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Formulation of the Internal Stress Equations of Pinned Portal Frames Putting Shear Deformation into Consideration

Okonkwo V. O, Onyeyili I. O, Aginam C. H., Chidolue C. A

Abstract- In this work the internal stress equations for pinned portal frames under different kinds of loading was formulated using the equilibrium method. Unlike similar equations in structural engineering textbooks these equations considered the effect of deformation due to shearing forces. This effect was captured in a dimensionless constant α, when α is set to zero, the effect of shear deformation is removed and the equations become the same as what can be obtained in any structural engineering textbook. An investigation into the effect of shear deformation on the internal stresses and its variation with the ratios of second moment of areas of the horizontal and vertical members of the frame ($^{I_2}/_{I_1}$) and the ratio of height to length of the portal frame ($^{h}/_{L}$) showed that the effect of shear deformation is generally small and can be conveniently neglected in manual calculations except for pinned portal frames under concentrated horizontal forces where the effect was considerable.

Keywords: Flexural rigidity, Pinned Portal frames, shear deformation, stiffness matrix

I. INTRODUCTION

Portal frames are generally low-rise structures that consist of columns and horizontal rafters connected rigidly. They are the mostly used structural forms for single storey buildings. Portal frames are usually made of steel but can be made of concrete or timber. It is estimated that around 50% of the hot-rolled constructional steel used in the UK is fabricated into single-storey buildings [1]. This no doubt shows the growing importance of this structural unit. Computer analysis of structures has taken the central stage in the analysis of structures [2] and few computers programs incorporate the effect of shear deformation [3]. However, many simple structures like portal frames are still analyzed manually using equations found in structural engineering textbooks and design manuals [4]. These equations are very useful and can be used to check computer results [5]. These equations were however formulated without any consideration for the effect of shear deformation on the internal stresses hence the need for the development of equations that capture the contribution of shear deformation in portal frames for different loading conditions.

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Okonkwo V. O, Department of Civil Engineering, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria.
Onyeyili I. O, Department of Civil Engineering, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria.
Aginam C. H., Department of Civil Engineering, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria.
Chidolue C. A, Department of Civil Engineering, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria.

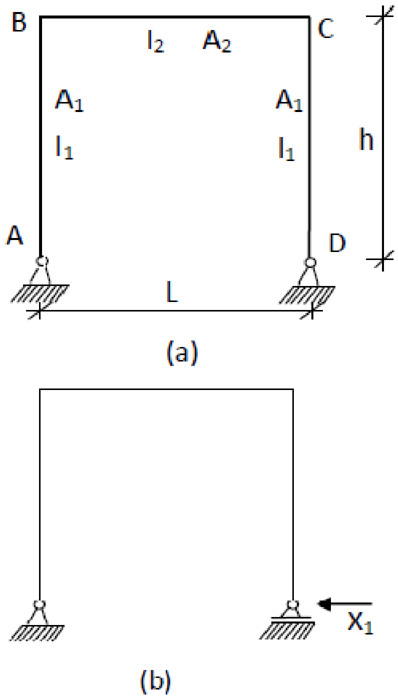

Figure 1 : The Basic System showing the removed redundant force

II. APPLICATION OF FLEXIBILITY METHOD

The basic system or primary structure for the structure in Figure 1a is given in Figure 1b. The removed redundant force is depicted with X_1.

The flexibility matrix of the structure can be determined using the principle of virtual work.

By applying the unit load theorem the deflection in beams or frames can be determined for the combined action of the internal stresses, bending moment and shearing forces with

$$D = \int \frac{\bar{M}M}{EI} ds + \int \frac{\bar{V}V}{GA_r} ds \quad . \quad . \quad . \quad . \quad (1)$$

$$A_r = \frac{A}{\aleph} \quad . \quad . \quad . \quad . \quad (1a)$$

Where $\bar{M}$ and $\bar{V}$ are the virtual internal stresses while M and V are the real/actual internal stresses.

E is the modulus of elasticity of the structural material

A is the cross-sectional area of the element

G is the modulus of elasticity in shear, $G = \frac{E}{2(1+v)}$ where v is poisson's ratio

The reduced area Ar of the section can be evaluated from

$$A_r = \frac{I^2}{\int_\Omega R^2 d\Omega} \quad . \quad . \quad . \quad . \quad . \quad (1b)$$

Ar is the reduced area, I is the second moment of area of the cross section.

$R = \frac{S}{b}$ . . . . (1c)

Where b is the width of the section, $\int_{c_2}^{c_1} y\, da$. c_1 is the distance of the topmost fibre from the neutral axis, c_2 is the distance of the bottom fibre from the neutral axis and y is the distance from the neutral axis to any infinitesimal area on the cross section da.

