# The Potential of Portable Technologies for Supporting Graphing Investigations


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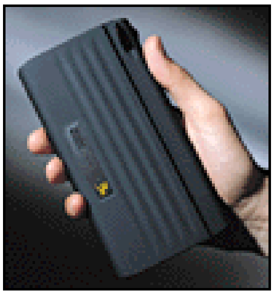


**ABSTRACT**

This article begins by reviewing the relevant research literature concerning the portable revolution in education. It outlines the advantages and disadvantages of portable forms of technology, with an emphasis on their potential impact upon student learning and attitudes. (The focus is on mathematics as far as possible, but writing is touched upon too as it is the main use for computers in school and hence the subject of much research.) A detailed discussion is then undertaken of the research on students' understanding of graphing and studies with graphic calculators and portable computers. We also discuss the related trend towards investigative learning in mathematics, the use of 'real world' activities, issues concerning collaborative use of portable computers and research on relevant gender issues. The review concludes that portable graphing technologies present an important opportunity to help students develop understanding and skills in the traditionally difficult curriculum area of graphing.

**Key words:** portable computers, graphic calculators, palmtops, graphing, multiple representations, real world problems, collaboration, gender

**CONTENTS**

## 1. The changing face of IT in education: the portable revolution

In this introductory section, the increasing paradigm shift away from conventional uses of desktop computing towards an emphasis on more compact, personal machines in schools is discussed. The common uses and perceived benefits and disadvantages of these powerful tools for facilitating student learning are considered; the benefits include flexibility, accessibility and personal ownership, apparently resulting in more successful, more independent and more investigative learning. The review concentrates on portable computers and graphic calculators; four-function calculators are not included. Graphic calculators are highly specialised tools, used primarily by older students for graphing; since the research into their use focuses on their impact on learning about graphical representations, it is discussed mainly in section 3. Nevertheless the same benefits of portability as outlined below for palmtop and laptop computers apply to the calculators too.

Today's school students are increasingly computer-literate, although it is astonishing that only just over a decade ago, over half of upper secondary students had never used a computer at school and a further quarter used them less than monthly (Fife-Schaw et al., 1986). Schools in the UK, US and Australia now average at least one computer per 10 students (British Educational Suppliers Association survey, 1995; Littleton, 1995; Fluck, 1995) and in British secondary schools, the ratio has leapt to 1:9 in 1995-6 from 1:60 in 1984-5 (DfEE, 1997). However, actual levels of both provision and usage of computers vary widely. A longitudinal study carried out by researchers at King's College London provides further information derived from 2300 students in primary and secondary schools throughout England and Wales. The ImpacT Report (Watson, 1993) showed that access to IT is markedly unequal between schools, classes and individuals, and each machine can be shared by up to 5 students at one time. The most recent (1995) Ofsted review of IT inspections (of 240 secondary schools) confirmed that hardware is aging badly and desktop machines are often used ineffectively. They involve too many students working together at one screen, reducing visibility and effective lesson time. In fact, the IT experience of many students is inadequate and contravenes National Curriculum requirements. The ImpacT study found that significant numbers of students aged 12-16 in 37% of classes had no use of IT during the 2-year investigation (1989-91), despite the project classes being selected for their good, innovative teaching practice. However, 53% used computers at least once a week and nearly half of these used them several times a week. About half of students also have access to a computer at home (Newman, 1994; Robertson et al., 1996) and many of these are more sophisticated than those in schools.[1]

The new generation of smaller, portable computers[2] could potentially make a significant impact: as more models are released, machine size, weight and prices are falling while processing power, battery life, peripheral adaptations and screen clarity increase. The notion of truly 'personal' computers is becoming more and more feasible. Indeed one in five personal computers sold in the U.K. is now a portable (Bowell, France & Redfern, 1994) and the proportion is likely to increase after the launch last year of a range of hand-held personal computers running the new Windows CE (consumer electronics) operating system.[3] Yet portables have until very recently remained entrenched in the workplace, where computers suitable for mobile employees support a range of working practices and offer a commercial advantage. Portable computers may soon be much more common in higher education too, if the recommendations of the recent Dearing Report are followed; the plan is for all students to have their own machines within the next 7 years.

Laptops are prevalent at school level in technology-oriented Australia, where most private schools persuade parents to buy all students a laptop. Much pioneering research has been carried out, starting with the Sunrise project in 1989. Loader (1993) reports how 1500 laptops were introduced into a

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[1] Geoff Strack, TES 10.10.97, Computer Update, p.21.

[2] These began to appear in about 1989 so are a relatively recent phenomenon.

[3] See The Independent 11.3.97, Tabloid, p.5.

girls' school, beginning with the entire year group of grade 5 students and culminating in a transformation of the school, its culture, curriculum and its teaching / learning paradigm.[4] In the U.S., where Apple predominate, the eMate (or 'lunchbox' computer), their radical new product aimed exclusively at the education market, is intended to revolutionise school computing. It became available in the U.K. during 1997 and is cheap enough to make the (alleged) goal of 'one computer per child' achievable.[5]

Portables presently remain uncommon in British schools (they accounted for only 7% of computers in seconadry schools in 1995-6: DfEE, 1997) although the policies of the current Labour government are to change this. The recent National Council for Educational Technology (NCET) "Portable Computers in Schools" pilot scheme, the largest of its kind, also took a major step in this direction. Over 6000 machines (notebook computers, hand-held palmtops, cheap word processors, graphic calculators) were issued to 250 primary, secondary and special schools in an attempt to increase and monitor their use and their support and suitability for learning in particular curriculum areas. [6] The fundamental aims were to stimulate innovation with IT in the school context and to improve the quality of work by developing IT capability across the curriculum and the motivation to apply it. The main subject areas of focus were English, science, mathematics and geography. A particular goal of the evaluation was to discover how portables enable schools to do what was previously unachievable with desktop computers; to this end, some projects incorporated control groups. (The scheme was evaluated by NFER, who produced a clear and informative report describing the scheme and its key findings: Stradling, Sims & Jamison, 1994. NCET produced its own companion report of the scheme's impact, with many inspiring examples of practice in individual schools: Bowell, France & Redfern, 1994. Note, however, that the claims made are based partly on a student survey and observation of a subset of projects, but mainly on teacher reports.) This review draws heavily on the results of the NCET scheme and on other portable computing initiatives in the U.K. and abroad, including previous work carried out by Open University (OU) researchers with palmtops.

*Accessibility and flexibility*

The availability of more powerful, cheap portables is expected to continue to grow. Our own experience of research in this area led us to postulate that the increasing use of such computers represents a radical and growing *paradigm shift* away from conventional uses of desktop computing in educational institutions (Fung, O'Shea and Hennessy, 1995). Until recently, most IT lessons were located in designated computer rooms (Ofsted, 1995), where 60% of computers in secondary schools are in fact located (DfEE, 1997). However, IT is increasingly perceived as a cross-curricular activity in school using content-free software rather than focused on learning about computers and programming them (Moss, 1992). We now expect the constraints of the traditional IT lab[7] (including its physical distance from classrooms and laboratories and the need for booking access), of queuing to use classroom computers, of power point locations, and even of rigid lessontime boundaries, to be gradually eroded by a new emphasis on small, personal machines. These are unobtrusive on desks during lessons and readily accessible at other times. Most importantly, then, portables can be used

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[4] Other examples of recent Australian research into high access to portables include Jones (1995) and Gus (1995).

[5] Between a laptop and a palmtop in size, the eMate presents an integrated software package (including a word processor, spreadsheet, drawing program and graphing calculator) and can be interfaced via keyboard or pen. It offers colour, 24-hour battery life, internet connectivity and built-in infrared technology allowing lessons to beam between machines, saving copying time. It has no hard disk or floppy disk drive, however; cost is about £500. (See Apple magazine, Spring 1997, p.14)

[6] The scheme was backed by £2.5 million of Department for Education funding and ran in 1993-4; a cross-section of institutions throughout England were selected through a competitive bidding process.

[7] Goldstein (1994) paints a depressing picture of 'computer lessons' in mathematics with students glued to machines arranged around the perimeter of a bleak room with the stark choice of staring at the screen or at a blank wall.

flexibly; they can easily be passed around, tilted or quickly set aside; this is useful when moving on to other activities, other classroom work areas or into groups for comparison, analysis and discussion of findings (Stradling, Sims & Jamison, 1994, ch.2). Students can take them to the teacher or friends to ask for help, rather than waiting for it to arrive. Students also have more choice over the location of their working environment. This tremendous flexibility is a real bonus since students can continue with a piece of work whenever they have the opportunity, rather than waiting for access to a free desktop machine at a specified time and going through a logging on and off procedure each time.

A practical advantage is that other facilities and resources in the base classroom - often lacking in the IT room or area - remain accessible. Indeed minimal organisational change is usually required to accommodate portable computers, compared with investment in desktop machines. Portables can also go where desktop computers cannot, being moved easily between lessons, classrooms or distant buildings, between the classroom, the field and home, and they can be used at any time of day. They occupy far less space than conventional machines, freeing space for other uses. Students can also spread out from their classrooms, conducting experiments in corridors and maintaining the momentum of an activity by entering data directly (Ainley & Pratt, 1995). Portables can be taken into the library or reading area to make notes. Where desktop computers are subject based rather than fixed in an IT room, portables still offer the advantage of easy mobility without trolleys or classes having to swap rooms (NCET, 1993). In sum, "the IT resources can accompany the user rather than the user having to go to the IT" (Bowell, France & Redfern, 1994, p.12).

Portables can obviously offer far higher access to IT than conventional facilities, where access is often sporadic and restricted. Indeed more widespread use of portables may not only benefit students but can ease demand on other IT facilities in the school (Fung, 1995). Worries about the cost implications of supplying portables may be dispelled with the realisation that fewer desktop machines are necessary because laptops are more easily shared (Thomas, 1991). Bradshaw and Massey (1996) point out that laptop computers can also be programmed to be compatible with a school's existing computer system, avoiding problems students might potentially face in using or adapting their home computers. Moreover, the knowledge developed in conventional lessons using computer facilities is not necessarily transferred back to the classroom, whereas portables potentially allow IT to be fully integrated into subject teaching (Jones, 1995).

Students seem to be more motivated to carry out additional projects or investigations of their own when using portables. They may also spend longer on their investigations: the year 9 and 11 students observed by Friedler and McFarlane (1997) became more deeply involved in refining and extending their investigations in response to the instant feedback they received via the dynamic graphs during data logging[8]. Similarly, a critical claim from the NCET projects was that the ability to analyse data on the spot enabled some students to test alternative hypotheses and approaches that otherwise would not have been possible and to think about interim results, hence developing a deeper understanding of underlying concepts (Stradling, Sims & Jamison, 1994, ch.2). Teachers also agreed that students recorded more information using portables in the field than when using pen and paper only, appearing more motivated to develop extra databases related to unforeseen phenomena arising. In sum, students showed considerable enterprise in exploiting the opportunities presented by portables. According to NCET, the flexibility of portables can offer students greater choice, control and independence, so that "IT becomes more supportive of the learning process rather than being seen as an end in itself" (Bowell, France & Redfern, 1994, p.3). Thus, the machines no longer dominate the learning process, lesson design or the workspace (Stradling, Sims & Jamison, 1994, ch.2). The accessibility of portables - both in school and at home - allows students much more than a 'taste' of computer use and renders them more likely to be perceived as an everyday tool for learning; students can apply IT as and when appropriate rather than when the teacher dictates. While

[8] This employs software which gathers, stores and displays measurements from physical sensors directly connected to a computer; the system is commonly referred to in USA as a Microcomputer Based Laboratory.

this new flexible style of working fits fairly well with the current organisation of primary classrooms, it has radical implications for secondary teaching, where lessons are often planned around IT facilities.[9]

Teachers' use of portables as a versatile and flexible resource evidently widens the range of subjects using IT and increases access to the curriculum for all students. It can allow work to be tackled from alternative angles and therefore earlier than planned; Stradling et al. (1994, ch.3) offer a couple of examples of this in relation to mathematics activities. They also maintain that flexible use of portables facilitates *links across the curriculum* (e.g. data from science experiments can be used in mathematics lessons), although evidence for transfer of skills between curriculum areas was mixed. Some student-initiated investigations where connections were spontaneously recognised were apparent, although opportunities for transfer generally need to be planned and coordinated.[10] Other authors offer inspiring examples of projects using portables throughout for a variety of different kinds of activities.[11]

### *Palmtop technology*[12]

Further (anecdotal) evidence that a paradigm shift is beginning comes from a couple of articles in the Acorn Education publication, *Arc* (Autumn 1994, p.15-21; Spring 1996, p.34-35), which focus on the Pocket Book palmtop computer. This machine was adapted from the Psion Organiser for the educational market; it contains a 256k RAM and a 512k ROM and comes with built-in menu-driven software including a word processor and spell check, database, spreadsheet with related graphing facility, plus calculator functions. The *Arc* articles depict the variety of uses which schools are currently getting students to make of the machines - e.g. using the word processing, spreadsheet and graphing facilities for mathematics activities, recording and analysing data from surveys and history projects, scientific experiments and environmental field studies, and writing up assignments. The emphasis is on practical settings and real life problems. Reported student benefits include involving students in the design and execution of the information recording process and encouraging them to see IT as a useful tool in these and related activities. Practical problems with suitable space in already crowded classrooms, and time taken to introduce new programs on conventional desktop machines to small groups of students in turn, are said to be eradicated by giving a group of students a small machine each, introducing a program to them and allowing them to teach their peers.

While the above benefits apply to all portables, the small hand-held palmtops overcome some of the practical difficulties previously observed with using laptops (Gardner, Morrison & Jarman, 1993; Peacock & Breese, 1990) which some people do not consider to be truly portable as they can be

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[9] The optimum number of portables for keeping the resources fully utilised varies too, from perhaps 8-10 machines (one third or one quarter of a class) at primary level, to a one-to-one or one-to-two basis at secondary level (Bowell et al., 1994).

[10] One example (p.23) was a project using data-logging machines with sensors to measure temperature, light etc. in science and geography, then downloading the data for analysis on sub-notebooks. The aim was to initiate Year 9 and 10 students to design and execute their own investigations using the equipment, including in other subjects such as PE and technology.

[11] For instance, Thomas (1991) describes a history of art project where laptops were used for sketching paintings in a gallery, taken home for improvising the sketches, used to create a story-board for a video sequence of the collective results, and ultimately used for desktop publishing of associated publicity material.

[12] Palmtops have a small QWERTY keyboard and a black and white screen typically displaying 8 lines of about 40 characters; the design resembles a folding 'clam shell' with a hinged lid. Solid-state memory disks can store user files and other applications can be purchased on cards for use on certain palmtops. Adapter kits are available for printing and PC links.

heavy and fragile (Mital, Genaidy & Fard, 1989; Yau, Ziegler & Siegel, 1990).[13] Palmtops work best in the field because of their longer battery life (10-50 hours) and small size (Bowell et al., 1994). They are also much cheaper, permitting provision of large numbers of machines at relatively low cost (around £200 each including a range of software versus £1000 for a laptop). Ironically, though, while the vast majority of teachers interviewed by Robertson et al. (1995a) considered portability of palmtops to be an important feature, a couple were afraid to entrust data to them, considering them easily stolen. The Acorn Pocket Book is particularly popular with schools because of its user-friendly interface, educational specification and availability of class packs.

There are advantages other than practical ones too. Studies of schools in the NCET scheme have highlighted the substantial benefits observed by teachers when students gain personal ‘ownership’ of portables and the feeling of working privately, particularly with the small palmtop machines:

> Work that students do on the portables remains private to them unless they choose to share it with their teacher or their classmates. They are not sitting at a desktop in the corner of the classroom where anyone can see the quantity or qualtity of their work. A portable is a non-threatening friend. (Bowell et al., 1994, p.20)

By contrast, conventional computers allow passers-by to read the screen before the author is ready, rendering spelling errors and so on more conspicuous.

## The London Docklands Learning Acceleration Project

One outstanding initiative which exploited palmtop technology to tackle the problem of low achievement within a low-income area of East London is the London Docklands Project (National Literacy Association, 1996). The scheme provided 600 students in 15 primary schools in three of the nation’s poorest boroughs with their own Pocket Book computers in an effort to improve reading and writing. The palmtops were used within the context of a broad programme of activities supporting literacy and numeracy, using the portables alongside other more traditional computer software to enhance and reinforce learning. Special attention was devoted to encouraging parental and community involvement in and support for the children’s education, including offering workshops on writing and word processing skills. Training for teachers in using the hardware and software was also provided.

The project met significant resistance initially as teachers accustomed to the low achievement of many of their students were not convinced that improvements were possible, and worries about loss and damage dominated. Nevertheless the results were very encouraging; the students were highly motivated by gaining possession of personal computers and there were significant gains in reading levels. Before the project began in 1995, only one school’s reading was at national average; in 1996, seven schools were progressing at or above average with improvement rates of up to 19 months in reading age over the year. Library use was also markedly increased. The conclusion was that the technology noticeably acted as a facilitator and leveller in the project, helping students to overcome many of the traditional barriers to learning. The project coordinator insists that the palmtops are not considered as recreational toys: “they are seriously interactive educational tools... They create an individual space for constructive authentic learning” (National Literacy Association, 1996, p.8).

## The OU Pocket Book pilot study

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[13] Laptop or notebook machines are typically A4 size, weighing about 3 kg. with a full-size sprung keyboard, hinged lid, backlit screen (colour is becoming increasingly common) and both hard disk and floppy disk drive. Some A5 machines are also available. Laptops are now as powerful (and costly) as their desktop equivalents; they use the same disks and can run much of the same software. Batteries typically last 3-5 hours.

The author and her colleagues undertook an evaluation of a pilot concerning the use of Pocket Books in a secondary school, as part of the NCET scheme. The Pocket Book pilot study (Fung, 1995; Fung, O'Shea & Hennessy, 1995) provided each of 235 students aged 15-16 with unlimited access to a palmtop for a 3-week period. The students composed, recorded, edited and printed sections of their 'National Records of Achievement' (RoAs), and teachers compiled the student files and added their own comments.

Overall, our observations showed that for the majority of participants, the experience of using a portable computer over even a short timescale proved stimulating. Their attitudes to using computers, both generally and for the specific tasks concerned with their RoAs, became more positive (as elaborated later on). The major benefits perceived of using a portable were the ease and rapidity of producing and editing written work (corroborating the findings of the Scottish Laptop Computer Project, where a class of primary students was issued with laptops for more than a year: Turnbull & Gilmour, 1991). Personal gains included increased feelings of confidence and empowerment, as elaborated in a later section. Half of the sample used the Pocket Books for purposes other than their RoA assignments. 65% of the group indicated that they would have liked to have kept their Pocket Book for longer.

While it could be argued that the students' enthusiasm was high because the machines' limitations may not have become fully evident over the relatively short timescale of the project, the findings of Turnbull and Gilmour (1991) refute this argument: they found that the novelty effect of using laptop computers did not wear off over a period of several months and that motivation continued to be very high. The short time period in our study was nevertheless a significant constraint and some staff felt that students were prevented from utilising their full potential. However, most students did complete some tasks (more than would usually have been expected) and a significant number of students benefited from a largely positive educational experience which they would not otherwise have encountered. People may even make better use of portables once the novelty has worn off, adapting to their own experiences and needs (Robertson et al., 1995a).[14]

The Pocket Book project

The positive outcomes of the Pocket Book pilot study led the school to apply (successfully) for further resources to continue the work, supplying every member of staff (70) and a class of first-year students (30) with a Pocket Book machine. The project's aims were expanded to include raising the IT awareness of staff, thereby enriching IT opportunites for students and increasing their capability and enthusiasm. The pilot work and the school's subsequent expenditure on Pocket Books led the OU to conduct a more extensive study of the impact of high access to palmtops, using similar methods. Robertson et al. (1995a; 1996; 1997) investigated the resulting organisational change and changes in knowledge of and attitudes to computers over 8 months. Questionnaire data was obtained from two classes of first-year (Year 8) secondary school students, one class having unlimited access to Pocket Books and one control group; 65 teachers plus 10 OU (Business) students. (Results pertaining to changes in attitude and gender differences are described later on.)

The results were confounded by the school's tutorial group system since the experimental class was mixed in with others for most lessons, leading to jealousy (even interference with files) and restricting the scope of computer-based activities. Nevertheless, interesting results emerged from measures of students' use over time. 5 weeks into the project, almost all students reported using the Pocket Book 'very frequently' for homework, schoolwork and personal use. Much of this use was directed towards exploring its functionality and the situation changed little over the first 5 months of

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[14] O'Shea (1995) has argued that technological change is so rapid that novelty effects can almost be relied upon as a motivational factor in the classroom and other educational settings.

the project. However, use for homework dropped considerably after 8 months and there was only half as much use in school, while personal use increased (Robertson et al., 1997). The picture for the adult users was quite different; the OU students' use in their studies, at work and particularly for personal use, decreased over the year. Explanations offered included the temporary nature of the study and consequent unwillingness to commit themselves to the machine, and their greater access to more powerful desktop machines with more memory and larger screens and keyboards, more suited to adult hands and touch typing. Students were not issued with PC links and all asserted they would otherwise have used it more. Teachers' experiences were similar. Robertson et al. concluded that the introduction of Pocket Book had been successful in many ways, including increased levels of IT skills by both teachers and students, particularly greater knowledge of several content-free applications such as spreadsheets which were new to some participants. These skills are ones sought after by employers. The questionnaire results from students, parents and OU students confirmed that familiarity with computers through frequent use promotes awareness of their future importance in education.

