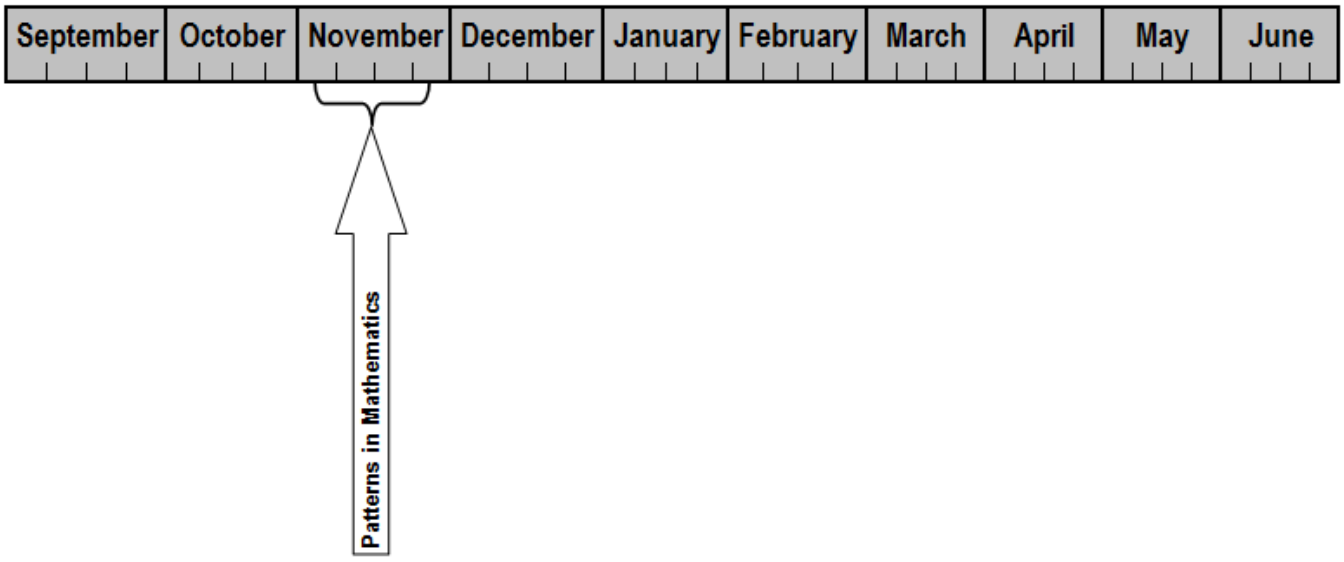
Patterns in Mathematics

Suggested Time: 3 Weeks



Unit Overview

Focus and Context

A solid foundation in analyzing and understanding patterns is fundamental to student success and progress throughout intermediate and high school algebra. The use of tables, such as graphic organizers, allow students to see the relationship between input and output values. Students need to recognize patterns within columns of a given table of values as well as the relationship between rows. This relationship is described through the use of algebraic expressions. The focus is to generate a table of values from an algebraic expression and to derive the expression from a given table of values. Through their understanding of representing patterns concretely, extending patterns, finding missing values and creating algebraic expressions, students are encouraged to use their knowledge of patterns to solve problems.

In Grade 6, the focus is on the preservation of equality within algebraic equations. The solving of these equations will be addressed in Grade 7. Students will use concrete models, primarily a balance scale to demonstrate preservation of equality, with emphasis on the two sides of the equation balancing. This can be done by changing each side of an equation in the same manner.

Math Connects

Patterns are an effective way to demonstrate the relationship between variables. Providing students with opportunities to analyze, model and extend patterns will help prepare them for algebra in higher grades. The concept of modelling equivalent equations, using a balance scale is also important. The skills acquired in Grade 6 contribute to the development of a deeper understanding of mathematics and will continue to be useful in real world situations.

Process Standards Key

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[ME]	Mental Mathematics and Estimation	[T]	Technology
		[V]	Visualization

Curriculum Outcomes

STRAND	OUTCOME	PROCESS STANDARDS
Patterns and Relations (Variables and Equations)	6PR1 Demonstrate an understanding of the relationships within tables of values to solve problems.	[C, CN, PS, R]
Patterns and Relations (Variables and Equations)	6PR3 Represent generalizations arising from number relationships, using equations with letter variables.	[C, CN, PS, R, V]
Patterns and Relations (Variables and Equations)	6PR4 Demonstrate and explain the meaning of preservation of equality, concretely and pictorially.	[C, CN, PS, R, V]

Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Demonstrate an understanding of the relationships within tables of values to solve problems.

[C, CN, PS, R]

Achievement Indicators:

6PR1.1 Create a concrete or pictorial representation of the relationship shown in a table of values.

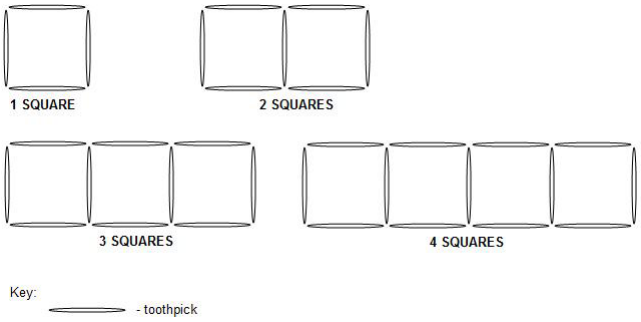
6PR1.2 Identify errors in a given table of values.

