELIGMAN & POLLARD MODEL COMPONENTS



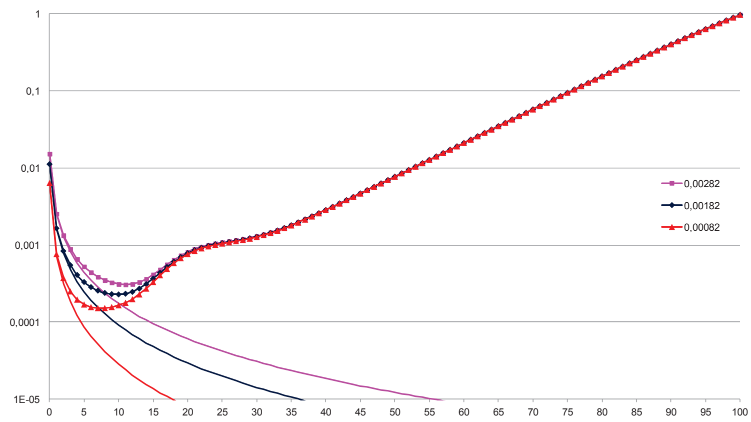
In what follows, the components are described separately with a sensitivity analysis of its parameters. The first three parameters A, B and C are related to the first component, which is important for child mortality and decreases in importance for adults and old age mortality:

$$A^{(x+B)^C}.$$

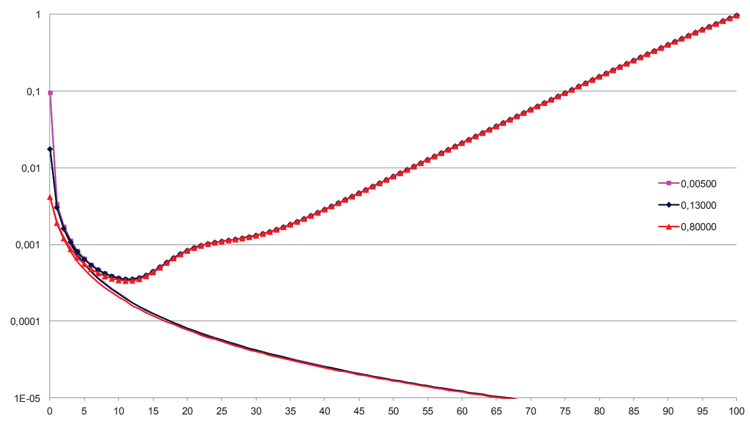
In fact, it has almost no effect for ages above the 70's.

Graph 22 to graph 24 depict sensitivity analyses of the Heligman & Pollard function to the three parameters A, B and C. The parameters A and C are scale parameters whereas B is a location parameter.

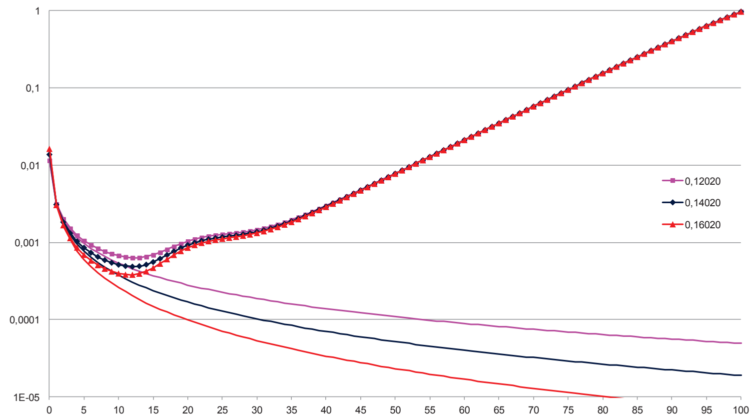
GRAPH 22. SENSITIVITY OF THE FUNCTION TO SCALE PARAMETER 'A', WITH OTHER PARAMETERS HELD CONSTANT



GRAPH 23. SENSITIVITY OF THE FUNCTION TO SCALE PARAMETER 'B', WITH OTHER PARAMETERS HELD CONSTANT



GRAPH 24. SENSITIVITY OF THE FUNCTION TO SCALE PARAMETER 'C', WITH OTHER PARAMETERS HELD CONSTANT

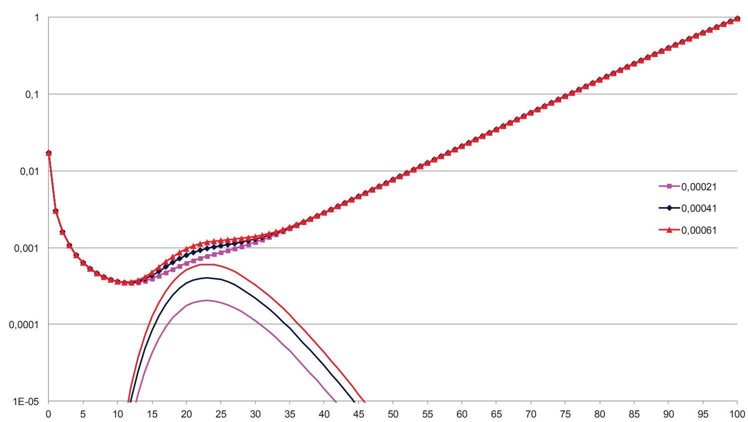


Parameters D, E and F are related to the second component, the "hump" for young adult mortality:

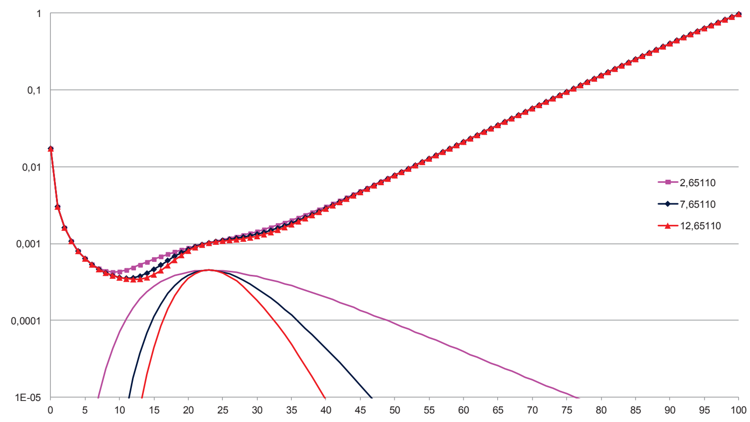
$$De^{-E(\ln x - \ln F)^2}$$

The parameter D is related to the level of the "hump" while E is related to the amplitude of the "hump". On the other hand, F is a location parameter which positions the "hump" (the mode of the "hump" occurs at *ln(F)*). Graph 25 to graph 27 show the sensitivity of Heligman & Pollard model to the parameters *D, E* e *F*.

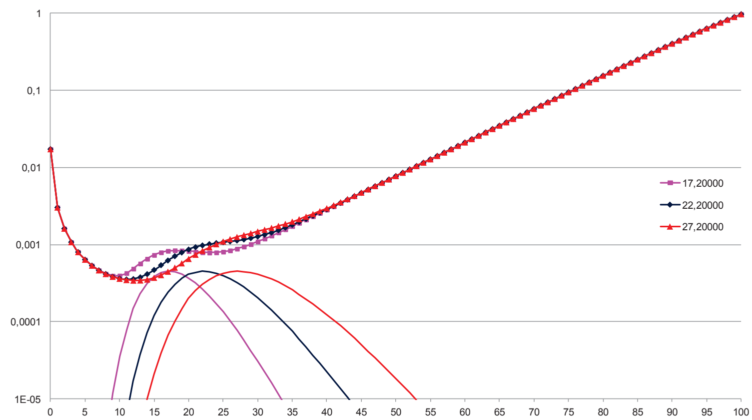
GRAPH 25. SENSITIVITY OF THE FUNCTION TO SCALE PARAMETER 'D', WITH OTHER PARAMETERS HELD CONSTANT



GRAPH 26. SENSITIVITY OF THE FUNCTION TO SCALE PARAMETER 'E', WITH OTHER PARAMETERS HELD CONSTANT



GRAPH 27. SENSITIVITY OF THE FUNCTION TO SCALE PARAMETER 'F', WITH OTHER PARAMETERS HELD CONSTANT

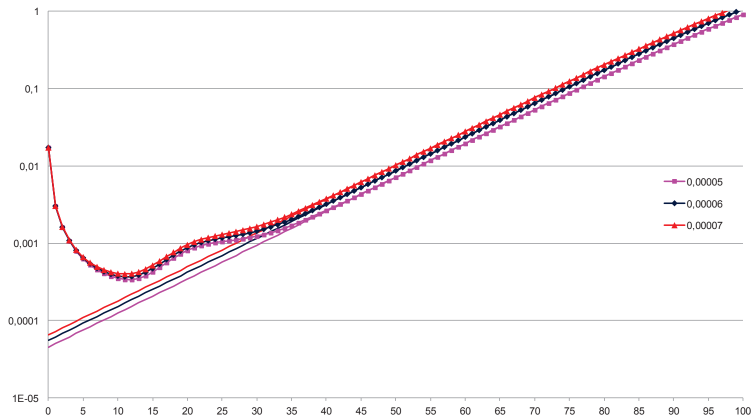


The last component can be also written as

$$\frac{GH^x}{(1+KGH^x)} = \frac{H^{x+\log_H G}}{(1+KH^{x+\log_H G})}$$

Therefore, the parameter G can be seen as a translation of $log_H G$ in the x of age scale. Graph 28 shows the sensitivity of Heligman & Pollard model to the parameters G.

GRAPH 28. SENSITIVITY OF THE FUNCTION TO SCALE PARAMETER 'G', WITH OTHER PARAMETERS HELD CONSTANT

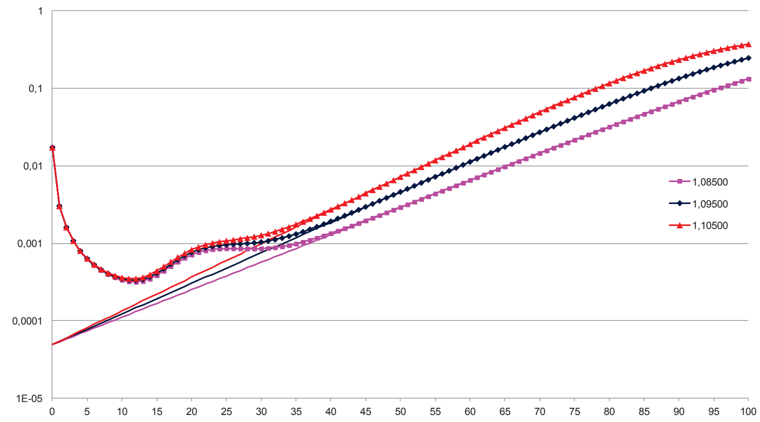


The parameter H concerns the change in curvature of the last component, from concave to convex since the inflection point occurs at age

$$x = \frac{-\ln(KG)}{\ln(H)} .$$

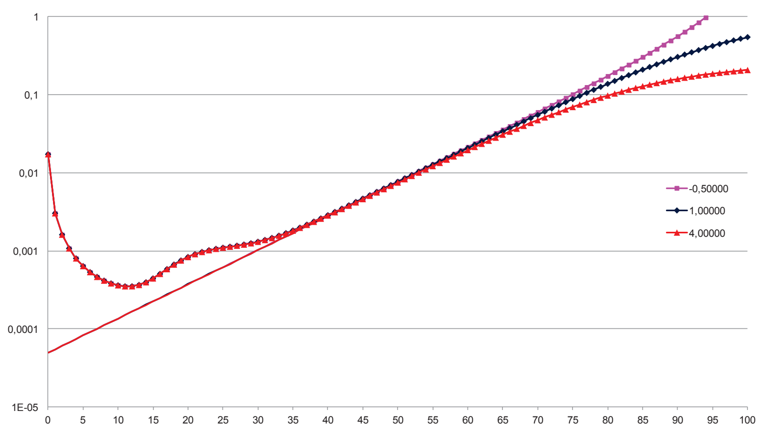
The greater the value of H, the earlier the inflection point occurs. Graph 29 shows the sensitivity analysis of the Heligman & Pollard curve to parameter H.

