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Joan Garfield; Andrew Ahlgren

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DIFFICULTIES IN LEARNING BASIC CONCEPTS IN PROBABILITY AND STATISTICS: IMPLICATIONS FOR RESEARCH

JOAN GARFIELD, *University of Minnesota*
ANDREW AHLGREN, *University of Minnesota*

There is a growing movement to introduce elements of statistics and probability into the secondary and even the elementary school curriculum, as part of basic literacy in mathematics. Although many articles in the education literature recommend how to teach statistics better, there is little published research on how students actually learn statistics concepts. The experience of psychologists, educators, and statisticians alike is that a large proportion of students, even in college, do not understand many of the basic statistical concepts they have studied. Inadequacies in prerequisite mathematics skills and abstract reasoning are part of the problem. In addition, research in cognitive science demonstrates the prevalence of some "intuitive" ways of thinking that interfere with the learning of correct statistical reasoning. The literature reviewed in this paper indicates a need for collaborative, cross-disciplinary research on how students come to think correctly about probability and statistics.

Elements of statistical reasoning have become requisite for a wide range of fields of study. A survey of offerings at the University of Minnesota, for example, turned up over 160 statistics courses—of which 40 were introductory—taught in 13 departments. This ubiquity is reflected in the popular press, where ordinary readers almost daily find reports of medical, economic, or psychological studies—reports that can be understood and evaluated only with some understanding of statistics.

At present very little statistics is typically introduced to students before they enter college. There is, however, a vigorously growing movement to introduce elements of statistics and probability into the secondary school curriculum, and even the elementary school curriculum, as part of the basic literacy in mathematics that all citizens in today's world should have (Scheaffer, 1984; Swift, 1982). Statistics and probability were major themes in recent publications of the National Council of Teachers of Mathematics (NCTM) (Shufelt, 1983; Shulte, 1981). The American Statistical Association (ASA) and the NCTM, through their Joint Committee on the Curriculum in Statistics and Probability, also have emphasized the desirability of such a curriculum in several recent publications, including a newsletter, *The Statistics Teacher Network*. The ASA-NCTM Joint Committee has published a document with recommended guidelines for teaching statistics within the K-12 mathematics curriculum (ASA-NCTM Joint Committee, 1985), which includes rudimentary statistics activities as early as Grades 1 to 3.

The enthusiasm for statistics in the curriculum among specialists in mathematics education is endorsed by the mathematics community in general. In recent reports concerning the curriculum, the Conference Board of the

Mathematical Sciences (1982, 1983) described elementary data analysis and statistics as "more important" than current advanced mathematics topics and recommended that these topics be included as early as the middle school curriculum.

Some elements of probability and data description have been included in major curriculum projects in the past (e.g., American Association for the Advancement of Science, 1974; Educational Services, 1968). Yet only a few materials have been written specifically to introduce probability and statistics at the precollege level. In the United States, a set of booklets (Mosteller, 1973) for high school mathematics students was prepared under the supervision of the ASA-NCTM Joint Committee. These numerically focused booklets are considered "challenging if not impossible" for average high school students and have not been used extensively (Boruch, Nowakowski, Zawojewski, & Goldberg, 1984).

In the United Kingdom, the Schools Council Project on Statistical Education has published materials for secondary students covering topics that illustrate how statistics are used in meaningful contexts in different subject areas. The emphasis of these materials is on developing concepts rather than carrying out calculations (P. Holmes, personal communication, 10 April 1985). Several current curriculum development projects, including the University of Chicago School Mathematics Project (1985) and the ASA-NCTM Quantitative Literacy Project, have produced and tested sets of materials for students in Grades 7 through 12 (Gnanadesikan, Scheaffer, & Swift, 1986; Landwehr & Watkins, 1986; Landwehr, Watkins, & Swift, 1987; Newman, Obremski, & Scheaffer, 1986). Yet, despite the enthusiastic development of new instructional materials, little seems to be known about how to teach probability and statistics effectively. For example, in the introduction to the working-group reports on probability and statistics from the Fifth International Congress on Mathematics Education (Bell, Low, & Kilpatrick, 1985), it is noted that many problems still remain in the teaching of probability and statistics.

