The great logarithmic and trigonometric tables of the French Cadastre: a preliminary investigation

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Important notice

This report is supplemented by 47 volumes of tables, each as a PDF file. These tables replicate the structure of the *Tables du cadastre* and are provided to ease further research. They can be found on the LOCOMAT site, or at the location `http://hal.inria.fr/inria-00543946/en`.

Contents

Introduction

Je ferai mes calculs comme on fait les épingles.[1]

Prony

As part of the French reform in the units of weights and measures, an effort was undertaken at the beginning of the 1790s at the *Bureau du cadastre* to construct tables of logarithms which would not only be based on the more convenient decimal division of the angles, but also would become the most accurate such tables ever created.

Gaspard de Prony had the task to implement this project, and he decided to split the computations among a number of computers. Use was made of only the simplest operations: additions and subtractions of differences.

Begun in 1793, these tables were completed around mid-1796, but, although they were supposed to, they were never printed. Eventually, in the 1830s, the project was totally abandoned.

This mythical endeavour of human computation nowadays lies forgotten in libraries in Paris and, apart from a 30-page description of the tables by Lefort in 1858,[2] very little has been written on them.

The work done cannot be merely described as interpolations using the method of differences. In fact, perhaps the main outcome of our investigation is that the picture is not as clear as the myth may have made it. It is actually much more complex. Additions and subtractions may seem simple operations, but so much appears to have been left unspecified. This has probably become clear to Prony and others, but only when it was too late.

This document summarizes a preliminary investigation of these tables, but the task is much more daunting than it appears at first sight. This study does in fact barely scratch the surface. It tries to give some impetus, but much still lies ahead.

[1] [Edgeworth (1894)]

[2] [Lefort (1858b)]

Chapter 1

The *Tables du cadastre*

1.1 The decimal metric system

The decimal logarithmic and trigonometric tables conceived by the French cadastre take their roots in the metric reform. The founding act was the law of 26 March 1791 which based the metric system on the measurement of the meridian.[3] As pointed out by Gillispie, decimalization became incorporated, or even "smuggled," into the metric system in corollary of that law, because the new unit was defined as the *10 millionth* part of a quarter of a meridian and a sexagesimal division would have corrupted the unity of the system.[4]

In other words, the decimal metric system was made complete by substituting a decimal or centesimal division to the old division of the quadrant.[5] In turn, the decimal division of the quadrant made it necessary to compute new tables. Such arose the need for tables of logarithms of trigonometric functions using the decimal or centesimal division of the quadrant.[6]

Moreover, it was felt that the publication of new tables would help the propagation of the metric system.[7]

The first decimal[8] tables made as a consequence of the new division of

[3][Méchain and Delambre (1806–1810)]

[4][Gillispie (2004), p. 244]

[5][Carnot (1861), p. 552]

[6]It should be remarked that there had been at least one table with a partial decimalization, namely Briggs' *Trigonometria Britannica* [Briggs and Gellibrand (1633)]. Briggs used the usual division of the quadrant in 90 degrees, but divided the degrees centesimally. Briggs also gave a small table of sines for a division of the circle in 100 parts, which is the division used by Mendizábal in 1891 [de Mendizábal-Tamborrel (1891)].

[7][Gillispie (2004), p. 484]

[8]Some early trigonometric tables are now called "decimal" for a slightly different reason, namely for a decimalization of the radius. Around 1450, Giovanni Bianchini introduced in Western mathematics tables of tangents in which the radius was 10^3. He also computed

the quadrant were those of Borda (1733–1799),[9] completed in 1792, but only published in 1801.[10] The 7-place tables of Callet computed after those of Borda were actually published before them in 1795.[11,12]

Given the context in which the decimal division was popularized, it was sometimes called the "French division of the circle."[13]

In 1799, Hobert and Ideler published in Berlin another table of logarithms based on the decimal division of the quadrant.[14] In their introduction, they defend the idea that the decimal division is a logical evolution that followed

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tables of sines in which the radius was $60 \cdot 10^3$, following the tradition of Ptolemy. The former tables can be called "decimal," while the latter are "sexagesimal." The angles themselves were in both cases sexagesimal, and not decimal or centesimal in the revolutionary meaning [Rosińska (1981), Rosińska (1987)]. Of course, the new decimal tables such as Borda's were decimal in both senses.

