Radar Systems Engineering
Lecture 6
Detection of Signals in Noise

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Guest Lecturer

Block Diagram of Radar System

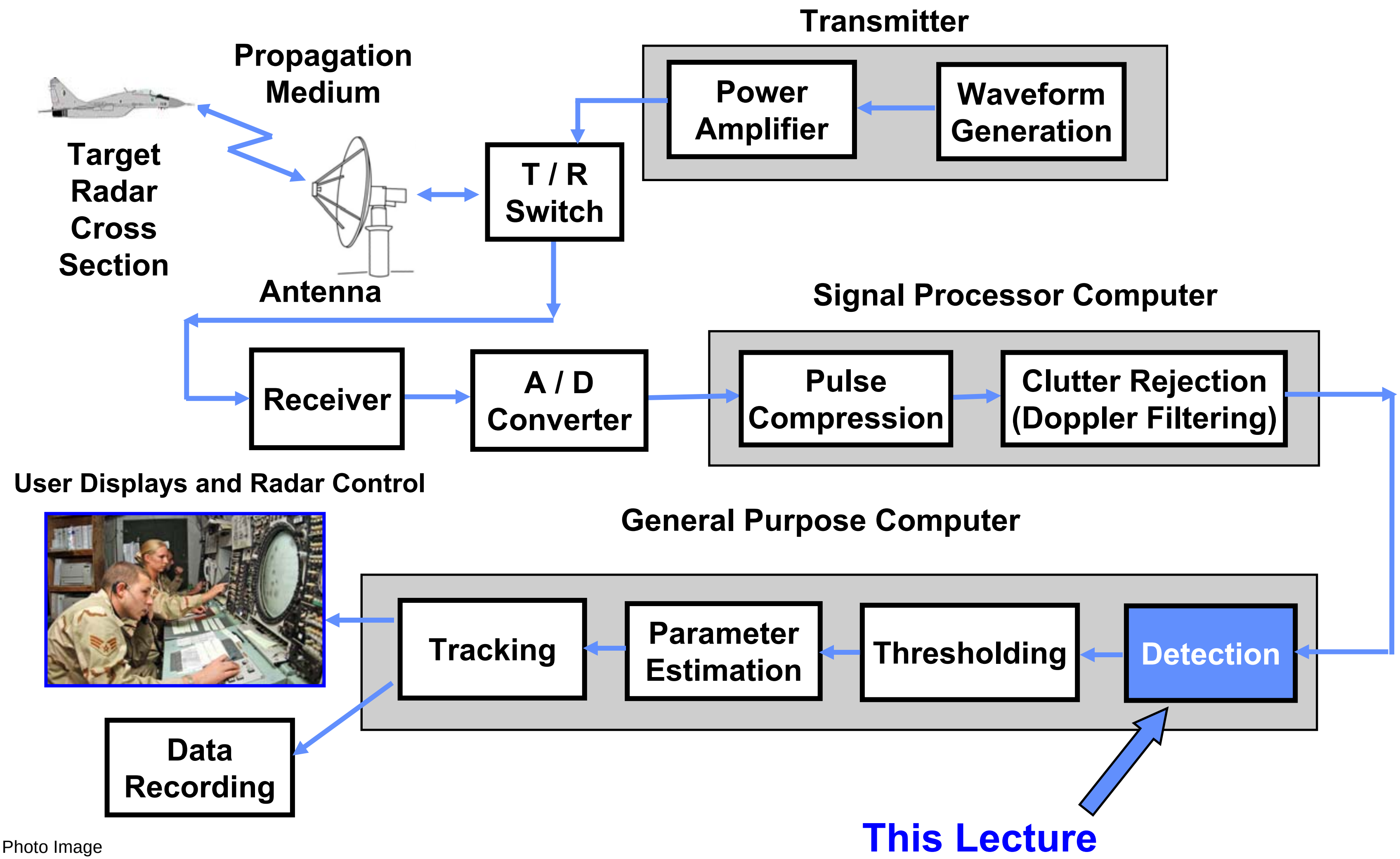

Photo Image
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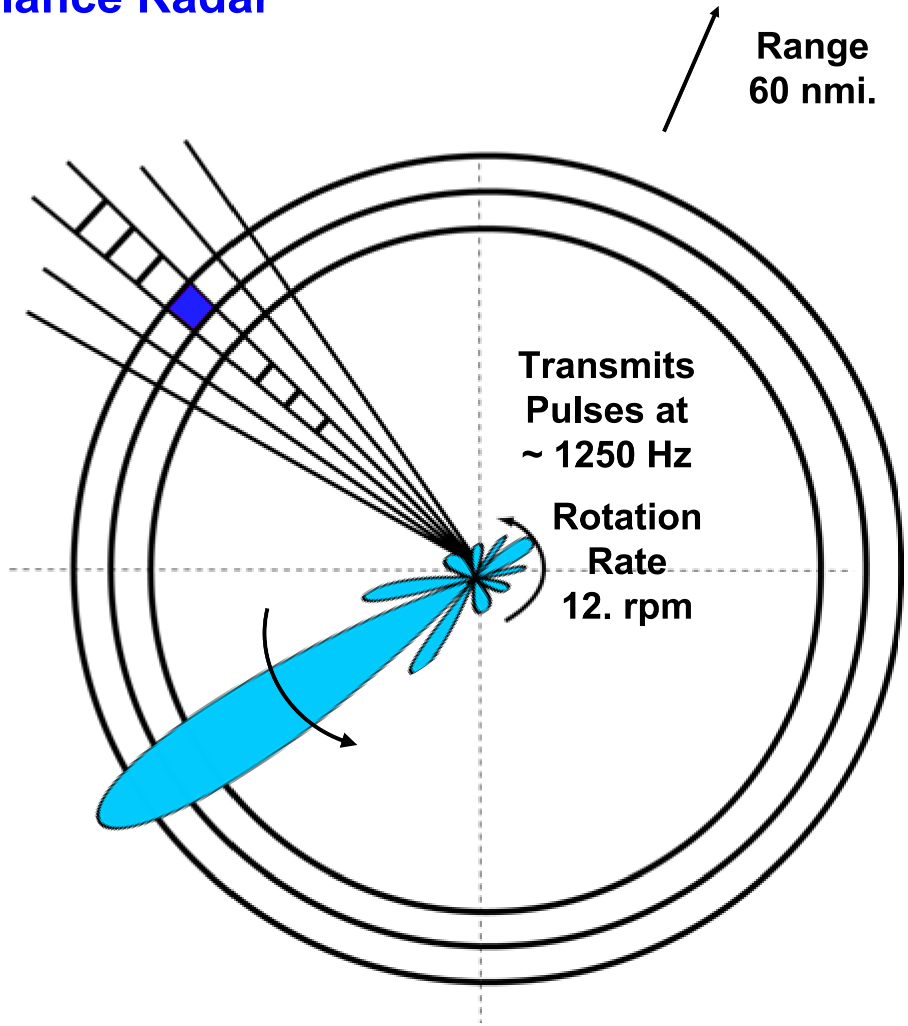
Outline

- **Basic concepts**
  - Probabilities of detection and false alarm
  - Signal-to-noise ratio
- Integration of pulses
- Fluctuating targets
- Constant false alarm rate (CFAR) thresholding
- Summary

Radar Detection – "The Big Picture"

Example – Typical Aircraft Surveillance Radar ASR-9



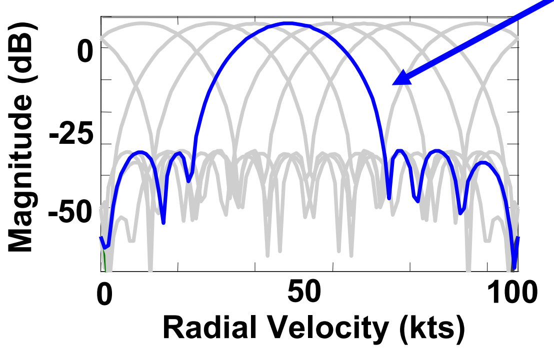
- Mission – Detect and track all aircraft within 60 nmi of radar
- S-band $\lambda \sim 10$ cm

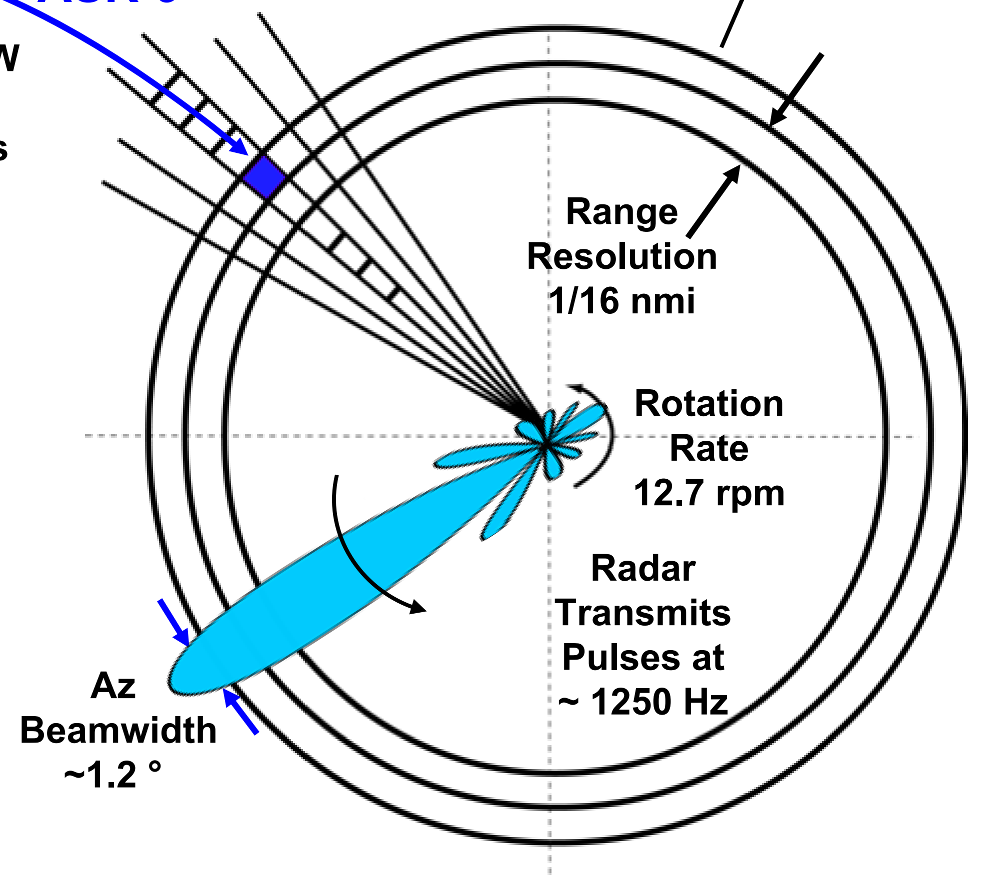
Range-Azimuth-Doppler Cells to Be Thresholded

Example – Typical Aircraft Surveillance Radar ASR-9

10 Pulses / Half BW
Processed into
10 Doppler Filters

Doppler Filter Bank



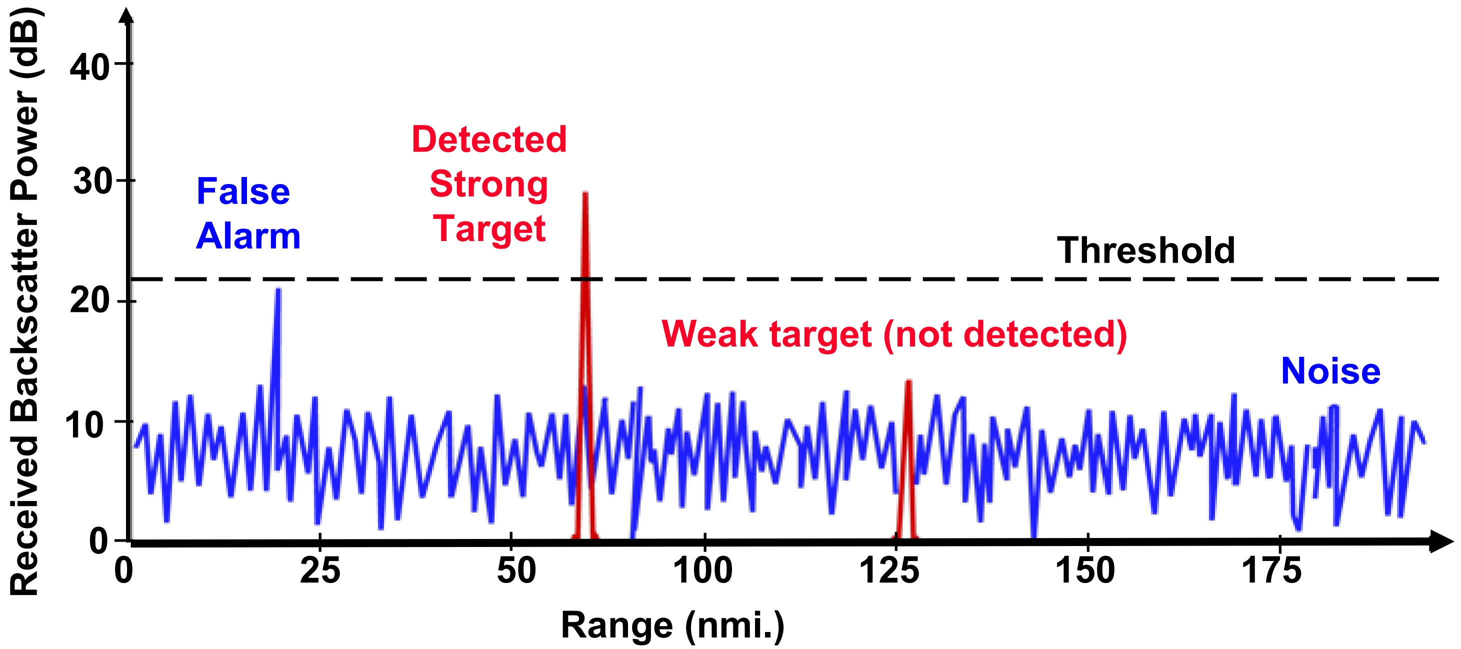


As Antenna Rotates
~22 pulses / Beamwidth

<u>Range - Azimuth - Doppler Cells</u>
~1000 Range cells
~500 Azimuth cells
<u>~8-10 Doppler cells</u>
5,000,000 Range-Az-Doppler Cells
to be threshold <u>every 4.7 sec.</u>

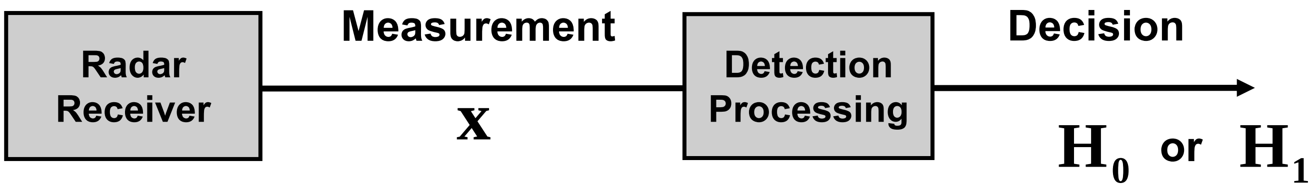
Is There a Target Present in Each Cell?

Target Detection in Noise



- Received background noise fluctuates randomly up and down
- The target echo also fluctuates…. Both are random variables!
- To decide if a target is present, at a given range, we need to set a threshold (constant or variable)
- Detection performance (Probability of Detection) depends of the strength of the target relative to that of the noise and the threshold setting
  - Signal-To Noise Ratio and Probability of False Alarm

The Radar Detection Problem



For each measurement There are two possibilities:	Measurement	Probability Density
Target absent hypothesis, H_0 Noise only	$x = n$	$p(x \mid H_0)$
Target present hypothesis, H_1 Signal plus noise	$x = a + n$	$p(x \mid H_1)$

For each measurement
There are four decisions:

Truth \ Decision	H_0	H_1
H_0	Don't Report	False Alarm
H_1	Missed Detection	Detection

Threshold Test is Optimum

Decision

		H_0	H_1
Truth	H_0	Don't Report	False Alarm
	H_1	Missed Detection	Detection

Probability of Detection: P_D — The probability we choose H_1 when H_1 is true

Probability of False Alarm: P_{FA} — The probability we choose H_1 when H_0 is true

Objective: Neyman-Pearson criterion

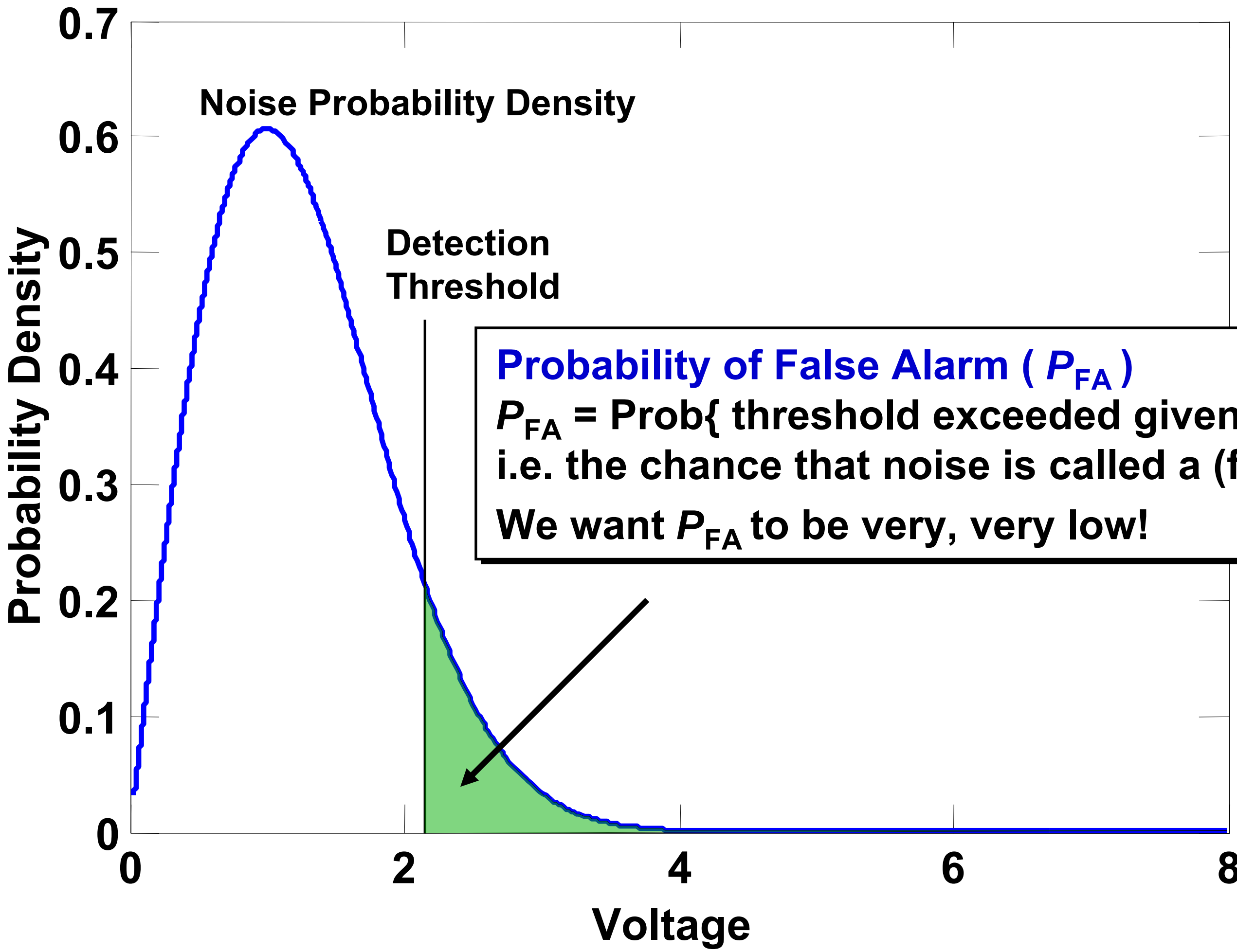
Maximize P_D subject to P_{FA} no greater than specified $(P_{FA} \le \alpha)$

