Teaching Mathematics Contextually

The Cornerstone of Tech Prep

CORD

Leading Change in Education

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Preface

Tech Prep has become the leading educational reform movement in the United States because it focuses on improving student achievement in academic subjects, particularly for the middle 60 percent . . . those we call the "neglected majority." We call them "neglected" because academic subjects such as mathematics, science, and English have traditionally been taught in a manner that benefits abstract learners—and most Tech Prep students are not abstract learners. In fact, research by learning theorist David A. Kolb and others concludes that less than one-fourth of our students are abstract learners; most students learn best when they can connect new concepts to the real world through their own experiences or experiences teachers can provide them.

Tech Prep opens the door to higher education as well as to high-skill jobs because it requires the blending of high-level academic skills with technical and employability skills. Employers are no longer looking for workers who can perform only a prescribed set of tasks or operate equipment; they want workers who can continuously grasp new information and acquire new skills, who can improvise, solve open-ended problems, and work effectively in teams. The key to the door of higher education is mastery of academic disciplines; the key to successful employment is mastery of "useful academics."

One fundamental change in curriculum essential to the successful implementation of Tech Prep curricula in grades nine through fourteen is the teaching of a foundation of basic knowledge and skills. This must be done before further knowledge and skills can be added to that foundation. Technical (career-specific) courses cannot be taught successfully without students first understanding the technical principles that underlie them (the academic or theoretical basis). We know it is realistic to have high expectations of academic achievement from nearly all students if we restructure our learning materials and teaching to match their learning styles, and use hands-on contextual teaching and learning methods to help them learn.

For nearly a decade (1985 to 1995), the “applied academics” movement demonstrated that students who had previously performed poorly in abstract math and science courses could achieve high levels in those subject areas if they were taught in an applied or hands-on format. Instead of watering down the content (as had been done in courses for general education), CORD’s applied academics courses maintained academic rigor but introduced real-world examples, applications, and problems, and engaged the students in laboratory activities using equipment familiar to life and actual work applications. In other words, it wasn’t easier; it was just easier to learn. More than 5.5 million students nationwide have enjoyed success through applied academics curricula. More than ten thousand teachers have been trained in applied academics methodology.

In the past five years, state and national standards for math and science have evolved; this has required adjustments to the content, organization, and pedagogy of specific courses and of assessments. In response to this, CORD, a not-for-profit education research and reform organization, has prepared this booklet to aid educators who are engaged in the vital and growing field of contextual learning.

Facing the Challenge

What is the best way to convey the many concepts that are taught in a particular class—or any other class—so that *all* students can use and retain that information? How can the individual lessons be understood as interconnected pieces that build upon each other? How can a teacher communicate effectively with students who wonder about the reason for, the meaning of, and the relevance of what they study? How can we open the minds of a diverse student population so they can learn concepts and techniques that will open doors of opportunity for them throughout their lives? These are the challenges teachers face every day, the challenges that a curriculum and an instructional approach based on *contextual learning* can help them face successfully.

The majority of students in our schools are unable to make connections between what they are learning and how that knowledge will be used. This is because the way they process information and their motivation for learning are not touched by the traditional methods of classroom teaching. The students have a difficult time understanding academic concepts (such as math concepts) as they are commonly taught (that is, using an abstract, lecture method), but they desperately need to understand the concepts as they relate to the workplace and to the larger society in which they will live and work.

As the need for higher-level academic and work skills increases, the challenge to help all students master these skills grows stronger. In many schools across the country, Tech Prep has become the agent for change in this area—opening doors for a fresh approach to teaching and learning.

Tech Prep curricula require not only a stronger academic foundation and a higher caliber of work skills, but also a better understanding of how academic concepts relate to the workplace and how vocational skills connect with these academic concepts. This higher level of learning is not usually taught even to the academically “above-average” student, much less to the average student who needs it most. Traditionally, students have been expected to make these connections on their own, outside the classroom.

However, growing numbers of teachers today—especially those frustrated by repeated lack of student success in demonstrating basic proficiency on standard tests—are discovering that most students' interest *and* achievement in math, science, and language improve dramatically when they are helped to make connections between new information (knowledge) and experiences they have had, or with other knowledge they have already mastered. Students' involvement in their schoolwork increases significantly when they are taught *why* they are learning the concepts and *how* those concepts can be used outside the classroom. And most students learn much more efficiently when they are allowed to work cooperatively with other students in groups or teams.

Dan Hull
President and
Chief Executive Officer
CORD
February 1999

The Contextual Approach to Learning

Contextual learning is a proven concept that incorporates much of the most recent research in cognitive science. It is also a reaction to the essentially behaviorist theories that have dominated American education for many decades. The contextual approach recognizes that learning is a complex and multifaceted process that goes far beyond drill-oriented, stimulus-and-response methodologies.

According to contextual learning theory, learning occurs only when students (learners) process new information or knowledge in such a way that it makes sense to them in their own frames of reference (their own inner worlds of memory, experience, and response). This approach to learning and teaching assumes that the mind naturally seeks meaning *in context*—that is, in relation to the person's current environment—and that it does so by searching for relationships that make sense and appear useful.

Building upon this understanding, contextual learning theory focuses on the multiple aspects of any learning environment, whether a classroom, a laboratory, a computer lab, a worksite, or a wheat field. It encourages educators to choose and/or design learning environments that incorporate as many different forms of experience as possible—social, cultural, physical, and psychological—in working toward the desired learning outcomes.

Are You Teaching Mathematics Contextually?

Take this self-test and see.

These standards appear to some degree in almost all texts. But *contextual* instruction is rich in all ten standards.

1. Are new concepts presented in real-life (outside the classroom) situations and experiences that are familiar to the student?
2. Are concepts in examples and student exercises presented in the context of their use?
3. Are new concepts presented in the context of what the student already knows?
4. Do examples and student exercises include many real, believable problem-solving situations that students can recognize as being important to their current or possible future lives?
5. Do examples and student exercises cultivate an attitude that says, “I need to learn this”?
6. Do students gather and analyze their own data as they are guided in discovery of the important concepts?
7. Are opportunities presented for students to gather and analyze their own data for enrichment and extension?
8. Do lessons and activities encourage the student to apply concepts and information in useful contexts, projecting the student into imagined futures (e.g., possible careers) and unfamiliar locations (e.g., workplaces)?
9. Are students expected to participate regularly in interactive groups where sharing, communicating, and responding to the important concepts and decision making occur?
10. Do lessons, exercises, and labs improve students’ reading and other communication skills in addition to mathematical reasoning and achievement?

In such an environment, students discover meaningful relationships between abstract ideas and practical applications in the context of the real world; concepts are internalized through the process of discovering, reinforcing, and relating. For example, a physics class studying thermal conductivity might measure how the quality and amount of building insulation material affect the amount of energy required to keep the building heated or cooled. Or a biology or chemistry class might learn basic scientific concepts by studying the spread of AIDS or the ways in which farmers suffer from and contribute to environmental degradation.

Curricula and instruction based on this strategy will be structured to encourage five essential forms of learning: **R**elating, **E**xperiencing, **A**pplying, **C**ooperating, and **T**ransferring (Figure 1).

R*elating*
E*xperiencing*
A*pplying*
C*ooperating*
T*ransferring*

Figure 1. Essential elements of the REACT strategy

Relating. Learning in the context of life experience, or relating, is the kind of contextual learning that typically occurs with very young children. For toddlers, the sources of learning are readily at hand in the form of toys, games, and everyday events such as meals, trips to the grocery store, and walks in the neighborhood.

As children grow older, however, providing this meaningful context for learning becomes more difficult. Ours is a society in which the workplace is largely separated from domestic life, in which extended families are separated by great distances, and in which teens lack clear societal roles or responsibilities commensurate with their abilities.

Under ideal conditions, teachers might simply lead students from one community-based activity to another, encouraging them to relate what they are learning to real-life experience. In most cases, however, given the range and complexity of concepts to be taught and the limitations of our resources, life experiences will have to be evoked through text, video, speech, and classroom activity.

The curriculum that attempts to place learning in the context of life experiences must, first, call the student's attention to everyday sights, events, and conditions. It must then *relate* those everyday situations to new information to be absorbed or a problem to be solved.

Experiencing. Experiencing—learning in the context of exploration, discovery, and invention—is the heart of contextual learning. However motivated or tuned-in students may become as a result of other instructional strategies such as video, narrative, or text-based activities, these remain relatively passive forms of learning. And learning appears to "take" far more quickly when students are able to manipulate equipment and materials and to do other forms of active research.

In contextual academics texts, laboratories are often based on actual workplace tasks. The aim is not to train students for specific jobs, but to allow them to experience activities that are directly related to real-life work. Many of the activities and skills selected for labs are cross-occupational; that is, they are used in a broad spectrum of occupations.

Applying. Applying concepts and information in a useful context often projects students into an imagined future (a possible career) and/or into an unfamiliar location (a workplace). In contextual learning courses, applications are often based on occupational activities.

As noted above, young people today generally lack access to the workplace; unlike members of previous generations, they do not see the modern-day counterpart of the blacksmith at the forge or the farmer in the field. Essentially isolated in the inner city or outer suburbia, many students have a greater knowledge of how to become a rock star or a model than of how to become a respiratory therapist or a power plant operator. If they are to get a realistic sense of connection between schoolwork and real-life jobs, therefore, the occupational context must be brought to them. This happens most commonly through text, video, labs, and activities, although, in many schools, these contextual learning experiences will be followed up with firsthand experiences such as plant tours, mentoring arrangements, and internships.

Cooperating. Cooperating—learning in the context of sharing, responding, and communicating with other learners—is a primary instructional strategy in contextual teaching. The experience of cooperating not only helps the majority of students learn the material, it also is consistent with the real-world focus of contextual teaching.

Research interviews with employers reveal that employees who can communicate effectively, who share information freely, and who can work comfortably in a team setting are highly valued in the workplace. We have ample reason, therefore, to encourage students to develop these cooperative skills while they are still in the classroom.

The laboratory, one of the primary instructional methods in applied academics, is essentially cooperative. Typically, students work with partners to do the laboratory exercises; in some cases, they work in groups of three or four. Completing the lab successfully requires delegation, observation, suggestion, and discussion. In many labs, the quality of the data collected by the team as a whole is dependent on the individual performance of each member of the team.

Students also must cooperate to complete the many small-group activities that are included in the applied academics courses. Partnering can be a particularly effective strategy for encouraging students to cooperate.

T*ransferring*. Learning in the context of existing knowledge, or transferring, uses and builds upon what the student already knows. Such an approach is similar to relating, in that it calls upon the familiar.

As adults, many of us are adept at avoiding situations that are unfamiliar—the part of town we don't know, the unusual food we've never eaten, the store we haven't shopped. Sometimes we also avoid situations in which we have to gain new information or develop a new skill (especially if there are likely to be witnesses)—using a new type of computer software or coping in another country with our fledgling foreign-language skills.

Most traditionally taught high school students, however, rarely have the luxury of avoiding new learning situations; they are confronted with them every day. We can help them retain their sense of dignity and develop confidence if we make a point of building new learning experiences on what they already know.

Correcting False Assumptions About Learning

Contextual learning offers more than a tool for defragmenting the American educational system; it provides a more effective approach to teaching the majority of students because it is specifically geared to the way these students learn.

In recent years, cognitive science and studies of the relationships between structured learning and the work environment have given us a better basis for evaluating the effectiveness of various methods of teaching and learning. Many educators, however, tend to interpret the learning environment according to their own experience as students. In other words, they teach the way they have been taught—usually through traditional abstract lecture methods.

But, while the traditional classroom model is valid, it is not necessarily the most effective strategy for teaching the majority of students. To increase their effectiveness in the classroom, many educators may need to change some of their basic assumptions about how people learn.

Dr. Sue Berryman of the Institute on Education and the Economy at Columbia University has isolated five common misconceptions about the ways people learn:

1. People predictably transfer learning from one situation to another.

2. Learners are passive receivers of wisdom—empty vessels into which knowledge is poured.

3. Learning is the strengthening of bonds between stimuli and correct responses.

4. What matters is getting the right answer.

5. Skills and knowledge, to be transferable to new situations, should be acquired independent of their contexts of uses.

These are assumptions that may well be blocking many students from an effective learning experience. In each case, the contextual learning approach can help correct the false assumption and the inefficient educational processes that grow out of the assumptions.[1]

False Assumption #1. People predictably transfer learning from one situation to another.

