Initialization of Magneto-rotational Simulation of Core-Collapse Supernovae

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TABLE OF CONTENTS

ABSTRACT

Initialization of Magneto-rotational Simulation of Core-Collapse Supernovae. TONY Y. LI (Cornell University, Ithaca, NY) SHIZUKA AKIYAMA (Kavli Institute for Particle Astrophysics and Cosmology, Stanford Linear Accelerator Center, Menlo Park, CA)

The full mechanism behind core-collapse supernovae is currently not fully understood, as computer simulations have been unable to produce explosions for massive progenitor stars. Strong magnetic fields may play a significant role during the process by driving the stalled shock, transporting angular momentum, and possibly accounting for observed features of supernovae such as bipolar jets. However, the simulation of magneto-rotational core collapse is an inherently three-dimensional problem. In this paper, we present in detail our initialization of a fully three-dimensional core-collapse simulation, which we run using the GRMHD code COSMOS++. In our initial model, we incorporate a polytropic density distribution, a rotation profile with variable degree of differential rotation, and a poloidal magnetic field. We describe key features that we have implemented in our simulation code, including a logarithmically spaced mesh and a magnetic field inclined to the axis of rotation. In our continuing work we aim to analyze the results of a completed simulation and perform a comprehensive parameter study of magneto-rotational core collapse.

INTRODUCTION

The full physical mechanism behind core-collapse supernovae has been a long-standing problem in astrophysics for several decades. Sufficiently massive stars ($\gtrsim 8M_\odot$), toward the end of their stellar evolution, acquire a central iron core. When this core exceeds a threshold mass at roughly the Chandresekhar limit, gravity dominates over all other outward pressures, causing the core to collpase until the matter inside reaches nuclear densities, forming a proto-neutron star (PNS) and sending an outward shockwave to the rest of the star. However, the mechanism by which this core implosion transforms into an overall stellar explosion is still not fully understood, despite the inclusion of increasingly sophisticated physics in supernovae models. At the heart of the problem is the outward shock, which necessarily stalls within the stellar envelope. Without reviving the stalled shock, the star fails to explode. Currently, neutrino heating is believed to account for a significant fraction of that driving energy. However, neutrino-driven models, when simulated, will only yield explosions for small mass progenitors, and in larger stars ($\gtrsim 15M_\odot$), the shock still stalls within stellar envelope and the star fails to explode. In all likelihood, the present model of core collapse is incomplete and missing crucial additional physics.

It is possible that magnetic fields, amplified by rotation, play an integral part in this mechanism. This suggestion is not new, but it has gained traction in recent years, owing to a proliferation of research into the magnetorotational instability (MRI), a magnetohydrodynamic shear instability previously invoked in the study of accretion disks. Combined with magnetic winding, the MRI provides a potential mechanism for rapidly amplifying the magnetic fields to saturation on a short enough timescale ($\sim$10 ms) to be significant within the context of core collapse [1]. Even if magnetic fields do not provide the complete solution to the problem of core collapse, there are strong reasons to believe in their importance. The amplified fields may exert enough magnetic pressure to help drive the stalled shock. They can aid in angular momentum transport, and may account for asymmetries in observed supernovae, such as bipolar jet explosions. Additional motivation comes from observations of anomalous X-ray pulsars and soft gamma-ray repeaters, which have been suggested to be magnetars — neutron stars with exceptionally strong magnetic fields.

Numerous magneto-rotational core collapse simulations have been performed in previous studies, but most have been done in only two dimensions [2, 3, 4]. While two dimensions breaks the restriction of spherical symmetry, it cannot incorporate non-axisymmetric features such as magnetic fields inclined to the rotation axis. Such magnetic fields are believed to be a feature of pulsars, and a complete treatment of magneto-rotational core collapse requires three-dimensional simulations. Such simulations are relatively new, since adequate computng power for such simulations has only recently become a reality. In this paper, we present in detail our initialization and equations for an ongoing magneto-rotational core collapse simulation, which is indeed designed to be run in three dimensions.