GRAPH 29. SENSITIVITY OF THE FUNCTION TO SCALE PARAMETER 'H', WITH OTHER PARAMETERS HELD CONSTANT



The parameter K has two roles: it regulates the curvature for advanced ages (depending on its sign) and it may define the asymptotic value, 1/K, of the curve, when it is positive. Conversely, when it is negative, the curve diverges to infinity, thus having no asymptotic value. The larger the value of K, the more concave the curve becomes and the smaller the increases in the old age mortality. Negative values of K imply a convex curve with larger gains in mortality as age increases. Graph 30 shows the sensitivity analysis of the Heligman & Pollard curve to parameter K.

GRAPH 30. SENSITIVITY OF THE FUNCTION TO SCALE PARAMETER 'K', WITH OTHER PARAMETERS HELD CONSTANT



6.2. Methodology for the parameters estimation

For a given age x, the number of deaths can be seen as a random variable with binomial distribution $B(N_x, q_x)$, with parameters N_x and q_x, where N_x is the number of individuals in the population with age x and q_x is the probability of death at age x. The unknown parameter is q_x which is to be estimated using the Heligman & Pollard (1980) model.

The estimation problem consists in finding the parameter values which minimize the quadratic objective function

$$\sum_x \frac{\left(q^{o}_{c,s}(x) - q^{hp}_{c,s,k}(x)\right)^2}{\operatorname{var}\left(q^{hp}_{c,s,k}(x)\right)}$$

where

$q^{o}_{c,s}(x)$ is the observed value for the subpopulation of age x, sex and coverage;

$q^{hp}_{c,s,k}(x)$ is the adjusted Heligman & Pollard value for the subpopulation of age x, sex and coverage, at iteration k; and

var($q^{hp}_{c,s,k}(x)$) is the variance of the corresponding Heligman & Pollard estimate.

According to the binomial hypothesis, this variance is given by

$$\operatorname{var}(q^{hp}_{c,s,k}(x)) = \frac{q^{hp}_{c,s,k}(x)(1 - q^{hp}_{c,s,k}(x))}{N_{x,c,s}}$$

where $N_{x,c,s}$ is the number of individuals in the population with age x, coverage c and sex s.

This is a non-linear regression problem with weights depending on the parameter values to be estimated. This problem has to be solved iteratively: at each step k, the parameters are estimated and are used to recalculate weights (inverse of the estimated variances) for the following step (k + 1). The SPSS *(Statistical Package for the Social Sciences) non-linear regression procedure* was used.

This process resembles the methodology of Generalized Linear Models (see Dobson, 1983 or MacCullagh & Nelder, 1983), except that in GLM the usual packages perform this internally, whereas in this case the weights of each step have to be fed manually. One could wonder whether using the Poisson or the Normal approximations would lead to a simplified process. But this is not true since both distributions have age dependent variances heteroscedasticity, which would not allow a closed solution.

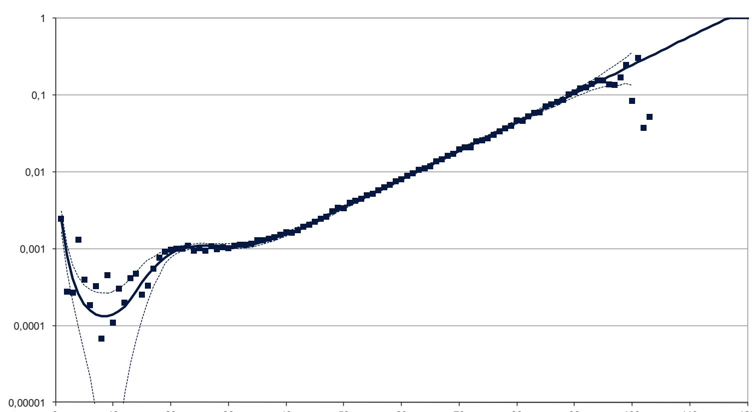
6.3. Curve fitting

The fitting process was conducted in two stages for each sex.

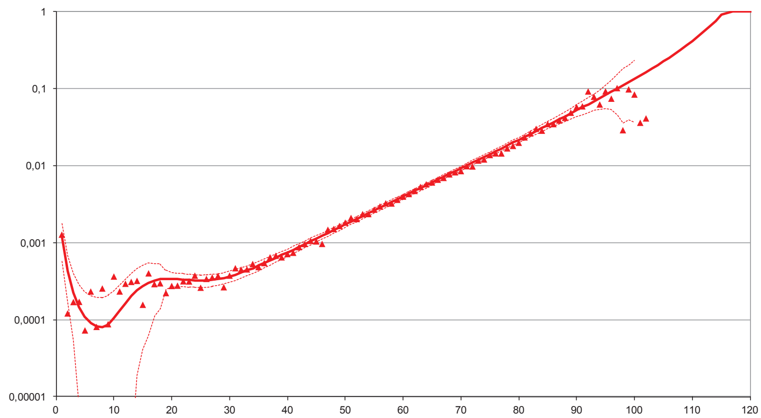
In the first stage, the fitting was conducted, separately for men and women, for the population independently of the coverage type (survivorship and death). For both sexes, the adjusted curves presented the three Heligman & Pollard components. It was decided to estimate the first two Heligman & Pollard components up to the age of 35 years, independently of the last component. The third component was estimated afterwards, taking the first two estimated components as given. For this third component the minimization process was conducted for all ages up to 100 years. It should be noted that the estimation of the third component parameters is quite independent of the first two components.

Estimated infant mortality is higher for men but with a declining difference up to 12 years of age. The second component (associated with external causes) starts to appear at younger ages for women, although it is less pronounced and shows shorter amplitude as compared to men. For the country as a whole, this second component is less pronounced for women, but starts showing up at a later age as compared to men.

GRAPH 31. CRUDE AND ADJUSTED MORTALITY RATES WITH 95% CONFIDENCE INTERVALS – INSURED BRAZILIAN POPULATION – MALE – 2004 TO 2006



GRAPH 32. CRUDE AND ADJUSTED MORTALITY RATES WITH 95% CONFIDENCE INTERVALS – INSURED BRAZILIAN POPULATION – FEMALE – 2004 TO 2006

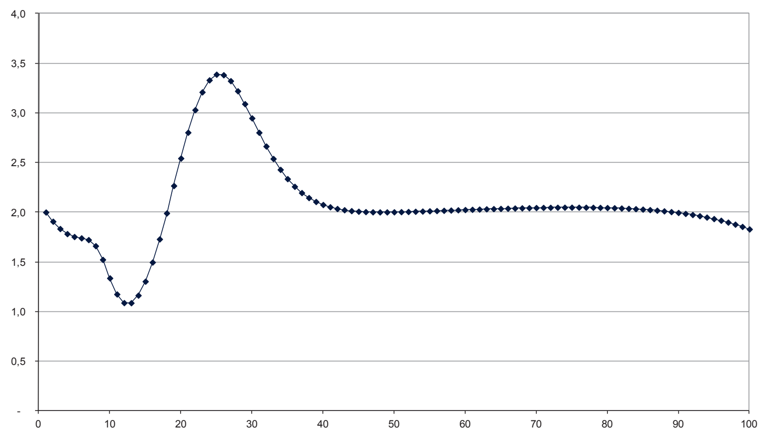


Both men and women present a peculiar mortality behavior for older ages. Part of it may be explained by the rarefied population for ages above 90. The female population presents a negative value for the parameter K and an accelerated mortality rate for old ages. The male population presents the value zero as a possible estimate for K, since zero stands in the confidence interval of this parameter. Since the parameter H, which defines the growth rate for most of the adult life, is very similar for both sexes, the male and female mortality curves are approximately parallel from age 40 to 90.

TABLE 5. PARAMETER ESTIMATES FOR THE HELIGMAN & POLLARD MODEL FOR THE WHOLE INSURED POPULATION, FOR BOTH SEXES		
Parameter	Males	Females
A	0,002604761	0,001310736
B	0,085846127	0,094227517
C	0,257875929	0,22549935
D	0,000703699	0,000213665
E	6,996195901	5,989195613
F	23,335424984	17,857979158
G	0,000052027	0,000028359
H	1,089064831	1,087142882
K	0	-1,46752

Graph 33 presents the male over mortality of the adjusted rates, between ages 0 and 120, showing a convergence for ages over 115. This graph highlights the accentuated mortality differences for younger ages and an almost parallel behavior from the ages 40 to 90.

GRAPH 33. MALE OVER MORTALITY FOR ADJUSTED VALUES – 2004 TO 2006



In the second stage, the adjustment was conducted for all combinations of coverage and sex. In this stage, due to the scarce data in the earlier ages, it was not possible to estimate the first component separately for men and women. Therefore, it was decided to adjust the second and third components for each sex and, afterwards, to fit the first components parallel to the components estimated in the previous stage. In order to do so, one adopted a suggestion proposed by Brass et al, calculating the constant *logit* distance

$$\left(\log\text{it}(q) = \ln\left(q \big/ (1-q) \right) \right)$$

between the male and female curves (these curves already had the second and third components). These authors found that when mortality curves undergo a *logit* transformation, there is an almost linear function connecting any two of these *logit* transformations.

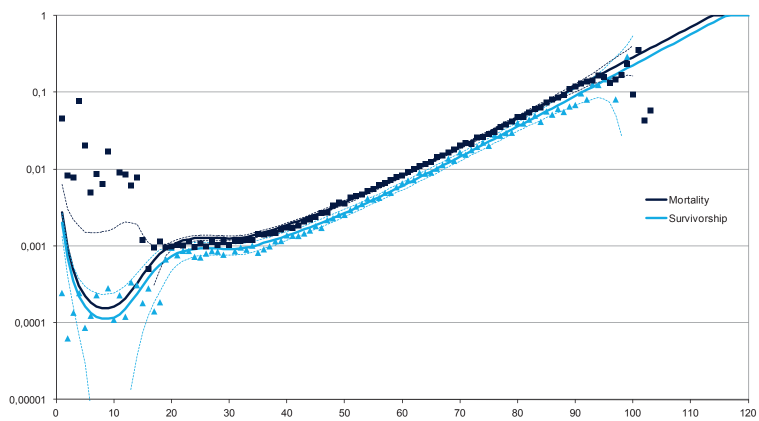
The adjusted models did not include neither the first component nor the second one, but just a constant parameter D:

$$q_{c,s,k}^{hp}(x) = D + \frac{GH^x}{(1+KGH^x)} \cdot$$

This model fitting was conducted for ages in the interval between 20 to 100 years. It is worth noting those ages below and above this interval were not considered in the adjustment process but were incorporated afterwards by an analysis of the adherence of the estimated curves to the data, taking into consideration confidence intervals for the death rates.

Graph 34 presents the crude death rates, the adjusted curves for adult ages and estimate of the infant mortality and corresponding confidence intervals for the male population.

GRAPH 34. CRUDE AND ADJUSTED MORTALITY RATES WITH 95% CONFIDENCE INTERVALS – BRAZILIAN INSURED POPULATION – MALE ACCORDING TO COVERAGE – 2004 TO 2006



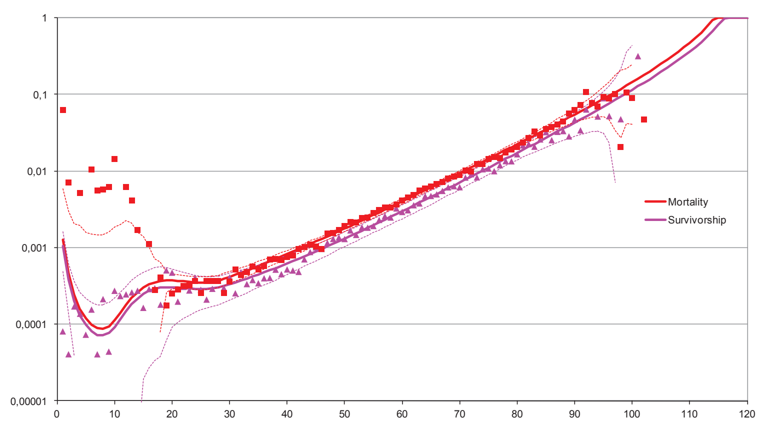
Is worth noting that the great majority of the observed rates in the central ages lie within the confidence intervals and clearly shows the separation of curves according to type of coverage. For this male sub-population, the parameter K was also zero. The other parameters were re-estimated for the K=0. The final values are shown in the next table.