Berryman questions, for example, whether most people actually use in everyday practice the knowledge, skills, and strategies they acquired during their formal education. For instance, a student training to be a radiology technician may have difficulty relating the theories she learned in physics class to the technical skills she is learning in her electronics courses.

False Assumption #2: Learners are passive receivers of wisdom—empty vessels into which knowledge is poured.

Each student approaches the task of learning equipped with a matrix of acquired skills, knowledge, and experience—and a set of expectations and hopes. The most effective learning happens when the student is invited (and taught) to make connections between past learning and future actions. But teaching techniques that require an essentially passive response from students, such as lecturing, deprive them of this opportunity to actively involve themselves with the material. They may miss the most important means of learning—exploration, discovery, and invention. Passive learners who are dependent upon the teacher for guidance and feedback may also fail to develop confidence in their own intuitive abilities.

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[1] Sue E. Berryman and Thomas Bailey, *The Double Helix of Education and the Economy* (New York: Institute on Education and the Economy, Columbia University, 1992), 45-68.

False Assumption #3: Learning is the strengthening of bonds between stimuli and correct responses.

This misconception is based on a behaviorist approach to education, which tends to reward response instead of understanding. Education based on behaviorist theory typically leads to breaking down complex tasks and ideas into oversimplified components, unrelated subtasks, repetitive training, and an inappropriate focus on the “right answer.” It does not help students learn to solve problems on a more systemic level.

False Assumption #4: What matters is getting the right answer.

Students who focus primarily on getting the right answer tend to rely on memorized shortcuts instead of acquiring the problem-solving skills they will need in a real-life setting.

False Assumption #5: Skills and knowledge, to be transferable to new situations, should be acquired independent of their contexts of uses.

The process of abstracting knowledge, or taking it away from its specific context, has long been thought to make that knowledge more useful to a number of situations; this philosophy underlies much of our current educational system. However, Berryman points out that such *decontextualization* can easily rob students of a sense of motivation and purpose. They may have difficulty understanding why a concept is important and how it relates to reality, and this may make the material more difficult to retain. For example, the definition of a term may be difficult to learn and retain without an understanding of the context of its use.

The Context of the Workplace

In 1991, the United States Department of Labor initiated the Secretary's Commission on Achieving Necessary Skills (SCANS) to analyze the future skills that would be needed by the American workforce. The commission, led by Dr. Arnold Packer of Johns Hopkins University, prepared a report on developing world-class standards for educational performance.[2] The SCANS report reinforces the need for a more effective structure of learning that responds to the changing needs of the new workforce—and contextually based teaching methods are especially effective in making this kind of connection.

The SCANS report duly notes that traditional basic competencies such as reading, writing, and arithmetic have been and continue to be a key part of the total skills required of the workforce. However, the SCANS report strongly emphasizes two other sets of competencies as critical for the current and future workforce:

- personal qualities: the ability to relate to others in and out of the classroom and develop individual responsibility and self-esteem; and
- thinking skills: the ability to think and problem-solve an entire system rather than working with isolated tasks and problems.

These two sets of abilities are now seen not only as skills that should be learned in combination with the three Rs, but also as the basis for strategies that all teachers should consider using to enhance the learning capacity of their students.

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[2] Secretary's Commission on Achieving Necessary Skills, *What Work Requires of Schools: A SCANS Report for America 2000,* a letter to parents, employers, and educators (Washington, DC: Government Printing Office, 1992).

The process of learning interpersonal skills, for instance, requires students to work on teams, teach others, lead, negotiate, and work well with people from culturally diverse backgrounds. But these techniques, in addition to helping students learn to get along with others, also help them learn content more effectively. The math students working together on a project not only learn interpersonal skills; they also learn more math.

Similarly, students acquire thinking skills best in a learning environment that requires them to be creative, make decisions, solve problems, and know how to learn and reason.[3] And, once again, this kind of environment will facilitate the learning of the course content.

The adoption of the SCANS report as a structure for learning can help students transfer knowledge from school to career and understand the context and meaning in which the curriculum is taught. It is therefore an important part of the contextual-learning model for Tech Prep curricula.

Changes in the development of the workforce require employees who have multiple skills and abilities. Similarly, the changes in the educational system must reflect the fact that students cannot continue to learn in an isolated fashion. If educational reform reshapes the way students learn, the outcome could enhance the abilities of the future workforce.

More Than One Kind of Intelligence

In a sense, there is nothing new about contextual learning. There have always been teachers who intuitively understood how to teach concepts so that all learners could grasp them—through example, illustration, and hands-on application. But even these naturally effective teachers can benefit from

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[3] *What Work Requires of Schools,* 4-5.

understanding the findings of relatively recent cognitive research and from learning how to put these findings to use. The results of this seminal research explain the success of contextual teaching and learning approaches in the classroom.

Cognitive science, similar in purpose and methodology to educational psychology, poses two important questions about the teaching and learning process:

- How do the human mind and body work in their learning capacity?
- How can an understanding of the mind/body's way of learning be used in educational settings?

In addressing the first question, Howard Gardner, professor of education at Harvard University, has challenged traditional thinking by questioning whether intelligence is a single, measurable capacity. Gardner posits instead that the human capacity for learning is much broader than traditional measurements of intelligence would indicate.

Alfred Binet, the inventor of the IQ test, standardized the assessment of intelligence through two measurements, verbal and analytical. Gardner argues, however, that people have as many as seven forms of intelligence: linguistic, logical/mathematical, musical, spatial, kinesthetic, interpersonal, and intrapersonal (Figure 2). He bases this theory on his observance of the wide range of capabilities of adolescents.

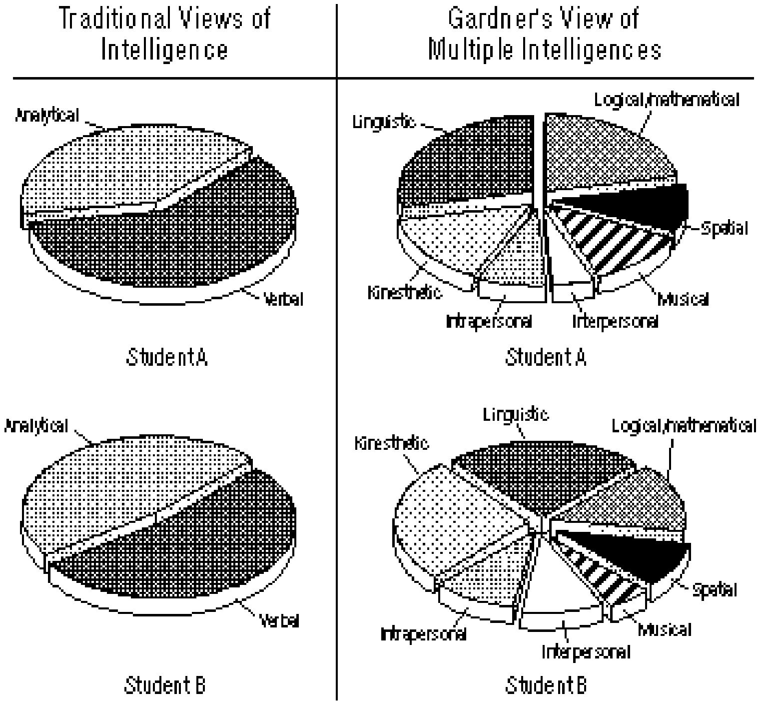

Figure 2. Traditional versus contemporary views of intelligence

For instance, Gardner describes "a fourteen-year-old adolescent in Paris, who has learned how to program a computer and is beginning to compose works of music with the aid of a synthesizer." This young Parisian displays both musical and logical/mathematical intelligence, a combination of abilities that traditional educational assessment does not usually measure or even acknowledge.

Gardner makes another important observation regarding multiple intelligences: While everyone has some measure of each of the seven intelligences, the specific strengths and combinations vary according to the individual. No two people have the same kind of mind. (His observations of adolescents strongly support this assertion by showing that not one of the adolescents could possibly master every intelligence.) For this reason, Gardner argues against a uniform school system that does not allow students to make choices about what to learn and, even more important, how to learn.[4]

More Than One Way to Learn

Gardner's theory that people have multiple intelligences helps answer the second question of cognitive science—how an understanding of the mind/body's way of learning can be used in educational settings. The "learning styles" movement of the last twenty years also helps answer this question through an abundance of material it has produced concerning the various approaches students take to learning and the teaching techniques that are best suited to reaching certain students.

In his discussion of the variety of learning styles, learning theorist David Kolb observes that learners tend to perceive information either abstractly (by conceptualizing/thinking) or concretely (by experiencing/feeling) and then process that information either actively (by experimenting/doing) or reflectively (by observing/watching). Kolb (as well as other learning theorists) has typically set each of these four learning styles on an axis as a way to understand the entire realm of students' learning tendencies.

Kolb's construction, like Gardner's, clearly indicates that most students do not fit neatly into one category or the other. Almost all students can learn by and benefit from all four experiences (thinking, feeling, doing, and watching). And no one type of learning is superior to another; all contribute to the process of effective learning. Nevertheless, most students will show a preference for one or two particular kinds of learning, and this preference will indicate the individual's primary learning style(s).

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[4] Howard Gardner, *Frames of Mind: The Theory of Multiple Intelligences* (New York: Basic Books, 1983), 4-6.

The emphasis for contextual learning is to use this process for effective learning to reach the strengths of all students. However, as Kolb's studies indicate, most students have a tendency to learn in a concrete manner (with an emphasis on feeling and doing), while the school system tends to teach in an abstract manner (with an emphasis on thinking and watching) (Figure 3).

Kolb's study of student responses found that relatively few students tend to learn by thinking and watching—the learning style catered to in the commonly used lecture method. Most students tend to perceive and process information through some kind of concrete experiences and/or experimentation. Most people, in other words, are extroverted learners; they learn best through interpersonal communication, group learning, sharing, mutual support, team processes, and positive reinforcement.

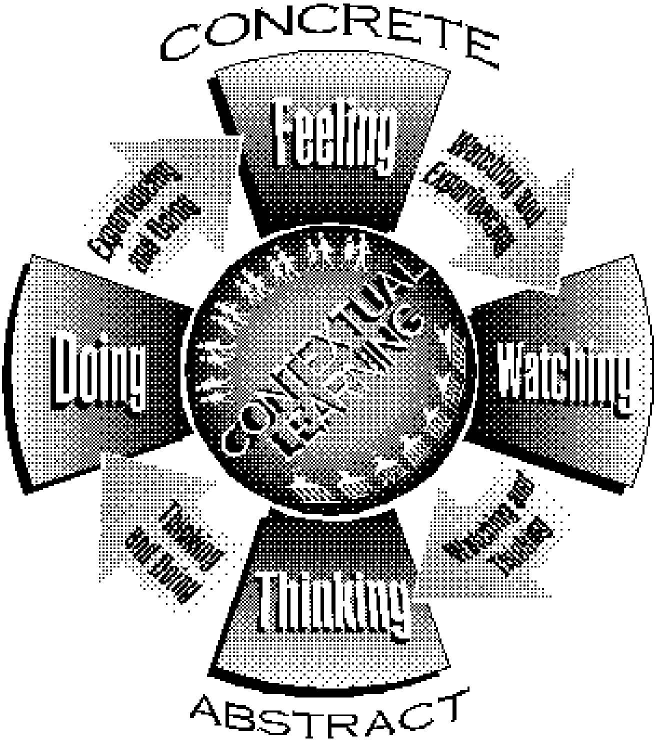

Figure 3. Contextual learning should encompass all styles of learning.[5]

Nevertheless, Kolb encourages the use of teaching methods that develop all four learning styles so that students can broaden their learning abilities beyond natural inclination. After all, upon entering the workforce, even those few students who learn best by thinking and watching will be required to experience and act. And hands-on learners must be able to take the conceptual information they receive in traditional teaching and learning methods and transfer it into practice.

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[5] Adapted from David A. Kolb, *Experiential Learning: Experience As the Source of Learning and Development* (New Jersey: Prentice-Hall, 1984).

Striving for Connectedness

Despite the individual differences in learning styles and intelligences, all learners strive for connectedness. Learning is cumulative. Isolated bits of information normally are not processed and retained by the mind for meaningful use unless connections are made and points of reference or relationships are established between what is known and what is not known.

Teaching through illustrations and examples is a classical form of imbuing a learning experience with understanding and meaning. Many adults, for example, still remember at least one field trip or nature walk they took in kindergarten or elementary school. The children pick up leaves or study plants and insects, learning terms and concepts while seeing and experiencing nature for themselves. Instead of just memorizing a list of names, they are allowed to touch or smell the objects that go with the names.