METHODS

Computational Method

The simulations are run using COSMOS++, a massively parallel, general relativistic magnetohydrodynamic (GRMHD) code. It has previously been used in the study of accretion disks, but its application to core collapse situations is fairly new. In our implementation, the COSMOS++ code solves an internal energy formulation of the GRMHD equations with artificial viscosity to incorporate shock heating (see [5] for details).

Our simulations were performed on a computational mesh in spherical coordinates (r, θ, ϕ). The mesh contains 300 radial zones, in the domain $0 \leq r \leq 2R$, where R is the radius of the stellar core. The zones are logarithmically spaced so that the increase in size between adjacent zones is $\sim 2.5\%$. The logarithmic spacing was previously a feature of COSMOS++ in two dimensions, and for the purposes of this study we have extended it to three dimensions. It ensures a fine enough grid to provide adequate resolution around both the pre-collapse initial core and the much smaller PNS formed during collapse. In the angular directions, we have included 60 uniform zones in θ, $0 \leq \theta \leq \frac{\pi}{2}$, and 120 uniform zones in ϕ, $0 \leq \phi < 2\pi$, translating to 3° of resolution in each angular coordinate.

Polytropic Initial Model

We begin by setting up the initial conditions for the stellar core, just before collapse. We assume that the core is initially rotating slowly enough to be approximated by a spherically symmetric polytrope. As such, we can obtain the density profile by specifying an initial adiabatic index γ_0 for the core and solving for the function $\theta(\xi)$ in the Lane-Emden equation, given in [6] as

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n \tag{1}$$

with boundary conditions

$$\theta(\xi = 0) = 1$$
$$\left.\frac{d\theta}{d\xi}\right|_{\xi=0} = 0.$$

The polytropic index n is given by $n = 1/(\gamma_0 - 1)$, and the variables θ and ξ are related to physical variables of density ρ and spherical radius r by

$$\xi = r\left(\frac{4\pi G\rho_c^2}{(n+1)P_c}\right)^{1/2} = r\left(\frac{\xi_{\rm surf}}{R}\right) \quad , \quad \rho = \rho_c\theta^n$$

where G is the gravitational constant, ρ_c and P_c are central density and pressure, respectively, ξ_{surf} is the first zero of $\theta(\xi)$, corresponding to the surface radius R. The pressure within the star is given as

$$P = K\rho^{\gamma_0} \tag{2}$$

where K is the so-called polytropic constant. Since this is a second-order ODE, we obtain a straightforward numerical solution via 4th order Runge Kutta.

We have used $\gamma_0 = 4/3$, corresponding to a relativistic degenerate gas, and $\rho_c = 10^{10}$ g cm^{-3} and $R = 1.7\times10^8$ cm, approximately following the stellar model of [7] for a 15 $M_\odot$ progenitor.

Rotational Profile

Having obtained an initial density distribution, we add in the rotational profile and magnetic field of the core. For the rotational profile, we set up our simulation with the option of three profiles from [8]: rigid rotation, or the so-called v-const and j-const laws, given by

$$\Omega(\varpi) = \Omega_c \qquad \text{rigid} \tag{3}$$
$$\Omega(\varpi) = \frac{\Omega_c}{1 + \frac{a}{\varpi}} \qquad v\text{-const} \tag{4}$$
$$\Omega(\varpi) = \frac{\Omega_c}{1 + \left(\frac{a}{\varpi}\right)^2} \qquad j\text{-const} \tag{5}$$

where ϖ is the cylindrical radial coordinate, Ω_c is the central angular velocity of the core, and a is a constant with dimensions of length which may be varied to adjust the degree of initial differential rotation. For sufficiently large a, $a \gg \varpi$, the three above equations are very nearly rigid rotation.

We have chosen to use the j-const law of Eq. 5, using $a = 5.0 \times 10^9$ cm and $\Omega_c = 1\ \mathrm{s}^{-1}$ for our initial run.

Magnetic Field

For the magnetic field, we have implemented in our code the option of setting up a poloidal, toroidal, or purely vertical field directed in the $+z$ direction. The initial characteristic strength B_0 of the field can be varied.