[9][de Borda and Delambre (1801)] Delambre writes that the manuscript had been completed in 1792 [de Borda and Delambre (1801), p. 39]. These tables were based on those of Briggs [Briggs (1624)] and Vlacq [Vlacq (1628)] (trigonometric part), see [de Borda and Delambre (1801), pp. 40 and 114].

[10]These tables can be viewed as a superset of those of Callet published six years before. They gave the logarithms of numbers from 10000 to 100000 with 7 decimals, the logarithms of the sine, cosine, tangent and cotangent with 11 decimals every ten centesimal seconds from 0 to 10 centesimal minutes, then every 10 centesimal minutes until 50 centesimal degrees, and finally the logarithms of the six trigonometric functions with 7 decimals every 10 centesimal seconds (every 100000th of a quadrant) from 0 to 3 centesimal degrees, and every minute (every 10000th of a quadrant) from 3 centesimal degrees to 50 centesimal degrees. [de Borda and Delambre (1801)]

[11]Callet writes that his decimal tables can be viewed as an abridged version of Borda's tables [Callet (1795), p. VI]. Callet has possibly used Borda's tables as his source, but he is not explicit about it. Delambre writes that Callet had Borda's manuscript in his hands [de Borda and Delambre (1801), pp. 113–114]. Prony wrote that Callet based his tables on the *Tables du cadastre* [Riche de Prony (1824), pp. 39–40], but Callet actually only made a comparison, still leaving errors [de Borda and Delambre (1801), pp. 113–114]. Delambre, instead, compared the logarithms of sines and tangents of Borda's table with the *Tables du cadastre* [de Borda and Delambre (1801), p. 114].

[12]In addition to the logarithms of sines and tangents in the sexagesimal division, Callet's 1795 tables also gave the logarithms of the sines, cosines and tangents with 7 decimals every 10000th of the quadrant. Moreover they gave the natural sines and cosines with 15 decimals and the logarithms of the sines and cosines with 9 decimals every 1000th of the quadrant. [Callet (1795)] In the first edition of Callet's tables, published in 1783, the decimal division was not yet used [Callet (1783)].

[13][Keith (1826), p. X]

[14][Hobert and Ideler (1799)] Hobert and Ideler gave the sines, cosines, tangents, cotangents and their logarithms with 7 places, as well as the first differences, for the arcs $0^q.00000$ to $0^q.03000$ (by steps of $0^q.00001$) and from $0^q.0300$ to $0^q.5000$ (by steps of $0^q.0001$). There are also several auxiliary tables and corrections to Callet's decimal tables. Hobert and Ideler's arc values happen to be the same as those used by Plauzoles in 1809 [de Plauzoles (1809)].

the steps of the computation by chords, by sines, Briggs' decimal division of the degrees, then that of the quadrant. The authors mention the ongoing reform in France and explain the necessity for all non French mathematicians to get acquainted with this system:

> "Jeder nicht französische Mathematiker wird alsdann genöthigt seyn, sich mit der neuen Kreiseintheilung vertraut zu machen, sey es auch nur, um die Resultate französischer Messungen und Rechnungen benutzen zu können."[15]

In an 1811 review of Dealtry's *Principles of Fluxions* (1810), the author attributes the first objection to the sexagesimal system to the mathematicians Oughtred and Wallis. He refers to John Newton's centesimal trigonometric table from 1659 (sic).[16] According to the reviewer, Hutton's idea of using the arc whose length is equal to the radius as the unit (later called the radian[17]) awakened the attention of the French to the subject,[18] and this—so the reviewer—is what set the French to "instantly" prepare more extensive tables, and in particular those of Callet and Borda:

> "From this period the French always speak of the centesimal division of the quadrant as theirs; English authors also speak of the 'new *French* division of the quadrant;' although the original