Elaborations—Strategies for Learning and Teaching

Give students a table of values and encourage them to visualize the change in each variable, as well as model it pictorially (drawing pictures) or concretely (using manipulatives). Patterns can be modelled using any available manipulatives. E.g., students could model the table of values below using toothpicks.

Number of Squares	Number of Toothpicks
1	4
2	7
3	10
4	13

Students should first identify that the number of squares starts with 1 and increases by 1. They will then determine that the number of toothpicks starts with 4 and increases by 3. This means that each new square in the pattern must be constructed using only 3 additional toothpicks. Based on this identified relationship students must construct each new square by starting with one side of a pre-existing square.



It is important that students are able to identify errors in a given table of values so that they do not extend the pattern incorrectly. They should be able to support their answer. E.g., Sam has a weekly paper route. He gets paid $30 a week. The following table of values shows his earnings over a 5 week period. Identify the error in this table.

Weeks	Earnings ($)
1	30
2	60
3	90
4	100
5	130

General Outcome: Use Patterns to Describe the World and to Solve Problems

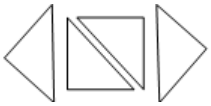
Suggested Assessment Strategies

Performance

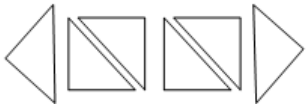
- Provide the following table of values. Ask students to use the green triangles from the set of pattern blocks to represent what the worm would look like on day 3, 4 and 5.

Age/Days	Number of Triangles
1	4
2	6
3	8
4	10
5	12

E.g., This is a one-day-old worm:



This is a two-day old worm:



(6PR1.1)

- A train has 8 wheels and pulls cars each having 4 wheels. The table of values below shows the number of wheels on a train with different numbers of cars being pulled. Draw or make a model of the train with each number of cars being pulled.

Number of cars	Number of wheels
0	8
1	12
2	16
3	20
4	24

(6PR1.1)

Paper and Pencil

- Create tables containing an error in the right column and ask students to try to identify the value that does not fit the pattern. Ask students to explain why the value in question is incorrect. Ask them to justify their choice.

(6PR1.2)

Resources/Notes

Math Focus 6

Lesson 1: Identifying Number Patterns

6PR1

TG pp. 13 - 17

Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicators:

6PR1.3 Describe the pattern within each column of a given table of values.

6PR1.4 Create a table of values to record and reveal a pattern to solve a given problem.

Elaborations—Strategies for Learning and Teaching

When describing patterns in a given column of a table of values, students sometimes overlook stating the starting value of the pattern. The table below shows changes in the height of a plant over time.

Week	Plant Height (cm)
1	6
2	10
3	14
4	18
5	22

Most students would describe the change in weeks as "going up by 1" and the change in plant height as "going up by 4". Both of these descriptions do not acknowledge the starting value. A more appropriate description of each would be, "Weeks start at 1 and increase by 1 each time. Height starts at 6 and increases by 4 each time".

Students have previously extended patterns concretely and pictorially. Creating a table of values to record the pattern is new to Grade 6. At this point, students are simply extending the pattern in a given column to solve a problem rather than using a pattern rule that relates one column to the other.

Students are expected to create their own table of values. They may have difficulty determining how each column should be labelled. Remind students that the values in the second column result from (or is dependent on) the change in the first column. For instance, in the previous example, plant growth is determined by the number of weeks that have passed.

General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Journal

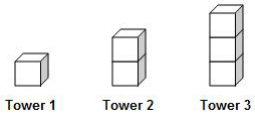
- Ask students to identify the pattern within each column of the given table:

Hour	Snowfall (cm)
1	7
2	12
3	17
4	22
5	27

(6PR1.3)

Performance

- Tell students that a statue is shaped like a tower and is made of a single column of cubes (see diagram below). A painter has been hired to paint all the cube faces that are visible. This would include the side and top faces as well. The bottom face of each cube is not visible. Students may build the model using multilink cubes or blocks. Ask students to create a table of values to record the number of faces that need to be painted on towers 1, 2, 3, 4 and 5 blocks high. Ask students to find the number of faces that would have to be painted for a tower containing 10 blocks. 20 blocks.



(6PR1.4)

Resources/Notes

Math Focus 6

Lesson 1 (Cont'd): Identifying Number Patterns

6PR1

TG pp. 13 - 17

Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicators:

6PR1.5 Generate values in one column of a table of values, given values in the other column and a pattern rule.

6PR1.6 State, using mathematical language, the relationship in a given table of values.

6PR1.7 Predict the value of an unknown term, using the relationship in a table of values, and verify the prediction.

6PR1.8 Formulate a rule to describe the relationship between two columns of numbers in a table of values.

Elaborations—Strategies for Learning and Teaching

The focus is to identify patterns that increase or decrease between values within and between the columns of a table of values chart and write the pattern rule. It is important that students first identify the value at which the pattern begins and then indicate the amount they increase or decrease from that given value.