In this paper we review the literature related to difficulties that students at the precollege and college level have in understanding probability and statistics and discuss the research that is beginning to illuminate the exact nature of the difficulties, their causes, and what can be done about them. Our intent is not so much to catalog exhaustively the accumulated findings as to identify and characterize the nature of the wide range of relevant endeavors. We conclude with a proposal for a new approach to research—one that is cross-disciplinary and involves experts from several disciplines.

TEACHING PROBABILITY AND STATISTICS

College Level

Stochastics as a scientific discipline is usually first taught at the college level. (We will use the term *stochastics* to refer to the study of probability and statistics, as is common in Europe.) The introductory course is usually divided into three areas: descriptive statistics, probability theory, and inferential statistics (Borovcnik, 1985). The topics typically included in each area are listed in Table 1. Over the past 20 years, much of the literature on teaching stochastics has been at the college level. This literature has been filled with comments by instructors about students not attaining an adequate understanding of basic statistical concepts and not being able to solve applied statistical problems (Duchastel, 1974; Jolliffe, 1976; Kalton, 1973; Urquhart, 1971). The experience of most college faculty members in education and the social sciences is that a large proportion of university students in introductory statistics courses do not understand many of the concepts they are studying.

Table 1
Typical Content of an Introductory Statistics Course (College Level)

- Descriptive statistics
  - Measures of central tendency (mean, median, mode)
  - Measures of variability (range, variance, standard deviation)
  - Measures of position (percentiles, *z* scores)
  - Frequency distributions and graphs
- Probability theory
  - Rules (addition, multiplication)
  - Independent and mutually exclusive events
  - Random variables
  - Probability distributions
  - The binomial distribution
  - The normal distribution
  - Sampling
  - Central limit theorem
- Inferential statistics
  - Estimating parameters (mean, variance, proportion, correlation coefficient)
  - Testing hypotheses

Studies in the research literature confirm this impression. Students often tend to respond to problems involving mathematics in general by falling into a "number crunching" mode, plugging quantities into a computational formula or procedure without forming an internal representation of the problem (Noddings, Gilbert-MacMillan, & Lutz, 1980). They may be able to memorize the formulas and the steps to follow in familiar, well-defined problems but only seldom appear to get much sense of what the rationale is or how concepts can be applied in new situations (Chervany, Collier, Fienberg, & Johnson, 1977; Garfield, 1981; Kempthorne, 1980). Within the conceptual underpinnings, the details they have learned or memorized, for whatever use they might be, therefore quickly fade.

Recently attention has focused on the processes involved in solving statistical problems (Chervany et al., 1977; Stroup, 1984) and the need for basing statistics courses on problem solving (Garfield, 1981; Kempthorne, 1980). (Courses based on problem solving introduce problems from the beginning and use real data sets, with statistical concepts being introduced as the need for them arises.) There does not appear to be substantial evidence yet for improved practice.

Precollege Level

Most of the recent literature about precollege stochastics instruction, which is more diverse than the college literature, falls into four categories: statements concerning the *need* for instruction at the precollege level (Pereira-Mendoza & Swift, 1981), suggestions for *how to teach* at the precollege level (Duncan & Litwiller, 1981; Ernest, 1984; Swift, 1983); descriptions of the curricular *role* stochastics can play (Richbart, 1981), and descriptions of the *difficulties* secondary school students have understanding the concepts (Carpenter, Corbitt, & Kepner, 1981; Hope & Kelly, 1983; Shaughnessy, 1981). Only the last category is of interest here.

Research on students' understanding of probability is more extensive than research on statistics and has developed as an area separately from those studies cited above, which focus on stochastics in general. There have been two distinct lines of research on probabilistic understanding: One has focused on school children (e.g., Fischbein, 1975; Green, 1983; Piaget & Inhelder, 1978), and the other has focused on college students and adults (e.g., Konold, 1983; Tversky & Kahneman, 1982).

