Quantum gravity as a group field theory: a sketch

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Abstract. We give a very brief introduction to the group field theory approach to quantum gravity, a generalisation of matrix models for 2-dimensional quantum gravity to higher dimension, that has emerged recently from research in spin foam models.

1. Introduction: sum-over histories quantum gravity and the 3rd quantization idea

Quantum gravity remains the elusive dream of fundamental theoretical physics. The multitude of approaches that are currently pursued is vast [1]. Some of these approaches attempt to realise on solid grounds the idea of defining quantum gravity as a sum-over-histories of the gravitational field. This would work roughly as follows. Consider a compact 4-manifold (spacetime) with trivial topology $\mathcal{M}$ and all the possible geometries (spacetime metrics up to diffeomorphisms) that are compatible with it. The partition function of the theory would then be defined [7] by an integral over all possible 4-geometries, with a diffeomorphism invariant measure, weighted by the exponential of the action of General Relativity. For computing quantum gravity transition amplitudes, one would instead consider a manifold $\mathcal{M}$, again of trivial topology, with two disjoint boundary components S and S' and given boundary data, i.e. 3-geometries, on them: $h(S')$ and $h'(S')$, and define the transition amplitude by:

$$Z_{QG}\left(h(S), h'(S')\right) = \int_{g(\mathcal{M}|h(S), h'(S'))} \mathcal{D}g \, e^{i \, S_{GR}(g, \mathcal{M})} \tag{1}$$

i.e. by summing over all 4-geometries inducing the given 3-geometries on the boundary. The expression above is purely formal, in absence of a rigorous definition of a suitable measure in the space of 4-geometries; also, its physical interpretation is challenging, given that the formalism seems to be bound to a cosmological setting, where our usual interpretations of quantum mechanics are not applicable. This has not prevented physicists to propose generalisations. Why not to include also spacetime topology into the set of dynamical variables and allow for spatial topology changing processes? One could just extend the sum over geometries above to include a sum over different manifolds, but faces the impossibility of classifying topologies in 4 dimensions, and no clearcut criterion could be found for assigning a weight to each topology in the sum. A "3rd quantization" formalism [8, 9] was then proposed, in which the topology changing processes are described as a field theoretic 'interaction of universes'. The idea is to define a (scalar) field $\phi(^3h)$ in superspace $\mathcal{H}$, i.e. in the space of all possible 3-geometries (3-

metrics $^3h_{ij}$ up to diffeos), with action:

$$S(\phi) = \int_{\mathcal{H}} \mathcal{D}^3h \, \phi(^3h) \Delta \phi(^3h) + \lambda \int_{\mathcal{H}} \mathcal{D}^3h \, \mathcal{V}\left(\phi(^3h)\right) \tag{2}$$

with Δ being the Wheeler-DeWitt operator of canonical gravity here defining the free propagation of the field, while $\mathcal{V}(\phi)$ is a generic, e.g. cubic, and possibly non-local (in superspace) interaction term, governing topology change. The partition function $Z = \int \mathcal{D}\phi e^{-S(\phi)}$, produces, in perturbative expansion in Feynman graphs, the quantum gravity path integral for trivial topology, representing a sort of one particle propagator, thus a Green function for the Wheeler-DeWitt equation, plus a sum over topologies with definite weights. Note two features of this formalism: 1) the classical field equations will be a non-linear extension of the Wheeler-DeWitt equation of canonical gravity, due to the interaction term in the action, i.e. due to topology change; 2) the perturbative 3rd quantized vacuum of the theory will be the "no spacetime"state, and not any state with a semiclassical smooth geometric interpretation, e.g. Minkowski space.

2. Modern approaches: matrix models, dynamical triangulations, spin foams
These "3rd quantization"ideas were realised rigorously, although in a much simpler context, in matrix models for 2-d Riemannian quantum gravity [10]. Consider the action

$$S(M) = \frac{1}{2} \operatorname{tr} M^2 - \frac{\lambda}{3!\sqrt{N}} \operatorname{tr} M^3 \tag{3}$$