Technical and practical issues

The Pocket Book project highlighted the difficulties as well as the benefits of equipping a year group with portable computers and attempting to integrate them with classwork and these have implications for all such initiatives. Technical problems during the pilot project were minimal: the machines proved very robust overall and the battery failure and loss of files characterising some previous studies with portables (e.g. Peacock & Breese, 1990) were not a problem (other studies similarly reported no more difficulties than those experienced with manual or written work: Friedler & McFarlane, 1997). Those problems emerging during the follow-up project were mainly avoidable: a quarter of students experienced problems with lost files (they persistently failed to back up their work though) and with printing. A minority of students experienced problems with the computer's memory, sometimes because too many files were open. These problems were used occasionally as fabricated excuses for missing homework. They may have arisen because the computer's file and memory management system is not intuitive and because students had limited access to manuals; moreover the help system may not always be instructional. Robertson et al. (1995a) point out that manuals could usefully be supplemented with separate texts for adults and students with numerous examples and opportunities for practicing saving and backing up files. Some students were demotivated by the spell checker which could not cope with phonetic spellings; a common problem is that spell checkers intended for adults do not always address young children's needs (NCET, 1993). Thus while most students used the Pocket Book frequently throughout the project, a danger is that it may be the students with weaker IT skills who become demotivated if sufficient training in using the machines is not given.

Robertson et al. found that availability of printers is an important issue for students and teachers; students in the Pocket Book project had to leave the classroom to print out their work, either in their own time or during lesson time. Even if printers are available in the classroom, queuing may still be problematic and dot matrix printers are noisy. The small screen size of palmtops was not perceived as problematic by students but a larger screen is often needed to view the text more clearly for editing extended documents. This requires a PC link, again not always available. Easier interconnectivity with other systems and peripherals would certainly be advantageous. Indeed, schools using Pocket Books during the NCET scheme found it difficult or even impossible to get the 'A-link' working which enables printing out. Redrafting often involved a time-consuming process of printing hard copies for correction by the teacher, then incorporating the corrections electronically and printing another copy. The screen size also makes clear viewing of punctuation difficult and this appeared to have some impact on quality of written work. (However, the Pocket Book II overcomes several problems with its clearer screen, increased memory and phonetic spell check.) A security access code would also help prevent interference; and other problems would be minimised if teachers were able to organise time and opportunities for redrafting and printing.

Note that most of the above issues also govern the use of laptop computers, as other authors have documented.[15] However laptops tend to be less robust and although their screens are obviously larger, they can be difficult to see from an angle, hence inhibiting groupwork (Cloke, 1996). Their cost is still high relative to desktop equivalents, although their educational advantages and portability seem to outweigh the disadvantages.

*Impact on written work*

The NCET "Portable Computers in Schools" initiative and a handful of other studies undertaken with laptops have produced encouraging results in terms of improving students' motivation and intensity of effort as well as the quality and quantity of their written work; these improvements were dramatic in some cases and related to frequent use (Yau, Ziegler & Siegel, 1990). Typically, students' speed of writing and technical accuracy improved and they wrote significantly more when using a laptop computer; teachers in the Scottish Laptop Computer Project found it difficult to keep up with marking their output (Turnbull & Gilmour, 1991).

Peacock and Breese (1990) pointed out that efficient text editing on a computer is most useful to writers who revise their work, whereas many students do not. However, Stradling, Sims and Jamison (1994, ch.2) found that students became more aware of the writing process, perceiving it as requiring continual review and revision. Redrafting became less onerous and students were more willing to refine their work on a portable, continuing work in their own time and space. Many teachers reported seeing more purposeful work, more attention to detail and greater accuracy with better spelling, punctuation and grammar. Students seemed pleased with and proud of their end products, frequently describing their work as 'professional', and their grades improved (Stradling, Sims & Jamison, 1994, ch.3). These results override the reservation expressed by a couple of teachers in the Pocket Book project, namely that increasing use of computers may hinder the development of other skills such as writing and social skills. In direct contrast, other teachers felt that the Pocket Book was especially good for those students whose writing and spelling skills were poor. Furthermore, their unobtrusive nature means that portables encourage students to communicate face to face with each other much more than desktop machines do (Bowell et al., 1994). (This issue is followed up in section 6 on cooperative working.) Teachers in one school reported that the benefits of portables extended beyond writing to stimulate high levels of participation in discussions and better recording of the ideas emerging.

However there was a lack of agreement between teachers about improved content of written work. Portables offer greater spontaneity and the opportunity to record ideas immediately without the compulsion to concentrate on presentation and worry about mistakes (NCET, 1993). However, the security this offers can be counter-productive; more planning may take place when writing by hand (Peacock & Breese, 1990). Where improvements were perceived, they were attributed to greater thoughtfulness of students when redrafting their work on portables (Bowell et al., 1994). Some students were successfully encouraged to experiment with different ways of expressing an idea when the original remained easily accessible; choosing the clearest and most appropriate way is an important skill to learn and some successfully restructured their assignments to improve the clarity and flow of text. By contrast, students with limited access to computers tended to confine themselves to corrections and improving presentation. Some less able primary children showed a similar preoccupation when using portables, leading staff to examine ways of developing the thought

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[15] Bowell et al., 1994, provide further information about technical issues and offer examples of enterprising strategies used successfully by schools in the NCET scheme for overcoming problems with printing and downloading to desktop machines. Gardner, Morrison, Jarman, Reilly & McNally, 1994, also discuss in some detail the operational issues around using portables.

processes associated with editing and redrafting; a number of potential strategies were identified but these often need to be individually tailored. Length of time may be a critical factor for less able children, as the controlled study of 56 learning disabled students by Yau, Ziegler & Siegel (1990) demonstrated. The researchers found some development of thinking strategies after 1 year but limited revision and evidence of reorganisation and refinement of ideas. Nevertheless these kinds of improvement did occur during the second year of the study, suggesting that for this population at least, a couple of years of portable computer experience may be necessary before “the technology is completely at their service as thinkers and writers” (*ibid.,* p.31). However, other studies of students of mixed ability using laptops confirm that substantial improvement in their ability to organise ideas effectively and communicate through writing do occur within a year (e.g. McMillan & Honey, 1993).

Further related information comes from a study in 9 primary and secondary schools by a research team in Northern Ireland. The Pupils’ Learning and Access to IT (PLAIT project) set out to assess the impact of high access to computers - in the form of laptops - on learning in three core disciplines. Gardner, Morrison, Jarman, Reilly and McNally (1994) reported that quantitative measures of the impact on academic performance after extensive use over one school year yielded minimal differences between experimental and matched control groups (of about 200 students each) in mathematics and English. A non-significant difference between the secondary school groups on APU assessments and creativity measures suggested that the use of portables did not significantly enhance writing quality and may even “encourage a somewhat structuralist approach to the detriment of creativity and style” (Gardner, Morrison and Jarman, 1993, p.15), although the reverse trend was true for primary students. By contrast, the qualitative data were more encouraging: the English teachers observed by Gardner et al. (1994) were unequivocal in their view that content and presentation had improved and that quality and effort were exceptional (some students were working two levels above that expected of them). As in the NCET evaluation, students were reported to be more prepared to experiment with their writing and showed greater confidence in expressing themselves. They learned to write straight onto the machine. The authors proposed that the computer-based spell-checking and redrafting activities sufficiently increased students’ awareness of and practice in these skills to transfer to handwritten work.

Some students - particularly at primary level - find writing by hand tiring and key pressing much easier. Nevertheless some others type more slowly than they write, and may take much longer to complete work on the portable if they are not given sufficient opportunity to develop their own rapid keyboard approaches (albeit two-finger ones), as did the primary students observed by Turnbull and Gilmour (1991). There may be a case for regular typing tutoring to overcome the preference for handwriting short tasks which persisted in the small study of 11-year-olds undertaken by Peacock and Breese (1990). Although Anderson-Inman, Knox-Quinn and Horney (in press) found that good keyboarding skills were linked to levels of adopting portable computing innovations, the ability to produce legible assignments far outweighed any frustration of slow keyboarding. Most teachers in the NCET evaluation agreed that production of second drafts is usually much quicker anyway, and most secondary school teachers reported that assignments were longer if completed on a portable. Moreover, keeping background material on working files in the portable led students to offer more evidence substantiating their conclusions. These findings related mainly to English lessons but they have implications for all curriculum areas, including mathematics, where students might experiment with different means of solving problems, knowing they could easily return to previous working.

### *Impact on mathematics activities*

The evaluators of the NCET scheme indeed considered mathematics to offer great scope for using different kinds of portables, often in conjunction with software appropriate for National Curriculum (NC) work, such as *Logo* or *Derive*, which is relevant to algebra work at Key Stages 3 and 4 (ages 12-16). Some of the practical advantages of portables outlined above were particularly true in mathematics. For example, the minimal demand on workspace left students free to work with other materials such as workbooks, multi-link cubes and 3-D shapes. Some teachers reported that students were introduced to the new software more quickly and effectively using portables than if it had been done in the computer room or on a classroom microcomputer. Most significantly, though, teachers in one secondary school using *Derive* on palmtops noticed that the students began to conduct their own investigations, asking 'What if...?' questions once they had the facilities available to pursue the answers. The software was pushed to its limits. Students were able to exploit opportunities to gain insights into mathematical processes by testing their intuitive notions and predictions using such software, whose dynamic visual displays help to introduce complex and challenging mathematical ideas and processes. The spreadsheet, database and graphing facilities of notebook and palmtop computers offered similar benefits and were reported to be useful in many curriculum areas. They helped students to integrate quantitative data more effectively into their essays in Humanities. The spreadsheet function provided a flexible, convenient and interesting means of recording and analysing data in several subjects, but also offered good opportunities for involving the learner in actively interpreting mathematical ideas.

Less enthusiastic results emerged from the PLAIT project by Gardner, Morrison and Jarman (1993), where no significant improvement in performance on general mathematics tests was detected after a year of using laptops. However, this may reflect the inappropriateness of the test; specific IT-related skills such as data handling were not tested whereas performance on science tests comprising data handling, especially graphical interpretation questions reflecting the content-oriented use of portables and assimilation of scientific principles, was significantly better in the experimental group. By contrast performance was poor on the higher level process-oriented demands of mathematical reasoning despite a focus on process-based activities, such as *Logo* problem solving, in the classroom. Gardner et al. (1994) pointed out that IT-related NC statements of attainment (which the study used as its framework) generally have a process focus, with few concessions to content, yet an evaluation of NC assessment by the School Examinations and Curriculum Council in 1992 indicated that process work in the classroom may not be perceived as normal class work and few teachers actually assess content during investigations. The suspicion that delivery of IT-related mathematics tends to be contrived outside the normal mathematical context was endorsed. Further support comes from the information offered by the researchers - without explanation or comment upon teaching quality - that laptop activities such as spreadsheet work were not perceived as being particularly mathematical (Gardner, Morrison & Jarman, 1993). Students were apparently unable to understand underlying principles or to articulate a rationale behind these activities (in contrast with IT facilities related to their English and science work: word processing, data logging and graph production). This appears to conflict with an attitude survey linked to the study, which indicated that a greater proportion of the experimental students in fact perceived mathematics as relevant to the real world (Morrison et al, 1993) and the conflict is not satisfactorily explained.

On a more optimistic note, the qualitative data obtained during the Primary Laptop Project undertaken by Ainley and Pratt (1995) showed learning gains after only 3 weeks of laptop use. In one case, pairs of Year 5 students worked on an activity with a data handling component, investigating variables affecting a toy car rolling down a slope. The researchers described the *movement* they observed as a consequence of portability of the machines: not only physical movement as the students collected their own data, but data flow between different software components. The students naturally combined text, graphics, tables and graphs in their written reports and were motivated by being able to illustrate a document with data presented in multiple forms. This fluidity of data enabled them to reorganise and re-present their ideas very easily; examples were offered to illustrate how this helped children gain new understanding and also highlighted misunderstandings, offering opportunities for teacher intervention. Ainley and Pratt propose that this process led to *portable mathematics*, enabling students to transfer mathematical understanding to new contexts. They described, for example, how students who had never previously drawn a scattergram appeared to access the concept directly from the portable computer after the facility was introduced in the meaningful context of the car activity. The students interpreted the graphs easily and extended their use to a follow-up investigation of the diameter and circumference of wheels, quickly noticing that the points were roughly linear and using the graphics tool to add the line, intuitively understanding its significance. The authors explain how the laptop facilitated portable mathematics here through allowing easy movement between different representations of the data and making the powerful concept of scattergrams accessible as a simple technique which generated something concrete and useful. The children's natural inclination to learn about the computer and to share their ideas facilitated the process, and careful planning of the sequence of activities offered the chance to test the new technique in a different context. (Further examples of graphing activities using IT are described in section 3 of this review.)

*IT skills*

Most of the NCET-funded projects yielded considerable evidence of rapid development of IT skills (by both students and teachers) through using portables, particularly where accessibility was high. Examples were offered of students of all ages and abilities learning to apply these skills effectively, again illustrating the fluidity of data: students combined artwork with text, incorporated graphs and tables into records, used databases and spreadsheets with ease, copied and pasted from one piece of software to another (Bowell et al., 1994). There was also evidence that primary students had become able to work directly on screen rather than drafting ideas on paper first. However there were some doubts about the suitability of portables at Key Stage 1 and Stradling et al. (1994, ch.3) concluded that more research into appropriate machines for IT novices in this age group is needed. Teachers also acknowledged that students who had most extended their IT skills and explored the machines thoroughly - through trial and error and self-help tutorial programs - were already the most proficient. Students with limited IT skills appeared to need more structured induction and continuing support. Encouraging the most able IT users to act as mentors to other students (and even staff) seemed fairly successful. Bowell et al. (1994) stress the importance of teaching IT skills generally rather than allowing students to learn about software by discovery. Able students can display confidence but be using inefficient approaches which may go undetected unless students are observed while they work.

*Home - school links*

*Arc* (Autumn 1994, p.16-17) points out that portable computers can bridge the gap between home and school, not only by being affordable by parents but by supporting children not in school owing to illness or mobility problems, for example. They can also play a major role in homework, apparently helping to improve writing, spelling, presentation and fine motor skills. (Indeed the parents surveyed by Robertson et al., 1995a, reported that they were allowed greater involvement in homework, which had also become more enjoyable.) Home use of Pocket Books in one school was said to improve home/school relationships and raise parental interest when students (about half of the pilot sample) were able to take machines home (*Arc*, Autumn 1994, p.21).[16] This opened up a much wider educational discussion with parents, exploring school policies on all aspects of managing children's learning. It also enabled parents to see for themselves that students acquire IT skills quite differently from most adults, i.e. through trial and error and learning from others.

NCET (1993) endorse the general idea of workshops for parents to help develop their IT awareness and to discuss any concerns about introducing portables computers into the home environment. Such a scheme was remarkably successful in the case of the London Docklands Project where parents were also ask to sign contracts indicating their interest and commitment to borrowing the machines (National Literacy Association, 1996). In some schools 90% of parents attended the training workshops, increasing their own motivation and their involvement in the children's learning process.

[16] Concerns about damage and loss of machines were largely unfounded.

The ability of students to quickly become experts in manipulating portable technology and to teach other family or self-esteem and others' expectations of them were observed. The project coordinators concluded that literacy can thereby become a communal activity and a focus of the home as well as the school, as community members was another benefit highlighted by the project; consequent increases in children's those around the student become involved in helping with writing and other learning activities.

A final example comes from a recent intiative in the US: the 18-month-old 'Anytime Anywhere Learning' scheme involving provision of constantly accessible personal laptops to 8000 teachers and students in 52 schools (John, 1997). The cost is borne jointly by the parents and the schools; parental involvement is attributed to their financial investment along with workshops provided for parents and their desire to improve their own and their children's competence with IT.

*Independence, ability and differentiation*

The privacy afforded by a personal machine encourages experimentation, self-help and individually paced work, playing a critical role in enhancing learning. The NCET scheme evaluators considered the portable to complement and support the individualised learning approach so common in secondary school mathematics. Individual work may need to be suspended briefly while students are familiarised with the machines, but they can then be used as and when needed. Taking the machines home seems to allow students to complete work more quickly, to develop and build on the mathematical ideas and procedures they have already grasped. Thus "the portability supports differentiated teaching: reinforcement of learning for some, extension of learning for others" (Stradling, Sims & Jamison, 1994, p.8). Some teachers found it easier to analyse where individuals were making mistakes, not only identifying them on the screen but taking the learner back through the contributing processes (Stradling et al., 1994, ch.3).

Ability was a critical factor for teachers to consider: 'average' ability students tended to lose interest first. Most able students persisted in using portables and extended their use where possible, whilst slower learners appreciated the improvements regarding presentation, spelling and calculations and continued using portables with support from staff. A similar willingness of more able students to redraft and reorder as opposed to correcting text was noted by Turnbull and Gilmour (1991). Stradling et al. (1994, ch.3) maintained that a differentiated approach - with teachers taking the initiative in monitoring individuals - combined with hands-on experience offered greater challenges than usual to average and below average students (for example where students connected sensors or probes to notebook computers). It also proved crucial in avoiding demotivation of the minority of learners with handwriting and motor skill problems; these students were easily daunted by early setbacks. Bowell et al. (1994) found that in some subjects such as mathematics, portables (spreadsheets, geometry software and graphic calculators) can enable teachers to make higher demands on the most able students, helping them to understand more at an earlier stage. However, one project which issued very able students with palmtops found that the students' high initial expectations meant that they soon outgrew the systems, despite a significant increase in IT capability.

Anderson-Inman, Knox-Quinn and Horney (in press) have investigated the issue of ability differences in levels of adopting portable computing innovations in some detail. They studied a class of learning disabled students using laptops in an American secondary school. The students were taught a variety of computer-based independent study strategies for information recording, organisation and manipulation. Three levels of adopting these strategies were identified: (a) skilled Power Users who used the technology extensively, independently and appropriately; (b) skilled Prompted Users with positive attitudes but requiring prompting and assistance to move beyond the basic applications known to be useful for assignments; (c) Reluctant Users with basic skills but dependence upon teacher supervision. (About a quarter of users fell into each of categories (a) and (c), the other half being Prompted Users.) The Power Users were described as highly motivated and persistent in the face of obstacles; they completely 'partnered' with their computer (Saloman, Perkins & Globerson, 1991), relying on it to help them realise their academic potential. By contrast, Reluctant Users failed to apply what they knew. Since the three levels were significantly related to intelligence and reading ability, the authors concluded that these may be good predictors of (learning disabled) students who will adopt and effectively use computer-based strategies.

Further investigation is needed to establish whether this relationship generalises to the wider student population, but it is corroborated by Robertson et al. (1995a) who found a correlation between non-verbal reasoning tests and attitudes to computers. Two possible explanations offered were: (a) students with low scores may perceive computers as complex, incomprehensible machines or (b) students with high scores may belong to a higher socio-economic group with greater access to computers at home. In accordance with (a), which is quite feasible, the indication from the work reviewed above that high ability students make more use of portable computers could reflect not greater previous experience but instead these students' realisation that such a tool can - initially at least - provide an interesting challenge at their own level of ability and can help them achieve more. Although it has been notoriously difficult to show cognitive gains after exposure to computers used for programming purposes (e.g. *Logo*), there might be a two-way relationship between effective use of portables in other ways and the development of higher order cognitive skills. This deserves some empirical investigation.

Primary teachers in the NCET scheme also used portables to support differentiation by task. They often preferred to limit the number of machines used in the classroom as this fitted better with small-group work and reduced classroom management problems. Speed of operation was another key factor in facilitating differentiated learning, as discussed below under 'Graphic Calculators'. However, in most projects portables were not used to support differentiation by task or learning needs, partly because familiarisation of teachers and students with hardware and software often took longer than expected.

Several projects moved their focus outside the classroom and investigated how portables might support independent, flexible and enquiry-based learning in a variety of curriculum areas; apart from the more conventional scientific field studies, portables were used in public libraries, museums etc. (Stradling et al., 1994, ch.3 & 6). Unsurprisingly, schools committed to open and enquiry-based learning were quicker to realise the potential of incorporating portables. A few projects explored their use (a) for supported self-study, with students receiving special training for using the machines for home-based work, or (b) in open learning areas, where students could use the machines individually or in small groups. They could also book out machines for individual use in the classroom or at home. (In future, school libraries may lend out laptops, as in one school in Melbourne: Thomas, 1991). In open learning areas, use of portables may be more effective simply because students can operate with a number of files and computers at the same time, taking notes, entering and analysing data and reorganising material as they work. Open access to portables facilitates independent work by helping students to make more effective choices between alternative ways of tackling a task; ideally, this involves planning but in practice, self-initiated use of portables is often inefficient. Stradling et al. pointed out that in some cases using a micro or doing work by hand would have been easier and quicker. There was some evidence for more discriminating use of portables in the second term of the evaluation period.

The report concludes that innovative use of portables requires structured opportunities and support for both students and teachers to explore the potential of portables in an open-ended way, although this may be time consuming. Teachers need to monitor usage and ensure that it remains purposeful and task-related, to help students plan effectively, and to invest time in developing quality activities. Another objective central to one project was self-sufficiency in the use of appropriate software and in managing and operating the machines (Bowell et al., 1994). This makes a lot of sense since it minimises organisation for teachers as well as contributing to students' confidence and autonomy (although it is harder when machines are shared and ownership is temporary, as in most of the NCET projects).

Other key factors in creating opportunities for independent learning at secondary level appeared to comprise frequency of access both for classroom and home use and time to experiment. The portables allowed basic tasks concerned with data logging, spreadsheet analysis, graph production and other ways of presenting data to be done much more quickly than by hand. This left more time to explore and compare different ways of tackling a task, to interpret, discuss and reinforce what had been learned, and to extend learning by hypothesising about new variables or data. At primary level, some teachers observed that exploration and experimentation led to an increase in confidence with using IT. Key factors here were the small size of machine and encouragement from seeing peers using the machines simultaneously; students felt less exposed than when using the only desktop machine in the classroom. In some cases their greater willingness to experiment meant that they attempted tasks not usually encountered until Key Stage 3.