Likelihood Ratio Test

Likelihood Ratio → $L(x)$; Threshold → η

$$L(x) = \frac{p(x|H_1)}{p(x|H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \eta$$

Basic Target Detection Test



Noise Only
Target Absent

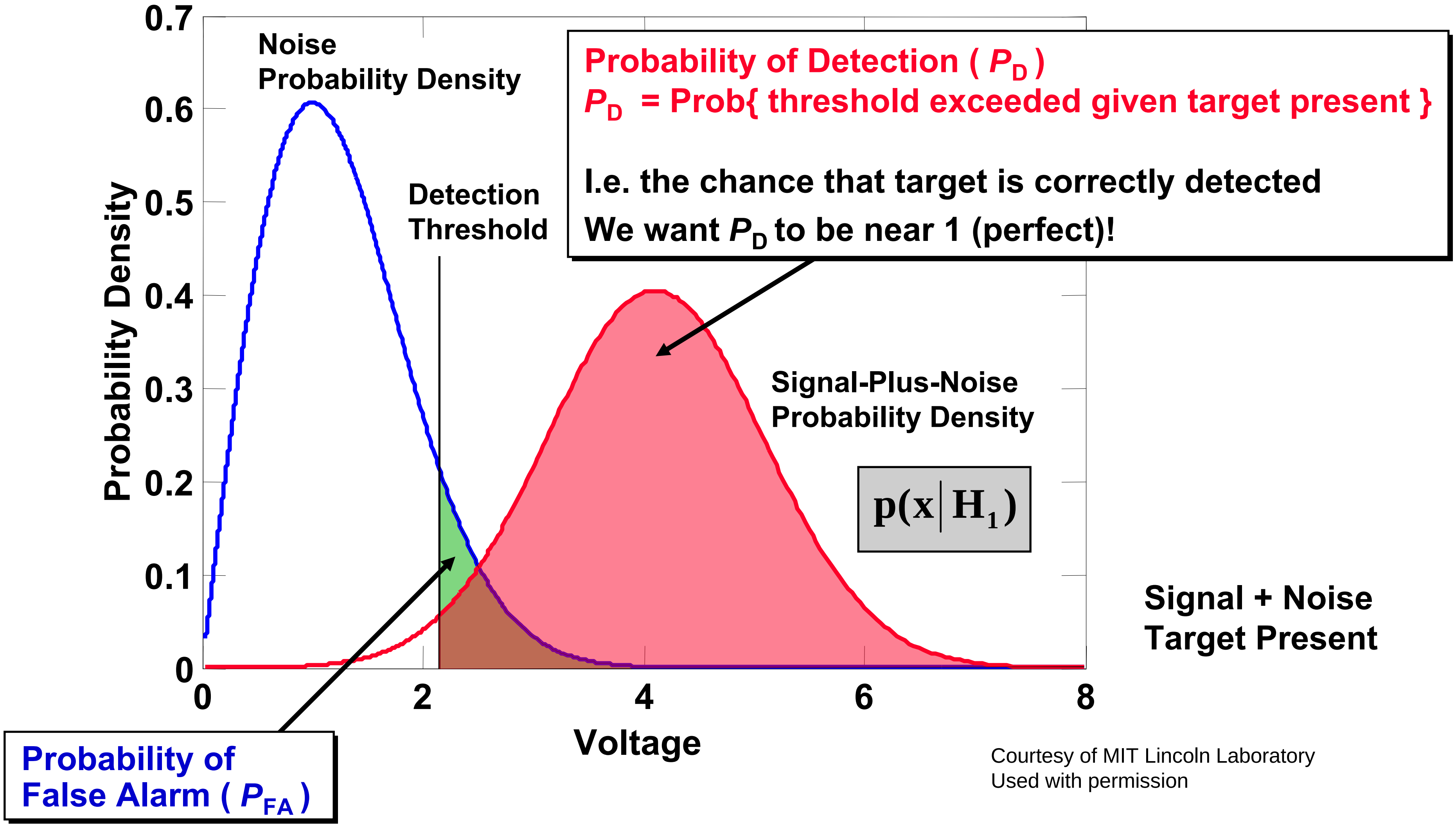
Probability of False Alarm (P_{FA})

P_{FA} = Prob{ threshold exceeded given target absent }

i.e. the chance that noise is called a (false) target

We want P_{FA} to be very, very low!

Basic Target Detection Test



Signal + Noise
Target Present

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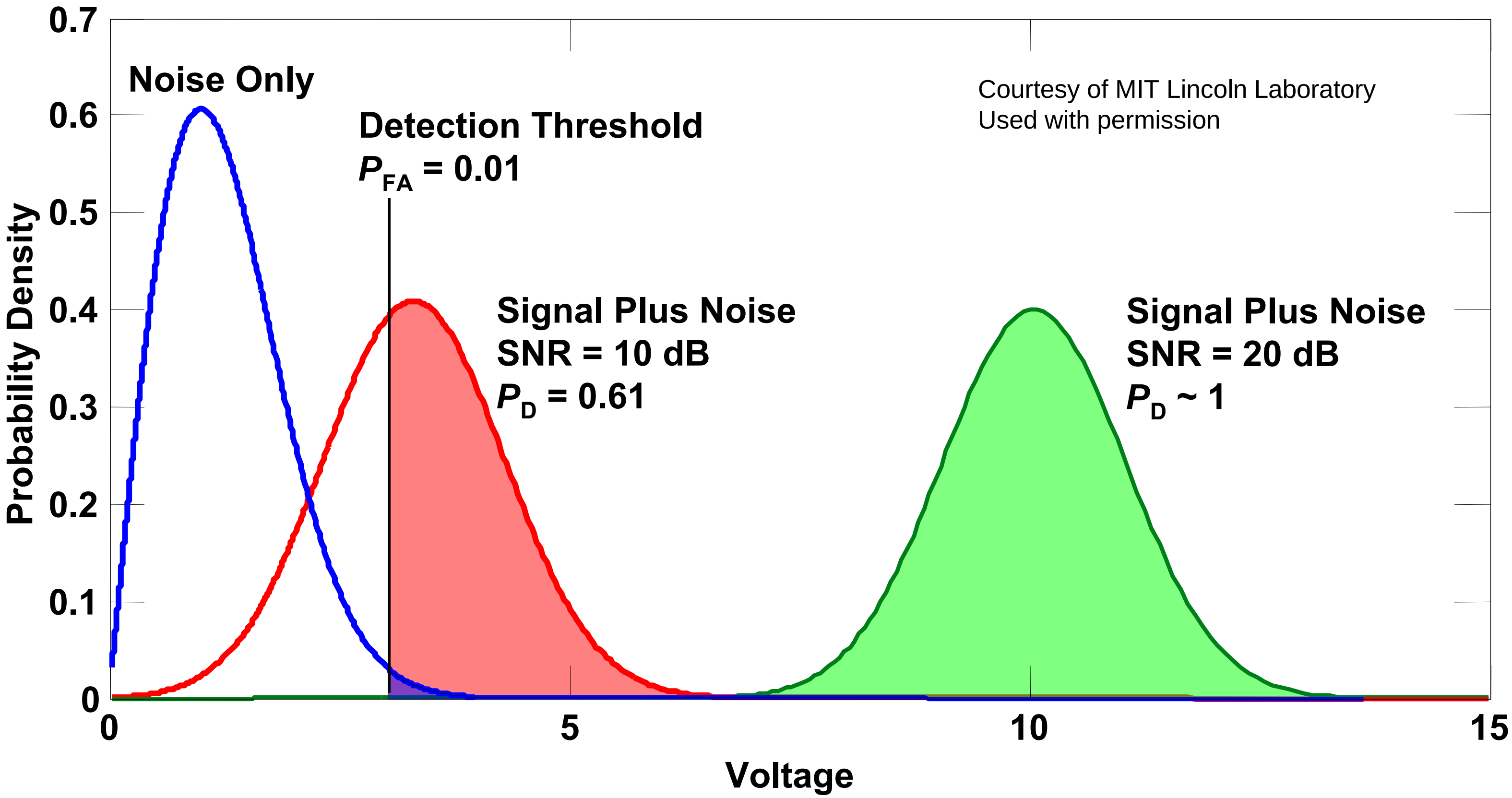
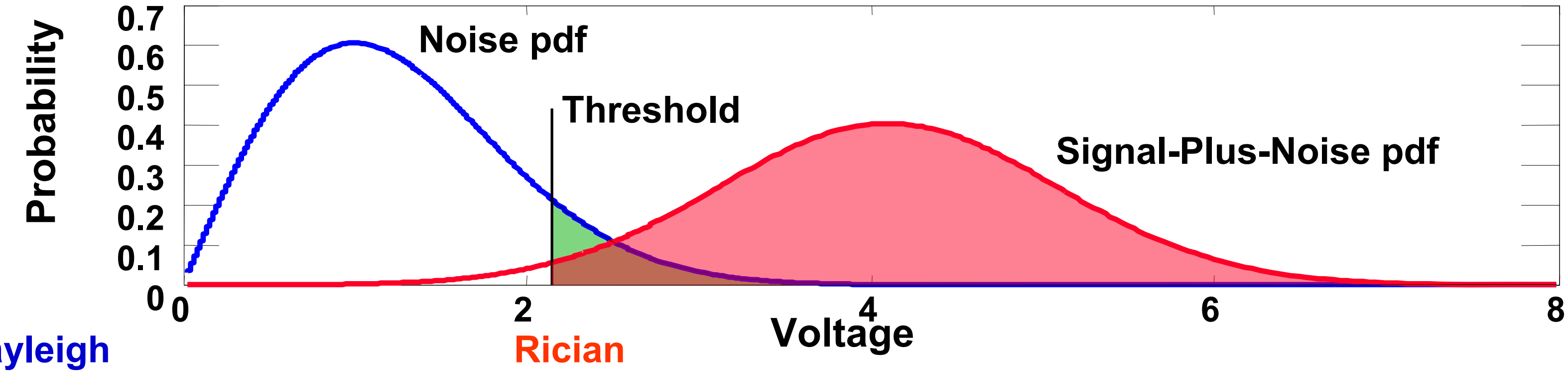
Detection Examples with Different SNR



- P_D **increases with target SNR for a fixed threshold (**P_{FA}**)**
- **Raising threshold reduces false alarm rate and increases SNR required for a specified Probability of Detection**

Non-Fluctuating Target Distributions



Rayleigh

$$p(r|H_o) = r\exp\left(-\frac{r^2}{2}\right)$$

Rician

$$p(r|H_1) = r\exp\left(-\frac{r^2+R}{2}\right) I_o\left(r\sqrt{R}\right)$$

$$SNR = \frac{R}{2}$$

Set threshold r_T based on desired false-alarm probability

$$r_T = \sqrt{-2\log_e P_{FA}}$$

Courtesy of MIT Lincoln Laboratory
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Compute detection probability for given SNR and false-alarm probability

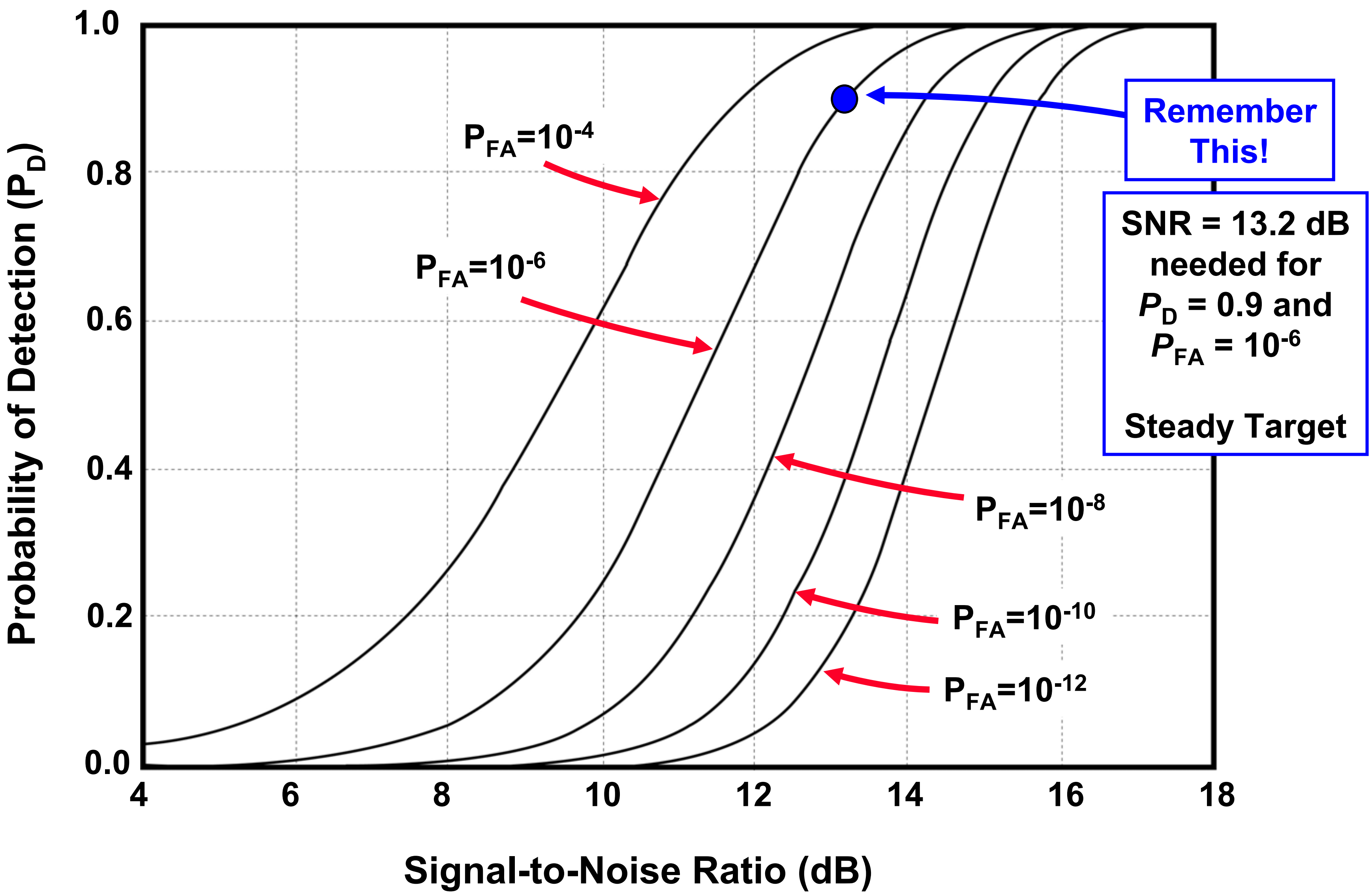
$$P_D = \int_{r_T}^{\infty} p(r|H_1)\,dr = Q\left(\sqrt{2(SNR)}, \sqrt{-2\log_e P_{FA}}\right)$$

where

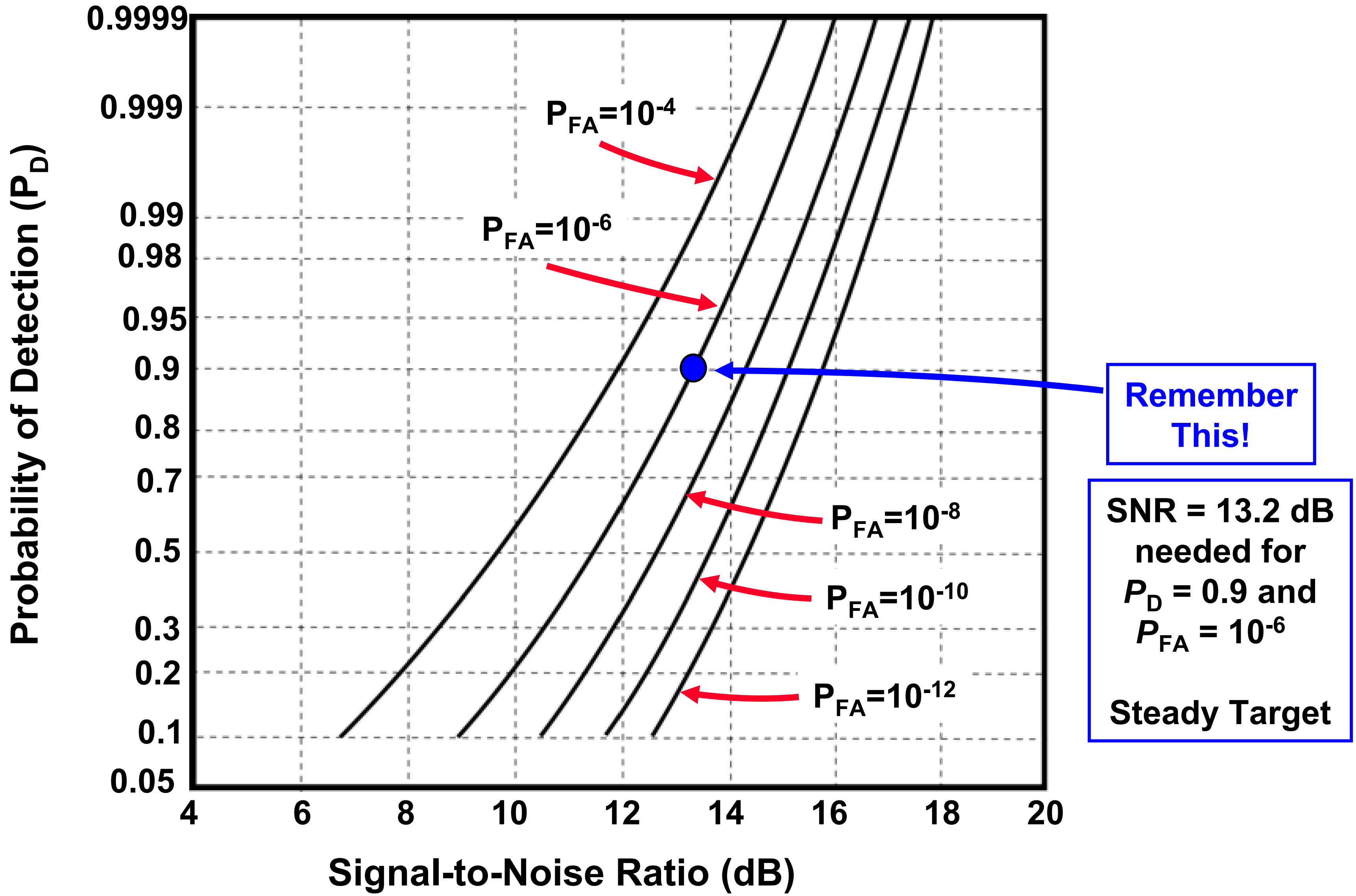
$$Q(a,b) = \int_b^{\infty} r\exp\left(-\frac{r^2+a^2}{2}\right) I_o(a\,r)\,dr$$

