CRITICAL SHEAR STRESS OF NATURAL SEDIMENTS

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ABSTRACT: The critical shear stress of individual fractions τ_{ci} in unimodal and weakly bimodal sediments shows little variation with grain size and depends only on the mean grain size of the mixture. For strongly bimodal sediments, τ_{ci} increases with grain size, an apparent result of a lateral segregation of the finer and coarser fractions on the bed surface that causes τ_{ci} to deviate from size independence in the direction of unisize (Shields) values. A quantitative definition of mixture bimodality may be used to estimate the degree of mixture bimodality beyond which a substantial size dependence of τ_{ci} is observed and to predict the variation of τ_{ci} with grain size in bimodal sediments. Because properties of sediment grain-size distributions other than bimodality appear to have little influence on τ_{ci}, the trends presented here for 14 unimodal and bimodal sediments may be quite general.

INTRODUCTION

Previous observations of incipient motion in mixed-size sediments suggested that relatively simple relations might exist for the critical shear stress of individual grain sizes τ_{ci}. For example, Parker et al. (1982) demonstrated that all sizes in the bed of Oak Creek, Oregon begin moving at nearly the same bed shear stress. A number of authors have suggested that the critical shear stress of the entire mixture can be estimated using an index grain size (most commonly the mean or median size) and the Shields criterion (Lane 1955; Pantelopoulos 1957; Neill 1968; Vanoni 1975). Wilcock and Southard (1988) found that both these results held for a wide range of size distributions, including skewed, lognormal, and rectangular unimodal distributions and weakly bimodal distributions. Subsequently, however, observations in flume and field demonstrated that this simple result does not apply to all sediments. Laboratory experiments with two-component (very strongly bimodal) mixtures show that τ_{ci} varies with grain size in some, but not all, bimodal mixtures (Misri et al. 1984; Wilcock 1992a; Worman 1992). Kuhnle (1992) found that τ_{ci} consistently increased with grain size for samples from Goodwin Creek, Mississippi. The Goodwin Creek sediment is more strongly bimodal than those previously examined by Wilcock and Southard (1988). These observations led the writer to conduct additional experiments with an extremely bimodal sediment in order to obtain observations of τ_{ci} for a range of mixture bimodality that would include almost all natural sediments. This paper has three purposes:

1. To present these additional observations of τ_{ci} and a physical interpretation for the differences in τ_{ci} observed between unimodal and bimodal mixed-size sediments.
2. To develop a simple parameter defining mixture bimodality and evaluate its utility in describing the variation of τ_{ci} with bimodality and in defining

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the degree of mixture bimodality at which τ_{ci} begins to deviate from the simpler unimodal behavior.

3. To determine if consistent trends may be found that describe τ_{ci} for both unimodal and bimodal sediments.

Because bimodal sediments are common, particularly in gravel-bed rivers (Shaw and Kellerhals 1982), a consistent means of predicting τ_{ci} for these sediments is of some importance. Because other parameters defining the shape of sediment grain-size distributions (e.g., mixture standard deviation and skewness) appear to have little influence on τ_{ci} (Rakoczi 1975; Wilcock and Southard 1988), a method of predicting τ_{ci} for both unimodal and bimodal mixed-size sediment offers the possibility of being of quite general.

AVAILABLE DATA

The primary requirement used to select the data examined here is that the fractional transport rates were well measured over a wide range, including very low transport rates near incipient motion. In practice, this requirement limits the data to cases where all transport was sampled (generally through a slot sampler spanning the full width of the bed). A further requirement was that sufficient sampling was performed to account for variability of fractional transport rates over a variety of time scales.

Table 1 provides a summary of all mixtures discussed here. The reader is referred to the original papers for further detail. Grain-size distributions for all mixtures are shown in Fig. 1. The distributions are plotted as histograms of $1/4\phi$ fractions (fraction width equals a factor of $\sqrt[4]{2}$) to provide the most direct comparison of the distribution shapes. Because the published grain-size distributions were given in a variety of fraction widths, the proportion in each $1/4\phi$ fraction was determined where necessary from cumulative curves of the published values. The incipient motion behavior of many of these mixtures have been discussed previously (Day 1980; Parker et al. 1982; Wilcock and Southard 1988; Wilcock 1992a).

Because previous observations suggest that τ_{ci} depends on mixture bimodality, transport measurements with a new, extremely bimodal, mixture (LH-50) were performed for this paper to provide incipient motion observations for a mixture that, together with previously studied sediments, span the range of mixture bimodality that might be expected in nature. These experiments were made using methods similar to those described in more detail elsewhere (Wilcock and Southard 1988; Wilcock 1992a) and are only summarized here. Both water and sediment were recirculated in 60-cm-wide flume. The bed and transport were allowed to come to an equilibrium transport condition before hydraulic and transport sampling was conducted. Flow depth in all runs was held within a narrow range around 11 cm. Many runs were conducted at different discharges, with a focus on runs producing transport near incipient motion. Transport was sampled through a slot in the sediment bed that removed all of the moving sediment and a small amount of water to the sediment recirculating system. All sediment in motion was sampled with the exception of runs at the highest flow strengths, during which a small portion of the sand overpassed the sampling slot and passed into the main tail box of the flume, where it was recirculated by the main pumps. Because at these flows the transport rates of the fine fractions were far in excess of incipient motion, the unsampled transport had a negligible effect on our estimates of τ_{ci} for these fractions.