In previous grades students completed tables of values given simple expressions involving one operation, addition or subtraction. In Grade 6 they will be exposed to expressions that involve two operations, most commonly multiplication followed by addition or subtraction. E.g., Judy pays $10 for her cell phone. Each time she uses the Internet it costs $2. This relationship can be represented by the pattern rule $2n+10$. Use this pattern rule to complete the table shown here.

Number of Internet Visits	Cost per Month
1	12
2	
3	
4	
5	

In describing the relationship between or within the columns of a given table of values, students should be encouraged to use appropriate mathematical language. Invite students to share how they arrived at their conclusions. To create a comfortable environment for students to justify their reasoning, model appropriate language. Brainstorm, with students, other words or phrases that could be used and display these mathematically, appropriate words.

As students explore relationships in and between numbers in columns of a table of values, they will predict what missing values might be and figure out values for numbers not covered in the table.

Previously, students found relationships (pattern rules) within a column and then used that rule to predict terms not in the table of values. This strategy works well when the numbers in the column are in sequence, and when they are asked to find larger terms in the sequence in which extending the pattern is not practical (E.g., find the 50th term in this sequence). Problems such as these will require developing a pattern rule which can be used to determine the value in the second column based on the corresponding value in the first column.

The objective is that students are able to derive a pattern rule that relates one column of a table of values to the other column.

Students will initially see the pattern in each column of the table but may have difficulty identifying the relationship between the input and output columns. This concept can be introduced using guess and check to derive a pattern rule.

General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

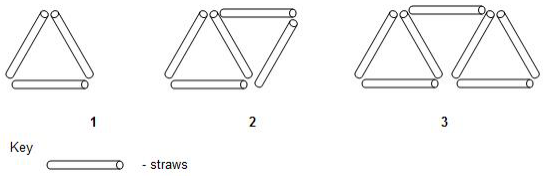
Performance

- Give students the following situation: You are going to play paintball with your friends. It costs $20 for admission and an additional $5 for every round of balls. This relationship can be represented by the expression $5b + 20$. Use this pattern rule to complete the table of values below.

Number of Rounds	Total Cost
1	25
2	
3	
4	
5	

(6PR1.5)

- Ask students to create a table of values to represent the pattern below.



  i) Write a pattern rule to describe the change within each column.

  ii) Predict the number of straws for diagram 10.

(6PR1.6, 6PR1.7)

Journal

- Ask students to find the missing values in the table below, based on the patterns observed. Ask: How many sandwiches would each person get? Following this pattern, predict how many sandwiches would be needed if 60 people attended the picnic. How many could attend if 90 sandwiches were provided?

Number of people	Number of sandwiches
3	6
6	
	18
12	24
15	
	36

(6PR1.7)

Resources/Notes

Math Focus 6

Lesson 2: Describing Relationships in Tables

6PR1

TG pp. 18 – 22

Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Continued

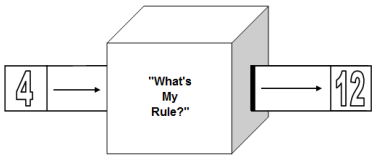
Achievement Indicator:

6PR1.8 Continued

Elaborations—Strategies for Learning and Teaching

"What's My Rule" box - Prepare sentence strips with numbers on both ends as shown below. Insert the sentence strip so students can see input and output numbers. Show students and challenge them to come up with several different operations or combinations of operations that could be used to get the output from the initial input.

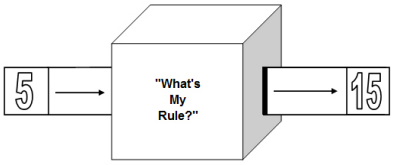
E.g., 4 is inserted into the box and 12 comes out.



Ask "What rule could have been used to get an output of 12 from an input of 4"?

Students may reply "We multiplied by 3", "We added 8", "We doubled the input and added 4", etc.

Record all the students' suggestions and then present another input/output situation based on the same pattern rule. For example, 5 is put in and 15 comes out.



Ask students, "Which rule from the first input/output situation could also be used to describe an input of 5 and an output of 15"?

Students will likely reply "Adding 8 will not work, because 5 + 8 = 13. However, multiplying by 3 will work. Therefore the rule for this pattern must be multiply the input by 3".

Verify that this is the correct rule by presenting another input/output scenario from the same pattern.

General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Performance

- Use the "What's my Rule?" box to model the input and output for each row in the tables below. Ask students to come up with a pattern rule that can be used to describe the relationship between all the input/output combinations in the table.