Is Marcum's Q-Function (and $I_0(x)$ is a modified Bessel function)

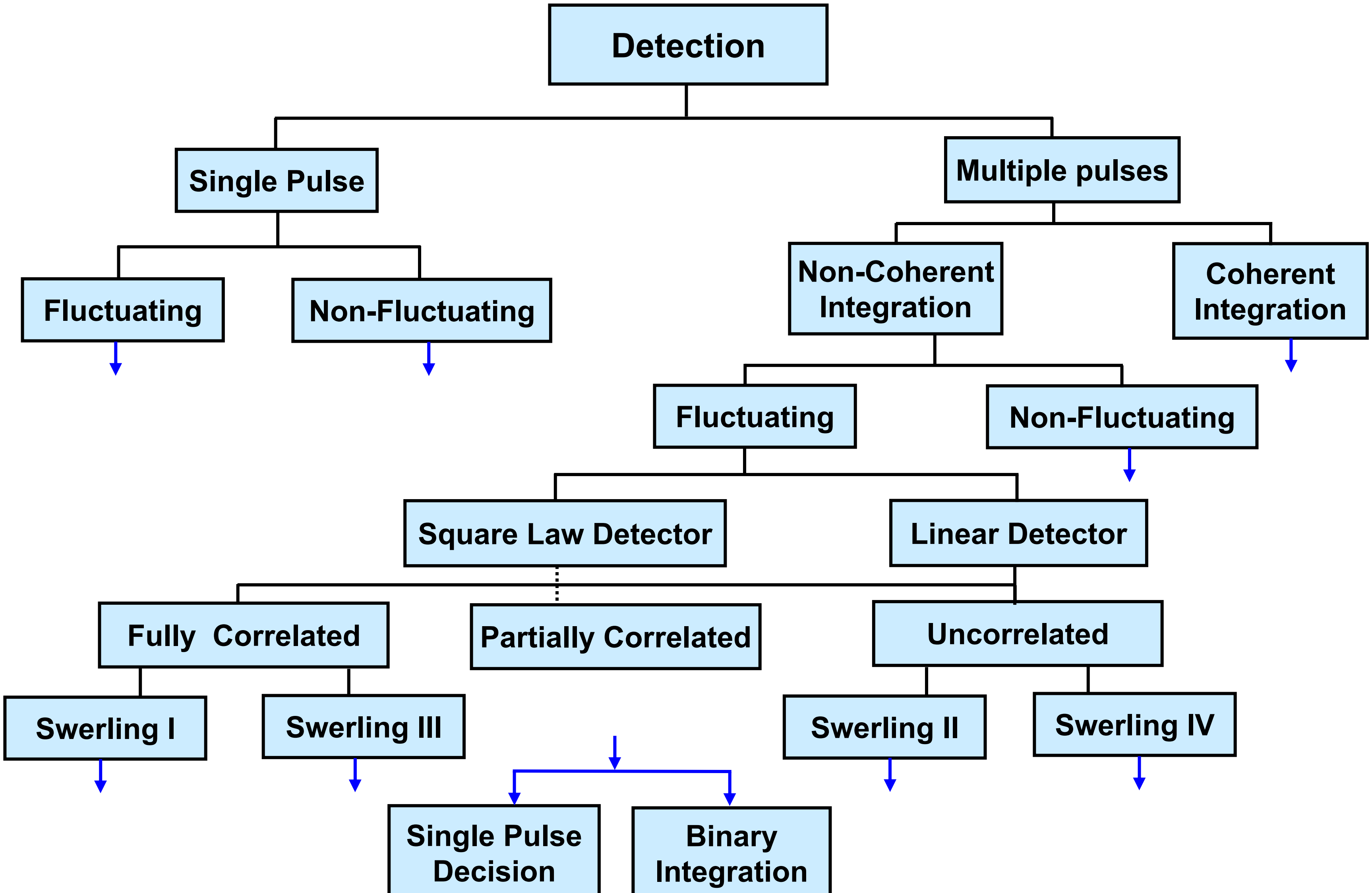
Probability of Detection vs. SNR



Probability of Detection vs. SNR

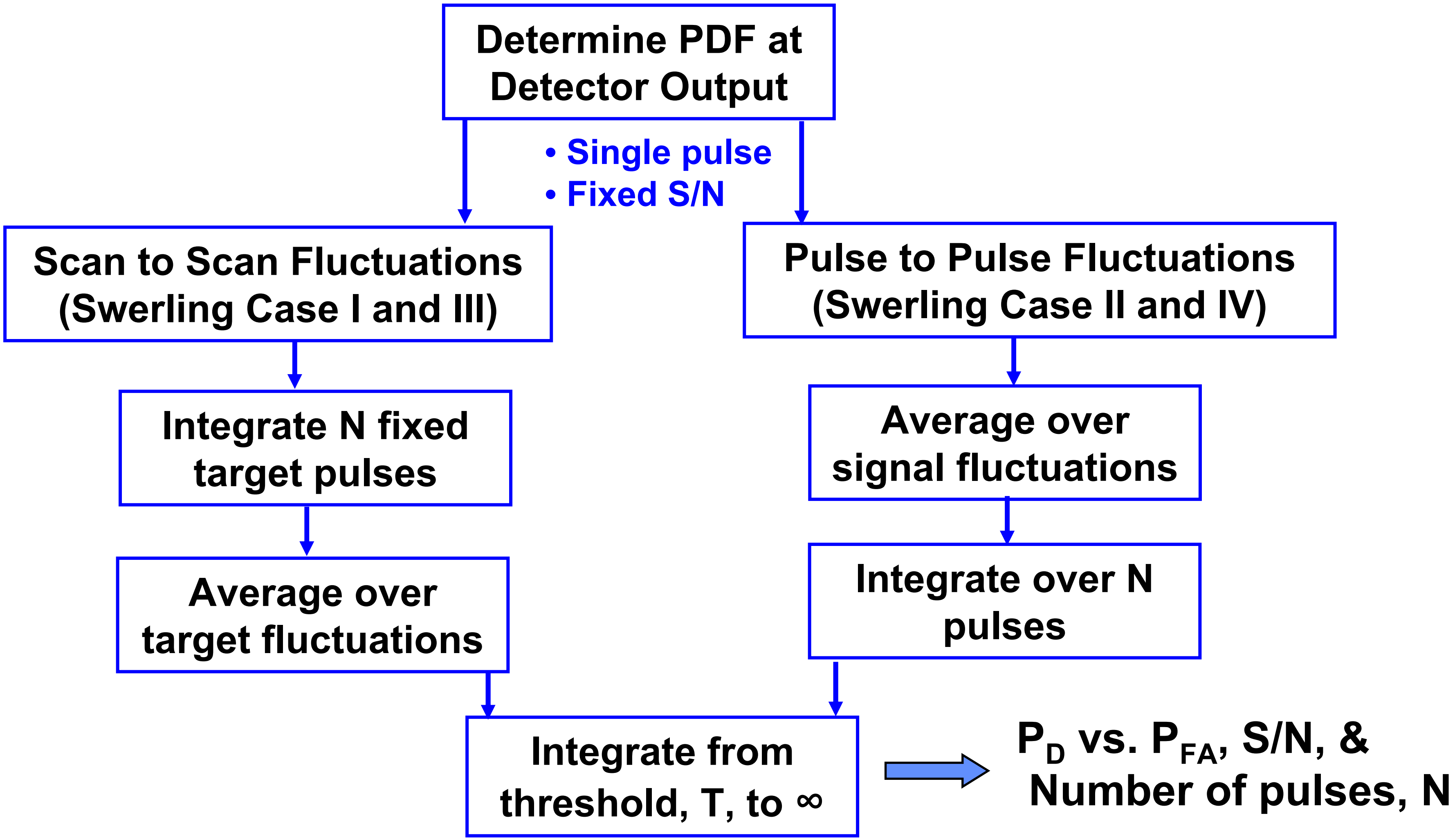


Tree of Detection Issues



Detection Calculation Methodology

Probability of Detection vs. Probability of False Alarm and Signal-to-Noise Ratio

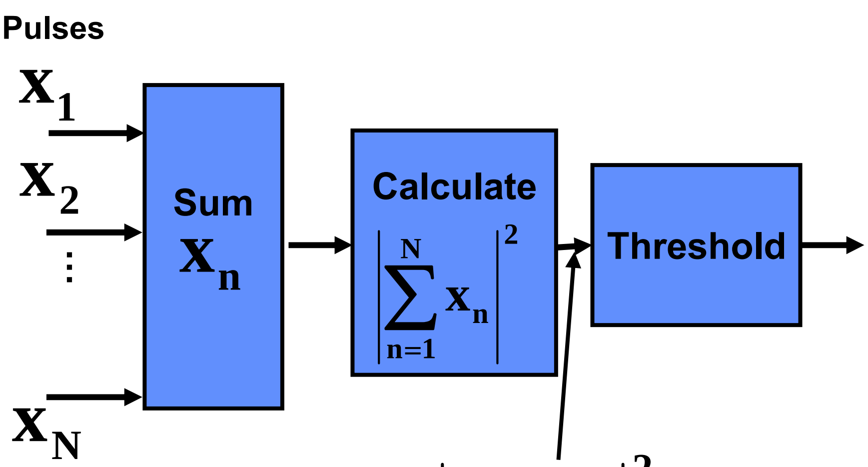


Outline

- Basic concepts
- Integration of pulses
- Fluctuating targets
- Constant false alarm rate (CFAR) thresholding
- Summary

Integration of Radar Pulses

Coherent Integration

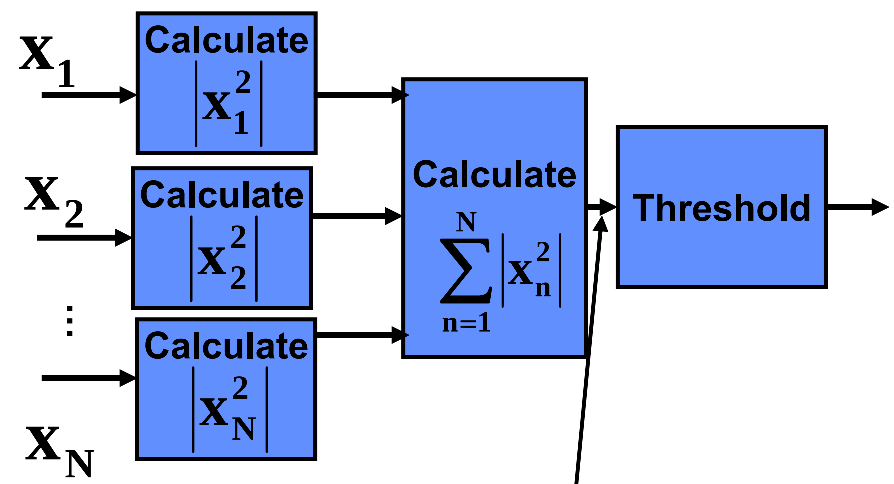


Target Detection Declared if

$$\frac{1}{N}\left|\sum_{n=1}^{N} x_n\right|^2 > T$$

- Adds 'voltages' , then square
- Phase is preserved
- pulse-to-pulse phase coherence required
- SNR Improvement = 10 $\log_{10} N$

Noncoherent Integration



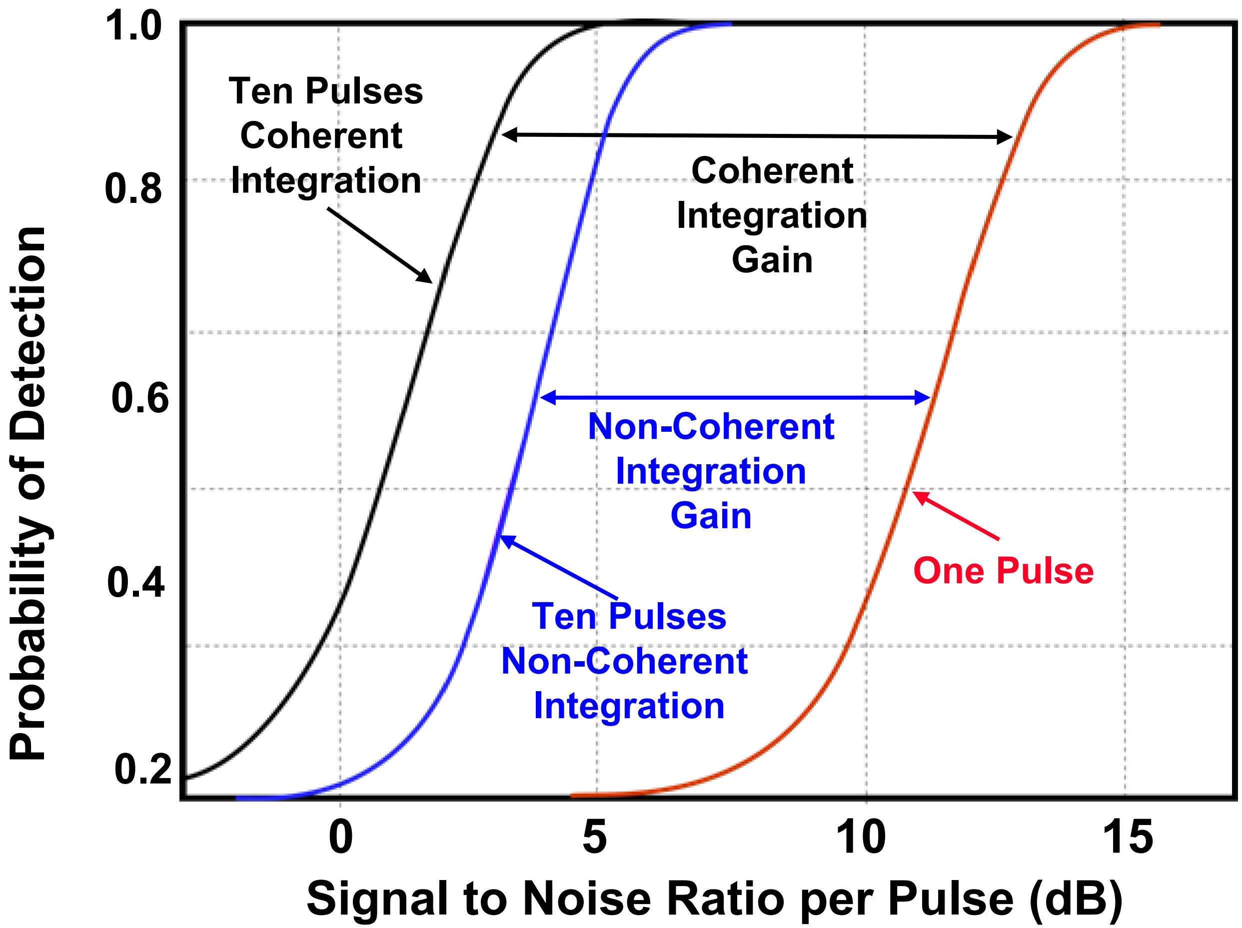
Target Detection Declared if

$$\frac{1}{N}\sum_{n=1}^{N}\left|x_n^2\right| > T$$

- Adds 'powers' not voltages
- Phase neither preserved nor required
- Easier to implement, not as efficient

Detection performance can be improved by integrating multiple pulses

Integration of Pulses



Steady Target

$P_{FA}=10^{-6}$

For Most Cases, Coherent Integration is More Efficient than Non-Coherent Integration

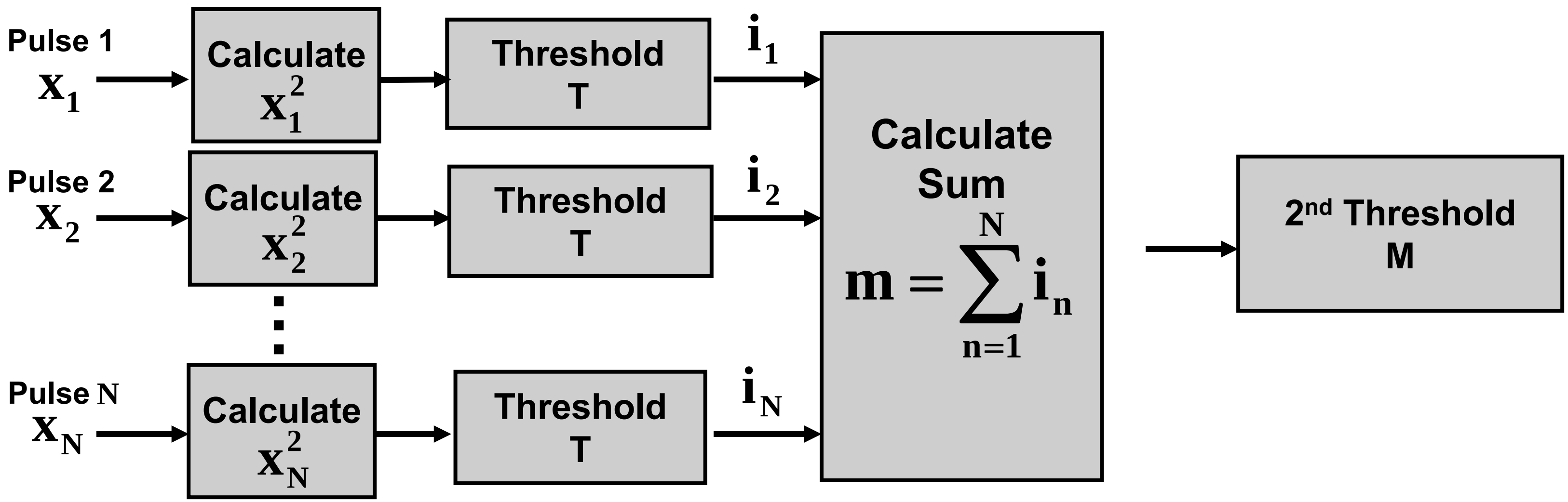
Different Types of Non-Coherent Integration