*Perceptions, attitudes and motivation*

Fung, O'Shea and Hennessy (1995) explored the impact of the Pocket Book project experience upon attitudes of students, teachers and parents to computers and to the usefulness of portables in producing RoAs. First, the most striking finding from our observations was the enthusiasm for using the Pocket Books expressed by almost all students consulted; we detected the same highly motivated and purposeful yet relaxed and unselfconscious attitude to the use of computers in the classroom described by Turnbull and Gilmour (1991). Secondly, the ease which which they came to grips with the machines' facilities was notable. Manuals had been issued with the machines but they were rarely in evidence in the classroom and the preferred mode of learning appeared to be through exploration and discovery. Most students found the machines so user-friendly that they were able to use them productively after less than half an hour of unguided interaction with them. The results from 40 matched pre- and post-questionnaires yielded marginal differences in attitudes in some cases, but confirmed our observations about perceived ease of use and simplicity of the Pocket Book interface after only 3 weeks; there was a clear change in students' assessment of how easy or difficult it was to use computers generally. There was some increase in the numbers of students finding computers 'useful' and 'essential' too, another encouraging outcome. The enormous excitement of anticipation inevitably dissipated, but the level of enjoyment was higher than predicted. Likewise, most of the staff (n=8) considered that the experience had been worthwhile for themselves as well as for the students. Their expectations concerning general benefits of Pocket Book computing and the role of the technology in raising standards of RoAs were met on the whole.

Further information comes from the longer term study reported by Robertson et al. (1995a). As before, students generally found the computers relatively easy to use, particularly the spell check, database and calculator facilities (over 90% students when questioned after 5 weeks) and the word processor (86%). Ratings for the spreadsheet and its graphing facility were lower; slightly fewer were familiar or confident with the spreadsheet (43%) than the graphing (52%), possibly reflecting concentration on this aspect of spreadsheet use. The latter two figures increased over 5 months although all figures dropped again after 8 months. A likely reason was the students' realisation with experience that their earlier estimates of knowledge of the applications had been unrealistic. This was confirmed by an experiment in which all 10 subjects failed to deal successfully with an error message on the Pocket Book; learners are not always aware of shortcuts and may not understand why common errors arise. Similarly, computer attitude scores dropped for the experimental class and remained constant for the control group; this difference was not significant but nevertheless unexpected. It may reflect a better appreciation of what using a computer actually involves, especially since this was their first experience of IT teaching in school and their previous view of computers was probably as a form of entertainment. This warrants further investigation.

Students' enthusiasm and willingness to teach themselves how to use the machines were again evident (Robertson et al., 1995a; 1997). The majority of the staff similarly found the Pocket Book interesting, enjoyable and easy to use after 4 months and 60% used it frequently, constant accessibility being a major reported benefit. Many became dependent on it and several claimed it improved their efficiency and self-confidence (although a few machines remained in their original boxes!). They considered it a useful tool for students and a potential aid to higher-order thinking skills. Parents made no negative comments and generally believed their children had learned from using the machines. They reported improvements in children's knowledge about computer applications and in computer skills, and greater confidence in computers. OU students had the most positive attitudes to computers, reflected in their more frequent use for certain purposes, although they portrayed as little anxiety as the school students. Teachers and parents portrayed significantly more. One implication is that school and University students feel more strongly that computers are a natural part of our culture and of their lives and one which they can appropriate more easily; they expect to be able to understand and enjoy using them. By contrast, teachers desire a specific reason for using computers and may worry about time taken up by training. Robertson et al. (1995a, p.53) propose:

> Pupils may see them as no different from a hi-fi or video recorder, even as hi-tech toys. The teachers, on the other hand, may be perceiving them in a pedagogical context where computers are relatively unexplored tools that they are as yet inadequately prepared to use.

The researchers also found a positive correlation at the beginning of the project between high frequency of use (by students and teachers) and low computer anxiety. Previous experience was also correlated with intention to use computers and enjoyment, although not with confidence or competence ratings (Robertson et al., 1995b). Students saw themselves as more likely to use computers in future than teachers did, and (related to the anxiety difference) they rated their own enjoyment of computers more highly than did teachers although they assessed their own competence similarly.

Studies of attitudes to laptops also produce consistently positive results. NCET (1993) report that laptops were more popular than networked machines with less frequent IT users in a Welsh study of three schools. In a survey of over 200 year 7 students in an Australian school, almost all students described learning with these portable tools as "fun" and reported that they had obtained new skills (Loader, 1993). The students were appreciative of the computer's portability and agreed that it had allowed them to do things that could not be done in other ways. They felt more organised and more independent; 95% noted that teachers trusted them to work alone on projects. Attitudes to specific subject areas are less clear-cut, however. Morrison et al (1993) reported that positive effects of high access to IT on student attitudes to the three disciplines of English, mathematics and science, and towards school in general, were relatively marginal on the whole. Nevertheless the one-off questionnaire highlighted many encouraging non-significant trends, and significantly more students in the experimental group portrayed positive attitudes to English and perceptions of classwork as important. The authors report that positive impacts were confined to instances where the students' process-based IT work (e.g. mathematical problem solving using *Logo*) transferred to content learning, which it mostly failed to do.

Further important information regarding students' own perspectives on portables was derived from the questionnaire survey of over 500 students by Stradling, Sims and Jamison (1994, ch.3). Their responses to a question about what they liked about using portables illustrated a marked difference between attitudes of primary and secondary students; the former group focused heavily on technical and operational features of the machines while the latter focused much more on portability (between classes, to the home and the field). Significantly more secondary students also mentioned the impact on their learning and quality of work. The survey found a strong positive relationship between frequency of use and emphasis on the impact on learning, including improved school work and IT skills; occasional users tended to focus on portability instead. Another interesting difference emerging was that more regular users took the initiative in deciding when to use a portable; they also agreed more often that portables saved them time. This is linked to the observation that some students became less motivated to use the portables if it took them longer to complete a piece of work than doing it by hand (the evaluators pointed out that some such students were not yet very familiar with the machines).

Conversely, an advantage of portable computers arising from evaluation of the NCET scheme (and the study by Turnbull & Gilmour, 1991) was the greater willingness of students to spend their own time and effort on completing work and on presentation (Stradling, Sims & Jamison, 1994, ch.3). Portables can facilitate sustained effort and concentration on the content of work by eradicating boring and repetitive handwriting tasks. Thus, portables were especially motivating for students whose attention span was short; some of these students undertook more substantial pieces of work when using the machines. Teachers in our own and other studies have reported that students with poor writing and spelling skills or special educational needs find the experience of using a portable particularly motivating. Difficulties in forming and recognising letters are eased and the usual frustration of not being able to produce legible written work dissipates. Obtaining better results with much less effort on the computer, and checking one's own spellings, subjects students to far fewer teacher corrections and increases enjoyment, self-esteem and motivation (Fung, O'Shea and Hennessy, 1995). According to Bowell, France and Redfern (1994), self-esteem of students with messy writing is also increased because word-processed work can be displayed in the classroom. Moreover the hurdles of getting started and of producing words in the correct order are overcome by the knowledge that later amendments are possible. The unique drafting potential of writing on a portable computer also boosted the confidence of students with learning difficulties during the London Docklands Project; the programme proved so absorbing that the concentration span of some students increased from 5 to 30 minutes (National Literacy Association, 1996).

Access to desktop machines can produce similar benefits for low-attaining students in terms of building self-confidence and overcoming isolation. This is exemplified by the powerful story of Tanya, an 11-year-old apparently unable to spell or write, whose transformation after encountering a computer culminated in presentation to the school library of a volume of her own poems (Turkle, 1984, ch.3). However, the privacy which portables afford increases the beneficial effects of the technology as a tool even further and offers students greater control. In addition, using portables actually provides students with special needs enhanced status with their peers and overcomes embarrassment at coming out of class for special lessons. Studies with students with learning disabilities tend to converge on the conclusion that using portable computers is beneficial in several different ways: it increases independence, encourages more positive attitudes towards work, and improves writing, notetaking and keyboarding skills (e.g. NCET, 1993; Price, 1994).

The majority of project leaders in the NCET scheme reported improved attitudes to school work and homework, plus increased confidence in using IT, through regular use of portables by students across the ability range. Some teachers even reported that increased motivation led to improvements in behavioural problems such as punctuality and attendance (Bowell, France & Redfern, 1994). Time and effort saved with portables were not the only motivating factors; portability and compactness helped to overcome feelings of intimidation by allowing private use or consultation of peers as desired. Accessibility also helped improve motivation by overcoming the frustration of waiting for turns on desktop computers.

Similar observations have been made of students using graphic calculators in the mathematics classroom. Ruthven (1992a) found that despite the occasional frustration during familiarisation with the new technology, knowing that the resource would be available on demand was apparently a powerful incentive to use it. Indeed most of the 'A' level students who participated in the "Graphic Calculators in Mathematics" project were described as making confident and spontaneous use of the calculator facilities by the end of their first term and virtually all who had personal access to a GC preferred using it to their previous calculator, even for calculating. Another survey of American high school students' attitudes to graphic calculators found significantly more agreement after a few months of use on items portraying calculators as challenging, enjoyable and helpful in learning difficult mathematical topics (Dick & Shaughnessy, 1988). Studies with college students (e.g. Paschal, 1995; Scott, 1994) have also found that introducing graphic calculators improves attitudes toward mathematics, helps to relieve mathematics anxiety and equips many students with new confidence, although accompanying increases in achievement are not always evident.

Another key factor in raising motivation and quality of work produced during the NCET scheme was the sense of privilege and responsibility for an expensive machine and for managing their own learning. At one school, girls in particular saw themselves as 'business ladies'![17] The students developed a mature outlook to caring for the equipment. This sense of responsibility resulted in some students producing work consistently for the first time (Bowell et al., 1994). However, this study and most of the others reviewed here lasted less than 1 year so it is unknown whether enhanced motivation would persist over the longer term. Indeed some schools in the NCET scheme reported a plateau effect during the second term, when the novelty began to wear off; frequency of use and enthusiasm for home use declined (although there were identifiable reasons in most cases). Nevertheless, while some degree of the increased motivation oberved by researchers and teachers is probably due to the novelty of access to a new, personal techology, the benefits described above for students with poor writing and spelling, and those due to privacy, accessibility and time saving are likely to persist after any novelty effect has worn off.

Moreover, Stradling et al. (1994, ch.3) assert that steps can be taken to improve and sustain motivation, as shown by their observations of secondary school students using graphic calculators for limited periods of time; the work was purposeful and the value apparent to students. Good induction programmes were critical, particularly those encouraging exploration of the machines' features. Differentiated support by a teacher, technician or another student could help; several projects offered effective peer group support. The most able students generally tended to be self-motivated but those with less developed IT skills needed more support and input from teachers to sustain initial motivation, particularly where they experienced widely different reactions from teachers in different classrooms; less able students were less resistant to the effects of computer illiteracy in teachers. Finally, most positive impacts on student motivation and attitudes were evident where effective induction programmes for staff had been established, including opportunities for them to use and experiment with the portables at home over the summer before the study began. (This was the case in the school we investigated, where support for teachers from the IT department was considerable and varied: Fung, O'Shea and Hennessy, 1995). Widespread discussion amongst staff about different uses of portables was also important.

*Student empowerment*

A major conclusion to emerge from the results of the Pocket Book project is that the experience of having a portable computer of one's own contributes to a feeling of empowerment. This feeling probably explains the improvements in attitudes and motivation reported in many similar studies. It gives students an experience of personal computing which is under their own control and accessible whenever they choose. Students rarely get the opportunity to appropriate sophisticated equipment of any kind for themselves and the very fact of being entrusted with a valuable machine increases their self-esteem. On the whole, those in our studies and elsewhere responded by looking after the machines with care. The use of portables proved particularly appropriate in supporting compilation of RoAs; the self-assessment RoA system itself incorporates encouragement of students to take more responsibility for their own learning (Fung et al., 1995).

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[17] St. Theresa's RC School: Final Report to NCET.

Another important factor in the students' positive attitudes is that they learnt how to use the Pocket Books mainly by themselves and through working collaboratively. It was a pleasant change for the students not to have to rely on a teacher for information; for once, they were largely autonomous and this gave them a sense of ownership, albeit temporary. The fact that the portable computers could be used at home and for purposes other than schoolwork supported this special relationship with their Pocket Books. For a few individuals with extensive computer skills, the project provided an opportunity to take a more active role in class and to take on a teaching role - not only with their peers but in some cases with their teachers![18] These findings were corroborated by reports from other projects in the NCET scheme and obviously have implications for use of other portables in various contexts. Students occasionally becoming more proficient than their teachers is somewhat inevitable and has certainly been observed by other researchers. Some schools are already harnessing student expertise and organising accreditation for training skills.

One Swedish research project has designed its own palmtop computer with a pen interface specifically for students age 10-12. Its innovation is in a wireless link to the school database and thereby to resources on the Internet, through an intelligent agent who collects and stores information on the child's personal preferences (Pargman, 1995). This kind of facility lends credibility to the researcher's claim that "the teacher will become less of an omniscient source of knowledge and more of a team leader" although "more and not less important in the school of tomorrow" (*ibid.,* p.4). However, there is a conflict between the investigative, empowering culture of the computer and that of the traditional classroom, as outlined by McFarlane (1995a). Some of the research reviewed here reports that some teachers exercise strict control over use of portables in the classroom and are reluctant to permit students a significant degree of autonomy, with unfortunate consequences for student learning and motivation.

*Active participation and investigative learning*

In some cases, widespread use of portables simply extends opportunities for more students to gain regular hands-on experience of using IT, as the NCET scheme demonstrated. Overcoming difficulties with writing and spelling, and enhancing presentation through facilities for editing and visual display (including graphs), could have been achieved using conventional hardware, for example. However, the general view of the project teachers and some students was that there was insufficient time, access or privacy to experiment to the same degree and so to carry out the extra activities observed as effectively using a desktop machine, especially when portables could be used at home (Stradling, Sims & Jamison, 1994, ch.2, 3). Students were also able to spend their time on the portables doing work for subjects which rarely employed IT. While teachers could have supported differentiated learning through interacting with individuals in the computer room, they often preferred to work with portables in their own classrooms. Moreover, instead of passively watching a teacher demonstration or huddling around one or two desktop machines, students can all have a chance to try out something new for themselves.

In other cases, there are specific benefits related only to use of portables. Work was observed that could not be done on desktop machines, and additional learning developments took place. For instance, one science investigation of the effects of gravity with Year 5 students led to the observation that some students showed a surprising ability to generalise from trends shown in their graphs and links between variables in their experiments (Stradling et al., 1994, ch.3). Other examples are mentioned throughout this review and in NCET (1993). This phenomenon of self-discovery and students spontaneously taking the initiative in extending projects in their own directions was also observed in the Scottish Laptop Computer Project, despite its stated aim of

[18] One boy in particular was able to show his teacher, a self-confessed computerphobe, how to port text from Pocket Books to a laptop for marking.

providing an additional resource within the class without altering the normal curriculum. (For example, a number of students collaborated in producing their own novel on the laptops.) The issue of relationships between portable computing and other learning activities, IT-based and otherwise, would benefit from further exploration. The NCET scheme illustrated that it is perfectly possible - indeed advantageous - to use several kinds of computer in a classroom, when each is suited to different tasks (Bowell et al., 1994).

A fundamental distinction from more typical curriculum activities is that students with access to portables - whether individually or in groups - become actively and productively involved in their own learning activities from beginning to end. All of the researchers in this area in fact concur that this first-hand experience is a key benefit of portables. Various research studies indicate that knowledge is less likely to remain inert when acquired in problem-solving mode rather than in factual knowledge mode (e.g. CTGV, 1992). A literature survey by Aitkenhead (1996) highlighted the importance of a reflective style of teaching and learning for improving understanding, namely one where students are required to predict and justify their predictions before exploring, recording results and drawing conclusions. Portable computers can be highly useful for promoting this kind of investigative learning. As well as offering students opportunities for involvement in setting up, running and reporting on the whole investigation, the speed of operation also means that students often have more time for exploration and hypothesis testing. Stradling et al. (1994, ch.3) concluded that the portable computer and graphic calculator can thereby help to promote investigative ways of thinking and several examples within this review illustrate this. However, the degree of student involvement is critical. The work of Ainley and Pratt (1995) warns us that the power of portable technology can backfire, as it did when Year 7 students used it to produce ‘quick but meaningless graphs’, indicating that they had no real grasp of the nature of the data from historical sources which they were manipulating. Subsequent activities with Year 5 students (as described earlier) were more carefully planned to involve students in collecting the data initially and resulted in greater understanding of it.

Portables can enable students to decide for themselves when it is appropriate to use computers, i.e. when they will help with a particular task. This recognition develops after the novelty value has worn off; it is highly motivating and encourages independent working (Bowell et al., 1994). However it depends on portables being appropriated for home as well as school use and on machines being generally available in the classroom. Ideally, they should be available during every lesson (with ready access to printers too); a minimum degree of access may be necessary to overcome the technical challenge of learning to use them (Watson, 1993). In practice, there is wide variation in the ways in which portables are employed in schools. While some schools restrict their use to specific activities, others endorse the notion of the computer as a readily available and portable, personal resource and they offer the freedom to use portables in any subject. One teacher view of the Pocket Book is as a timetabled, shareable school resource like books. Another view was expressed by the teacher who stated: “I’d like the girls to take them for granted as part of normal school life, just like their pen or calculator, rather than feeling that they’re something special” (Arc, Spring 1996, p.34). This relates to Papert’s vision of the computer as pencil, used “for scribbling as well as for writing, doodling as well as drawing, for illicit notes as well as for official class assignments” (1980, p.210). Portable computers used as an educational resource may similarly become ‘transparent’ and may well help to foster this productive aspect of computer use.

*Changing teaching and learning styles*

NCET concluded that “Personal ownership of the less expensive kinds of portables will increase far faster than many people imagine, probably to the point where they are as commonplace as calculators” (*Arc*, Autumn 1994, p.21). They propose that soon every teacher and student who wants a computer will have one, and that combining portability with electronic communications, new media and interactive learning systems can give rise to revolutionary new learning styles (Bowell, France & Redfern, 1994). Some secondary projects in the NCET scheme similarly viewed portables as helping to develop a *new learning culture* within the school, namely one of active, independent and investigative learning using IT extensively but most importantly, appropriately.[19] Linking back to our discussion of a paradigm shift, this moves away not only from the typically constrained association by students of computing with IT lessons, but also from the currently all too common use of portables as analogues of desktop machines, i.e. for traditional IT tasks.

We must remember, however, that portable technologies are likely to be used alongside other forms of IT and desktop machines will continue to be used. The latter have their own advantages, including larger screens and the ability to run more software, for example. There is no reason why various forms of technology should not be used as appropriate for different tasks and different users. Portable computers themselves may be used in different ways, including using a display screen with an overhead projector for whole class teaching; this technique is commonly employed already with graphic calculators. In the UK there is currently a political move towards encouraging whole class interactive teaching; this is reflected in the proposed ICT National Curriculum which requires students to learn to use a computer projected onto a large screen via engaging in class discussion. There is much controversy surrounding whether such whole class discussion can be effective, and the role of individual experimentation, as discussed above, should not be lost. Nevertheless there is probably a role for this kind of class use upon occasion, at least for introducing the interface to a new technological tool, and possibly for introducing or reinforcing complex mathematical concepts or procedures. This is a further avenue for research.

All kinds of classroom learning with portable technology necessitate careful planning and management and the role of the teacher remains critical. That role is described by Ruthven (1992a, p.9) as being to create the situations "from which important concepts and relations are likely to emerge, and, through sensitive intervention, to support students in exploring these situations and clarifying and organising their ideas". For this new teaching style to be effective, the curriculum must clearly develop in parallel, through collaboration between IT and subject specialist staff. For instance, Gardner et al. (1994) have warned that IT cannot simply be 'bolted on' to an already contrived separation of process skills from understanding content if it is to be truly integrated into the core subjects or the attempt to raise IT literacy overall could be undermined. Unfortunately IT has to date had a far greater impact on society than on the school curriculum or on teaching methods, despite generally positive attitudes of teachers towards computers (Dupagne & Krendl, 1992; Watson, 1993).

Stradling et al. point out that the evolution of a new learning culture takes time; it entails raising the IT awareness, capability and confidence of staff and students and often significant changes of teaching style, using a more open-ended approach. Specific practical suggestions for teachers made by project coordinators for encouraging a new IT culture are outlined (Stradling et al., 1994, p.19). Key factors found to influence teacher motivation and confidence were (a) the level of *training* and support provided and (b) the time available for *familiarisation*. Indeed 2/3 of teachers do not use computers in the classroom because of a lack of opportunity for practice and development of their own confidence. The lack of computer literacy among some teachers and their slowness to recognise the potential uses of portables constitutes a major obstacle to their innovative use, leading to unnecessary restrictions on students (*ibid.,* ch.3). This resistance to new forms of technology has been observed by others (including during our own pilot study with palmtops: Fung, O'Shea and Hennessy, 1995). It highlights the fact that most of the abovementioned advantages of portables

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[19] This is reminiscent of the results of the landmark Calculator-Aware Number (CAN) Curriculum Project which offered 4000 primary students unlimited access to numerical calculators (abandoning standard algorithms). Clear mathematical gains and a significant impact on the curriculum (large numbers, decimal, negative numbers were introduced earlier and became the subject of investigations) were evident, as were changes in teachers' roles and classroom organisation (Shuard et al., 1991).

depend upon teachers' willingness to use them flexibly and to confer greater responsibility upon students. Yet the review by Dupagne and Krendl (1992) shows that some teachers perceive a threat to their role as educators. The demands on teachers are indeed great: along with developing confidence in using new technologies, they must also embrace sometimes radical curriculum changes and teaching styles. Resistance to innovation is well-established (Olson, 1988) and may reflect social, cultural or political barriers within a school or responses to perceived external demands, as in the case of the CAN project, whose ultimate impact in the primary classroom was limited.[20] Current external factors include the time pressures resulting from an emphasis within the National Curriculum, Ofsted inspectorate and national league tables on teaching the 'basic' language and mathematical skills. The first battle, though, is to overcome the common inadequacies in basic IT skills of teachers (Ofsted, 1995).