We use the poloidal magnetic field in our simulation, and it derives from the vector potential $\mathbf{A}(\mathbf{r})$ of a circular current loop of radius r_{mag} as given in [9]. Here, B_0 is the magnitude of the magnetic field at the origin. In the most general case, we assume that the magnetic moment of the loop is tilted at an angle α to the axis of rotation. To calculate the magnetic field, then, we define two coordinate frames: an unprimed frame spanned by (x, y, z) or (r, θ, ϕ), in which the z-axis aligns with the rotation axis, and a primed frame spanned by (x', y', z') or (r', θ', ϕ'), in which the z'-axis aligns with the magnetic moment. As such, the two frames are related by the transformation:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{6}$$

Note that the simulation itself is carried out in the unprimed frame. However, in the primed frame, the only nonzero component of $\mathbf{A}'$ is the azimuthal component A'_ϕ, which can be expressed as

$$A'_\phi(\mathbf{r}') \propto \sum_{n=0}^{\infty} \frac{(-1)^n (2n-1)!!\ r'^{2n+1}_<}{2^n (n+1)!\ r'^{2n+2}_>} P^1_{2n+1}(\cos\theta') \tag{7}$$

where $(2n-1)!! = 1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)$, $r_< = \min(r, r_{\mathrm{mag}})$, $r_> = \max(r, r_{\mathrm{mag}})$, and P^n_m denotes the Legendre polynomials.

The magnetic field in the primed frame is then calculated from a discrete approximation of $\mathbf{B}' = \nabla \times \mathbf{A}'$, though in order to avoid any of the inherent singularities in spherical coordinates (i.e. at $r = 0$, $\theta = 0$, and $\theta = \pi$), we perform this calculation by converting to Cartesian coordinates. We then transform the magnetic field back to the original, unprimed coordinate frame and scale it by B_0.

For our initial run, we have chosen $r_{\rm mag} = R/4$ and $B_0 = 10^{12}$. It is worth noting that this magnetic field is significantly greater than the pre-collapse fields calculated in [10], but we motivate the magnitude of this field by noting that we aim to study the effect of high field strengths, but we do not expect to resolve the MRI on our grid, nor should amplification through collapse and magnetic winding be dynamically significant within the time window of our simulation.

Equation of State

We use an approximate parametric equation of state, given in [11] and previously adopted in such studies as [4, 3], which sets the pressure P as a sum of a polytropic component and a thermal component:

$$P = P_p + P_{th} \tag{8}$$

The polytropic component P_p is given by

$$P_p = K_p\, \rho^{\gamma_p} \tag{9}$$

where the adiabatic index γ_p and the polytropic constant K_p are defined as

$$\gamma_p = \begin{cases} \gamma_1 & \text{if } \rho < \rho_{\rm nuc} \\ \gamma_2 & \text{if } \rho \geq \rho_{\rm nuc} \end{cases} \qquad \text{and} \qquad K_p = \begin{cases} K_1 & \text{if } \rho < \rho_{\rm nuc} \\ K_2 & \text{if } \rho \geq \rho_{\rm nuc} \end{cases} \tag{10}$$

This piecewise definition reflects the phase transition which occurs at nuclear density $\rho_{\rm nuc}$. Initially, the matter in the core is at sub-nuclear densities, and the pressure is primarily due to relativistic degenerate electrons, giving $\gamma_1 \lesssim 4/3$. However, as the core collapses, the matter within reaches a far more incompressible $\rho_{\rm nuc}$, and the adiabatic index jumps to $\gamma_2 \approx 2.5$. Because the pressure is continuous at $\rho_{\rm nuc}$, we can relate the polytropic constants K_1 and K_2 as

$$K_2 = K_1\ \rho_{\rm nuc}^{\gamma_1 - \gamma_2} \tag{11}$$

The thermal component of pressure P_{th} is of the form

$$P_{th} = (\gamma_{th} - 1)\epsilon_{th} \tag{12}$$

where ϵ_{th} is the thermal internal energy. This component of the pressure accounts for thermal heating from shocks.

Following the work of [4], we have chosen $\gamma_{th} = 1.5$ to reflect a partially relativistic gas. Additionally, we have chosen $\gamma_1 = 1.28$ and $\gamma_2 = 2.5$. That γ_1 is reduced from the initial adiabatic index $\gamma_0 = 4/3$ reflects the absorption of electrons occuring within the core, softening the equation of state and initiating gravitational collapse.