At any level, students appear to have difficulties developing correct intuition about fundamental ideas of probability for at least three reasons. First, many students have an underlying difficulty with rational number concepts and proportional reasoning, which are used in calculating, reporting, and interpreting probabilities (Behr, Lesh, Post, & Silver, 1983). Results reported by the National Assessment of Educational Progress (NAEP) from the second and third mathematics assessments indicated that students were generally weak in rational number concepts and had difficulties with basic concepts involving fractions, decimals, and percents (Carpenter, Corbitt, & Kepner, 1981; Carpenter, Lindquist, Matthews, & Silver, 1983). Low percentages of students had correct responses to exercises involving complex concepts and skills requiring understanding of underlying mathematics principles. Difficulties in translating verbal problem statements plague stochastics as they do the rest of school mathematics (Hansen, McCann, & Myers, 1985). Second, probability ideas often appear to conflict with students' experiences and how they view the world (Kapadia, 1985). We discuss this conflict below in the section on misconceptions in statistical reasoning. Third, many students have already developed a distaste for probability through having been exposed to its study in a highly abstract and formal way. For this reason, Freudenthal (1973) cautioned against teaching any technique of "mathematical statistics" even to college freshmen.

RECOMMENDATIONS FOR OVERCOMING DIFFICULTIES

Several recommendations from teachers for overcoming difficulties in learning stochastics can be generalized from the literature cited above. Teachers should—

1. introduce topics through activities and simulations, not abstractions;
2. try to arouse in students the feeling that mathematics relates usefully to reality and is not just symbols, rules, and conventions;
3. use visual illustration and emphasize exploratory data methods;
4. teach descriptive statistics alone without relating it to probability;
5. point out to students common misuses of statistics (say, in news stories and advertisements);
6. use strategies to improve students' rational number concepts before approaching proportional reasoning;
7. recognize and confront common errors in students' probabilistic thinking;
8. create situations requiring probabilistic reasoning that correspond to the students' views of the world.

The articles cited that recommend instructional methods contain different mixes of logical argument, teacher intuition, classroom anecdotes, and exhortation on the importance of stochastics topics in schooling. An excellent collection of this genre is the 1981 NCTM yearbook (Shulte, 1981). These recommendations are based almost entirely on experience of what has not worked and speculation about what might work. However, little empirical research has focused on the effectiveness of the different instructional methods or teaching approaches in developing statistical and probabilistic reasoning.

Recently, some research on problem solving has shown that students receiving deliberate instruction in how to solve problems do become better problem solvers and are better able to "think mathematically" (Schoenfeld, 1985). What may be needed is similar research on statistical instruction and students' ability to "think statistically." In an attempt to help students think statistically there has been some interesting experimentation with the role of computers in learning stochastics.

USE OF COMPUTERS IN TEACHING STOCHASTICS

The increasing prevalence of computers in schools has already had some influence on the teaching of stochastics and is producing its own literature. Computers have been used in several ways to aid in the teaching of introductory courses in college. Students may access large mainframe computers and use statistical packages, such as SPSS (Norusis, 1986) or MINITAB (Ryan, Joiner, & Ryan, 1976), to do the number-crunching operations for them (Bialaszewski, 1981) or have problem sets assigned by the computer and then get assistance from the computer in working the problems (Edgar, 1973). Attempts have also been made to use large-system programs to have students run simulations (Stockburger, 1982). Some mainframe applications have made use of special graphics terminals to give the students graphic displays in addition to numerical output (Nygard, 1983). Mainframe computer application is very limited, however, because of the high cost of terminals and a lack of access to special ports.

Microcomputers are more promising than mainframe computers in teaching statistics because they are less expensive and have fairly good graphic capability. Some recent classroom applications have made use of graphics to display histograms (Vandermeulen & DeWreede, 1983) or to compute and display distributions of means of samples drawn from a specific population (Thomas, 1984). Graphics-oriented software is now commercially available to use in supplementing instruction in introductory courses at the college level (Doane, 1985; Elzey, 1985). The Quantitative Literacy Project has also made microcomputer activities an important component of its materials.

The literature contains suggestions as to how microcomputers in schools provide opportunities to incorporate stochastics into secondary mathematics courses (Collis, 1983; Fey, 1984). There is, nonetheless, little research on the efficacy of computers in guiding the design of optimal instruction.