- **Non-Coherent Integration – Also called (“video integration”)**
  - Generate magnitude for each of N pulses
  - Add magnitudes and then threshold

- **Binary Integration (*M*-of-*N* Detection)**
  - Separately threshold each pulse
    - 1 if signal > threshold; 0 otherwise
  - Count number of threshold crossings (the # of 1s)
  - Threshold this sum of threshold crossings
    - Simpler to implement than coherent and non-coherent

- **Cumulative Detection (*1-of-N* Detection)**
  - Similar to Binary Integration
  - Require at least 1 threshold crossing for N pulses

Binary (*M*-of-*N*) Integration



Individual pulse detectors:

$$|x_n|^2 \geq T, \quad i_n = 1$$
$$|x_n|^2 < T, \quad i_n = 0$$

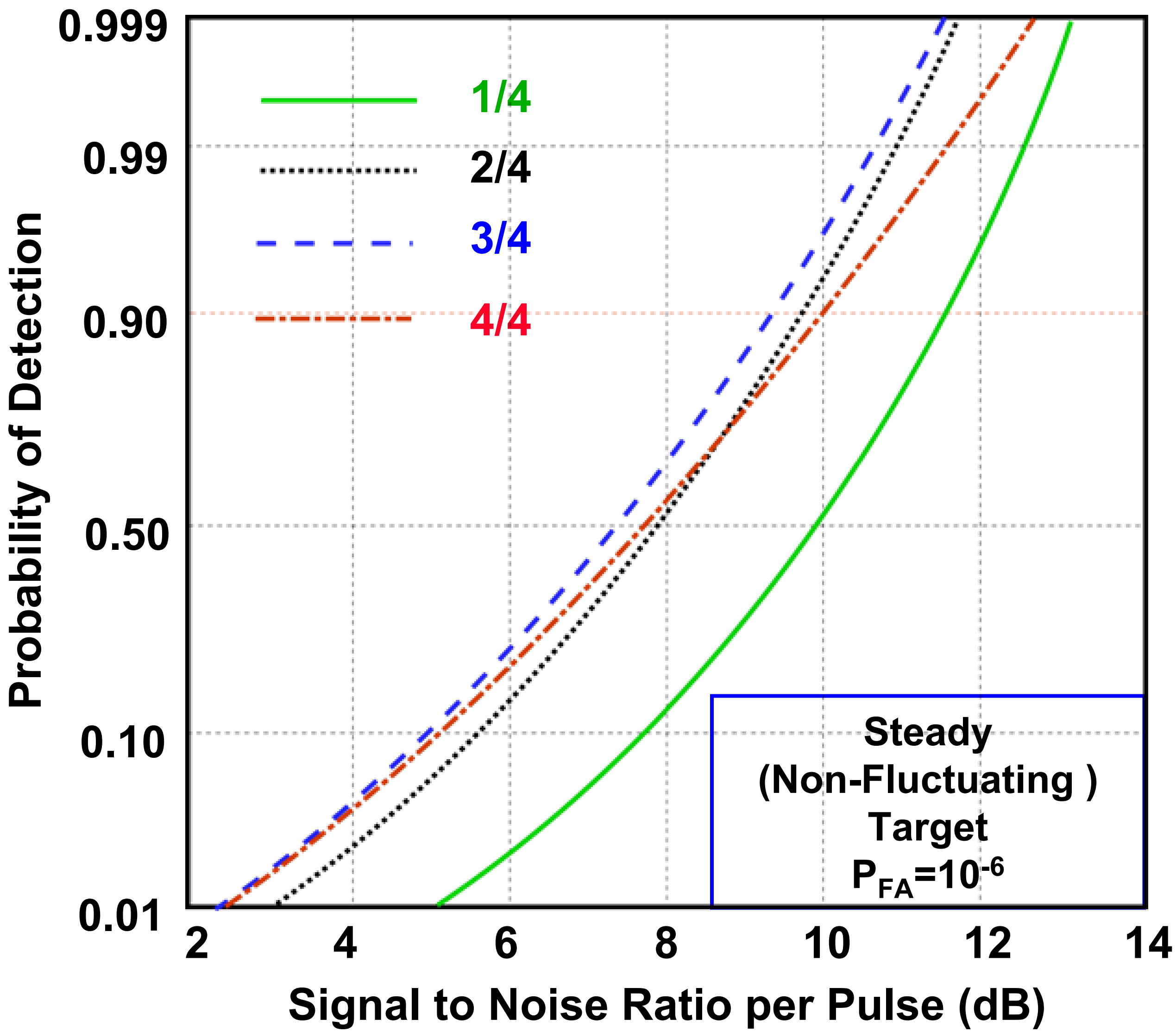
2nd thresholding:

$m \geq M$, target present
$m < M$, target absent

Target present if at least *M* detections in *N* pulses

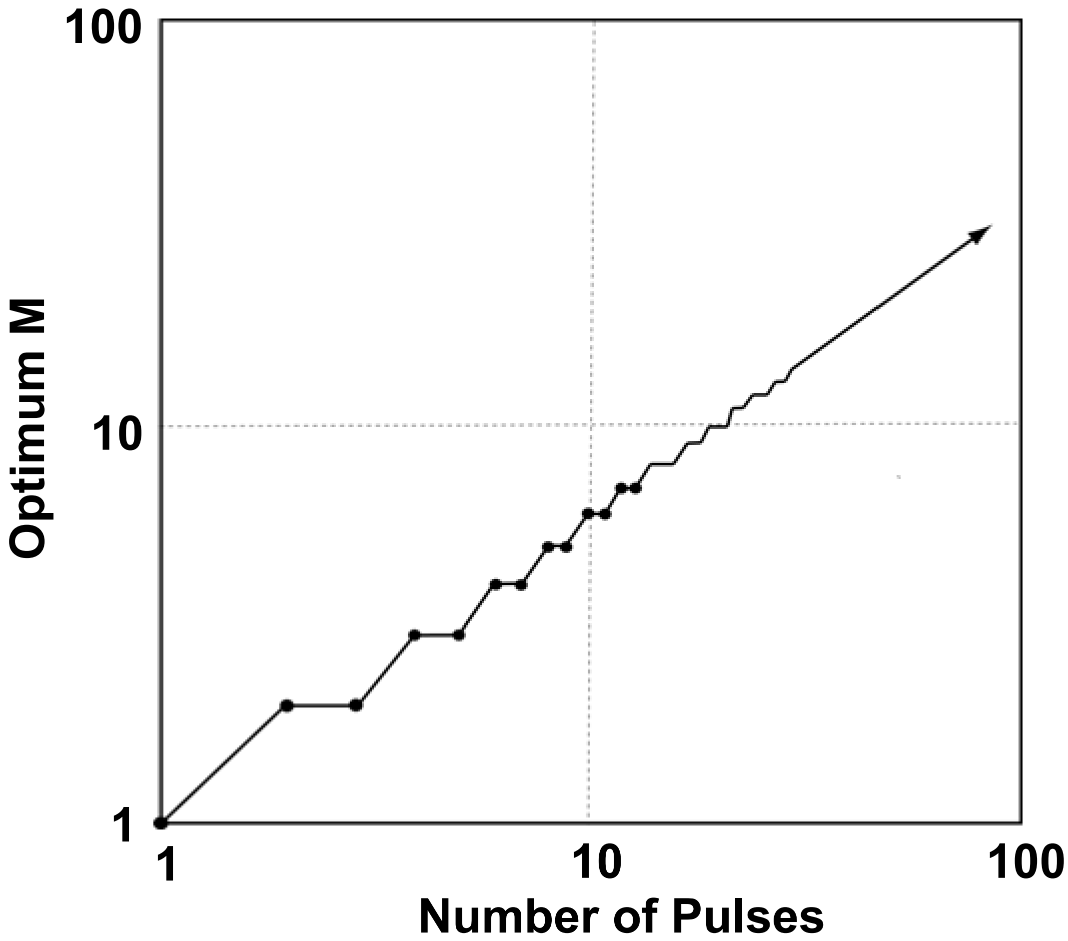
Binary Integration		Cumulative Detection	
At Least M of N Detections	$P_{M/N} = \sum_{k=M}^{N} \frac{N!}{k!(N-k)!} p^k (1-p)^{N-k}$	At Least 1 of N	$P_C = 1 - (1-p)^N$

Detection Statistics for Binary Integration



"1/4"
= 1 Detection
Out of 4
Pulses

Optimum M for Binary Integration



Steady
(Non-Fluctuating)
Target

P_D=0.95

P_{FA}=10^{-6}

For each binary Integrator, M/N, there exists an optimum M

M (optimum) ≈ $0.9\ N^{0.8}$

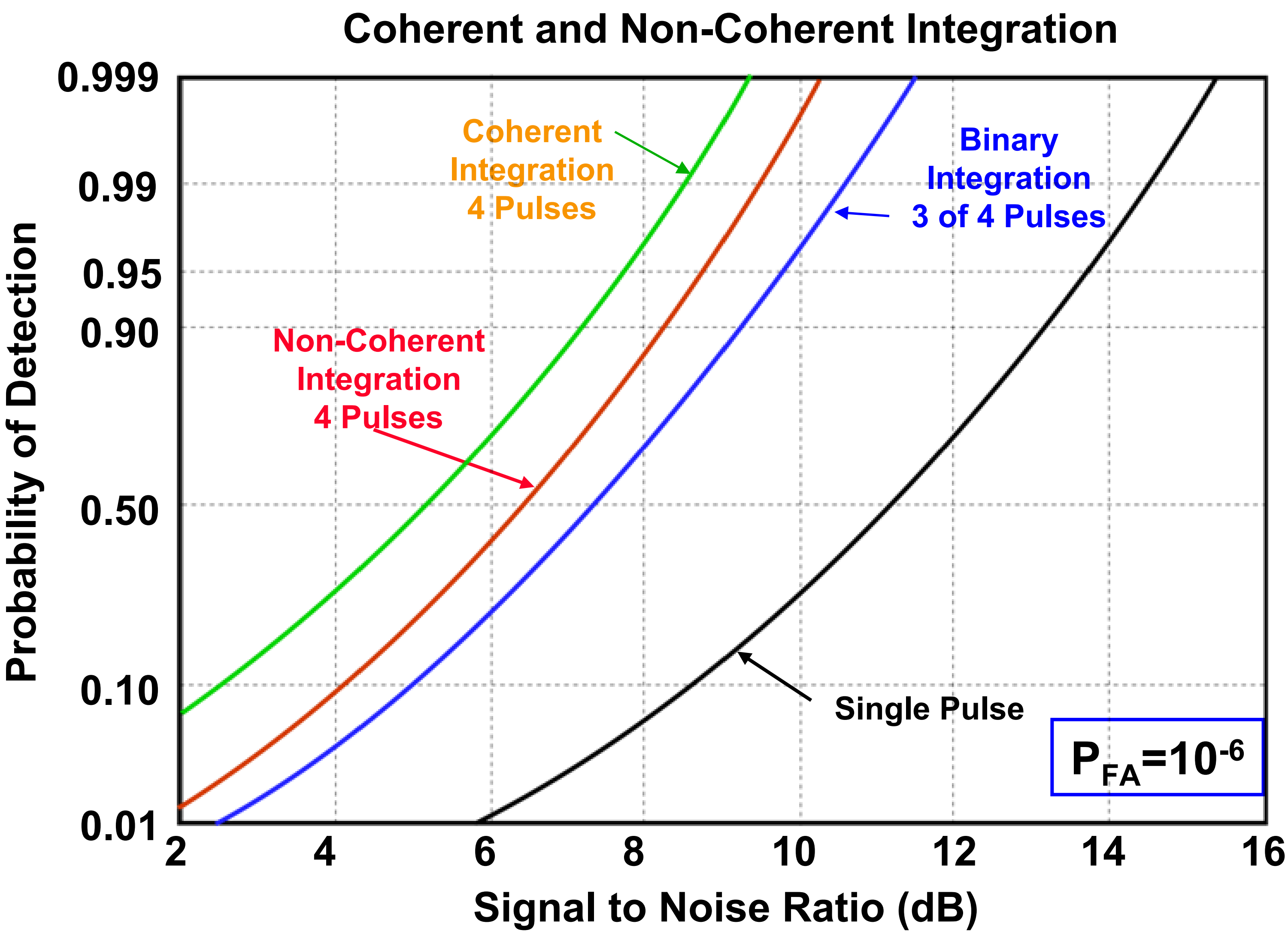
Optimum M for Binary Integration

- Optimum M varies somewhat with target fluctuation model, P_D and P_{FA}
- Parameters for Estimating $M_{OPT} = N^a \, 10^b$

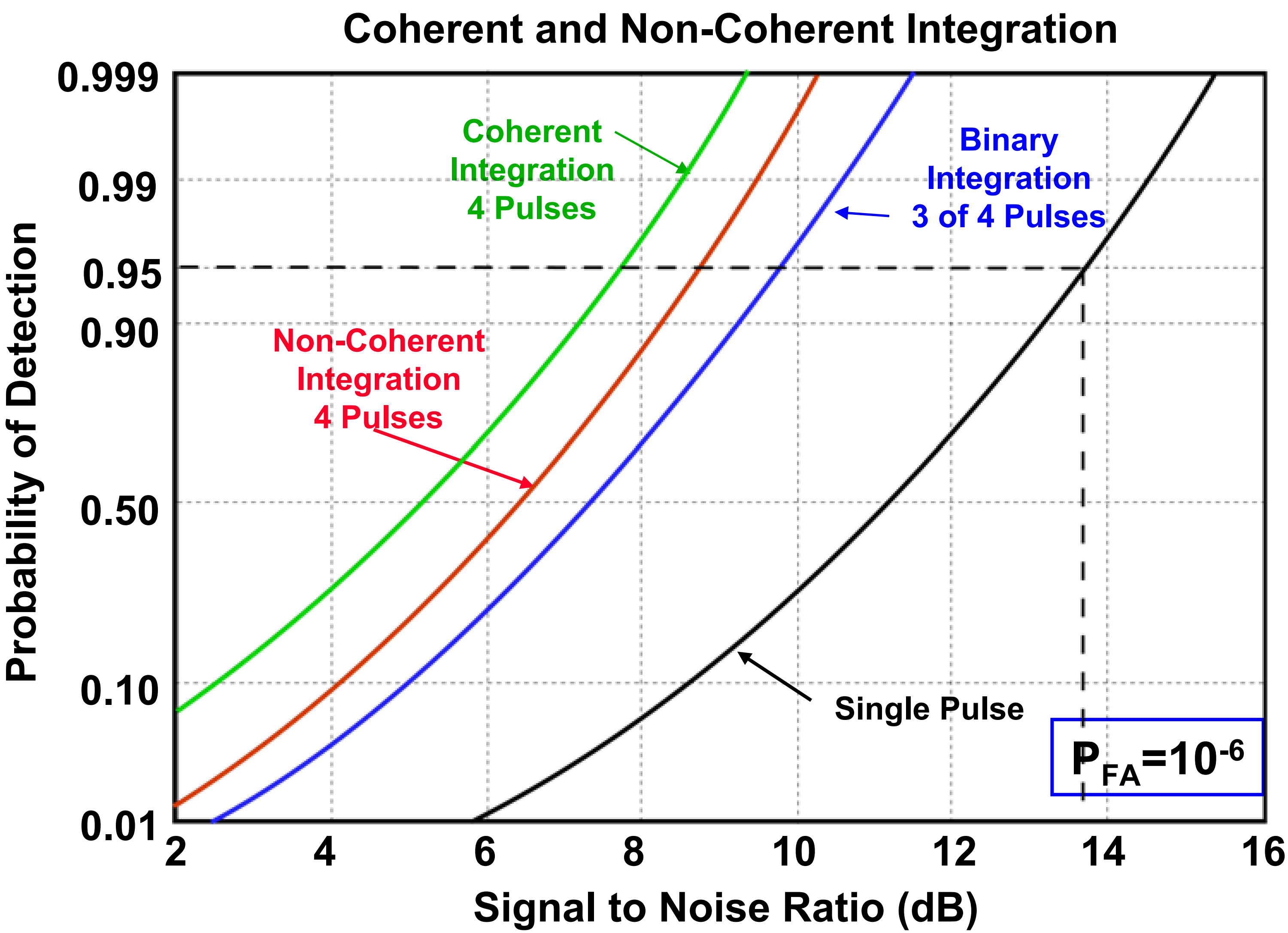
Target Fluctuations	a	b	Range of N
No Fluctuations	0.8	- 0.02	5 – 700
Swerling I	0.8	- 0.02	6 – 500
Swerling II	0.91	- 0.38	9 – 700
Swerling III	0.8	- 0.02	6 – 700
Swerling IV	0.873	- 0.27	10 – 700

Adapted from Shnidman in Richards, reference 7

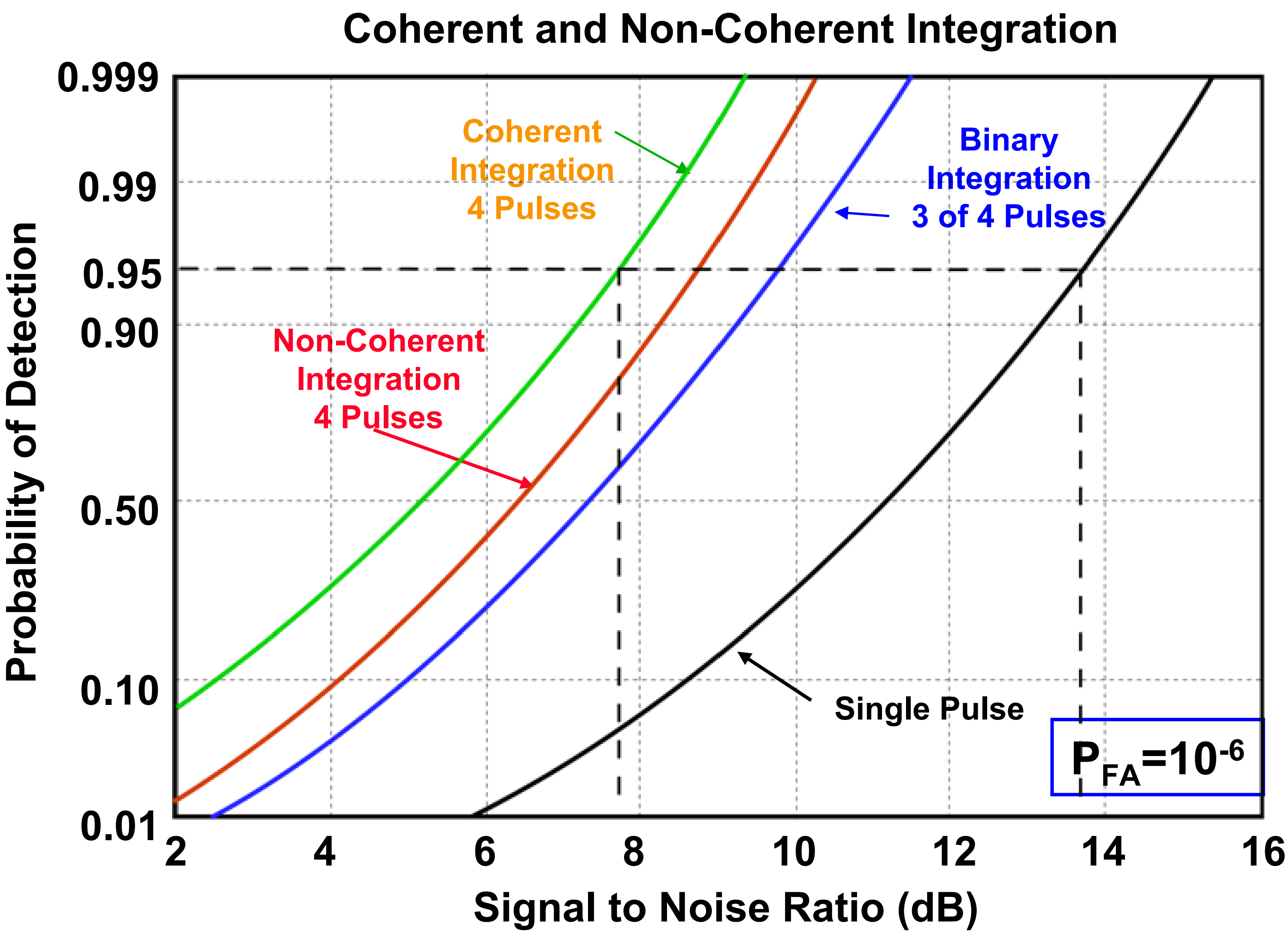
Detection Statistics for Different Types of Integration