This concrete experience introduces meaning to the knowledge the children are acquiring. Later they will be able to rely on their experiences and their knowledge of nature to learn more about science, the environment, and the role played by nature in literature, art, and history.

Learning theorists Renate Nummela Caine and Geoffrey Caine explain this "connectedness" theory by pointing out that all knowledge is "embedded" in other knowledge.[6] Academic "subjects" such as English, math, and chemistry are at best artificial distinctions within a single body of knowledge. Allowing these subjects to overlap and integrating them into a single curriculum can therefore provide a better, more connected understanding.

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[6] Renate Nummela Caine and Geoffrey Caine, *Making Connections: Teaching and the Human Brain* (Alexandria, Virginia: Association for Supervision and Curriculum Development, 1991), 92-97.

Caine and Caine suggest that any robot can be programmed to do rote memorization or acquire surface knowledge about a subject. However, using knowledge that a student already has in the context of new knowledge helps the student acquire deeper understanding as well as basic facts.

In addition to making connections between different school subjects, teachers can enhance the learning process by engaging students in hands-on activities and concrete experiences as other ways of reinforcing the usefulness of the knowledge. Lab activities, experiments, and projects that require students to be actively involved in the community usually stimulate interest and motivation to learn. Integrating work-based learning with school subjects is another effective way to ground learning in actual experience.

What Do We Know About the Learning Process?

The convergence of intelligence theories and learning theories suggests similar methods for more effective teaching and learning. For instance, if we accept Gardner's theory that the mind's capacity for learning is much broader than traditionally assumed, we can probably go along with Kolb's assertion that individuals have a natural ability to learn through a variety of methods. We can further conclude from the studies of Caine and Caine that connectedness is a key to effective learning. The following summary statements about effective learning are a distillation of the theories of intelligence and learning that have been addressed here:

- Most people learn best in a concrete manner involving personal participation, physical or hands-on activities, and opportunities for personal discovery.
- Learning is greatly enhanced when concepts are presented in the context of relationships that are familiar to the student.

- Most people relate better to concrete, tangible examples and experiences than to abstract conceptual models.
- Most students learn best through some sort of personal interaction with other students—through study groups, team learning, and so on.
- Rote memorization of isolated fragments of knowledge is a relatively inefficient and ineffective learning strategy for most students.
- Transfer of learning from one situation to another is not consistently predictable, and the ability to do so is a skill that must be learned.

In the last forty years, only a few alterations in the content of elementary and secondary education have been necessary. Aside from a knowledge of computers, globalization, recent history, and environmental change, students need the same sound, solid education they needed four decades ago.

Instead, the major changes needed in today's educational system center around processes. We need to

- provide students with compelling reasons to remain in school,
- use the discoveries of cognitive science to help them achieve enhanced learning, and
- create learning environments that open their minds and enable them to become more thoughtful, participative members of society and the workforce.

Clearly, if the thrust of educational reform is on the classroom (and laboratories), the emphasis should be upon empowering teachers to facilitate these processes. The "neglected majority" can be successful learners, and they can be vital elements of America's new workforce. But, for this to happen on a national level, teachers must be helped to understand how students learn, teachers and students must be provided with suitable resources that contain career-oriented motivational elements, and teachers must be provided with sufficient institutional support (including in-service training) to allow them to use new material and equipment effectively. In short, teachers and students must be immersed in contextual teaching and learning, and supported in their use of the REACT strategy. Tech Prep is the most successful framework for contextual teaching and learning, and can help educators bridge the gap between what *is*—often static classrooms with bored, “tuned out” students—and what *should be*—classrooms alive with dynamic students actively engaged in learning.

Tools for Implementing the REACT Strategy

A prime reason for the struggle to transform the classroom is that most of today’s textbook designers are stuck in the rut of traditional educational philosophies and see contextual learning as just another fad that will soon fade away. As a result, they take advantage of the word *contextual* in their advertising and book cover designs, but analysis of their instructional designs quickly shows minimal adherence to the REACT strategy or anything similar.

CORD is different. CORD believes strongly in contextual learning and its ability to elevate the achievement of all students. This belief is not based on mere speculation, but rooted in years of experience and testing. CORD builds the REACT strategy into the substance and content of its educational products. Through CORD’s reforms, the potential of contextual learning has been realized. CORD’s research and reform initiatives have led to the development of new contextual learning materials in biology, applications in biology/chemistry, mathematics, and principles of technology. These materials provide mathematics teachers the tools they need for reaching *all* students, and in particular those in the middle 60 percent. The way these contextual-learning texts implement the REACT strategy is outlined in the pages that follow.

CORD Bridges to Algebra and Geometry

CORD Bridges prepares students for first-year algebra and formal geometry using an interactive, real-world approach to teaching math concepts. *CORD Bridges* provides the content (Figure 4) for a rigorous course for eighth- or ninth-grade students, a course based on a contextual, hands-on approach to learning.

Chapter 1	Decimals and Problem Solving
Chapter 2	Working with Data
Chapter 3	Integers
Chapter 4	Solving Equations
Chapter 5	Rational Numbers
Chapter 6	Ratio, Proportion, and Probability
Chapter 7	Percent
Chapter 8	Graphing on the Coordinate Plane
Chapter 9	Introduction to Geometry
Chapter 10	Powers and Roots
Chapter 11	Measurement
Chapter 12	Surface Area and Volume

Figure 4. Table of contents of *CORD Bridges*

CORD Bridges contains the following features:

- a motivational "Why should I learn this?" introduction to each chapter that gives students insight into how the mathematics in the chapter can be used to answer real-life questions
- concepts that are introduced by putting the student into a realistic scenario that stimulates learning
- development of new concepts through student-led discovery activities

- numerous examples that clarify concepts and provide students models for solving problems
- student exercises that include many real, believable problem-solving situations recognizable as being important to students' current or future lives
- cultivation of positive attitudes such as "I need to learn this" and "I can learn this"
- laboratory activities that support hands-on learners through measurement, data collection, and real-world simulation
- regular student participation in interactive groups where sharing, communicating, and decision making occur
- lessons, exercises, and labs that improve students' reading and other communication skills
- a four-step problem-solving plan, reinforced in each chapter with a different problem-solving strategy
- technology integration that makes effective use of scientific and graphing calculators and spreadsheet programs
- a teacher's annotated edition and a teacher resource book that supplement the text with alternative teaching strategies, answers, additional assessments, practice problems, reteaching activities, and project-based learning ideas

CORD Bridges implements the REACT strategy in meeting the needs of a widely diverse population of learners. Examples from the text that correspond to each REACT element are shown in the following sections.

Relating

Relating is learning in the context of life experiences. *CORD Bridges* uses relating as a tool by presenting situations completely familiar to the student and extracting new concepts or developing deeper understanding of concepts from those situations.

For example, the lesson on ratio and proportion begins with a familiar situation—making fruit punch from frozen concentrate. From the directions for the fruit punch, ratio is defined (Figure 5).

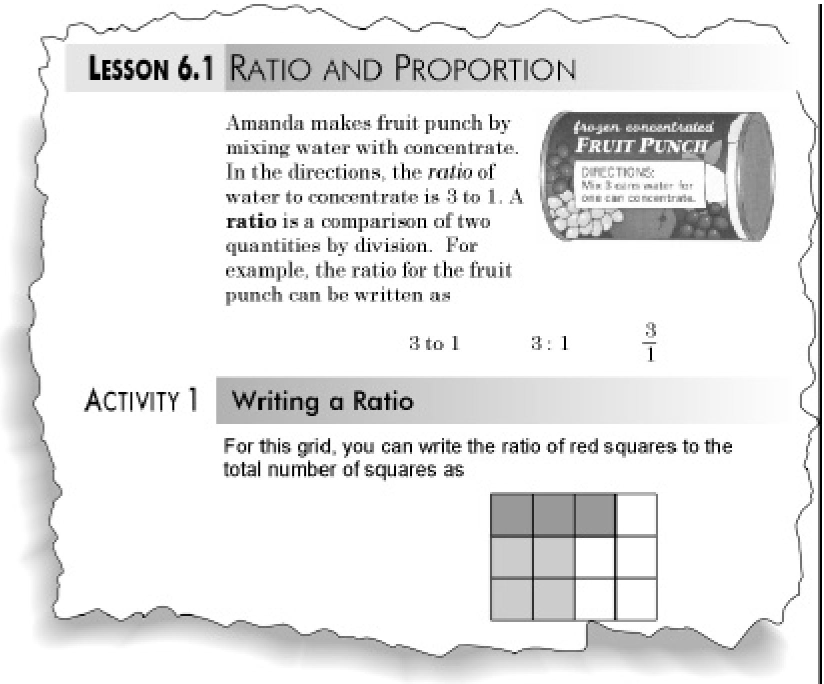


Figure 5. Students use a familiar situation to learn about ratios.

The rest of the lesson explores writing ratios for other situations and equating ratios in proportions. The lesson ends with an example of using ratios and proportions to decide how much water and concentrate are needed to make a large batch of punch (Figure 6).

EXAMPLE Using the Cross-Products Property

Amanda's fruit punch will have 3 cans of water per can of concentrate. Which of the following ratios for $\frac{\text{Water}}{\text{Concentrate}}$ should Amanda use to make 20 cans of punch?

a. $\frac{12}{8}$ b. $\frac{12}{4}$ c. $\frac{15}{5}$

SOLUTION

The $\frac{\text{Water}}{\text{Concentrate}}$ must be in the ratio $\frac{3}{1}$. The total amount of water plus concentrate must be 20 cans. Write a proportion and use the Cross-Products Property.

a. $\frac{3}{1} \stackrel{?}{=} \frac{12}{8}$, $3 \cdot 8 \neq 12 \cdot 1$ The ratio $\frac{12}{8}$ does not form a proportion.

b. $\frac{3}{1} \stackrel{?}{=} \frac{12}{4}$, $3 \cdot 4 = 12 \cdot 1$ The ratio forms a proportion, but the total amount of water plus concentrate is 16 cans.

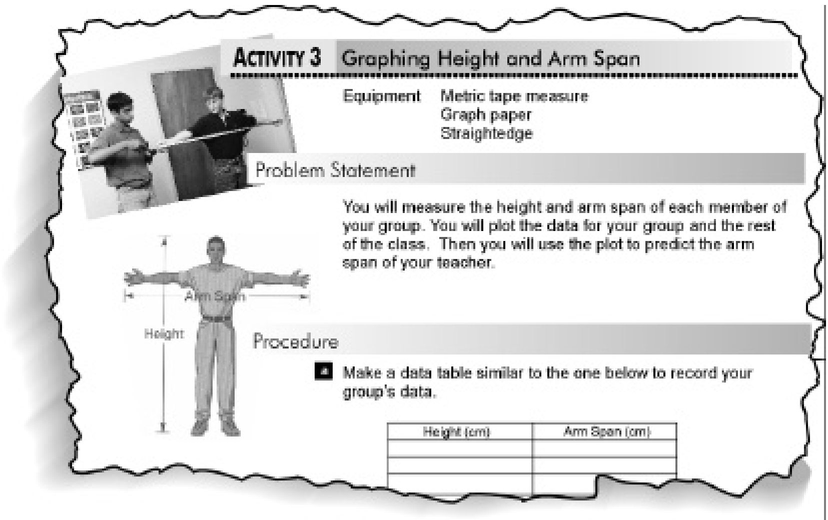
Figure 6. Examples provide students models for solving problems.

Experiencing

Experiential learning takes events and learning out of the realm of abstract thought and brings them into the realm of concrete exploration, discovery, and invention.

In addition to developmental activities in the lessons, *CORD Bridges* has three laboratory activities per chapter. These labs explore new concepts or enrich the students' understanding of fundamental concepts presented in the lessons. Students work in groups to collect data, record data in tables formatted for the lab, analyze data, and answer discussion questions.

In the lab from Chapter 2 shown in Figure 7, for example, students discover the power and utility of a correlation. They measure and record their heights and arm spans. After combining their group's data with those of the rest of the class, students plot the data on a coordinate plane and draw a line of best fit. At the end of the lab, students measure their teacher's height and use the fitted line to predict his or her arm span. The concepts of ordered pairs, scatter plots, and lines of best fit come alive when students use their own data in analysis and discussion.


ACTIVITY 3 Graphing Height and Arm Span

Equipment Metric tape measure
Graph paper
Straightedge

Problem Statement

You will measure the height and arm span of each member of your group. You will plot the data for your group and the rest of the class. Then you will use the plot to predict the arm span of your teacher.