For rectangular sections $\varkappa$ is 1.2; for a circular cross section it is 1.185 [6] and for a circular tube it is 1/6 [7]. The values of $\varkappa$ for other cross sections are given in [8].

$\varkappa$ is as defined earlier. [9], [10].

If d_{ij} is the deformation in the direction of i due to a unit load at j then by evaluating equation (1).

$$d_{11} = \frac{h(2h^2 I_2 GA_{1r} + 3hlI_1 GA_{1r} + 6EI_1 I_2)}{3EI_1 I_2 GA_{1r}} \quad . \quad (2)$$

The structure's compatibility equation can be written thus

$$d_{10} + X_1 d_{11} = 0. \quad . \quad . \quad . \quad (3)$$

Where X_1 is the redundant force and d_{10} is the deformation due to external load on the basic system (reduced structure) .

$$X_1 = -d_{10}/d_{11} \quad . \quad . \quad . \quad . \quad (4)$$

Equation (4) is evaluated to get the redundant force and this is substituted into the structure's force equilibrium (superposition) equation to obtain the internal stress at any point.

$$M = M_o + M_1 X_1 \quad . \quad . \quad . \quad (5)$$

Where M is the required stress at a point, M_o is the stress at that point for the reduced structure, M_1 is stress at that point when the redundant force X_1 =1 acts on the reduced structure.

For the loaded portal frame of Figure 2, the deformation of the reduced structure due to external load is

$$d_{10} = -\frac{ql^3 h}{12EI_2} \quad . \quad . \quad . \quad . \quad (6)$$

By substituting the value of equation (6) into equations (4)

$$X_1 = \frac{wl^3(I_1 GA_{1r})}{4(2h^2 I_2 GA_{1r} + 3hlI_1 GA_{1r} + 6EI_1 I_2)} \quad . \quad . \quad (7)$$

Evaluating equation (5) for point B and C on the structure using the force factor obtained in equation (7)

$$M_B = M_C = \frac{wl^3 h(I_1 GA_{1r})}{4(2h^2 I_2 GA_{1r} + 3hlI_1 GA_{1r} + 6EI_1 I_2)} . \quad . \quad (8)$$

For the loaded portal frame of Figure 3, the deformation of the reduced structure due to external load is

$$d_{10} = \frac{-whl^3}{24EI_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

By substituting the value of equation (9) into equations (4)

$$X_1 = \frac{wl^3 I_1 GA_{1r}}{8(2h^2 I_2 GA_{1r} + 3hlI_1 GA_{1r} + 6EI_1 I_2)} \quad . \quad . \quad . \quad (10)$$

Evaluating equation (5) for point B and C on the structure using the force factor obtained in equations (10)

$$M_B = M_C = -\frac{whl^3 I_1 GA_{1r}}{8(2h^2 I_2 GA_{1r} + 3hlI_1 GA_{1r} + 6EI_1 I_2)} . \quad . \quad (11)$$

This process was repeated for other loaded portal frames and the results are presented in Table 1.

III. DISCUSSION OF RESULTS

The internal stress (Bending Moments) on the loaded portal frames is summarized in table 1. The effect of shear deformation is captured by the dimensionless constant α and is taken as the ratio of the end translational stiffness to the shear stiffness of a member.

$$\propto_1 = \frac{12EI_1}{h^3} \cdot \frac{h}{GA_{1r}} = \frac{12EI_1}{h^2 GA_{1r}} \quad . \quad . \quad (12)$$

$$\propto_2 = \frac{12EI_2}{l^3} \cdot \frac{l}{GA_{2r}} = \frac{12EI_2}{l^2 GA_{2r}} \quad . \quad . \quad . \quad (13)$$

When $\propto_1 = 0$, the effect of shear deformation in the columns is ignored and likewise when $\propto_2 = 0$, the effect of shear deformation in the beams is ignored.

The internal stress equations enable an easy calculation of the internal stresses on pinned portal frames under different kinds of loads but this time putting shear deformation into consideration.

The contribution of shear deformation is calculated by evaluating the equations in table 1 less the equivalent values when shear deformation is neglected. This was expressed as a percentage of the moment values when shear is neglected.