[15][Hobert and Ideler (1799), p. X] In addition, Hobert and Ideler state that Schulze had intended to publish decimal tables as soon as 1782, but that this publication never took place. According to Sarton, Schulze's tables had a centesimal division of the degree, like Briggs' tables [Sarton (1935), p. 199]. Eventually, Schulze only suggested a way to compute such tables [Hobert and Ideler (1799), p. XI]. These suggestions were actually followed between 1785 and 1791 by Schmidt of Schwerin who was not able to complete them. Hobert and Ideler obtained Schmidt's work, but decided against using it for their own computations. Instead, they used the method of differences which was suggested to them by reading Cagnoli's book in trigonometry (*Traité de trigonométrie rectiligne et sphérique*, 1786) [Cagnoli (1786)]. Their very interesting method is described in details in their introduction and many parallels can be drawn with Prony's work. The Δ^n, however, were not computed analytically. Hobert and Ideler state that the computations went very quickly and that on certain days, they obtained 300 results, compared to the 600 results obtained daily by Prony's 15 computers, as reported by Bode [Bode (1795), p. 215] [Hobert and Ideler (1799), p. XXXIII].

[16]This is probably Newton's *Trigonometria Britannica* (1658) [Newton (1658)].

[17]The name "radian" was first used in print by James Thomson in 1873 [Cajori (1928–1929), vol. 2, p. 147]. Hutton's idea of using this unit was published in 1783 [Hutton (1812)], but the concept of radian actually goes back at least to Cotes' *Logometria* (1714), which was described by Hutton.

[18]On the other hand, Gabriel Mouton proposed a decimal system of measures in 1670 [Hellman (1936), p. 314], and this was mentioned in the *Encyclopédie*, before the French Revolution.

> idea is undoubtly English, and a table, as we have observed, was published here in 1659, nearly 150 years before our neighbours thought of any such division."[19]

One should however remember that these lines were written in 1811, during the war between France and Britain.[20]

But even though some British were considering the decimal division as their invention, there were also opponents to the reform. Thomas Keith gives for instance a summary of the reasons opposing the introduction of a decimal or centesimal division of the quadrant:

> "The advantages of this new division of the circle, should it be generally adopted in practical calculations, are few and trifling, when compared with the confusion and perplexity it would occasion. It is true that degrees, &. would be more readily turned into minutes or seconds, *et vice versá*, and some other advantages of minor importance would be obtained, were the new division to be *universally* adopted; at the same time all our valuable tables would be rendered useless; the many well-established trigonometrical and astronomical works, which from time to time have been published, would be little better than waste paper; the most valuable mathematical instruments, which have been constructed by celebrated artists, must be considered as lumber in the different observatories of Europe; the latitudes and longitudes of places must be changed, which change would render all the different works on Geography useless; or otherwise the Astronomers, and those in the habit of making trigonometrical calculations, must be perpetually turning the old division of the circle into the new, or the new into the old. (...) The logarithmic tables of sines, tangents, &c. which were originally constructed by the British mathematicians, have passed through so many hands, and have been so often examined, that they may be depended upon as correct; whilst the new tables would require great caution in using them."[21]

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[19][Anonymous (1811), p. 344] According to Sarton, decimally graduated instruments were made and sold in London in 1619 [Sarton (1935), p. 189].

[20]One might also contrast this opinion with the failure of the joint publication effort initiated in 1819, probably mainly because of the centesimal structure of Prony's tables. The British wanted to convert the tables to the sexagesimal division and this would have meant that all computations should have been redone, see [Anonymous (ca. 1820)] and section 1.4.6 in this document.

[21][Keith (1826), pp. X–XI]

In spite of this, the decimal or centesimal division of the quadrant did not die. In 1905, for instance, it was made compulsory for the entrance examinations to the French *École polytechnique* and *Saint-Cyr* schools, and other decrees prescribed its use for various examinations.[22]

1.2 The need for more accuracy

The requirement to have tables with more decimals, or smaller intervals between consecutive values,[23] was also felt more and more. Although Briggs and Vlacq gave logarithms of numbers and trigonometric functions with 10 to 15 places, their editions were not very practical, they had many errors, and they were excessively rare. Smaller and yet accurate tables were needed and they appeared little by little. John Newton's tables (1671),[24] for instance, gave logarithms with five or six places. There were few seven-place tables and among the first such tables, we can name those of Vega (1783),[25] Hutton (1785),[26] Callet (1795),[27] Borda (1801),[28] Babbage (1827)[29] and Sang (1871).[30] Most of these tables were derived from Vlacq's tables.[31]