Procedure

Make a data table similar to the one below to record your group's data.

Height (cm)	Arm Span (cm)

Figure 7. Students discover the power and utility of a correlation.

Applying

Learning is enhanced when concepts are presented in the context of their use. Students are motivated to learn math when examples and student tasks include real-life problems that students can recognize from their current or possible future lives. These problems inherently answer the legitimate question, "Why do I have to learn this?"

An example from one of the problem-solving features in *CORD Bridges* is shown in Figure 8.

PROBLEM SOLVING Using the Four-Step Plan

You are a computer engineer for a satellite launch. The launch time has been delayed from 9:30 p.m. to 11:45 p.m.

You need to reschedule the beginning and ending times of three programs that the satellite's computer will automatically run before and after the launch. The beginning times and the durations for each program are shown in the table.

Program	Beginning Time	Program Duration
Computer memory test	T – 15 minutes	3 minutes, 15 seconds
Launch safe hold	T – 8 minutes	15 minutes, 0 seconds
Separation from launch vehicle	T + 28 minutes	1 minute, 45 seconds

The beginning times of the programs are given relative to liftoff time *T*. The minutes and seconds before liftoff are negative.

Figure 8. A sample problem-solving exercise from *CORD Bridges*

Students plot positive and negative integers on a number line—the lesson's central theme—to solve this real satellite-launch problem. Even a student who does not aspire to become a computer engineer or to work with satellites can see and understand the applications of the mathematics concepts in solving this real-world problem, thus gaining confidence in his or her ability to solve similar problems outside the classroom.

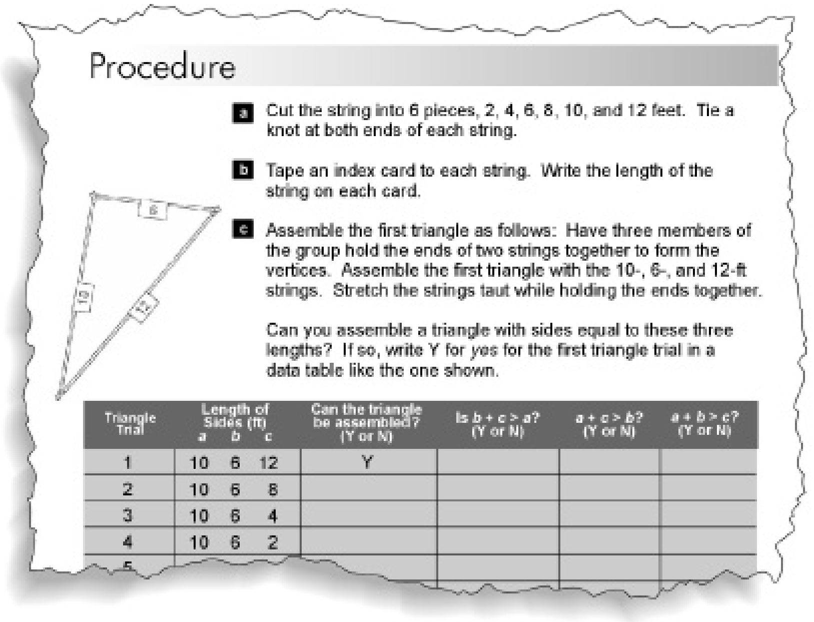
The best application problems and examples satisfy two criteria: They are real, not artificial or contrived, and they are important in some area of a student's life, current or future. *CORD Bridges* presents a great number of diverse situations, so all students will find scenarios applicable to their lives outside the classroom, for example, as consumers, family members, recreationists, sports competitors, workers, and citizens.

CORD Bridges cultivates two attitudes in students: “I can learn this math,” and “I need to learn this math.” Taken together, these attitudes motivate students to reach their fullest academic potential.

Cooperating

Cooperating is learning in the context of sharing, responding, and communicating with other learners. Cooperative learning has a positive effect on students’ achievement, interpersonal relationships, and communication skills. It also improves students’ attitudes toward the opposite gender and toward other racial and ethnic groups.

Laboratory activities in *CORD Bridges* involve cooperative learning. They get the student involved in the learning process. For example, in a lab on triangle inequalities, three group members act as vertices of a triangle. A fourth acts as data recorder (Figure 9).



Procedure

a Cut the string into 6 pieces, 2, 4, 6, 8, 10, and 12 feet. Tie a knot at both ends of each string.

b Tape an index card to each string. Write the length of the string on each card.

c Assemble the first triangle as follows: Have three members of the group hold the ends of two strings together to form the vertices. Assemble the first triangle with the 10-, 6-, and 12-ft strings. Stretch the strings taut while holding the ends together.

Can you assemble a triangle with sides equal to these three lengths? If so, write Y for *yes* for the first triangle trial in a data table like the one shown.

Triangle Trial	Length of Sides (ft) a	b	c	Can the triangle be assembled? (Y or N)	Is $b + c > a$? (Y or N)	$a + c > b$? (Y or N)	$a + b > c$? (Y or N)
1	10	6	12	Y			
2	10	6	8				
3	10	6	4				
4	10	6	2				
5							

Figure 9. A cooperative learning activity involving triangle inequalities

The group must decide which combinations of string lengths will form triangles and which will not. Then, as a group, they study their results and make a conjecture about the sum of the lengths of any two sides of a triangle. The group uses its conjecture to predict other combinations of lengths that will form triangles, and tests the predictions by trying to form the triangles with strings.

In addition to lab activities in *CORD Bridges*, many of the developmental activities in the lessons provide opportunities for cooperative learning. In the example shown in Figure 10, students use a calculator or spreadsheet program to generate sequences of random numbers to simulate basketball free throws.

ACTIVITY 1 Simulating Free Throws

1 Use a calculator, spreadsheet program, or table to generate a sequence of 20 random numbers between 1 and 10. Write the numbers in the order they are generated in a table similar to the one shown.

Simulated Free Throws	1	2	3	...	20
Random number	7	3	9	...	...
M = Made X = Missed	M	M	X	...	...

2 The list simulates Janet's 20 free throws. She has a 7 out of 10 chance of *making* each free throw. Label each number in the random sequence that is less than or equal to 7 as a free throw *made* (M). Label each number greater than 7 as a free throw *missed* (X).

3 Your list simulates one trial of 20 free throws attempted. Does your trial include at least five free throws made in a row? If so, you have simulated a favorable outcome.

4 Combine your trial with those from the rest of the class. How many favorable outcomes are there? How many trials are there?

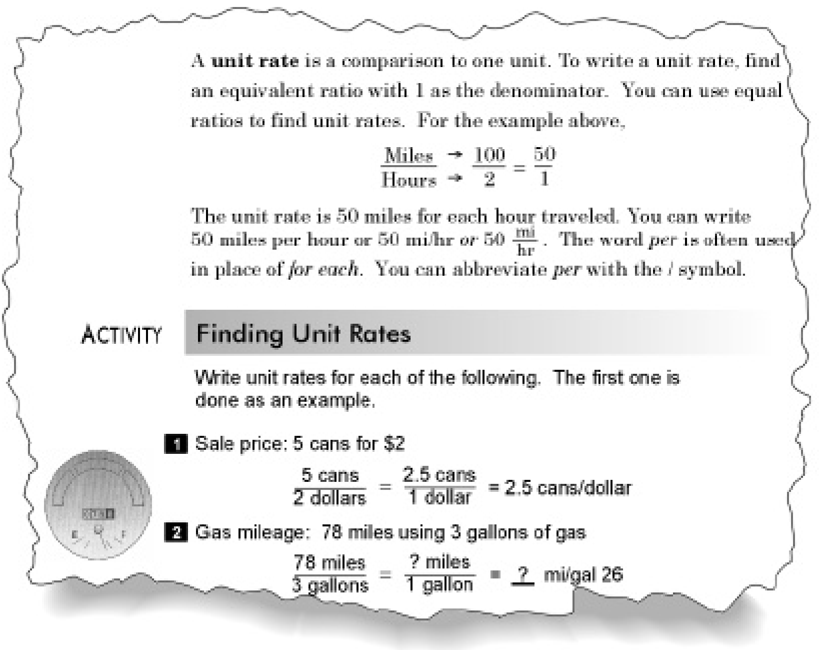
5 Use the class's data to estimate Janet's probability of making 5 free throws in a row when she attempts 20.

Figure 10. Students generate sequences of random numbers to simulate basketball free throws.

Many students will not understand immediately why a number less than or equal to 7 should represent a free throw made. If the activity is done in a group, someone in the group will probably be able to explain the rationale. In addition, the 20 trials recorded by each group must be combined with those from the rest of the class to have enough data to estimate the final probability. This is an opportunity for a larger group interaction.

Transferring

Transferring is using knowledge—existing or newly acquired—in a new context or situation. For example, when introducing dimensional analysis, *CORD Bridges* starts with students' existing knowledge of the distance formula, ratios, and proportions. The familiar relationship between distance and time is used to find the speed (or rate) of an automobile. Rate is generalized as a ratio that compares two unlike quantities (such as miles and hours). A unit rate is then defined as a comparison to one unit. A method of finding unit rates is given using the automobile example and proportions. Students are very familiar with speed as miles per hour. At the beginning of the lesson shown in Figure 11, they learn that it is also a unit rate.



A **unit rate** is a comparison to one unit. To write a unit rate, find an equivalent ratio with 1 as the denominator. You can use equal ratios to find unit rates. For the example above,

$$\frac{\text{Miles}}{\text{Hours}} \begin{matrix}\rightarrow\\\rightarrow\end{matrix} \frac{100}{2} = \frac{50}{1}$$

The unit rate is 50 miles for each hour traveled. You can write 50 miles per hour or 50 mi/hr *or* 50 $\frac{\text{mi}}{\text{hr}}$. The word *per* is often used in place of *for each*. You can abbreviate *per* with the / symbol.

ACTIVITY **Finding Unit Rates**

Write unit rates for each of the following. The first one is done as an example.

1 Sale price: 5 cans for $2

$$\frac{5 \text{ cans}}{2 \text{ dollars}} = \frac{2.5 \text{ cans}}{1 \text{ dollar}} = 2.5 \text{ cans/dollar}$$

2 Gas mileage: 78 miles using 3 gallons of gas

$$\frac{78 \text{ miles}}{3 \text{ gallons}} = \frac{? \text{ miles}}{1 \text{ gallon}} = \underline{?} \text{ mi/gal } 26$$

Figure 11. Students are introduced to the concept of unit rate through the familiar concept of miles per hour.

Students are tasked immediately with using the new knowledge in writing unit rates in different situations. Thus, the foundation for using unit rates in dimensional analysis and conversion factors in a broad range of situations is established through the transference of existing knowledge in conjunction with the introduction of two new concepts—rate and unit rate.

CORD Algebra 1

CORD Algebra 1 employs an interactive, workplace-centered approach to teaching algebra concepts. This approach is consistent with the design philosophy of the *CORD Applied Mathematics* series from which *CORD Algebra 1* was derived. *CORD Algebra 1* provides the contents (Figure 12) for a rigorous secondary-level first-year algebra course and an instructional design that is based on a contextual, hands-on approach to learning.

Chapter 1	Integers and Vectors
Chapter 2	Scientific Notation
Chapter 3	Using Formulas
Chapter 4	Solving Linear Equations
Chapter 5	Linear Functions
Chapter 6	Nonlinear Functions
Chapter 7	Statistics and Probability
Chapter 8	Systems of Equations
Chapter 9	Inequalities
Chapter 10	Polynomials and Factors
Chapter 11	Quadratic Functions
Chapter 12	Right-Triangle Relationships

Figure 12. Table of contents of *CORD Algebra 1*

CORD Algebra 1 contains the following features:

- a “Why should I learn this?” segment whose answer gives students insights and motivation into how the mathematics in a chapter can positively affect their lives and future careers
- workplace and other real-life scenarios that introduce concepts by putting students into contextually rich environments that stimulate learning by drawing on past experiences
- activities that encourage students to actively engage with the concepts presented in the text and make discoveries individually or in groups
- numerous examples that clarify concepts and provide students models for solving problems
- real-world applied problems that provide students an interesting and relevant way of translating mathematics from an abstract, theoretical approach to a concrete, applied approach
- laboratories that support hands-on learners through measurement, data collection, and real-world simulation
- videos that bring the workplace into the classroom and demonstrate the importance of mathematics as an essential workplace skill
- technology integration that makes effective use of graphing calculators and computers
- a teacher resource book that supplements the text with additional assessments and practice problems, reteaching activities, and project-based learning ideas

CORD Algebra 1 prepares students for more advanced mathematics courses and helps diverse learners master algebraic concepts that are essential for success in today’s high-tech workplace. The REACT strategy is a key element in meeting student learning needs and helping students reach their fullest intellectual potential. The following demonstrates how *CORD Algebra 1* implements this strategy.