TABLE 6. ESTIMATES OF HELIGMAN & POLLARD PARAMETERS FOR THE SIMPLIFIED MODEL ACCORDING TO SEX AND COVERAGE TYPE				
Parameter	Males		Females	
	Survivorship	Death	Survivorship	Death
D	0,00048519	0,00064511	0,00013377	0,000113
G	2,3647E-05	3,5241E-05	1,5425E-05	2,53E-05
H	1,09690236	1,09510716	1,09246271	1,089157
K	0	0	-1,46752	-1,46752

Graph 35 presents the same information for the female population. Notice that the estimated parameter values are shown in the table above. It is worth noting that the parameter H presents similar values for all subpopulations considered and also to the first stage estimates.

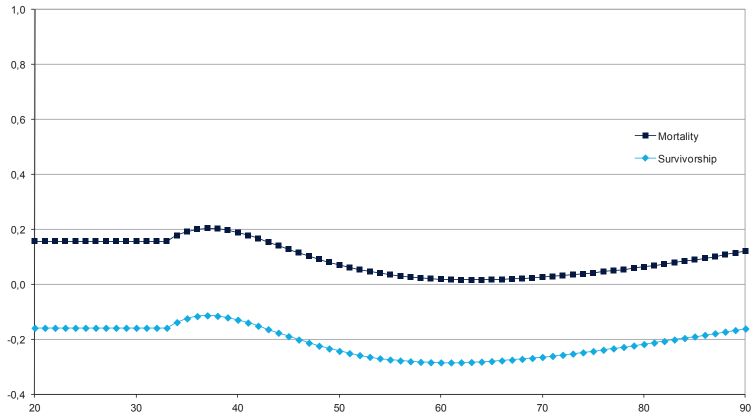
In a similar manner to the male population, most of the crude rates for the female population appear inside the confidence intervals for the estimated rates. For the two female subpopulations, death and survivorship, estimatives for parameter K were negative. They indicate a large growth in the estimated death rates for older ages. Since both values were in the confidence interval for the parameter estimated in the first stage, this value of K was adopted and fixed. The other parameters were re-estimated.

GRAPH 35. CRUDE AND ADJUSTED MORTALITY RATES WITH 95% CONFIDENCE INTERVALS – BRAZILIAN INSURED POPULATION – FEMALE ACCORDING TO COVERAGE – 2004 TO 2006

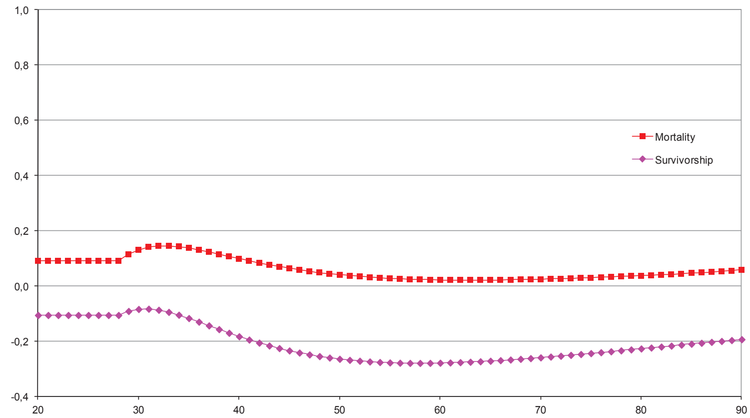


The concatenation of the adjusted curves for adults in the second stage (ages from 20 to 100) and the curve adjusted in the first stage, was done using the ideas of Brass et al. The *logit* difference between the two stages was calculated for both sexes. Graph 36 and graph 37 present these differences.

GRAPH 36. LOGIT DIFERENCE OF ADJUSTED RATES, ACCORDING TO COVERAGE, TO THE MALE GLOBAL RATES – 2004 TO 2006



RAPH 37. LOGIT DIFERENCE OF ADJUSTED RATES, ACCORDING TO COVERAGE, TO THE FEMALE GLOBAL RATES – 2004 TO 2006



One sees that these differences are reasonably constant from 20 to 90 years of age for both sexes. Therefore, it can be assumed that this difference can be extended for ages below 20, since in the first stage the mortality curves for children were estimated for all individuals independently of coverage types. Hence the mortality rates for the beginning of the curve were established through the use of the following equation:

$$\log\text{ito}\left(q_{c,s}^{hp}(x)\right) = \log\text{ito}\left(q_{s}^{hp}(x)\right) + \alpha_{c,s}$$

where

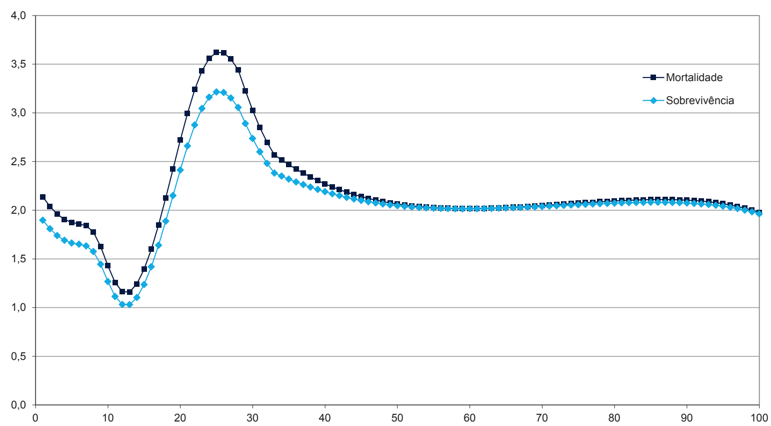
$q_{c,s}^{hp}(x)$ is the adjusted mortality rate for those with coverage c and sex s for ages x in the interval from 0 to 20;

$q_{s}^{hp}(x)$ is the adjusted mortality rate for those with sex s and age x in the interval from 0 to 20, estimated in the first stage;

$\alpha_{c,s} = \log\text{ito}\left(q_{c,s}^{hp}(x)\right) - \log\text{ito}\left(q_{s}^{hp}(x)\right)$ is a translation constant for those with coverage c and sex s, calculated as the *logit* difference of the adjusted rates at a concatenation age x of the adjusted curves (x = 33 for men and x = 28 for women).

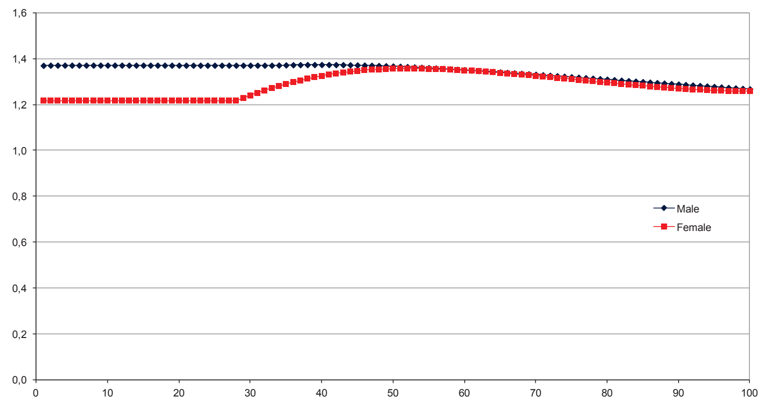
Graph 38 presents the male over mortality for death and survivorship coverage. One notes that the death coverage presents a higher male over mortality.

GRAPH 38. MALE OVER MORTALITY FOR ADJUSTED VALUES ACCORDING TO COVERAGE – 2004 TO 2006



Graph 39 presents, for both sexes, the over mortality of death over survivorship coverage. One notes that it is higher for the male population.

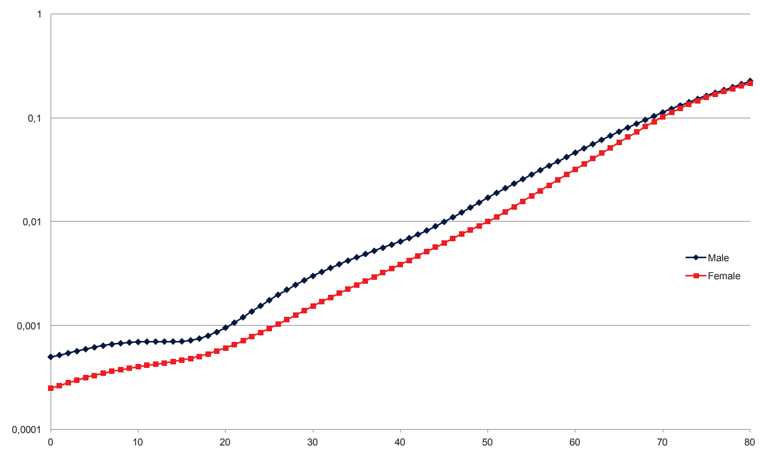
GRAPH 39. OVER MORTALITY OF POPULATION WITH DEATH COVERAGE TO POPULATION WITH SURVIVORSHIP COVERAGE BY SEX – ADJUSTED VALUES – 2004 TO 2006



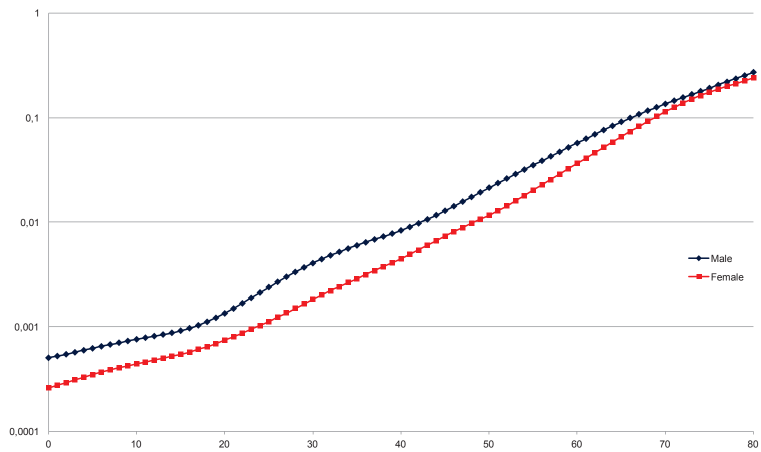
Special adjustments for older women

It is known that in the *AT83 and AT2000* tables there is a convergence of the male and female mortality curves as age increases. Graph 40 and graph 41 show this fact.