In 1794, Vega published 10-place tables based on Vlacq's calculations,[32] but apart from them, by the time the *Tables du cadastre* were set up, there were very few 8, 9, or 10-place tables. After Newton's *Trigonometria Britannica* (1658),[33] the next 8-place tables were those of the *Service géographique*

[22] See the foreword in Bouvart and Ratinet's tables [Bouvart and Ratinet (1957)] and also the note about the use of the centesimal division at the *École polytechnique* [Anonymous (1901)]. See also Archibald [Archibald (1943a), pp. 36–37].

[23] The number of decimals and the step are related, in that if more decimals are sought for a logarithm, then one will also need to compute logarithms of numbers with greater number of decimals located between two values of a table, and a smaller step will make interpolation easier. See for instance [Vincent (1825–1826)] for a contemporary account of the errors resulting from using a mere linear interpolation.

[24] [Newton (1671)]

[25] [Vega (1783)]

[26] [Hutton (1785)]

[27] [Callet (1795)]

[28] [de Borda and Delambre (1801)] This table was a decimal table.

[29] [Babbage (1827)]

[30] [Sang (1871)]

[31] [Vlacq (1628), Vlacq (1633)]

[32] [Vega (1794)] It should be remarked that Vega's tables are not exactly those of Vlacq, because Vega computed new values between 0° and 2° by interpolation. He also corrected many errors.

[33] [Newton (1658)]

de l'armée (1891),[34] of Mendizábal (1891),[35] and of Bauschinger and Peters (1910–1911).[36] Nine-place tables are extremely rare and none are listed in Fletcher's index.[37]

One may wonder if there really was a need for such accurate tables. In fact, they were more and more required by the increased accuracy of measurements. Pondering the need for 9-place tables in 1873 following Edward Sang's project, Govi gave the example of an accurate scale which can be sensitive to a difference of 1 milligram for a weight of 20 kilograms in each plate, hence a sensitivity of $5 \cdot 10^{-8}$ relatively to the weight of the load. In order to use such values in calculations, logarithms of 8 or 9 places are necessary. The measurement of time, or of lengths, are other examples requiring accurate computations. In most cases, the computations could be done differently, but it would be slower and more complex than using adequate tables. Govi also pointed out that it was the astronomers, who are great users of tables of logarithms, but who at the same time have data with only a few accurate digits, who worked against more accurate tables. In tables of compound interest, there is also a need for logarithms with more than 10 places.[38]

It is interesting to recall Ernest W. Brown's comments written in 1912, when reviewing Henri Andoyer's tables of logarithms.[39] After having observed that the accuracy of observations had increased very much in the previous fifty years, Brown stressed that

> "[The] problem is not so much that of getting the numerical value of a single function [...] in such cases one can usually adopt devices which grind out the result at the cost of trouble and time. Many of the present day problems are on a large scale. The calculations are turned over to professional computers [...] Extended tables and, if possible, mechanical devices are more and more sought after in order to economize time and money in scientific work, just as in business."[40]

[34][Service géographique de l'Armée (1891)]

[35][de Mendizábal-Tamborrel (1891)]

[36][Bauschinger and Peters (1910–1911)]

[37][Fletcher et al. (1962), p. 160] Edward Sang had a project of building a nine-place table of logarithms from 100000 to one million, but this project never saw the light of day [Roegel (2010a)].

