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# A MODEL FOR THE CALCULATION OF SOLAR GLOBAL INSOLATION

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**Abstract**—A theoretical model is described that is designed to give the total global insolation falling on the earth's surface and the transmission of the atmosphere. It is compared to a model by Braslau and Dave[1] and found to agree to within a few percent in all cases. Climatogical values of total pricipitable water, turbidity, and surface albedo are required as the model inputs, and the sources of these data are described. The model has been applied to 26 stations in the National Weather Service (NWS) pyranometer network, where measured true solar noon atmospheric transmission values are available, as part of the NOAA program to rehabilitate the old pyranometer observations. For three of these stations where reliable true solar noon irradiance and transmission values are available, the model calculations and observations are compared. At 18 locations the calculated and measured daily mean insolation values are compared for clear days. At one location (Boulder, Colorado) calculated and measured radiation climatologies for all weather conditions are compared. In all comparisons the model and observations differ by no more than 2.7 per cent, which is within the experimental accuracy (± 5 per cent) of the pyranometers. Possible sources of errors are discussed.

## 1. INTRODUCTION

As interest in applications of solar energy increases, accurate values of solar insolation at the earth's surface are needed. The National Weather Service (NWS) has been gathering global insolation data at a number of locations in the United states since the early 1950s. Unfortunately the data gathered by the NWS pyranometer network have not always been completely satisfactory for a number of reasons (see Riches *et al.*[2] for details). There is a need to improve both the data quality at individual stations and the consistency of measurements among all stations. In 1976 the Energy Research and Development Administration (ERDA) asked the National Oceanic and Atmospheric Administration (NOAA) to rehabilitate the NWS network measurements. The work described in this paper is one aspect of the resulting effort to improve the quality of the NWS measurements.

Atmospheric transmission is defined as the fraction of the extraterestrial solar radiation falling on a horizontal plane at the earth's surface. Measured values of atmospheric transmission on clear days at true solar noon are used for quality control[3]. These values are adjusted to agree with theoretical calculations of the true solar noon atmospheric transmission. Obvious errors in the measurements, such as discontinuities associated with instrument changes or long-term drifts due to instrument degradation, are revealed by this technique.

This paper describes the theoretical model for calculating true solar mean atmospheric transmission values that was used in the rehabilitation of the NWS insolation measurements. Transmission values are calculated for each of 26 NWS stations, for each day of a climatologically mean year, and may be used either as standard values to correct the measured values to or as a basis for such standard values. True solar noon insolation values, the clear day mean insolation values, and the radiation climatology at Boulder, Colorado, are calculated in addition to the true solar noon atmospheric transmission values, and these are compared to observations as a check on the validity of the model. The model values of insolation are always within 2.7 per cent of the measured values when they are on an absolute radiation scale and therefore within the ± 5 per cent accuracy[4] of the measurements. Model calculations are also compared to the theoretical model values given by Braslau and Dave[1].

## 2. THE MODEL FOR CLEAR DAY GLOBAL INSOLATION VALUES

The solar radiation model used in the calculation of the global irradiance values is a further development upon the two stream models of Katayama[5], Sasamori *et al.*[6] and Hoyt[7]. The major difference in the need model is the treatment of Rayleigh and dust scattering.

If one considers an air column through the entire atmosphere of unit cross-section parallel to the solar zenith angle, the relative depletion of the incident solar radiation by scattering and by absorption in the air column are known respectively as the scattering ratio (reflectivity) and absorption ratio (absorptivity). The following notation will be used for the scattering and absorption ratios:

$s_a$, $s_d$ Scattering ratios for air and dust respectively.
$\alpha_i$ Absorption ratios for water vapor ($i = 1$), carbon dioxide ($i = 2$), ozone ($i = 3$), oxygen ($i = 4$), and dust ($i = 5$).

The absorption ratios for the gaseous components of the atmosphere are approximated using empirical relations based on Yamamoto[8] for water vapor and oxygen, on Burch, Gryvnak and Williams[9] for carbon dioxide, and on Manabe and Strickler[10] for ozone. The formulas have appeared in earlier studies by Sasamori *et al.*[6] and Hoyt[7] and are reproduced for convenience as follows:

For water vapor:

$$\alpha_1 = 0.110\,(u_1 + 6.31\times10^{-4})^{0.3} - 0.0121 \qquad (1)$$

where $u_1$ is the total pressure-corrected precipitable water ($\mathrm{gcm^{-2}}$) in the path.

For carbon dioxide:

$$\alpha_2 = 0.00235\,(u_2 + 0.0129)^{0.26} - 7.5\times10^{-4} \qquad (2)$$

where $u_2$ is the pressure-corrected path length of carbon dioxide (cm at STP; $u_2 = 126$ cm for air mass equals one).

For ozone:

$$\alpha_3 = 0.045\,(u_3 + 8.34\times10^{-4})^{0.38} - 3.1\times10^{-3} \qquad (3)$$

where $u_3$ is the ozone path length (cm at STP).

For oxygen:

$$\alpha_4 = 7.5\times10^{-3}(m^*)^{0.875} \qquad (4)$$

where $m^*$ is the pressure-corrected air mass, which is used in the equations below.