By contrast, Stradling et al. (1994) observed that some teachers initially nervous about using conventional forms of IT became much more confident and enthusiastic after experiencing portables. This was often because the machines could be used privately, particularly at home, for learning through trial and error and without embarrassment in front of others (e.g. Cloke, 1996). This again tallies with our own study and with the correlation found by Robertson et al. (1995a) and a number of others (Dupagne & Krendl, 1992) between teachers' frequency of computer use and positive attitudes to computers. The opportunity for home experimentation made the teachers more willing to initiate innovations in their teaching (Bowell et al., 1994). Teacher reluctance to allow technology to 'overrun the classroom' was also overcome with experience in the American 'Anytime Anywhere Learning' scheme (John, 1997).

Using portables may also help overcome the concern of some teachers that students will possess more expertise than themselves, which of course is often true![21] In conclusion, Stradling et al. expect increased levels of skill and confidence to occur as more teachers gain significant experience with IT and portable computers specifically. A feeling of involvement is also crucial in introducing new technology initiatives so teacher consultation is essential. As with integrating conventional calculators into teaching, teachers benefit particularly from support and guidance in planning activities which have the potential for advancing learning (Ainley & Pratt, 1995; Hembree & Dessart, 1986). In sum, greater access to personal technology is only the first step.

Teacher input was also singled out as the most important influence on the impact of IT on learning in the ImpacT Report (Watson, 1993). While IT was found to increase concentration and motivation, its actual contribution to learning was inconsistent across subjects and age groups. This is an important finding, showing that we cannot assume that positive effects on motivation and attitude will improve actual performance consistently and significantly. Factors inhibiting the effective use of IT identified by both ImpacT and the PLAIT report (Gardner et al., 1994) included teachers'

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[20] An Ofsted report (1993) into mathematics teaching found that calculators were used in only 10% of lessons and most had no policy for their use. (Duffin, 1991) reports how extraordinary success during the first three years of CAN was tempered when the first cohort of students transferred to secondary school; teachers perceived the teaching of standard algorithms (abandoned by the project) as an expectation of parents and secondary schools and few were able to resist the pressure. Schools who participated in the CAN project are teaching standard methods these days, as Ruthven, Rousham and Chaplin (1997) discovered. They prefer to encourage mental calculation rather than calculator use, which is no longer taught effectively. Sadly, their study of 3 post-CAN schools showed that no long-term effects were visible in terms of students' grasp of number concepts, compared to control groups.

[21] However, some students, particularly boys, over-estimate their own competence in this area; see Sanger, 1997, p.15 for a wonderful example.

difficulties in understanding and learning to use software, and in troubleshooting students' problems. Understanding the pedagogic principles underlying certain programs was a significant problem for teachers, especially when the software required students to use an open-ended approach, e.g. generating and testing their own hypotheses was pre-empted by teachers' leading suggestions (Watson, 1993). Both reports indicate that these difficulties may have led to the choice of inappropriate software for the intended activities.

Other factors included students' difficulties in learning to use software; in contrast with reports from the NCET scheme, the PLAIT report claimed that teachers needed to spend considerable time explaining technical procedures. Teachers' problems nevertheless tend to be more pronounced than students' and the investment of time and energy required to familiarise themselves with both hardware and software before planning purposeful lessons must not be underestimated (Hammond, 1994). For example, the Pocket Book project (Robertson et al., 1995a) showed that teachers experienced more difficulties after four months than their students after one month's experience! Another obstacle to the influence of IT on learning described in the ImpacT report was that student cooperation was sometimes restricted to conversation about using the computer and software rather than about the topic being studied. The report concluded that unsuccessful cooperation concealed the value of some activities (Watson, 1993). The report also argued for - but, disappointingly, did not specify - a minimum threshold of access to IT, which students need to cross in order to become independent users of machines. IT may initially be labour inducing rather than labour saving. This issue needs further investigation.

Where IT is integrated into classroom activities it apparently has a positive effect on students' learning. Yet the research literature consistently shows that teachers often feel - and are - inadequately trained to integrate IT into their teaching (see Moss, 1992, and reviews by Dupagne and Krendl, 1992; Hammond, 1994). One-day INSET courses in IT are typical and can stimulate interest but fail to significantly change the culture of a school (Bowell et al., 1994). Moreover the many demands on teachers' time may preclude take-up of INSET opportunities offered, as acknowledged by several teachers during the Pocket Book project, where only a small minority attended initiation sessions (Robertson et al., 1995a). Several teachers admitted that they were unwilling to learn about the machines, which obviously does not bode well for their students. The conclusion drawn was that IT training needs a higher priority, including more time and commitment within the school. Of course, training needs to emphasise fruitful uses of computers (Underwood and Underwood, 1990). Learning to use portables has the advantage that training could be carried out in teachers' familiar surroundings. Unfortunately the most useful type of INSET perceived by teachers is most difficult to deliver: one-to-one tuition with an experienced individual. One Australian school did manage to tailor its training programme to individual needs and starting points; this paid off handsomely and staff developed new skills, greater confidence and interest in computers (Loader, 1993). Robertson et al. (1995a) proposed that other ways of sharing information could be sought, e.g. hints within a newsletter. Allowing staff to take portables home regularly for experimentation can also make a big difference (Bowell et al., 1994).

NCET is currently addressing the issue of teacher confidence and experience with IT in a new scheme which has already provided 1150 teachers with laptops containing CD-ROMs and training in using them.[22] The machines possess all the functionality of state-of-the-art desktop machines with the added advantage of being designated a personal resource for teachers themselves. The "Multimedia Portables for Teachers Pilot Scheme" investigated the impact of giving teachers the

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[22] Nearly £5.5 million was spent on portables (mainly PC-compatible) and teacher training in 575 U.K. primary, secondary and special schools (The Independent 4.11.96, Network Supplement, p.10, and NCET, personal communication). The teacher group was carefully balanced for age, gender, subject area and prior degree of confidence with IT. The initial scheme ran from June 1996 - June 1997 and was again externally evaluated. It was extended via an additional £1 million funding during 1997, supplying an extra 300 teachers with portables.

hardware (accompanied by various productivity tools and Internet access) on their levels of confidence and competence in use of IT. The results indicated that giving teachers portable computers significantly increases their productivity and use of IT in the classroom. The portability also allowed teachers to take the machines home for additional work; they spent many voluntary hours each week on their own professional development. A positive evaluation of the pilot project led to the recent Government announcement of a follow-up scheme to hand out a further 10,000 machines.[23]

Finally, students themselves are important potential agents of change. They are growing up in a new world which increasingly exploits technology in novel ways and they typically respond with enthusiasm rather than anxiety. Portable computers offer them opportunities for more personal, independent, cooperative and effective learning (Bowell et al., 1994). In our view, these opportunities are now ripe for exploitation.

*Summary*

The key factors which seem to affect the impact of portables on students' attitudes and progress in learning are:

- accessibility, flexibility and frequency of use - within and outside school
- portability and compactness of equipment
- interconnectivity with desktop machines and printers
- access to previous work; time- and effort-saving facilities and ease of use
- time, opportunities and encouragement for both private experimentation and peer support for students and teachers
- keyboarding skills and availability of technical training for students
- teachers' computer literacy, confidence and awareness of portables' potential
- teachers' receptiveness to innovation and student autonomy
- carefully designed, open-ended activities with active student involvement and opportunities for cross-curricular work
- differentiated teaching and support

The focus of our research project is on how portable technologies might enhance students' skills in and understandings of graphing, so we now turn to the literature from mathematics education which is concerned with graphing. The next section outlines the common difficulties which students experience in learning to produce and understand graphs, and discusses some ways in which they might be overcome.

## 2. Students' understanding of graphing: common difficulties

Graphs are deployed in many curriculum areas (e.g. geography and science) as well as being common in mathematics and in everyday life to present relational information; they are an extremely

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[23] The follow-up scheme will cost £23 million (TES, 24.4.98, p.10).

useful tool for examining patterns.[24] However, most students apparently leave school without any underlying understanding of graphs. Although they can read information from them and draw them proficiently, they are inexperienced in interpreting graphs (Kerslake, 1981; McFarlane, 1995a). The growing body of research on students' understanding and skill in graphing indicates that there are some typical areas of difficulty. A comprehensive review by Leinhardt et al. (1990) identified a range of misconceptions for upper elementary school students, including erroneous concepts of variable and of what constitutes a graph of a function; tendency towards linearity; a pointwise focus; translation between graphical/ algebraic representations; interval/point confusion; slope/height confusion; iconic interpretations; difficulties in choosing, constructing and scaling axes. The most commonly researched problems are elaborated below.

A critical source of difficulty is the fact that graphing uses one symbolic system to understand another (data patterns or algebraic functions). Students find it hard to apprehend graphs as abstractions and to develop a link between the algebraic and the graphical/visual forms. Learning to see this connection and to interpret correctly information provided by graphs is crucial to understanding the concepts of calculus, without which calculus is merely a collection of useful algorithms. Experienced students may understand the rule-oriented process of solving inequalities but not the underlying concept. One study of 400 college students found the more complex solution processes for solving an algebraic inequality to be considerably easier than visually interpreting a graph, after participation in 3 weeks of intensive instruction concerning graphs and functions (Dunham & Osborne, 1991).[25] The authors argue that secondary school experience with graphing and functions is typically limited and superficial. "Moreover, the process of learning to visualize and make interpretations about functions in the coordinate plane is remarkably complex and little understood by teachers of mathematics" (*ibid.,* p.36). Fortunately there has been an increasing emphasis on graphical representation (of functions and more generally) in the mathematical curriculum over recent years and a movement away from techniques such as solving equations by reading off points of intersection of two graphs. However literacy in graphical representations is a complex skill requiring careful instruction (McDermott, Rosenquist & vanZee, 1987).

Manipulating symbolic representations is difficult in itself for many students because it is an indirect reformulation of the situation rather than a means to reconceptualising mathematical relations. Algebraic symbolism tends to separate a mathematical argument from its original context, so that the meaning of the underlying situation is lost and the symbols can become objects in themselves (Ruthven, 1992a). This makes it very hard to understand relationships between tabular numerical data, graphs and symbolic equations and to translate in both directions between different representations.

It is believed that linking multiple representations provides a promising environment for developing understanding of notoriously difficult symbolic ideas and techniques (Kaput, 1989). However, doctoral work by Scanlon (1990) questions this. She studied the responses of 35 science learners (ranging from 16-year-old novices to Ph.D physicists) to a number of problems in Newtonian mechanics, one of which required students to take a graphical approach. The less successful solvers were found to switch between algebraic and graphical approaches. As a result, a 'cognitive economy' hypothesis was developed, suggesting that multiple representations of a problem were not necessarily helpful for novice problem solvers. This is supported by the work of Ainsworth, Wood and Bibby (1997) showing that mixing different types of representation which users cannot coordinate should be avoided. Furthermore, certain representations make certain inferences easier: tables make patterns easier to recognise and graphs supply the variation that is hidden in an equation

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[24] The ability to interpret graphs also appears to be a strong predictor of success in general mathematics at university level, reflecting a critical underlying cognitive structure (Gill, 1998).

[25] This work was carried out for the Ohio State University Calculator and Computer PreCalculus Project.

and make structure visible (Cox and Brna 1995; Kaput, 1989). However, different types of representations are appropriate - and need to be learned - for different contexts; for example, the scatter graph is a very useful data interpretation tool in certain contexts but inappropriate in another whole range of situations which may appear similar to the learner (O'Reilly, Pratt and Winbourne, 1997). A doctoral study by Hart (1992) confirmed that individuals portray definite preferences for certain representations in different situations.

The above findings indicate that the popular rhetoric about multiple representations needs empirical investigation. This could encompass development of a perspective about the appropriate circumstances in which graphical representations might be used and the best way of constructing a solid understanding of a relationship between numerical, graphical and symbolic representations in interpreting graphs. Dick (1992) emphasises that differences in the quality of students' use of multiple representations must be assessed; namely their interpretation and translation skills.

Much research finds that students tend to focus on the perceptually obvious features of graphs; after all, the path of a line is the salient visual feature, not the relationship of relative changes. For example, results of the APU work on difficulties in interpreting graphs confirmed that students perceive line graphs to be a picture of a situation and a direct model of how a single variable changes. Swatton and Taylor (1994) assessed over 700 students from 181 schools throughout England; they concluded that most students aged 11 and 13 do not appreciate the power of graphs as models of underlying relationships between variables (i.e. the data, which can be concrete or abstract, discrete or continuous) presented symbolically. Friedler and McFarlane (1997) assert that students are usually not taught interpretation skills explicitly - linking the events depicted and their symbolic representation - and neither the NC for Science or Mathematics encourage change in this approach. Consequently, students plot graphs mechanically and cannot articulate why they use them to represent data. Support for this view comes from a study of 1800 13-to 15-year-olds by Kerslake (1981) showing that the vast majority of lower secondary school students cannot link equations with straight line graphs. A smaller scale survey by Preece (1983b) confirmed that only 11% of 14- to 15-year-olds knew that a graph depicted the relationship between two variables.

Preece conducted an in-depth doctoral investigation of students' ability to interpret and sketch graphs, with a particular emphasis on understanding of gradient, where many interpretation errors occur. She identified four conceptual levels in students' understanding of gradient, manifest in their graph interpretation and sketching:

(1) the graph is viewed iconically

(2) points are interpreted/marked, and then joined by a line

(3) discrete changes in gradient are considered and comparisons made between intervals

(4) continuous changes in gradient are interpreted; descriptions of gradual speed changes are given

Students find it particularly difficult to distinguish between a quantity and the rate of change in it, although even comparing two curves is challenging for many (Beichner, 1990; Preece, 1983b). A common error is confusion of the slope of a line with a point on the line (e.g. the steepest slope is equated with the point of highest value). Iconic interpretations are also rife, and Preece observed that the highest line was often associated with fastest motion or increase in magnitude. Those who successfully interpreted graphs representing changes in temperature or animal populations used a high percentage of gradient and speed words and a low percentage of direction, position and shape words. The reverse was true for poor interpreters (Preece, 1983d).

Kerslake's (1981) work corroborated the above findings and highlighted similar difficulties in understanding and applying the concept of gradient. It provided further examples of misconceptions about the significance of a graph's visual appearance. Incorrect spatial interpretations particularly concerned distance/time graphs and where the axes and scale of a graph were altered. They were most typical of students who were strong visualisers. Speed/time graphs are also frequently drawn incorrectly, with students representing the path of an object or a person's journey instead of speed; they may view graphs as concrete objects rather than as indicators of abstract trends (Beichner, 1990; Mokros and Tinker, 1987). Additional evidence for difficulties specifically with distance/time and velocity/time graphs comes from the work of McDermott and her colleagues. They employed written problems, laboratory experiments and a diagnostic computer program, *GRAPHS AND TRACKS*, which simulates the motion of a ball on a set of tracks and presents or requests a corresponding graph. These tools were used to document a range of difficulties and errors in connecting graphs to physical concepts and events (McDermott, 1990; McDermott, Rosenquist & vanZee, 1987). These included discriminating between slope and height of a graph, interpreting changes in height and slope, relating position/time and velocity/time graphs, representing continuous motion, separating graph shape from path of motion, representing negative velocity and constant acceleration, and distinguishing between different types of motion graphs.

Some of the above findings were also supported by our work for the Conceptual Change in Science project (Driver & Scanlon, 1988). In particular, Hennessy observed that 12- to 13-year-olds were limited in their ability to generate speed/time sketch graphs representing the simulated horizontal motion under friction they observed during pilot tests. This (unpublished) work highlighted several of the common kinds of confusion. It again indicated that students need much support in understanding relationships between variables and in learning which measurements to take. Qualitative interpretation is usually associated with global features of graphs that represent situations and is more difficult than quantitative interpretation (Preece, 1983b). While this is an under-represented area in the mathematics curriculum (e.g. Bell & Janvier, 1981), it is especially important for science learning, and indeed research on this emerges from a tradition which employs mathematics to help explain physical phenomena (e.g. McDermott, Rosenquist & vanZee, 1987; Mokros & Tinker, 1987).

The nature of the **context** of a graph - and the setting in which it is used - influences both the type of variables used and the interpretation of results. Leinhardt et al. (1990) maintain that students' understanding of graphing is partly based on intuitions about graphs from experience and prior teaching. Some intuitive and everyday knowledge (e.g. that time only increases) serves as a useful basis for learning how to interpret graphs, however students may become distracted or overwhelmed by situational knowledge that interferes with graph abstraction (see also Janvier, 1981). Further insight may be derived from another study by Preece (1993) which investigated the role of context in students's interpretations of graphs of biological phenomena. She found that many students aged 14-15 offered graphical interpretations focused on graph syntax - and influenced by shape - without relating them to task context in a meaningful way. Those interpretations which were integrated with context were either pictorial, dominated by particular conceptions related to the context or other world knowledge, or confused and unanalysable, and these categories were fairly robust. Thus most of the knowledge students bring to the task of mentally reconstructing a phenomenon or situation is irrelevant or misleading, whereas successful interpretation requires gradual integration of accurate and appropriate knowledge of the context. Preece noted that the most difficult graphs to interpret have unfamiliar variables within a familiar context and a similar form to a real object; this entices students to give non-graphical interpretations (Preece, 1983c). For example, the thermometer simulation used for diagnosis formed an analogue with its graph; as the temperature rose and fell, the graph corresponded. This encouraged students to develop notions of gradient based upon physical features, whereas a display of a random population of animals proved much easier (Preece, 1983d). (The issue of real world applications in task contexts is discussed later on in section 5 under 'Context and authenticity').

Further detail about difficulties concerning graphing is provided by the studies of college students carried out by Dunham and Osborne (1991). They identified three major (interrelated) areas of difficulty and analysed the behaviours and understandings required to build "better visual intuition". The first problem is failure to make **connections between symbolic and graphical representations**, as mentioned above. Three aspects of understanding the nature of graphs and functions were deemed necessary for making these connections:

(1) the notion of an ordered pair in the context of a function; students particularly fail to grasp the association between an ordered pair and corresponding *x*- and *y*- values;

(2) understanding that a graph is composed of an infinite collection of points (both on a line and between any two points) rather than being a single physical object - or alternatively, as the majority of schoolchildren believe - a represention of the discrete quantities plotted (Kerslake, 1981); understanding that manipulating a graph affects the original function;

(3) ability to infer characteristics of a function from more than one point, understanding the complex connections between what is seen on a graph, the behaviours of *x*- and *y*-values and their relationships; students frequently focus on a small number of distinct points of a graph (e.g. vertex or intercept), failing to relate visual characteristics with interval behaviour.

To elaborate further, the mathematics curriculum overemphasises local, i.e. pointwise interpretation (students plot graphs from tables of ordered pairs and tackle questions involving looking up specific information) rather than elaborating the properties of the underlying situation (Janvier, 1981). This may result in the conception of a graph as a collection of isolated points rather than a conceptual entity; teaching one element of graphing does not produce capability in other areas, particularly qualitative interpretation of graphs (Stein, Baxter & Leinhardt, 1990).

The second area of difficulty identified by Dunham and Osborne (1991) is in understanding **scales** and choosing appropriate ones, a persistent problem for students as other researchers have observed (e.g. Hart, 1981; Kerslake, 1981; Lindquist, 1989). Students misunderstand that issues of both relative order and measure need addressing (Leinhardt et al., 1990), and they tend to assume single unit intervals and equal scaling on both axes (Dunham, 1990). Dunham and Osborne point out that this inattention to scale leads to difficulties when students have to graph data in science classes; their attempts are confounded by mathematics teaching which presents examples of easy-to-draw graphs where scale is symmetric and therefore insignificant.

Thirdly, **transformations** of functions and **scale changes** are little understood, even by those college students who can plot and identify points, scale axes and make good use of graphing space (Dunham & Osborne, 1991). A complete understanding of graphical representations includes realising what visual features of the graph will change (geometrical angles between lines and axes) and what will not (*x*- and *y*-intercepts) under the change of scales. Difficulties with scaling arise in both constructing and interpreting graphs (Kerslake, 1981). A critical problem is confusion of geometric transformation with scale change, i.e. not perceiving a connection between a graph display and the reference system provided by the coordinates. Axes and graph are consequently treated as changing independently in scaling tasks. Traditional graph plotting tasks tend toportray functions as collections of individually calculated points joined together by straight line segments; this results in students' failure to understand, for example, $y = 2x^2 - 5$ as a stretch of $y = x^2$. Ansell (1988, p.53) points out that most importantly, traditional methods encourage narrowness of thought, with students seeing the problem as a graph to be plotted, as an end in itself, rather than constructing functions as a means of problem solving.