An exceptional research effort to use microcomputer graphics to demonstrate statistical *processes* to high school students is the design and use of an Apple II program to illustrate the central limit theorem (Johnson, 1985). The graphic display of Johnson's program shows samples being taken one element at a time from a discrete population (one of the several selectable), indicates the position of the mean for each sample after it is complete, and moves that mean down to an accumulating distribution of the sample means at the bottom of the screen. The rationale was that the entire sampling and distributing story is presented concretely and continuously, with no steps that the student has to imagine or fill in. Johnson gave microcomputer presentations to two stratified random samples of students, one group receiving the dynamic presentation and the other receiving a very similar but static microcomputer presentation. Delayed retention tests after 2 and 6 weeks showed the dynamic presentation group was significantly better at identifying likely samples and likely distributions of sample means. Johnson also noted an interesting side effect: Students watching the dynamic presentation showed particular interest in whether successive sample means would fill gaps or balance asymmetries that remained in the accumulating distribution, sometimes even cheering when this happened. There was some evidence that these students may have (by selective memory) come to believe that such "missing" means are more likely to occur. Subtleties like this effect demonstrate the need for integrating research and instructional development.

RESEARCH IN COGNITIVE DEVELOPMENT

In general terms, we know what some of the important sources of the difficulty in learning stochastics are. One already mentioned is students' level of specific mathematics skills. There is also students' general mental maturity. A variety of studies of the cognitive development of students in senior high school indicate that perhaps as many as half cannot think on a formal operational level (e.g., Herron, 1978; Smith, 1978). For example, they do not completely grasp proportionality, hypothetical argument, or the concept of controlling variables (Cantu & Herron, 1978; Green, 1983; Milakofsky & Patterson, 1979; Stonewater & Stonewater, 1984). Green (1983) also reports that pupils' verbal ability is often inadequate for accurately describing probabilistic situations.

For many students, a considerable improvement of skills in dealing with abstractions may be necessary before they are ready for much of the probabilistic reasoning and hypothesis testing that underlie basic statistical inference. For some students, teachers may have to be content to forgo abstraction and to convey what statistical ideas they can in simpler, concrete terms. It is noteworthy that introductory units in the ASA-NCTM Quantitative Literacy Project have avoided virtually all computations, using instead plotting and counting methods and using medians rather than means, even for fitting prediction lines in bivariate plots.

Beyond the underlying skills problems, however, there is an even more serious source of difficulty: the students' intuitive convictions about statistical phenomena. The second NAEP mathematics assessment produced evidence that students' intuitive notions of probability seemed to get stronger with age but were not necessarily more correct (Carpenter et al., 1981). Fischbein (1975) also found decrements in probabilistic performance with increasing age, which he attributed to school experience and to scientific reductionism. Students' intuitive ideas, presumably formed through their experience, may be reasonable in many of the contexts in which students use them but can be distressingly inconsistent with the statistics concepts that we would like to teach them. Green (1983) found that performance in recognizing randomness declines with pupils' age. He hypothesized that two opposing tendencies are involved: maturation on the one hand and a dominance of scientific deductivism on the other, which "stifles the appreciation of randomness by seeking to codify and explain everything" (p. 774). As has been found in recent research on students' intuitive ideas about natural science, students' ideas cannot be ignored or dismissed. Indeed, it is difficult to drive such ideas out even with concentrated efforts to do so.

MISCONCEPTIONS IN SCIENCE

There is already a sizable body of research on pedagogical implications of students' misconceptions in science, and some of the insight accumulating from it is relevant to our current inquiry. We cite below several studies that make particular points about the persistence of preconceptions despite instruction, the lack of correspondence between understanding of a topic and belief in it, developmental stages of concept understanding, and attempts to change misconceptions.

Some recent research in physics education is suggestive of the kind of difficulties students may have and of how to investigate them (Clement, 1982; diSessa, 1982; Fredette & Lochhead, 1980; McCloskey, 1983; McCloskey, Washburn, & Felch, 1983; McDermott, 1984). These investigations, which involved children, college physics students (including secondary school teachers in training), and adults with no science training, show that some misconceptions are quite widespread and can persist in spite of relevant instruction. Moreover, the evidence shows that misconceptions may exist despite an individual's ability to use appropriate terminology or to answer questions correctly on a typical test.

Studies of misconceptions of gravity (Sneider & Pulos, 1983) are particularly interesting because they have led to a developmental scheme of stages through which children appear to progress in an invariant fashion. By recognizing the stage at which a learner is thinking, the researchers suggest, teachers may be able to tailor instruction to be optimally helpful in making the transition to the next stage.