Input	Output
1	5
2	9
3	13
4	17

Input	Output
1	2
2	3
3	4
4	5
5	6

  Remind students that the pattern rule has to apply to all the input/output combinations in the table, not just the first one. (6PR1.8)

- Ask two student volunteers to model how a "What's my Rule?" box (as described on the previous page) works. Give one student a card with an operation written on it (E.g., +2, × 3, -1 etc.). No one else should know what operation is on this student's card. Have the other volunteer put in an input number between 1 and 10. The student with the operation card will perform the operation mentally on this input value and put the resulting output card through the output slot and say it aloud to the class. Ask the to class identify what operation was performed on the input. Repeat the process using the same operation but a new input to verify the class's answer. Once students become proficient at identifying a single operation ask another volunteer to come forward and give them a second operation card. The first student will say another input aloud. The student with the first operation card will mentally perform his/her operation and say the output aloud. The student with the second operation card will perform their operation on that output mentally and say the resulting number aloud. Have the class deduce the two operations performed. Verify using another input number. Once students gain proficiency at identifying two operations have the student with the first operation card whisper their output to the student with the second operation card so that the class is only hearing the final output of both operations and not the output of each. This will require more verification using different inputs to deduce the pattern rule. (6PR1.8)

Resources/Notes

Math Focus 6

Lesson 2 (Cont'd): Describing Relationships in Tables

6PR1

TG pp. 18 – 22

Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

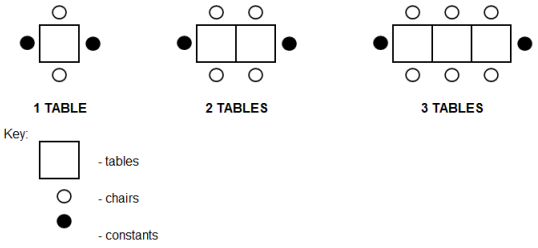
6PR1 Continued

Achievement Indicator:

6PR1.8 Continued

Elaborations—Strategies for Learning and Teaching

From this point, concrete examples can be used. For example, four students could be seated around a square table, with one on each side. Tables could be added with another chair on either side in the pattern shown below. This could be concretely modelled using actual tables and chairs if available. Pattern blocks or interactive whiteboard graphics could also be used to create the model.



This could be represented in the following table of values.

Number of Tables	Number of Chairs
1	4
2	6
3	8
4	10
6	12

Once this table has been generated, attention should be drawn to how the increase in the second column (constant change) relates to the operation(s) performed on the values in the first column to find the corresponding value in the second.

Number of Tables	Number of Chairs	
1	4	
2	6	+2
3	8	+2
4	10	+2
5	12	+2

With 1 table there will be 4 chairs.

With 2 tables there will be 4 + 2 = 6 chairs

With 3 tables there will be 4 + 2 + 2 = 8 chairs

With 4 tables there will be 4 + 2 + 2 + 2 = 10 chairs

Notice that the number of chairs is increasing by 2. This would be logical because for every table added, two new chairs are added (one on each side of the table). However, there will always be two extra chairs (one on each end of the line of tables) that are constant.

Thus, the pattern rule for this scenario would be $2n + 2$, where n = the number of tables.

General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies	Resources/Notes
	Math Focus 6 **Lesson 2 (Cont'd):** Describing Relationships in Tables **6PR1** TG pp. 18 – 22

Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicator:

6PR1.8 Continued

Elaborations—Strategies for Learning and Teaching

By extension, I can use this rule to predict the number of chairs needed for 9 tables. Double the number of tables (9 × 2 = 18), and add the two constant chairs at the ends (18 + 2 = 20). Therefore, we would need 20 chairs for 9 tables.

Students should recognize that the amount of constant change in the output is the number by which the input is being multiplied. If the product of this amount of change and a given input is not equal to its corresponding output, we must then determine the amount of increase or decrease required to arrive at that output. For example, in the table below students should first determine the amount of constant change in the output values (second column).

INPUT	OUTPUT
1	5
2	8
3	11
4	14
5	17

Draw attention to the output values. Ask "How much do the output values increase by each time"? Students will observe a constant increase of 3 in the outputs. This means that each input must have been initially multiplied by 3. Thus, the first part of our pattern rule is $3n$, where n represents any given input value.

Starting with the first input, multiply by 3.

$1 \times 3 = 3$

Now ask, "What did we add to or subtract from this product to get the output of 5?" Students should recognize that 2 was added. So, this means our pattern rule is $3n + 2$, or in words, "multiply the input by 3 and add 2." Confirm that this is correct by evaluating the expression using the other inputs in the table of values.

$3(2) + 2 = 8$ correct

$3(3) + 2 = 11$ correct

$3(4) + 2 = 14$ correct

$3(5) + 2 = 17$ correct, $3n + 2$ must be the pattern rule.

It is important to note that using the difference in the outputs to determine a pattern rule in this manner will only work if the inputs are consecutive numbers. (e.g. 1, 2, 3, 4, 5…). Therefore, **when assessing this outcome be sure to use consecutive input numbers.**

General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Journal

- Present the following scenario to the class:

  Ted is having a pot-luck dinner. He has prepared 4 dishes of food for the dinner and told all his invited guests to bring two dishes each. The number of dishes at the party depends on the number of guests that come.

  i) Write a pattern rule that could be used to determine the number of dishes that will be at the dinner for any number of guests that might attend.

  ii) Use this pattern rule to complete the table of values below.