GRAPH 40. PROBABILITIES OF DEATH – AT2000



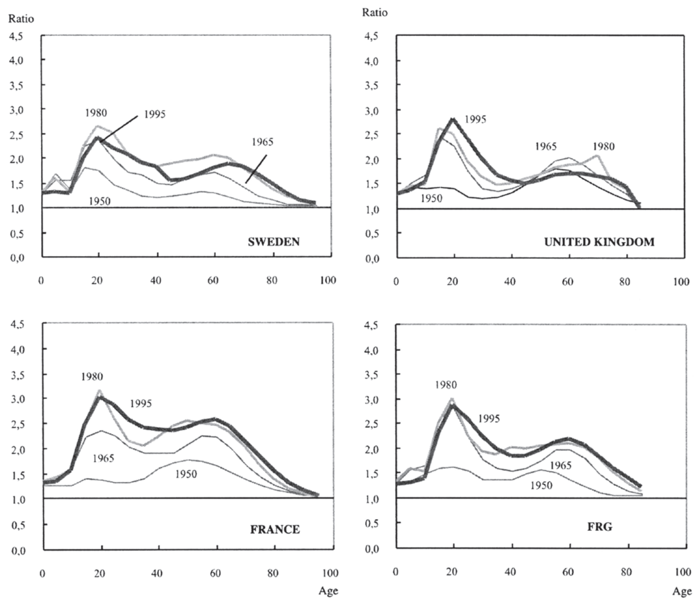
GRAPH 41. PROBABILITIES OF DEATH – AT83



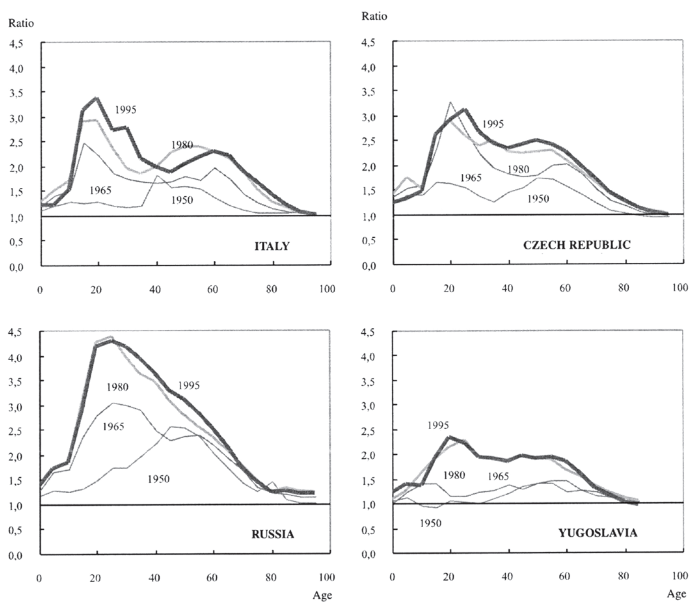
Moreover, despite the fact that life expectancy at birth for women is higher than for men, with older ages (90 and over), the mortality rates for both sexes tend to converge in most developed societies, for example France, Germany, United Kingdom, Russia, Czech Republic and Italy since 1950. As it occurs in the AT tables (see graph 42 and graph 43)[1].

1 Vallin, J., Meslé, F.; *Trends in mortality and differential mortality*, Population studies Nº 36, Council of Europe Publishing, ISBN 92-871-4725-6, December 2001 – www.coe.int/t/e/social_cohesion/population/N%C2%B036_Trends_in_mortality.pdf

GRAPH 42. AGE-SPECIFIC EXCESS MALE MORTALITY IN 1950, 1965, 1980 AND 1995 IN EIGHT SIGNIFICANT COUNTRIES – PART 1

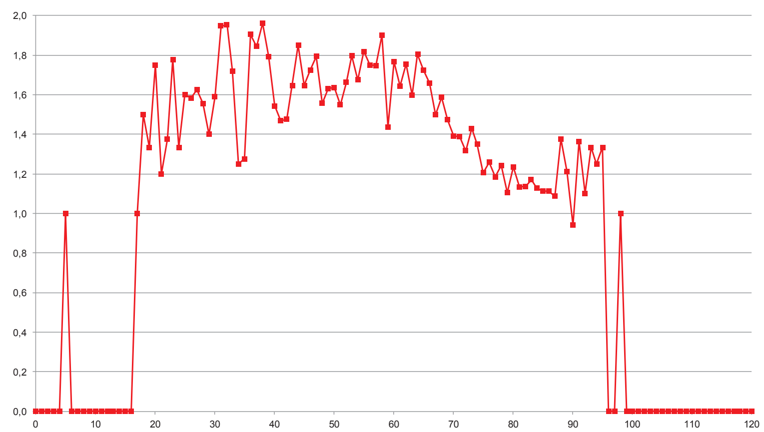


GRAPH 43. AGE-SPECIFIC EXCESS MALE MORTALITY IN 1950, 1965, 1980 AND 1995 IN EIGHT SIGNIFICANT COUNTRIES – PART 2



There is reason to believe that the countercheck of death registration using the CNIS (National Registry of Social Data) and SISOBI (National Death Registry System) data was less efficient for elderly women as opposed to younger women. Graph 44 shows the levels of female death corrections with mortality coverage, for all ages in 2004. It can be seen that the correction level is slightly lower for ages 70. This could possibly be due to tax legislation, which did not require women to have their own tax ID before 2005.

GRAPH 44. FEMALE DEATH CORRECTIONS WITH MORTALITY COVERAGE – ALL AGES IN 2004



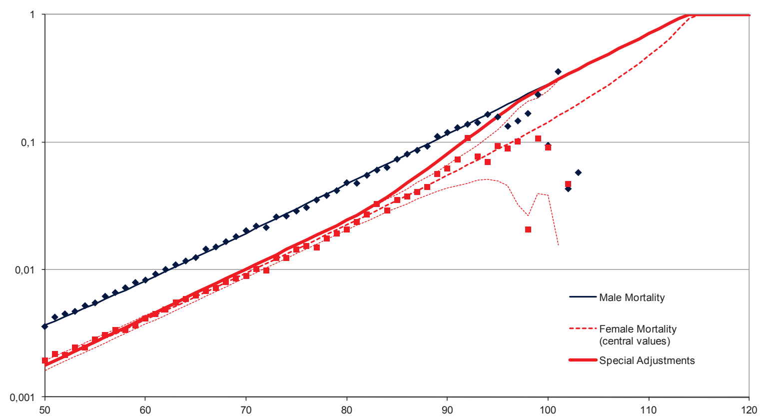
As mentioned previously, the number of deaths at a given x is a binomial random variable $B(N_x, q_x)$, where N_x, is the known number of individuals with age x and q_x is the unknown parameter. This q_x was estimated using the Heligman & Pollard model. This estimate comprised a 95% confidence interval. In theory, any value of q_x in this interval is statistically equivalent to any other.

It is well-known that gains in mortality are lower for women than for men. There are many possible causes for this fact to be found in the literature. For example, increase in female participation in the labor market and the stress associated with the double work shift, increase in the number of women smokers etc.

However, if one considers the central values of q_x (the Heligman & Pollard estimates) the female curve does not converge to the male curve for older ages. Therefore, accepting the convergence hypothesis, and since the data for older women was rather sparse, the higher interval bound for the female q_x was assumed for ages above 80.

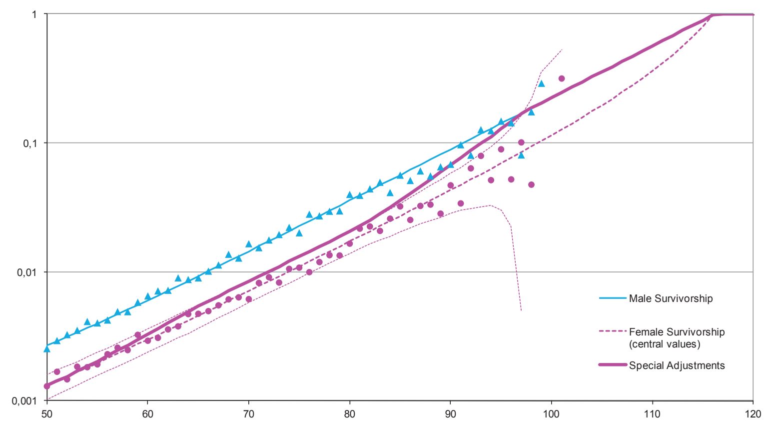
Graph 45 shows the crude and estimated values for q_x for male and female death coverage. The female component detaches from the central values starting at 60 years of age, following the upper limit of the 95% confidence interval, and meeting the male component at the age 100.

GRAPH 45. CRUDE AND ADJUSTED MORTALITY RATES WITH SPECIAL ADJUSTMENTS FOR OLDER WOMEN – BRAZILIAN INSURED POPULATION – DEATH COVERAGE ACCORDING TO SEX – 2004 TO 2006



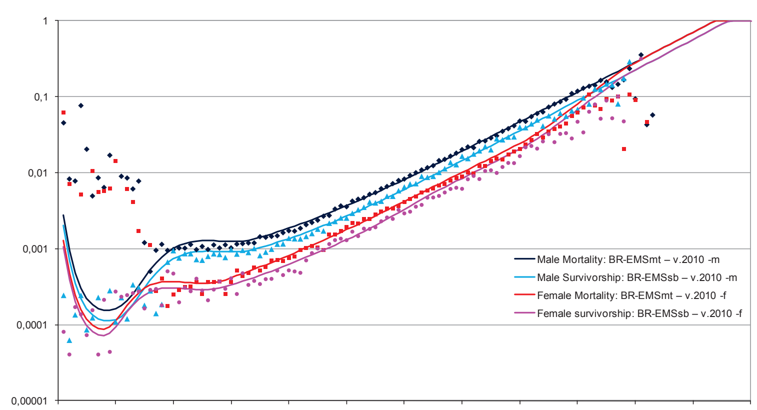
Graph 46 shows the crude and estimated values for q_x for male and female survivorship coverage. The female component detaches from the central values starting at 55 years of age, following the upper limit of the 95% confidence interval, and meeting the male component around the age of 100.

GRAPH 46. CRUDE AND ADJUSTED MORTALITY RATES WITH SPECIAL ADJUSTMENTS FOR OLDER WOMEN – BRAZILIAN INSURED POPULATION – SURVIVORSHIP COVERAGE ACCORDING TO SEX – 2004 TO 2006



Graph 47 shows the crude and estimated values for q_x for men and women and for both coverage types. Note that each dot corresponds to a given crude rate. These crude rates were calculated based on different numbers of insureds. In particular, for older ages, it's usual for these numbers to be rather small. As a consequence, confidence intervals tend to be very large.

GRAPH 47. CRUDE AND ADJUSTED MORTALITY RATES ACCORDING TO SEX AND COVERAGES WITH SPECIAL ADJUSTMENTS FOR OLDER WOMEN – 2004 TO 2006



7 | BR-EMS Tables

The adjusted life table, denominated "**Experiência do Mercado Segurador Brasileiro – BR-EMS**", presents variants classified according to coverage type and sex. The given names of these variants follow the usual life table nomenclature, coverage type – "sb" for survivorship and "mt" for mortality – and sex.

Although the life tables relate to the death experience for the years 2004 to 2006, centered on 2005, they were officially adopted as standard tables by SUSEP as the 2010 version of the BR-EMS, due to the year of its publication (SUSEP Circular nº 402, 18.03.2010 – D.O.U.: 19.03.2010).

Hence, we have the following variants of the **BR-EMS.**

- **BR-EMSsb-v.2010-m**
- **BR-EMSsb-v.2010-f**
- **BR-EMSmt-v.2010-m**
- **BR-EMSmt-v.2010-f**

Each variant comprises the following parameters:

- q_x – probability (or rate) of death between ages x and $x+1$;
- l_x – number of living at age x;
- e_x – life expectancy at age x;
- +IC (95%) – upper bound of 95% confidence interval and
- -IC (95%) – lower bound of 95% confidence interval.

Confidence intervals were obtained using the normal approximation to the binomial distribution. Hence negative values are not to be considered.

7.1 Male survivorship: BR-EMSsb-v.2010-m

Table 7 presents the "**BR-EMSsb-v.2010-m**" life table for males with survivorship coverage type, from ages 0 to 116. Extreme values of the 95% confidence intervals (IC) are given in red for the younger ages and in black otherwise.