The scattering ratio for pure air is given by

$$s_a = 1 - f(m^*)^{m^*} \qquad (5)$$

where $f(m^*)$ is a function of air mass given in Table 1. The function $f(m^*)$ was calculated using Elterman's[11] values of the Rayleigh optical depth and the solar spectrum of Labs and Neckel[12]. The generally increasing values of $f(m^*)$ as $m^*$ increases reflect the initial depletion of the shorter wavelength radiation and therefore the decreasing fractional scattering per unit air mass for the longer path lengths.

The scattering ratio for dust is given by:

$$s_a = 1 - g(\beta)^{m^*} \qquad (6)$$

where $\beta$ is the Angstrom turbidity coefficient or aerosol optical depth at 1 $\mu$m. The aerosol optical depth, $\tau_D$, as a function of wavelength is given by

$$\tau_D = \frac{\beta}{\lambda^{\alpha}} \qquad (7)$$

Table 1. The variation of the function $f(m^*)$ in the expression for the scattering ratio for pure air with air mass $m^*$

| m* | f(m*) |
|---|---|
| 0.0 | 1.000 |
| 0.5 | 0.909 |
| 1.0 | 0.917 |
| 1.5 | 0.921 |
| 2.0 | 0.925 |
| 2.5 | 0.929 |
| 3.0 | 0.932 |
| 3.5 | 0.935 |
| 4.0 | 0.937 |

where $\lambda$ is the wavelength in $\mu$m. The wavelength exponent $\alpha$ is equal to one in this study. The function $g(\beta)$ was calculated using the above form for the aerosol optical depths and the extraterrestrial solar spectrum of Labs and Neckel[12]. Values of $g(\beta)$ are tabulated in Table 2. Formulation of the scattering ratio in this way allows the Volz sunphotometer network values (e.g. Flowers *et al.*[13]) to be incorporated into the calculations.

Table 2. The variation of the function $g(\beta)$ in the expression for the scattering ratio for dust with aerosol optical depth at 1 $\mu$m, $\beta$

| β | g(β) |
|---|---|
| 0.0 | 1.000 |
| 0.02 | 0.972 |
| 0.04 | 0.945 |
| 0.06 | 0.919 |
| 0.08 | 0.894 |
| 0.10 | 0.870 |
| 0.12 | 0.846 |
| 0.14 | 0.824 |
| 0.16 | 0.802 |
| 0.18 | 0.780 |
| 0.20 | 0.758 |
| 0.24 | 0.714 |
| 0.28 | 0.670 |
| 0.32 | 0.626 |

The absorption ratio for dust is given by:

$$\alpha_5 = (1-\omega)g(\beta)^{m^*} \qquad (8)$$

where $\omega$ is the albedo for single scattering and is taken to equal 0.95.

At any solar zenith angle, the direct solar radiation is given by:

$$I = S_0 \cos Z\left\{1 - \sum_{i=1}^{5}\alpha_i\right)(1-s_a)(1-s_d) \qquad (9)$$

and the diffuse solar radiation is given by:

$$D = S_0 \cos Z\left(1 - \sum_{i=1}^{5}\alpha_i\right)(0.5s_a + 0.75s_d) \qquad (10)$$

where $S_0$ is the extraterrestrial solar total irradiance (1353 $\mathrm{W\,m^{-2}}$ at one astronomical unit), $\cos Z$ is the cosine of the solar zenith angle, and the other symbols have been explained previously. One half of the Rayleigh scattered radiation and three-fourths of the dust scattered radiation is in the forward direction. There is an additional component of diffuse solar radiation arising from the interreflection radiation between the earth's surface and the atmosphere. Letting $R_\uparrow$ equal the quantity of solar radiation reflected from the earth's surface on the first pass of the radiation through the atmosphere, the amount of radiation $G_\downarrow$ reflected back to the earth's surface is:

$$G_\downarrow = R^\uparrow\left(1 - \sum_{i=1}^{5}\alpha_i'\right)(0.5s_a' + 0.75s_d') \qquad (11)$$

where $s_a'$ and $s_d'$ are evaluated for 1.66 times the minimum optical air mass ($P/P_0$) to account for the Lambert reflectivity of the surface, and the absorption are evaluated for air mass values of $m^* + 1.66\,P/P_0$ where $P$ is station surface pressure and $P_0$ is the sea level surface pressure of 1013.25 mb. These values of the air mass are chosen to account for the depletion of radiation by absorption initially passing through the atmosphere and to account for the Lambert reflectivity of the surface.

The total global irradiance at any one instance therefore is:

$$G = I + D + G_{\downarrow}. \qquad (12)$$

The transmission of the atmosphere $T$ is:

$$T = G/S_0 \cos Z. \qquad (13)$$

To calculate the global irradiance over an entire day the above quantities are summed in 15 min increments from sunrise to sunset.

### 3. THE MODEL FOR AN OVERCAST SKY

A model similar to the above is used for the case of the sky being totally overcast.