It is important to note that our equations of state do not include the effects of neutrino heating, which is believed to play an essential role in energizing the stalled outward shock. As such, our simulation results are restricted to the first several tens of milliseconds, before neutrinos become dynamically important.

RESULTS

A visualization of our computational mesh is shown in Figure 1. Near the center of the mesh, around the expected radius of ~ 50 km of the PNS, we obtain a resolution in the radial direction of about $\Delta r \approx 1$ km.

The density profile of our polytropic model, alongside the core profile of a 15 $M_\odot$ progenitor from [7], is displayed in Figure 2. Though the two curves diverge from each other near the surface of the core, they are very close in the central regions, and are qualitatively similar in that they model a rapid dropoff in density outside of a region of relatively consistent central density. Both models have been used in previous core collapse simulations.

A vector field visualization of our initial magnetic field, as viewed in the xz-plane, is displayed in Figure 3.

As is evident in the figure, the field is purely poloidal, tilted by 45 °, and is strongest near the center of the core. We expect that, as the simulation progresses and differential rotation becomes more pronounced during collapse, the magnetic field lines will wrap around each other and a significant toroidal field component will develop.

DISCUSSION AND CONCLUSIONS

Because of the intrinsically asymmetric properties of magnetic fields, three dimensional simulations are a crucial tool in obtaining a full understanding of magneto-rotational effects in core-collapse supernovae. In our work, we have set up the initialization for a 3D magneto-rotational core collapse simulation. Our initial conditions specify a polytropic model of the core that reflects a relativistic degenerate electron gas, a j-constant rotation profile with very slight differential rotation, and a poloidal field inclined to the axis of rotation. The values of all parameters and constants have been specified for the initial run of the full simulation.

However, because this is an ongoing study in that we are currently in the midst of running full simulations, there is ample room for further work. We aim to perform a comprehensive and detailed investigation of all magneto-rotational parameters, in which we would compare results from varying such parameters as the strength of the initial rotation, the degree of initial differential rotation, and the initial magnetic field strength, geometry, and angle of inclination. Accomplishing this will require significant computing resources, but as the logical next step in our research, we look forward to analyzing the results of such a study.