TABLE 7. MALE SURVIVORSHIP: BR-EMSsb-v.2010-m					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
0	0,00200	0,00090	0,00210	1.000.000	81,9
1	0,00069	0,00030	0,00080	997.997	81,0
2	0,00035	0,00020	0,00060	997.308	80,1
3	0,00022	0,00010	0,00050	996.957	79,1
4	0,00016	0,00010	0,00050	996.735	78,1
5	0,00013	0,00010	0,00050	996.573	77,1
6	0,00012	0,00010	0,00040	996.440	76,1
7	0,00011	0,00010	0,00040	996.322	75,2
8	0,00011	0,00010	0,00040	996.210	74,2
9	0,00012	0,00010	0,00050	996.098	73,2
10	0,00013	0,00010	0,00050	995.981	72,2
11	0,00015	0,00010	0,00050	995.853	71,2
12	0,00019	0,00010	0,00060	995.702	70,2
13	0,00024	0,00020	0,00070	995.516	69,2
14	0,00031	0,00020	0,00080	995.279	68,2
15	0,00039	0,00020	0,00080	994.975	67,3
16	0,00048	0,00030	0,00090	994.591	66,3
17	0,00057	0,00030	0,00090	994.118	65,3
18	0,00066	0,00040	0,00090	993.554	64,3
19	0,00074	0,00040	0,00090	992.902	63,4
20	0,00080	0,00050	0,00090	992.171	62,4
21	0,00085	0,00050	0,00090	991.375	61,5
22	0,00089	0,00050	0,00080	990.527	60,5
23	0,00092	0,00050	0,00080	989.644	59,6
24	0,00093	0,00050	0,00080	988.738	58,6

TABLE 7. MALE SURVIVORSHIP: BR-EMSsb-v.2010-m					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
25	0,00093	0,00050	0,00080	987.822	57,7
26	0,00093	0,00050	0,00080	986.903	56,7
27	0,00092	0,00060	0,00080	985.988	55,8
28	0,00092	0,00060	0,00080	985.079	54,9
29	0,00091	0,00060	0,00090	984.176	53,9
30	0,00092	0,00060	0,00090	983.275	53,0
31	0,00093	0,00070	0,00090	982.374	52,0
32	0,00094	0,00070	0,00100	981.465	51,0
33	0,00099	0,00070	0,00100	980.541	50,1
34	0,00103	0,00080	0,00100	979.574	49,1
35	0,00109	0,00080	0,00110	978.561	48,2
36	0,00115	0,00080	0,00110	977.497	47,2
37	0,00121	0,00090	0,00120	976.377	46,3
38	0,00128	0,00090	0,00120	975.196	45,4
39	0,00136	0,00100	0,00130	973.948	44,4
40	0,00144	0,00110	0,00140	972.627	43,5
41	0,00153	0,00110	0,00140	971.225	42,5
42	0,00164	0,00120	0,00150	969.735	41,6
43	0,00175	0,00130	0,00160	968.149	40,7
44	0,00187	0,00140	0,00170	966.458	39,7
45	0,00200	0,00150	0,00190	964.651	38,8
46	0,00215	0,00160	0,00200	962.719	37,9
47	0,00231	0,00170	0,00220	960.648	37,0
48	0,00249	0,00190	0,00230	958.427	36,1
49	0,00268	0,00200	0,00250	956.042	35,1
50	0,00290	0,00220	0,00270	953.477	34,2
51	0,00313	0,00240	0,00300	950.715	33,3
52	0,00339	0,00260	0,00320	947.740	32,4
53	0,00367	0,00280	0,00350	944.531	31,5
54	0,00398	0,00310	0,00380	941.067	30,7
55	0,00431	0,00340	0,00420	937.326	29,8

TABLE 7. MALE SURVIVORSHIP: BR-EMSsb-v.2010-m					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
56	0,00468	0,00370	0,00460	933.283	28,9
57	0,00509	0,00400	0,00500	928.911	28,0
58	0,00554	0,00440	0,00550	924.182	27,2
59	0,00603	0,00480	0,00600	919.064	26,3
60	0,00656	0,00530	0,00660	913.524	25,5
61	0,00715	0,00580	0,00730	907.527	24,6
62	0,00780	0,00640	0,00800	901.035	23,8
63	0,00851	0,00700	0,00880	894.007	23,0
64	0,00929	0,00760	0,00970	886.400	22,2
65	0,01014	0,00840	0,01060	878.169	21,4
66	0,01107	0,00920	0,01160	869.266	20,6
67	0,01210	0,01010	0,01280	859.639	19,8
68	0,01323	0,01110	0,01400	849.237	19,1
69	0,01446	0,01220	0,01530	838.005	18,3
70	0,01581	0,01340	0,01680	825.887	17,6
71	0,01730	0,01460	0,01850	812.826	16,9
72	0,01893	0,01590	0,02030	798.763	16,2
73	0,02072	0,01740	0,02220	783.643	15,5
74	0,02268	0,01910	0,02420	767.408	14,8
75	0,02483	0,02080	0,02650	750.005	14,1
76	0,02719	0,02260	0,02890	731.384	13,4
77	0,02977	0,02460	0,03150	711.500	12,8
78	0,03261	0,02670	0,03430	690.315	12,2
79	0,03573	0,02890	0,03730	667.802	11,6
80	0,03914	0,03110	0,04060	643.944	11,0
81	0,04289	0,03330	0,04430	618.740	10,4
82	0,04700	0,03530	0,04840	592.204	9,9
83	0,05150	0,03750	0,05270	564.373	9,3
84	0,05645	0,03970	0,05720	535.306	8,8
85	0,06187	0,04180	0,06200	505.090	8,3
86	0,06782	0,04380	0,06730	473.841	7,8

TABLE 7. MALE SURVIVORSHIP: BR-EMSsb-v.2010-m					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
87	0,07434	0,04540	0,07310	441.706	7,4
88	0,08150	0,04740	0,07870	408.869	6,9
89	0,08935	0,04840	0,08540	375.547	6,5
90	0,09796	0,04910	0,09260	341.992	6,1
91	0,10741	0,04910	0,10040	308.490	5,7
92	0,11777	0,04880	0,10860	275.357	5,3
93	0,12913	0,04700	0,11820	242.929	4,9
94	0,14160	0,03980	0,13320	211.559	4,6
95	0,15527	0,02670	0,15390	181.603	4,2
96	0,17027	0,00470	0,18350	153.405	3,9
97	0,18672		0,21710	127.285	3,6
98	0,20477		0,25560	103.518	3,4
99	0,22457		0,33780	82.320	3,1
100	0,24628		0,30070	63.834	2,8
101	0,27010		0,25860	48.113	2,6
102	0,29622		0,30290	35.118	2,4
103	0,32488		0,41370	24.715	2,2
104	0,35632		0,45540	16.686	2,0
105	0,39080		0,49240	10.740	1,8
106	0,42862		0,51440	6.543	1,6
107	0,47011			3.738	1,5
108	0,51562			1.981	1,3
109	0,56553			960	1,2
110	0,62029			417	1,0
111	0,68035			158	0,9
112	0,74623			51	0,8
113	0,81849			13	0,7
114	0,89776			2	0,6
115	0,98471			–	0,5
116	1,00000			–	0,5

7.2 Male mortality: BR-EMSmt-v.2010-m

Table 8 presents the "**BR-EMSmt-v.2010-m**" life table for males with mortality coverage, from ages 0 to 113. Extreme values of the 95% confidence intervals (IC) are given in red for the younger ages and in black otherwise.

TABLE 8. MALE MORTALITY: BR-EMSmt-v.2010-m					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
0	0,00274	–	0,00518	1.000.000	78,4
1	0,00095	–	0,00259	997.256	77,6
2	0,00048	–	0,00240	996.311	76,7
3	0,00030	–	0,00231	995.831	75,7
4	0,00022	–	0,00218	995.528	74,7
5	0,00018	–	0,00232	995.306	73,8
6	0,00016	–	0,00259	995.124	72,8
7	0,00015	–	0,00244	994.962	71,8
8	0,00015	–	0,00251	994.808	70,8
9	0,00016	–	0,00268	994.655	69,8
10	0,00018	–	0,00308	994.495	68,8
11	0,00021	–	0,00324	994.319	67,8
12	0,00026	–	0,00304	994.113	66,8
13	0,00033	–	0,00273	993.858	65,9
14	0,00042	–	0,00197	993.534	64,9
15	0,00053	–	0,00149	993.118	63,9
16	0,00065	0,00041	0,00122	992.593	62,9
17	0,00078	0,00053	0,00121	991.946	62,0
18	0,00090	0,00076	0,00106	991.174	61,0
19	0,00101	0,00083	0,00105	990.282	60,1
20	0,00110	0,00086	0,00105	989.283	59,1
21	0,00117	0,00087	0,00106	988.195	58,2
22	0,00122	0,00088	0,00105	987.037	57,3
23	0,00125	0,00087	0,00104	985.830	56,3

TABLE 8. MALE MORTALITY: BR-EMSmt-v.2010-m					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
24	0,00127	0,00086	0,00103	984.594	55,4
25	0,00127	0,00085	0,00102	983.343	54,5
26	0,00127	0,00087	0,00104	982.090	53,6
27	0,00126	0,00089	0,00106	980.842	52,6
28	0,00126	0,00091	0,00109	979.602	51,7
29	0,00125	0,00094	0,00112	978.371	50,8
30	0,00126	0,00097	0,00115	977.144	49,8
31	0,00127	0,00100	0,00118	975.916	48,9
32	0,00129	0,00103	0,00122	974.678	47,9
33	0,00135	0,00107	0,00126	973.421	47,0
34	0,00142	0,00111	0,00131	972.105	46,1
35	0,00149	0,00116	0,00136	970.726	45,1
36	0,00157	0,00122	0,00142	969.277	44,2
37	0,00166	0,00127	0,00149	967.753	43,3
38	0,00176	0,00134	0,00156	966.145	42,3
39	0,00186	0,00141	0,00164	964.447	41,4
40	0,00198	0,00150	0,00172	962.649	40,5
41	0,00211	0,00159	0,00182	960.743	39,6
42	0,00225	0,00169	0,00193	958.720	38,6
43	0,00240	0,00180	0,00206	956.567	37,7
44	0,00256	0,00192	0,00219	954.273	36,8
45	0,00275	0,00206	0,00234	951.826	35,9
46	0,00295	0,00221	0,00251	949.212	35,0
47	0,00317	0,00238	0,00270	946.414	34,1
48	0,00341	0,00257	0,00290	943.418	33,2
49	0,00367	0,00278	0,00313	940.206	32,3
50	0,00396	0,00301	0,00339	936.757	31,5
51	0,00427	0,00326	0,00367	933.052	30,6
52	0,00462	0,00355	0,00398	929.067	29,7
53	0,00499	0,00387	0,00433	924.779	28,8

TABLE 8. MALE MORTALITY: BR-EMSmt-v.2010-m					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
54	0,00541	0,00422	0,00471	920.162	28,0
55	0,00586	0,00461	0,00514	915.188	27,1
56	0,00635	0,00504	0,00562	909.826	26,3
57	0,00690	0,00551	0,00615	904.044	25,5
58	0,00749	0,00604	0,00673	897.808	24,6
59	0,00814	0,00663	0,00738	891.081	23,8
60	0,00886	0,00728	0,00810	883.824	23,0
61	0,00964	0,00800	0,00889	875.996	22,2
62	0,01049	0,00880	0,00976	867.553	21,4
63	0,01143	0,00968	0,01073	858.449	20,6
64	0,01246	0,01066	0,01180	848.637	19,9
65	0,01358	0,01175	0,01298	838.067	19,1
66	0,01481	0,01295	0,01427	826.687	18,4
67	0,01616	0,01426	0,01572	814.444	17,6
68	0,01763	0,01570	0,01731	801.286	16,9
69	0,01925	0,01729	0,01907	787.158	16,2
70	0,02102	0,01902	0,02101	772.008	15,5
71	0,02295	0,02091	0,02317	755.783	14,8
72	0,02508	0,02299	0,02553	738.436	14,2
73	0,02740	0,02530	0,02808	719.919	13,5
74	0,02994	0,02783	0,03087	700.194	12,9
75	0,03273	0,03060	0,03393	679.228	12,3
76	0,03578	0,03359	0,03729	656.997	11,7
77	0,03912	0,03681	0,04098	633.489	11,1
78	0,04278	0,04032	0,04500	608.706	10,5
79	0,04679	0,04410	0,04936	582.664	10,0
80	0,05118	0,04817	0,05411	555.401	9,4
81	0,05598	0,05255	0,05924	526.977	8,9
82	0,06125	0,05720	0,06483	497.474	8,4
83	0,06701	0,06212	0,07088	467.005	7,9