It is possible to express all the internal stress equations in terms of the ratios h/L and I_2/I_1. This way the variation of shear contribution with these parameters was investigated. Figure 4 shows a plot of the percentage (%) change in internal stresses (shear deformation contribution) of frames 1, 2 and 7 versus the ratio of beam second moment of area to column second moment of area (I_2/I_1). Figure 5 shows a similar plot of the percentage (%) change in internal stresses (shear deformation contribution) versus the ratio of height to length of portal frame (h/L). From the plots it can be inferred that the percentage (%) contribution of shear deformation in vertically loaded pinned portal frames is

i. Constant for any value of the ratios I_2/I_1 and h/L.
ii. Negative ie the effect of shear deformation reduces the expected internal stresses (bending moment).
iii. Decreases progressively with increasing values of the ratio h/L.

Figure 6 shows a plot of the percentage (%) change in internal stresses (shear deformation contribution) of frames 3,4,5,6 and 8 versus the ratio of beam second moment of area to column second moment of area (I_2/I_1). From the plot it can be deduced that the % contribution of shear deformation in these frames

i. Range from 1% to -4%. The extreme value (of about -4%) occurring only in frame 8 at low values of I_2/I_1 $(at\ I_2/I_1 < 0.5)$.
ii. Tended to a constant value dependent on the kind of external loads at values of $I_2/I_1 > 5$.

Figure 7 shows a plot of the percentage (%) change in internal stresses (shear deformation contribution) of frames 3, 5, 6 and 8 versus the ratio of height to length of portal frame

(h/L). From the plot it can be deduced that the % contribution of shear deformation in these frames

i. Varies between -1.5% to 3.5% and only rose up to 10% at the ratio $h/_L = 1.0$ for frame 6 indicting a critical ratio for such portal frames.
ii. Dropped in absolute values at values of $h/_L > 1.5$.

IV.CONCLUSION

The use of the flexibility method simplified the analysis of pinned portal frames and the results are presented in table 1. Table 1 contains simple equations that can be used for the computation of the internal stresses (bending moments) in pinned portal frame putting the effect of shear deformation into consideration.

An investigation of the variation of the effects of shear deformation with the ratios of beam second moment of area to column second moment of area ($I_2/_{I_1}$) and the ratio of height to length of portal frame ($h/_L$) shows that the effect the shear deformation is very small and can be conveniently neglected in manual calculations. However for pinned portal frames under appreciable concentrated horizontal forces (frame 6) the effect of shear deformation is significant (10%) when the height of the frame is equal to its length (h = L). The effect of shear deformation should be put into consideration in the analysis of such structures.

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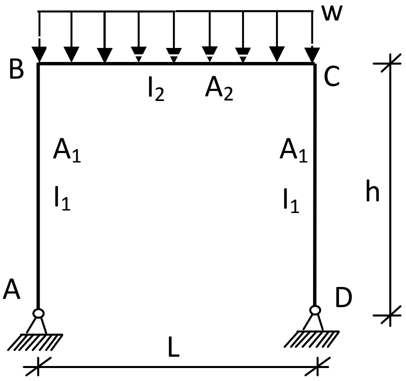