*Overcoming students' difficulties with graphing*

Dunham and Osborne (1991) offer suggestions for appropriate activities to help overcome the difficulties they identified. These focus particularly on the use of mathematical language and metaphor in different contexts (e.g. the metaphor of stretching and shrinking), and on activities which help students develop their own intuitions about what a graph reveals by supplying them with rich experiences in visualisation. Leinhardt et al. (1990) also present detailed evidence from the literature and their own analysis of guidelines for effective teaching and activities. The emphasis is on the teacher guiding and presenting, through drawing on and transforming their subject matter knowledge of graphs and functions. Issues of curriculum sequencing are also discussed in some detail; the consensus is that movement between qualitative and quantitative presentation of graphs is most appropriate. Instructional explanations and the burden of notational conventions are also covered.

Further pedagogic implications arise from the above discussion of research into students' difficulties with understanding graphs. Students seem to need more practice in interpreting trends in data and graphs as well as reading values, constructing graphs and doing more mathematical tasks. They need more guidance and activities focused on scale and on graphical representation of functions. To assist students in making links between different representations, working directly with a range of examples in numeric or graphic form, particularly when addressing new situations, can be very helpful before moving to a more generalised symbolic form (Ruthven, 1994). Paper-and-pencil methods encourage movement from an algebraic equation towards generating data points and eventually creating the graph of a function, but not in the reverse direction, which is considered to be harder. Most importantly, graphing activities need to take a qualitative approach emphasising exploration of mathematical relationships - intuitions about two simultaneously changing variables and their pattern of covariation - rather than symbolic manipulation for its own sake.

## 3. The role of technology in supporting graphing

The author believes that the new portable technologies can make a significant impact on a traditional area of the mathematics curriculum, namely students' understanding of graphing. New graphing tools - computer plotters and graphic calculators - are continually appearing and research is clearly needed to understand best how to deploy them. Some research in this area has begun to emerge during the last decade, and this is the focus of this section of the article. The research indicates that portable computers and graphic calculators may help to overcome some of the conventional difficulties students experience, not least because they encourage an investigative problem-solving approach. The new interactive tools enable students to evaluate or simplify an expression, solve an equation or produce the graph of a function at the touch of a button. They are particularly useful for comparing multiple graphs simultaneously and for showing specific features of related graphs (Coles, 1990). Graphing tools can replace traditional construction procedures with more rapid and reliable ones, removing the need for calculating, arduous manual plotting and drawing, and axis scaling skills (e.g. McFarlane et al., in press). Thus students first meet graphs as a representation of the relationship between changing variables.

Graphing technologies are particularly suited to students who lack confidence in their symbol manipulation skills (Hart, 1992). Several authors report that by reducing the burden of numerical and algebraic computation, they allow more time for instruction and for students to concentrate on analysing problems and solutions, i.e. on mathematics rather than algebraic manipulation (e.g. Dunham, 1993); Dick (1992) asserts that the time available is doubled.

> With powerful numeric, graphical, and symbolic computational tools in hand, the student can see the "carry out the plan" stage of problem solving as the least daunting step. Students appreciate more the relative importance of heuristic processes, mathematical modeling, and the interpretation of results. (Dick, 1992, p.152)

According to Dunham and Osborne (1991), graphing technologies offer a potential opportunity to increase the role of graphing and visualisation throughout the curriculum, addressing the critical need to combine visual and algebraic techniques. Dunham's (1993) comprehensive review of research into the use of graphing technologies (in conjunction with curriculum change) highlighted a number of other benefits, including higher levels of understanding graphs and functions (see discussion of Browning, 1989, below), greater motivation for and success on problem-solving tasks and more flexible approaches (e.g. Boers-van-Oosterum, 1991), greater skill in interpreting graphs and relating graphs with equations (e.g. Boers-van-Oosterum, 1991; Rich, 1990)[26], and better understanding of connections between different representations generally (e.g. Browning, 1989; Ruthven, 1990). With regard to the latter, graphing technologies allow the student to examine many more graphs - and their corresponding equations - more quickly with an extremely high degree of accuracy. Leinhardt et al (1990) have pointed out that this may help to illustrate the relationship between graphical entailments and algebraic parameters, although some unconventional informal conceptions seem to arise.

Hart's (1992) study indicates that graphing tools may overcome problems with multiple representations, as identified by Scanlon (1990), because working simultaneously with at least two linked representations (graph + equation) becomes more manageable. Technology-generated graphs can now be used to determine important numerical and algebraic properties, reversing the traditional trend from equation to graph (Demana & Waits, 1990). Note that some technologies also display the table of data on which the graph builds. In multiple representation software, changes in one representation trigger automatic changes in another, so alterations to a graph immediately promote a change in the table of values and/or the algebraic description of the function. O'Reilly, Pratt and Winbourne (1997) point out that this two- or even three-way change offers a physical and educational advantage over spreadsheet graphing since the student has control over where the links are made. They introduce a note of caution, though: the student is constrained to using other people's representations, running the risk that the difficulties of reading a representation are multiplied by the number of modalities represented on the screen simultaneously. The student must make sense of each modality in turn <u>and</u> the links between them. The authors point to the expressive *Boxer* software environment as one which offers structures allowing the student to program links for herself; they distinguish between this 'constructive representation' and traditional 'instructive representations' such as graphic calculators (which they claim are used as tools to uncover pre-existing truths).

Evidence for the benefits of multiple representation software for the understanding of functions comes from the work of Confrey and Borba and their colleagues in Brazil. They have developed a computer graphing system, *Function Probe,* which allows a graph to be transformed into another through direct actions using translation, stretch or reflection icons. The system has been tested individually with school students, who were asked to transform a graph, to predict the effect on coordinate values of a given point, then to explore the relationships between actions in the graph and coefficients of algebraic expressions. The results demonstrated that students can develop effective strategies of inquiry using an environment supporting multiple representations, plus carefully formulated tasks and teaching approaches. One case study showed that the student, Ron, was able to expand the notion of change in the axes, incorporating vertical and horizontal translations and stretches (Borba, 1995). He was also able to coordinate the numerical changes in coefficients with the transformations on the graph. The student actually had to build his own notion of translation, overcoming the limits of the software, which cannot allow for transformations in the axes in a dynamic manner. Although discrepancies among representations generated a problem for Ron initially, this was overcome; Borba and Confrey (1996) proposed that the strength of Ron's investigation lay in his ability to coordinate visual actions with changes in other representations.

Another case study (Borba, 1994) offers an example of a student posing a "why" question and trying to explain the rule he detected in visual terms inspired by the activities he developed in the graph window. He investigated the way a given point moved horizontally when a graph is stretched and generated a powerful rule to link a stretched graph with the original one. This process included

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[26] Students using GCs tend to choose graphical solutions to algebraic problems (Rich, 1990), although some do have greater confidence in algebra (see O'Reilly, Pratt & Winbourne, 1997).

combining paper and pencil with the computer medium and comparing graph and table representations; it subsequently helped him to understand stretching better, enabling him to make a wider variety of predictions. Borba describes how the important "why" question in a multiple representation environment is usually motivated by discrepancies among patterns in different representations or by regularities found in one representation but lacking any kind of justification; the why question provides critical reassurance. New why questions and new connections with other representations and knowledge about other phenomena may arise during an investigation. This provides endless scope for allowing different students to choose their own problem-solving methods and to answer different questions, posed by the teacher or the students themselves.

The study by Browning (1989) associated with the Ohio State Calculator and Computer Precalculus Project provides additional important evidence for increases in students' understanding of graphing through using graphic calculators and graphing software on microcomputers. Browning asserted that four hierarchical levels of graphing understanding exist:

Level 1. Recognition of graph of a parabola; simple graph interpretation; development of initial vocabulary

Level 2. Simple interpolation; translation of verbal information into a simple sketch graph; use of initial vocabulary

Level 3. Recognition of the connection between a graph and its algebraic representation; using properties of functions to construct graphs

Level 4. Using given information to construct a graph; deducing information from a graph

Pretest scores showed 78% of students operating at levels 0, 1 or 2. The (non-technology) control groups continued to function at similar levels, but 73% of the experimental groups had attained level 3 or 4 by the end of the year. Their understanding of graphing and function concepts, understanding of connections between algebraic and visual representations of problems and their knowledge of graphing strategies had improved.

Specific examples of benefits related to *dynamic graphing of experimental data* come from the NCET scheme where programs such as *Investigate* proved highly stimulating. They enable students to see graphs being plotted in real time as experiments progress, helping them to associate cause and effect. One controlled study reported that data logging produced measurable improvements in skills of interpretation of line graphs in Years 3, 4 and 9 compared with a traditional approach (Bowell et al., 1994). Another recent study employed a "Calculator-Based Laboratory" (CBL) to confront American college students' misconceptions of graphs of motion; the innovative system consisted of a graphic calculator, a motion detector and a device which transforms data to the calculator (Hale, 1996). The author points out that it is cheaper and more portable than the conventional Microcomputer-Based Laboratory. Student understanding was improved when the activities were followed up by whole class discussion. The Texas CBL can be linked with over 20 different sensors and probes; practical issues concerning its use in British schools are described by Oldknow (1997a).

Further evidence, this time concerning graphing on the Pocket Book, comes from Robertson et al. (1995a, p.23), who quote an example of the use of Pocket Books attached to (temperature, light, humidity etc.) sensors for data collection in science. Information was downloaded straight into the Pocket Books, students then generated and printed graphs and spreadsheets. Immediate collection of data and production of graphs from sensors allowed more time for observation and motivated students to ask exploratory 'what if' questions (e.g. "what happens if I put my figure on the tip of the temperature sensor?"). These questions were spontaneously generated as a consequence of the Pocket Book freeing the student from making tricky and time-consuming measurements and having to generate graphs by hand.

These assertions are substantiated by extensive work on data logging by Rogers and his colleagues. They extol the virtues and flexibility of graphing software such as *INSIGHT* (Rogers, Morris & Wheelhouse, 1994), which displays data in an extremely versatile manner and contributes towards interpreting it. Rogers et al. point out that the computer can provide an experience which progresses from qualitative to quantitative. Immediate display of a graph provides students with a qualitative overview of the data being collected without the need for handling numbers or tabulating data, thus making the graph a starting point rather than an endpoint in evaluating the results. They can inspect and select what is of interest on a graph and appraise its significance, being introduced to the quantitative properties of the data later when the need arises. This allows the low-level skills underlying investigative work to become automatic. It particularly empowers students with limited graph plotting skills by avoiding interruptions to the problem-solving process which can lead to errors (Underwood & Underwood, 1990). Unfortunately, data logging is only used by a small minority of schools at present.

Real time computer plotting portrays graphs as dynamic relationships rather than static pictures and helps to reduce “graph as picture” errors (Mokros & Tinker, 1987). Preece’s pilot work with the *SKETCH* program she developed indicated that offering students an opportunity to view a simulation (of temperature changes or animal population size) and then to sketch and interpret the graph of the changes seen helped students to gain an intuitive understanding of gradient (Preece, 1983a; 1983d). The simultaneity of dynamic graphing is commonly suggested to facilitate a mental linking between a real physical event and its graphical representation. However, passive simulation produced no educational impact in Beichner’s experiments with video recreation of a motion event alongside its graph; these showed that simultaneous perception, i.e. visual juxtaposition, of motion and graph, is insufficient to promote graph interpretation skills. Active control over the physical event and its graphical representation, with instantaneous feedback, may be the significant feature of real time graphing. It may be more motivating, for a start. The questionnaire survey by Robertson et al. (1995a) illustrated how experience with the graphing facility appeared to be responsible for a dramatic doubling in the percentage of students who considered the graphs easy to use (from 31% to 62%) when questioned five weeks and five months into the project respectively.

Graph plotting software is also useful for exploration of which mathematical functions fit a set of data. Students can simply try out different functions and instant visual feedback allows corrections/ conjectures to be tried out immediately (Ansell, 1988). Another area where computer graphing utilities can potentially help students is in acquiring a feel for a graph being a collection of points. However, the capability for graphing a few points of a function and then adding other points is helpful. The typical left-to-right continuous sweep across the domain does assist with the notion of continuity but it presents the graph as a completed picture and along with attention to variation of a function’s parameters can obscure the fact that $x$ varies as well (Goldenberg, 1988). More random plotting could emphasise the role of discrete points as pairs of numbers; using paper-and-pencil techniques instead can convince students of this but makes continuity hard to grasp.

Graphing technology additionally assigns greater importance to scale, through facilitating the use of more realistic numbers whose graphs require attention to scale. Indeed the dynamic nature of computer- or calculator-generated graphs focuses much more attention on scale changes than do static paper-and-pencil graphs (Dunham & Osborne, 1991). In particular, the viewable portion of a graph changes frequently within the constantly-sized display window. Moreover students can explore the effects of (a) scale and (b) geometric transformation on shape by using graphing technologies, for example to compare different shapes of a single graph from different display windows and finding transformations producing different shapes in a single window or adjacent windows (as in the *Resolver* software developed by Yerushalmy, 1989, to help students to evaluate their algebraic transformations). Dunham and Osborne assert that this is crucial in overcoming ideas generated by prior experiences where scale was insignificant in graphing and for developing awareness that shifts in scale can alter what is seen even where the function remains the same. Language is again important and teachers must recognise that metaphors associated with graphing utilities (e.g. 'zooming' in and out) require clarification as they can influence students' ideas (in this case, about what happens to scale). The authors advocate addressing problems of shape and scale through using multiple views of a graph and training students in recognising critical visual features of a graph, then tracking them under transformations and scale changes. Using graphing technology does not always overcome difficulties in scaling axes (Coles, 1990). However it makes plotting more complicated algebraic functions much easier than by hand and as they possess more identifiable features than linear functions, they offer a richer base for visual intuition (Dunham & Osborne, 1991). Complex functions are traditionally introduced later on, but could now come earlier in the curriculum.

Several research studies highlight the importance of technology in introducing graphing skills and concepts to primary and lower secondary school students. McFarlane's work illustrates how students unfamiliar with algebraic techniques can explore complex computer-generated graphs and engage in related problem-solving activities, so graphing can be introduced at a much earlier stage (cf. Leinhardt et al, 1990). She used probes linked to a portable computer allowing corresponding real time line graphs to be drawn. Friedler and McFarlane (1997) reported that the technology had a significant impact on graphing skills at age 14, including the ability to sketch line graphs and curves predictive of behaviour in a new context. This included labelling, accuracy of plotting and sketching - skills more usually associated with manual plotting. However, 16-year-olds were less susceptible to the advantages of data logging and found it difficult to make a link between a graph and the variables represented, possibly due to an added layer of abstraction in the investigations. Another controlled study used a similar approach with 7- to 8-year-olds, introducing graphs as a representation of the relationship between the variables of temperature and time (McFarlane et al., in press); the results showed an increased ability in the experimental group to read and interpret line graphs and again to sketch predictive curves.

The NCET publication 'Approaches to IT Capability' for KS3 highlights the use of spreadsheets as a medium for experimenting with and becoming familiar with symbolisation; the graphing facility can simultaneously be used to provide another view of the situation. This is exemplified by Ainley and Pratt's recent studies with primary school students, referred to earlier, using data (e.g. reporting various body measurements) collected in spreadsheets on laptop computers. This allowed students to bypass the process of drawing by hand. The results showed that students as young as 8-9 years quickly learned to handle line graphs with intuitive ease (Ainley, 1995). However a comparative study of 9-to 10-year-olds found no difference between performance by a computer group and a 'paper' group - using a table of data and a hand-drawn graph - on a task to produce and interpolate from a line graph using new data. Performance by both groups was impressive and all students who could plot points and construct sensible axes could also interpolate successfully; they gained a sense of the conventions of graphing. The researchers believe that the two tasks shared features which were significant in supporting the children's understanding, such as presentation of a complete image. Experiencing a number of similar graphs enabled them to assimilate some features of scale, which they could then use to produce their own graphs. Focusing on meaningful tasks using the graphs for a clear purpose - rather than on the representations themselves - was critical in successfully introducing difficult but powerful graphing skills. Traditional approaches focus instead on construction skills for their own sake, often concealing the context and purpose of drawing a graph. Normally, computers offer the only access to immediate graphical images drawn from purposefully collected data. Also instrumental was the process of 'active graphing'[27] described by Pratt (1994): the students collected their own 'real' data, entered it into a spreadsheet and viewed a

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[27] The review of research into analytical reasoning with external representations by Cox and Brna (1993) supports this notion in its assertion that external representations including graphs must be actively constructed by the user; passive use of pre-formulated materials is less effective.

numerical representation of the phenomenon; the graph then showed up gaps in the data requiring further experimentation and predictions could be made from outside the data range. This meaningful interaction between graph and data offered the students greater control and enabled some to develop better understanding of their graphs. The teacher confirmed that the children's ability to interpret and discuss the graphs illustrated their grasp and application of mathematical ideas normally introduced much later on (Stradling, Sims & Jamison, 1994, p.7).[28]

The Pocket Book II palmtop introduces a sophisticated graph 'Plotter' facility which overcomes the more limited graphing capability of the original model and could bring more complex graphing concepts within the reach of younger students. The machine offers a range of complex functions, allowing the user to type in equations and plot Cartesian, polar and parametric functions, inequality graphs and coordinate pairs. Plotter can also display multiple graphs or families of functions; it can trace functions and report derivatives. The user can rescale the axes, zoom in and out, pan in all directions and change the viewing window. The simple interface to all of these functions coupled with the other functionality the machine offers (spreadsheet, word processing) makes this a potentially powerful tool for students across the age range; research is needed to establish how it could most usefully be exploited.

As with other portable technologies, little is known about the endurance over time of the benefits of graphing technologies. A handful of studies report little benefit during short periods of exposure to the technology. Martin (1994) found that one semester of use by college students was insufficient for novices to become sophisticated users or to display enhanced conceptual knowledge. Nevertheless students in the same study who had used the technologies subsequently showed significant differences in their approach to graphing problems, continuing to draw on a graphing perspective even when use of the technology was discouraged or prohibited. Thus graphing technologies can have a lasting impact.

Finally, a number of teachers in the Pocket Book project pointed out that drawing graphs by hand is a necessary skill and part of the NC. The concern is that using a computer prevents learning of that skill, but Robertson et al. (1995a) point out that it could still be taught, using the Pocket Book graphing facility more appropriately, e.g. for investigating changes in temperature or humidity. These tasks are not mutually exclusive and serve different pedagogic functions.

*Graphic Calculators*

Research showing the benefits of using graphic calculators (GCs) - custom-made tools for graphing - is beginning to emerge (e.g. Dunham & Dick, 1994). These overlap with the benefits of graphing on portable computers and a number of the points made in this section follow up aspects of the discussion in section 1. Although GC screens are more limited in size, the calculators are even more

[28] Another example of active graphing comes from the science investigation of the effects of gravity with Year 5 students, mentioned earlier and described by Pratt (1994) and in the NCET evaluation report (Stradling, Sims & Jamison, 1994, p.4); spreadsheets were used to record distances travelled by a toy car down a slope. Scattergrams and lines of best fit highlighted emerging patterns and non-conforming results, helping the students to decide where more data was necessary.

portable and easily affordable by individual students, making them even more controllable by students. The calculators are robust and highly personal, offering the same opportunities for private and informal use as small portable computers. Many schools also consider them to offer better value since at £30-£40 each[29], a class set together with a teacher's model with OHP display pad, currently costs no more than one or two desktop PCs (Oldknow, 1998). Resembling scientific calculators in appearance, size and price, the 'supercalculators' are fast overtaking them as the standard for 'A' level mathematics students, about 60% of whom have their own machines (Oldknow, 1997b).[30] They are in common use at college/university level and in schools although use below the age of 16 is variable and relatively recent. The'A' level Mathematics core curriculum does not mention them specifically. It 'encourages the appropriate use of calculators and computers' and insists that 25% of assessment must be carried out without any technological aid (SCAA, 1997).[31] By contrast, provision of GCs is expected for American high school students; their availability has been assumed at high school level since 1989 (Dunham & Dick, 1994). Shumway (1988) reports that the feeling of educators around the world has recently been converging towards greater use of calculators with accompanying major changes in curriculae to de-emphasise computation and emphasise earlier, deeper, conceptual learning.

Oldknow (1997b) has carried out an extensive survey which provides further information about GC use in schools within 26 countries in Europe, North America, Asia and Australia . He concludes that two different routes are being taken regarding the use of IT in secondary school mathematics. In some countries the pressure is on to build up a greater provision of desk-top computers and associated software. Others have succumbed to the developing power of personal hand-held products, particularly GCs, as offering an alternative solution with obvious advantages. Because of the practicability of using GCs in an examination room, some of those countries taking the second route have undergone a rapid transition from (a) banning such calculators, to (b) allowing their use, through (c) encouraging their use and finally to (d) making them compulsory.

GCs are potentially an ideal intermediate technology in terms of cost and power between traditional (pen and paper) and advanced (computer) technology (Ruthven, 1994). Calculators offer less flexibility and lower capability of data storage; for example, when one graph is erased on some models, all graphs must be erased. Nevertheless calculators could be said to be more widely integrated than computers into the curriculum at secondary school and college level (Sharp, 1994), precisely because they offer access to computing facilities without the administrative burden and at a fraction of the previous cost (the cost is generally borne by the students). GCs provide multiple representations of mathematical ideas, offering sophisticated environments for graphing mathematical expressions and modelling using real data, and sometimes for manipulating symbolic expressions. They also have facilities for calculating and mathematical programming (Ruthven, 1992a, p.1).[32] (This review concentrates on the use of graphing facilities, which teachers appear to find most useful, e.g. Dick & Shaughnessy, 1988.)

The most significant research initiative in this area was the "Graphic Calculators in Mathematics" project (funded by NCET). This involved 1000 'A' level students in 24 schools across 6 regions and offered unrestricted individual access to calculators over 2 years, either as a personal or as a

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[29] This puts the calculators almost in the realm of Christmas stocking fillers!