Cloud types are divided into six different groups: stratus and stratocumulus (St, Sc); cumulus (Cu); nimbostratus (Nb); cumulonimbus (Cb), altostratus and altocumulus (Ac, As), cirrus, cirrostratus, and cirrocumulus (Ci, Cs, Cc). Absorptivities of the clouds and vapor are given by eqn 1 with the effective path length of water being:

$$u_1 = m^* u_1' + \eta u_1'' + 1.66 u_1''' \qquad (14)$$

where $u_1'$, $u_1''$ and $u_1'''$ are the total pressure corrected water vapor amounts above the cloud, in the cloud, and beneath the cloud, assuming the cloud to be completely saturated. $\eta$ has a value of 8.1 based on Fritz[14] and accounts for the water droplets in the clouds. Absorptivities by other components of the atmosphere are given by eqns (2–4 and 8). All radiation beneath the clouds is diffuse radiation.

Transmission by the clouds is taken to be:

$$T = t_i/[(Z_u - Z_l)m^*]^{0.33} \qquad (15)$$

where $t_i$ is a constant for each cloud type given in Table 3, $Z_u$ and $Z_l$ are the upper and lower cloud heights in km and $m^*$ is the air mass. The approximation is based upon the work of Haurwitz[15] and a previous radiation budget study by Sasamori *et al.*[6].

Table 3. Parameter used in model calculations of transmission of cloud, $t_i$(km$^{1/3}$)

| Cloud type | $t_i$ (Km$^{1/3}$) |
|---|---|
| St, Sc | 0.125 |
| Cu | 0.320 |
| Nb | 0.200 |
| Cb | 0.200 |
| As, Ac | 0.470 |
| Ci, Cs, Cc | 1.000 |

The transmitted radiation beneath the clouds is corrected for multiple scattering between the clouds and the surface by the formula[16]:

$$D = D'/(1 - \alpha d) \qquad (16)$$

where $D$ is the actual downward transmitted diffuse radiation at the earth's surface, $D'$ is the transmitted radiation at the earth's surface after the first pass through the atmosphere calculated by the method above, $\alpha$ is the surface albedo for diffuse radiation, and $d$ is the cloud albedo which is taken to equal one minus the transmitted radiation. This formulation corrects for multiple scattering between cloud and ground and is particularly important in the arctic regions whre whiteouts occur. Effectively, the clouds become more transparent.

### 4. COMPARISON OF MODEL RESULTS TO THOSE OF BRASLAU AND DAVE[1]

There are many models described in the literature for calculating the solar global irradiance at the earth's surface (e.g. Paltridge and Proctor[17] or Albrecht[18]). The model described in the paper by Braslau and Dave[1] seems to be the most detailed and thorough and for that reason the results of this model are compared to the result obtained by Braslau and Dave.

Braslau and Dave (henceforth $B/D$) have compiled atmospheric transmission values using a radiative transfer model with a model standard atmosphere, several solar zenith angles, and several underlying surface albedos. The transmission values of that model can be compared to the transmission values of this model for equivalent conditions. For 2.925 cm of precipitable water (2.383 cm of pressure-corrected precipitable water), 0.318 cm of ozone, and $\beta$ equal to 0.04, the present model was run for solar zenith angles of 0, 30 and 60° and for surface albedos of 0.0, 0.1, 0.2, 0.3, 0.4, 0.6 and 0.8. The atmospheric transmission values for the two models are tabulated in Table 4.

For low surface albedos the present model gives 0.5–1.3 per cent less transmission than $B/D$'s model probably because of the improper treatment of the overlapping absorption by water vapor and carbon dioxide in this model and because $B/D$ treat radiation to 2.5 $\mu$m only and thus will obtain overall higher transmission values. For high surface albedos the present model gives lower transmission values because only the first order reflections between the earth's surface and the atmosphere are taken into account for a clear sky and thus the diffuse irradiance is underestimated. Since, most often, surface albedos in the range 0.1–0.2 prevail, this underestimation is not a serious problem. The ratio of transmission value at solar zenith angles 60° to that at 0° are nearly identical for both models indicating the daily curve of global insolation vs time will be properly reproduced in the two models although they may be offset relative to one another. The transmission values calculated in this model are usually no more than 1–2 per cent lower than the values calculated using the more

Table 4. Atmospheric transmission values for present model ($P$) with $\beta = 0.04$ and Braslau and Dave model ($B/D$) with average aerosols for three zenith angles and seven surface albedos

| | Z=0° | | Z=30° | | Z=60° | |
|---|---|---|---|---|---|---|
| albedo | P | B/D | P | B/D | P | B/D |
| 0.0 | 0.818 | 0.825 | 0.804 | 0.811 | 0.743 | 0.748 |
| 0.1 | 0.823 | 0.833 | 0.809 | 0.819 | 0.748 | 0.755 |
| 0.2 | 0.828 | 0.841 | 0.814 | 0.827 | 0.752 | 0.762 |
| 0.3 | 0.834 | 0.850 | 0.819 | 0.836 | 0.757 | 0.770 |
| 0.4 | 0.839 | 0.859 | 0.824 | 0.845 | 0.761 | 0.778 |
| 0.6 | 0.850 | 0.878 | 0.835 | 0.863 | 0.771 | 0.794 |
| 0.8 | 0.860 | 0.899 | 0.845 | 0.884 | 0.778 | 0.813 |

detailed radiative transfer model at a typical midlatitude location.

## 5. COMPARISON OF THE MODEL TO OBSERVATION

Since the initial impetus of this study was to produce true solar noon atmospheric transmission values to which the NWS pyranometer measurements could be corrected, it is important to determine how accurate the calculations are. Furthermore, the measurement of the observed spring maximum in atmospheric transmission needs a theoretical explanation.