[38][Govi (1873)]

[39][Andoyer (1911)]

[40][Brown (1912)]

1.3 Prony and the cadastre

At the dawn of the French Revolution, at a time when the Treasury needed money, setting up a general cadastre was seen as the only efficient remedy to assign land taxes in a non-arbitrary way.[41,42] All taxes were abolished by the law of 1 December 1790 and replaced by a single property tax.[43] Then, a decree of 16 September 1791, which became a law on 23 September 1791, proclaimed the establishment of a "cadastre général de la France" and on 5 October Gaspard Riche de Prony (1755–1839) became director of the *Bureau du cadastre*. Prony, as he was called, remained in that position until the cadastre was terminated in 1799.[44]

Prony (figure 1.1) graduated from the *École Royale des Ponts et Chaussées*[45] in 1780 and became the leading engineer and engineering educator of his days, as famous as Lagrange and Laplace.[46] Among other things, in 1794 he became professor at the newly founded *École Centrale des Travaux Publics* (later, the *École Polytechnique*) where he remained professor until 1815. He was also director of the *École des Ponts et Chaussées* between 1798 and 1839.[47]

In a report he submitted on 10 October 1791, Prony described all the tasks involved in establishing a cadastre, in particular the need to revise the geodetic triangles of the Cassini map. New measurement devices would enable surveyors to make their calculations by measuring angles on the land, rather than on paper. In addition, Prony anticipated the measurement reform and planned to use several units, including an estimated value of the meter.[48]

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[41] See [Noizet (1861), pp. 13–14]. It would however take years to implement this cadastre fully. In 1807, Napoleon passed a law in order to measure and evaluate precisely every parcel of property. It took until 1850 to complete the survey of the entire France.[Herbin and Pebereau (1953), pp. 21–24]

[42] [Herbin and Pebereau (1953), p. 17]

[43] [Kain and Baigent (1992), p. 225]

[44] [Berthaut (1902), p. 322], [Konvitz (1987), pp. 47–48] See [Grinevald (2008), de Oliveira (2008)] for overviews of the beginnings of the French cadastre. At the beginning, there were only five employees, in addition of Prony. Jean-Henri Hassenfratz was assistant director in 1791–1792 [Grison (1996)], and then followed in that position by Charles-François Frérot d'Abancourt (1758–1801) [Grison (1996), p. 21].

[45] On the history of the *Ponts et chaussées*, see [Brunot and Coquand (1982)].

[46] [Grattan-Guinness (1990b), p. 110] For other biographical elements on Prony, see in particular [Tarbé de Vauxclairs (1839)], [Parisot (no year), pp. 399–405], [Moigno (1847)], [Walckenaer (1940)], [McKeon (1975)], [Picon et al. (1984)], [Bradley (1994)], and [Bradley (1998)]. A biographical note by C. Perrin and dated 1895 is contained in the Archives of Chamelet, Prony's birthplace (Archives du Rhône, Série C25, Dossier 1).

[47] [Konvitz (1987), pp. 47–48]

[48] [Konvitz (1987), p. 48]

Figure 1.1: Gaspard Riche de Prony (1755–1839) (Source: Wikipedia) A copy of this engraving is also contained in Prony's file in the Archives of the *Académie des Sciences*.

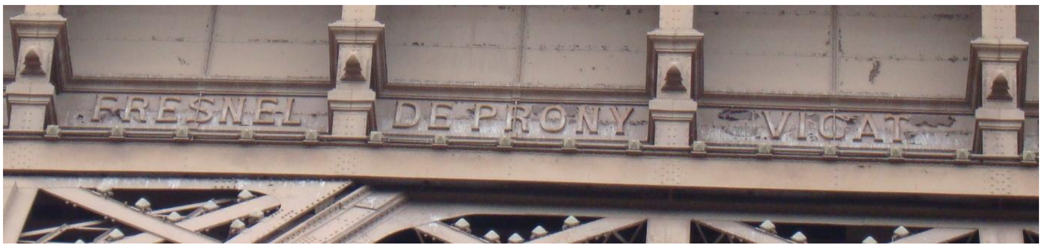

Figure 1.2: Prony's name on the Eiffel tower, between those of Fresnel and Vicat. (Photograph by the author.) The names were concealed by paint from the beginning of the 20th century until their restauration in 1986. See also [Chanson (2009)].