There have been some projects directed specifically at changing misconceptions in physics (e.g., Champagne, Gunstone, & Klopfer, 1983; Hewson & Hewson, 1983; Posner, Hewson, Gertzog, & Strike, 1982). Success has been elusive. Some important lessons may be drawn from Minstrell (1982), who attributed his moderate success in changing students' conceptions to (a) a classroom setting that encourages expression of beliefs without criticism, (b) an emphasis on observation rather than authority, (c) juxtaposition of several different demonstrations of a phenomenon, and (d) encouragement to identify simple and consistent explanations.

Similar studies have begun on misconceptions in biology, as exemplified by an excellent study of misconceptions about evolution. Bishop and Anderson (1986) report that number of years of high school and college biology made no difference in prevalence of misconceptions in college underclassmen—80% of whom had serious misconceptions about fundamentals of natural selection. This study also testifies to the resilience of misconceptions: Even after a college biology course in which the misconceptions were systematically challenged, only 50% of the students had relinquished them by the end of the course. Moreover, students' understanding of natural selection was not correlated with their belief in it—either before or after the course. It is important to note that Bishop and Anderson also developed a diagnostic test for identifying students' difficulties, to serve as a basis for designing instruction.

MISCONCEPTIONS IN STATISTICAL REASONING

Identifying Misconceptions

Most of the research on inappropriate statistical reasoning has been done not by educators, but by psychologists. Piaget and Inhelder (1975) are often cited for initiating the developmental research that has helped reveal difficulties students have with theoretical conceptions of probability. They have been criticized, however, for using a solely classical approach to probability and for basing their experiments on theoretical concepts and proportional reasoning—which often appeared to be games comparing fractions, with little connection to ideas of chance (Kapadia, 1985).

More recently, an area of inquiry referred to as *judgment under uncertainty* has emerged. Key studies in this area were brought together in a book edited by Kahneman, Slovic, and Tversky (1982). Several articles address the idea of *representativeness*, which refers to varieties of the idea that an occurrence is probable to the extent that it is "typical." In the combinational form of this misconception, for example, the probability of a man's being black *and* being unemployed is judged to be higher than the probability of his being black. In the sampling form of the representativeness misconception, small samples are generally believed to be more reliable representatives of a population than is statistically likely. (For example, a preference for Brand X over Brand Y in four out of five people would typically be believed to be clearly indicative of a general preference—although the probability of getting such an extreme sample of 5 just by chance is 3/8.)

Other types of judgment analyzed include *availability* (a heuristic that judges a categorization is probable to the extent that instances of it can be easily brought to mind), and *the inference of causality from correlation*. Another relevant topic is *covariation and control*, in which a central finding is that people are not likely to spot a correlation unless they expect it—and if they do expect it, they tend to see one even if it is not there. The clear story that runs through the articles in the Kahneman et al. (1982) book is that inappropriate reasoning is (a) widespread and persistent, (b) similar at all age levels, (c) found even among experienced researchers, and (d) quite difficult to change.

There has recently been a good deal of research by educators exploring the probabilistic intuitions of students from elementary through college level. Much of this research is being done in Europe, in connection with the Centre for Statistical Education in Sheffield, England, or with the Institut für Didaktik der Mathematik der Universität Bielefeld, in the Federal Republic of Germany. Results of several studies around the world were presented at the First International Conference on Teaching Statistics, held in Sheffield, England. Among these are preliminary attempts to describe stages in the development of probabilistic thinking (Falk, 1983; Fischbein & Gazit, 1983; Green, 1983).

Misconceptions Involving Descriptive Statistics

Less extensive attention has been given by psychological and educational researchers to putatively simple statistical ideas such as distribution, average, sample, and randomness. But there is evidence that conceptual difficulties abound for these topics, too. Indeed, investigators in the Quantitative Literacy Project have discovered, in beginning with the activity of constructing a simple "line plot," that many junior high school students do not yet understand what a number line is and that even simple tables of data are incomprehensible to many students (J. Landwehr, personal communication, 10 January 1985). Pollatsek, Lima, and Well (1981) report on students' difficulties in understanding the need to weight data in computing a mean. College students asked to combine two grade point averages that were based on different numbers of courses into a single average were unable to do the task correctly. Pollatsek et al. believe that "for many students dealing with the mean is a computational rather than a conceptual act" (p. 191) and that knowledge "of a computational rule not only does not imply any real understanding of the basic underlying concept, but may actually inhibit the acquisition of more adequate (relational) understanding" (p. 202).