The model is applied to the calculation of the global insolation and atmospheric transmission at true solar noon for 26 locations in the contiguous United States, to daily mean values of insolation on clear days only at 18 locations, and to a complete radiation climatology at Boulder, Colorado. These calculations are discussed and compared to the available observations in the subsections below.

### *True solar noon transmission and irradiance values*

As part of the effort to rehabilitate the National Weather Service (NWS) pyranometer solar radiation values, the true solar noon atmospheric transmission values were calculated for each day of the year for 26 locations in the contiguous United States[19]. The true solar noon values are used in the data rehabilitation following a suggestion by Case[3] that this parameter is useful in the quality control of the data. Climatological values of total precipitable water, total ozone, turbidity, and surface albedo are required as input parameters to the model. Clear days without clouds are the only days for which observations and calculations are considered in this section.

Latitudinal and seasonal values of total ozone based on data from *Ozone Data of the World* (e.g. Hoyt[7]) are used to calculate the total ozone amounts for each day at each station by linear interpolation. Total precipitable water amounts are derived from radiosonde values. Monthly mean values for each station to 500 mb were supplied by Korshover[20] and converted to monthly mean values on clear days only to 250 mb by multiplying by a factor of 0.93[21]. To determine the total precipitable water to 250 mb, the values to 500 mb were multiplied by 1.148. To convert these values to those for clear days only, they were multiplied by 0.81. The product of these two factors is 0.93. Turbidity values are derived from published data[22–25] when available for the station in question or either from a nearby station or the values of Flowers *et al.*[13]. These climatological monthly mean values of total precipitable water and turbidity have been tabulated for 26 stations by Hoyt[19].

Following the review of surface albedo data of Kondratyev[26], summertime surface albedos are taken to be 0.14 for forested and city regions, 0.18 for steppe or dry regions, and 0.28 for desert regions. Winter snow-covered regions are taken to have an albedo of 0.66. Based upon the number of days with a surface temperature below freezing and the amount of snow per month[27], the number of days of snow cover per month were estimated. A weighting using the summertime and wintertime surface albedos and the fraction of time that the surface was snow-covered was made to determine the monthly mean values of the surface albedo. The resultant climatological mean values for 26 stations are listed by Hoyt[19]. Uncertainties in the turbidity and surface albedo climatologies are major sources of uncertainty in the present calculations, contributing to an uncertainty in the transmission values of about ±0.5 per cent from the turbidity values and about ±1 per cent from surface albedo values in most cases. Uncertainties in the total precipitable water may give rise to another systematic error of about 1 per cent or less in the atmospheric transmission values.

The transmission values at true solar noon on clear days for 26 stations are tabulated in the publication of Hoyt[19]. A sample plot of the typical transmission values for the course of a year is shown in Fig. 1 for Boston, Massachusetts. The spring maximum in transmission occurs because the true solar noon air mass values are becoming smaller but the total precipitable water is not increasing rapidly enough to cause a decrease in transmission. By midsummer the amount of water vapor increases sufficiently to cause a minimum in transmission. A deeper minimum occurs in the winter when the true solar noon air mass values reach their maximum values. A slight maximum in transmission may occur in the autumn between these two minimums. Most stations follow a pattern of this nature, but the secondary maximum in transmission does not always occur.

The annual amplitudes between maximum and minimum transmission values vary between 3.1 per cent at Albuquerque, New Mexico and 8.0 per cent at Seattle, Washington. The more northerly stations generally have the larger annual amplitudes because of the influence of the larger variation in air mass.

A comparison of the true solar noon calculations to the observations is summarized in Tables 5–7. Observations are available for Albuquerque, Bismarck and Raleigh using The Eppley Laboratory, Inc., Model 2