[30] The most advanced models with "Computer Algebra Software" or CAS, such as the Texas T1-92 (which is effectively a specialised palmtop computer incorporating symbolic algebraic manipulation, interactive geometry software, a 3-D surface grapher, data analysis and a text editor, at a cost of about £200) have recently been banned in UK schools, however, presumably because they allow students to factor complex algebraic expressions.

[31] This ruling indicates that the T1-92 developers' call for abandoning the teaching of 'obsolete paper and pencil algebra and calculus manipulations' (TI-TIME, Autumn 1996, p.2) is unlikely to be realised in the near future.

[32] Shoaf-Grubbs (1995) points out that GCs overcome a disadvantage of computer algebra systems which may not offer in one software package all of the mathematics that a teacher desires.

classroom resource. The main goal was to develop, trial and evaluate teaching approaches which exploited the potential of the GC. The teachers planned their own calculator work within the normal syllabus constraints. The project culminated in a resource pack for schools: *Graphic Calculators in Advanced Mathematics* (NCET, 1992), offering a wide range of short calculator activities on workcards and classroom accounts of using GCs in seven mathematical areas. The teachers' accounts illustrate how the technology can stimulate and support problem solving and empower students, giving them greater scope to generate, test and develop their own ideas, supported by their teacher and in collaboration with peers (Ruthven, 1992b). Examples included using the calculator to generate varied instances of some mathematical phenomenon, supporting a process of seeking patterns and relationships and helping students to formulate and test conjectures. Ruthven (1992a, p.8) maintains that these conjectures and explanations can later be translated into more formal algebraic language, for consideration at a greater level of generality and abstraction; algebraic treatment is thus deferred until students are familiar enough with the underlying mathematical principles.

Experience with the calculator was found to erode students' initial views of GCs offering automatic procedures, replacing them with sophisticated judgments about the efficiency and appropriateness of procedures. Most significantly, students using GCs generated strategic trial-and-improve approaches. These are a progressive refinement of guesses using evidence generated by the GC; they depend on interpreting graphic feedback in the light of key mathematical principles.[33] Ruthven (1992a) argues that devolving mathematical responsibility to students and moving towards student generation and evaluation rather than teacher presentation of new ideas helps students grasp those ideas more firmly and develops their capacity for investigation and problem solving. This is essential in coming to grips with algebraic symbolism.

Clearly, the exploratory learning activities devised for the *Graphic Calculators in Advanced Mathematics* project were central in achieving the effects observed. By contrast, an American project with students of a similar age studying algebra, trigonometry and calculus found significantly less agreement - after a few months of use - that there are many different ways to solve most mathematical problems (Dick & Shaughnessy, 1988). This was the case despite the teachers involved reporting taking a more exploratory teaching approach to graphing topics. The male students exhibited a greater tendency than females to view mathematics as a rule-dominated discipline. (It is unknown whether this shift is natural with continued mathematical study, however, or whether open-ended activities can pre-empt it.)

The kind of investigative approach employed by Ruthven can help teachers gain both critical insights into students' thinking and difficulties, and fresh mathematical insights for themselves. As with four-function calculators, use of the GC may itself throw up important mathematical issues and motivate discussion about them. Ruthven (1994) illustrates how calculator utilities may themselves embody novel concepts and make new mathematical approaches possible, notably those exploiting the calculator's numeric or graphic processing capacities. An example is the facility to zoom in and out on a section of graph. According to Ruthven, this facilitates working with ideas of approximation and limit, supporting a more simple - and cognitively direct - approach to the generalised notion of gradient than the traditional approach. Using the symbolic language of the calculator, itself a mathematical symbol system, can also increase confidence and competence with using symbolism in other contexts.

Note that while graphic calculators may make sophisticated graphing skills redundant, in practice teachers and curriculae usually expect students to be able to perform manually most of the tasks previously carried out with pencil and paper: scaling, calculating, tabulating and plotting points, interpolating and sketching. Thus there remains a minimum level of competence that is expected

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[33] See Ruthven, 1994, p.161 for examples.

without the use of a calculator, just as mental arithmetic strategies are demanded alongside the use of a four-function calculator. Graphing on a calculator actually involves different activities to graphing an expression by hand, including a different kind of scaling, considerable selecting and entering, but its great advantage is that it bypasses the other manual activities (Monaghan, 1997). Ruthven (1992a) observed that the graphing facilities rapidly replaced mental and written methods (although the underlying mathematical approaches employed tended to be conventional ones). The project results showed that using GCs to sketch a graph quickly and reliably or find numerical solutions to an equation reduces drudgery and frees students to focus on the strategic and tactical elements of a problem-solving task (Ruthven, 1988). One study implied a broader approach to the use of the graphing facility with the GC than on computer (Ruthven, 1990). This controlled study compared the performance of project students in four schools with that of students in parallel classes without access to GCs or computer graphing. The project group scored significantly more highly on a reverse task to that automated by the calculator, namely that of describing a given graph in algebraic terms, or 'symbolising'. Three fundamentally different approaches to symbolising a graph were identified, although all began by identifying the type of graph. The project group were significantly better both at recognising the type of graph appropriately in verbal or symbolic terms and at building up a precise symbolic description of the graph (refinement). They exploited salient information (e.g. orientation, position of extreme values, zeros) and knowledge of relationships between these features of a graph and its symbolisation.

Most importantly, *using a GC extensively is likely to reinforce relationships between particular symbolic/algebraic and graphic forms* since the calculator itself operates through such relationships (Ruthven, 1990, p.447). Access to GCs is also said to encourage both students and teachers to make more use of graphic approaches in solving problems, rehearsing more general relationships between graphic and symbolic forms and leading to new mathematical ideas being developed. These may derive from visual understandings being exploited to guide and support a symbolic treatment later on (Ruthven, 1992a). Ruthven cites an example showing the value of students using their manipulative skills and comparing graphs in the course of determining the equivalence of different symbolisations of a relationship *(ibid.,* p.7). Another set of factors operating during symbolisation concerns the improved quality of information available to students using a GC (Ruthven, 1990). This was observed to facilitate more frequent checking of formulae by graphing in the project group and to enable a graphic-trial approach; this repeatedly modifies a symbolic expression using information from comparing successive expression graphs with a given graph. This approach increases chances of success and reduces uncertainty and anxiety among less confident students.

There was no treatment effect on interpretation items entailing extraction of information from a verbally contextualised graph in Ruthven's study. The results were explained by the different competences involved in symbolisation and interpretation and the fact that teachers used a cautious, fairly traditional approach to their teaching of calculus. Borba (1996) suggests that when less traditional approaches are used in the classroom (i.e. less reliance on textbook sequences and the blackboard), students can use the graphic calculator for tasks which go beyond the interpretation of graphs.

While some may argue that GCs have their limitations and that a more expressive software environment on a desktop computer allows more active student involvement (O'Reilly, Pratt & Winbourne, 1997), evidence is accumulating in favour of the benefits afforded by the calculator's handling of multiple representations. Shoaf-Grubbs (1995, p.227) suggests that the visual, self-paced exploration process created through using a GC enables students to use concrete imagery to further abstraction of mathematical concepts. Spatial visualisation is important not only in mathematical reasoning but is believed to promote the creative and insightful use of mathematical concepts that can transfer to other areas of learning. Shoaf-Grubbs asserts that students using GCs are more likely to construct their own mathematical understanding through conscious reflection, although an exploratory and interactive learning environment which offers students greater control is again essential. Her controlled study with female college students provided an emphasis on making mathematical connections, explicit teaching of concepts, multiple representations, support for student problem solving, and encouragement to develop positive attitudes in a traditionally difficult area of mathematics. The findings showed that the GC had a significantly positive influence on performance in the calculator group and that spatial visualisation and mathematical understanding were strengthened.

In the same vein, Smart (1995) observed a class of 13-year-old girls using GCs, again with unlimited access, over the course of a term. The class teacher believed in encouraging active student involvement, self-directed exploration and discussion of discoveries made; this was undoubtedly a critical factor. Their investigations included fitting a line or curve to data points and the effects of varying a constant. The data collected in this context showed that access to the GCs helped the girls to develop strong visual representations of functions given in their algebraic form and simultaneously developed their confidence in talking about mathematics and in investigating new problem areas. They were able to picture and describe a graph when given its equation in symbolic form and to use feedback from the graph, employing their visual knowledge to help perceive patterns and make generalisations. The girls' written reports of their explorations were coherent and indicated that all had been stretched until they were happy solving problems that cause difficulties to older students who employ only algebraic methods; some students were working confidently at 'A' level mathematics problems. They were articulate about the predictions they had generated and tested and about their prior misconceptions. The girls' views of their experience were divergent but they generally found graphing quicker and less arduous, although the calculator took time to get used to.

Smart is currently conducting a follow-up study in another school, a girls' technology college, to see whether using the GC can provide students in Years 9-11 with a higher level of achievement based on a stronger concept of the relationship between algebraic form, tabular numerical form and graphical form (as the work by Hart, 1992, suggests it does). The GC technology may help because it allows translation between algebraic and visual forms - in both directions. (Ideally, then, after producing a graph from data points, further data could be extracted from the graph and then used to try to create an equation. This in turn could be used to create data or data could be used to check the algebraic rule. This complex process may take several weeks of experience with related activities, perhaps beginning with manual methods and predictions but moving on to integrate these with calculator use: Smart, personal communication.) For instance, Smart's activities use GCs to investigate parallel and perpendicular line graphs, moving from symbolic to visual representations and then linking back to the equations.

The work of Borba and his colleagues discussed in section 3 employed a similar graphical visualisation approach which inverts the traditional approach to translations of functions. Their activities began with visualisation of graphs, then focused on the relationship between graphs and tabular values and finally, focused on the relationship between graphs and algebraic expressions (Borba and Confrey, 1996).

> We believe that the emphasis on visualisation, in addition to restoring the historic emphasis on visual aspects of the study of transformations, allows students to move easily into the algebraic symbolism while containing some visual meaning for the symbolism. (*ibid.,* p.320)

The empirical work they present describes the final transition to algebraic symbolism and how previous experiences in visualisation and data manipulation facilitate that transition. The story of Ron illustrates the power of visual reasoning; the authors call for adequate time, opportunities and resources for students to make constructions, investigations, conjectures and modifications.

More generally, the GC's encouragement for visual reasoning drawing on graphic representations means that using it can render mathematics and science work a more visual activity, even for older students tackling complex concepts, as these too are often facilitated through visual solutions. Ruthven (1994) speculates that in the future, the supercalculators will link numeric and graphic representations to symbolic expressions, so that any standard operation on an expression is associated with a corresponding transformation in a numeric table and graph plot. (Graphs linked to tables in vertical split screen mode are already available on models such as the Texas T1-83.)

It must be emphasised here that the instructional approach is critical and students' understanding is not necessarily improved by simply adding a technological component to an existing curriculum. A doctoral study of 36 students by Porzio (1995) found little difference between traditional students and those using GCs in their ability to make connections between different representations. Evidence was provided that students are better able to use, and make connections between representations when the instructional approach they experience emphasises different representations *and* has students solve and interpret problems specifically designed to explore, establish, or reinforce connections between representations. This is the key to successful activities like those discussed above. Moreover, there may be a threshold level of understanding below which students will not benefit greatly. Giamati (1990) observed that the benefits of graphic calculators were much stronger for students who already had solidly formed links between graphs and equations. She also proposed that constructing tables of functional pairs of numbers was essential for developing conceptual links.

Further advantages of GCs were apparent from the positive responses of teachers and students in the NCET pilot scheme (Stradling, Sims & Jamison, 1994, ch.2). Students quickly familiarised themselves with the calculators. Teachers reported marked increases in the speed, quality and quantity of work although some were uncertain whether students' understanding of underlying principles had improved. They acknowledged, however, that the same uncertainty had previously surrounded the use of scientific calculators in the classroom. Some teachers explored ways of providing differentiated support. As with other portable and graphing tools, a key factor was the increased time available; by speeding up certain mathematical processes (e.g. allowing several data sets to be processed quickly), the GC offered students with difficulties more time to assess their work and their understanding with the teacher and to repeat difficult tasks. At the same time it allowed more able students to do extension work and to see if they could transfer their learning to new kinds of problems. The NCET scheme showed that GCs can be used in other subjects, for example in geography for analysis of demographic data (Stradling et al., 1994, ch.3).

An innovation at university entry level is the recent introduction of course MU120 ("Open Mathematics") at the Open University, which was studied by about 2000 students in its first year (1996). The course materials exploit the graphic calculator technology through specially designed activities accompanying each course unit, based on the Texas T1-80.[34] The materials attempt to develop understanding of graphing and its rationale through exploration and information handling. There is an emphasis on skills for using mathematics - including manipulation of graphical representations - in investigating questions from other contexts as well as on mathematical techniques. Research in progress by the author indicates that the calculator is extremely popular with the students and perceived as a valuable tool for learning a range of mathematical concepts.

While GCs are used increasingly by 'A' level and older students, mathematics educators now believe that they can also be used by younger students (Key Stages 3-4, i.e. 11-16). In fact, the National Curriculum states that students of this age should use GCs and computers to understand the behaviour of functions (DFE/WO, 1995b). The NCET teacher resource document for KS3 ('Approaches to IT capability') points out further uses of GCs: exploring functions and graphs, analysing statistics, studying sequences and iteration. They suggest exploring the possibility of combining functions to build a more complex formula using models allowing data tabulation. Post-16 may even be too late to introduce GCs as younger students are more susceptible to improving

[34] The Texas T1-80 is a widely available machine aimed at the educational market. Facilities include a screen display of 8 lines by 16 characters and pull-down menus; graphing of 4 rectangular functions, 3 parametric equations and 4 plot types; 7 interactive zooms; tabular data input of 6 columns; statistical drawings including scatterplot, box plots, xy lines and histograms. The calculator costs about £30.

their graphing abilities. Research by Friedler and McFarlane (1997) indicates that using data logging has a significant impact on graphing skills at age 14 but much less so at 16. Smart (1995) similarly believes that KS3 is a crucial age for the formation of mathematical ideas and confidence. NCET asserts that the technology (like computer graph plotting) may nevertheless be less appropriate for students younger than KS3 since they need to understand the symbolic nature of a mathematical formula first. However there is a spectrum of diverse activities which can be performed with graphic calculators. MacKernan (1995) reports on his work with 10-year-olds exploring the functions of sine, cosine and tangent and subsequent activities with 7- to 9-year-olds. The younger students were able to program the calculators to produce mathematical shapes, explore parallel line graphs, plot graphs from coordinates and predict missing points.

The NCET scheme employed GCs widely with the age group below ‘A’ level. At Key Stage 4 they were used for helping students understand a range of mathematical principles and procedures (including converting data into graphs, solving equations, points of intersection, comparing families of graphs, transforming graphs, iterative processes, measuring projectile paths: Stradling, Sims & Jamison, 1994, p.7). Many teachers initially assumed the calculators would be used mostly at Key Stage 4 and for GCSE mathematics but were surprised by their potential to introduce advanced mathematical concepts to younger age groups. Calculators were used competently by students as young as Year 6. The evaluators illustrated this by describing how a class of Year 7 secondary school students carried out a whole range of mathematical investigations using GCs. They made predictions about graph shape and explored how it can alter when the range is changed. This led to a keen interest in graphs; students brought in graphs from newspaper cuttings, wanting to emulate them on the calculator. This in turn provoked discussion about different types of graph, described by their teacher as a valuable spin-off which would not have been possible without the calculators.

As with other portable tools, then, GCs were used in some NCET projects for work which would not otherwise have been done. Stradling, Sims and Jamison (1994, p.22) describe another example where Year 10 students went beyond the work set to seek patterns in their results and to make predictions about new expressions. The calculators permitted a faster speed of progress and led all groups to extend their knowledge and understandings of graphs, which would have taken much longer and been less enjoyable if every student had had to draw each graph. Three schools used the calculators to investigate number patterns, iterations, coordinates, simultaneous and linear equations. (One teacher reported that student motivation was high and that calculators provided easy access to graphs, shifting emphasis away from drawing skills to an understanding of shape and form which is increasingly important for Key Stage 4 examinations.) This experience led to greater reliance of mathematics teachers on the calculators. Together with the “Graphic Calculators in Mathematics” project (at which time GCs cost at least £70 each and up to £300 for some models), the scheme raised the awareness of teachers in general of the calculators’ capabilities and increased access to them.

Smart (1995) concluded that “the arrival of the graphic calculators has provided the opportunity to develop a secondary school curriculum that is calculator-aware” (p.198).[35] The research literature indicates that use of GCs has already significantly changed the climate of the mathematics classroom (e.g. Dunham & Dick, 1994). As we might expect, teachers using GCs tend to lecture and explain less. Students have become more active, exerting greater control over mathematical processes; more groupwork, investigations, explorations and problem solving have been observed (Farrell, 1990). Teachers have been forced to relinquish some control, acting as consultants and dealing flexibly with the uncertainty inevitably arising in the course of using GCs, but have generally responded well. Rich (1990) observed that those teaching with GCs asked more higher order questions and stressed the importance of graphs in problem solving. One teacher - and his students - exhibited more exploration, conjecturing and generalising (compared to behaviour measured in the same classroom

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[35] This refers to the CAN Project (Shuard et al., 1991).

earlier in the year and in a control classroom). Like the microcomputer, GCs can now be viewed as a third agent in the classroom, with students consulting both the technology and the teacher.[36]

Research Issues

The review of research on the use of GCs by Dunham and Dick (1994) points out that the research evidence from controlled studies is mixed, although encouraging. As with studies of other kinds of technology, difficulties in matching content and instruction between experimental and control groups and the issue of whether the calculator is made available during final testing can confound the data.[37] Hammond (1994) exemplifies how difficulties in measuring the impact of IT on learning may arise as a consequence. Certain programs are intended to be used in different ways and contexts to the traditional activities used by non-computer groups and means that assessment of both groups using the same tests - which normally return to traditional methods - may be inappropriate. For instance, one reason for the apparently low impact of IT in the ImpacT and PLAIT reports discussed above could have been the restricted scope of the subject-specific tests used, which may say more about transferability - rather than development - of skills. While the changes in activities which accompany innovative uses of GCs preclude direct comparisons with traditional approaches, they are both desirable and inevitable. For example, the ‘multiple representation’ approach necessitates the tools to use numeric and graphic strategies as well as paper-and-pencil algebraic techniques and demands that problems are open-ended; GCs have served as a catalyst for curriculum change in this direction and consequently for mathematical learning, according to Dunham and Dick (1994). However, some controlled studies (e.g. Quesada & Maxwell, 1994) offer no insight into underlying reasons for improved performance linked to calculator use.

In-depth studies of students’ understanding of graphs can offer most insight and with a couple of exceptions, the results are generally positive for GCs. However, many studies are descriptive and for guidance about curriculum development, further information is needed about the process of using calculators and how it may influence mathematical thinking. Dunham and Dick (1994) call for data concerning (a) which aspects of GCs promote greater understanding (e.g. presence or dynamic creation of a graph, ability to easily manipulate or generate multiple graphs, immediate feedback and checking facility); (b) the role of multiple representations and student preferences; (c) what paper-and-pencil skills retain their importance (e.g. point plotting or scaling); (d) which factors determine success in implementing use of GCs (user friendliness, availability of suitable course materials, teacher training and attitudes). Ruthven (1992a, p.12) supplements this list with a call for more information about whether the use of trial-and-improve methods can support the development of more direct strategies - and the corresponding powerful concepts - by improving students’ discrimination of relevant features of a problem situation. Alternatively, those methods may inhibit such development by providing an indirect but simple strategy adequate for many situations?

Dick (1992) expands upon the cognitive issues affecting the pedagogical use of graphic calculators for understanding and translating between multiple representations. He asks what the cognitive obstacles are, how students manage conflict between multiple representations, whether easy external access to multiple representations deters students from creating for themselves a ready internal access, and how the role of symbolic computation will change? In sum, there is a great deal of work to be done before we can judge the true potential of graphic calculators for facilitating mathematics learning.

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[36] There have been some extraordinary recent developments in GC technology, including the “Flash ROM” facility to download programs and data to the calculator, so that curriculum materials could potentially be accessed via the Web. Digital control technology can also now be used with the GC instead of the PC (Oldknow, 1998).

[37] Positive findings tend to derive from studies where students have access to calculators during testing, e.g. Quesada & Maxwell (1994); Ruthven (1990).

*Summary: Aims of using portable graphing technologies*

The aims of graphic calculator use identified by the “Graphic Calculators in Mathematics” project (Ponte, Nunes & Veloso, 1991, p.158) can be extended as follows in light of the above discussion and applied more widely to encompass the aims of other portable technologies for promoting understanding of graphing:

- to promote spontaneous use of computing facilities within normal mathematical activity
- to foster students’ mathematical thinking; to encourage an active role in learning and evaluation of students’ own ideas and conjectures through problem solving and investigations; to create more time for investigative work
- to assist visualisation and exploration of mathematical relationships
- to encourage a variety of approaches for the same problem - numeric, algebraic, graphical - and translation back and forth between different representations (particularly using graphs to solve equations)
- to develop graph recognition and interpretation skills, and generalisation using feedback from graphs
- to develop understanding of transformations, scale changes and relationships between graphs

The above discussion has made it obvious that while portable technologies offer many advantages, on their own they can have little impact upon learning. Supportive teaching and mathematically rich activities in which graphs are used both meaningfully and purposefully are essential to develop students’ understanding and skills in this complex area. The notion of active graphing (Cox & Brna, 1993; Pratt, 1994) is critical to this aim.