TABLE 1. Properties of Sediment Mixtures

Mixture name (1)	Mixture type (2)	D_m (mm) (3)	Standard deviation (ϕ) (4)	Standard deviation (geometric) (mm) (5)	α (Eq. 3) (6)	β (Eq. 3) (7)	B (Eq. 4) (8)	Reference (9)
$1/2\phi$	Lognormal	1.82	0.50	1.41	0.98	−0.04	0.67	Wilcock and Southard (1988)
1ϕ	Lognormal	1.85	0.99	1.99	1.00	0.01	0.37	Wilcock and Southard (1988)
Misri N1	≈Log-normal	2.37	1.00	2.00	1.32	0.00	0.22	Misri et al. (1984)
Oak Creek	Long fine tail	13.1	2.10	4.29	1.22	0.07	0.39	Milhous (1973)
MC-50	Pure bimodal	3.19	0.79	1.73	1.05	0.00	1.7	Wilcock (1992a)
FC-50	Pure bimodal	1.88	1.51	2.85	0.83	0.40	2.7	Wilcock (1992a)
FC-70	Pure bimodal	1.30	1.45	2.73	0.65	0.32	2.7	Wilcock (1992a)
LH-50	Pure bimodal	1.90	2.77	6.81	0.57	0.72	6.7	This paper
DAY A	Weakly bimodal	1.50	1.77	3.41	1.13	0.17	1.9	Day (1980)
BOMC	Bimodal	4.1	2.44	5.42	0.57	0.51	3.0	Wilcock and McArdell (*in press*)
Misri M1	Pure bimodal	2.71	0.69	1.62	1.10	−0.06	1.6	Misri et al. (1984)
Misri M2	Pure bimodal	3.61	0.69	1.61	1.10	−0.06	1.6	Misri et al. (1984)
Misri M3	Pure bimodal	2.04	0.70	1.62	1.10	−0.06	1.6	Misri et al. (1984)
Misri M4	Pure bimodal	3.13	0.69	1.62	1.10	−0.06	1.6	Misri et al. (1984)

To include additional samples at subthreshold transport rates, the subset of the Oak Creek data examined here (34 samples) is larger than that used previously [22 samples; Parker et al. (1982); Wilcock (1992a)]. Both subsets are taken from the winter 1971 data set, which was noted by Milhous (1973) as representing the highest quality observations of general transport. All samples for a discharge greater than 0.5 m³/s are used here. The additional data form a smooth and consistent extension of the trend between total transport and shear stress in the data previously analyzed. The remaining, unanalyzed samples of the winter, 1971 data set correspond to discharges smaller than 0.45 m³/s and show considerable scatter, a result of contami-
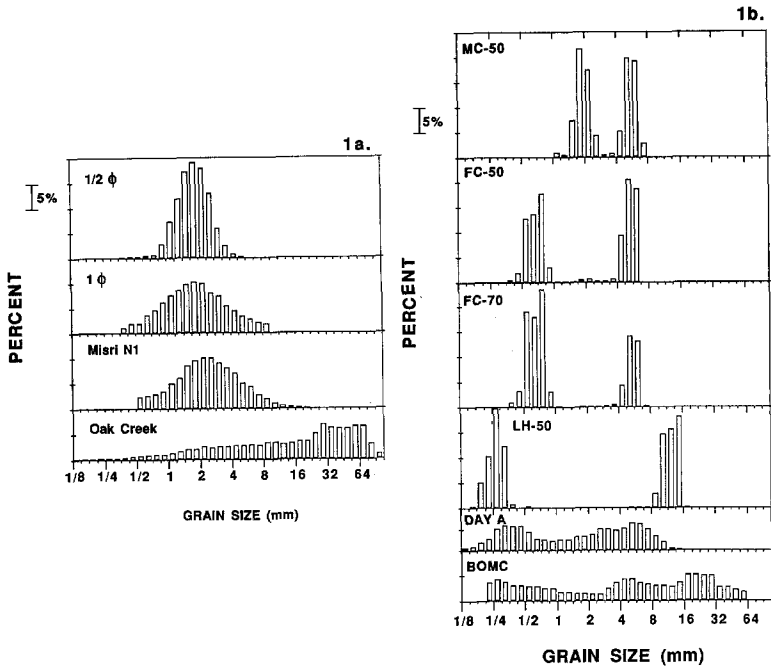
FIG. 1.   Grain-Size Distributions of Sediments Examined in This Paper: (a) Unimodal Size Distributions; (b) Bimodal Size Distributions

nation by organic debris, bed consolidation, and the difficulty in accurately sampling very small transport rates ($q_b < 10^{-5}$ kg/ms).