Number of Guests	Number of Dishes
0	
1	
2	
3	
4	

(6PR1.5, 6PR1.6, 6PR1.8)

Resources/Notes

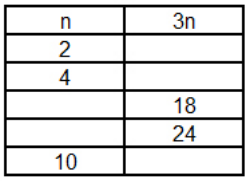
Math Focus 6

Lesson 2 (Cont'd): Describing Relationships in Tables

6PR1

TG pp. 18 – 22

Strand: Patterns and Relations (Patterns)

Outcomes	Elaborations—Strategies for Learning and Teaching
Students will be expected to **6PR1 Continued** **Achievement Indicators:**	
6PR1.9 Identify missing elements in a given table of values.	Students are familiar with using a given pattern rule to complete the right column of a table of values. However, they will now be expected to find missing values in either column of an incomplete table using a given pattern rule. Inverse operations may be addressed here as a possible strategy. E.g.,  Students will complete the first two missing values by simply applying the pattern rule, $3n$. To find the third and fourth missing values they will have to work backwards (use the inverse operation). For example, to determine the third missing value the student should think "Three groups of an unknown number gave me 18. If I share 18 into three equal groups, how many will be in each group?" (18 divided by 3).
6PR1.4 Continued	Students will now be expected to develop an expression and table to solve a given problem. Consider the following example: Gloria is going to a community celebration. Admission is $5 and each activity costs $2. i) Use words to describe how to find the total amount of money Gloria will spend for any number of activities that she may participate in. (Multiply the number of activities by 2 and add 5.) ii) Write an expression to represent the above situation. iii) Use your expression to create a table of values showing how much Gloria will spend if she takes part in 0 to 5 activities.

n	3n
2	
4	
	18
	24
10	

General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Performance

- Present the following situation to students:

  (i) Jill works in a store for a wage of $9 per hour. Help Jill complete the following table to show her total earnings after each hour worked in a day. Some values were omitted on each side of the table. Find the missing values:

Hours Worked	Total Earnings
2	
	36
6	
	72

  Ask students to explain how they derived each omitted value using the pattern rule.

  (ii) Jill wants to buy two pairs of jeans that cost $46.00 each. How many hours does she need to work in order to buy the jeans?

  (6PR1.9)

- Present the following situation:

  Sheila works in a computer repair shop. She gets paid $75 a day plus $5 for every computer she fixes.

  i) Create a table to display the total amount of money Sheila could make in a day for any number of computers she might fix.

  ii) Write a pattern rule that you could use to find the total amount of money Sheila could make in a day for any number of computers she might fix.

  iii) Use your rule to determine how much money Sheila would make if she fixed 12 computers in one day. (6PR1.4)

Resources/Notes

Math Focus 6

Lesson 3: Using Expressions to Create Tables

6PR1

6PR3

TG pp. 23 – 27

Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR3 Represent generalizations arising from number relationships, using equations with letter variables.

[C, CN, PS, R, V]

Achievement Indicators:

6PR3.1 Describe the relationship in a given table, using a mathematical expression.

6PR3.2 Represent a pattern rule, using a simple mathematical expression such as 4d or 2n + 1.

Elaborations—Strategies for Learning and Teaching

Students have worked on writing a word rule to describe the relationship in a given table. Students will now extend this skill and write the pattern that is found in the table using a mathematical expression, or numbers and variables.

Students will take a word rule and use it to generate a mathematical expression using variables. E.g., the cost to join minor hockey is $120.00 per player. Each player must pay an additional fee of $5.00 for each practice. To represent the total cost for any given player, the word rule would be to multiply the number of practices by $5.00 and add $120.00. This can be now written as a mathematical expression, $5p + 120$, where 'p' represents any number of practices. Students could then use this expression to generate a table of values showing total costs for various possible numbers of practices.

Number of Practices p	Total Cost 5p + 120
5	145
10	170
15	195
20	220
25	245

General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Journal

- Ask students to create a situation for each expression:

  (i) $p - 3$

  (ii) $3 - p$

  Explain how the position of the variable changes the meaning of the expression in each case.

  Would this also apply to the following expressions?

  (i) $p + 3$

  (ii) $3 + p$ (6PR3.1)

- Ask students to write an algebraic expression to represent each pattern rule below:

  i) Double a number

  ii) Five more than a number

  iii) Three less than a number

  iv) A number less than ten

  v) Six more than twice a number

  vi) One less than triple a number (6PR3.2)

Performance

- Ask students to match each of the following situations with the correct expression:

  - Harry is twice as old as Noel — $4n + 2$
  - Susan has a bag of candy and gave away four — $2n$
  - Margot has four packages of hockey cards and two individual cards — $n + 2$
  - Harry is two years older than Noel — $4 - n$
  - Susan has 4 dolls and gives some away — $n - 4$

  (6PR3.1)

Resources/Notes

Math Focus 6

Lesson 3 (Cont'd): Using Expressions to Create Tables

6PR1

6PR3

TG pp. 23 – 27

Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicators:

6PR1.8 Continued

6PR1.3 Continued

6PR3 Continued

Achievement Indicator:

6PR3.2 Continued

Elaborations—Strategies for Learning and Teaching

The focus is to compare patterns created by different expressions. It should become apparent to students that expressions that share the same numbers and variables will not necessarily produce the same pattern. For example, $2n + 3$ will not produce the same pattern as $n + 3$. Neither will $4n + 2$ create the same pattern as $4n + 3$. Although each expression has a common feature, each generates a different table of values.

This concept should be introduced using manipulatives to generate two different patterns. From these patterns two separate tables of values would be generated and two different mathematical expressions (pattern rules) derived.

Give students counters. Students will start with 3 counters placed in a row. Each student will then create rows where each is increased by any desired constant amount. Encourage students to keep their amount of increase small so as not to run out of counters. Students will continue their pattern up to five rows.