## 4. Portable graphing technologies: some elements of caution

Portable technologies may create their own unique problems, motivational and conceptual as well as practical ones, and these are the subject of this section. Some general problems associated with computer use, particularly those related to over-reliance on technology, are raised, followed by problems specific to graphic calculator interfaces.

*Motivation and 'magic'*

The potential problems which may derive from using portable technologies call for further research (Leinhardt et al., 1990). First, the individual's motivation for using a particular form of technology must be considered, particularly that of adults for using a palmtop (cf. Robertson et al., 1995a). Some of these machines have keys too small for easy use even by young primary students and a keyboard size encouraging a one-finger approach. With students, lack of a mouse can be a hindrance since control of editing and other features depends on the child's ability to use function keys. (Other technical problems were outlined in an earlier section.) Another consideration is the initial learning curve of the chosen software and hardware. In the Australian laptops project reported by Gus (1995), teachers complained that students spent too much time learning to use the software rather than grappling with the mathematics content. The next section indicates that GCs are especially complex.

We particularly need to look out for the new errors or misconceptions which new technologies may possibly introduce. One potential pitfall highlighted by Hennessy and O'Shea (1993) is the attribution of 'magic' to results produced on a computer which can create or reinforce misconceptions. There may be a tendency, particularly among younger students, to regard information from computers as more authoritative, objective and accurate than that from other people (Howe, Tolmie & MacKenzie, 1995). There may be a sex difference here: Macleod and his colleagues have continually noticed that male undergraduates demonstrate much more unquestioning faith in the power of IT than females (Macleod, personal communication and Siann, Macleod, Glissov & Durndell, 1990). Leinhardt et al. (1990) and Smart (1995) have similarly warned of the consequences of over-reliance on graphing technology and of students perceiving computer- or calculator-generated graphs as unquestionable. According to Leinhardt et al., these may include not understanding the underlying patterns and principles that drive the production of graphs: "The authority may be in the hands of the computer in subtle ways that do not support learning" (p.7). Hart (1992) reported that college students were least critical of results obtained through routine methods of computation. Their confidence in graphical information appearing on the screen was linked to having a priori information.

Some of Smart's students actually felt that working with the calculator was cheating! This observation of "calculator guilt", particularly by females, was also made by Dick (1992); Dunham (1991) and Foxman (1997). Smart's students recognised that some graphing activities extended their capability further than they could have achieved alone and this provoked insecurity and unease at having access to or being dependent on a kind of magic technology. Similarly, Dunham's students preferred the satisfaction of solving a problem algebraically. A handful of students in the "Graphic Calculators in Mathematics" project were initially reluctant to use any kind of calculator, considering that it led to a loss of responsibility and control of the mathematics; they preferred to retain awareness of every detail of a mathematical process (Ruthven, 1992a). (While this view became increasingly impractical and calculators were ultimately used, it did serve to encourage more critical interpretation of results.) Some student teachers of mathematics also under-utilise available technology, even in examinations; they worry that mathematics may become 'de-skilled' (Aitkenhead, 1996).

Our own experience indicates that while GCs are motivating for the majority of students, a minority are technophobic; they tend to be put off by syntax errors and to blame the calculator for problems experienced. (This is probably true of all forms of IT and the consequence is that a few students will inevitably experience enough difficulty with the technology that its use does not positively impact their learning.) One teacher we observed pointed out that calculators can indeed be fallible occasionally or at best, confusing, especially as students tend not to check their answers for sense (which is not specific to technology-based mathematics). Dick (1992) points that we should make a quick mental sketch of the graph a derivative as a check on the reasonableness of a GC's symbolic presentations (just as we should estimate the result of a numerical computation on a calculator).

A common view among teachers and researchers involved with GCs is that so-called ‘errors’ or ‘problems’ arising are actually challenges which provide an opportunity for learning. They can even be deliberately evoked, e.g. by comparing calculator responses to the same input in different modes. This strategy helps to overcome the tendency to simply input expressions, and along with sketching predicted graphs on paper first, it may encourage thinking about the relationship between an equation and its graph. Dick (1992) reports that ‘confrontational examples’ can be experienced by having students change the viewing window slightly; by drawing students’ attention to GC limitations explicitly, his teachers hope to breed a healthy skepticism.

Finally, young children can succumb to the novelty of the visual impact made by computer graphing rather than perceiving its relevance to the data. Nonsensical graphs can nevertheless be of benefit if students are brought into conflict with their own criteria for judging these graphs (Pratt, 1994). The mathematical richness of a situation may not emerge spontaneously from activities where students are expected to find patterns and generalise since spotting patterns sometimes becomes “an activity in its own right and not a means through which insights are gained” (Hewitt, 1992, p.7). Could technological assistance in seeing and comparing patterns exacerbate the problem?

*Problems specific to graphic calculators*

A question posed by Dunham and Dick (1994) was whether the calculator technology itself poses obstacles to expertise and understanding and there is some evidence that it can. Certainly, GCs have a lot of buttons and sometimes three or four functions attached to each key; hence perhaps more than other portables, they require some induction. Graham (1992) noted that students given the calculators with only a hefty manual to help them were noticeably more critical and less able to exploit its special features, despite having access to a particularly interactive and intuitive machine. Those provided with calculator worksheets and specially designed activities progressed more quickly and enthusiastically. Evidence for conceptual difficulties comes from Giamati (1990), who observed that students with poorly and partially formed conceptual links between graphs and equations were cognitively distracted by having to learn how to use the graphing utility on the calculator. Their understanding of stretches, shrinks and translations was not aided by using GCs. Giamati concluded that unfamiliarity with certain calculator characteristics may affect its effectiveness as an instructional tool. Reliance on certain features may evoke conceptual difficulties: Lagrange (1996) reported that many students using a calculator initially connected the concept of limit primarily with the corresponding key of the device, relying on the key even for very simple limits like $1/x$ when $x$ approaches infinity rather than finding the limits by reflection.

A complication in scaling tasks is that shape of a graph is an artifact of scale and depends on the viewing window, whose dimensions are chosen by the student. This is not recognised by students trained to draw a single view of a graph, choosing a reasonable scale to fit within the paper’s boundaries (Dunham & Osborne, 1991), but it creates a conceptual demand that may affect the kind of mental images a student can construct (Leinhardt et al., 1990). Dick (1992) stresses that since different graphical operations can lead to a similar visual picture (e.g. simple translation of axes gives the same picture as both changing the centre of the viewing window and translating the function), underlying causes of graphical changes can be masked. A specific problem is that the part of a graph displayed may be misleading in predicting the behaviour of the whole graph. Zooming in obscures global information and zooming out obscures local information about the graph *(ibid.)*. Many of the girls observed by Smart developed problems when they could see less of one graph than another on the graphic calculator screen but failed to recognise that it was a partial view of a parabola.

A survey of high school students by (Dick & Shaughnessy, 1988) indicated that remembering key sequences and correct syntax for entering information were the most frustrating aspects of calculator use. Similarly, Ruthven (1992a) describes in some detail the practical problems arising during the "Graphic Calculators in Mathematics" project and confirms that users initially experienced difficulties in coping with the density of information and in finding particular symbols or operations on the keyboard. The different calculator models varied in the format of mathematical notation; all required adaptation from conventional notation but reverse notation increased the cognitive load on users. Lack of confidence in working with the new notation persisted in a few cases (particularly where the GC was a classroom resource and where the teacher showed reluctance to use it). Some operations did not behave as users expected and a graphing algorithm intended to create visual continuity may have provoked confusion. Machines whose graphing facilities (e.g. axis scaling and magnification) matched paper-and-pencil methods more closely, and those whose screen was larger, proved easier to use. Models offering symbolic manipulation were unsuccessful owing to a mismatch between users' informal concepts and the formal 'language' of the calculator. The similar mismatches in encoding mathematical expressions and specifying graphic ranges were more successfully overcome: new schemata were incorporated into students' thinking; the machine's operation was modified to be more conventional; or novel approaches were generated which portrayed cognitive change stimulated by the structures implicit in the machine's primitive operations (*ibid.,* p.6).

Teachers we have questioned have detected idiosyncratic features of different calculator models and various technical problems and visual illusions which cause difficulties (e.g. students' attempt to explain an apparent difference in width between the graphs of $x^2 + 5$ and $x^2 - 5$). Our observations show that some problems are due to students' investigational techniques, e.g. drawing too many lines simultaneously on the calculator without remembering which were linked to which equation. Obstacles also arise where students omit to specify the range or attempt to simply input an equation like $2y = 3$ without first rearranging it to the required form '$y = ...$' Other problems emerge when the calculator connects up points which should not be connected or as discussed above, when it displays a partial view of a graph. Finally, the calculator may not always be the best tool for a job, especially where more than one approach is required; for example, teachers may use a spreadsheet to explore convergence because it allows a graph to be viewed alongside numerical data. The GC clearly lends itself to certain topics in the mathematics curriculum more readily than others.

## 5. Curriculum change and the influence of technology

The discussion in this section focuses on the impact of technology upon curriculum development and introduces the notion of the portable computer as a catalyst in supporting the investigational approach to mathematics learning in school. The controversy surrounding problems set in real world contexts is also discussed.

Computers and calculators are enhancing many areas of the school curriculum, but Goldstein (1994) believes that they are having a radical and continual effect in mathematics, changing the curriculum itself, at all levels. This is essential since ready access to new technologies cannot in itself improve mathematical performance and understanding. Noss and Hoyles (1996, chapters 3, 5, 6) elaborate by portraying the computer as an agent of cultural change and as a context which facilitates construction of mathematical connections:

> Our vision of mathematical learning includes the computer as an important part of the material realities of the learner and teacher, with the potential to catalyse fundamental changes in them. (*ibid.* p.53)

Noss and Hoyles' thesis is that the computer offers a *'window on the construction of mathematical meanings'* through the interaction of internal and external resources between learners' activities, teachers' practices and mathematical knowledge. Thus, it is mathematical cultures and not just technologies which shape meaning and facilitate knowledge development. In fact, the mathematical needs of society and the mathematics which is accessible to students are both changing rapidly; the curriculum must keep up with social and cultural forces. For instance, as technology progresses, decisions need to be taken about which of the traditional skills taught may become redundant. The order of curriculum topics may also need to be reconsidered (e.g. continuity may need to be addressed earlier and in a different way: Leinhardt et al., 1990). One implication of the above discussion is the notion of a shift in direction of graphing activity, towards interpretation of global behaviour of graphs and toward generalisation from a number of related graphs to a wider class of functions.

We have also discussed examples of how students can spot patterns and explore difficult concepts and relationships using technological tools to generate and process large quantities of data rapidly. However they still need to think analytically: planning operations, interpreting results and contextualising them (Goldstein, 1994). As with four-function calculators, teachers must devise activities which enable students to make sense of results obtained on a graphic calculator or computer program (Shuard et al., 1991). They must help students learn to exploit personal technologies, making effective choices about when it is appropriate to employ a machine and which application to use. New tools obviously generate new solutions; we already have evidence from students working with GCs that they can adapt their approaches and methods to exploit the strengths of the machine, e.g. using a graphical method rather than symbols to solve a trigonometric identity. Goldstein expects all students to have access to personal technologies in future and asserts that students and teachers will need to adapt their approaches to the curriculum accordingly.

*The investigational approach to mathematics learning in school*

Curriculum innovation tends to lag behind technological progress, probably owing to a combination of factors including already over-loaded curriculae, students' lack of access to new technology and teachers' lack of time to develop expertise and confidence in using it. Indeed a range of complex cultural factors may affect the actual impact of proposed curriculum change, as Noss and Hoyles (1996, ch.7) elaborate. Penetration of new mathematical tools - such as the readily available four-function calculator - has certainly been slow in the past[38], despite overwhelming evidence that students using calculators achieve as much or more in comparison (e.g. Suydam, 1986). A meta-analysis of 79 research studies by Hembree and Dessart (1986) indicated that use of calculators improves the average student's basic skills with paper and pencil too, both in working exercises and in problem solving, as well as improving attitudes towards mathematics. Where such tools are accepted, however, their use has usually been confined to traditional mathematical practices. Ruthven (1994) fears that the graphic calculator may meet the same fate. Encouragingly, however, the latest version of the NC for mathematics (DFE/WO, 1995b, p.15) expects Key Stage 3 and 4 students to use graphic calculators and computers to understand the behaviour of graphs.

Other curriculum developments are also encouraging; an important one is the inclusion of 'investigations' throughout the mathematics curriculum. This trend began with the landmark Cockroft Report in 1982, which resulted in considerable pressure to introduce investigative learning and "real" tasks. Unfortunately the result in practice was that investigations became separated from mainstream mathematics and tasks became less and less real. While experimentation and inquiry have not traditionally been routine tools in the teaching and learning of secondary school mathematics in most countries (e.g. Colgan, 1992), a curriculum change is nevertheless beginning in the U.K. This partly reflects the introduction of new technology and complements its use very well. The shift is most significant at GCSE where assessment includes written reports detailing investigational processes (e.g. planning) as well as their results, although investigations are not

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[38] They are still banned until upper secondary level in many Far Eastern countries such as Japan, Singapore, Hong Kong, Korea (Foxman, 1997).

necesarily open-ended. They can be algorithmic too (Morgan, 1994); our own observations confirm this.

Encouraging progress has nevertheless been made in terms of student attainment through investigative mathematics. The latest report by the Third International Maths and Science Study overturns to some extent their previous suggestions that English students ranked towards the bottom of the international league tables in mathematics. Tasks administered to 450 students aged 13 in 50 schools for the study by the National Foundation for Educational Research required students to employ a range of equipment and materials. They tested skills such as weighing and measuring in 'real-life' contexts, as well as graphing and scale drawing. Evidence of specific skills and thinking processes was obtained, including those involved in designing and executing investigations and analysing the data produced. The results showed that the NC's emphasis on using and applying mathematics has apparently been successful: English students came seventh in the world and only one other country performed substantially better (Harmon et al., 1997; TES, 26.9.97, p.1).

The trend towards investigations is now visible in 'A' level syllabuses, where the old emphasis on algebra and calculus has diminished recently, the latter in line with demand from the world of employment (although algebra is important in many fields). With respect to graphing, GCs are becoming more prevalent and teachers are incorporating them in their teaching.[39] The calculators are even becoming the norm in the new innovative 'A' level syllabuses (SMP 16-19 and Nuffield) and examinations. A key development is the Nuffield Advanced Mathematics course, which was initiated in 1990 partly in response to a perceived need to respond to changes in technology. Burns (1994) describes the course's aims as being to encourage appropriate use of technology, to attract more students to 'A' level mathematics, and to encourage more active learning.[40] In more detail, the intention was to build "good intuitive understanding of concepts, through experimentation and through using graphical and numerical tools to support analytical approaches" with students working cooperatively and becoming increasingly independent (*ibid.,* p.204). The observable success of previous graphic calculator projects - including teachers' enthusiasm and the manageability of inservice training - encouraged the developers to require access to GCs for all students, as well as provision within each institution of a computer running a graphing spreadsheet. (The latter had proved to be a powerful general modelling tool, a key feature of the Nuffield course, along with the use of algorithms.)

The advent of the new generation of graphing tools, and calculators in particular, is resulting in a welcome shift away from tasks involving merely drawing and interpreting given graphs towards explorations of ideas about graphs and how they can be transformed (for example, by investigating the effect on a graph of changing a coefficient). According to Ruthven (1992a, p.11), this area of understanding and expertise is the key to many of the graphic strategies enabled by the calculator and it reflects the importance of visual methods in mathematics. Ready access to GC graphing potentially allows graphic ideas (e.g. knowledge of particular graph forms, solving equations using graphical methods and using transformational ideas to describe relationships between graphs) to be addressed earlier in the curriculum (Ruthven, 1994). The notion of graphic zoom is useful and could be progressively developed. The latest 'A' level syllabuses place less emphasis on analytic methods, notably marginal techniques for symbolic manipulation, and their associated examination questions no longer require routine mental and written procedures for which an automatic calculator alternative exists. There is still room for considerable change; Browning (1989) points out that the greatest task for educators is to allow students opportunities for making their own connections without providing shortcuts early on.

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[39] Calculators are likely to have an impact in other curriculum areas too, for example plotting data from experimental work and fitting a trend line (Ruthven, 1992a).

[40] Note that post-16 mathematics curriculae around the world are moving in a similar direction towards incorporating technology, real data and more active learning (Burns, 1994).

*'Real world contexts'*

Recent mathematics textbooks and schemes (e.g. the very popular SMP 11-16) have increasingly attempted to use real world settings as educators realised they were failing to engage students with traditional algorithmic approaches and that knowledge acquired in the abstract tends to remain inert (Collins, Brown and Newman, 1989). However, real world applications are often designed to motivate students to improve their understanding of abstract mathematical concepts. Unfortunately, simply introducing a shopping or other real world context into school problems serves only to disguise mathematical relations, according to Lave (1988). Certainly the almost random inclusion of such contexts has proved unsuccessful (as in the famous example of the typical response of a large sample of students to a calculation of the number of 36-seater buses needed to carry 1128 soldiers, namely '31 remainder 12': Schoenfeld, 1987). Problems of this kind are not the same as those faced in life and are more simplistic; according to Maier (1980, p.21), "they are school problems, coated with a thin veneer of 'real world' associations" and school students recognise this. 'Real world' mathematics is infinitely complex and does not easily enter the classroom, which has its own cultural perspective, values and goals.

Moreover, embedding tasks 'in context' does not necessarily provide learners with any extra handle on mathematical meanings; it might well evoke confusion in the form of more than one set of meanings, practices and and understandings (Noss and Hoyles, 1996, ch.2). The strategy of introducing real world contexts assumes general familiarity or understanding of the context and is problematic also because no single task context can be familiar and meaningful to all students. It ignores the intricate relationship between an individual's own social and cultural values and previous experience, and his or her mathematical goals and beliefs; it continues to underestimate the importance of task context and the myriad of individual interpretations and forms of interaction with tasks as students struggle to construct their own meanings in different situations (Boaler, 1993; Murphy, 1995). Task context is incorrectly expected to influence motivation but not mathematical procedures and performance. Learning mathematics in realistic contexts is assumed to be easier and to promote transfer to the students' everyday lives. Boaler indicates that this assumption is problematic too because students must learn to ignore certain factors which would be pertinent in the real life situation whilst attending to others. Their tendency to bring in world knowledge deemed 'irrelevant' to the task is supported by other work, including the study of graphing by Preece (1993) referred to in section 2. A review of research into science learning by Murphy (1995) offers further evidence and contends that task context determines the link between students' knowledge and its relevance in a particular situation. In sum, the intended task is often very different from that received. According to Boaler, transfer often fails because students do not perceive underlying connections; contexts can enhance transfer by encouraging such perceptions but only through stimulating interest in the mathematical idea or generating discussion and negotiation of the activity and its underlying structure.

> What is important is the appreciation and understanding of the potential generalisability of what is learned and the resemblances to future problems. This appreciation can only come from an examination of the underlying structures and *processes* which connect experiences. (Boaler, 1993, p.15)

Thus contexts can provide an important source of motivation but will only enhance learning transfer to the extent that they make mathematics more meaningful to the individual. This demands that teachers recognise students' own methods, values and experiences (social and cultural as well as cognitive), encourage students to analyse mathematical situations, and take account of individual differences in selecting contexts. Also of prime importance in making a context meaningful is spending some time on elaborating and discussing the context with students, perhaps even engaging them in prediction and speculation about the data. A successful example of this is Ansell's (1988) investigation of variation insunrise patterns throughout the year with an enthusiastic group of year 11 students; the qualitative approach used was successful in motivating the students. In sum, achieving personal authenticity means that students must perceive the underlying rationale and value of their activity. Boaler (1993) points out that 'real world' tasks are devised by adults and usually based on adult everyday life and workplace practice rather than student daily life. In this case, activities are authentic in both senses from the teacher's or employee's view but less 'real' to students.

There are a handful of exceptional curriculum development projects which build on the situated cognition framework and apply its notion of flexible, authentic problem solving to school mathematics. The exemplary Middle-School Mathematics through Applications Project (MMAP) uses various technologies in activities based on real world applications, e.g. students take the roles of designers of a research station in Antarctica (Goldman, 1994). Another is the Jasper Problem-Solving Series of 'contextually anchored' mathematics and science problem-solving tasks, a computer system incorporating interactive video of rich problem-solving situations; learners identify or generate their own problems and subgoals collaboratively (CTGV, 1992).

Brenner (1996) reports a study of seventh and eighth graders engaged in a 5-week investigation of student preferences for different pizzas, involving assessment of orders and invoices for equipment, advertising, nutritional content, profit and loss (the "Pizza Unit"). The mathematics in context included data collection, graph and table construction and interpretation, equation generation and formulae for area. The students demonstrated their everyday knowledge of mathematics and this sometimes formed an effective candidate for more formal mathematical reasoning. However the results highlight the problematic and challenging aspects of trying to incorporate everyday mathematics into the curriculum; it is irrelevant to students unless teachers bring it to the fore. They need to make informal mathematics an explicit part of classroom discourse, with enthusiasm, modeling and elaborating written material using examples of how problems relate to personal experience and local circumstances.