DEFINING AND MEASURING τ_{ci}

The critical shear stress of individual fractions in a mixture is difficult to both define and measure. Different approaches to the problem have been discussed and compared elsewhere (Ashworth and Ferguson 1989; Diplas 1992; Ferguson 1992; Hammond et al. 1984; Komar 1987; Parker et al. 1982; Petit 1990; White and Day 1982; Wilcock 1988, 1992a, 1992b; Wilcock and Southard 1988) and are not treated in detail here. τ_{ri} is estimated in this paper as the bed shear stress τ_{ci} that produces a small reference transport rate for each fraction. This method offers several particular advantages. Because it defines a shear stress that produces a particular transport rate, it is directly applicable to predictions of fractional transport rates. It is based on many observations of transport rate for each fraction and provides a relatively stable, well-defined, and repeatable estimate of τ_{ci}. Finally, the reference transport approach can be determined for all fractions in a mixture, regardless of the distribution of τ_{ci}, thereby allowing the same method to be consistently applied to all fractions in any sediment mixture.

The reference transport rate is defined as $W_r^* = 0.002$, and W^* is defined for each fraction as:

$$W_i^* = \frac{(s-1)gq_{bi}}{f_i \left(\dfrac{\tau_0}{\rho}\right)^{3/2}} \quad \dotfill \quad (1)$$

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where s = ratio of sediment and water density ρ_s/ρ; f_i = proportion of fraction i in the sediment bed; q_{bi} = fractional transport rate computed as $p_i q_b$; p_i = proportion of fraction i in the transport; and q_b = volume transport rate per unit width. Because grain size is not present in W^*, plots of the resulting τ_{ri} versus grain size are undistorted with respect to grain size, a considerable advantage when comparing τ_{ri} among different size fractions (Parker et al. 1982).

In this paper, estimates of τ_{ri} are made only for mixtures with measured transport rates below $W_i^* = 0.002$ for most or all fractions. No estimates of τ_{ri} were made for individual fractions with a minimum value of $W_i^* > 0.02$. This was done to avoid the uncertainty involved in extrapolating a curve fitted to the fractional transport rates, which is especially important with strongly bimodal sediments, for which both the value of τ_{ri} and the general trend of the fractional transport rates can vary with grain size. Estimates of τ_{ri} for most fractions were determined by interpolation about the reference value and are essentially independent of the overall trend of the transport relation. In the case of fractions whose smallest W_i^* falls in the range $0.002 < W_i^* < 0.02$, τ_{ri} was estimated using a trend similar to an adjacent fraction with measured $W_i^* < 0.002$. The fractional transport curves for several bimodal mixtures and the reference transport criterion are given in Fig. 2, which is plotted in dimensional units to provide a direct illustration of the variation in transport from fraction to fraction and from mixture to mixture. Plots of other fractional transport rates may be found in the original
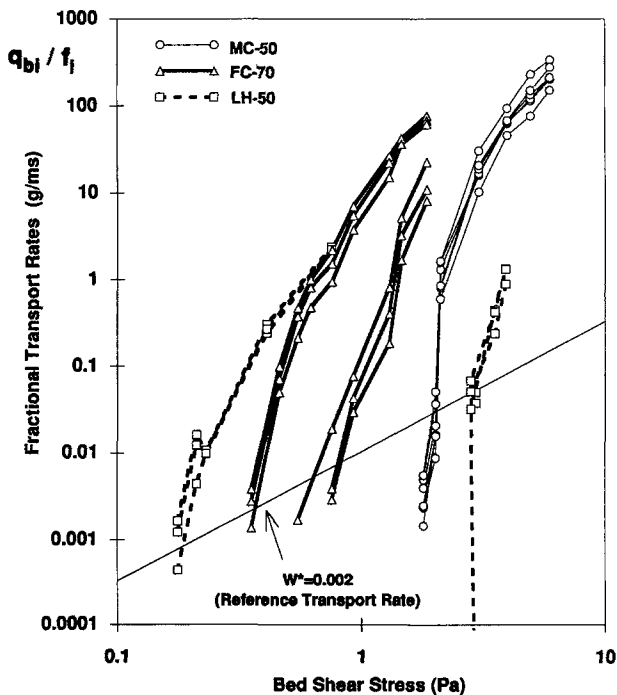

FIG. 2. Example of Fractional Transport Rates for Three Bimodal Sediments. See Table 1 for Sediment Properties. Also Shown is Reference Transport Line $W^* = 0.002$

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references (Parker et al. 1982; White and Day 1982; Wilcock 1992a; Wilcock and McArdell, in press, 1993).