Figure 2

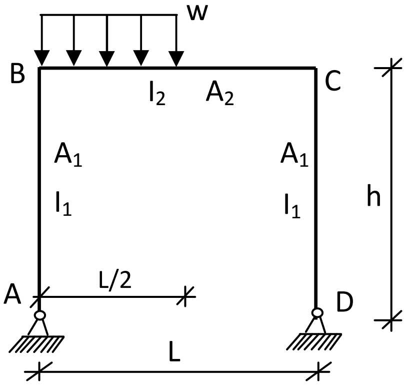
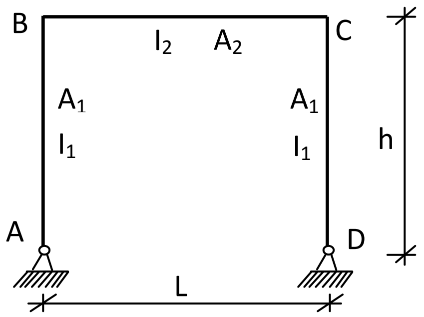
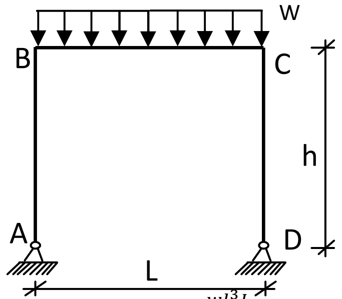
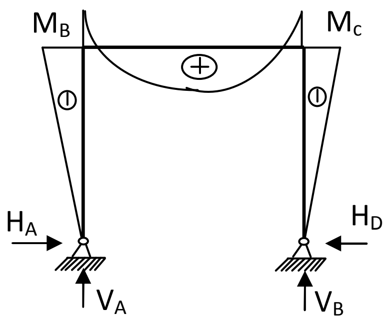
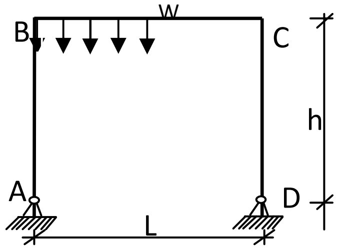
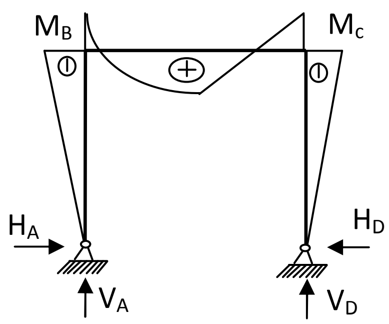
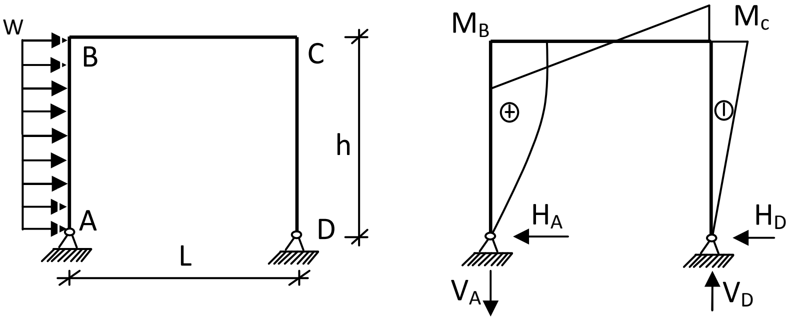
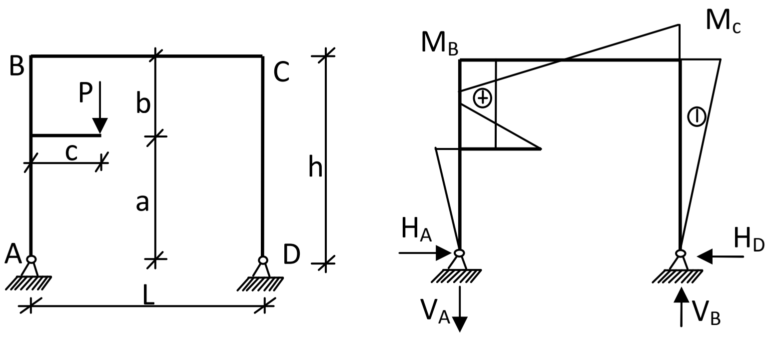
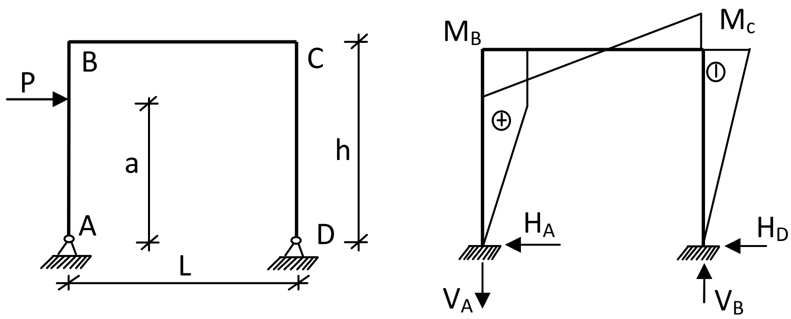


Figure 3

Table 1: Internal stresses for a loaded rigid frame

	A_1 = Cross-sectional area of the columns I_1 = Second moment of area of the column cross-section A_2 = Cross-sectional area of the beam I_2 = Second moment of area of the beam cross-section $\propto_1 = \frac{12EI_1}{h^2 GA_{1r}}$ $\propto_2 = \frac{12EI_2}{l^2 GA_{2r}}$ $\beta = \frac{\alpha_2}{\alpha_1}$
Frame No	LOADED FRAME
1	  $M_B = M_C = \frac{-wl^3 I_1}{2(4hI_2+6lI_1+\propto_1 hI_2)}$ $V_A = V_D = \frac{wl}{2}$ $H_A = H_D = \frac{-wl^3 I_1}{2(4h^2 I_2+6hlI_1+\propto_1 h^2 I_2)}$
2	  $M_B = M_C = \frac{wl^3 I_1}{4(4hI_2+6lI_1+\propto_1 hI_2)}$ $V_A = \frac{3wl}{8}$ $V_D = \frac{wl}{8}$ $H_A = H_D = \frac{wl^3 I_1}{4(4h^2 I_2+6hlI_1+\propto_1 h^2 I_2)}$