Relating

The curriculum that attempts to place learning in the context of life experiences must, first, call the student's attention to everyday sights, events, and conditions. It must then *relate* those everyday situations to new information to be absorbed or a problem to be solved.

In Chapter 3 of *CORD Algebra 1,* the familiar problem of finding the cost of doing a job is related to the mathematical concepts of variables and expressions (Figure 13). First, students are provided all the basic information needed to find the cost of a job, i.e., on-site fees, hourly rate, and total hours worked. Then a table is provided that enables students to discern the relationships between these parameters. Finally, students develop algebraic expressions that model these relationships. To check the validity of their expressions, students can relate their everyday experiences to the results generated by these expressions. For instance, students know intuitively that the longer an hourly employee works the more money she or he makes. They can relate this fact to their mathematical expressions to see if they generate commensurate results.

Algebraic Expressions

Every time you hire an electrician, it costs $50 for the house call. The total bill, however, will depend on the number of hours the electrician is on the job. A table is useful in finding the cost for the total number of hours on the job when the hourly cost is $30.

Hours	Expression	Cost
1	$50 + 30 \cdot 1$	80
2	$50 + 30 \cdot 2$	110
3	$50 + 30 \cdot 3$	140

The table shows that the cost depends on the number of hours the electrician works on the job. How can you find the cost after any

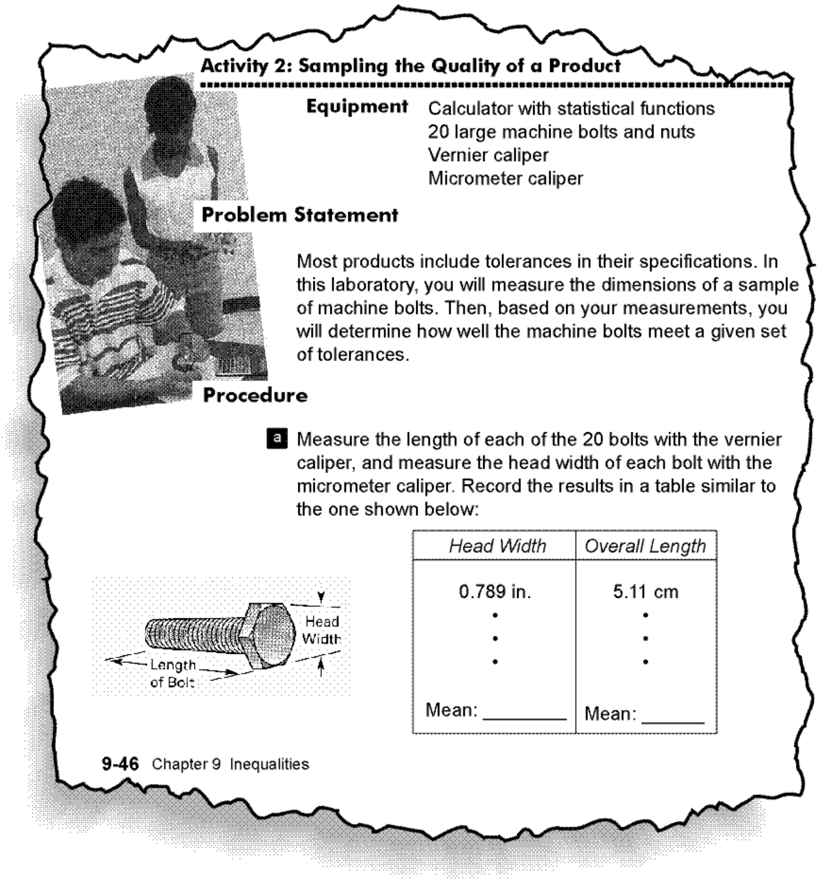
3.1 Variables and Expressions **3-5**

Figure 13. Finding the cost of doing a job is related to the mathematical concepts of variables and expressions.

By relating common experiences to new problems, students gain an appreciation for the power of mathematics and its ability to model the real world. This appreciation eventually manifests itself as motivation, and motivation is a key ingredient for enhancing learning.

Experiencing

Using *CORD Algebra 1* is a true hands-on learning experience. Through the thirty-five laboratories, students are placed in team situations in which they must make measurements, record data, and find graphical representations for discerning mathematical patterns. This experience actively engages students in the learning process and provides an alternate means for them to master mathematical concepts. For instance, in Chapter 9 of *CORD Algebra 1*, students are placed in a "you are there" scenario in which they are responsible for monitoring the quality of a product (Figure 14). During the laboratory they *experience* the variation in a sample of supposedly "identical" bolts. Through this experience they learn that manufacturing processes all lead to certain degrees of variation and that these variations can be represented as mathematical inequality statements. By doing this laboratory, students learn to work together, gain experience and confidence in making accurate measurements, and develop confidence in using mathematics for solving work-related problems.



Activity 2: Sampling the Quality of a Product

Equipment Calculator with statistical functions
20 large machine bolts and nuts
Vernier caliper
Micrometer caliper

Problem Statement

Most products include tolerances in their specifications. In this laboratory, you will measure the dimensions of a sample of machine bolts. Then, based on your measurements, you will determine how well the machine bolts meet a given set of tolerances.

Procedure

a Measure the length of each of the 20 bolts with the vernier caliper, and measure the head width of each bolt with the micrometer caliper. Record the results in a table similar to the one shown below:

Head Width
Length of Bolt

Head Width	*Overall Length*
0.789 in.	5.11 cm
•	•
•	•
•	•
Mean: ________	Mean: ______

9-46 Chapter 9 Inequalities

Figure 14. Students are placed in a "you are there" scenario in which they are responsible for monitoring the quality of a product.

Applying

Job-related *applications* may be based on devices and systems used in occupations or based on occupational activities. In *CORD Algebra 1,* Chapter 8, the "Workplace Communication" feature challenges students to solve a real occupational problem related to the Internet (Figure 15). To solve this problem, students form teams and develop a strategy for dealing with the given data. They find that the statistical techniques presented in the chapter are useful tools for formulating a solution. Students know of the Internet and see it as a part of their future. Thus, this problem is motivational and provides a forum for showing the relevance of mathematics to state-of-the-art technologies.

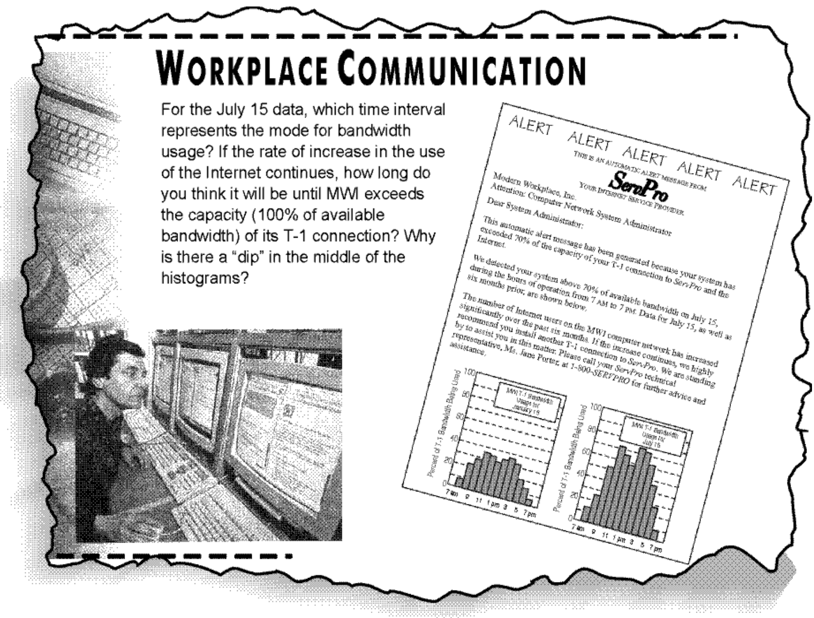
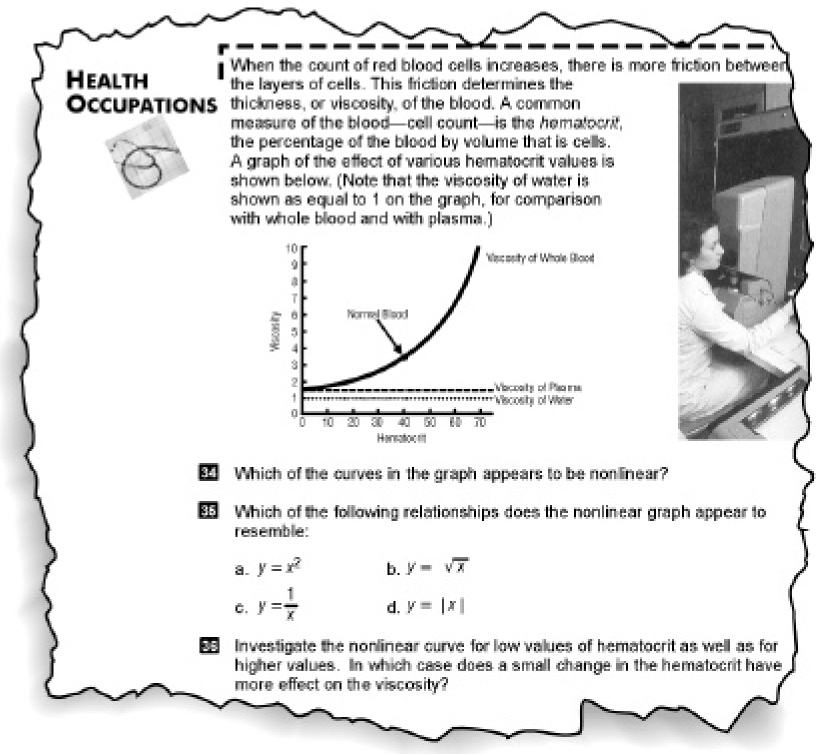


Figure 15. Students are challenged to solve a real occupational problem related to the Internet.

CORD Algebra 1 contains a feature called "Math Applications," which appears in each chapter. These applications cover the areas of business and marketing, industrial technology, agribusiness and agriculture, health occupations, and family and consumer sciences. This feature highlights occupational activities. For instance, in Chapter 6, students are asked to interpret a nonlinear curve that displays the hematocrit value of the blood and how it is related to blood's thickness or viscosity (Figure 16).



HEALTH OCCUPATIONS

When the count of red blood cells increases, there is more friction between the layers of cells. This friction determines the thickness, or viscosity, of the blood. A common measure of the blood—cell count—is the *hematocrit*, the percentage of the blood by volume that is cells. A graph of the effect of various hematocrit values is shown below. (Note that the viscosity of water is shown as equal to 1 on the graph, for comparison with whole blood and with plasma.)

34 Which of the curves in the graph appears to be nonlinear?

35 Which of the following relationships does the nonlinear graph appear to resemble:

a. $y = x^2$ b. $y = \sqrt{x}$

c. $y = \frac{1}{x}$ d. $y = |x|$

36 Investigate the nonlinear curve for low values of hematocrit as well as for higher values. In which case does a small change in the hematocrit have more effect on the viscosity?

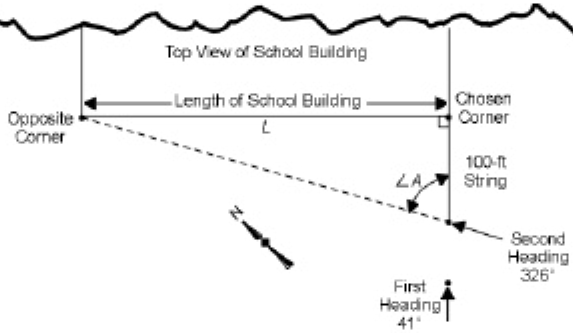
Figure 16. Blood flow is used as the context for explaining nonlinear functions.

Through this problem, students gain a contextual foundation for understanding the difference between nonlinear and linear effects. Students begin to understand that, at some point in a nonlinear process, small inputs can cause large changes. Because of the context of this problem, they can rationalize this behavior by picturing the density of the cells in an artery or vein and the interaction between these cells as the density increases. This picture provides a concrete example of how nonlinear effects occur and the power of mathematics in describing these effects. As seen by this problem, context adds a dimension to learning that allows students to use their senses as learning aids. For some students, this added dimension overcomes their inability to understand concepts from a purely conceptual perspective.