An estimate of the error in estimating τ_{ri} was determined by defining a minimum and maximum conceivable fit to the data near $W_i^* = 0.002$. These estimates were made by eye to facilitate the emphasis used here on points near W_i^* and to minimize the effect of obvious outliers. Because this procedure introduces an element of subjectivity in determining τ_{ri}^*, these error bounds were made as large as possible. The error bounds were typically on the order of 10% or less for estimating the value of τ_{ri} for an individual fraction. The error in estimating the variation of τ_{ri} with grain size within a particular mixture is considerably smaller, however, if the fit is made consistently from fraction to fraction. As discussed later, the effect of a 10% error in individual τ_{ri} estimates has only a minor effect on the variation of τ_{ri} with grain size (which is the central topic of this paper).

Bed Shear Stress

Because bed forms were present in some runs, particularly those with an abundance of sand in the mixture, the drag-partition formula of Einstein (1950) was used to estimate the skin-friction portion τ_0' of the total bed shear stress. This model computes the skin friction as $\tau_0' = \rho g R' S$, where R' is the portion of the flow hydraulic radius that corresponds to the skin friction and is computed iteratively from a Keulegan-type profile

$$\frac{U}{\sqrt{gR'S}} = \frac{1}{\kappa} \ln\left(\frac{R'}{2.718 z_0}\right) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2)$$

where U = mean channel velocity, g = the acceleration of gravity; S = the water slope; κ = von Karman's constant, taken here to be 0.4; and z_0 = a roughness length characteristic of the sediment bed. Values of z_0 for each sediment were determined using the total hydraulic radius in (2) for flows with plane beds and little or no transport. A value of $z_0 = D_{65}/30$ was found to give a good approximation of the roughness for the unimodal sediment beds. For bimodal sediments, the mean grain size of the coarse mode was found to provide a good estimate of the bed roughness. Because the primary use of the bed shear stress data in this paper is to determine a critical shear stress, no corrections were made for an increased roughness attributable to moving grains. Bed forms in the flume runs were generally long and low [Wilcock (1992a); Bettess, personal communication (1984); regarding experiments of Day (1980)], and form drag varied between 0% and 30% of the total bed shear stress, with the smaller value generally pertaining to the incipient motion conditions discussed here. Form drag corrections for the field cases investigated for this paper were much larger. The form drag estimated for Oak Creek varied slightly as a function of discharge about a value of 55% of the total bed shear stress. This magnitude of form drag corresponds well with an average Manning's n of 0.05 back-calculated for the study reach, which is nearly twice that predicted by the Manning–Strickler formula for the bed-material size (Milhous 1973).

It is important to note that estimates of skin friction apply equally to all fractions in a mixture and do not, therefore, influence the variation of τ_{ri} with grain size, which is the primary focus of this paper. Skin friction estimates do, however, directly influence any index value of reference shear stress for the entire mixture. This matter will be taken up once the comparison of τ_{ri} from mixture to mixture is made.

Observations of τ_{ci}

Fig. 3 presents values of τ_{ri} as a function of grain size for four different unimodal mixed-size sediments. Also shown on the figure is the Shields
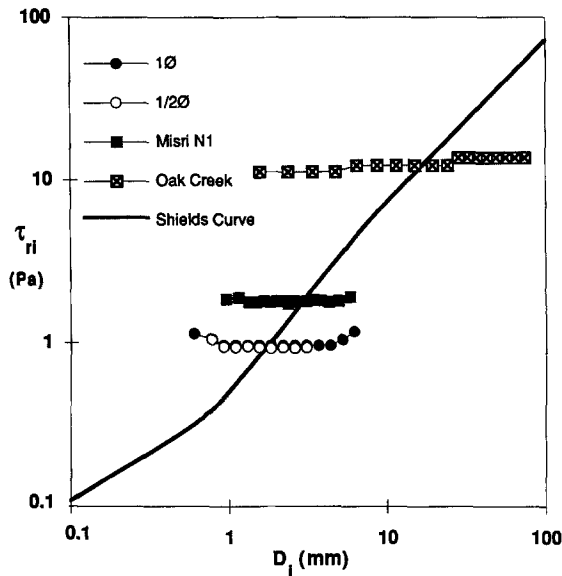

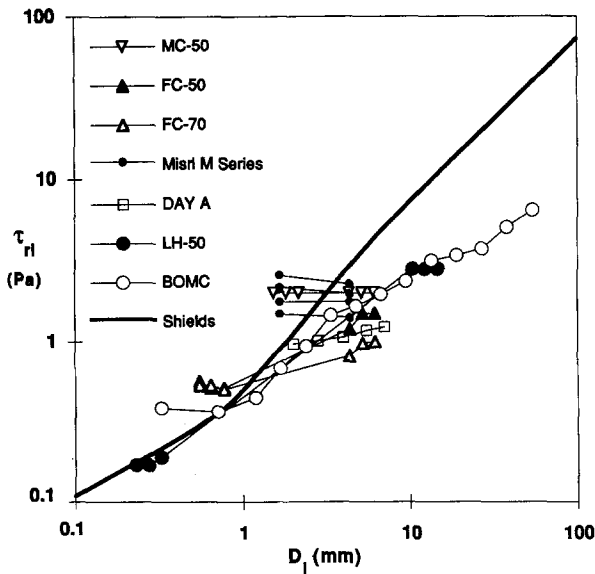FIG. 3. Variation of τ_{ri} with Grain Size for Unimodal Sediments