TABLE 8. MALE MORTALITY: BR-EMSmt-v.2010-m					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
84	0,07332	0,06730	0,07744	435.710	7,5
85	0,08024	0,07256	0,08468	403.763	7,0
86	0,08781	0,07800	0,09252	371.366	6,6
87	0,09609	0,08370	0,10084	338.759	6,2
88	0,10517	0,08952	0,10980	306.206	5,8
89	0,11511	0,09486	0,11995	274.001	5,4
90	0,12600	0,09990	0,13107	242.460	5,0
91	0,13792	0,10521	0,14255	211.910	4,7
92	0,15098	0,11004	0,15506	182.682	4,4
93	0,16528	0,11254	0,17039	155.101	4,0
94	0,18093	0,11254	0,18863	129.467	3,7
95	0,19808	0,11246	0,20725	106.042	3,5
96	0,21686	0,11059	0,22788	85.037	3,2
97	0,23742	0,10511	0,25223	66.596	2,9
98	0,25994	0,10061	0,27561	50.784	2,7
99	0,28460	0,08615	0,30886	37.583	2,5
100	0,31161	0,05509	0,35850	26.887	2,3
101	0,34118			18.509	2,1
102	0,37357			12.194	1,9
103	0,40904			7.639	1,7
104	0,44788			4.514	1,5
105	0,49042			2.492	1,4
106	0,53700			1.270	1,2
107	0,58801			588	1,1
108	0,64387			242	1,0
109	0,70505			86	0,9
110	0,77204			25	0,8
111	0,84540			6	0,7
112	0,92575			1	0,6
113	1,00000			–	0,5

7.3 Female survivorship: BR-EMSsb-v.2010-f

Table 9 presents the "**BR-EMSsb-v.2010-f**" life table for females with survivorship coverage type, from ages 0 to 116. Extreme values of the 95% confidence intervals (IC) are given in red for the younger ages and in black otherwise.

TABLE 9. FEMALE SURVIVORSHIP: BR-EMSsb-v.2010-f

Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
0	0,00038	0,00060	0,00160	1.000.000	87,2
1	0,00038	0,00020	0,00060	999.618	86,3
2	0,00020	0,00010	0,00050	999.236	85,3
3	0,00013	0,00010	0,00040	999.034	84,3
4	0,00010	0,00010	0,00040	998.903	83,3
5	0,00008	–	0,00040	998.805	82,3
6	0,00007	–	0,00030	998.725	81,3
7	0,00007	–	0,00040	998.652	80,4
8	0,00008	–	0,00040	998.581	79,4
9	0,00009	0,00010	0,00040	998.503	78,4
10	0,00012	0,00010	0,00050	998.411	77,4
11	0,00015	0,00010	0,00060	998.295	76,4
12	0,00018	0,00010	0,00060	998.149	75,4
13	0,00022	0,00010	0,00060	997.968	74,4
14	0,00025	0,00010	0,00060	997.753	73,4
15	0,00027	0,00010	0,00060	997.506	72,4
16	0,00029	0,00010	0,00060	997.235	71,5
17	0,00030	0,00010	0,00050	996.946	70,5
18	0,00031	0,00010	0,00050	996.646	69,5
19	0,00030	0,00010	0,00040	996.342	68,5
20	0,00030	0,00010	0,00040	996.038	67,5
21	0,00030	0,00010	0,00030	995.737	66,6
22	0,00029	0,00010	0,00030	995.441	65,6
23	0,00029	0,00010	0,00030	995.150	64,6
24	0,00029	0,00010	0,00030	994.861	63,6

TABLE 9. FEMALE SURVIVORSHIP: BR-EMSsb-v.2010-f					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
25	0,00029	0,00010	0,00030	994.575	62,6
26	0,00029	0,00010	0,00030	994.286	61,7
27	0,00030	0,00010	0,00030	993.994	60,7
28	0,00032	0,00010	0,00030	993.694	59,7
29	0,00033	0,00010	0,00030	993.379	58,7
30	0,00035	0,00010	0,00030	993.047	57,7
31	0,00037	0,00020	0,00030	992.696	56,8
32	0,00040	0,00020	0,00030	992.326	55,8
33	0,00042	0,00020	0,00040	991.934	54,8
34	0,00045	0,00020	0,00040	991.518	53,8
35	0,00047	0,00020	0,00040	991.076	52,8
36	0,00051	0,00020	0,00050	990.605	51,9
37	0,00054	0,00030	0,00050	990.104	50,9
38	0,00058	0,00030	0,00050	989.568	49,9
39	0,00062	0,00030	0,00060	988.996	48,9
40	0,00066	0,00040	0,00060	988.383	48,0
41	0,00071	0,00040	0,00070	987.726	47,0
42	0,00077	0,00040	0,00070	987.022	46,0
43	0,00083	0,00050	0,00080	986.264	45,1
44	0,00089	0,00050	0,00080	985.450	44,1
45	0,00096	0,00060	0,00090	984.573	43,2
46	0,00104	0,00060	0,00100	983.628	42,2
47	0,00112	0,00070	0,00100	982.608	41,2
48	0,00121	0,00080	0,00110	981.508	40,3
49	0,00131	0,00080	0,00120	980.319	39,3
50	0,00142	0,00090	0,00130	979.034	38,4
51	0,00155	0,00100	0,00150	977.642	37,4
52	0,00169	0,00110	0,00160	976.129	36,5
53	0,00185	0,00120	0,00170	974.481	35,6
54	0,00203	0,00130	0,00190	972.680	34,6
55	0,00223	0,00140	0,00210	970.709	33,7

TABLE 9. FEMALE SURVIVORSHIP: BR-EMSsb-v.2010-f					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
56	0,00245	0,00160	0,00230	968.546	32,8
57	0,00271	0,00170	0,00250	966.170	31,8
58	0,00299	0,00190	0,00270	963.556	30,9
59	0,00330	0,00200	0,00300	960.678	30,0
60	0,00365	0,00220	0,00320	957.507	29,1
61	0,00403	0,00240	0,00360	954.014	28,2
62	0,00445	0,00270	0,00390	950.171	27,3
63	0,00491	0,00290	0,00430	945.945	26,5
64	0,00541	0,00320	0,00470	941.301	25,6
65	0,00593	0,00350	0,00510	936.211	24,7
66	0,00648	0,00380	0,00560	930.663	23,9
67	0,00710	0,00420	0,00610	924.628	23,0
68	0,00775	0,00460	0,00670	918.065	22,2
69	0,00843	0,00510	0,00730	910.951	21,3
70	0,00919	0,00560	0,00800	903.267	20,5
71	0,01006	0,00610	0,00880	894.963	19,7
72	0,01102	0,00660	0,00970	885.956	18,9
73	0,01204	0,00730	0,01060	876.193	18,1
74	0,01313	0,00800	0,01160	865.642	17,3
75	0,01433	0,00880	0,01270	854.278	16,5
76	0,01566	0,00970	0,01400	842.035	15,8
77	0,01714	0,01070	0,01540	828.849	15,0
78	0,01876	0,01170	0,01690	814.642	14,3
79	0,02055	0,01290	0,01860	799.358	13,5
80	0,02264	0,01410	0,02060	782.932	12,8
81	0,02516	0,01530	0,02280	765.203	12,1
82	0,02817	0,01660	0,02540	745.952	11,4
83	0,03176	0,01800	0,02840	724.942	10,7
84	0,03577	0,01960	0,03160	701.918	10,0
85	0,04042	0,02130	0,03520	676.813	9,4
86	0,04582	0,02320	0,03930	649.454	8,8

TABLE 9. FEMALE SURVIVORSHIP: BR-EMSsb-v.2010-f					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
87	0,05219	0,02490	0,04420	619.698	8,2
88	0,05928	0,02700	0,04960	587.353	7,6
89	0,06734	0,02930	0,05570	552.533	7,0
90	0,07651	0,03160	0,06290	515.324	6,5
91	0,08727	0,03380	0,07140	475.896	6,0
92	0,09906	0,03600	0,08130	434.365	5,5
93	0,11227	0,04250	0,08870	391.338	5,1
94	0,12800	0,03890	0,10810	347.401	4,7
95	0,14641	0,03240	0,13280	302.934	4,3
96	0,16835	0,02030	0,16610	258.582	3,9
97	0,18672		0,21530	215.049	3,6
98	0,20477		0,29470	174.894	3,4
99	0,22457		0,32950	139.081	3,1
100	0,24628		0,35270	107.848	2,8
101	0,27010			81.287	2,6
102	0,29622			59.332	2,4
103	0,32488			41.756	2,2
104	0,35632			28.190	2,0
105	0,39080			18.146	1,8
106	0,42862			11.054	1,6
107	0,47011			6.316	1,5
108	0,51562			3.347	1,3
109	0,56553			1.621	1,2
110	0,62029			704	1,0
111	0,68035			267	0,9
112	0,74623			85	0,8
113	0,81849			22	0,7
114	0,89776			4	0,6
115	0,98471			–	0,5
116	1,00000			–	0,5

7.4 Female mortality: BR-EMSmt-v.2010-f

Table 10 presents the "**BR-EMSmt-v.2010-f**" life table for females with mortality coverage, from ages 0 to 113. Extreme values of the 95% confidence intervals (IC) are given in red for the younger ages and in black otherwise.

TABLE 10. FEMALE MORTALITY: BR-EMSmt-v.2010-f

Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
0	0,00128		0,00850	1.000.000	85,2
1	0,00046		0,00430	998.716	84,3
2	0,00025		0,00370	998.252	83,4
3	0,00016		0,00390	998.007	82,4
4	0,00012		0,00360	997.847	81,4
5	0,00010		0,00360	997.728	80,4
6	0,00009		0,00360	997.630	79,4
7	0,00009		0,00370	997.542	78,4
8	0,00009		0,00420	997.455	77,4
9	0,00011		0,00470	997.361	76,4
10	0,00014		0,00490	997.249	75,4
11	0,00018		0,00490	997.108	74,4
12	0,00022		0,00410	996.930	73,5
13	0,00026		0,00300	996.711	72,5
14	0,00030		0,00250	996.449	71,5
15	0,00033		0,00170	996.150	70,5
16	0,00035	0,00020	0,00120	995.820	69,5
17	0,00037	0,00030	0,00100	995.469	68,6
18	0,00037	0,00050	0,00070	995.105	67,6
19	0,00037	0,00050	0,00060	994.735	66,6
20	0,00037	0,00040	0,00060	994.366	65,6
21	0,00036	0,00040	0,00050	994.001	64,7
22	0,00036	0,00040	0,00050	993.642	63,7
23	0,00035	0,00040	0,00050	993.288	62,7
24	0,00035	0,00040	0,00050	992.938	61,7

TABLE 10. FEMALE MORTALITY: BR-EMSmt-v.2010-f					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
25	0,00035	0,00040	0,00050	992.589	60,7
26	0,00036	0,00040	0,00050	992.239	59,8
27	0,00037	0,00040	0,00050	991.884	58,8
28	0,00039	0,00040	0,00050	991.520	57,8
29	0,00041	0,00040	0,00050	991.134	56,8
30	0,00044	0,00040	0,00060	990.723	55,9
31	0,00047	0,00040	0,00060	990.286	54,9
32	0,00050	0,00050	0,00060	989.820	53,9
33	0,00054	0,00050	0,00060	989.323	52,9
34	0,00057	0,00050	0,00070	988.792	52,0
35	0,00062	0,00050	0,00070	988.223	51,0
36	0,00066	0,00060	0,00070	987.615	50,0
37	0,00071	0,00060	0,00080	986.962	49,1
38	0,00076	0,00070	0,00080	986.261	48,1
39	0,00082	0,00070	0,00090	985.509	47,1
40	0,00088	0,00080	0,00090	984.700	46,2
41	0,00095	0,00080	0,00100	983.830	45,2
42	0,00103	0,00090	0,00110	982.892	44,2
43	0,00111	0,00090	0,00120	981.882	43,3
44	0,00120	0,00100	0,00120	980.793	42,3
45	0,00130	0,00110	0,00130	979.618	41,4
46	0,00140	0,00120	0,00140	978.349	40,4
47	0,00152	0,00130	0,00150	976.979	39,5
48	0,00164	0,00140	0,00170	975.498	38,6
49	0,00178	0,00150	0,00180	973.897	37,6
50	0,00193	0,00160	0,00190	972.166	36,7
51	0,00209	0,00180	0,00210	970.292	35,8
52	0,00228	0,00190	0,00230	968.261	34,8
53	0,00248	0,00210	0,00250	966.058	33,9
54	0,00270	0,00230	0,00270	963.665	33,0