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FIGURES

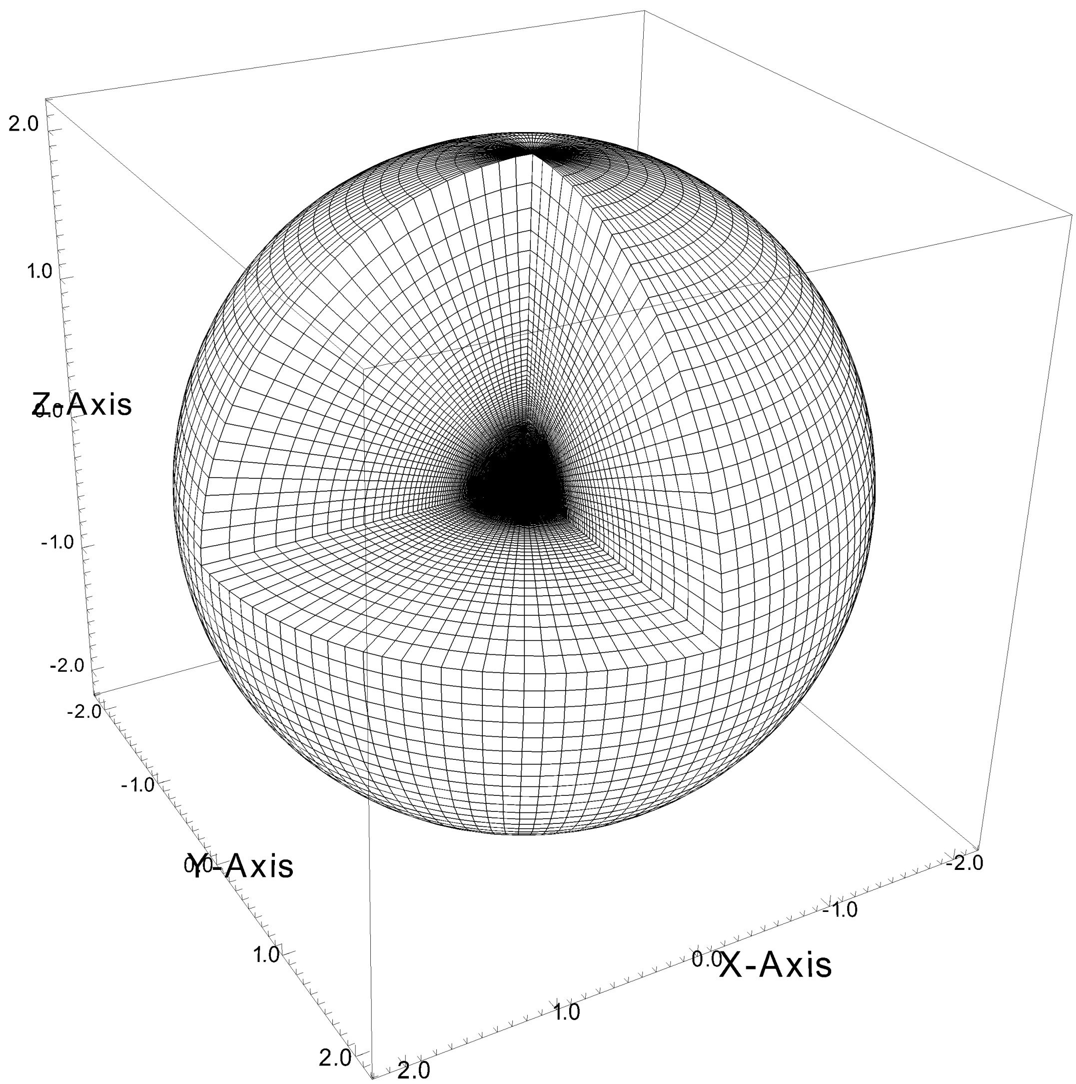

Figure 1: Visualization of the spherical mesh used in the simulation. Units for each axis are normalized by R.