for an $N \times N$ hermitian matrix M_{ij}, and the associated partition function $Z = \int dM e^{-S(M)}$. This can be expanded in Feynman diagrams; propagators and vertices of the theory can be represented diagrammatically, and Feynman diagrams, obtained as usual by gluing vertices with propagators, are given by *fat graphs* of **all** topologies. Moreover, propagators and vertices can be understood as topologically dual to edges and triangles of a 2-dimensional simplicial complex dual to the whole fat graph; one can then define 2d quantum gravity, via the perturbative expansion for the matrix model above, as sum over **all 2d triangulations** T of **all topologies**.
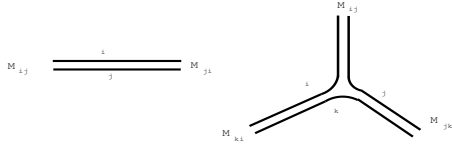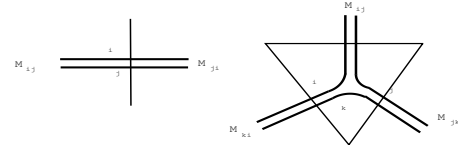

Figure 1. Propagator and vertex   **Figure 2.** Dual picture

Each Feynman diagram amplitude can be related to the Regge action for simplicial gravity for fixed edge lengths N and positive cosmological constant, and the partition function is:

$$Z = \int dM e^{-S(M)} = \sum_T \frac{1}{sym(T)} \lambda^{n_2(T)} N^{\chi(T)} \tag{4}$$

where $sym(T)$ is the order of symmetries of the triangulation T, n_2 is the number of triangles in it, and χ is the Euler characteristic of the same triangulation. Many results have been obtained over the years for this class of models, for which we refer to the literature [10], among which the link with continuum formulations of 2d quantum gravity. Matrix models manage to treat topology as a dynamical variable in a simplicial context, while rigorously defining a simplicial path integral formulation of quantum gravity for given topology. This raised the hopes that similar techniques and structures could be used to define a path integral for gravity also in higher dimensions and possibly for the Lorentzian signature. The dynamical triangulations

approach [11] is defined exactly on these bases. A path integral for gravity (for fixed topology) can be given meaning in a simplicial setting, modelling D-dimensional spacetime as the simplicial complex with fixed edge length a, thus encoding the degrees of freedom of the gravitational field in the combinatorics of the simplicial complex only, and defining the partition function as a sum over all triangulations with fixed topology weighted by the Regge action for gravity:

$$Z(G, \lambda, a) = \sum_T \frac{1}{sym(T)} e^{iS_R(T,G,\Lambda,a))} \tag{5}$$

where G is the gravitational constant and Λ is a cosmological constant. In the Lorentzian case one also distinguishes between spacelike and timelike edges, and imposes some additional restrictions on the topology considered and on the way the triangulations are constructed. This leads to a well-defined partition function of gravity, that can be dealt with both analytically and numerically to extract physical predictions. In particular, one may look for a continuum limit of the theory, corresponding to the limit $a \to 0$ accompanied by a suitable renormalisation of the constants of the theory Λ and G. And exciting recent results [11] seem to indicate that, in the Lorentzian context and for trivial topology, a smooth phase with the correct dimensionality is obtained even in 4 dimensions, which increases the confidence in the correctness of the strategy adopted. Spin foam models [2, 3] are yet another implementation of the path integral idea. Here spacetime is represented by a 2-complex (a collection of vertices, edges joining them and faces bounded by these edges), the histories of the gravitational field (4-geometries) are given by **spin foams**, i.e. by these 2-complexes labelled with irreps ρ of the Lorentz group assigned to their faces, and boundary data (3-geometries) are **spin networks**, i.e. graphs (boundary of the 2-complexes) labelled by irreps of the same type, assigned to the links of the graph.
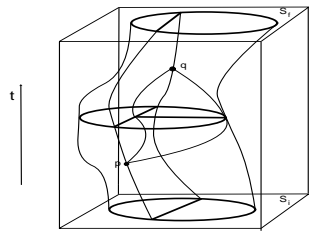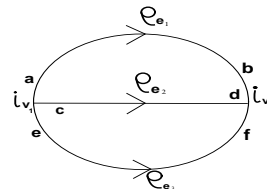

Figure 3. A spin foam

Figure 4. A spin network

The geometric degrees of freedom are thus encoded in purely combinatorial and algebraic data, and the model is defined by an assignment of a quantum probability amplitude (here factorised in terms of face, edge, and vertex contributions) to each spin foam σ, and by a sum over both 2-complexes and representations, for given boundary spin networks Ψ, Ψ':

$$Z = \sum_{\sigma|\Psi,\Psi'} w(\sigma) \sum_{\{\rho\}} \prod_f A_f(\rho_f) \prod_e A_e(\rho_{f|e}) \prod_v A_v(\rho_{f|v}).$$

The crucial point is how to choose the quantum amplitudes, e.g. from some discretization of a classical action for gravity. Whatever the starting point, one would have a rigorous implementation of a sum-over-histories for gravity in a combinatorial-algebraic context, and should then prove that one can both analyse fully the quantum domain, and recover classical and semi-classical results in some appropriate approximation. Spin foam models have grown to a promising approach to quantum gravity only recently, but a multitude of results have been already obtained in this context, for which we refer to the literature [2, 3].