Cooperating

Cooperative learning is based on the belief that learning is inherently social. Since common inquiry is basic to the learning process, a teacher must challenge students to see that they will “sink or swim together.” The *CORD Algebra 1* laboratories provide this level of challenge. Having been adequately prepared to understand the mathematics concepts underlying the laboratories, students are challenged to validate these concepts through measurement and data analysis. Given the uncertainties of some laboratory processes, students often must pool their thoughts, experiences, and expertise to troubleshoot the sources of erroneous results or poor experimental techniques. The synthesis of thought that results from these efforts shows students the power of working in teams and sets the environment for developing teamworking strategies.

For instance, in Chapter 12 of *CORD Algebra 1,* students are asked to perform a surveyor task using right-triangle relations to find the length of one side of their school building (Figure 17). This laboratory requires a team of students to coordinate the necessary activities for collecting the required data, which involve angle and distance measurements. To get reasonable results, students must determine a method for accurately sighting the compass in the desired directions and keeping the string level over a 100-foot run. Since there is no set way for making these angle measurements or ensuring the levelness of the string, students are left with an opened-ended situation that invites brainstorming and cooperation.



c Subtract the two compass headings. If the difference between the compass headings is less than 90°, this difference is the measure of $\angle A$ in the drawing. If the difference is greater than 90°, subtract the difference from 360° to get $m\angle A$.

For example, suppose the compass headings were 41° and 326°, as shown in the drawing. This gives $326° - 41° = 285°$. Since 285° is greater than 90°, subtract 285° from 360° to get 75°. The measure of is $\angle A$ 75°.

d Use the formula below to calculate the distance between the two corners.

$$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{\text{length of side of school building}}{\text{Length of string}}$$

$$\tan A = \frac{L}{100 \text{ feet}}$$

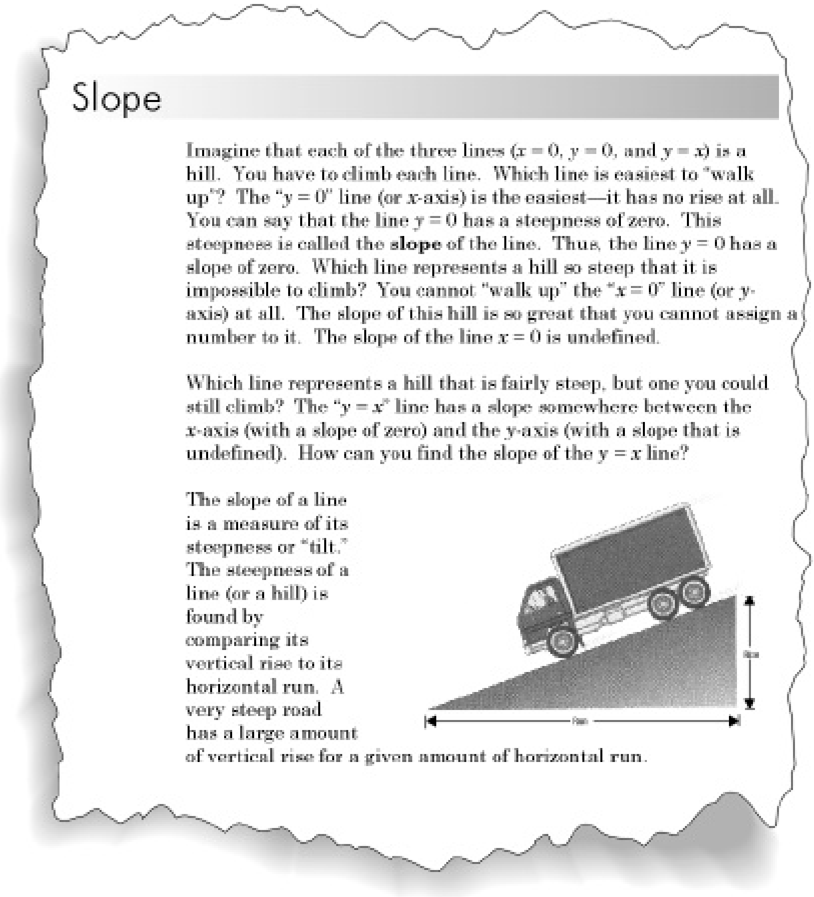
$$L = 100 \tan A$$

e Describe how you could use this method to measure the distance between two points on opposite sides of the Grand Canyon.

Figure 17. Using right-triangle relations to find the length of one side of their school building, students become familiar with tasks performed by surveyors.

Transferring

Sometimes students have knowledge of which they are not aware, simply because what they know has not been called to their attention or named or otherwise given a value. For example, virtually every student has experienced walking up and down hills. Yet chances are that no one has pointed out to them that they therefore have practical knowledge of the mathematical concept of slope. Transferring this practical knowledge requires making linkages between students' experiences and the mathematical concepts underlying those experiences. In Chapter 5 of *CORD Algebra 1*, these linkages are made by relating the steepness or tilt of a hill to the slope parameters of rise and run (Figure 18). Using these linkages, students are able to relate their knowledge of steep and shallow hills to mathematical lines with large and small slopes. Slope is no longer an abstract mathematical concept, but rather an intuitive reality of their everyday lives.



Slope

Imagine that each of the three lines ($x = 0$, $y = 0$, and $y = x$) is a hill. You have to climb each line. Which line is easiest to "walk up"? The "$y = 0$" line (or x-axis) is the easiest—it has no rise at all. You can say that the line $y = 0$ has a steepness of zero. This steepness is called the **slope** of the line. Thus, the line $y = 0$ has a slope of zero. Which line represents a hill so steep that it is impossible to climb? You cannot "walk up" the "$x = 0$" line (or y-axis) at all. The slope of this hill is so great that you cannot assign a number to it. The slope of the line $x = 0$ is undefined.

Which line represents a hill that is fairly steep, but one you could still climb? The "$y = x$" line has a slope somewhere between the x-axis (with a slope of zero) and the y-axis (with a slope that is undefined). How can you find the slope of the $y = x$ line?

The slope of a line is a measure of its steepness or "tilt." The steepness of a line (or a hill) is found by comparing its vertical rise to its horizontal run. A very steep road has a large amount of vertical rise for a given amount of horizontal run.

Figure 18. Students make linkages between mathematics and everyday experiences by relating the steepness of a hill to the slope parameters of rise and run.

CORD Geometry

CORD Geometry employs the same interactive, real-world approach to teaching math concepts as *CORD Bridges* and *CORD Algebra 1*. *CORD Geometry* provides the content (Figure 19) for a rigorous course that is based on a contextual, hands-on approach to learning.

Chapter 1	Discovering Geometry
Chapter 2	Reasoning and Proof
Chapter 3	Parallel Lines
Chapter 4	Congruent Triangles
Chapter 5	Quadrilaterals
Chapter 6	Similar Triangles
Chapter 7	Coordinate Geometry
Chapter 8	Area
Chapter 9	Circles
Chapter 10	Surface Area and Volume
Chapter 11	Transformational Geometry

Figure 19. Table of contents of *CORD Geometry*

CORD Geometry contains the same features as *CORD Algebra 1*:

- a "Why should I learn this?" segment whose answer gives students insights and motivation into how the mathematics in a chapter can positively affect their lives and future careers
- workplace and other real-life scenarios that introduce concepts by putting students into contextually rich environments that stimulate learning by drawing on past experiences
- activities that encourage students to actively engage with the concepts presented in the text and make discoveries individually or in groups
- numerous examples that clarify concepts and provide students models for solving problems
- real-world applied problems that provide students an interesting and relevant way of translating mathematics from an abstract, theoretical approach to a concrete, applied approach

- laboratories that support hands-on learners through measurement, data collection, and real-world simulation
- videos that bring the workplace into the classroom and demonstrate the importance of mathematics as an essential workplace skill
- technology integration that makes effective use of graphing calculators and computers
- a teacher resource book that supplements the text with additional assessments and practice problems, reteaching activities, and project-based learning ideas

CORD Geometry implements the REACT strategy in meeting the needs of a diverse population of learners. Some examples from the text that correspond to REACT elements are shown in the following sections.

Relating

Relating is learning in the context of life experiences. *CORD Geometry* uses relating as a tool by presenting situations familiar to the student and extracting new concepts or developing deeper understanding of concepts from those situations.

For example, the lesson on deductive reasoning begins with a familiar situation—a dead car battery (Figure 20).

These three statements can be logically combined in the following way:

> When the headlight switch is on, the headlights are draining energy from the battery. *If the headlights are on all night,* the battery will have no energy the next morning. With no energy in the battery, *the car will not start.*

The highlighted words can be written in the form of a **conditional statement** or simply a **conditional,** as follows:

> If the headlights are on all night, then the car will not start.

The phrase *the headlights are on all night* is the hypothesis of the conditional. The phrase *the car will not start* is the conclusion.

EXAMPLE 1 Writing the Hypothesis and Conclusion

Identify the hypothesis and conclusion of each conditional.

a. If $2x = 8$, then $x = 4$.

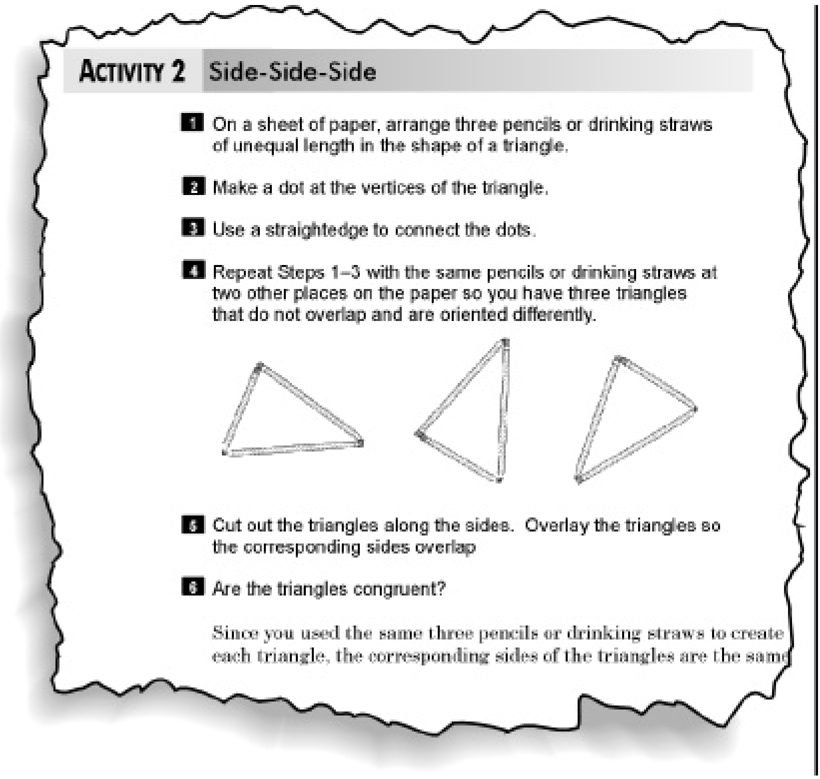
b. If a roof is properly constructed, then the building will not leak when it rains.

Figure 20. Students learn about deductive reasoning through a familiar situation.

Linking known facts logically to reach conclusions is deductive reasoning, and this is the cornerstone of constructing the geometric system. By relating this process to a common experience such as diagnosing the cause of a dead battery, students can more easily apply the process in other situations, including algebra and geometry. Relating is a key ingredient in fostering an attitude that says confidently, "I can learn this."

Experiencing

CORD Geometry contains interactive discovery activities in which students *learn* the important geometry concepts by *doing* geometry. For example, in a lesson on triangle congruence postulates (Figure 21), students discover side-side-side congruence in an activity.



ACTIVITY 2 Side-Side-Side

1. On a sheet of paper, arrange three pencils or drinking straws of unequal length in the shape of a triangle.
2. Make a dot at the vertices of the triangle.
3. Use a straightedge to connect the dots.
4. Repeat Steps 1–3 with the same pencils or drinking straws at two other places on the paper so you have three triangles that do not overlap and are oriented differently.
5. Cut out the triangles along the sides. Overlay the triangles so the corresponding sides overlap
6. Are the triangles congruent?