In surveying the statistical ideas of mathematically able senior high school students, Johnson (1985) found that most students regard an average as the usual or typical value. (For example, when asked about the usefulness of the average temperature for a city, many responded that it would tell you what to wear if you went there.)

Attempts to Change Misconceptions

The literature that describes misconceptions about statistics and probability is much more extensive than literature on what can practically be done to ameliorate them. Fischhoff (1982) reviewed 40 articles in a subfield he calls *debiasing theory*, which is devoted to explicit attempts to give professional adults some heuristics that will help to override their faulty intuitions. He concluded that the debiasing attempts have had very limited success. Nisbett, Krantz, Jepson, and Kunda (1983) have attempted to explain why people develop misconceptions and maintain them even with exposure to contradictory experience. They report that subjects can be taught to give better answers to probabilistic textbook problems but admit that they are unsure about whether the subjects are using a formal model to do so and about the likelihood of transfer to the real world.

There are, however, reports of success. Shaughnessy (1977) reported that his activity-based course and small-group work helped college students overcome some misconceptions in representativeness and availability. Mevarech (1983), having found that high school students err in solving problems about means because they believe that means have the same properties as simple numbers, reported that it is helpful to provide corrective-feedback instruction. Ojemann, Maxey, and Snider (1965a, 1965b, 1966) claimed to have successfully trained children at the fifth-grade level to make significant improvements in making probability judgments. Fischbein, Pampu, and Minzat (1970) claimed that, after instruction, fourth-grade students were able to think accurately about probability. Fischbein and Gazit (1984) made a similar claim for students in Grades 6 and 7.

Because these studies assessed the accuracy of judgments without probing the students' means of arriving at them, it is not clear to what extent the students' ideas of probability changed and to what extent they merely learned rules. Indeed, Fischbein and Gazit (1984) reported that the children appeared to do much better on some probability tasks than they were able to do on proportional reasoning—which one might expect to be requisite for understanding probability calculations. When students were given a test of transfer to intuitive ideas about probability, the instructed group of students did distinctly worse on some parts than the uninstructed group. Fischbein and Gazit hypothesized that "by emphasizing (via systematic instruction) specific probability viewpoints and procedures one may disturb the subjects' proportional reasoning, still fragile in many adolescents" (p. 23).

In a review of selected psychological and pedagogical research studies, Hawkins and Kapadia (1984) identified key questions concerning how children understand probability. These questions included (a) What conceptions of probability do children of various ages have? (b) How might these conceptions be changed? (c) Are there optimum teaching and learning techniques? Hawkins and Kapadia believed that because of differences in terminology or research methods used, research has tended to generate debate rather than answer these questions. In their thorough discussion of classroom implications of the research on children's ideas of probability, they cited a nationwide study in the United Kingdom in which 11-year-olds first predicted, and then made a trial of, the number of heads in 12 tosses of a coin and the pattern of results in 24 rolls of a die. Hawkins and Kapadia quoted this passage from the study report:

> In general the results show a disparity between the quality of predictions and the justifications given for them. The vast majority of pupils gave mathematically sound predictions. However, when they found that these predictions were not confirmed by the data, many pupils reverted to past experience, hunches or cynicism. . . . Pupils become confused, either clinging to what they believed must be the mathematically correct answer despite contrary results, or abandoning theory altogether. (p. 363)

Hawkins and Kapadia questioned whether the children's original answers revealed anything of their thinking about probability.

These reports of attempts at instruction and assessment raise the important issue of the relative merits of retraining students' *intuition* about probability—say, by extensive exposure to probabilistic simulations on a computer or, on the other hand, by giving them *rules* for improving their answers. It is likely that the real issue will be how to optimize the interplay of experience and rules. The design of optimal instruction is complicated further by the possibility of instruction's having unexpected side effects that may even interfere with the development of students' understanding—as Johnson (1985) has found.

How Students Think

A major reason for the limited success of remediation is that the research, although rich in detail on the *incidence* of misjudgment, has not attended to underlying psychological mechanisms. A number of researchers are beginning to produce useful information about how misconceptions operate that should be useful in designing remediation.