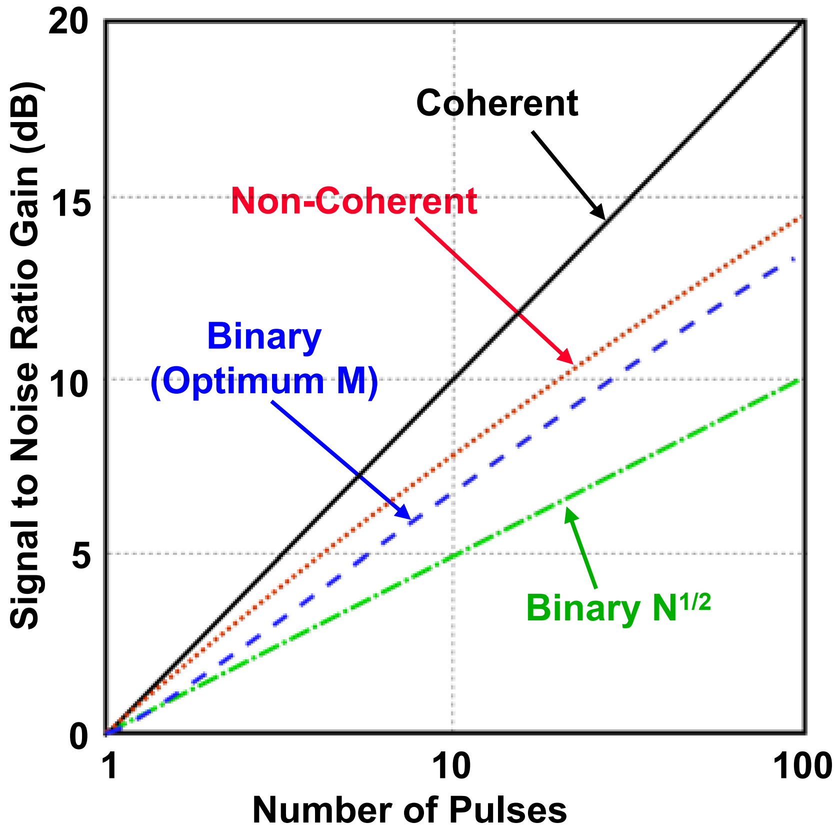
Detection Statistics for Different Types of Integration

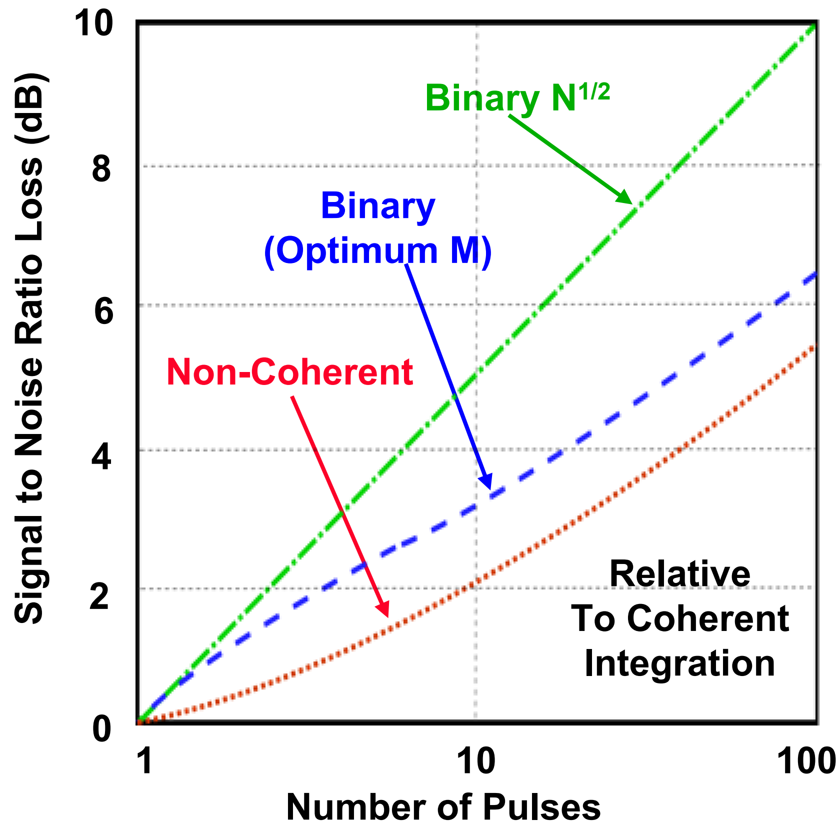


Detection Statistics for Different Types of Integration



Signal to Noise Gain / Loss vs. # of Pulses

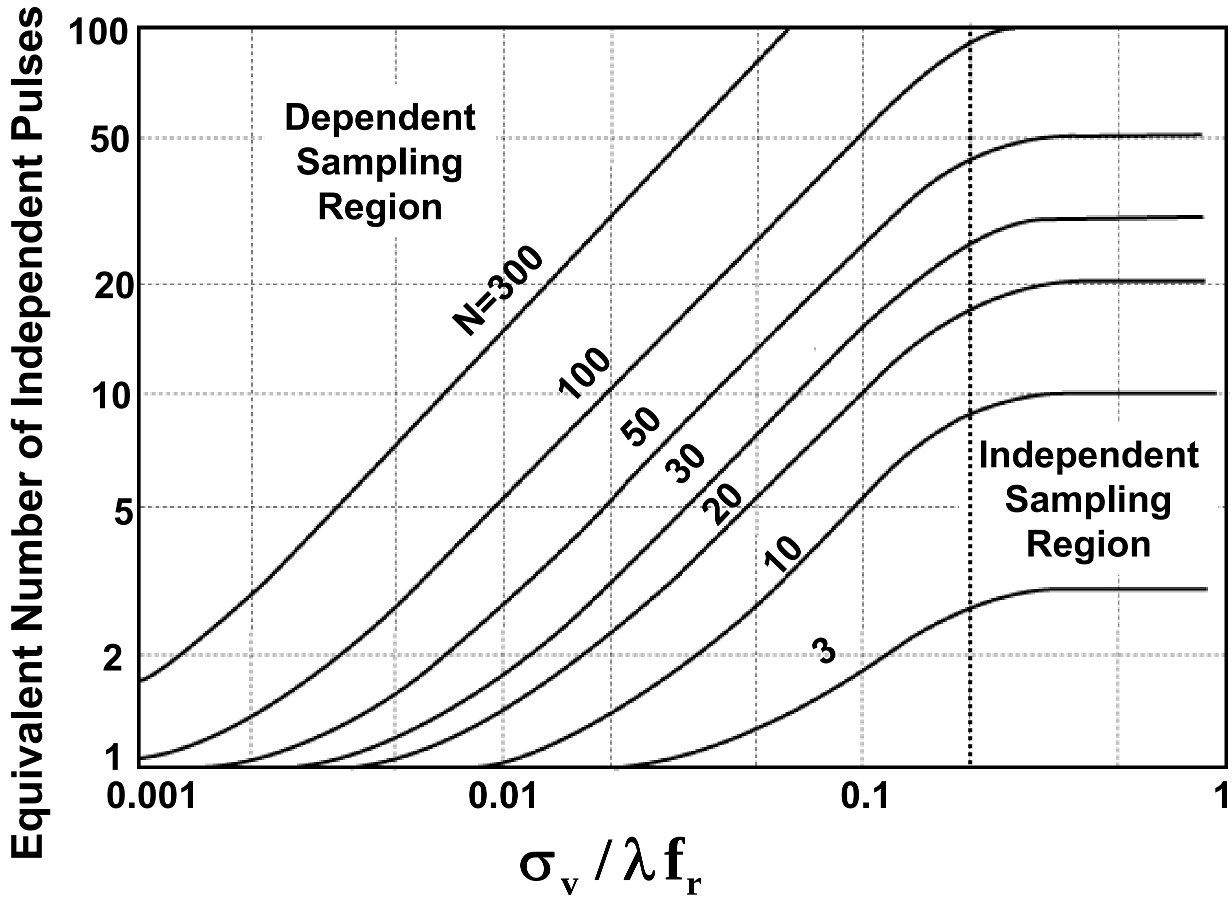




Steady Target
P_D=0.95
P_{FA}=10^{-6}

- Coherent Integration yields the greatest gain
- Non-Coherent Integration a small loss
- Binary integration has a slightly larger loss than regular Non-coherent integration

Effect of Pulse to Pulse Correlation on Non-Coherent Integration Gain



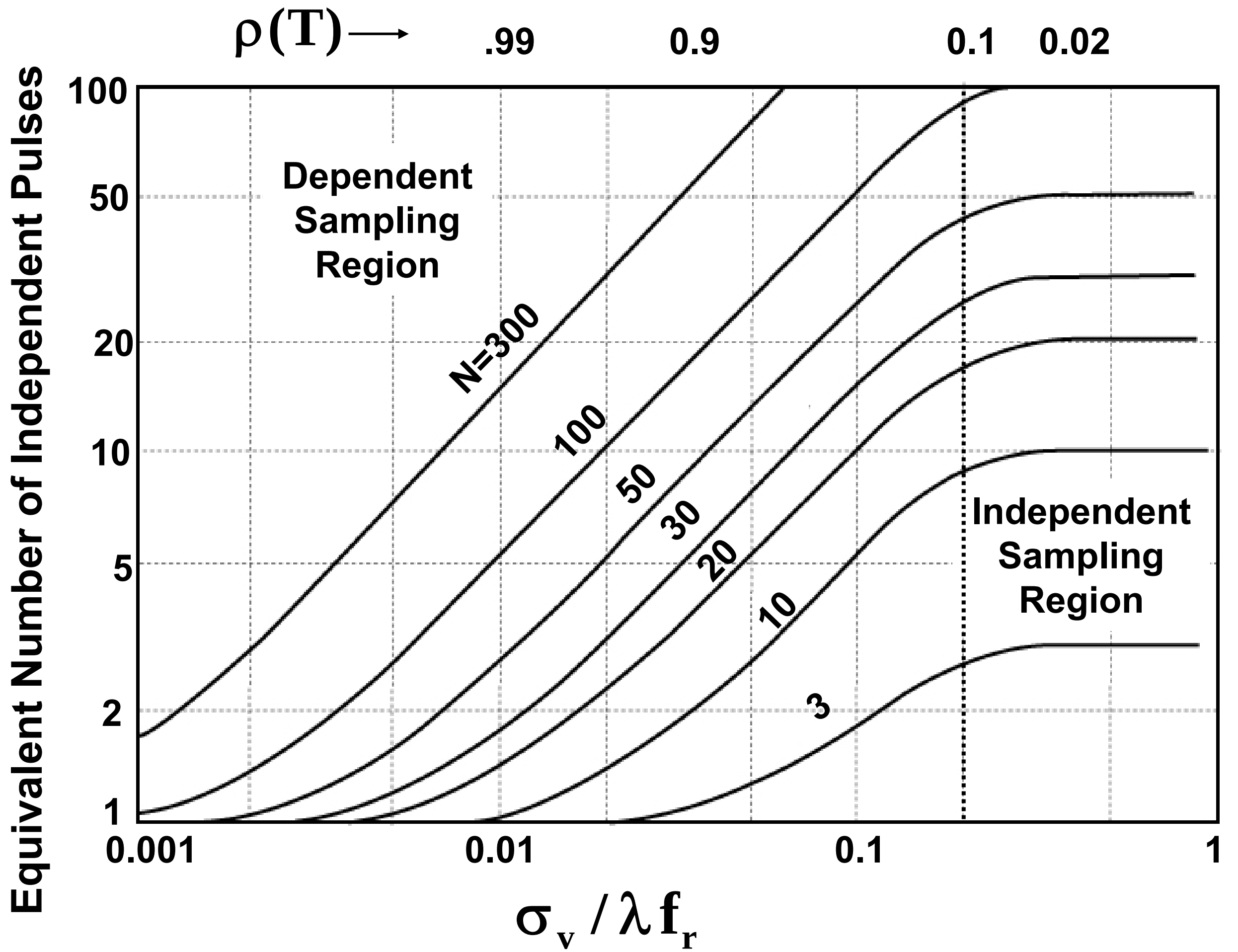
σ_v Velocity Spread Of Clutter

$$f_r = \frac{1}{T}$$

Pulse Repetition Rate

- Non-coherent Integration Can Be Very Inefficient in Correlated Clutter

Effect of Pulse to Pulse Correlation on Non-Coherent Integration Gain



σ_v Velocity Spread Of Clutter

$$f_r = \frac{1}{T}$$

Pulse Repetition Rate

- **Non-coherent Integration Can Be Very Inefficient in Correlated Clutter**

Adapted from nathanson, Reference 8

Albersheim Empirical Formula for SNR
(Steady Target - Good Method for Approximate Calculations)

- **Single pulse:** $$\mathrm{SNR(natural\ units)} = A + 0.12\, A\, B + 1.7\, B$$
  - Where: $A = \log_e\left(\dfrac{0.62}{P_{FA}}\right)$ $B = \log_e\left(\dfrac{P_D}{1 - P_D}\right)$
  - Less than .2 dB error for:

    $10^{-3} > P_{FA} > 10^{-7}$ $0.9 > P_D > 0.1$
  - Target assumed to be non-fluctuating
- **For n independent integrated samples:**

$$\mathrm{SNR}_n(\mathrm{dB}) = -5 \log_{10} n + \left(6.2 + \frac{4.54}{\sqrt{n + 0.44}}\right) \log_{10}\left(A + 0.12\, A\, B + 1.7\, B\right)$$

SNR Per Sample

  - Less than .2 dB error for:

    $8096 > n > 1$ $10^{-3} > P_{FA} > 10^{-7}$ $0.9 > P_D > 0.1$
  - For more details, see References 1 or 5

Outline

- Basic concepts
- Integration of pulses
- **Fluctuating targets**
- Constant false alarm rate (CFAR) thresholding
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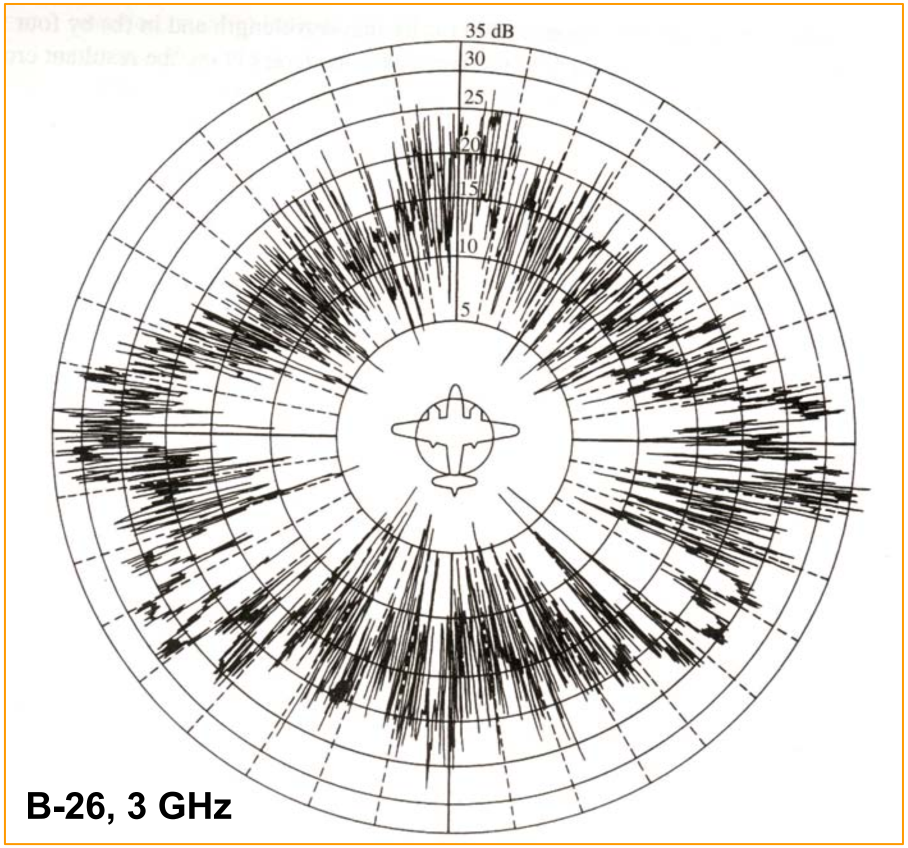
Fluctuating Target Models

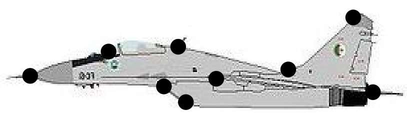
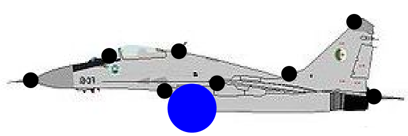
RCS vs. Azimuth for a Typical Complex Target



RCS versus Azimuth

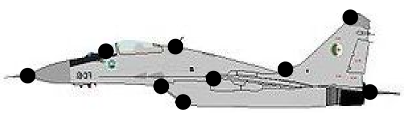
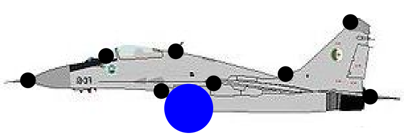
- For many types of targets, the received radar backscatter amplitude of the target will vary a lot from pulse-to-pulse:
  - Different scattering centers on complex targets can interfere constructively and destructively
  - Small aspect angle changes or frequency diversity of the radar's waveform can cause this effect
- Fluctuating target models are used to more accurately predict detection statistics (P_D vs., $P_{FA,}$ and S/N) in the presence of target amplitude fluctuations