3	 $M_C = -\frac{wh^2(5hI_2+6lI_1+\propto_1 hI_2)}{4(4hI_2+6lI_1+\propto_1 hI_2)}$ $M_B = \frac{wh^2(3hI_2+6lI_1+\propto_1 hI_2)}{4(4hI_2+6lI_1+\propto_1 hI_2)}$ $V_A = V_D = \frac{wh^2}{2l}$ $H_D = \frac{wh(5hI_2+6lI_1+\propto_1 hI_2)}{4(4hI_2+6lI_1+\propto_1 hI_2)}$ $H_A = wh - H_D$
4	 $M_C = -\frac{3Pc[bI_2(h+a)+hlI_1]}{h[4hI_2+6lI_1+\propto_1 hI_2]}$ $M_B = Pc\left(1 - \frac{3[bI_2(b+a)+hlI_1]}{h(4hI_2+6lI_1+\propto_1 hI_2)}\right)$ $V_A = \frac{P(l-c)}{l}$ $V_A = \frac{Pc}{l}$ $H_A = H_B = \frac{3Pc[bI_2(h+a)+hlI_1]}{h^2[4hI_2+6lI_1+\propto_1 hI_2]}$
5	 $M_B = Pa\left[1 - \frac{2I_2(3h^2-a^2)+6hI_1+\propto_1 h^2 I_2}{2h(4hI_2+6lI_1+\propto_1 hI_2)}\right]$ $M_C = -\frac{Pa[2I_2(3h^2-a^2)+6hI_1+\propto_1 h^2 I_2]}{2h(4hI_2+6lI_1+\propto_1 hI_2)}$ $V_A = V_D = \frac{Pa}{l}$ $H_D = \frac{Pa[2I_2(3h^2-a^2)+6hI_1+\propto_1 h^2 I_2]}{2h^2(4hI_2+6lI_1+\propto_1 hI_2)}$ $H_A = P - H_D$

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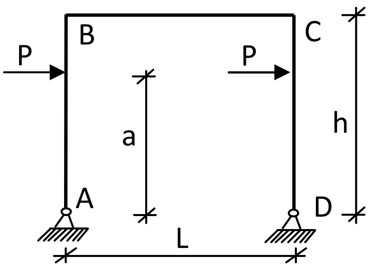

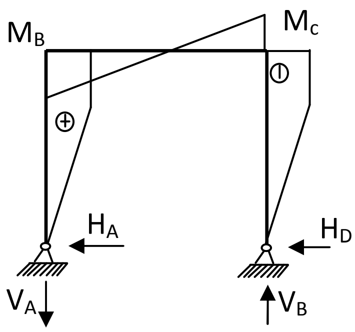

$$M_B = P\left[(h+a) - \frac{2(4h^3 - ah^2 + 2a^2h - a^3)I_2 + 12h^2 lI_1 + (3h-a)\propto_1 h^2 I_2}{2h(4hI_2 + 6hI_1 + \propto_1 hI_2)}\right]$$

$$M_C = P\left[(h-a) - \frac{2(4h^3 - ah^2 + 2a^2h - a^3)I_2 + 12h^2 lI_1 + (3h-a)\propto_1 h^2 I_2}{2h(4hI_2 + 6hI_1 + \propto_1 hI_2)}\right]$$

$$V_A = V_B = \frac{2Pa}{l}$$

$$H_D = \frac{P\left[2(4h^3 - ah^2 + 2a^2h - a^3)I_2 + 12h^2 lI_1 + (3h-a)\propto_1 h^2 I_2\right]}{2h^2(4hI_2 + 6hI_1 + \propto_1 hI_2)} \qquad H_A = 2P - H_D$$
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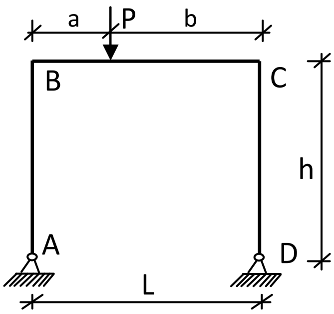