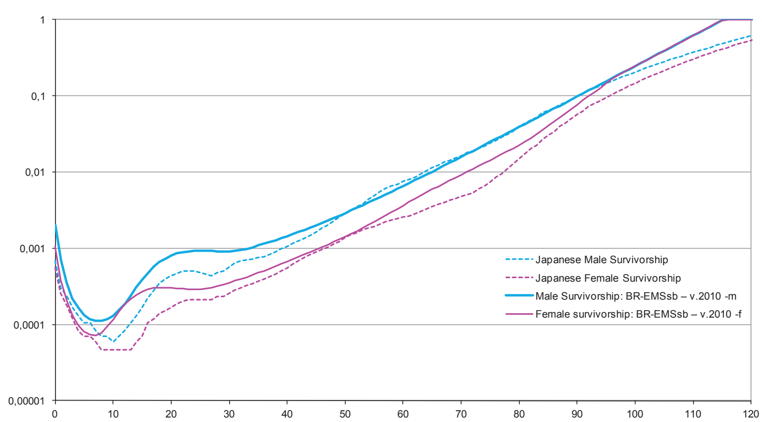
TABLE 10. FEMALE MORTALITY: BR-EMSmt-v.2010-f					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
55	0,00294	0,00250	0,00290	961.064	32,1
56	0,00321	0,00270	0,00320	958.234	31,2
57	0,00351	0,00290	0,00340	955.155	30,3
58	0,00384	0,00320	0,00370	951.802	29,4
59	0,00420	0,00350	0,00410	948.149	28,5
60	0,00459	0,00380	0,00440	944.171	27,6
61	0,00501	0,00420	0,00480	939.841	26,7
62	0,00548	0,00460	0,00530	935.130	25,9
63	0,00599	0,00500	0,00580	930.008	25,0
64	0,00654	0,00540	0,00630	924.438	24,2
65	0,00714	0,00600	0,00690	918.389	23,3
66	0,00778	0,00650	0,00750	911.836	22,5
67	0,00850	0,00720	0,00830	904.742	21,6
68	0,00927	0,00780	0,00900	897.055	20,8
69	0,01009	0,00860	0,00990	888.740	20,0
70	0,01100	0,00940	0,01090	879.768	19,2
71	0,01202	0,01030	0,01190	870.090	18,4
72	0,01312	0,01130	0,01310	859.633	17,6
73	0,01430	0,01240	0,01440	848.354	16,9
74	0,01558	0,01360	0,01580	836.227	16,1
75	0,01699	0,01500	0,01740	823.196	15,4
76	0,01856	0,01650	0,01910	809.208	14,6
77	0,02030	0,01810	0,02110	794.191	13,9
78	0,02221	0,01990	0,02330	778.066	13,2
79	0,02431	0,02180	0,02580	760.789	12,4
80	0,02674	0,02400	0,02850	742.296	11,7
81	0,02962	0,02640	0,03160	722.449	11,0
82	0,03307	0,02890	0,03520	701.050	10,4
83	0,03711	0,03180	0,03920	677.863	9,7
84	0,04185	0,03500	0,04370	652.705	9,1

TABLE 10. FEMALE MORTALITY: BR-EMSmt-v.2010-f					
Age	q_x	-IC (95%)	+IC (95%)	l_x	e_x
85	0,04749	0,03830	0,04900	625.387	8,4
86	0,05413	0,04190	0,05510	595.689	7,8
87	0,06170	0,04600	0,06200	563.447	7,2
88	0,07040	0,05070	0,06980	528.684	6,7
89	0,08096	0,05500	0,07970	491.464	6,2
90	0,09310	0,05980	0,09120	451.677	5,7
91	0,10647	0,06520	0,10440	409.624	5,2
92	0,12110	0,07170	0,11970	366.010	4,7
93	0,13857	0,07670	0,13990	321.687	4,3
94	0,15795	0,08120	0,16510	277.111	3,9
95	0,17998	0,08490	0,19670	233.342	3,6
96	0,20594	0,07780	0,24630	191.345	3,3
97	0,23015	0,08330	0,29240	151.939	3,0
98	0,25194	0,11390	0,32580	116.969	2,7
99	0,27912	0,12730	0,39340	87.500	2,5
100	0,31072	0,12880	0,49710	63.077	2,3
101	0,34118			43.478	2,1
102	0,37357			28.644	1,9
103	0,40904			17.943	1,7
104	0,44788			10.604	1,5
105	0,49042			5.855	1,4
106	0,53700			2.983	1,2
107	0,58801			1.381	1,1
108	0,64387			569	1,0
109	0,70505			203	0,9
110	0,77204			60	0,8
111	0,84540			14	0,7
112	0,92575			2	0,6
113	1,00000			–	0,5

7.5 Comparison of BR-EMS with well-known tables

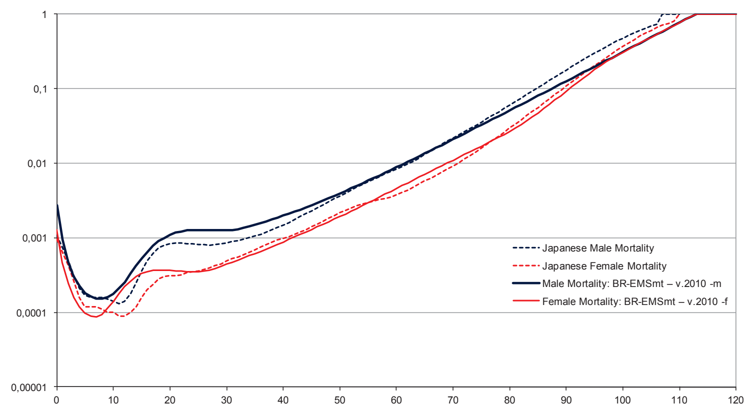
In this section, BR-EMS tables are compared with other well-known tables used by actuaries and with a particular table constructed for Japanese insurance companies for the year 2007, the SMT2007 table. The population considered for these Japanese tables were insureds from 32 Japanese insurance companies for the 3 year period 1999/2000/2001, classified according to sex and coverage type (Yamakawa, 2007). Therefore, the scope of this Japanese work is very similar to the BR-EMS table construction, since both are standard tables to be adopted by all insurance companies and are classified according to coverage type. Note that there is a 5 year difference between the Japanese and Brazilian data. For comparison purposes, the Japanese tables are presented in raw data form. Comparing these survivorship tables with the BR-EMS one notes that for males, Japanese mortality is lower than Brazilian mortality, except for ages between 50 and 90 where they almost coincide. For female survivorship, Japanese mortality is lower for all ages (see graph 48).

GRAPH 48. BR-EMS AND JAPANESE SMT2007 PROBABILITIES OF DEATH FOR THE SURVIVORSHIP COVERAGE – ACCORDING TO SEX



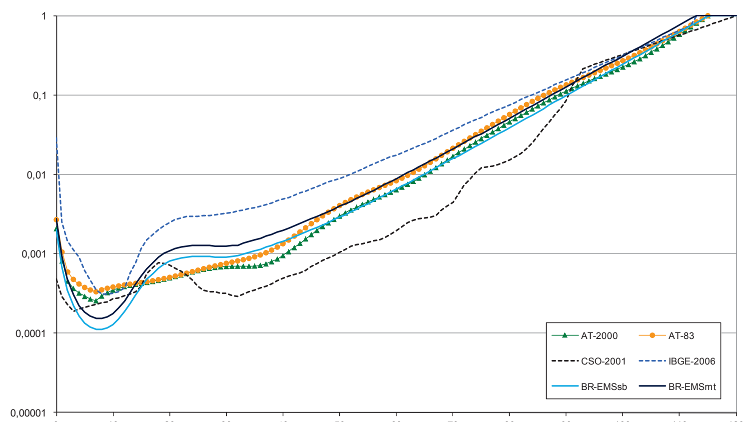
In contrast, the subpopulation with death coverage tables for both countries appear similar. For older ages, death rates in these tables are slightly higher in Japan than in Brazil (see graph 49).

GRAPH 49. BR-EMS AND JAPANESE SMT2007 PROBABILITIES OF DEATH FOR THE DEATH COVERAGE – ACCORDING TO SEX



Graph 50 presents a comparison of the BR-EMS tables for death and survivorship coverages for males with several other well-known male tables. The comparison of the survivorship BR-EMS table with the AT2000 shows that the male curve is very close from age 50 onwards. As far as death coverage is concerned, the BR-EMS male curve is parallel, if slightly above the AT2000. For ages between 20 and 90 years, the Brazilian survivorship table is much higher than the CSO-2001. From there onwards, it falls below until the age of 105. It is worth noting that the Brazilian mortality curve is close to the AT83 for most relevant ages.

GRAPH 50. MALE PROBABILITIES OF DEATH – WELL-KNOWN TABLES AND BR-EMS

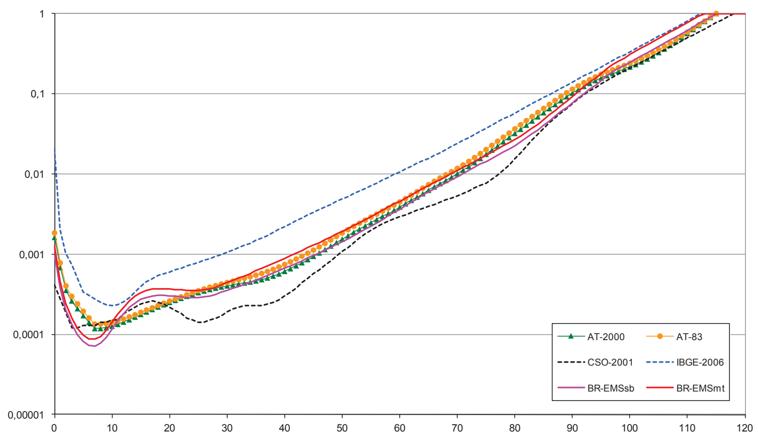


Graph 51 presents a comparison of the BR-EMS tables for death and survivorship coverages for females with several well-known female tables.

Notice that the survivorship curve for women is above the CSO-2001 for all ages. For ages between 86 and 94 years the curves are very close to each other.

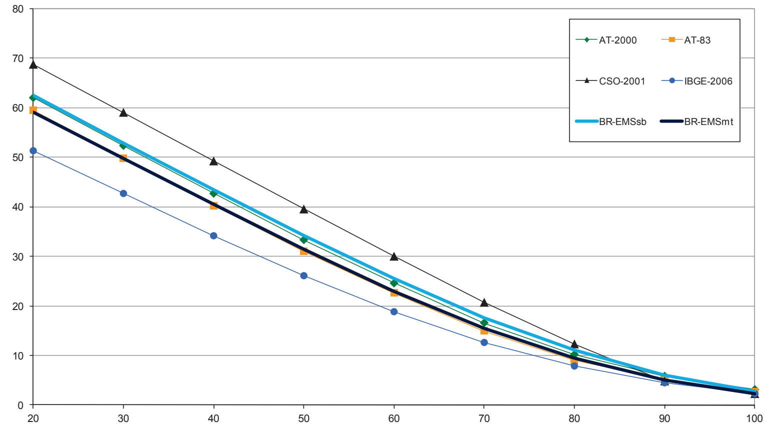
For women, the BR-EMS survivorship curve is below the AT2000 from 75 years on. It is well-known that the AT2000 was built from the AT83, applying *Scale G* for both sexes, but in such a way that the women death improvement is half that of men. As a consequence both curves meet at age 91, and from there on the curves coincide. For ages between 34 and 97 years, the Brazilian female curves are quite close to the AT2000.