FIG. 4. Variation of τ_{ri} with Grain Size for Bimodal Sediments

curve for unisize sediment (Miller et al. 1977). All τ_{ri} for a particular mixture fall within a factor of 1.2, and generally within a much smaller range.

Fig. 4 presents the values of τ_{ri} as a function of grain size for twelve bimodal sediments. For some, but not all, of the mixtures, values of τ_{ri} show an increase with grain size. Comparison with Fig. 1 suggests that the mixtures showing a size dependence in τ_{ri} are the more strongly bimodal sediments. Quantification of this observation requires a consistent definition of the bimodality of a grain-size distribution.

Variation of τ_{ci} with Grain Size

Because τ_{ri} appears to vary little with grain size in unimodal and weakly bimodal sediments, and appears to follow relatively loglinear trends in bimodal sediments, it should be possible to describe the variation of τ_{ri} with grain size with a relatively simple relation. The relation used here is

$$\frac{\tau_{ri}}{\tau_{sm}} = \alpha \left(\frac{\tau_{si}}{\tau_{sm}}\right)^\beta \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (3)$$

where τ_{si} and τ_{sm} = the Shields values for D_i and D_m, respectively; and α and β may be expected to vary with the mixture grain-size distribution within the following limits. For unimodal sediments, $\beta \approx 0$, so $\tau_{ri} = \alpha\tau_{sm}$ for all fractions. For strongly bimodal sediments, $\beta > 0$, with a possible upper limit of β on the order of unity, so that τ_{ri} approaches τ_{si} in the limit of extremely bimodal mixtures (see physical interpretation section herein). For any size distribution, $\tau_{ri} = \alpha\tau_{sm}$ at $D_i = D_m$, so (3) predicts both a particular value of τ_{ri} (for D_m), as well as the variation of τ_{ri} with grain size. Values of α and β fitted to the data of Figs. 3 and 4 are tabulated in Table 1.

An alternative form of (3) would use D_i/D_m in place of τ_{si}/τ_{sm}. The basic functional difference between the two alternatives is that a grain Reynolds number ($R_{*i} = u_* D_i/\nu$) effect is present at the fractional scale in the term τ_{si} of (3). Above $R_{*i} \approx 500$, τ_{si}/τ_{sm} is directly proportional to D_i/D_m and no difference in the computed value of β should occur. At smaller R_{*i}, τ_{si}/τ_{sm} is proportional to D_i/D_m raised to a power less than one and a different value of β could be obtained by substituting D_i/D_m for τ_{si}/τ_{sm} in (3). Both independent variables were investigated. Differences in α and β between the two were generally negligible, presumably because R_{*i} for D_m in all mixtures was in the fully rough flow range and differences between D_i/D_m and τ_{si}/τ_{sm} were negligible for most fractions.

Defining Mixture Bimodality

To quantify the effect of mixture bimodality on τ_{ci}, an index representing the salient attributes of a bimodal grain-size distribution is needed. Surprisingly, no appropriate parameter describing the degree of bimodality appears to have been proposed in the sedimentation literature. At a minimum, the degree of bimodality (and its influence on transport) should depend on the separation in grain size between the two modes and the proportion of sediment contained in the modes. A useful expression for mode separation is the ratio of the size of the two modes D_c/D_f, which distinguishes between mixtures with widely separated modes and those with closely separated modes that tend to behave as unimodal mixtures. For example, the mixtures MC-50, M1, M2, M3, and M4 all are two-component mixtures with relatively closely spaced modes (small values of D_c/D_f) that do not show significant variation of τ_{ri} with D_i. In analogy with the definition

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of the mixture standard deviation for a lognormal sediment $\sigma_g = (D_{84}/D_{16})^{1/2}$, the square root of D_c/D_f is used here to represent mode separation.

The proportion of sediment in each mode in a bimodal sediment depends on mode width. In this paper, a mode is assumed to have a width of one ϕ unit (factor of two). Each mode is defined as the four contiguous $1/4\phi$ units containing the largest proportion. The proportion in the modes is expressed here as the sum in both modes ΣP_m, which can take a maximum value of unity for a purely bimodal mixture. ΣP_m helps to distinguish between mixtures with two prominent modes and those with one strong mode and a very weak mode, such as Oak Creek; the latter apparently behave as unimodal mixtures.