Eg.

Student A

Starts with 3 counters

○○○

chooses to increase rows by 1 each time

Row 1 ○○○

Row 2 ○○○○

Row 3 ○○○○○

Row 4 ○○○○○○

Row 5 ○○○○○○○

Student B

Starts with 3 counters

○○○

chooses to increase rows by 2 each time

Row 1 ○○○

Row 2 ○○○○○

Row 3 ○○○○○○○

Row 4 ○○○○○○○○○

Row 5 ○○○○○○○○○○○

Ask students to record the number of counters used to make each row in a table for values as shown:

Eg.

Student A (started with 3, increased by 1)

Row #(n)	# of counters
1	3
2	4
3	5
4	6
5	7

Student B (started with 3, increased by 2)

Row #(n)	# of counters
1	3
2	5
3	7
4	9
5	11

Ask students to analyze their table of values and create a word rule and an expression that could be used to find the number of counters for any row number.

General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies	Resources/Notes
	Math Focus 6 **Lesson 4:** Comparing Expressions **6PR1** **6PR3** TG pp. 28 – 31 **Curious Math Activity:** Clock Number Patterns **6PR1** TG pp. 32-33 **Additional Reading:** Small, Marion (2008), *Making Math Meaningful to Canadian Students K-8*, pp. 582 - 588

Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicators:

6PR1.8 Continued

6PR1.3 Continued

6PR3 Continued

Achievement Indicator:

6PR3.2 Continued

Elaborations—Strategies for Learning and Teaching

Eg.

Student A

Row #(n)	# of counters	
1	3	
2	4	+1
3	5	+1
4	6	+1
5	7	+1

Rule: n + 2

Student B

Row #(n)	# of counters	
1	3	
2	5	+2
3	7	+2
4	9	+2
5	11	+2

Rule: 2n + 1

After analysis of the expressions students developed, they should come to the conclusion that even though everyone started with the same number, they did not all get the same expression because their patterns increased by different amounts.

Next, ask the class to start their rows with any desired number of counters (advise students to keep their numbers small). They will then create rows where each increases by 2.

Eg.

Student A

Chooses to start with 1 counter

○

Each row is increased by 2 counters

Row 1 ○

Row 2 ○○○

Row 3 ○○○○○

Row 4 ○○○○○○○

Row 5 ○○○○○○○○○

Student B

Chooses to start with 2 counters

○○

Each row is increased by 2 counters

Row 1 ○○

Row 2 ○○○○

Row 3 ○○○○○○

Row 4 ○○○○○○○○

Row 5 ○○○○○○○○○○

Ask each student to generate a table of values to show the change in the number of counters in each row.

E.g.,

Fg

Student A's table

Row #(n)	# of counters
1	1
2	3
3	5
4	7
5	9

Student B's table

Row #(n)	# of counters
1	2
2	4
3	6
4	8
5	10

Ask students to analyze their table of values and create a word rule and an expression that could be used to find the number of counters for any row number.

After analysis of the expressions students developed, they should come to the conclusion that even though everyone increased their rows by the same amount, they did not all get the same expression because their patterns began with different amounts.

Eg.

Student A

Row #(n)	# of counters	
1	1	
2	3	+2
3	5	+2
4	7	+2
5	9	+2

Rule: 2n - 1

Student B

Row #(n)	# of counters	
1	2	
2	4	+2
3	6	+2
4	8	+2
5	10	+2

Rule: 2n

General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies	Resources/Notes
	Math Focus 6 **Lesson 4:** Comparing Expressions **6PR1** **6PR3** TG pp. 28 – 31

Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR4 Demonstrate and explain the meaning of preservation of equality, concretely and pictorially.

[C, CN, PS, R, V]

Achievement Indicators:

6PR4.1 Model the preservation of equality for addition, using concrete materials (e.g., a balance, pictorial representations), and explain and record the process.

6PR4.2 Model the preservation of equality for subtraction, using concrete materials (e.g., a balance, pictorial representations), and explain and record the process.

6PR4.3 Model the preservation of equality for multiplication, using concrete materials (e.g., a balance, pictorial representations), and explain and record the process.

6PR4.4 Model the preservation of equality for division, using concrete materials (e.g., a balance, pictorial representations), and explain and record the process.

Elaborations—Strategies for Learning and Teaching

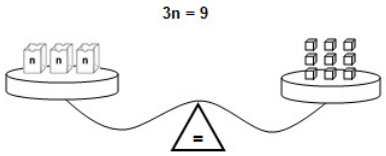
In order for equations to be equivalent, the same operations have to be performed on each side where the value of the variable does not change. E.g., $3n + 1 = 7$ and $3n = 6$ are equivalent equations because 1 is subtracted from each side in the first equation to make the second equation. This is called 'preservation of equality'.