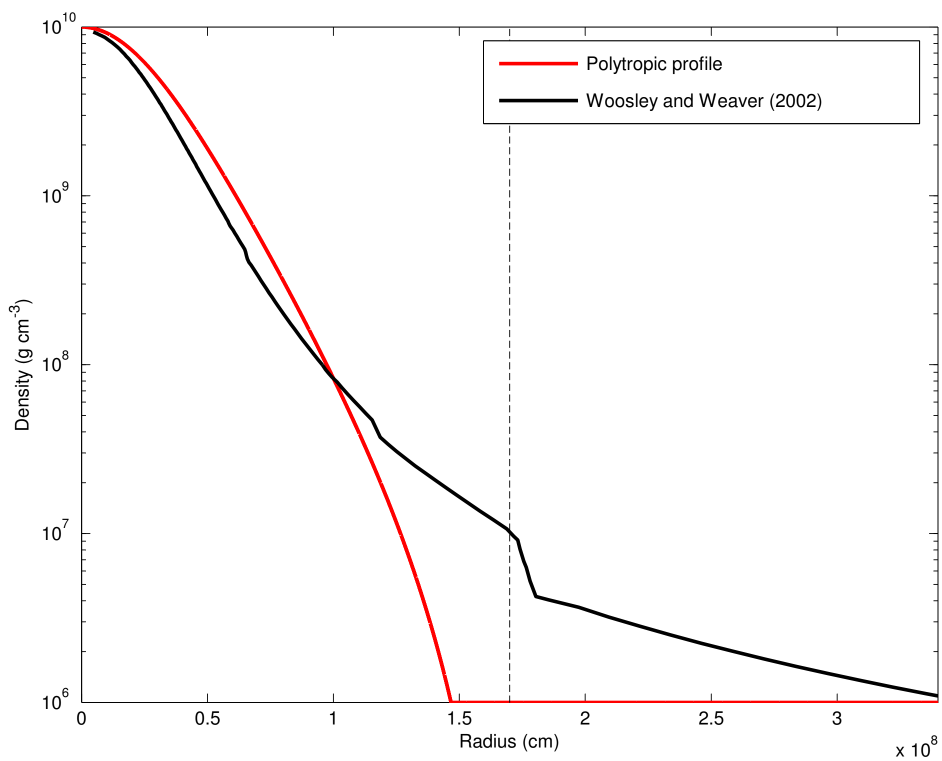

Figure 2: Comparison between our polytropic profile and that of [7] for the core of a 15 $M_\odot$ progenitor. The dotted line denotes the outer radius of the core in our simulation. Our profile ends slightly before stellar radius because we have imposed a minimum density throughout simulation.

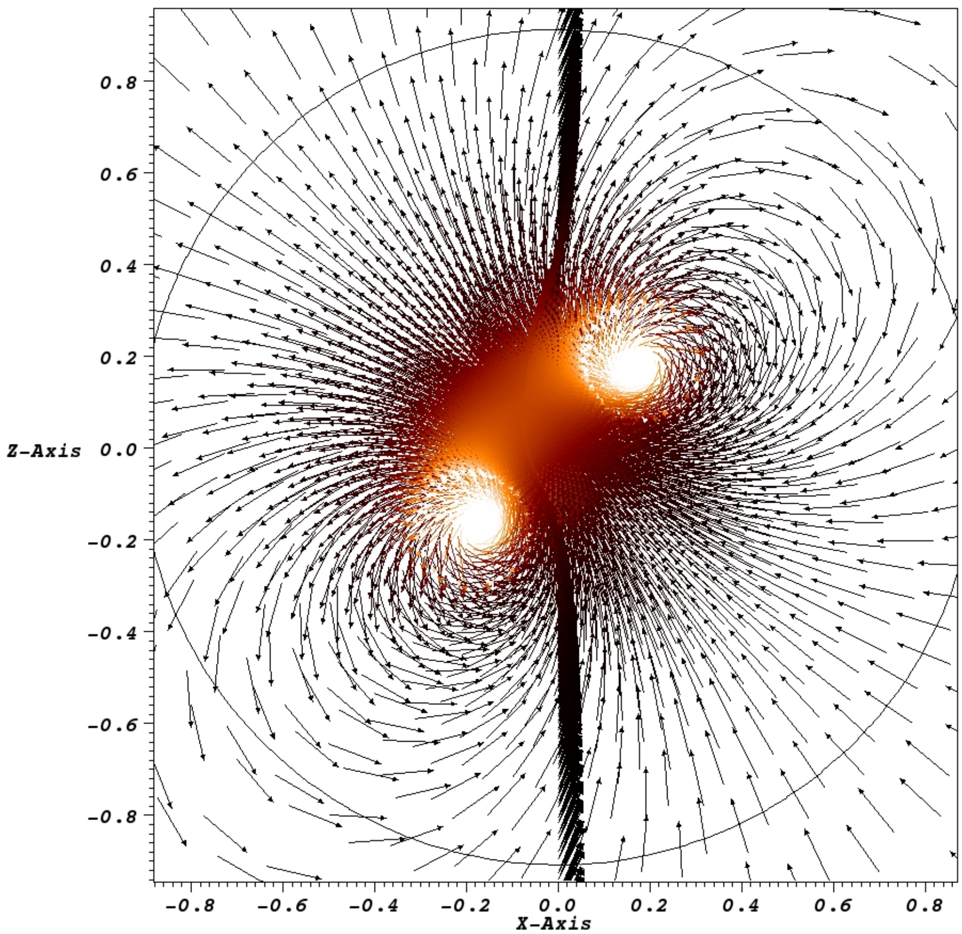

Figure 3: Vector field visualization of our tilted magnetic field. Colors represent increasing magnetic field strength from dark to light, and units on the axes are given in terms of R. The rotation axis is aligned with the z-axis, and the magnetic dipole moment is tilted at an angle $\pi/4$ from the rotation axis. The circular outline represents the approximate boundary of the initial stellar core. The unusual density of vectors along the z-axis merely arises from the increased density of mesh zones in that region—a byproduct of our spherical mesh—and is not reflective of a singularity in our magnetic field.