The simplest combination of $(D_c/D_f)^{1/2}$ and ΣP_m as descriptors of τ_{ci} would be as a product or a sum. Because of the numerical range these two parameters may take, a product is more likely to provide an effective index and a bimodality parameter B is defined as

$$B = \left(\frac{D_c}{D_f}\right)^{1/2} \Sigma P_m \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (4)$$

Values of B are tabulated in Table 1 for the sediments considered in this paper.

Grain Size, Mixture Bimodality, and τ_{ci}

Values of β were determined from a least-squares fit between the logarithm of τ_{ri}/τ_{sm} and τ_{si}/τ_{sm} for each sediment mixture. These are plotted as a function of B in Fig. 5. Values of B for unimodal sediments were taken as $B = \Sigma P_m$, assuming $D_c/D_f = 1$ for these sediments. Values of β fall about a value of zero for B less than about 1.7. At higher values of B, β demonstrates a consistent increase with B. Values of β computed after substituting D_i/D_m for τ_{si}/τ_{sm} in (3) are nearly identical to those plotted in Fig. 5 and are not shown. If the limits of β are taken to be $\beta \approx 1$ for very
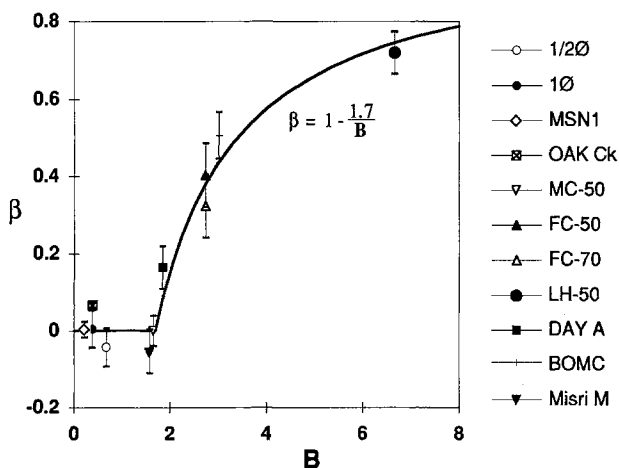

FIG. 5. Variation of the Exponent β from Eq. (3) with Mixture Bimodality $B = (D_c/D_f)^{1/2}(\Sigma P_m)$

large B and $\beta = 0$ for $B < B_r$, where B_r is the minimum value of B for which bimodality effects on τ_{ri} are evident, a simple relation for β is

$$\beta = 0, \qquad B < B_r \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (5a)$$

$$\beta = 1 - \frac{B_r}{B}, \qquad B > B_r \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (5b)$$

Eq. (5), with $B_r = 1.7$, is plotted in Fig. 5. An indication of the confidence in the individual calculated values of β is given by error bars on Fig. 5. These error bars represent 95% confidence limits on the least-squares slope fitted between the logarithm of τ_{ri}/τ_{sm} and τ_{si}/τ_{sm}, with the exception of the values for the Misri M series. Because only two points were available for each of these four sediments, Fig. 4 shows the minimum, mean, and maximum of the β values for these four mixtures. Confidence limits for the calculated values of β were also estimated by calculating values of β using different combinations of the minimum and maximum estimates of τ_{ri} for individual fractions in each size mixture. As discussed previously, the error bounds on individual τ_{ri} values were typically less than 10%. In all cases, the range in β possible when calculated using these limits was smaller than the error bars in Fig. 5.

Grain Size and τ_{cm}

Values of α from (3) are plotted as a function of B in Fig. 6. Error bars on the figure are 95% confidence limits on the predicted mean value of τ_{ri}/τ_{sm} for the least-squares relation between the logarithm of τ_{ri}/τ_{sm} and τ_{si}/τ_{sm}. All values of α fall within a factor of 1.8 of the Shields value, which is comparable to the range of scatter typically observed in plots of the critical shear stress of unisize sediments. All values of α fall within a factor of 1.6 about a mean value of 0.9. It is not likely that incipient motion of mixed size sediments can be determined with better accuracy. There is some indication that values of α for more strongly bimodal mixtures are smaller than those for less bimodal sediments. The two most bimodal mixtures, one (BOMC) a continuous size distribution and the other (LH-50) a two-part size distribution, both have $\alpha = 0.57$.
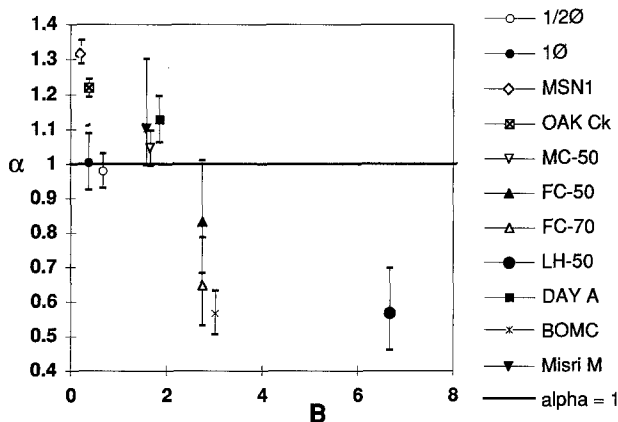