Students will now be introduced to equations involving multiplication as well as equations involving two operations, pictorially and concretely. Students will learn the difference between an equation and an expression in Grade 7 but it is important for them to hear the correct terminology modelled. An equation is a complete number sentence stating that two amounts are the same. Equations must contain an equal sign, e.g., $2 + 3 = 5$. A number sentence with a variable is an algebraic equation, e.g., $p + 2 = 3$ reads 'Two more than a number is equal to 3'. This is an algebraic equation because it is stating that an unknown amount plus 2 is the same as 3. An algebraic expression on the other hand is simply a statement without the notion of equivalency. For example, $p + 2$ reads 'A number plus 2'. In this case, the variable p could be any value. The emphasis here is on modelling these equations and showing that if you add or subtract the same amount to/from each side, the equality is preserved. **At this point students are not expected to solve equations although some students may be inclined to do so.**

Use pan balance scales and number lines when introducing this concept.

When modelling equations, using the pan balance, bags can be used to represent variables (unknown amounts) and multi-link cubes or blocks used to represent numbers.

Model a simple equation on a pan balance such as $3n = 9$.



Students can now add a constant amount to each side. No matter how much they add, the scale will remain balanced as long as they add the same amount on each side. This will help the student observe how the equality of the two pans (sides of the equation) is preserved.

(continued)

General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Performance

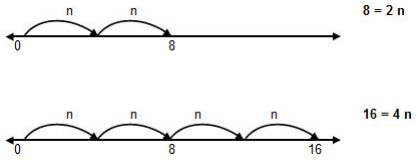
- Ask students to draw or model (using a two pan-balance or number line) each set of equations below. Determine if each pair of equations are equivalent or not. Explain how you know.
    - $n + 2 = 6$ and $n + 3 = 7$
    - $2m + 1 = 9$ and $2m + 2 = 8$
    - $5p + 3 = 18$ and $4p + 3 = 18$
    - $4y = 20$ and $8y = 40$
    - $3k = 12$ and $9k = 24$ (6PR4.1)

Journal

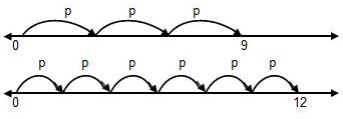
- Ask students if $2n + 2 = 6$ and $2n + 4 = 6$ are equivalent equations. Explain why or why not? Use a model to represent each equation. (6PR4.1)

Student - Teacher Dialogue

- Tell students that Dave says that the following number lines show that $8 = 2n$ and $16 = 4n$ are equivalent equations. Is he correct? Ask them to justify their answer.



- Present the following: Victoria says that the following number lines show that $3p = 9$ and $6p = 12$ are equivalent equations. Is she correct? Explain how you know. (6PR4.1)



- Provide cards with the following equations:

$15 - s = 9$	$11 = 22 - s$	$17 - s = 11$	$4s = 24$
$3s = 12$	$4s + 2 = 26$	$3s - 4 = 8$	$21 - s = 14$
$2 + s = 7$	$14 - s = 7$	$2 + 2s = 12$	$18 = 16 + s$
$7 + s = 15$	$15 = 13 + s$	$9 + s = 17$	$0 = 11 - s$

Lay out the equation cards (face up) so students can see what is on the cards. Match the equation card with its corresponding equivalent equation. When all equation cards are matched, choose one equation and create a word problem. Ask students to switch word problems and solve one belonging to a classmate. (6PR4.5)

Resources/Notes

Math Focus 6

Lesson 5: Equivalent Equations

6PR4

TG pp. 34 - 41

Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR4 Continued

Achievement Indicators:

6PR4.1 Continued

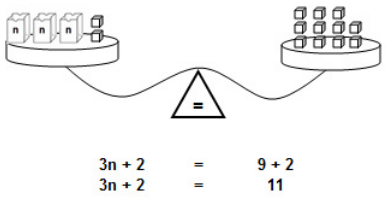
6PR4.2 Continued

6PR4.3 Continued

6PR4.4 Continued

6PR4.5 Write equivalent forms of a given equation by applying the preservation of equality and verify using concrete materials, e.g., 3b = 12 is same as 3b + 5 = 12 + 5 or 2r = 7 is the same as 3(2r) = 3(7).

Elaborations—Strategies for Learning and Teaching



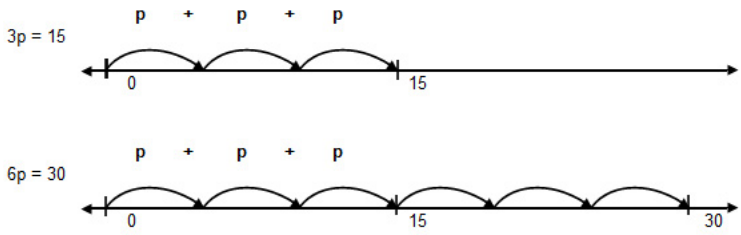
Therefore, $3n = 9$ and $3n + 2 = 11$ are equivalent equations.

Once students have had experience creating equivalent equations using the pan balance, additional activities may be modelled using a digital balance from the National Library of Virtual Manipulatives or another on-line applet but this should not be a substitute for students using the actual pan balance as a hands on activity.

Equivalent equations can also be modelled on a number line. E.g.,

Is $3p = 15$ equivalent to $6p = 30$?

Show students a model of $3p=15$ and $6p=30$.



These equations are equivalent because the "jumps" are all the same size.

To ascertain preservation of equality for multiplication it must be determined whether each side of the equation was multiplied by the same amount. E.g., $3n + 2 = 8$ and $6n + 4 = 16$ would be equivalent equations because all terms on both sides were doubled (×2).