Swerling Target Models

Nature of Scattering	RCS Model	Fluctuation Rate	
		Slow Fluctuation “Scan-to-Scan”	Fast Fluctuation “Pulse-to-Pulse”
Similar amplitudes 	Exponential (Chi-Squared DOF=2) $p(\sigma)=\frac{1}{\bar{\sigma}}\exp\left(-\frac{\sigma}{\bar{\sigma}}\right)$	Swerling I	Swerling II
One scatterer much Larger than others 	(Chi-Squared DOF=4) $p(\sigma)=\frac{4\sigma}{\bar{\sigma}^2}\exp\left(-\frac{2\sigma}{\bar{\sigma}}\right)$	Swerling III	Swerling IV

$\bar{\sigma} =$ Average RCS (m^2)

Swerling Target Models

Nature of Scattering	Amplitude Model	Fluctuation Rate	
		Slow Fluctuation “Scan-to-Scan”	Fast Fluctuation “Pulse-to-Pulse”
Similar amplitudes 	Rayleigh $p(a)=\frac{2\,a}{\overline{\sigma}}\exp\left(-\frac{a^{2}}{\overline{\sigma}}\right)$	Swerling I	Swerling II
One scatterer much Larger than others 	Central Rayleigh, DOF=4 $p(a)=\frac{8\,a^{3}}{\overline{\sigma}^{2}}\exp\left(-\frac{2\,a^{2}}{\overline{\sigma}}\right)$	Swerling III	Swerling IV

$\overline{\sigma} =$ Average RCS (m^2)

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Other Fluctuation Models

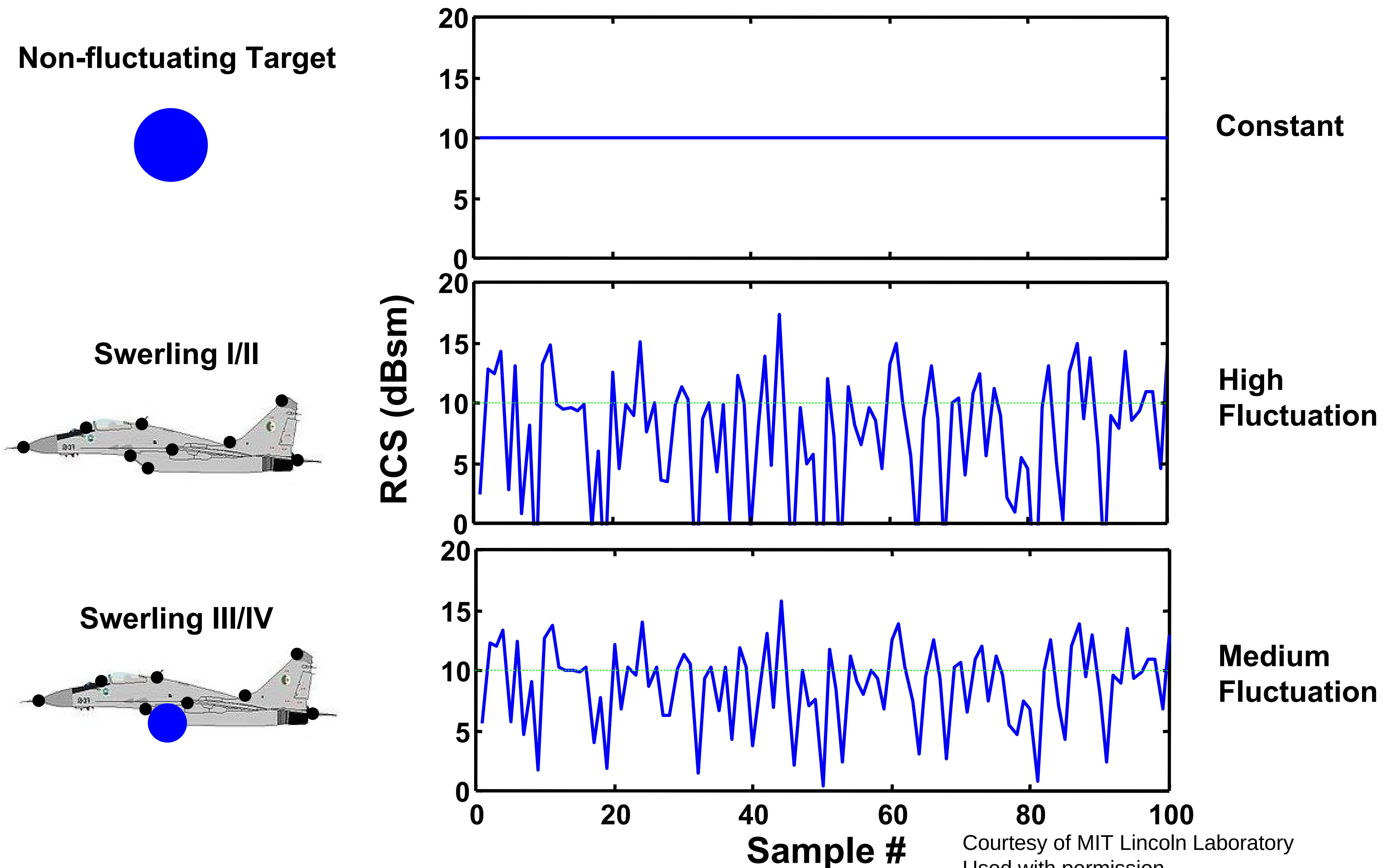
- Detection Statistics Calculations
  - Steady and Swerling 1,2,3,4 Targets in Gaussian Noise
  - Chi- Square Targets in Gaussian Noise
  - Log Normal Targets in Gaussian Noise
  - Steady Targets in Log Normal Noise
  - Log Normal Targets in Log Normal Noise
  - Weibel Targets in Gaussian Noise
- Chi Square, Log Normal and Weibel Distributions have long tails
  - One more parameter to specify distribution
    - Mean to median ratio for log normal distribution
- When used
  - Ground clutter — Weibel
  - Sea Clutter — Log Normal
  - HF noise — Log Normal
  - Birds — Log Normal
  - Rotating Cylinder — Log Normal

RCS Variability for Different Target Models

Non-fluctuating Target

Swerling I/II

Swerling III/IV



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Fluctuating Target Single Pulse Detection : Rayleigh Amplitude

$$H_1: \; x = a e^{j\phi} + n$$
$$H_0: \; x = n$$

Rayleigh amplitude model
$$p(a) = \frac{2a}{\overline{\sigma}} \exp\left(-\frac{a^2}{\overline{\sigma}} \right)$$

Detection Test
$$z = |x| > T \quad H_1$$
$$|x| < T \quad H_0$$

Average SNR per pulse
$$\overline{\xi} = \frac{\overline{\sigma}}{\sigma_N^2}$$

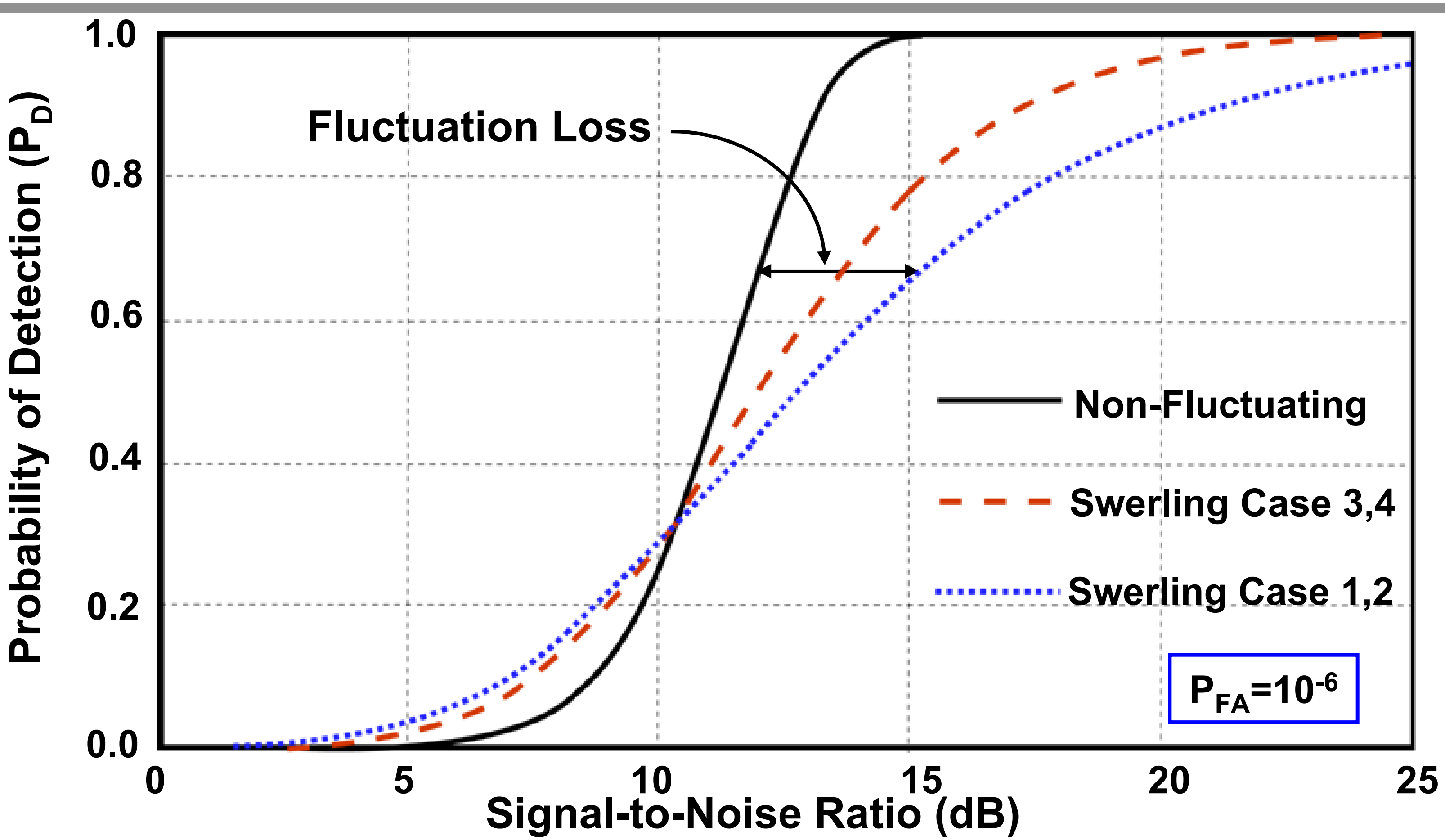
Probability of False Alarm (same as for non-fluctuating)
$$P_{FA} = \exp\left(-\frac{T^2}{\sigma_N^2} \right)$$

Probability of Detection Test
$$P_D = \int p(z > T \,|\, H_1, a)\, p(a)\, da\, dz$$

$$P_D = \exp\left(-\frac{T^2}{\sigma_N^2} \left(\frac{1}{1+\overline{\xi}} \right) \right)$$

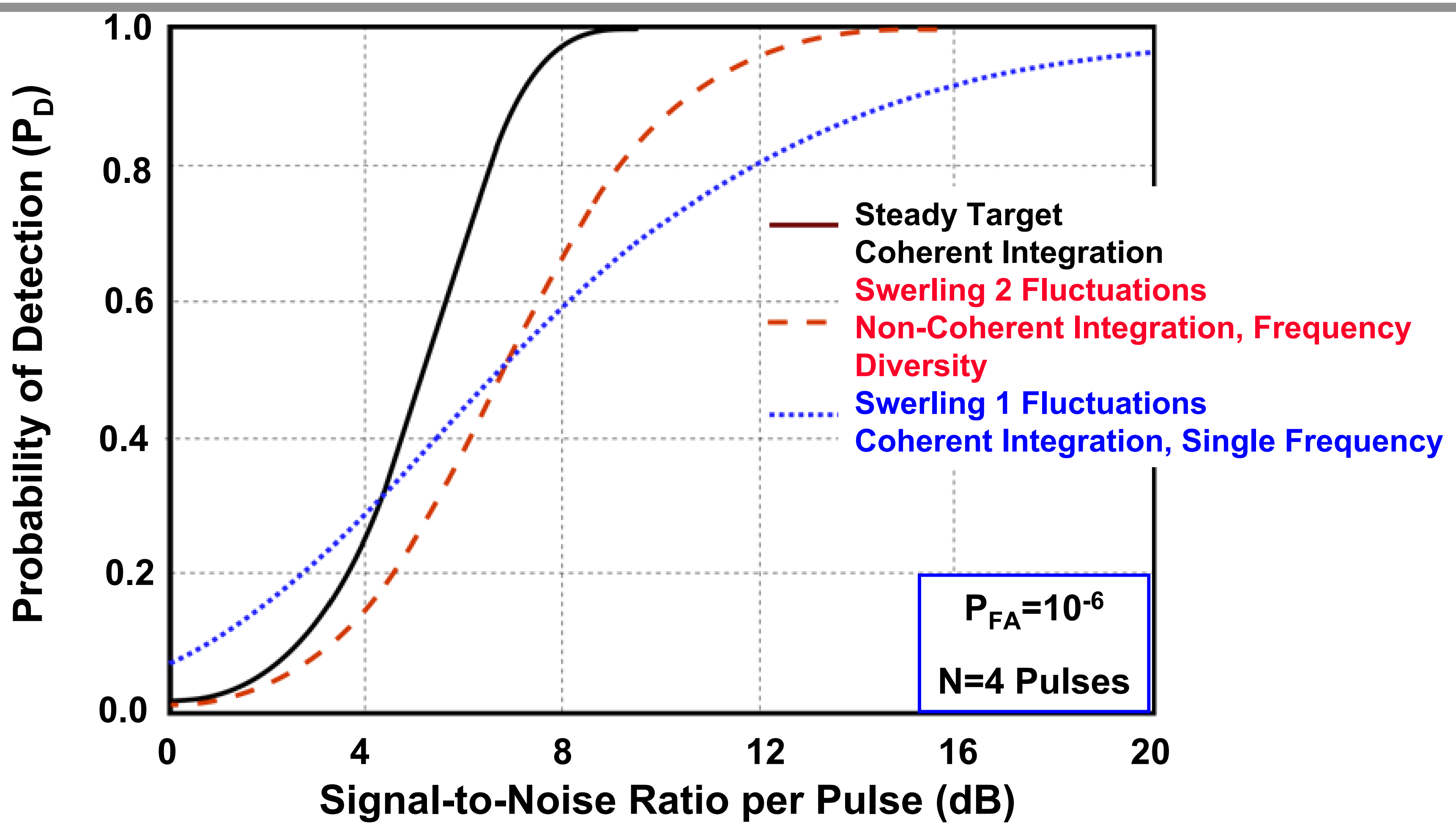
$$P_D = P_{FA}^{\left(\frac{1}{1+\overline{\xi}} \right)}$$

Fluctuating Target Single Pulse Detection



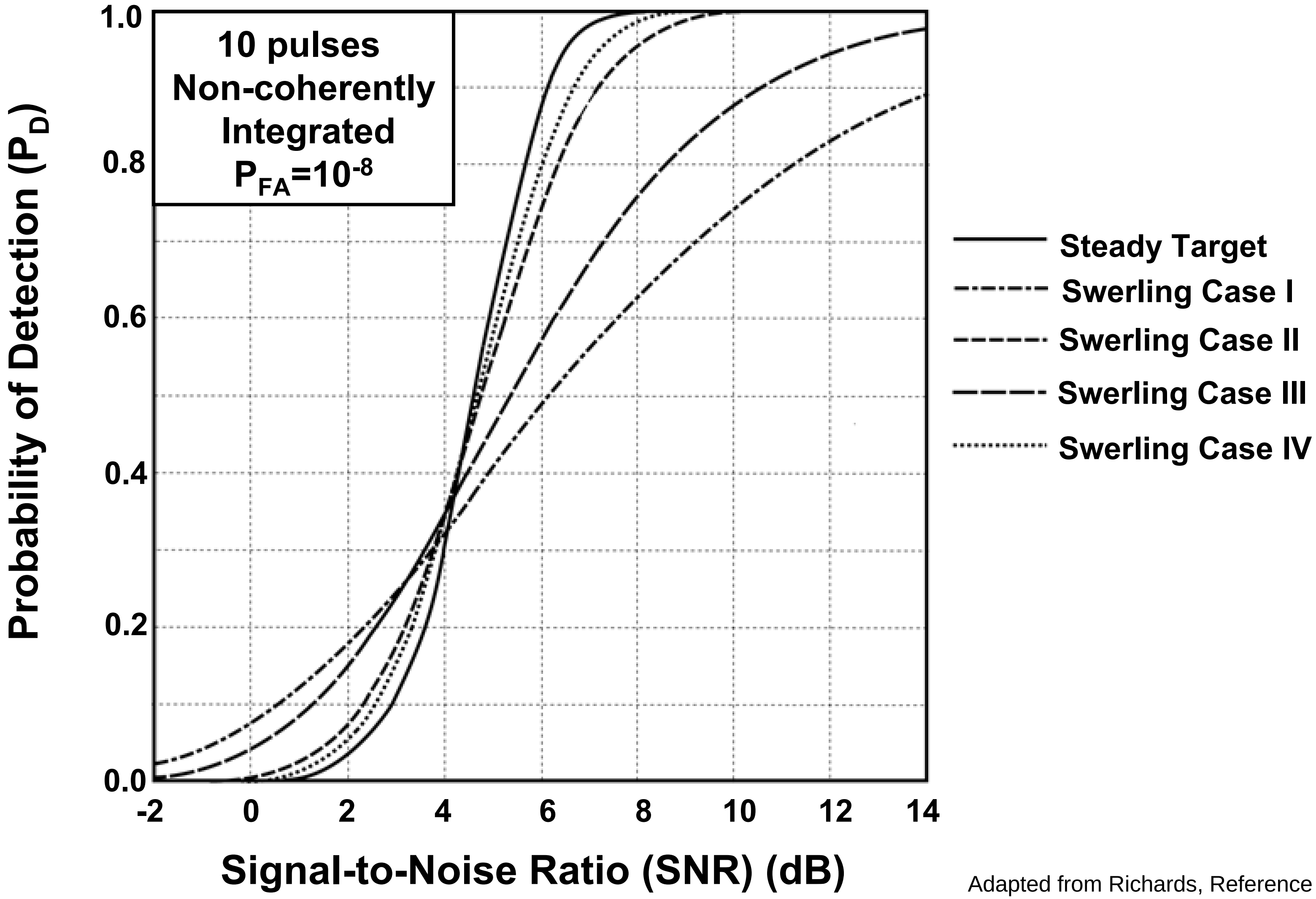
For high detection probabilities, more signal-to-noise is required for fluctuating targets.
The fluctuation loss depends on the target fluctuations, probability of detection, and probability of false alarm.