Since you used the same three pencils or drinking straws to create each triangle, the corresponding sides of the triangles are the same

Figure 21. Students use ordinary materials in an activity on triangle congruence postulates.

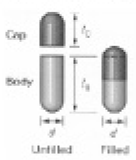
In this activity, students create the physical environment for discovering an important geometric concept: “If three sides of one triangle are congruent to three sides of another triangle, the triangles are congruent.” Using three pencils of fixed length, students can see that any arrangement using the same three sides produces congruent triangles. After demonstrating the concept for themselves with the pencils, students make a conjecture that the congruence relationship holds for any two triangles. The concept is then summarized as the side-side-side postulate (SSS).

Applying

Students are motivated to learn math when examples and student exercises include real-life problems that students can recognize as being relevant in their current or possible future lives. These problems inherently answer the familiar question, "Why do I have to learn this?"

Many of the real-life examples and exercises in *CORD Geometry* put students in workplace scenarios. In addition to geometry and problem solving, students learn something about the world of work in exercises such as the one shown in Figure 22.

Montgomery is a compounding pharmacist at a pharmaceutical manufacturing plant. He is responsible for ensuring the company's products contain the correct amounts of active and inert ingredients. He is also responsible for selecting the correct capsule sizes for specified dosages. When a compound is prepared, the dosage is determined by the capsule size. The company uses eight capsule sizes. The body length l_B, cap length l_C, and diameter d of the capsules are shown below.



Capsule Size	Body Length (mm)	Cap Length (mm)	Diameter (mm)
000	22.98	13.44	8.52
00	20.50	12.00	8.50
0	18.86	11.04	7.62
1	16.51	9.65	6.86
2	15.35	9.10	6.25
3	13.60	8.13	5.47
4	12.30	7.20	5.10
5	9.84	5.76	4.08

47 The volume of a filled capsule consists of a cylinder of length $(l_B - \frac{1}{2}d)$ and two hemispheres of diameter d. Calculate the volume of a filled number 0 capsule.

48 Montgomery must select a capsule size for production of a 25-milligram dosage of an antidepressant. Each capsule must contain 650 ± 10 mm^3 of the compound. Which capsule should Montgomery select for the production?

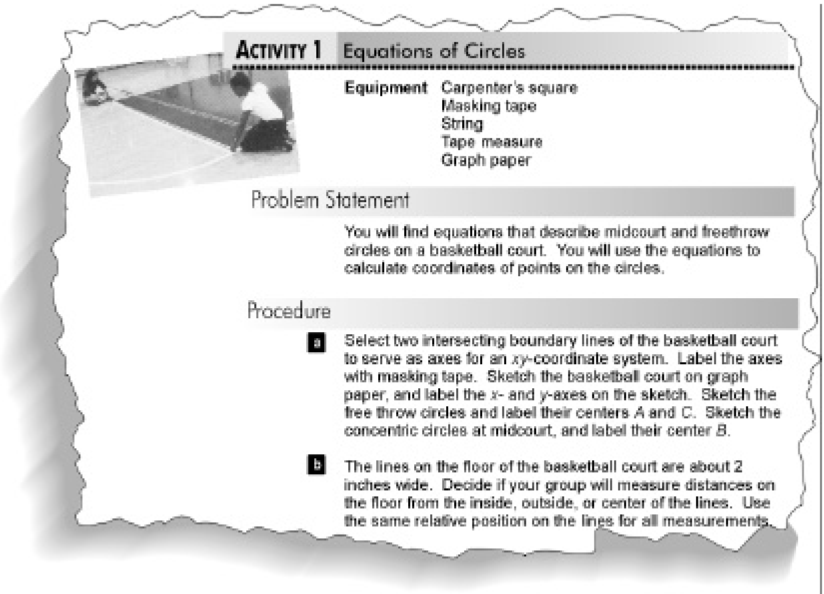
Figure 22. Students are placed in a workplace scenario.

In the exercise shown in the preceding figure, students learn that, in addition to chemistry, a pharmacist must use geometry and algebra on the job. To find the volume of a capsule, the pharmacist uses formulas for the volumes of a cylinder and a sphere. To decide which capsule is appropriate, the pharmacist uses a problem-solving plan.

Cooperating

In addition to the lesson activities, there are thirty-three laboratory activities in *CORD Geometry*. These labs provide hands-on learning experiences in which students work in groups to explore new concepts or enrich their understanding of fundamental concepts presented in the lessons. Student groups collect data, record data in tables formatted for the labs, analyze data, and answer discussion questions.

Many labs get the students out of the classroom, where more realistic measurements can be made. For example, in a lab on equations of circles (Figure 23), groups establish an *xy*-coordinate plane on a basketball court. The groups then determine the equations of the free throw circles and center circles on the court.



ACTIVITY 1 Equations of Circles

Equipment Carpenter's square
Masking tape
String
Tape measure
Graph paper

Problem Statement

You will find equations that describe midcourt and freethrow circles on a basketball court. You will use the equations to calculate coordinates of points on the circles.

Procedure

a Select two intersecting boundary lines of the basketball court to serve as axes for an *xy*-coordinate system. Label the axes with masking tape. Sketch the basketball court on graph paper, and label the *x*- and *y*-axes on the sketch. Sketch the free throw circles and label their centers *A* and *C*. Sketch the concentric circles at midcourt, and label their center *B*.

b The lines on the floor of the basketball court are about 2 inches wide. Decide if your group will measure distances on the floor from the inside, outside, or center of the lines. Use the same relative position on the lines for all measurements.

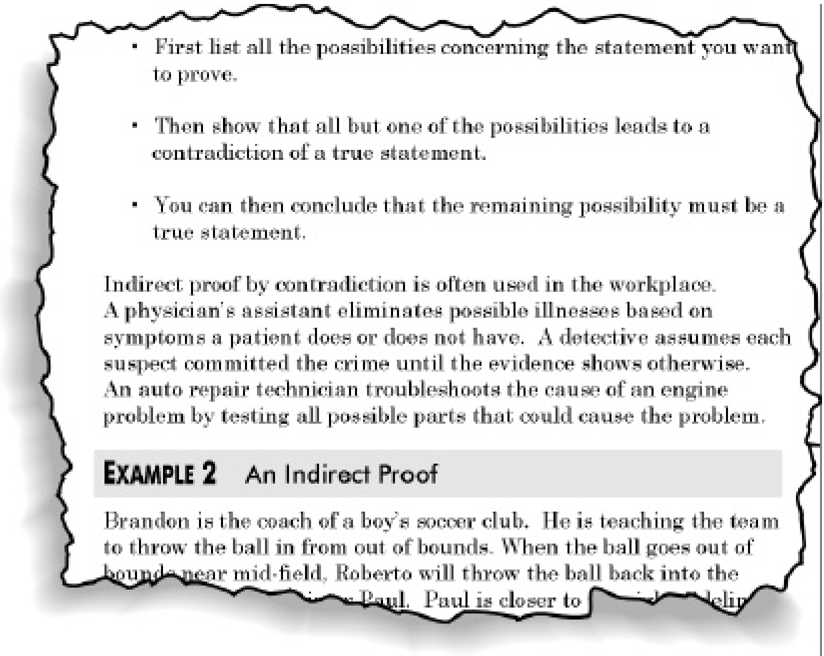
Figure 23. Students establish an *xy*-coordinate plane on a basketball court.

The group must decide where and how to establish the origin of the coordinate system, how to measure the coordinates of the centers of the circles, and how to write the equation of each circle. Then, as a group, students use their equations to predict the coordinates of points marked on the circles and test their predictions by measuring the coordinates.

Transferring

An indirect proof is one of the more difficult topics in mathematics. *CORD Geometry* introduces this method by comparing it to something many students already know—the process of elimination.

The method is used in an example that proves the opposite angle-side theorem in the context of a throw-in play in soccer (Figure 24).



- First list all the possibilities concerning the statement you want to prove.
- Then show that all but one of the possibilities leads to a contradiction of a true statement.
- You can then conclude that the remaining possibility must be a true statement.

Indirect proof by contradiction is often used in the workplace. A physician's assistant eliminates possible illnesses based on symptoms a patient does or does not have. A detective assumes each suspect committed the crime until the evidence shows otherwise. An auto repair technician troubleshoots the cause of an engine problem by testing all possible parts that could cause the problem.

EXAMPLE 2 An Indirect Proof

Brandon is the coach of a boy's soccer club. He is teaching the team to throw the ball in from out of bounds. When the ball goes out of bounds near mid-field, Roberto will throw the ball back into the ... Paul. Paul is closer to ...

Figure 24. *CORD Geometry* introduces students to a difficult topic—indirect proofs—through familiar concepts.

CORD Applied Mathematics—*A Contextual Approach to Integrated Mathematics*

CORD Applied Mathematics is a set of forty modular units (Figure 25) prepared to help students develop and refine job-related mathematics skills. These modules are based on an instructional design that emphasizes a contextual, hands-on approach to learning.

A	Getting to Know Your Calculator	16	Solving Problems That Involve Linear Equations
B	Naming Numbers in Different Ways	17	Graphing Data
C	Finding Answers with Your Calculator	18	Solving Problems That Involve Nonlinear Equations
1	Learning Problem-Solving Techniques	19	Working with Statistics
2	Estimating Answers	20	Working with Probabilities
3	Measuring in English and Metric Units	21	Using Right-Triangle Relationships
4	Using Graphs, Charts, and Tables	22	Using Trigonometric Functions
5	Dealing with Data	23	Factoring
6	Working with Lines and Angles	24	Patterns and Functions
7	Working with Shapes in Two Dimensions	25	Quadratics
8	Working with Shapes in Three Dimensions	26	Systems of Equations
9	Using Ratios and Proportions	27	Inequalities
10	Working with Scale Drawings	28	Geometry in the Workplace 1
11	Using Signed Numbers and Vectors	29	Geometry in the Workplace 2
12	Using Scientific Notation	30	Solving Problems with Computer Spreadsheets
13	Precision, Accuracy, and Tolerance	31	Solving Problems with Computer Graphics
14	Solving Problems with Powers and Roots	32	Quality Assurance and Process Control 1
15	Using Formulas to Solve Problems	33	Quality Assurance and Process Control 2
		34	Spatial Visualization
		35	Coordinate Geometry
		36	Logic
		37	Transformations

Figure 25. *CORD Applied Mathematics* units

CORD Applied Mathematics contains the following features:

- an integrated approach to presenting mathematics
- mathematics materials that support specific occupational studies in such areas as *health occupations, industrial*

technology, home economics, agriculture, or business and marketing

- motivational videos that introduce each mathematics unit and set the stage for the relevance of mathematics in the world of work
- concepts presented within a contextual setting
- student-led discovery activities
- numerous examples that clarify concepts and provide students models for solving problems
- student exercises that include many real-world problem-solving situations recognizable as being important to students' current or future lives
- laboratory activities that support hands-on learners through measurement, data collection, and real-world simulation
- interactive group activities where sharing, communicating, and decision making occur
- lessons, exercises, and labs that improve students' reading and other communication skills
- a four-step problem-solving process that students can build on and refine throughout their lives
- technology integration that makes effective use of scientific and graphing calculators and spreadsheet programs
- teachers' guides that include complete student editions; teacher notes; solutions and expected outcomes for video problems, laboratory activities, and problem-solving exercises; and blackline masters for end-of-unit tests, transparencies, and student handouts
- supplementary materials that include multiple-choice question banks, an implementation resource guide, skill

drill practice problems, a test database, study guides, and a teacher's resource guide

CORD Applied Mathematics is structured using the REACT strategy to meet the needs of the contextual learner. Some examples from the *CORD Applied Mathematics* materials that correspond to the five essential elements of the REACT strategy are shown in the following sections.

Relating

Relating is learning in the context of life experiences. *CORD Applied Mathematics* links everyday experiences to new skills or concepts.

For example, in Unit 28, an application of geometry in industrial technology (Figure 26) begins with an object familiar to everyone—a television.

The students are given two of the standards of the television industry—*aspect ratio* and *picture-tube size*. The students link these standards to the mathematical concepts of hypotenuse and Pythagorean theorem. Students are then challenged to find other parameters related to the design of a television screen.

Standard dimensions for TV picture tubes

All geometry in the industrial workplace does not necessarily involve mechanical items. The drawing below is something most of us see every day—the television.

The television industry has set a standard for the size of picture tubes. The ratio of *height-to-width* is 3:4. This ratio is called the *aspect ratio.* It means, for example, that a television picture tube 30 inches high will be 40 inches wide. Television *picture-tube size* is given in accordance with the *diagonal length* of the tube face. Thus, a nineteen-inch television set has a face with a 19-inch diagonal.