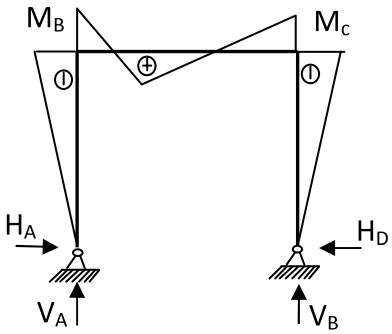

$$M_B = M_C = -\frac{3PabI_2}{4hI_2 + 6lI_1 + \propto_1 hI_2}$$

$$V_D = \frac{Pa}{l} \qquad V_A = P - V_D$$

$$H_A = H_B = \frac{3PabI_2}{h(4hI_2 + 6lI_1 + \propto_1 hI_2)}$$
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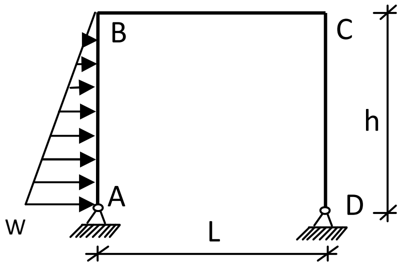

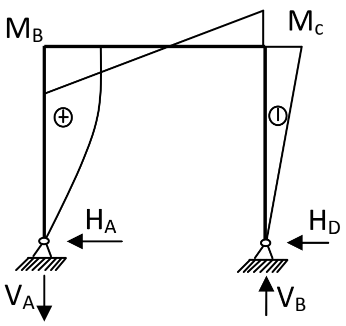

$$M_B = \frac{wh^2(13hI_2 + 30lI_1 + 5\propto_1 hI_2)}{60(4hI_2 + 26lI_1 + \propto_1 hI_2)}$$

$$M_C = -\frac{wh^2(27hI_2 + 30lI_1 + 5\propto_1 hI_2)}{180(4hI_2 + 26lI_1 + \propto_1 hI_2)}$$

$$V_A = V_B = \frac{wh^2}{6l}$$

$$H_D = \frac{wh^2(27hI_2 + 30lI_1 + 5\propto_1 hI_2)}{60(4hI_2 + 26lI_1 + \propto_1 hI_2)} \qquad H_A = wh - H_D$$
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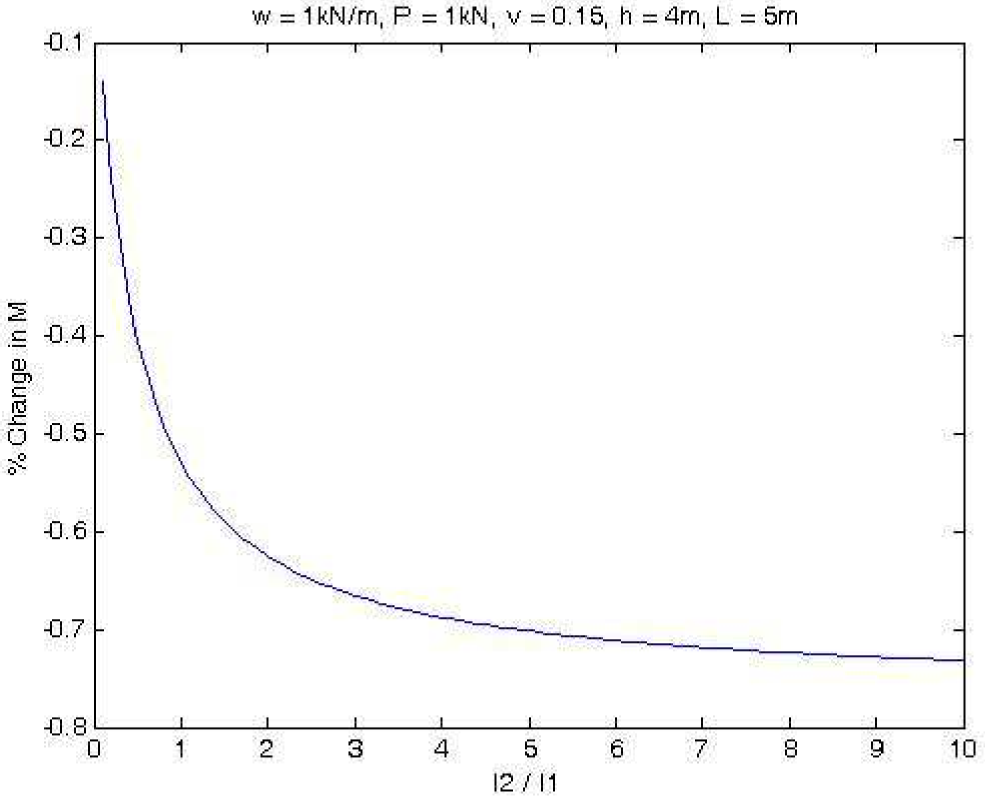

Figure 4: A plot of the percentage (%) change in internal stresses of frames 1, 2 and 7 versus the ratio of beam second moment of area to column second moment of area