Fluctuating Target Multiple Pulse Detection



- In some fluctuating target cases, non-coherent integration with frequency diversity (pulse to pulse) can outperform coherent integration

Detection Statistics for Different Target Fluctuation Models



Adapted from Richards, Reference 7

Shnidman Empirical Formulae for SNR (for Steady and Swerling Targets)

- Analytical forms of SNR vs. P_D, P_{FA}, and Number of pulses are quite complex and not amenable to BOTE* calculations
- Shnidman has developed a set of empirical formulae that are quite accurate for most 1st order radar systems calculations:

$$K = \begin{cases} \infty & \text{Non-fluctuating target ("Swerling 0 / 5")} \\ 1, & \text{Swerling Case 1} \\ N, & \text{Swerling Case 2} \\ 2, & \text{Swerling Case 3} \\ 2N & \text{Swerling Case 4} \end{cases}$$

$$\alpha = \begin{cases} 0 & N \leq 40 \\ \frac{1}{4} & N > 40 \end{cases}$$

$$\eta = \sqrt{-0.8\ln\left(4P_{FA}\left(1-P_{FA}\right)\right)} + \text{sign}\left(P_D - 0.5\right)\sqrt{-0.8\ln\left(4P_D\left(1-P_D\right)\right)}$$

Adapted from Shnidman in Richards, Reference 7

* Back of the Envelope

Shnidman Empirical Formulae for SNR

(for Steady and Swerling Targets)

$$X_\infty = \eta\left(\eta + 2\sqrt{\frac{N}{2}} + \left(\alpha - \frac{1}{4}\right)\right)$$

$$C_1 = \left(\left(\left(17.7006\,P_D - 18.4496\right)P_D + 14.5339\right)P_D - 3.525\right)/\,K$$

$$C_2 = \frac{1}{K}\left(e^{27.31\,P_D - 25.14)} + \left(P_D - 0.8\right)\left(0.7\ln\left(\frac{10^{-5}}{P_{FA}}\right) + \frac{\left(2N - 20\right)}{80}\right)\right)$$

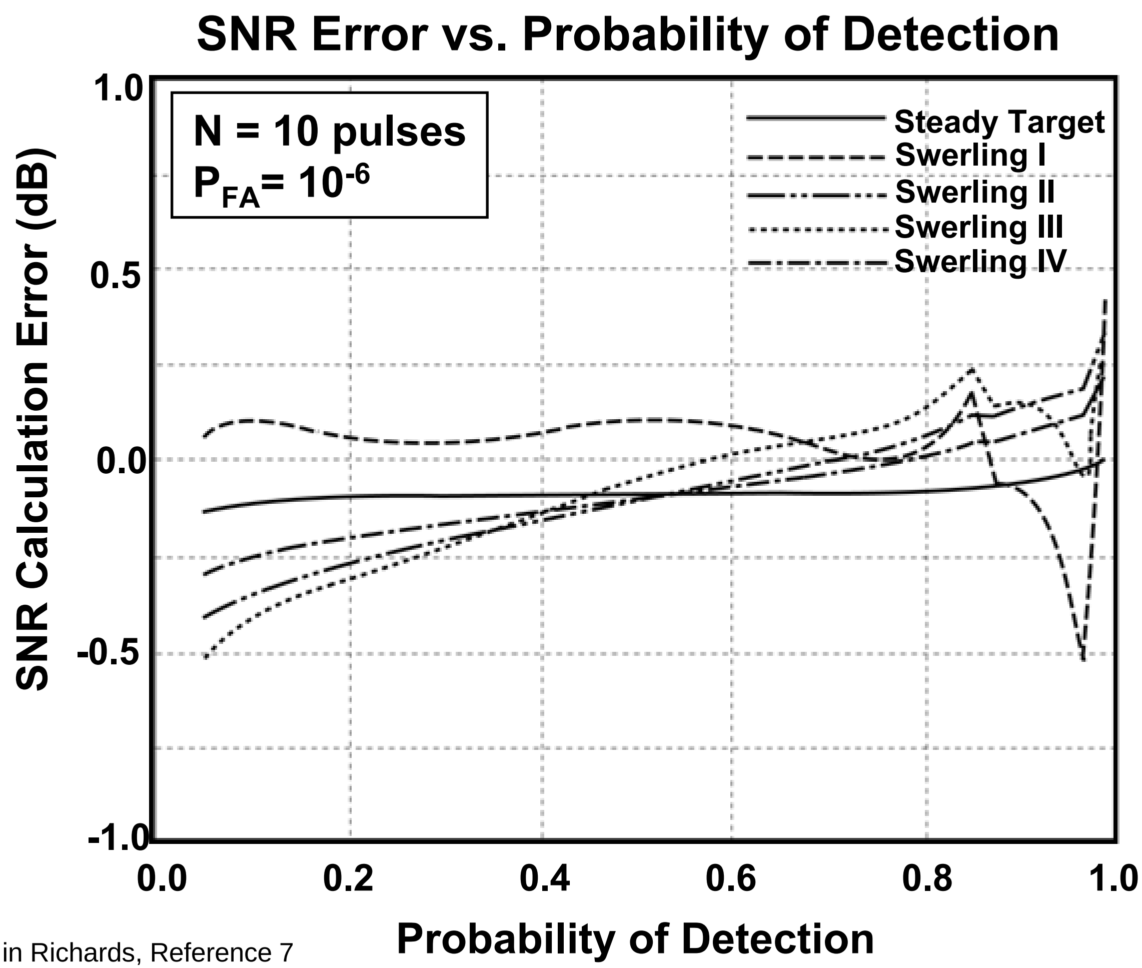
$$C_{dB} = \begin{cases} C_1 & 0.1 \le P_D \le 0.872 \\ C_1 + C_2 & 0.872 \le P_D \le 0.99 \end{cases} \qquad C = 10^{\frac{C_{dB}}{10}}$$

$$\text{SNR(natural units)} = \frac{C\,X_\infty}{N} \qquad \text{SNR}\left(\text{dB}\right) = 10\log_{10}(\text{SNR})$$

Adapted from Shnidman in Richards, Reference 7

Shnidman's Equation

- Error in SNR < 0.5 dB within these bounds
  - $0.1 \le P_D \le 0.99$ $10^{-9} \le P_{FA} \le 10^{-3}$ $1 \le N \le 100$



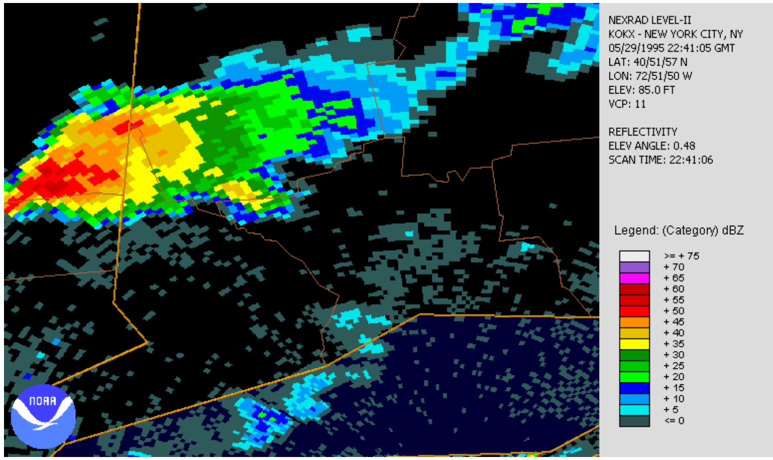
Adapted from Shnidman in Richards, Reference 7

Outline

- Basic concepts
- Integration of pulses
- Fluctuating targets
- Constant false alarm rate (CFAR) thresholding
- Summary

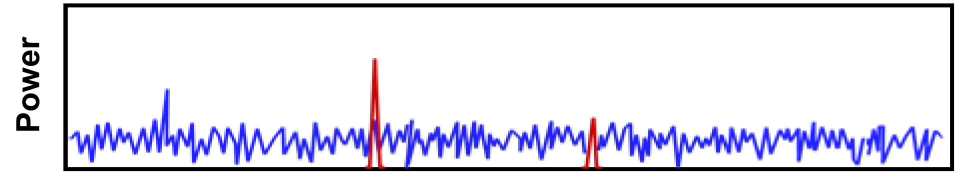
Practical Setting of Thresholds

Display, NEXRAD Radar, of Rain Clouds

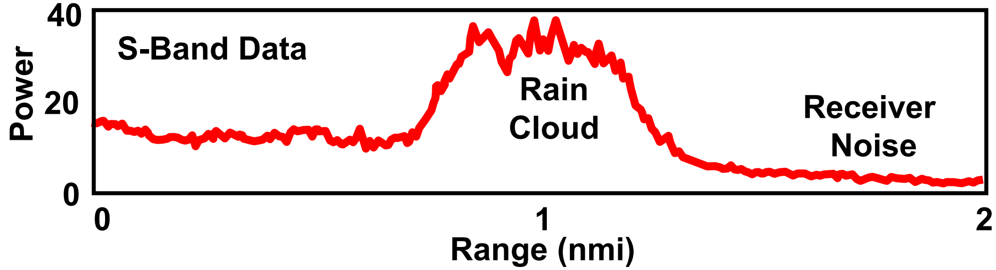


Courtesy of NOAA

Ideal Case- Little variation in Noise



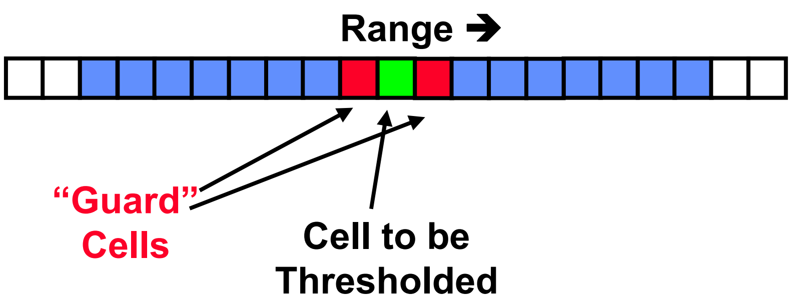
Rain Backscatter Data



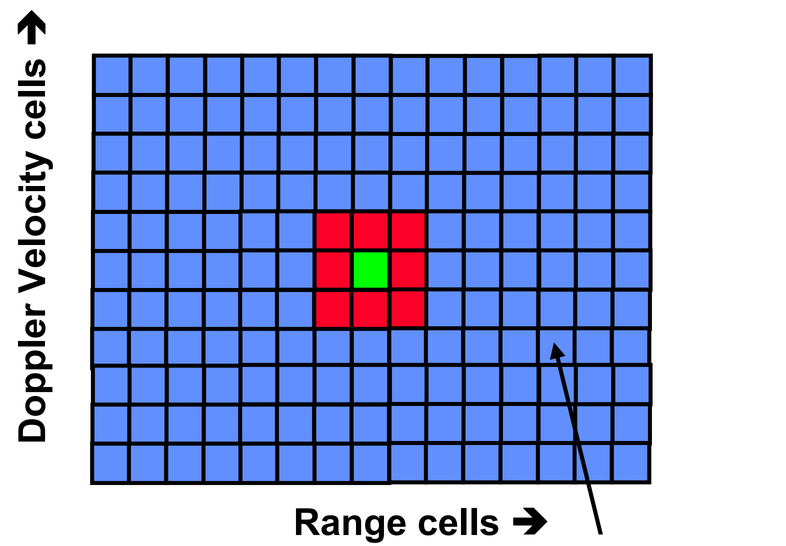
- **Need to develop a methodology to set target detection threshold that will adapt to:**
  - **Temporal and spatial changes in the background noise**
  - **Clutter residue from rain, other diffuse wind blown clutter,**
  - **Sharp edges due to spatial transitions from one type of background (e.g. noise) to another (e.g. rain) can suppress targets**
  - **Background estimation distortions due to nearby targets**

Constant False Alarm Rate (CFAR) Thresholding

CFAR Window – Range Cells



CFAR Window – Range and Doppler Cells

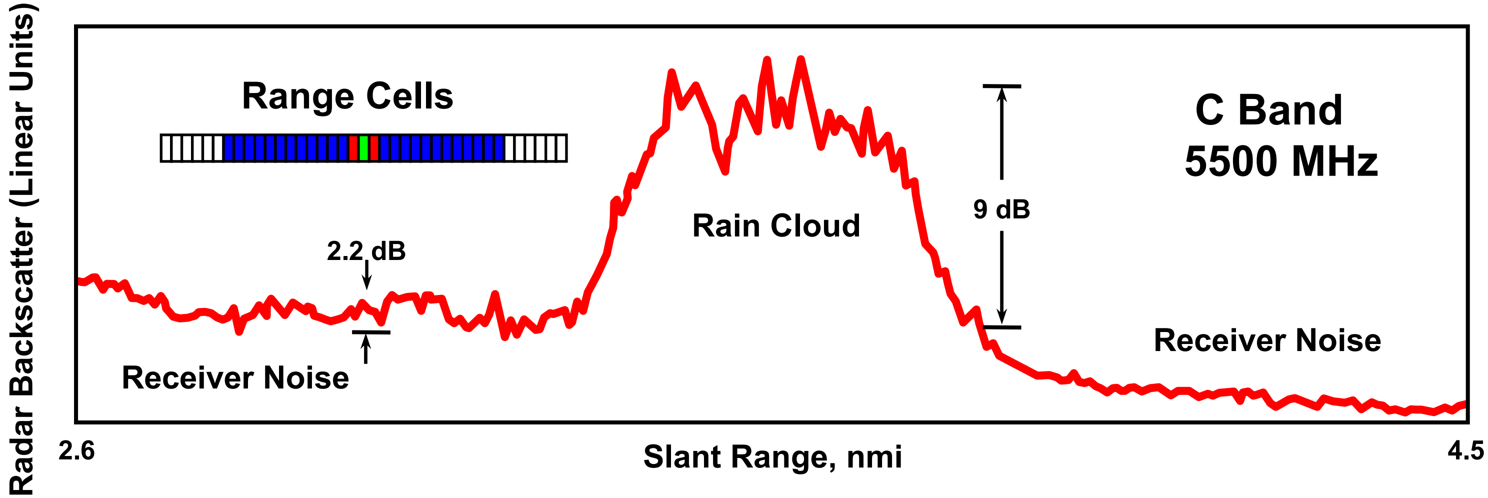


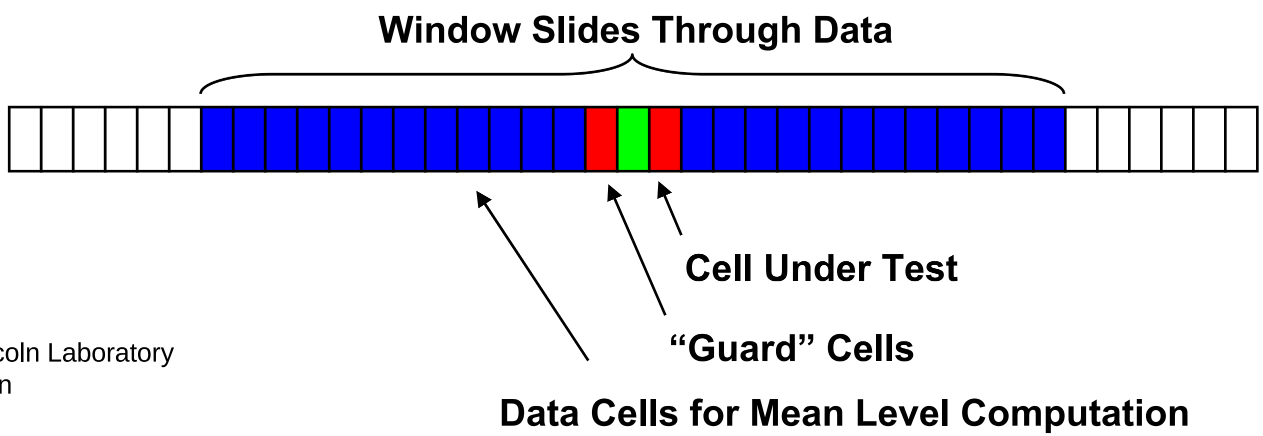
- Estimate background (noise, etc.) from data
  - Use range, or range and Doppler filter data
  - Set threshold as constant times the mean value of background

- Mean Background Estimate = $\frac{1}{N}\sum_{n=1}^{N} x_n$

Effect of Rain on CFAR Thresholding

Mean Level Threshold CFAR





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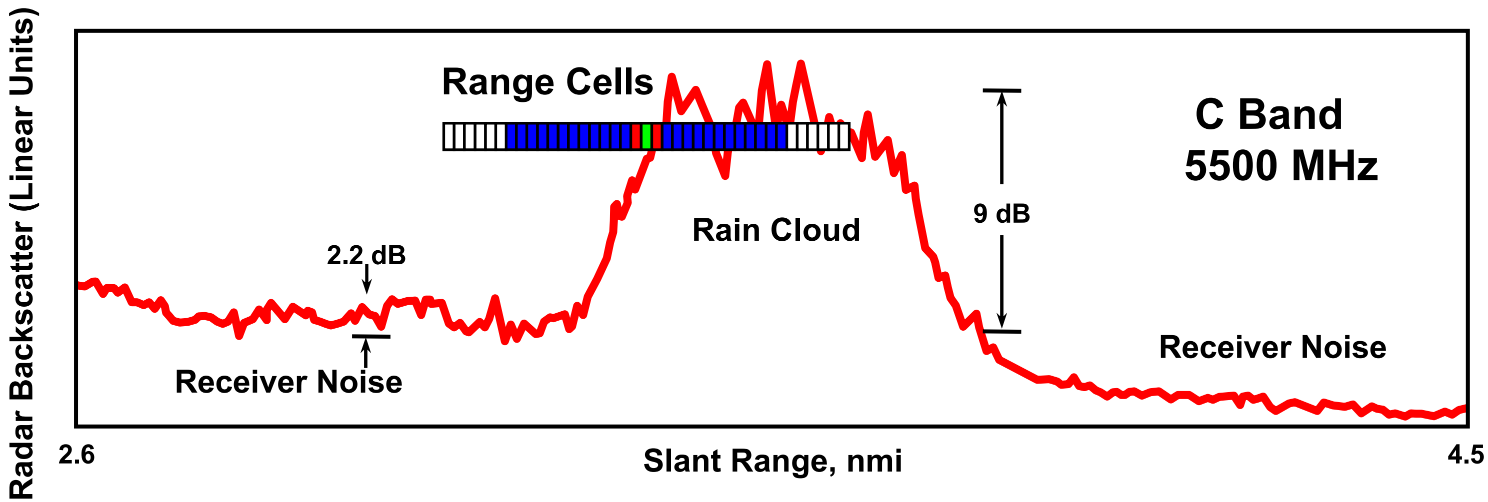
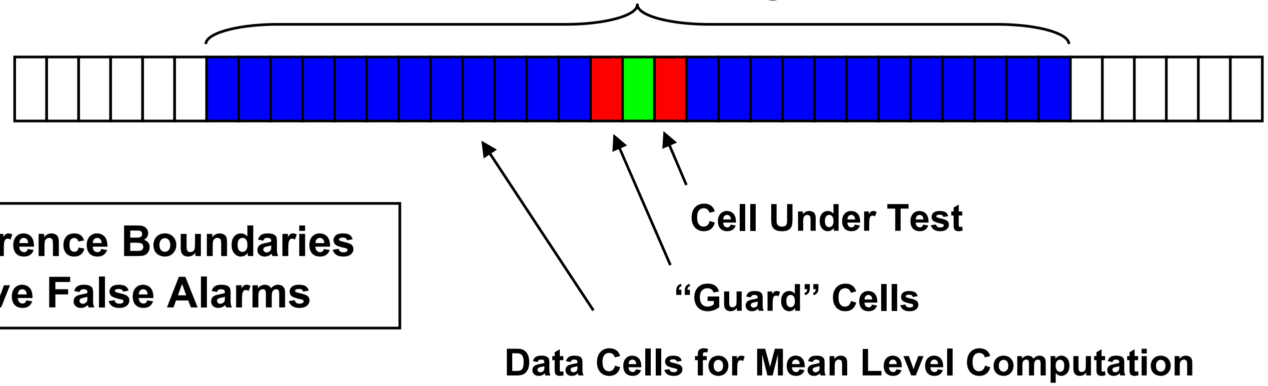
Mean Level Threshold CFAR