$2n + 3 = 7$ and $6n + 9 = 14$ would not be equivalent because the terms on the left side were tripled ($n \times 3$) but the terms on the right were doubled ($n \times 2$), therefore equality is not preserved. Use a balance scale to verify.

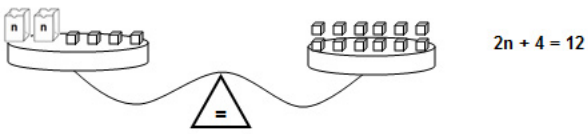
General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Performance

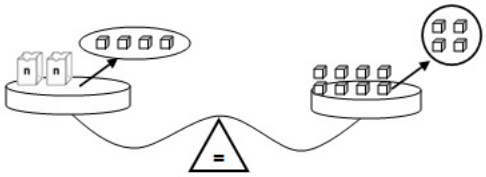
- Ask students to draw or model another equation (pan balance) that is equivalent to the first equation shown below. Explain why the model you created is equivalent to the original.

E.g.,



2n + 4 = 12

Answer:

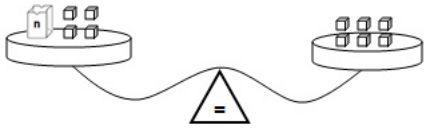


2n = 8

Are there other ways of showing equivalent equations using other operations? Explain. (6PR4.1, 6PR4.2, 6PR4.3, 6PR4.4)

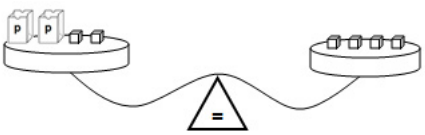
- Ask students to write an equation to represent the following situations:
  - (i) Bethany is 3 years older than Toby. Toby is 21 years old. Write and model an equation to represent the problem. Write an equivalent equation to represent the problem that preserves equality.
  - (ii) There are 11 muffins on a tray. There were 24 at the start. Some have been eaten. How many muffins are missing from the tray? Write and model an equation to represent the problem. Write an equivalent equation to represent the problem that preserves equality. (6PR4.1, 6PR4.2, 6PR4.3, 6PR4.4)
- Draw or model another equation (two pan balance) that is equivalent to each equation shown below using multiplication. Explain why the model you drew/made is equivalent to the original.

i)



n + 4 = 6

ii)



$2p + 2 = 4$ (6PR4.3

Resources/Notes

Math Focus 6

Lesson 5 (Cont'd): Equivalent Equations

6PR4

TG pp. 34 - 41

Other Resources:

National Library of Virtual Manipulatives

http://nlvm.usu.edu

Strand: Patterns and Relations (Patterns)

Outcomes

Students will be expected to

6PR1 Continued

Achievement Indicator:

6PR1.4 Continued

Elaborations—Strategies for Learning and Teaching

Patterns can be used to solve problems. Previously, students have extended a pattern within a column of a table of values and developed a pattern rule to represent the relationship between/within columns in a given table of values. At this point, students should be encouraged to try different methods to solve a single problem. In some instances it may be more practical simply to extend the pattern in a column. In other cases students may find it easier to use a pattern rule (expression) to solve a problem. Students will now have to determine when it would be more appropriate to take each approach.

Present the following situation. Allow students time to explore solving this problem using counters before attempting to create a table. Using a table to record data will allow students to see patterns that can then be used to solve problems. Tell students that when Mary visits her grandparents she likes to go to the beach to collect beach rocks. She collected 12 on the first visit. Students can use the table below to help them figure out how many beach rocks Mary would have collected after 4 visits. Encourage students to explore, without continuing the table, how many rocks Mary would have collected after 8 visits to her grandparents.

Visits	Number of Rocks Collected
1	12
2	24
3	36
4	

General Outcome: Use Patterns to Describe the World and to Solve Problems

Suggested Assessment Strategies

Performance

- Tell students that the average person's hair grows 3 cm in one month. Ask students to create a table that would help them determine how much Dan's hair would have grown after 6 months.

(6PR1.4)

Journal

- Give students the following prompt:

  The distance from Irishtown to Pouch Cove is 720 km. A bus left Pouch Cove at 8:00 am travelling west at an average speed of 90 km/hr. A car leaves Irishtown at 9:00 am travelling east at an average speed of 120 km. At what time will the bus and the car meet each other?

  Sample solution: In order to solve this problem, students should create a table of values for each vehicle and compare these to solve the problem:

  The bus and car will meet at 12:00 noon.

Bus

Hour h	Distance (km) 90h
8:00 AM	0
9:00 AM	90
10:00 AM	180
11:00 AM	270
12 noon	360

Car

Hour h	Distance (km) 120h
9:00 AM	0
10:00 AM	120
11:00 AM	240
12 noon	360
1:00 PM	480

(6PR1.4)

Resources/Notes

Math Focus 6

Lesson 6: Solving Problems Using Patterns

6PR1

TG pp. 42-45

Omit question 1 on p.26 of text.

Students have also had opportunities in lesson 3 to use a table of values to help them reveal a pattern and solve a given problem. You may wish to revisit some of the practice throughout this unit.

Math Game:

Rolling Equations

TG pp. 46 – 47