Of most relevance here is the indication that innovative uses of portables as described in the report by Stradling et al. (1994) tend to incorporate more meaningful activities. The authors give an example of newly generated enthusiasm when a spreadsheet activity was changed to a more practical exercise - a budget for the Christmas party; the primary school students relied less on written calculations and became more willing to extend the spreadsheet to other purposes. Several of the examples of work carried out in classrooms involved in the NCET scheme and some of the other studies outlined  throughout this review also demonstrate how portable technologies lend themselves to more authentic activities. We have seen how students' work with these tools, even where it begins with a relatively prescribed task, will often take off spontaneously in new directions; students can rapidly become engaged in extensive follow-up investigations. Unfortunately the mathematics NC is slow to develop in this direction, the only apparent concession being an attempt to make the Programme of Study for algebra at Key Stages 3 and 4 more palatable via frequent references to 'real-life situations' (DFE/WO, 1995b, p.15). The only reference to real-world situations in the latest specification for the 'A' level mathematics core curriculum concerns mathematical modelling (SCAA, 1997). It therefore remains up to imaginative teachers, students and researchers to continue the pioneering work which already offers considerable potential for engagement in meaningful activity.

A final and rare example of innovation is the course MU120 at the Open University, mentioned in section 3. This course takes a radically new approach to mathematics learning by introducing mathematical ideas in meaningful and accessible contexts. The course was designed for (and successfully attracted) those with rusty or limited mathematical knowledge; it aims to build confidence and engage students in using mathematics, and to encourage question posing rather than answering: questions are formulated before data from real world situations is collected or presented, analysed and interpreted (e.g. statistical investigations in contexts of health and economics; maps, art, baking, motion). The range of graphing contexts goes beyond the traditional restriction to the two categories of travel and growth. Some of the materials could be adapted for use by younger students too (as the conceptual level is similar to 'A' level syllabuses). Indeed a pilot evaluation of a small number of secondary school students who studied the course during 1996 (alongside their conventional 'A' level work) shows that it successfully heightened the students' awareness of the uses of mathematics, altering their perceptions of mathematics from numbers or textbook examples to "mathematics all around them" and how it "fits into everyday life" (Allen, 1997). The MU120 materials provide an exciting and wide range of applied activities for research purposes.

## 6. Cooperative use of portable computers

We turn now to another important issue in designing classroom activities for use with portable computers; the role of collaboration in facilitating learning. There is a wealth of research on the benefits of cooperative and collaborative learning and this has been reviewed at length elsewhere (including by the author in Hennessy & Murphy, 1998). Much of it focuses on computers, using the software either as a background context for student interaction (e.g. Moschkovich, 1996[41]), or specifically to structure collaboration, or as a critical mediator of social interaction (Noss & Hoyles, 1996, ch.6). Despite theoretical differences between these views, work in this area generally converges on the conclusion that computers can stimulate valuable discussion between students.

The research literature has clear implications for planning activities for students working with technology. For example, the group needs a shared goal and to work as a group, not merely alongside each other. The findings of the *Groupwork with Computers* project (Pozzi, Healy & Hoyles, 1993) imply that the optimal scenario for learning is for students first to engage in mutual discussion with peers whilst interacting with the computer, then coming across the perspectives of other students during whole group discussion. Having a public prediction phase (followed by checking) is believed to be motivating where conceptual development is the aim (e.g. Howe et al., 1995); students become interested to determine who is correct. Recent work by Hale (1996) draws on Roschelle's framework for how collaboration between students using graphical simulation software can promote 'convergent conceptual change': discourse can accomplish gradual convergence of meaning both between students and between informal and scientific concepts (Roschelle, 1992). Hale's work with the Calculator-Based Laboratory proved partially successful in improving understanding of graphs but indicated that discourse between students can actually reinforce or provoke new misconceptions unless it is followed by a well-organised teacher-led discussion giving students an opportunity to repair their misconceptions. Moschkovich's (1996) studies of the rich descriptions of the discourse between three pairs of high school students using graphing software to explore the connections between linear equations and their graphs concluded that students must address conflicts, resolve negotiations and maintain a focus on mathematical productive paths of investigation and questioning, for collaboration to be constructive.

The advent of portable technologies raises some unique issues regarding collaborative investigations and it is these we are particularly concerned with here. It must be acknowledged that portable computing requires a shift in classroom organisation since becoming carried away with the notion of one machine per student could mean that the benefits of student interaction are lost (Goldstein, 1994). Even though NCET (1993) happily predict that the ratio of students to computers will reach 1:1 during the 1990s, they express a small element of caution: "Not every technological advance means an advance in learning - 30 students sitting at 30 desks working at 30 'electronic slates' is no way forward" (p.29). Apart from educational and social considerations, most portable computer screens can only be viewed comfortably from particular angles; while this facilitates student privacy, groups can find them difficult to use (Cloke, 1996), although this will probably improve. Goldstein concludes that teachers will continue to need to work with a large group around a single computer sometimes. Desktop machines may still play a role in the classrooms of the future.

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[41] Moschkovich is one of the few authors who addresses the detailed processes underlying peer collaboration.

Our own and others' experience of the NCET scheme (*Arc*, Autumn 1994, p.21; Bowell et al., 1994) confirms that students working with portable computers spontaneously share information and expertise, both within and outside lesson time. (Technological problem solving in general appears to foster a culture based on cooperative learning far more successfully than ordinary book-based schoolwork does.) It is possible to teach a whole class a new technique by demonstrating it to one or two individuals (Bowell et al., 1994). In the Pocket Book pilot study there was a notable 'cascade effect' during every session observed whereby knowledge, skills and information about the machine's features were rapidly shared with other members of the group. On several occasions the observers witnessed small learning groups which had formed naturally to teach each other how to use the various Pocket Book facilities. Any student encountering a problem using the Pocket Book was more likely to consult peers rather than the teacher or the manual, except as a last resort (Fung et al., 1995). The degree of co-operative learning taking place within the tutor groups was in fact reflected in the finding that the Pocket Books were considered easy to use.

Peer tutoring was commonly observed in the primary and secondary schools in the NCET scheme. Bowell et al. (1994) pointed out that helping each other made students feel valued and raised their self-esteem. Some students previously viewed as disruptive or low achieving became motivated and respected members of the class as a consequence of their newly developed expertise. It is not only IT expertise which develops: some teachers found that students working with portables collaborated more and were more productive. Some noted that students, particularly those of primary age, were happier to work in pairs on the machines. The scheme also showed that students could write creatively in small groups by passing the palmtop between students (Stradling, Sims & Jamison, 1994, ch.2). While some of the learning activities described by teachers could have been undertaken using desktop machines, portables were preferred because they offered more opportunities for interaction. Bowell et al. (1994, p.24) concluded: "Far from discouraging verbal communication and collaborative working, the use of portables has provided opportunities for communication at a level that could not have been envisaged." Parents' and others' fears that portable computing would evoke social isolation have proven unfounded. Further evidence derives from the Australian experience. Loader (1993) asserts that peer learning played a new and major role when all students aged 9-10 were issued with personal laptops. He claimed that much informal classroom dialogue became work-focused rather than socially oriented.

As we will see below, a gender issue arises because research shows that working alone at computers can alienate girls, who tend to enjoy and benefit from collaborating (Underwood, 1994, ch.2); the superior social skills they develop from a young age offer them an advantage in working together in school (Murphy, 1998). Boys, by contrast, may fight for control, and they view collaboration as distracting from individual achievements (Sutherland and Hoyles, 1988). In the case of the smaller portable technologies - palmtops and GCs - which have smaller keys and screens than conventional computers, there may be a case for encouraging cooperation between students each working on their own machine. Smart (1992) observed that while this setup enables students to experiment and make mistakes in private, much involved mathematical discussion still develops and collaborative working can be encouraged through investigative tasks. An example is offered where collaboration is the most efficient way of working (using different calculators for different aspects of the task). However Smart (1995) successfully organised her research class into pairs sharing GCs and found that the girls not only shared the machines but collaborated in their planning and shared their findings with their partners and classmates. Underwood (1994) similarly observed that girls organised into pairs tended to work well as a team, whether they were instructed to collaborate or work individually.[42] With either kind of organisation, the cooperative element serves to pre-empt feelings of exclusion and incompetence by less confident students or indeed teachers. It may also overcome the

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[42] Pairs of boys were able to collaborate successfully but performed poorly when instructed to work individually.

perception, particularly by girls, that computing is an unsociable activity. Successful groupwork appears to allow students to transcend gender stereotypes (Pryor, 1995).

The next section presents further discussion of the gender issues which necessitate consideration in planning computer-based activities, including differential attitudes and approaches to using technology. Specifically, the potential benefits of portable computing for increasing girls' participation and engagement in computer activities are considered.

## 7. Gender issues in portable computing

*Use of and attitudes to computers*

Surveys of the literature (notably, Beynon, 1993; Hoyles, 1988; Kay, 1992; Robertson et al., 1995b) indicate a research focus on differences in *attitudes* and *access* to computers and in performance on computer tasks, generally converging on the conclusion that males of all ages are more dominant and more positive towards computers, although there are some exceptions (e.g. Loyd, Loyd & Gressard, 1987; Pozzi et al., 1993). Evidence for (physically and verbally) dominant behaviour by boys in mixed-sex groups and poor performance by girls indicates that boys may become more enthusiastic as a consequence (Culley, 1988; Kirkup, 1992; Underwood, 1994). Mixed-sex grouping undermines the confidence of girls and their perceived competence, however. As Underwood (p.9) put it, "Boys, girls and computers are a dangerous combination... because the boys see themselves as the rightful and superior users of the technology." Computing has become defined as an activity more appropriate to males (Culley, 1988).

Various different reasons for the gender gap have been proposed, including socialisation and gender stereotyping; consequent peer pressure, adult role models (most parents and teachers using computers are male: Culley, 1988), teacher bias and parental attitude; and an interaction with socio-economic status (Kirkman, 1993; Millard, 1997b). Resulting differences in *access* to computers may be the most important factor; boys certainly use them more often at home than girls, most computers are bought for males (e.g. Culley, 1988; Kay, 1992) and boys receive more parental encouragement than girls (Millard, 1997a). Although use within lesson time is equivalent, school computer clubs are dominated by boys too (Siann & MacLeod, 1986). Prior computer exposure, particularly home access (and therefore parental attitude), influences affective and cognitive attitudes towards computers more than gender (Levin & Gordon, 1989) and more than school use (Kirkman, 1993).[43] Thus attitudes formed at home probably dominate students' attitudes to computers, both generally and at school. Confident users - mainly boys - compete more strongly and more successfully for computer time.

The importance of home access and adult modelling of opportunity is corroborated by a recent study of 190 students aged 11-14 by Millard (1997b). Interviews determined that some boys acted as computer bouncers at home, restricting access to their sisters and expressing dismissal of girls using their machines. Questionnaires indicated that while actual computer provision appeared to be approximately equivalent, more than twice as many boys had computers situated in their own rooms. Millard claims that gender differences in the ownership and control of machines can be overcome by family shared machines and computer literate parents who use computer programs themselves for work; these are rare in less affluent households. In sum, students bring with them to school firm ideas on what aspects of computer literacy are most appropriate to themselves and boys' views are more intractable than girls' (Millard, 1997a).

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[43] Joiner and Norgate (1997) found that both computer experience and background knowledge about a problem-solving task were important in explaining better performance by boys than girls.

The nature of use may confound the apparent sex difference in use of computers if use for entertainment is included. The NCET questionnaire survey of over 500 students yielded no significant differences in the use of different features of the portables; both sexes used the machines for homework but boys were twice as likely to use them for purposes other than schoolwork such as playing computer games (Stradling, Sims & Jamison, 1994, ch.3). The questionnaire data from 61 students in the Pocket Book project indicated that boys and girls had equal use of a computer at home, once games machines were ignored. However boys were 10 times more likely to have sole access to computers and they used them significantly more often. Robertson et al. (1995b) interpreted this to mean that girls see computers as less effective instructional devices than boys. This finding is supported by Siann and Macleod (1986) and Turkle (1984), who suggest that girls are less interested in conventional uses of computers than boys. The gender difference begins to emerge at a very early age and persists through secondary school. However, Millard (1997b) claimed that when given free access and encouragement for using the technology, girls did not find the machine culture alienating but adapted it to their own purposes and interests. These girls were enthusiastic about using computers to support school work.

Portable technologies may provide a different picture in any case. Robertson et al. (1995a) found that girls and boys displayed the same overall pattern of use of the Pocket Book for homework, for personal use, for different applications and in different subjects. Similarly, the NCET survey found that girls were as positive as boys about the use of portables, so perhaps these personal machines are more appealing to girls than desktop machines. Additional support comes from the London Docklands Project; despite no significant differences between girls' and boys' attainment before the project, girls' reading levels improved significantly more than boys' did (National Literacy Association, 1996).

Girls also outperformed boys on a symbolising task used in the Graphic Calculators in Mathematics project - although boys in the control group performed better than girls, ostensibly because using GCs regularly reduces uncertainty and anxiety (Ruthven, 1990). The attitude survey by Dick and Shaughnessy (1988) provides further evidence that girls' initial anxiety about using GCs shifts over time towards a view of calculators as making mathematics enjoyable and easier. Calculator use offers greater exposure to symbolised graphic images which is believed to increase both competence and confidence of students, particularly females, on such tasks. Work by Smart (1992) confirms that GCs may benefit girls because they value the opportunity to use more personal forms of technology (concealing mistakes from their computer-literate male peers), and the processes of investigation and discussion lead to unusually high levels of confidence about their work. Similarly, Dunham (1991) observed an interaction between gender and confidence whereby low confidence females were the most likely group to use GCs and high confidence females the least likely. Furthermore, the final report from one school in the NCET scheme pointed out that boys dominate more where access to technology is limited. Male computer experts lose their power when portables are used since girls quickly match their expertise. These findings suggest at least that these new technologies offer no advantages to males, and they may be instrumental in overcoming girls' feelings of inadequacy and "learned helplessness" (Licht & Dweck, 1985).

Computer experience is clearly related directly to student attitudes to computers (e.g. Kirkman, 1993; Levin & Gordon, 1989; Loyd et al., 1987) but it is uncertain whether frequent use promotes a positive attitude, or vice versa. Robertson et al. (1995a) predicted that students' experience of using computers should lead to changes in attitude over time and this was supported by their questionnaire data. Pre-test scores indicated that girls felt significantly less confident about and less competent at using IT; they were less positive about the role and effectiveness of computers and would be less likely to work with computers and learn about them. These differences were probably due to the boys' greater previous experience of computers. In contrast with previous research, there were no differences in anxiety or enjoyment however. After 8 months of Pocket Book computing and similar experiences with other computers in school at least, there was a striking effect: gender differences had all disappeared except on the confidence subscale (even here, the difference was reduced).

Further evidence comes from the PLAIT project questionnaire results reported by Gardner et al. (1994), where the experimental girls enjoyed mathematics significantly more than their control counterparts while the boys showed an opposite trend. The authors raise the question of whether girls' typical dislike of mathematics is positively influenced through using portable computers; the experimental girls' perceptions of school were also significantly more favourable than in the control group of girls.

*Female-friendly technological activities*

Research in the area of IT and gender issues indicates that tasks may contain gender biases which can elicit attitudinal and performance differences (Littleton et al, 1992). While boys may be unaffected by task format and context, girls are considerably more sensitive to even slight changes; Littleton et al. found that girls engaged much better with a planning task involving 'Honeybears' than the same task with 'King's men and pirates' as characters. In a later study, girls preferred a female stereotyped version of the software ('Princesses') and performed better with it, whereas boys were motivated regardless of the software (Joiner et al., 1996). The authors concluded that software should be designed for girls. Unfortunately, teachers apparently make little effort to accommodate girls' interests in devising curriculum materials (Culley, 1988). Millard (1997b) confirms that the persistent gender differences in attitudes to computers around the world indicates a failure in the way gender issues are being addressed and tackled in schools.

There are some further hints about how to avoid gender stereotyping and bias, ensuring that technology-based tasks are female-friendly, from Burke et al. (1988), Carmichael et al (1985), Hoyles (1988), Licht and Dweck (1985), Pryor (1995) and Shoaf-Grubbs (1995): open-ended tasks with loosely-defined goals and similarly, teacher as participant rather than dictator; cooperation rather than competition; emphasis on process rather than outcome; minimal tension between computer and other school work, through curriculum-focused courses and computer activities which follow up previous non-computer work; lack of time pressure; peer evaluation. In the case of the complex GC, it is also important to provide user-friendly materials specially for the calculator and to encourage girls to familiarise themselves with the machines at their own pace (Smart, 1995). Building confidence is extremely important for girls, who unsurprisingly benefit from student autonomy, including in organising resources, and from positive female role models, as in the SMILE mathematics scheme (Burke et al., 1988).

Turkle's work (1995) implies that the nature of students' introduction to computers - as well as the content of software - should be carefully considered. One factor is that girls in single-sex schools demonstrate far more enthusiasm for and participation in computing activities; male domination and girls' apparent disadvantages (including cognitive ones: Siann et al, 1990) might thus be reduced if single-sex groupings are used (Culley, 1988). This is contentious, however (see Hughes et al., 1988, for a conflicting result); ability and personality factors may be more important (Pryor, 1995).

The new activities developed for the OU course MU120 (described earlier) certainly seem to appeal to women. The course team's deliberate attempt to broaden access to women (e.g. by gently building up complexity of carefully selected tasks and setting activities within everyday contexts) has been spectacularly successful: the proportion of women students enrolled in the first year more than doubled that of the previous foundation course (from 23% to 49%), and actual numbers of women increased by about 28%. Moreover, women achieved more and persevered for longer: only 28% failed the course or dropped out compared with 42% of men, and as with the previous course, twice as many women (20% versus 10% men) gained a distinction. However, the numbers of men who registered for MU120 dropped significantly - to just under half of the previous number - and the proportion of men who failed or dropped out increased during the new course (from 29% to 42%). Men may have been attracted to other more traditional or higher level courses instead. It would be worrying if the women's success was instead directly at the expense of the men's; this is an issue which needs addressing.

Some evidence that real world contexts are particularly attractive to females comes from the first small group of secondary school students to study MU120. The female students asserted that the "stress on real uses of mathematics" motivated them most to study the course whereas males reportedly "needed the mathematics for other courses" (Allen, 1997). However this sex difference was not apparent in a larger survey of 96 adult MU120 students (average age 35 years) over the first two years of the course. The results showed that "wanting to know more about mathematics" was the predominant source of attraction to the course in both sexes and "real uses of mathematics" was rated third (Allen & Mason, 1997). These findings do not explain why the course appeared far more accessible to women than its predecessor and this would benefit from further investigation. (Use of the GC does not seem to be the influencing factor since preliminary results of a questionnaire survey by the author and her colleagues indicate that the calculator was viewed as equally user-friendly and supportive of mathematical learning by male and female MU120 students.)

*Gender and engagement in computer activities*

Anecdotal evidence from a number of authors (Carmichael et al., 1985; McFarlane, 1995b; Pryor, 1995; Smart, 1992; Turkle, 1995), including our own informal observations during the Pocket Book project, confirm that there is a striking gender difference in the *nature* of computer use: girls will use computer technology only if it has clear relevance and a recognisable advantage. Boys, on the other hand, are more interested in exploring and discovering new facilities. (This phenomenon will be explored further by Hennessy in a forthcoming paper.) Turkle's research with college students shows that many women (and some men) have in fact felt alienated from the computer (Turkle, 1984; Turkle and Papert, 1990). Support comes from several authors (e.g. Culley, 1988) who have commented that many girls do not perceive computer activities and games as related to their own interests.

The educational research findings reported here are reflected in the wider technological society, where there is an overwhelming bias towards production of software which is specifically male-interest orientated (Frenkel, 1990). The software industry believes that targetting the female market through advertising would have an undesirable effect upon the male market. This culture understandably produces feelings of unease for women.

Siann and Macleod (1986) stress that if students are to familiarise themselves with technology and view it as a useful resource that will continue to play an increasing role in their lives, then we must not alienate girls any further. Parents have a role to play here too - in providing girls with access to home computers and software tools, as well as role models and encouragement. Fortunately, the computer culture may now be changing - away from a requirement for programming skills towards an emphasis on exploratory interaction; it seems to be rapidly becoming more accessible to women (Turkle, 1995; Kirkup, 1992). The limited evidence concering portable computers corroborates this proposed shift, which warrrants further empirical investigation. Studies of gender differences in perceptions of and interactions with portable technologies may provide some insight or highlight some new issues.

## 8. Conclusion

Our starting point was the proposition by Leinhardt et al. (1990, p.7) that "More than perhaps any other early mathematics topic, technology dramatically affects the teaching and learning of functions and graphs". This review of the literature indicates that graphing technologies, portable ones in particular, present a compelling opportunity to help students develop understanding and skills in a traditionally difficult curriculum area.

Portables are becoming increasingly popular in schools for a variety of reasons: they offer tremendous flexibility; increased access to IT and significant practical advantages over conventional desktop computing; personal ownership and opportunities for more independent, investigative learning across the curriculum; improvements in the quality of work produced by students; increased student motivation; a supportive environment for female styles of working and stimulation of productive collaboration between students. They are a potential catalyst for curriculum change, already forcing educators to re-evaluate what and how they teach with respect to graphing. The present National Curriculum in the UK will remain unchanged for 5 years but revisions are expected in 2000, when portable computers may perhaps play a greater role. Research is needed now to establish how the potential of this powerful tool may be exploited and to guide the introduction of future forms of portable technology into education.

The **PIGMI** (Portable Information Technologies for supporting Graphical Mathematics Investigations) Project currently being undertaken by the author and her colleagues, Pat Fung and Eileen Scanlon, at the Open University is addressing this challenge through a series of pilot studies with secondary school and Open University students. Using classroom observation, interviews, questionnaires and written records of graphing activities, we hope to shed some further light on the role which portable computers and graphic calculators can play in facilitating development of graphing concepts and skills. The main objectives are to investigate how these technologies mediate learning to handle graphical representations (establishing their strengths and weaknesses in this context), to identify the role of multiple representations in learning, and to develop and test appropriate activities involving investigations of real world problems.

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