Growing attention to the contextual aspects of probability judgments (Borgida & DeBono, 1985; Morier & Borgida, 1984; Nisbett et al., 1983) suggests that an important factor in misjudgment is a misperception of the question being asked. A tentative conclusion is that considerable advantage in comprehension may be realized by careful attention to the way in which questions are posed. For example, when asked what the probability is that a particular attractive, well-dressed woman is a fashion model (Beyth-Marom & Dekel, 1983), students typically give an answer of about 70%, and they give about that same value for the probability that she is an actress or the probability that she is a cosmetics distributor. Instead of answering with a probability about the woman's profession, the students apparently are estimating the probability that a fashion model (or actress or cosmetics distributor) would be attractive and well dressed. Insisting that the sum of probabilities must not exceed 1.0 (what Tversky and Kahneman call *coherence*) may therefore only be confusing to students. It is possible that a great deal of research, in focusing on the correctness of *answers*, has missed the subjects' perception of what the question was—and so misestimated the subjects' reasoning.

Tversky and Kahneman (1982) have characterized representativeness and availability not only as classifications of a number of misjudgment phenomena but as general features underlying intuitive reasoning. As appealing as these constructs are logically, it remains to be shown that they validly describe thinking. There is evidence both pro and con in some particularly interesting interview studies of college students' thinking when answering statistical questions (e.g., Hardiman, Well, & Pollatsek, 1984; Konold, 1983; Pollatsek, Lima, & Well, 1981). The useful integration of research and instruction is evident in a recent article (Pollatsek, Konold, Well, & Lima, 1984) on textbook attempts to ameliorate the "gambler's fallacy" by refuting students' belief in "active balancing," that is, that fluctuation in a random series will tend to be balanced by an opposite fluctuation:

> Our research suggests that such an approach is unfruitful because subjects do not have an incorrect process mechanism; indeed they have virtually no mechanistic way to think about random samples. To refute active balancing is to refute a belief that students actually do not

have, and this may confuse them. Since students' actual heuristic, representativeness, is so different in form from the appropriate mechanistic belief, it may not be easy to effect any lasting change in students' beliefs about random samples. (p. 400)

Findings like these among college students imply the need for great caution in making objectives for instruction in high school. More important, however, is the opportunity that the interview methods provide for testing our propositions of what is going on in students' minds. For example, Konold (1983) inferred from in-depth interviews that students typically have an "outcome orientation" model of probability that may be more fundamental than the representativeness model. The outcome orientation involves making yes-or-no decisions about single events. Konold compared the representativeness and outcome-orientation models in college students' responses to novel questions. For example, when students were asked what the outcome would be of selecting six marbles from a population that was 5/6 black and 1/6 white, most students thought it more likely that all six would be black (i.e., a black marble is the most likely outcome on every single trial) than that one would be white (the most representative result for the series).

In the same study, Konold found that when students observed frequencies in repeated trials, they inferred numerical probabilities by using an indirect, two-stage process: In the first stage, the observed frequencies led to a feeling of confidence; if a numerical representation was demanded of them, they then generated a numerical "probability" from the feeling—not from the frequencies. If true, this finding would imply considerable circumspectness in using simulations to change the way that students think about probability. Such in-depth exploratory research should be extended to other basic topics and to students at other levels of education.

IMPLICATIONS FOR FUTURE RESEARCH

The declared importance of introducing statistical concepts into the school curriculum, together with our limited knowledge about cognitive development, mathematics learning in general, and misconceptions in probability and statistics, indicates that the time is ripe for an intensive and coherent assault on the difficulty of learning basic concepts in stochastics. Indeed, the creation of the International Study Group on Probability and Statistics Concepts (ASA-NCTM Joint Committee, 1985) and international conferences on teaching and learning stochastics (Grey, Holmes, Barnett, & Constable, 1983) indicate that the learning of stochastics is an area of major interest and activity. Many researchers in education and psychology have explored parts of the misconceptions picture. Some researchers have attempted to design specific remedial treatments, and many insightful and creative teachers have suggested and tried improved techniques of instruction. Accounts of these continue to appear in the journal *Teaching Statistics* and the newsletter *The Statistics Teacher Network*.