Prony had valued accuracy and uniformity above all, and he believed that only a centralized administrative structure could guarantee them.[49]

Prony obtained for Jean-Guillaume Garnier (1766–1840) to become head of the geometrical section of the Bureau of the cadastre, that is, the section of computers.[50] Garnier remained at this position until the *1er Messidor an V* (19 June 1797)[51]. When the central office of the cadastre was complete, it was made of sixty employees, divided in two sections, one of geometers and calculators (headed by Garnier), and one of geographers and drawers.[52]

One of Prony's first tasks was to measure the total area of France from the original maps and it took him nearly a year.[53]

Instruments, in particular Borda's repeating circle conceived around 1787, were converted to the decimal division of the angles, and the need for tables based on that decimal division became more and more urgent.[54]

So, it is no surprise that in 1793 Lazare Carnot[55], Claude-Antoine Prieur ("from the Côte-d'Or"),[56] who were directing the war effort,[57] and Brunet (from Montpellier),[58] gave Prony the task of computing new tables of loga-

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[49][Konvitz (1987), p. 57]

[50][Garnier (1826), p. 118] Interestingly, it was the printer Firmin Didot to whom Garnier gave mathematics lessons who put him in touch with Prony. See Quetelet's notices on Garnier [Garnier and Quetelet (1841)], [Quetelet (1867), pp. 206–207], in which some dates may however be inaccurate. According to Michaud's biographical notice, Garnier did not benefit from this work as much as he hoped, and Prony took "the lion's share."[Michaud (1856), p. 594]

[51][Quetelet (1867), p. 207]

[52]This structure is reflected by the salary summaries, and other accounts, such as Garnier's [Quetelet (1867), p. 207].

[53][Konvitz (1987), p. 49] See also P.C., Ms. 2148 and 2402 for further details on the first activities of the *Bureau du Cadastre*.

[54][Konvitz (1987), p. 49]

[55]Carnot, who was trying to protect men of science, had actually first been in touch with Prony a few months earlier in 1793, when he sent him an anonymous note in order to warn him of possible problems resulting from Prony hiring some persons with non Republican views [Carnot (1861), p. 506]. Later, Prony considered that Carnot saved his life [Barral (1855), p. 591]. On Carnot's scientific work, see [Gillispie (1971)].

[56]Prieur (1763–1832) was an engineer and was in particular involved in the metrical system. He presented a *Mémoire* on the standardization of weights and measures in 1790 [Zupko (1990), p. 417], [Gillispie (2004), p. 229], [Bigourdan (1901)], [Hellman (1931), p. 278], [Hellman (1936), p. 315]. He was one of the main founders of the *École polytechnique* [Bouchard (1946)].

[57]In August 1793, Carnot and Prieur became members of the *Comité de Salut Public* (Committee of Public Safety) and had the responsibility of arming the soldiers. They were the only members with a scientific and technical background.

[58]Probably J.-J. Brunet, president of the *Commission des subsistances et approvisionnements*, together with Raisson and Goujon.

rithms.[59]

In a letter to Arago, Prony recalled his first encounter with Carnot. He was asked to come in an office of the *Convention* and Carnot gave him very detailed instructions of the work to accomplish. The tables had to be the most accurate and "the greatest and most imposing monument of computation ever made, or even conceived."[60]

According to Prony, the demand was in fact even more accurate. Prony was not only asked to compute the trigonometric functions and their logarithms with a great number of decimals and with a small step, but he also had to recompute the logarithms of numbers, with twice the accuracy of the greatest known tables.[61]

The work on the tables was begun in 1793 and probably completed around mid-1796.[62] The work was completed on the premises of the *Bureau du cadastre*, namely at the *Palais Bourbon* (figure 1.3), the building which is now the seat of the French National Assembly.

In Nivôse IV (December 1795-January 1796), almost at the time of the completion of the tables, Prony had an annual salary of 12000 *francs*,[63] a section chief earned 7500 *francs*, and a calculator 3750 *francs*.

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[59]Some sources, such as [Bradley (1994), Bradley (1998)], state that the tables were begun in 1792, but Prony makes it clear that it was Carnot who asked him to make the tables at the end of 1793. Bradley also puts the completion of the tables at 1801 [Bradley (1994), p. 244] which was merely the date of a report [Riche de Prony (1801)].

[60][Carnot (1861), p. 552]

[61][Riche de Prony (1824), p. 35] On the other hand, this notice contains some errors, so that one could also doubt Prony's account. On Carnot's approach, Juhel recently made the observation that for Carnot there was no difference between warfare and mathematics [Juhel (2010), p. 59].