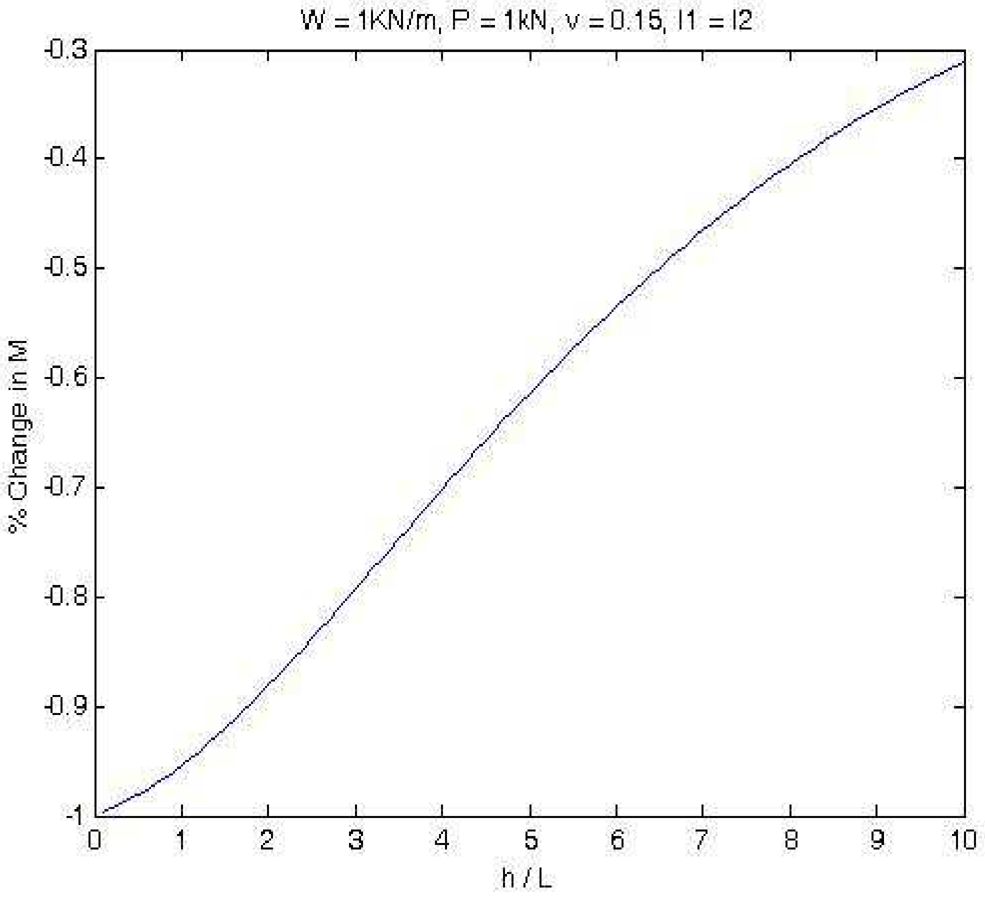

Figure 5: A plot of the percentage (%) change in internal stresses of frames 1, 2 and 7 versus the ratio of height to length of portal frame