FIG. 6. Variation of the Coefficient α from Eq. (3) with Mixture Bimodality $B = (D_c/D_f)^{1/2}(\Sigma P_m)$

A substantial part of the scatter evident in Fig. 6, particularly for $B < 2$, may result from uncertainty in the estimated values of skin friction (the value of α depends directly on the skin friction estimate; whereas its effect on β is negligible). For example, skin friction was estimated to be 45% of the total bed shear stress on Oak Creek; a moderate error in this estimate could produce a substantial difference in α relative to the other values on Fig. 6. This level of uncertainty will persist until a general and consistent means is developed to estimate skin friction in mixed-size sediment with bed forms. As a result, it is difficult to assess the degree to which the trends in Fig. 6 are due to error in the flow estimate and the degree to which they depend on true differences among sediments.

PHYSICAL INTERPRETATION OF OBSERVED τ_{ri}

Comparison of the Shields curve and the patterns of τ_{ri} shown on Fig. 3 clearly demonstrate the mixture effect on τ_{ri} discussed by many workers (Einstein 1950; Egiazaroff 1965; Parker and Klingeman 1982; Misri et al. 1984; Wilcock and Southard 1988). In general, τ_{ci} for coarse grains are reduced relative to unisize Shields values because the grains experience both increased flow exposure and reduced resistance to motion when placed in a mixture. For the finer fractions, observed values of τ_{ci} are increased relative to unisize Shields values because the grains experience both reduced flow exposure and increased motion resistance in a mixture. Semianalytical models have produced similar results using a force balance on individual grains, assuming a logarithmic velocity profile among the grains, and, in some cases, applying empirical expressions for frictional resistance based on the relative grain size of each fraction (Egiazaroff 1965; Wiberg and Smith 1987).

Comparison of Figs. 3 and 4 suggests that the effect of bimodality can be interpreted as a reduced mixture effect on τ_{ci}. Values of τ_{ri} for strongly bimodal sediments are intermediate between unisize (Shields) and unimodal ($\tau_{ri} \approx$ constant) values. Mixture bimodality appears to affect finer fractions more than coarse, in that they approach and, in the case of LH-50, are essentially equal to the Shields values. The coarse fractions approach, but do not exceed, the Shields values. It is difficult to explain these trends using only the mechanisms of flow hiding, flow exposure, and motion resistance used to explain the trends of τ_{ri} observed for unimodal sediments. For example, in an extremely bimodal mixture of medium sand and coarse gravel such as LH-50, it seems unlikely that the influence of flow hiding and increased motion resistance on the finer fractions should be entirely eliminated such that $\tau_{ri} \approx \tau_s$ (we observe that coarse grains remain exposed on the bed surface during low flows with transport of only the finer fractions).

Size segregation on the bed surface provides a mechanism that can explain the observed variation in τ_{ci} between unimodal and bimodal size distributions. Many previous observations of local lateral size sorting in mixed-size sediment beds have been made (Ferguson et al. 1989; Iseya and Ikeda 1987; Wolcott and Church 1991). Our observations of runs involving the bimodal sediments MC-50, FC-50, FC-70, and LH-50 suggest that the degree of surface size segregation increases with increasing mixture bimodality. For the least-bimodal mixture, MC-50, all fractions were observed to begin moving at once. Long, low dunes formed at most flows; the bed forms were composed of both modes and no obvious lateral-size segregation took place. For the more bimodal mixtures, a range of discharge existed over which the transport was composed almost entirely of the fine mode. This range increased with mixture bimodality. For runs with FC-50 and FC-70, the finer

mode tended to become segregated into flow-parallel streaks or low, isolated barchan-like dunes. For runs with LH-50, the sand fraction formed low, small ripples at any flow capable of moving sediment. These ripples tended to be roughly flow perpendicular; troughs of the coarse gravel fraction were exposed between the ripples.

If the finer sediment is segregated into relatively homogeneous zones, it is reasonable to expect that values of τ_{ri} for these fractions will approach those of unisize sediment, as observed on Fig. 4. A corresponding coarsening of the remaining bed surface, from which the coarse grains are entrained at higher flows, would result in an increase in τ_{ci} for the coarse fractions relative to a unimodal mix, a result of increased flow shielding and rolling resistance on a coarser bed. Although greater than the unimodal values (τ_{ri} = $\alpha\tau_{sm}$), τ_{ri} for coarse grains in laterally segregated bimodal mixtures should remain less than those for corresponding unisize beds (Shields curve) because incomplete size segregation will leave some finer grains among the coarse grains, thereby increasing flow exposure and decreasing rolling resistance in the vicinity of the coarse grains.