GRAPH 51. FEMALE PROBABILITIES OF DEATH – WELL-KNOWN TABLES AND BR-EMS



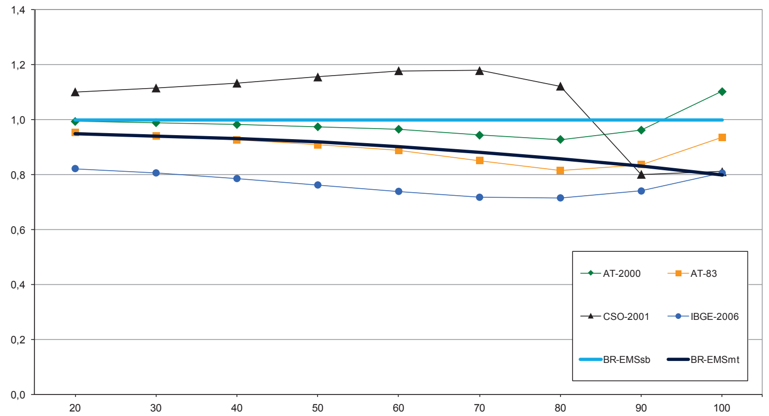
Graph 52 presents life expectancies for males from ages 20 to 100 for several well-known tables as well as the BR-EMS tables. Note that the AT2000 life expectancies lie between the BR-EMS survivorship and death coverage, being slightly closer to the survivorship branch for younger ages, while for older ages, the AT2000 approaches the BR-EMS death branch. As one would expect, the IBGE 2006 table is the lower limit for expected survivorship for all ages. In general, the CSO2001 table presents an upper limit on life expectancies for all ages, except for the very old.

GRAPH 52. MALE LIFE EXPECTANCIES – WELL-KNOWN TABLES AND BR-EMS



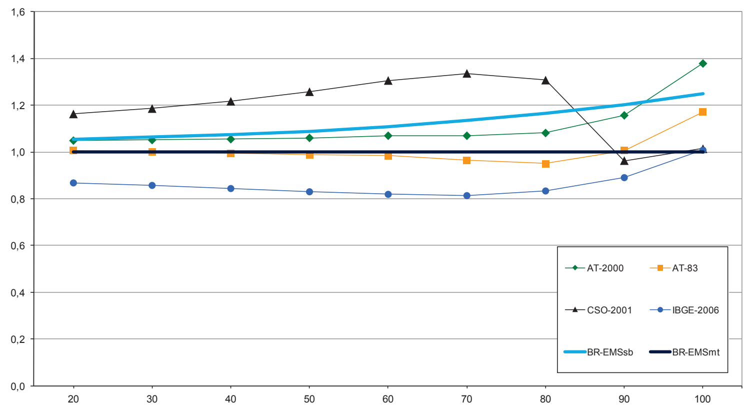
Graph 53 presents the ratio between the expected survival of well-known tables and the BR-EMS male survivorship, from ages 20 to 100. Here, as seen previously, the CSO2001 table is the upper limit for most ages except the very old, while IBGE2006 is the lower limit. The BR-EMS death coverage branch is very close to the AT83.

GRAPH 53. RATIO OF EXPECTED SURVIVORSHIP OF WELL-KNOWN TABLES TO BR-EMSsb – MALE



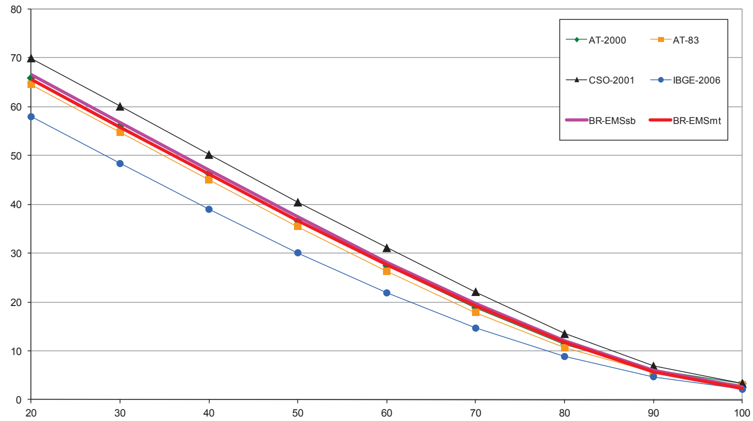
Graph 54 presents the ratio between the expected survival of well-known tables and the BR-EMS male death coverage table, from ages 20 to 100. Once again, one finds this curve to be very similar to the AT83.

GRAPH 54. RATIO OF EXPECTED SURVIVORSHIP OF WELL-KNOWN TABLES TO BR-EMSmt – MALE



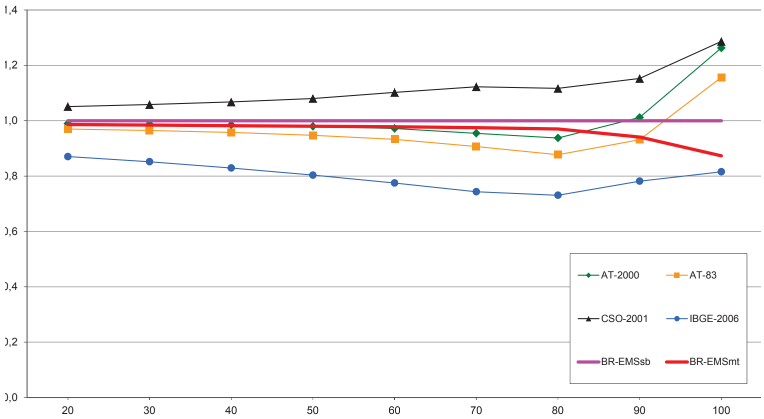
Graph 55 presents the ratio between the expected survival of well-known tables and the BR-EMS female survivorship, from ages 20 to 100. Here the BR-EMS lies below the CSO2001 curve with a constant distance of nearly 3 years. The upper limit is the CSO2001. The BR-EMS is very close to the AT2000.

GRAPH 55. FEMALE LIFE EXPECTANCIES – WELL-KNOWN TABLES AND BR-EMS



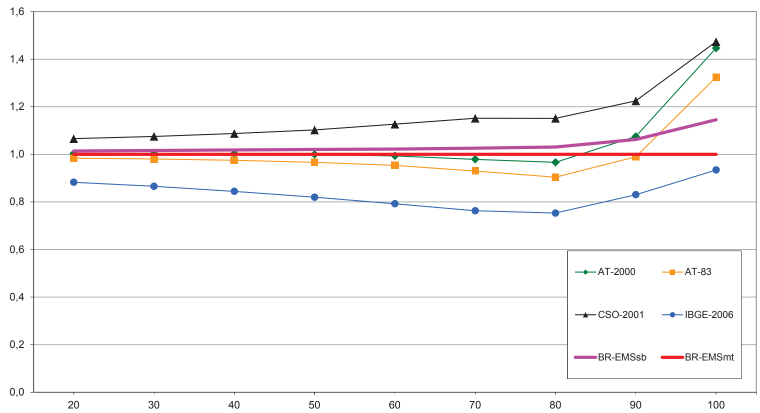
In a similar manner to graph 53, graph 56 presents the ratio between the expected survivorship of selected tables and the BR-EMS table for female survivorship coverage. Here, again, the BR-EMS is slightly lower than the CSO2001, around 8% at 30 years of age, with this percentage difference increasing with age, reaching nearly 15% at 80.

GRAPH 56. RATIO OF EXPECTED SURVIVORSHIP OF WELL-KNOWN TABLES TO BR-EMSsb – FEMALE



Graph 57 presents the ratio on expected survivorship of selected tables to the BR-EMS female death table. Here, also, the BR-EMS table lies very close to the AT2000.

GRAPH 57. RATIO OF EXPECTED SURVIVORSHIP OF WELL-KNOWN TABLES TO BR-EMSmt – FEMALE



7.6 Examples of the use of BR-EMS Tables

Consider an annual interest rate i and a discount factor v = 1/(1+i) and let the usual commutations for age *x*

$$C_x = d_x v^{x+1} \qquad M_x = \sum_{y \geq x} C_y$$

$$D_x = l_x v^x \qquad N_x = \sum_{y \geq x} D_y$$

For example, a 40-year-old woman wishes to acquire a whole life policy of value one, with indemnity paid at the end of the year of death. In this case, the variant to be used will be **BR-EMSmt-v.2010-f** (see Table 11 for selected ages). The pure premium for an anticipated single payment, using a yearly 6% interest rate, is given by:

$$\frac{M_{40}}{D_{40}} = \frac{8627,037}{95734,702} = 9,0114\ \%,$$

while the annual level premium for the whole life of the insured would be

$$\frac{M_{40}}{N_{40}} = \frac{8627,037}{1538902,083} = 0,56060\ \%$$

TABLE 11. COMMUTATIONS FOR ANNUAL i = 6% AND BR-EMSmt-v.2010-f BRANCH (SELECTED AGES)

Age x	q_x	l_x	e_x	C_x	M_x	D_x	N_x
39	0,00082	985.509	47,1	78,567	8.706,403	101.562,14	1.640.464,2
40	0,00088	984.700	46,2	79,478	8.627,836	95.734,69	1.538.902,1
41	0,00095	983.830	45,2	80,872	8.548,358	90.235,95	1.443.167,4

On the other hand, consider a 30-year-old man who wants to buy a unitary annuity, to be paid starting at age 55. Here the **BR-EMSsb-v.2010-m** branch will be used (see table 12 for selected ages). Considering i = 6%, his annual level pure premium between ages 30 and 54 is given by

$$\frac{N_{55}}{N_{30} - N_{55}} = \frac{526538,378}{(2819298,212 - 526538,378)} = 22,9653\ \%.$$

TABLE 12. COMMUTATIONS FOR i = 6% AND BR-EMSsb-v.2010-m BRANCH (SELECTED AGES)							
Age x	q_x	l_x	e_x	C_x	M_x	D_x	N_x
29	0,00091	984.176	53,9	155,933	11.773,600	181.636,31	3.000.934,4
30	0,00092	983.275	53,0	148,587	11.617,667	171.198,14	2.819.298,1
31	0,00093	982.374	52,0	141,570	11.469,080	161.359,68	2.648.099,9
44	0,00187	966.458	39,7	131,299	9.762,605	74.426,04	1.142.387,4
45	0,00200	964.651	38,8	132,230	9.631,306	70.081,97	1.067.961,3
46	0,00215	962.719	37,9	133,833	9.499,076	65.982,65	997.879,4
54	0,00398	941.067	30,7	151,943	8.372,553	40.467,26	567.005,6
55	0,00431	937.326	29,8	154,611	8.220,610	38.024,90	526.538,4
56	0,00468	933.283	28,9	157,698	8.065,999	35.717,82	488.513,5

Consider a 45-year-old man who takes out a loan of value 1 to be paid in 10 years in annual flat level payments, at the end of every year. He decides to buy a life insurance to cover the balance due in case of his death, with a single anticipated premium. In this case, either branch of the table could be used, depending on whether this man would be considered in the survivorship group or in the death group. Suppose that he is to be considered in the survivorship group. The single pure premium is given by

$$1-\frac{a_{45:10}}{a_{\overline{10}|}}=1-\frac{7.26814372}{7,360087}=1,05977\%$$

where

$$a_{45:10}=\frac{\left(N_{46}-N_{56}\right)}{D_{45}}=\frac{(997879.404 - 488513.466)}{70081.985}=7,268144$$

and

$$a_{\overline{10}|}=\frac{\left(1-v^{10}\right)}{i}=7,360087$$

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