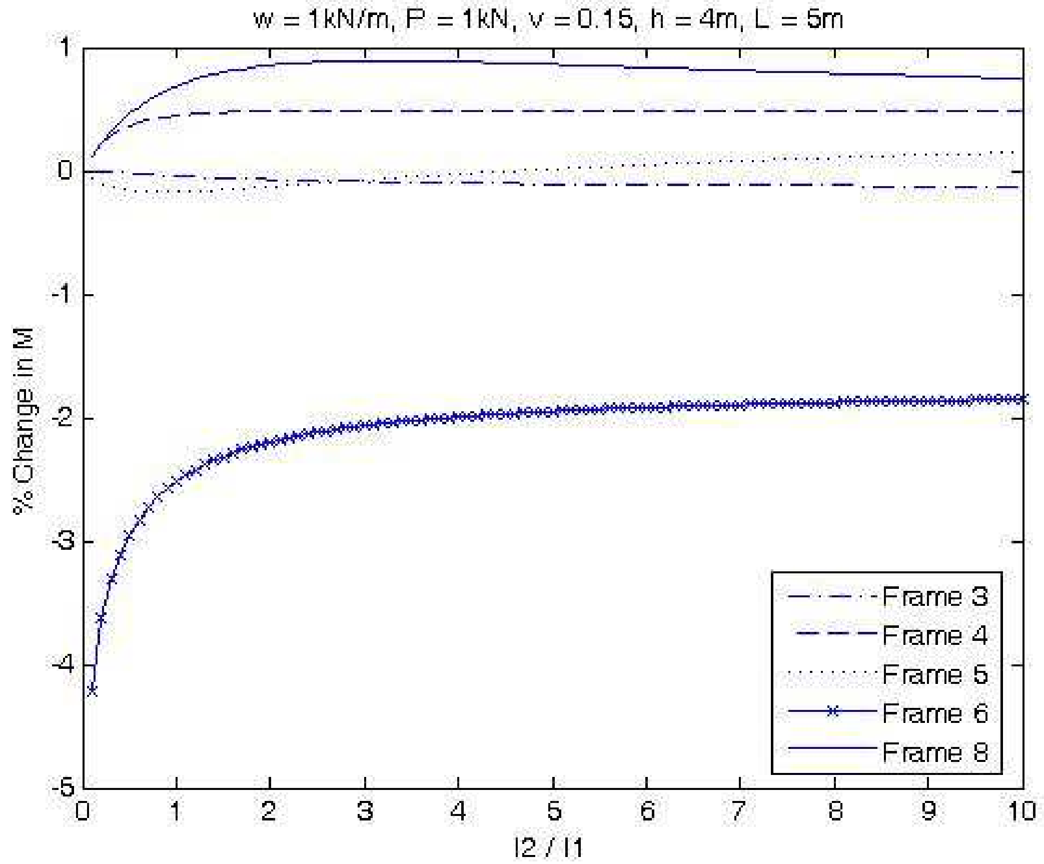

Figure 6: A plot of the percentage (%) change in internal stresses of frames 3,4,5,6 and 8 versus the ratio of beam second moment of area to column second moment of area

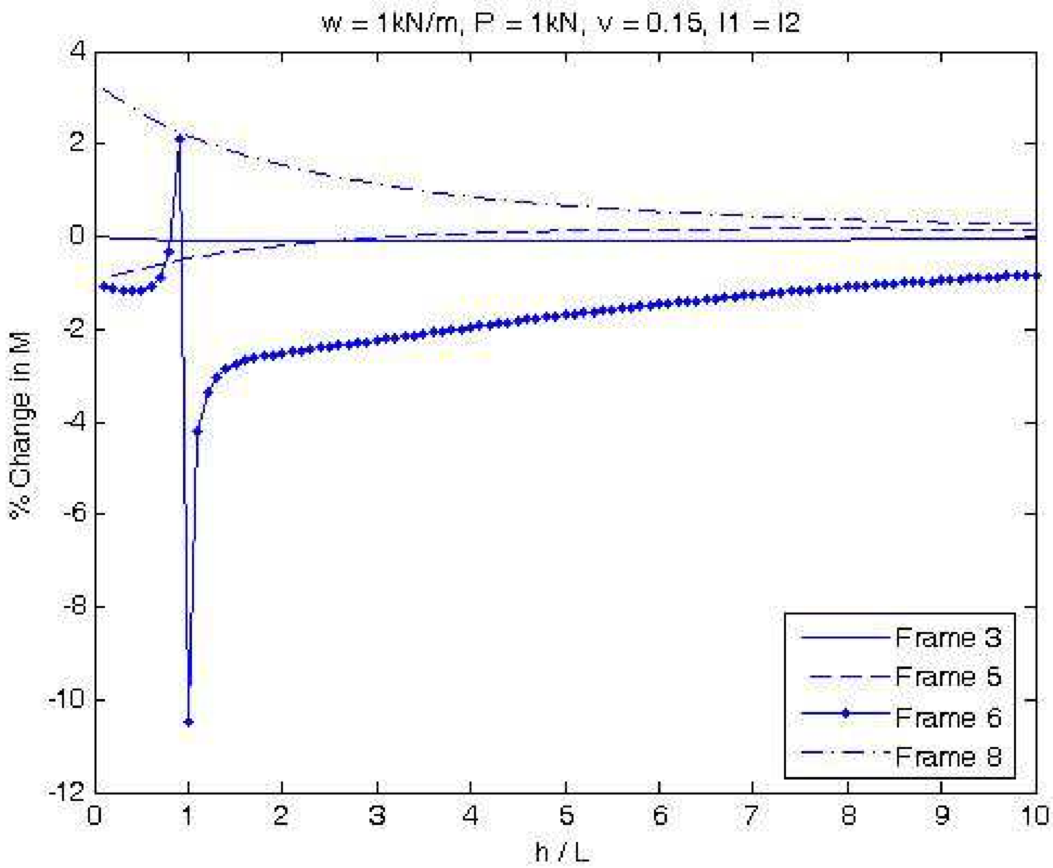

Figure 7: A plot of the percentage (%) change in internal stresses of frames 3,5,6 and 8 versus the ratio of height to length of portal frame.