CONCLUSIONS

Individual fractions in a range of sediment size distributions, including skewed, lognormal, and rectangular unimodal distributions and weakly bimodal distributions, begin moving at nearly the same flow strength. This simple result is not observed in sediments with more strongly bimodal size distributions, for which finer grain sizes begin moving at measurably smaller values of bed shear stress. A synthesis of the best available fractional transport data suggests that the degree of mixture bimodality at which size-dependent entrainment begins can be identified and that the critical shear stress of individual fractions in all mixed-size sediments can be estimated using the overall grain size of the mixture, as represented by its mean grain size, the size of each fraction, and the degree of bimodality of the mixture size distribution.

The product of two parameters representing mixture bimodality, the size ratio of the two modes D_c/D_f, and the proportion of the size distribution in both modes ΣP_m, can be used to account for the effect of mixture bimodality on τ_{ci}. For values of the bimodality product B [(4)] less than 1.7 (which includes all unimodal sediments), all values of τ_{ci} show little variation with fraction size. For larger values of B, τ_{ci} increases in a loglinear fashion with D_i/D_m. The slope of this relation [β in (3)] varies directly with B. Values of β may be estimated from B using (5), which is the simplest relation consistent with: (1) The observation of the limiting value $\beta = 0$ for unimodal sediments; (2) the existence of a threshold bimodality above which fine and coarse modes are entrained at different flows; and (3) an assumed limiting value of $\beta \approx 1$ at extreme values of B.

The absolute values of τ_{ci} for each fraction depend on the mean grain size of the mixture. For all of the data examined here, the critical shear stress for the mean size τ_{cm} falls within a factor of 1.8 of the critical Shields stress for that fraction, a range comparable to that observed for single-sized sediments (Miller et al. 1977). For mixtures with $B < 2$, the value of α in (3) may be taken to be unity as a first approximation. A value of $\alpha \approx 0.6$ may be more likely for strongly bimodal sediments. The potential error in any estimate of α must be evaluated based on the range of values shown in Fig. 6. The range in observed τ_{cm} may be attributed to several factors, some of which are indirectly related to the bed grain-size distribution. These

include error in estimating the skin friction shear stress and the influence on the critical shear stress of grain imbrication, bed consolidation, and vegetative debris.

Based on the data examined here, the relative variation of τ_{ci} within a mixture can be estimated with more accuracy than the actual value of τ_{ci} for a particular size fraction (e.g., τ_{cm}). To maximize accuracy, τ_{cm} can be estimated using local information concerning the flows that initiate any appreciable transport. The variation of τ_{ci} with grain size may then be more reliably estimated from (3), using β estimated from (5) and a calibrated value of α. Often, the value of β is of more interest than α, because the tendency of differential transport rates to produce sorting of a mixed-size sediment depends much more strongly on the value of β.

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APPENDIX II.   NOTATION

The following symbols are used in this paper:

B = bimodality parameters (Eq. 5);

B_r = value of B defining the transition between unimodal and bimodal behavior [(6)];

D = grain size;

D_{nn} = grain size for which $nn\%$ of mixture is finer;

D_c = grain size of coarse mode in bimodal sediments;

D_f = grain size of fine mode in bimodal sediments;

f_i = proportion of fraction i in bed sediment mixture;

g = acceleration of gravity;

P_m = proportion in mode in bimodal sediments;

p_i = proportion of fraction i in transport;

R' = skin friction portion of flow hydraulic radius;

R^* = grain Reynolds Number (u_*D/v);

q_b = volumetric transport rate per unit width of flow;

S = water surface slope (energy slope for uniform flow);

s = ratio of sediment to fluid density (ρ_s/ρ);

U = mean channel fluid velocity;

u_* = fluid shear velocity (τ_0/ρ)$^{1/2}$;

W^* = dimensionless sediment transport rate [(1)];

W_r^* = reference value of dimensionless sediment transport rate (0.002);

z_0 = hydraulic roughness in logarithmic velocity profile [(3)];

α = coefficient of (4);

β = exponent of (4);

ϕ = grain size scale, $\phi = -\log_2 D$ for D in millimeters;

κ = von Karman's constant;

v = fluid kinematic viscosity;

ρ = fluid density;

ρ_s = sediment density;

σ_g = geometric standard deviation of sediment mixture;

σ_ϕ = ϕ standard deviation of sediment mixture, $\sigma_g = 2^{(\sigma_\phi)}$;

τ_c = critical shear stress;

τ_0 = bed shear stress;

τ_r = reference shear stress, value of τ_0 that produces transport $W_r^* = 0.002$;

τ_s = critical shear stress from Shields diagram;

τ^* = shields dimensionless shear stress [$\tau_0/(s-1)\rho g D$]; and

τ_r^* = reference Shields stress, value of τ^* that produces transport rate $W_r^* = 0.002$.

Subscripts

i = of individual size fraction in sediment mixture; and

m = of mean grain size in sediment mixture.