3. The group field theory formalism
Let us now discuss how the dream of third quantization is realised (at least tentatively) in group field theories, by extending to higher dimensions the structures of matrix models.

3.1. General structure of GFTs

The general structure of a group field theory, independently of the spacetime signature, is as follows [12, 13, 2, 3]. Consider a (real or complex) scalar field over D copies of a group manifold G (for quantum gravity, the Lorentz group) whose classical dynamics is governed by the action:

$$S_D(\phi, \lambda) = \frac{1}{2} \prod_{i=1,..,D} \int dg_i d\tilde{g}_i \phi(g_i) \mathcal{K}(g_i \tilde{g}_i^{-1}) \phi(\tilde{g}_i) + \frac{\lambda}{(D+1)!} \prod_{i \neq j=1}^{D+1} \phi(g_{1j})...\phi(g_{D+1j}) \mathcal{V}(g_{ij} g_{ji}^{-1}),$$

where of course the exact choice of the kinetic and interaction operator is what defines the model. One usually imposes on the field invariance under simultaneous multiplication by a group element, and under their permutations (maybe only even ones). The quantum theory is coded in the partition function, defined again by its perturbative expansion in Feynman graphs:

$$Z = \int \mathcal{D}\phi \, e^{-S[\phi]} = \sum_\Gamma \frac{\lambda^N}{sym[\Gamma]} Z(\Gamma).$$

As in ordinary QFT, the field can be expanded in modes (momenta), and the Feynman amplitudes written in both configuration and momentum space; the modes of the field are labelled by representations of the group, whose elements define the configuration space of the field. As in matrix models Feynman graphs are represented by *fat graphs* given by D parallel lines for each propagators being re-routed at each vertex of interaction, and again one can give a dual interpretation of propagators and vertices in terms of (D-1)-simplices and D-simplices respectively. In this way, the Feynman graphs are cellular complexes (links identifying faces, that in turn close to form 2-cells, etc) that are topologically dual to D-dimensional triangulated (pseudo-)manifolds of **all topologies**. The Feynman amplitudes of the theory turn out to be given, when all fields are expanded in representations of the group G and thus the amplitude are given as a sum over these representations of appropriate functions of them, by spin foam models, with representation data assigned to the faces of the Feynman graph. When one restricts the sum over Feynman graphs to *tree level*, only manifolds with trivial topology are included [12], then boundary data and transition amplitudes acquire a canonical interpretation: boundary data define canonical quantum states of gravity and the transition amplitude between them defines a *projection* onto physical states, i.e. those satisfying the Hamiltonian constraint of canonical gravity, and thus the inner product of the canonical theory. The observables of the theory are gauge invariant (with respect to the symmetries of the action) functions of the field operators; for example, polynomial functionals can be expanded in spin networks. In particular, one defines transition amplitudes by inserting appropriately contracted field operators as observables in the partition function, as customary in field theory, and this produces (after perturbative expansion, and in momentum space) a sum over spin foams with spin network states on the boundary, reflecting the combinatorics of field operators in the observables whose expectation value is being evaluated. All this has a consistent quantum **geometric interpretation**: each field is understood as a 2nd quantized (D-1)-simplex, with its D arguments representing the (D-2)-simplices on its boundary; the evolution and interaction of these fundamental building blocks (quanta of space), that can in turn be phrased in terms of their creation/annihilation, and represented diagrammatically in Feynman graphs, is what generates a D-dimensional spacetime; depending on the actual graph considered (a possible spacetime history of interactions of the quanta of space), the resulting spacetime can have arbitrary topology and complexity, depending on the complexity of the states involved. The representations labelling the Feynman graphs and being summed over in the partition function are also interpreted geometrically: they represent the volume of the (D-2)-simplices they correspond to, while the group elements integrated over in configuration space correspond to holonomies of the gravity connection. In addition, the amplitude for each process, i.e. for each discrete spacetime, can be related to a discretization of the gravity action on that specific spacetime.