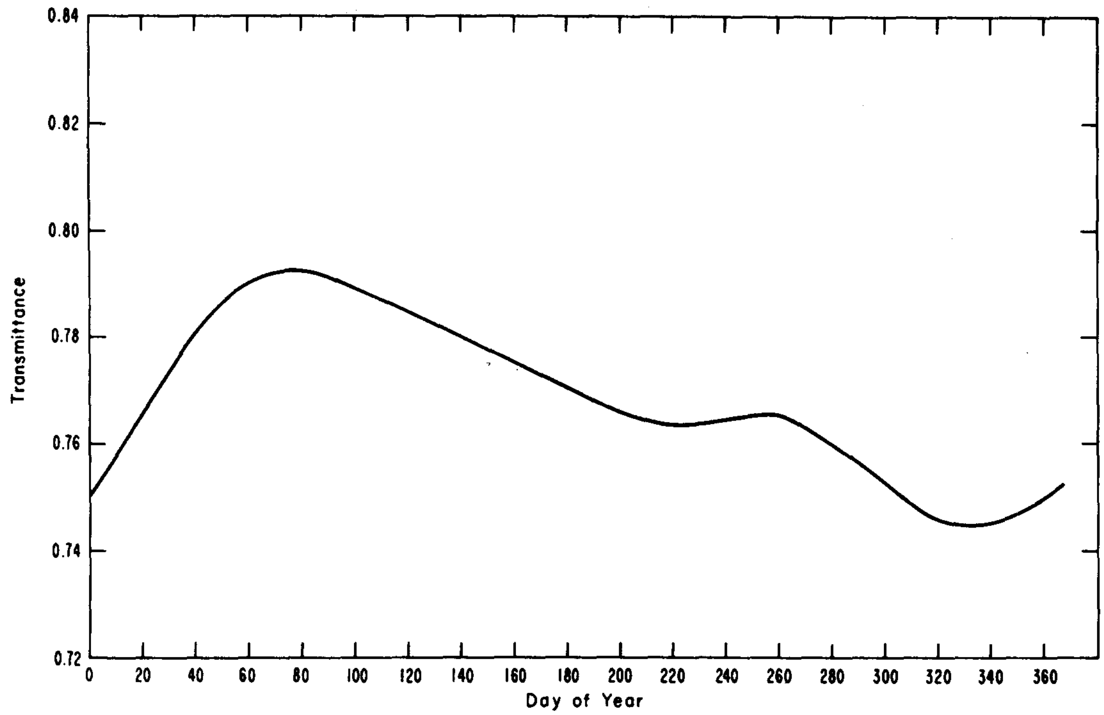


Fig. 1. Transmission at true solar noon for global radiation on a horizontal surface at Boston, Massachusetts, as a function of the day of the year. The general shape of this curve is typical for many stations.

pyranometers during the years 1972–74[20]. These observations have been corrected for any calibration errors and are on the radiation scale defined by the PACRAD (Primary Absolute Radiometer[29]). Thus, these observations of irradiances are some of the best now available. The measured transmission values are also tabulated for a solar constant equal to 1353 W m$^{-2}$[29]. The calculated transmission values are independent of the solar constant value.

The calculated irradiances exceed the measured values by 2.3 per cent at Albuquerque, 1.7 per cent at Bismarck, and 1.0 per cent at Raleigh on an annual average basis and averages 1.7 per cent for the three locations. The atmospheric transmission values follow a similar pattern. In general, the difference between the model calculations and observations is greatest in the autumn and early winter. The differences may be explained by one or more of the following reasons: (1) The model calculations may give atmospheric transmission values which are too high because of errors in the climatology such as an underestimation of the total precipitable water or overestimation of the surface albedo. (2) In the autumn and early winter, the Model 2 pyranometers may be underestimating the irradiance values because of the cosine response at the higher solar zenith angles which occur during the winter months. Because the present model compares favorably with *B/D*'s model at high solar zenith angles, the model calculations are not likely to be the causes of these differences. (3) The effect of the temperature response of the pyranometers has not been removed from the observations so this effect may be another source of error. It is found that the differences between calculations and observations cannot be explained by fitting them to linear regression equations with total precipitable water or air mass as independent variables. These regression equations are no better in reducing the residual deviation than assuming the differences are constants.

Whatever the causes of the differences, they are less than the $\pm 5$ per cent accuracy of the instruments[4]. On this basis and in concurrence with Paltridge and Proctor[17], it is difficult to justify large-scale solar radiation networks for the measurement of global insolation at the present instrumental accuracy. Network measurements can provide information on the variability of the global insolation at the earth's surface and can be used as ground-truth stations for satellite observations.

### *Daily means for clear days*

The instantaneous solar irradiance values on clear days provided by eqn (12) were averaged in 15-min increments for each day of the year from sunrise to sunset at 18 locations in the United States where measured values were available from Fritz[30] and a climatology was available from Hoyt[19].

The yearly mean measured and calculated insolation values were calculated and compared to those of Fritz. A summary of the yearly mean measured and calculated clear day solar irradiances (Wm$^{-2}$) at the 18 locations is given in Table 8. The calculated annual means exceed the measured means by an average 0.9 per cent. Of the 216 monthly mean values, the mean absolute difference between calculations and measurements is 39.1 Wm$^{-2}$ or 3.7 per cent of the measured values and all but 11 or 5 per cent of the cases fall within 98.0 Wm$^{-2}$ or 9.1 per cent of each other. The radiation scale upon which the measurements were based is not clearly identified by

Fritz but is probably the Smithsonian Scale of 1913 which is 2.6 per cent above the absolute radiation scale defined by the PACRAD[31]. Thus the calculations probably give irradiance values about 1.7 per cent below the measured values if they were placed on the absolute radiation scale. This difference compares favorably with the 1.7 per cent difference at true solar noon (Tables 5–7) but is in the opposite direction. The calculations are bracketed by the true solar noon and clear day mean insolation values.

Table 5. Monthly and annual mean values of the extraterrestrial insolation, the measured and calculated insolation at the earth's surface, and the percentage difference for Albuquerque. The mean measured and calculated atmospheric transmission and their percentage differences are also listed