Figure 26. In Unit 28, an explanation of the application of geometry in industrial technology begins with a familiar object.

Experiencing

CORD Applied Mathematics has three laboratory activities at the end of each unit that provide hands-on experience for learning mathematics in team situations. Students are actively engaged in concrete learning in the context of exploration, discovery, and invention. These labs introduce new concepts or enrich the students' understanding of fundamental concepts presented in the unit. Students work in groups to collect data, record data in tables formatted for the labs, make calculations, analyze data, discuss results, and solve special challenge problems. For example, in Unit 4, students measure the corresponding diameters and circumferences of several circular objects (Figure 27).

Activity 3: Graphing diameter and circumference values

Equipment

Vernier caliper
Cloth measuring tape
Ruler
One-pound coffee can
Soup can
Wooden dowel about 1 inch in diameter
BB
Five-gallon bucket
Drawing kit (Accu-Line™)

Statement of Problem

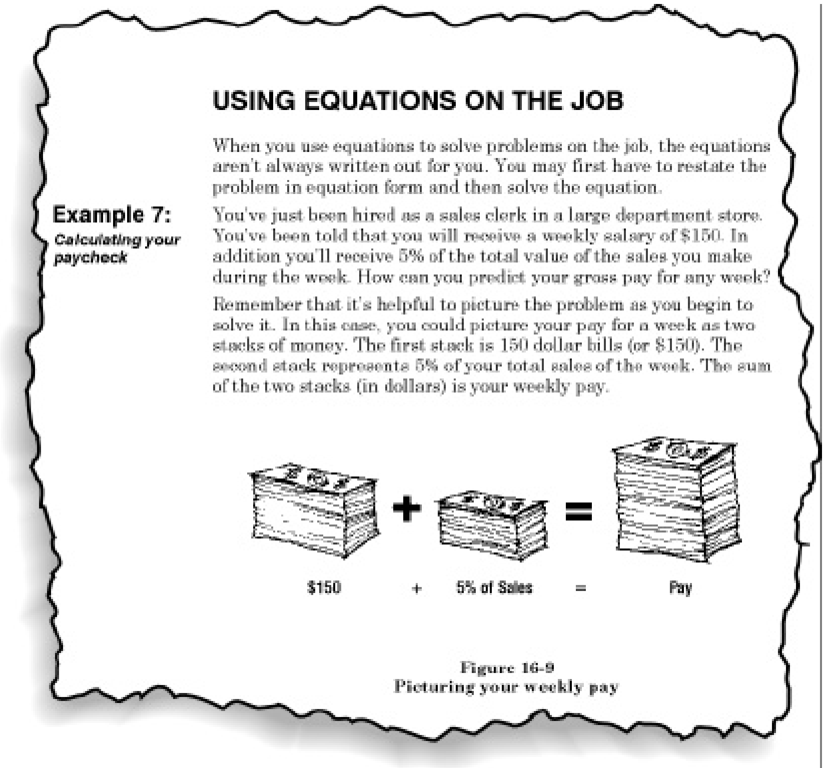
In this activity, you measure the corresponding diameter and circumference of several circular objects. Then you plot the corresponding measurements and draw a line graph. When you have drawn the line graph you will extend it to give the corresponding circumference and diameters of other circular objects you haven't measured.

Figure 27. In Unit 4, students gather data by measuring diameters and circumferences.

Students record the data as ordered pairs and then plot it on a coordinate plane. The students use the graph to determine the circumferences and diameters of other circular objects.

Applying

Applying concepts or information in a useful context enhances the learning process. Students are motivated to learn mathematical concepts when the information is presented in a setting that is relevant to them. An example problem from Unit 16 is shown in Figure 28.



USING EQUATIONS ON THE JOB

When you use equations to solve problems on the job, the equations aren't always written out for you. You may first have to restate the problem in equation form and then solve the equation.

Example 7:
Calculating your paycheck

You've just been hired as a sales clerk in a large department store. You've been told that you will receive a weekly salary of $150. In addition you'll receive 5% of the total value of the sales you make during the week. How can you predict your gross pay for any week?

Remember that it's helpful to picture the problem as you begin to solve it. In this case, you could picture your pay for a week as two stacks of money. The first stack is 150 dollar bills (or $150). The second stack represents 5% of your total sales of the week. The sum of the two stacks (in dollars) is your weekly pay.

$150 + 5% of Sales = Pay

Figure 16-9
Picturing your weekly pay

Figure 28. Students examine mathematical concepts in settings that are relevant to them.

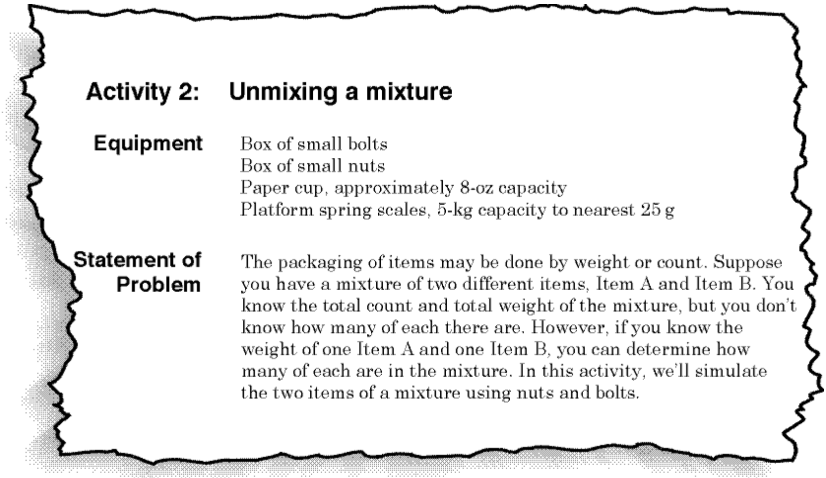
In the example, the information is presented in a familiar contextual setting—earning wages. Students find the application real and relevant to their lives. Thus, there is an intrinsic motivation to learn the mathematical concepts underlying the problem.

Cooperating

Cooperating is learning in the context of sharing, responding, and communicating. These skills are highly valued in the workplace.

CORD Applied Mathematics provides three laboratory activities in each unit. These laboratories are designed to encourage teamworking and develop students' interpersonal skills. Additionally, many of the activities and examples in *CORD Applied Mathematics* are very adaptable to the use of cooperative learning strategies.

In the example from Unit 26 shown in Figure 29, students use a system of equations to determine the number of nuts and bolts in a mixture.



Activity 2: **Unmixing a mixture**

Equipment

Box of small bolts
Box of small nuts
Paper cup, approximately 8-oz capacity
Platform spring scales, 5-kg capacity to nearest 25 g

Statement of Problem

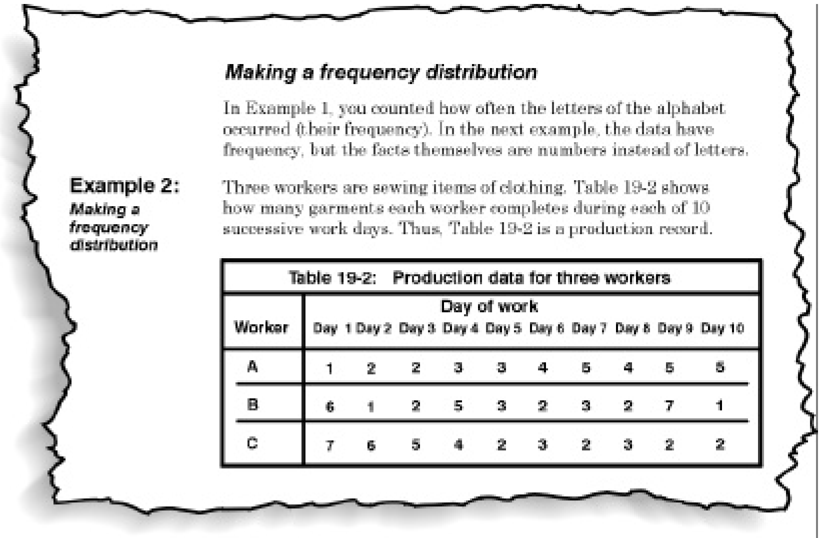
The packaging of items may be done by weight or count. Suppose you have a mixture of two different items, Item A and Item B. You know the total count and total weight of the mixture, but you don't know how many of each there are. However, if you know the weight of one Item A and one Item B, you can determine how many of each are in the mixture. In this activity, we'll simulate the two items of a mixture using nuts and bolts.

Figure 29. In Unit 26, students use the skills for solving a system of equations in a hands-on activity involving nuts and bolts.

Transferring

Students are motivated and develop confidence when a new learning experience uses and builds upon what they already know. This learning process is known as transferring.

For example, in the section *Measuring Central Tendency* from Unit 19, a table showing the production data for three workers is presented (Figure 30).



Making a frequency distribution

In Example 1, you counted how often the letters of the alphabet occurred (their frequency). In the next example, the data have frequency, but the facts themselves are numbers instead of letters.

Example 2: ***Making a frequency distribution***

Three workers are sewing items of clothing. Table 19-2 shows how many garments each worker completes during each of 10 successive work days. Thus, Table 19-2 is a production record.

Table 19-2: Production data for three workers

Worker	Day of work									
	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
A	1	2	2	3	3	4	5	4	5	5
B	6	1	2	5	3	2	3	2	7	1
C	7	6	5	4	2	3	2	3	2	2

Figure 30. In Unit 19, students learn how to make a frequency distribution by studying production data from a manufacturing setting.

The skill of reading charts and tables provides the background for teaching students how to make frequency distribution tables. From these tables students learn the three measures of central tendency—mean, median, and mode. The section concludes with students determining when the mean, median, or mode best represents the requested measure of central tendency. Thus, students build on their knowledge of graphs and use this knowledge as a foundation for learning the more complete concepts of central tendency.

How Can the Success of Contextual Learning Be Measured?

It is important to understand that student performance is only one factor to consider in determining the success of applied academics. If we are truly looking for different approaches to teaching and learning, our expectations and assessments should reflect these differences.

What should our expectations for success be?

The following is a list of criteria that provide some metrics for measuring the effectiveness of the REACT strategy.

- Students are able to transfer knowledge from academic content to vocational applications and from school to the workplace.
- Students are not afraid to take academic subjects such as math and science.
- Students are more interested and motivated and better understand the value of the subject and of school in general than they did in classes taught using traditional methods.
- The applied course is as challenging as the traditional "college-prep" course on the same subject—not low level or watered down.
- The student population that has traditionally done poorly in academic subjects displays improved performance.
- Applied courses receive the same recognition and acceptance from universities and colleges as do the traditional courses with the same content.

Educators value quantitative evaluations in determining the effectiveness of new teaching strategies. To accurately evaluate the impact of a contextually based strategy, educators must understand the environment in which this strategy is being presented. The following list of assumptions helps define this environment:

- Most of the students enrolled in an applied academics course traditionally have not been high achievers in that subject or discipline.
- Most of the students enrolled in applied academics courses do not have significant learning disabilities.

- Most of the teachers of applied academics curricula are certified in the academic subjects/disciplines in which they teach.
- Teachers have not necessarily used the pedagogy of applied/contextual methodology in the teaching of their courses in the past.
- All teachers of applied academics courses have received training in the different teaching methods, the lab equipment, and the overall management of materials and activities related to the academic course in question.

Empowering Teachers

Implementing contextual learning in the classroom does not simply require new tools such as applied academics courses. It also requires new teaching techniques. For contextual learning to have its maximum effect on students, teachers must be empowered to effectively implement the REACT strategy. This empowerment can come only through professional development. This professional development must acquaint teachers with the theory of contextual learning and its translation into classroom-specific practices. An additional component of this professional development, which is often overlooked, must include providing the information teachers need about the relationships of academic curricula to personal, societal, and especially occupational life.

We should not continue to expect teachers to do all the work of seeking out real-life applications and relating them to students through a variety of experiences. Instead, they need the help of employers and other community representatives to make the connection with the workplace that will enrich their teaching. They need adequate training and sufficient release time to learn new teaching methods and become familiar with new materials. (All of the research and field tests conducted on applied academics indicate clearly that the success of the curricula depends heavily on adequate teacher training prior to implementation of the curricula.)

Said in a different way, the most powerful tool is effective only when placed in skilled hands. CORD's applied academic approach is a powerful educational tool. For its full impact on education to be realized, teachers must be trained in its effective use.