3.2. An example: 3d Riemannian Quantum Gravity

An explicit realisation of the formalism will make clear the above picture. We consider explicitely the 3d Riemannian quantum gravity case (where the local gauge group is $SU(2)$), whose group field theory formulation was first given by Boulatov [5]. The other existing models in 3 and 4 dimensions have a very similar formulation [6, 14]. Consider the real field: $\phi(g_1, g_2, g_3) : (SU(2))^3 \rightarrow \mathcal{R}$, with the symmetry: $\phi(g_1 g, g_2 g, g_3 g) = \phi(g_1, g_2, g_3)$, imposed through the projector: $\phi(g_1, g_2, g_3) = P_g \phi(g_1, g_2, g_3) = \int dg \, \phi(g_1 g, g_2 g, g_3 g)$ and the symmetry: $\phi(g_1, g_2, g_3) = \phi(g_{\pi(1)}, g_{\pi(2)}, g_{\pi(3)})$, with π an arbitrary permutation of its arguments. In this specific case, the interpretation is that of a 2nd quantized triangle with its 3 edges corresponding to the 3 arguments of the field; the irreps of $SU(2)$ labelling these edges in the mode expansion of the field have the interpretations of edge lengths. The classical theory is defined by the action:

$$
\begin{aligned}
S[\phi] \;=\;& \frac{1}{2} \int dg_1 .. dg_3 [P_g \phi(g_1, g_2, g_3)]^2 \; - \\
-\;& \frac{\lambda}{4!} \int dg_1 .. dg_6 [P_{h_1} \phi(g_1, g_2, g_3)][P_{h_2} \phi(g_3, g_5, g_4)][P_{h_3} \phi(g_4, g_2, g_6)][P_{h_4} \phi(g_6, g_5, g_1)],
\end{aligned}
$$

whose structure is chosen so to reflect the combinatorics of a 3d triangulations, with four triangles (fields) glued along their edges (arguments of the field) pairwise, to form a tetrahedron (vertex term), and two tetrahedra being glued alog their common triangles (kinetic term). The partition function is defined in terms of perturbative expansion in Feynman graphs:

$$
Z \;=\; \int d\phi \, e^{-S[\phi]} \;=\; \sum_\Gamma \frac{\lambda^N}{sym[\Gamma]} \, Z(\Gamma).
$$

In order to construct explicitely the quantum amplitudes for the Feynman graphs, we need to identify propagator and vertex amplitude. These are read out from the action to be:

$$
\mathcal{P} \;=\; \mathcal{K}^{-1} = \mathcal{K} = \sum_\pi \int dg d\bar{g} \; \delta(g_1 g \bar{g}^{-1} \tilde{g}_{\pi(1)}^{-1}) \delta(g_2 g \bar{g}^{-1} \tilde{g}_{\pi(2)}^{-1}) \delta(g_3 g \bar{g}^{-1} \tilde{g}_{\pi(3)}^{-1}),
$$

$$
\mathcal{V} \;=\; \int dh_i \, \delta(g_1 h_1 h_3^{-1} \tilde{g}_1^{-1}) \delta(g_2 h_1 h_4^{-1} \tilde{g}_2^{-1}) \delta(g_3 h_1 h_2^{-1} \tilde{g}_3^{-1}) \delta(g_4 h_2 h_4^{-1} \tilde{g}_4^{-1}) \delta(g_5 h_2 h_3^{-1} \tilde{g}_5^{-1}) \delta(g_6 h_3 h_4^{-1} \tilde{g}_6^{-1})
$$

See the picture for a diagrammatic representation, with boxes representing the integration over the group. The Feynman graphs are obtained as usual by gluing vertices with propagators.
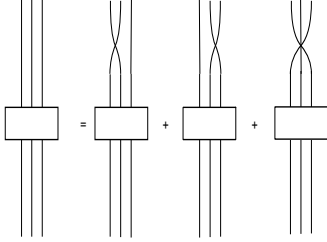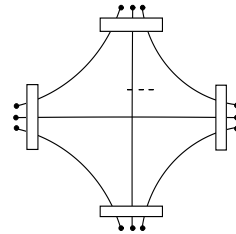