| | Insolation (W m$^{-2}$) | | | | Transmission | | |
|---|---|---|---|---|---|---|---|
| | Extraterrestrial | Measured | Calculated | % Difference | Measured | Calculated | % Difference |
| Jan. | 776 | 623 | 646 | 3.7 | 0.803 | 0.832 | 2.9 |
| Feb. | 923 | 774 | 782 | 1.0 | 0.839 | 0.847 | 0.8 |
| Mar. | 1085 | 916 | 931 | 1.6 | 0.844 | 0.858 | 1.4 |
| Apr. | 1212 | 1040 | 1040 | 0.0 | 0.853 | 0.858 | 0.5 |
| May | 1269 | 1073 | 1086 | 1.2 | 0.846 | 0.856 | 1.0 |
| June | 1283 | 1064 | 1090 | 2.4 | 0.829 | 0.850 | 2.1 |
| July | 1274 | 1054 | 1062 | 0.8 | 0.827 | 0.834 | 0.7 |
| Aug. | 1235 | 1010 | 1024 | 1.4 | 0.818 | 0.829 | 1.1 |
| Sept. | 1140 | 925 | 949 | 2.6 | 0.811 | 0.832 | 2.1 |
| Oct. | 991 | 786 | 826 | 5.1 | 0.793 | 0.834 | 4.1 |
| Nov. | 830 | 647 | 690 | 6.6 | 0.780 | 0.831 | 5.1 |
| Dec. | 734 | 576 | 609 | 5.7 | 0.785 | 0.830 | 4.5 |
| Annual | 1063 | 874 | 894 | 2.3 | 0.819 | 0.841 | 2.2 |

Table 6. Monthly and annual mean values of the extraterrestrial insolation, the measured and calculated insolation at the earth's surface, and the percentage difference for Bismarck. The mean measured and calculated atmospheric transmission and their percentage differences are also listed

| | Insolation (W m$^{-2}$) | | | | Transmission | | |
|---|---|---|---|---|---|---|---|
| | Extraterrestrial | Measured | Calculated | % Difference | Measured | Calculated | % Difference |
| Jan. | 524 | 411 | 410 | -0.2 | 0.784 | 0.782 | -0.2 |
| Feb. | 693 | 557 | 562 | 0.9 | 0.804 | 0.811 | 0.7 |
| Mar. | 894 | 717 | 742 | 3.5 | 0.802 | 0.830 | 2.8 |
| Apr. | 1069 | 875 | 882 | 0.8 | 0.819 | 0.825 | 0.6 |
| May | 1167 | 976 | 947 | -3.0 | 0.836 | 0.811 | -2.5 |
| June | 1202 | 960 | 959 | -0.1 | 0.799 | 0.798 | -0.1 |
| July | 1186 | 925 | 936 | 1.2 | 0.780 | 0.789 | 0.9 |
| Aug. | 1114 | 861 | 876 | 1.7 | 0.773 | 0.786 | 1.3 |
| Sept. | 972 | 742 | 770 | 3.8 | 0.763 | 0.792 | 2.9 |
| Oct. | 780 | 570 | 617 | 8.2 | 0.731 | 0.791 | 6.0 |
| Nov. | 582 | 435 | 458 | 5.3 | 0.747 | 0.787 | 4.0 |
| Dec. | 478 | 371 | 368 | -0.8 | 0.776 | 0.770 | -0.6 |
| Annual | 888 | 699 | 711 | 1.7 | 0.785 | 0.798 | 1.3 |

Table 7. Monthly and annual mean values of the extraterrestrial insolation, the measured and calculated insolation at the earth's surface, and the percentage difference for Raleigh. The mean measured and calculated atmospheric transmission and their percentage differences are also listed

| | Insolation (W m$^{-2}$) | | | | Transmission | | |
|---|---|---|---|---|---|---|---|
| | Extraterrestrial | Measured | Calculated | % Difference | Measured | Calculated | % Difference |
| Jan. | 760 | 584 | 588 | 0.7 | 0.768 | 0.774 | 0.6 |
| Feb. | 908 | 724 | 718 | -0.8 | 0.797 | 0.791 | -0.6 |
| Mar. | 1073 | 850 | 858 | 0.9 | 0.792 | 0.800 | 0.8 |
| Apr. | 1203 | 955 | 957 | 0.2 | 0.794 | 0.796 | 0.2 |
| May | 1264 | 980 | 990 | 1.0 | 0.775 | 0.783 | 0.8 |
| June | 1279 | 997 | 970 | -2.7 | 0.780 | 0.758 | -2.2 |
| July | 1270 | 956 | 960 | 0.4 | 0.753 | 0.756 | 0.3 |
| Aug. | 1228 | 907 | 923 | 1.8 | 0.739 | 0.752 | 1.3 |
| Sept. | 1130 | 841 | 858 | 2.0 | 0.744 | 0.759 | 1.5 |
| Oct. | 978 | 719 | 753 | 4.7 | 0.735 | 0.770 | 3.5 |
| Nov. | 810 | 598 | 621 | 3.8 | 0.738 | 0.767 | 2.9 |
| Dec. | 718 | 539 | 551 | 2.2 | 0.751 | 0.767 | 1.6 |
| Annual | 1052 | 804 | 812 | 1.0 | 0.764 | 0.773 | 0.9 |