Although strong arguments have been made that students learn best when instruction is couched in the context of students' "real world" knowledge (Freudenthal, 1973; Kapadia, 1985; Roth, 1985), there is still only a little published research on the effectiveness of this approach or any other. This lack of research is perhaps due, as Bentz and Borovcnik (1985) believe, to the difficulty of conducting this type of empirical research. Bentz and Borovcnik provide a catalog of problems that have limited the interpretation of empirical research on probabilistic concepts.

A related problem is a lack of research on the design and use of instruments to measure statistical understanding (Boveda, 1975; Chervany et al., 1977; Garfield, 1981). A few instruments have been designed to measure students' attitudes and anxiety toward statistics (Roberts & Saxe, 1982; Wise, 1985), and some research has appeared that shows the role of factors influencing general achievement in a statistics course (Harvey, Planke, & Wise, 1985). Green (1983) has recently developed a test for children ages 11 to 16 that measures three levels of attainment of probability concepts.

The literature makes it clear that far more research has been done on the psychology of probability than on other statistical concepts. In spite of this research, however, teaching a conceptual grasp of probability still appears to be a very difficult task, fraught with ambiguity and illusion. Accordingly, we make the pragmatic recommendation for two research efforts that would proceed in parallel: one that continues to explore the means to induce valid conceptions of probability, and one that explores how useful ideas of statistical inference can be taught independently of technically correct probability. The interpretation of many statistical results must deal in some way with whether or not we are being fooled by random variation, but it may be possible to rely on simulation to make semiquantitative arguments. This view entails accepting students' natural subjectivist view and endeavoring to get them to draw increasingly on theoretical and frequency information to set more appropriate belief levels.

As far as goals for instruction go, this view suggests a moratorium on the typical organization of statistics instruction: (a) descriptive statistics, (b) probability, and (c) inferential statistics. The intrusion of technical probability issues that are not likely to be understood will stall the learning process—and leave a distaste that could compromise subsequent instruction as well. This view is consistent with at least the beginning of the Quantitative Literacy Project curriculum, in which qualitative approaches are used as much as possible, computation is minimized, and probabilities are handled in large part by simulation. The view may not be consistent, on the other hand, with some of the ambitious goals for including probability in the K-12 curriculum, foreshadowed in preliminary drafts of the ASA-NCTM Joint Committee's (1985) preliminary guidelines. What is needed, however, is not debate but research.

The difficulty of learning probability and statistics seems not to be a simple problem that can be ameliorated by simple remedies. The misconceptions already identified are not isolated errors of information or arbitrary habits of thinking. It seems, rather, that misconceptions are part of a way of thinking about events that is deeply rooted in most people, either as learned parts of our culture or (in the extreme) even as brain functions arising from natural selection in a simpler time. Indeed, one direction for research would be in social anthropology—to attempt to deduce "misconceptions" from what would have been useful modes of thinking in prehistoric settings. The misconception problem is aggravated by difficulties in quantitative thinking in general and difficulties in hypothetical reasoning. These cognitive problems are complicated still further by affective obstacles—faintness of motivation for learning what is believed to be a useless, forbidding, and even deceptive topic.

A complete research assault on difficulties in learning stochastics would require attention to all these aspects of the problem. In reviewing the research on instruction in mathematics and science, Resnick (1983) encouraged collaboration between cognitive psychologists and discipline specialists to improve precollege instruction in mathematics and science. She recommended that improvement begin early, emphasize qualitative reasoning, build on what students already know, and confront naive intuition directly. In addition, we believe it will be essential to undertake longitudinal studies of how individual students actually develop in stochastic sophistication.

Although the accumulation of research findings from diverse current endeavors may eventually yield a substantial understanding of how to teach probability and statistics better, we would like to add the suggestion that progress will be realized more surely and efficiently by concentrated and cooperative projects at single sites, where there can be ongoing interaction among researchers from the several relevant fields. The sooner such projects begin the better, because once schools implement the curricula being developed, their commitment to these materials may make more optimal approaches difficult to sell. (Or, if the new curricula do not work adequately, the schools may develop resistance to stochastics altogether.) Only when we understand how people learn stochastics concepts will we be able to prescribe adequately those learning experiences that will be effective.

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AUTHORS

JOAN GARFIELD, Assistant Professor, General College, 106 Nicholson Hall, University of Minnesota, 216 Pillsbury Drive S.E., Minneapolis, MN 55455

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