Figure 5. Propagator

Figure 6. Interaction vertex

Let us see how they look like. Each line in a propagator goes through several vertices and for closed graphs it comes back to the original point, thus identifying a 2-cell; these 2-cells, together with the set of lines running parallel in each propagator, and the set of vertics of the graph, identify a 2-complex for each given Feynman graph. Each of these 2-compelxes is dual to a 3d triangulation, with each vertex correspondings to a tetrahedron, each link to a triangle and each 2-cell to an edge of the triangulation (see picture). The **sum over Feynman graphs** is thus equivalent to a **sum over 3d triangulations of any topology**.
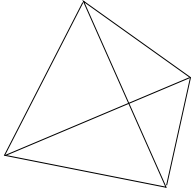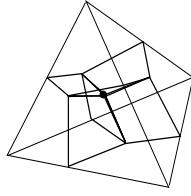
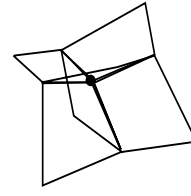**Figure 7.** Tetrahedron   **Figure 8.** Tetrahedron + dual 2-complex   **Figure 9.** Dual 2-complex

Let us now identify the quantum amplitudes that the theory assigns to the Feynman graphs. In configuration space the amplitude for each 2-complex is:

$$Z(\Gamma) = \left(\prod_{e \in \Gamma} \int dg_e \right) \prod_f \delta(\prod_{e \in \partial f} g_e)$$

which has the form of a lattige gauge theory partition function with simple delta function weights for each plaquette (face of the 2-complex) and one connection variables for each edge; the delta functions constraint the curvature on any face to be zero, as we expect from 3d quantum gravity [4]. To have the corresponding expression in momentum space, one expands the field in modes $\phi(g_1, g_2, g_3) = \sum_{j_1, j_2, j_3} \phi^{j_1 j_2 j_3}_{m_1 n_1 m_2 n_2 m_3 n_3} D^{j_1}_{m_1 n_1}(g_1) D^{j_2}_{m_2 n_2}(g_2) D^{j_3}_{m_3 n_3}(g_3)$, where the j's are irreps of $SU(2)$, obtaining, for the propagator, vertex and amplitude:

$$\mathcal{P} = \delta_{j_1 \tilde{j}_1} \delta_{m_1 \tilde{m}_1} \delta_{j_2 \tilde{j}_2} \delta_{m_2 \tilde{m}_2} \delta_{j_3 \tilde{j}_3} \delta_{m_3 \tilde{m}_3}$$

$$\mathcal{V} = \delta_{j_1 \tilde{j}_1} \delta_{m_1 \tilde{m}_1} \delta_{j_2 \tilde{j}_2} \delta_{m_2 \tilde{m}_2} \delta_{j_3 \tilde{j}_3} \delta_{m_3 \tilde{m}_3} \delta_{j_4 \tilde{j}_4} \delta_{m_4 \tilde{m}_4} \delta_{j_5 \tilde{j}_5} \delta_{m_5 \tilde{m}_5} \delta_{j_6 \tilde{j}_6} \delta_{m_6 \tilde{m}_6} \left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\}$$

$$Z(\Gamma) = \left(\prod_f \sum_{j_f} \right) \prod_f \Delta_{j_f} \prod_v \left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\}$$

where Δ_j is the dimension of the representation j and for each vertex of the 2-complex we have a so-called $6j - symbol$, i.e. a scalar function of the 6 irreps meeting at that vertex. The amplitude for each 2-complex is given then by a spin foam model, the Ponzano-Regge model for 3d gravity without cosmological constant, about which a lot more is known [4]. The full theory is defined by the sum over all Feynman graphs weighted by the above amplitudes:

$$Z = \sum_{\Gamma} \frac{\lambda^N}{sym[\Gamma]} \left(\prod_f \sum_{j_f} \right) \prod_f \Delta_{j_f} \prod_v \left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\}_v .$$

This gives an un-ambigous realisation, in purely algebraic and combinatorial terms, of the sum over both geometries and topologies, i.e. of the third quantization idea. More precisely, it is a simplicial third quantization, a *quantum field theory of simplicial geometry*, with fundamental classical dynamical objects being triangles, quantum states given by collections of quantum triangles represented as 3-valent spin networks, and histories given by 3d triangulations.