Table 8. Measured annual mean daily average global irradiance values ($Wm^{-2}$) from Fritz[30] and the corresponding calculated values and their difference in percent from the measured values

| | Measured | Calculated | % Diff. |
|---|---|---|---|
| Albuquerque, NM | 1212.8 | 1188.0 | -2.0 |
| Boulder, CO | 1045.5 | 1097.1 | 4.9 |
| Boston, MA | 1035.1 | 991.7 | -4.2 |
| Brownsville, TX | 1140.5 | 1194.2 | 4.7 |
| Charlestown, SC | 1084.7 | 1093.0 | 0.8 |
| Columbia, MO | 1086.8 | 1059.9 | -2.5 |
| El Paso, TX | 1225.2 | 1204.5 | -1.7 |
| Ely, NV | 1090.9 | 1138.4 | 4.4 |
| Fresno, CA | 1086.8 | 1107.4 | 1.9 |
| Great Falls, MT | 948.3 | 971.1 | 2.4 |
| Lake Charles, LA | 1070.2 | 1111.6 | 3.9 |
| Madison, WI | 981.4 | 1002.1 | 2.1 |
| Medford, OR | 1014.5 | 1006.2 | -0.8 |
| Miami, FL | 1150.8 | 1146.7 | -0.4 |
| Nashville, TN | 1024.8 | 1072.3 | 4.6 |
| Phoenix, AR | 1099.2 | 1157.0 | 5.3 |
| Seattle, WA | 944.2 | 900.8 | -4.6 |
| Washington, DC | 1043.4 | 1024.8 | -1.8 |
| MEAN | 1071.4 | 1081.5 | 0.9 |

*Radiation climatology at Boulder, Colorado*

As a test of how well the model treats radiation under all conditions including overcast skies, the radiation climatology at Boulder, Colorado, has been calculated and compared to the radiation climatology of Bennett[32]. Monthly mean values of total cloud cover are calculated using the percent of possible sunshine data in the *Climatic Atlas of the United States*[27] and the methodology of Hoyt[33]. For calculations of insolation at the earth's surface, all clouds at Boulder are taken to be altostratus and altocumulus. Cloud height come from the work of Hoyt[7]. The percentage of cloud cover and the calculated insolation in clear, overcast, and all weather conditions are summarized for each month in Table 9 along with the measured insolation values tabulated by Bennett in $Wm^{-2}$. The calculated values differ most from the observed values in the winter time but on an annual basis are 2.7 per cent less than the measurements. As in the previous section the measured daily totals exceed the calculated daily totals. No attempt is made to put these measurements on an absolute scale because of insufficient data on the calibration history of the field pyranometers. Two possible explanations for the differences are that the calculated values may be overestimating the absorption by the atmosphere or that the cloud climatology may be incorrect. The 2.7 per cent difference between calculations and measurements are within the limits of error of the measurements.

## 6. CONCLUSIONS

A theoretical model for the calculation of both clear sky and cloudy sky irradiance at the earth's surface has been described and compared to other model calculations and observations. The following are the major conclusions that were reached:

(1) In a comparison to the model results by Braslau and Dave, the present model typically gives 1–2 per cent lower atmospheric transmission values. The present model's improper treatment of overlapping gaseous absorption and its treatment of only first order surface-air interreflections combined with *B/D*'s wavelength coverage to only 2.5 $\mu$m are probably the major reasons for the differences between the two models.

(2) Climatological inputs to the model are total precipitable water, turbidity and the surface albedo. Total precipitable water values to 250 mb are used so an underestimation of about 0.1 cm of water is possible implying the model calculations will give atmospheric transmission values which are slightly too high ($\leq 1$ per cent). Uncertainties in the turbidity and surface albedo will lead to uncertainties in the model transmission values of about $\pm 0.5$ and $\pm 1$ per cent respectively.

(3) The true solar noon atmospheric transmission values are calculated for 26 locations as part of an effort to improve the quality of the National Weather Service pyranometer measurements. At three locations both reliable measurements and model calculations are avail-

Table 9. Calculated and measured radiation climatology at Boulder, Colorado ($Wm^{-2}$)

| | % Cloud Cover | Calculated insolation | | | Insolation |
|---|---|---|---|---|---|
| | | Clear | Overcast | Total | (Bennett) |
| Jan. | 33 | 371.4 | 74.2 | 445.6 | 526.9 |
| Feb. | 33 | 522.1 | 111.6 | 633.7 | 698.3 |
| Mar. | 35 | 699.6 | 171.1 | 870.7 | 900.8 |
| Apr. | 37 | 863.6 | 235.7 | 1099.3 | 1072.3 |
| May | 39 | 949.2 | 282.0 | 1231.2 | 1202.5 |
| June | 31 | 1127.2 | 232.2 | 1359.4 | 1351.2 |
| July | 32 | 1078.7 | 230.4 | 1309.1 | 1307.9 |
| Aug. | 32 | 975.2 | 208.1 | 1183.3 | 1223.1 |
| Sept. | 29 | 836.7 | 153.3 | 990.0 | 1020.7 |
| Oct. | 29 | 629.9 | 111.4 | 738.3 | 764.5 |
| Nov. | 33 | 413.4 | 83.7 | 497.1 | 541.3 |
| Dec. | 35 | 318.2 | 67.4 | 385.6 | 433.9 |
| Annual | 33 | 732.0 | 163.3 | 895.3 | 920.3 |