Window Slides Through Data



Sharp Clutter or Interference Boundaries Can Lead to Excessive False Alarms

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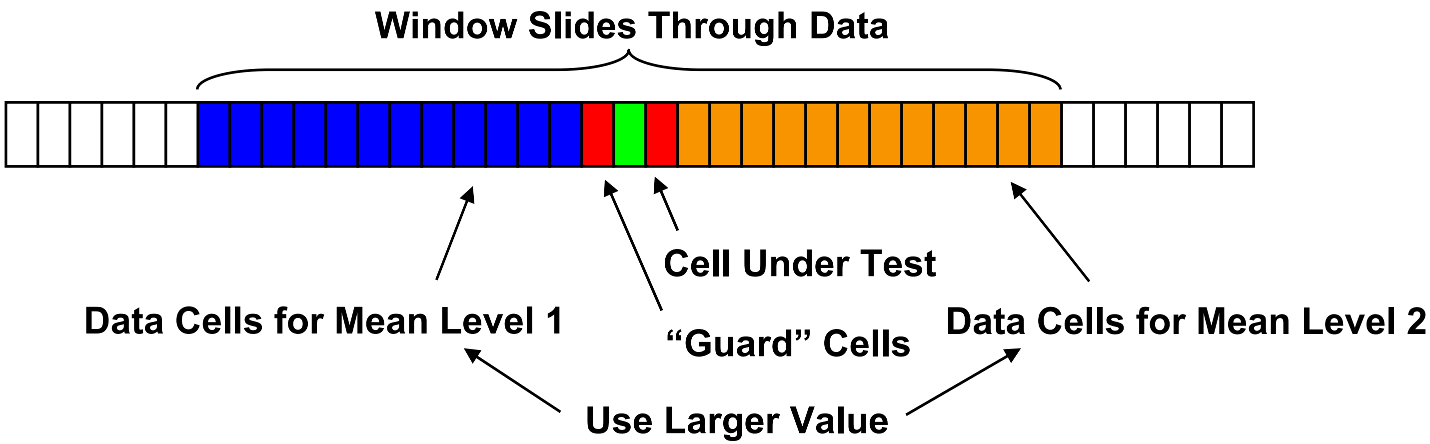
Greatest-of Mean Level CFAR

- Find mean value of N/2 cells before and after test cell separately
- Use larger noise estimate to determine threshold



- Helps reduce false alarms near sharp clutter or interference boundaries
- Nearby targets still raise threshold and suppress detection

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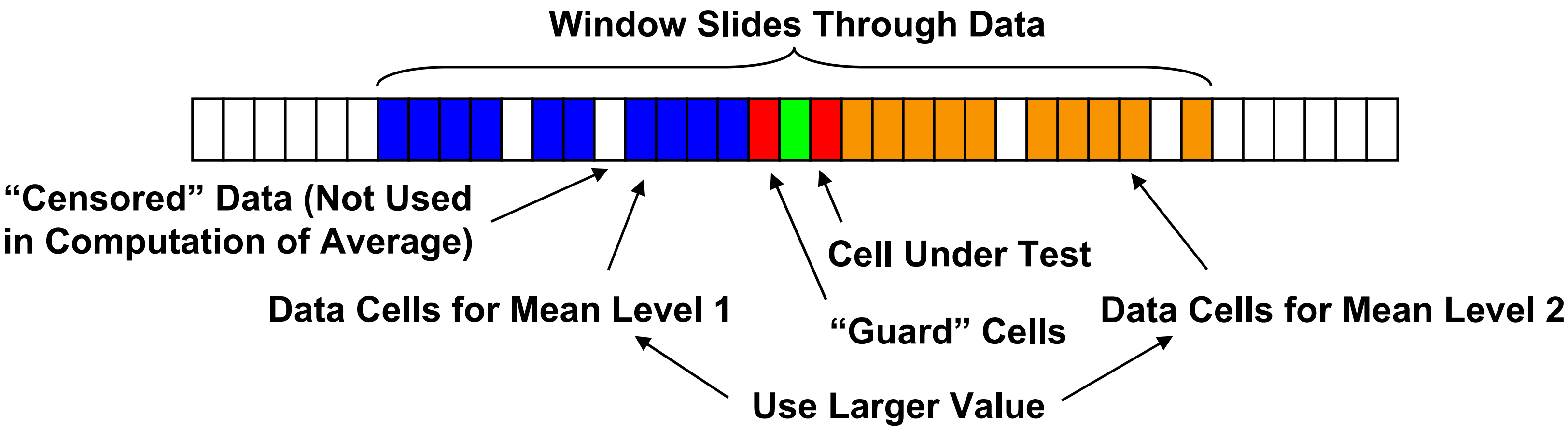
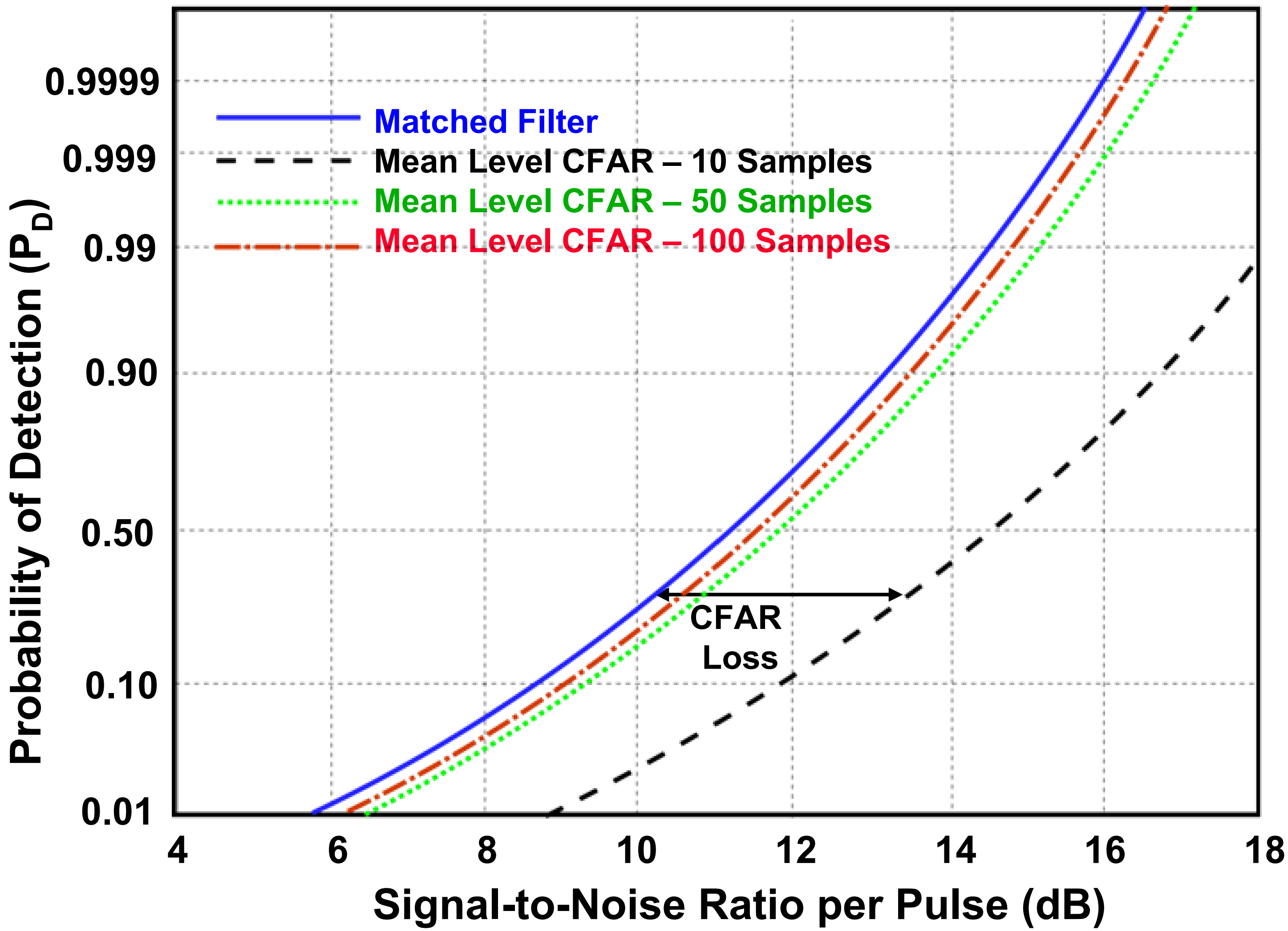
Censored Greatest-of CFAR

- Compute and use noise estimates as in Greatest-of, but remove the largest M samples before computing each average



- Up to M nearby targets can be in each window without affecting threshold
- Ordering the samples from each window is computationally expensive

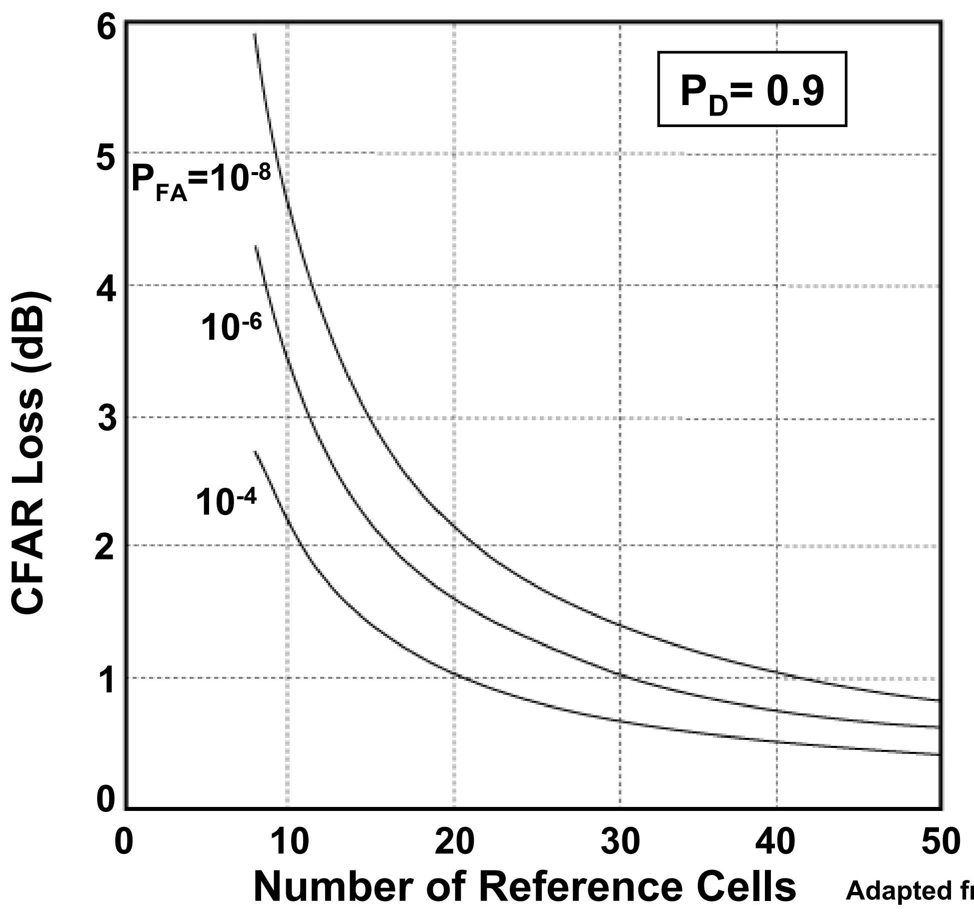
Mean Level CFAR Performance



$P_{FA}=10^{-6}$

1 Pulse

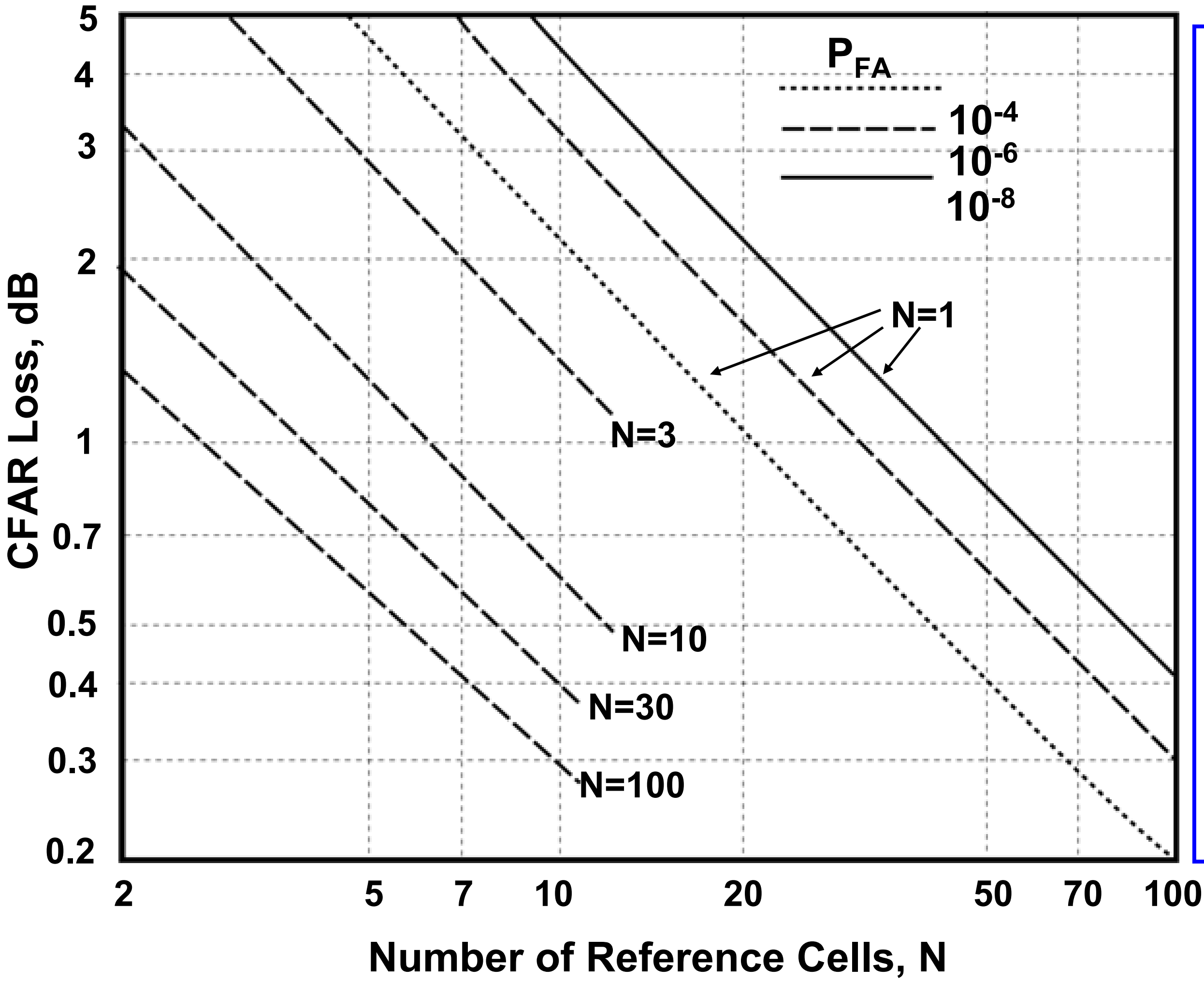
CFAR Loss vs. Number of Reference Cells



Adapted from Richards, Reference 7

The greater the number of reference cells in the CFAR, the better is the estimate of clutter or noise and the less will be the loss in detectability. (Signal to Noise Ratio)

CFAR Loss vs Number of Reference Cells



Adapted from Skolnik, Reference 1

For Single Pulse Detection Approximation

CFAR Loss (dB) = $-(5/N) \log P_{FA}$

Dotted Curve $P_{FA} = 10^{-4}$
Dashed Curve $P_{FA} = 10^{-6}$
Solid Curve $P_{FA} = 10^{-8}$

N = 15 to 20 (typically)

Since a finite number of cells are used, the estimate of the clutter or noise is not precise.

Summary

- Both target properties and radar design features affect the ability to detect signals in noise
  - Fluctuating targets vs. non-fluctuating targets
  - Allowable false alarm rate and integration scheme (if any)
- Integration of multiple pulses improves target detection
  - Coherent integration is best when phase information is available
  - Noncoherent integration and frequency diversity can improve detection performance, but usually not as efficient
- An adaptive detection threshold scheme is needed in real environments
  - Many different CFAR (Constant False Alarm Rate) algorithms exist to solve various problems
  - All CFARs algorithms introduce some loss and additional processing

References

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3. DiFranco, J. V. and Rubin, W. L., *Radar Detection*, Artech House, Norwood, MA, 1994.
4. Whalen, A. D. and McDonough, R. N., *Detection of Signals in Noise,* Academic Press, New York, 1995.
5. Levanon, N., *Radar Principles*, Wiley, New York, 1988
6. Van Trees, H.*, Detection, Estimation, and Modulation Theory, Vols. I and III, Wiley, New York,* 2001
7. Richards, M.*, Fundamentals of Radar Signal Processing,* McGraw-Hill, New York, 2005
8. Nathanson, F., *Radar Design Principles*, McGraw-Hill, New York, 2rd Ed., 1999.

Homework Problems

- From Skolnik, Reference 1
  - Problems 2.5, 2.6, 2.15, 2.17, 2.18, 2.28, and 2.29
  - Problems 5.13 , 5.14, and 5-18