3.3. GFT: the general picture
- GFT are thus a *local*, because one can easily consider bounded regions of space evolving or 'timelike 'boundaries, *discrete*, because it deals with discrete spacetimes, *algebraic and combinatorial*, because such are the variables in the theory, *3rd quantization of gravity*;
- in fact, in GFTs both geometry and topology are dynamical, with precise quantum amplitudes assigned to each possible geometric and topological configuration of spacetime;

- D-dimensional spacetime emerges via creation/annihilation of "chunks"of it, of spacetime quanta represented by (D-1)-simplices, as a Feynman diagram;
- spacetime is therefore purely virtual in the quantum theory: just as the trajectory of a quantum particle or any specific interaction process in particle physics; no single spacetime configuration is realised as the "truly existing"spacetime, but all of them should be summed over to obtain a physical quantity, that is the probability of a specific bundary configuration;
- Quantum Gravity is described by an (almost) ordinary QFT, although with peculiar structure, and one that uses even a background metric "spacetime" (although here interpreted as an internal space only), given by a group manifold;
- the GFT formalism has the potential to represent a unified framework for many current non-perturbative approaches to Quantum Gravity: Loop Quantum Gravity, Spin Foam models, Dynamical Triangulations, Quantum Regge Calculus, because its incorporates most of the basic ingredients on which these approaches are based, as one can easily realise: spin network states on the boundary, spin foam amplitudes for the histories, a dual sum over triangulations picture for its perturbative expansion, and a sum over geometric data, with amplitudes related to the Regge action for simplicial gravity.

4. What lies ahead

However fascinating the picture outlined above may be, we do not know enough about group field theories to see it clearly in all its detials, and therefore to fully believe. Even if lots is known about the Feynman amplitudes of the theory, in various models in 3 and 4 dimensions[2, 3, 12], in addition to what is known about matrix models in 2d, it is probably fair to say that at present we do not know what a group field theory is, and we can only deduce or guess some of its properties on the basis of its Feynman amplitudes. In particular the physical and geometric interpretation given above rests at present on intuition only and it is not solidly based on mathematical results.

We list here a few of the directions that need to be explored if one has to take GFTs seriously as a fundamental formulation of Quantum Gravity.

First of all, we do not know much about the *classical* field theories behind the perturbative expansion in spin foam we have described: what are the solutions, in symmetric reduced cases at least, of the classical equations of motion following from the above actions? and what is their geometric interpretation? Work on this in indeed in progress [15].

What are the symmetries of the above action and the corresponding Ward identities for Feynman graphs? Even in the simple 3d case it is not easy to identify at the GFT level the translation symmetry that we know it is present in the corresponding spin foam amplitudes [4]. Most important, what is the GFT analogue of the diffeomorphism symmetry of continuum gravity? what kind of other symmetries should we expect in a theory in which topology change is realised?

What is the physical meaning of the parameters of the action, i.e. in the model we described, of the coupling cnstant λ? It can be related to the cosmological constant in a simplicial gravity setting [16] and/or it has the interpretation of a parameter governing the strength of topology changing processes [12], but much remains to be understood.

At the quantum level, even though the picture of spacetime as a process of creation/annihilation of fundamental simplicial building blocks is appealing, it is at present only a tentative picture; in fact, the Fock structure of the theory has not been analysed in detail and rigorously, with a suitable definition of creation/annihilation operators, on the basis of a classical symplectic structure, and the definition of a 3rd quantized Fock vacuum.

The relation with a canonical theory based on spin network states is also unclear; while one can give a precise and well-posed definition of a canonical inner product between canonical states using a GFT [12], it would be interesting to be able to extract from this the corresponding

hamiltonian constraint operator and compare it to the existing proposals in loop quantum gravity; also, it would be interesting to compute the corrections to the hamiltonian constraint equation coming from topology changing terms, as in the formal continuum setting.

There is much more in a quantum field theory than its Feynman amplitudes in perturbative expansion, and all this has still to be unveiled for the GFT case; in particular, it is crucial for the issue of the continuum approximation of these quantum gravity models to develop non-perturbative techniques, probably after a suitable re-phrasing of them in statistical mechanical terms, that would allow to study the phase structure of the theory, and the emergence of a smooth spacetime in it, with continuum General Relativity as an effective description of the degrees of freedom of the theory in this phase.

The coupling of matter and gauge fields at the group field theory level, and the unification of these with gravity, is a whole area for future developments, and work on this has just started [17, 18].

Also, as in ordinary quantum field theory, it should be possible to define different types of transition amplitudes for the same group field theories, with different uses and interpretation, as seems to be confirmed by recent work [19].

Finally, it remains to be checked if the group field theory approach can maintain its promise of being a general framework for as different approaches to quantum gravity as loop quantum gravity, spin foam models, dynamical triangulations and Regge calculus; many details of the links with them have still to be understood and many gaps filled, but recent results give reasons to hope [19].

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