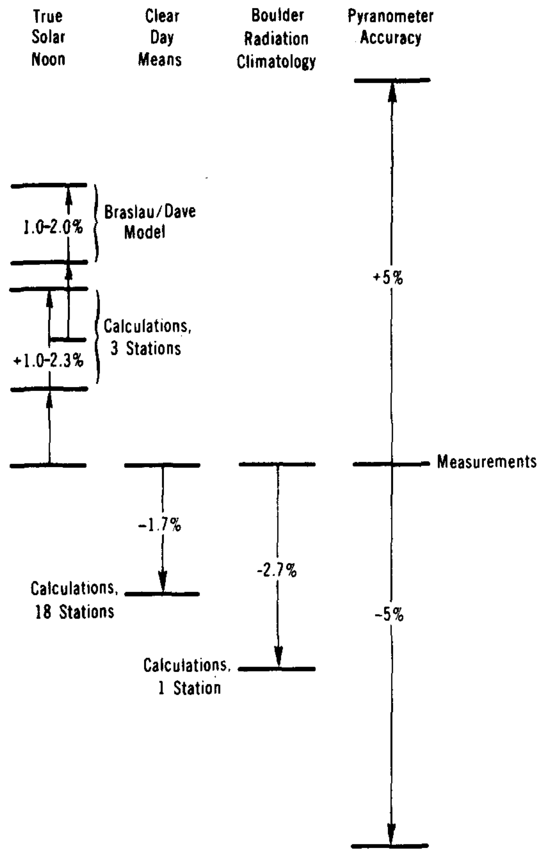


Fig. 2. Summary of where calculations of irradiance stand relative to measurements on the absolute radiation scale. See text for details.

able. On the average the calculated transmission values exceed the measured values at these three locations by 1.7 per cent. Several explanations are advanced to account for the differences, and one of the more likely ones is that the cosine response of the pyranometers results in an underestimation of the irradiance at the higher zenith angles.

(4) Measured true solar noon irradiance values on the absolute radiation scale are 1.0–2.3 per cent lower than the values derived from the model calculations but are within the limits of accuracy of the pyranometers.

(5) At 18 locations the measured annual average of the daily mean insolation on clear days only was compared to the model calculations. The calculated values exceed the measured values supplied by Fritz by an average of 0.9 per cent. If however the measured values are placed on the absolute radiation scale, the measured values exceed the calculated values by 1.7 per cent.

(6) As a test of the model including clouds, the radiation climatology of Boulder, Colorado, is computed and compared to the observations of Bennett. The annual mean calculated insolation is 2.7 per cent lower than the observations.

(7) In all the cases considered, the measured and calculated insolation values are within 2.7 per cent of each other when annual means are compared. Figure 2 summarizes where the calculated irradiance values stand relative to the measurements. Errors in the climatology used in the model calculations, errors in the model or measurement errors contribute to the differences between measurement and calculation. The differences are within the ±5 per cent accuracy claim for the pyranometers.

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**Resumen**—Se describe un modelo teórico diseñado para obtener le insolación totale incidente sobre la superficie de la Tierra y la transmisión en la atmósfera. Éste ha side comparado con un modelo de Braslau y Dave[1] y se encontró que coincide dentro de pequeños porcentajes en todos los casos. Se requiere entrar al modelo con los valores climatológicos de la precipitación total de agua, turbiedad y albedo superficial, y se describen las fuentes de estos datos. El modelo ha sido aplicado a 26 estaciones de la red de piranómetros del National Weather Service (Servicio Meteorológico Nacional), donde se dispone de los valores reales de transmisión atmosférica al mediodía solar, come parte del programa de la National Oceanic and Atmospheric Administration (Administración Nacional Oceánica y Atmosférica) para rehabilitar las viejas observaciones con piranómetros. Para tres de estas estaciones de donde se dispone de la radiación solar real del nediodía y de los valores de transmisión confiables, se compararon las observaciones y los cálculos con el modelo. En 18 localidades fueron comparadas las insolaciones medias calculadas y medidas para dias claros. En una localidad (Boulder, Colorado) las climatologías de radiación medidas y calculadas para todas las condiciones cimátical fueron comparadas, y las diferencias no han side mayores al 2,7%, las que están dentro de la exactitud experimental (± 5%) de los piranómetros. Se discuten las posibles fuentes de errores.

**Résumé**—On décrit un modèle théorique qui est prévu pour donner le rayonnement global arrivant sur la surface terrestre et la transmission de l'atmosphère. On le compare au modèle de Braslau et Dave[1] et on trouve qu'il convient dans tous les cas, à quelques pour près. Il est nécessaire de disposer comme entrées du modéle, de valeurs climatogiques sur les précipitations totales d'eau, la turbidité et l'albedo de la surface, et on décrit les sources où on a pris ces données. Ce modèle a été appliqué à 26 stations dans le réseau du pyranomètre du "National Weather Service (NWS)", pour lequel les valeurs de la transmission atmosphérique mesurée à midi temps solaire vrai sont disponibles, en tant que partie du programme NOAA qui vise à réhabiliter les observations du vieux pyranomètre. Pour trois de ces stations, pour lesquelles on dispose des valeurs de la transmission et du rayonnement à midi solaire vrai, on compare les calculs donnés par le modèle aux observations. Pour 18 lieux, on compare les valeurs moyennes du rayonnement journalier mesurées et calculées, pour des jours clairs. Pour un lieu (Boulder, Colorado), on compare les climatologies du rayonnement mesurées et calculées pour toutes les conditionsde temps. Dans toutes ces comparaisons, le modèle et les observations ne diffèrent pas de plus de 2,7%, ce qui est compris dans l'erreur expérimentale (± 5%) des pyranomètres. On discute des diverses sources possibles d'erreur.