[62]Since the tables were never published, and hopes still appeared at various stages of a process that lasted 40 years and in which reports were occasionally published to support the publication, the completion of the tables is sometimes reported with some uncertainty. Grattan-Guiness writes for instance that the tables were completed in 1801 [Grattan-Guinness (1990b), p. 179], but this is merely the date of Prony's note on the project [Riche de Prony (1801)]. The tables were waiting to be printed and published since their completion mid-1796. This date is supported by various facts, such as the completion of the trigonometric tables in 1795, that of more auxiliary tables (such as that of multiples of sines) after 1795, and a mention in the tables of logarithms of numbers showing that they had not been completed in 1795. Prony apparently announced in 1796 that the tables had been completed, and it was echoed abroad [Anonymous (1796b)]. It is possible that some other tables, such as the 8-place tables, were computed after mid-1796. See also the 1820 note [Anonymous (1820 or 1821), p. 8]. Various authors wrote that the tables had been completed in two years (for instance Parisot [Parisot (no year), p.400]), but usually copying on each other.

[63]The *franc* replaced the *livre* by the law of 18 Germinal III (7 April 1795) [Gillispie (2004), p. 244].

In 1795, the *École des Géographes* was created by the law of 30 Vendémiaire IV (22 October 1795), at the same time as the *École polytechnique* (the former *École Centrale des Travaux Publics*) and other schools.[64] This *École des Géographes* was to have about twenty students who could apply to it after having studied at least for one year at the *École polytechnique*. The director of the cadastre was attached to the school, implicitely being its director. The students of the school would be able to work at the cadastre, or at other administrations that needed them, and the students were to become *ingénieurs-géographes*. The law of 1795 explicitely stated that the number of students would initially be fifty, so as to stimulate the work of the cadastre.[65]

The school was of course instituted for the instruction of surveyors, anticipating field work that would begin in 1795, when Delambre and Méchain would have completed the measure of the meridian between Dunkerque and Barcelona.[66] This measure would provide a definition for the meter, which was to be a 10 millionth of a quarter of a meridian. As mentioned earlier, Delambre and Méchain had instruments graduated decimally, and there was a real need for decimal trigonometric tables.[67]

Unfortunately, the measure of the meridian was only completed in 1798 and the metric system was eventually adopted in 1799. Because of these delays, the geodetic section of the cadastre concentrated on matters other than surveying, and in particular on the tables of logarithms.[68]

By 1799, when measurement reform was complete, budgetary pressures led to the elimination of the cadastre. Prony complained that he and his staff had been asked to do too much and had been underfunded for too long.[69]

The Great Tables then became orphans. Funding was gone, and the tables remained in manuscript form. Decimal tables were not extinct, though, for the first readily accessible decimal tables were published by Borda and Delambre in 1801.[70]

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[64] [Denisart et al. (1807), p. 161], [Rondonneau (1818), pp. 631–632] The *École des Géographes* was to be associated with another school, the *École nationale aérostatique*, see [Bret (1990–1991)].

[65] [Rondonneau (1818), p. 631]

[66] [Konvitz (1987), p. 50] and [Gillispie (2004), p. 481] For more on Delambre and Méchain's journey, see Ken Alder's account [Alder (2002)]. The results of Méchain and Delambre's computations were published in three volumes [Méchain and Delambre (1806–1810)].

[67] [Gillispie (2004), p. 487]

[68] In 1808, in his *Manuel de l'ingénieur du cadastre*, Pommiés considered that Prony was able to devote himself to the computations, because there was little else to do for the cadastre due to the revolutionary wars [Pommiés (1808), p. x].

[69] [Konvitz (1987), p. 52]

[70] [Gillispie (2004), p. 487]

1.4 History of the tables

It is now possible to draw a fairly accurate picture of the history of the *Tables du cadastre*. When Prony was given the task to produce new tables of logarithms, he must have naturally thought of the method of differences, in particular since he had written about differences in 1790 in the context of interpolation to determine gas expansion laws.[71] The problem was therefore to find out how these differences should be computed.

Figure 1.3: The Palais Bourbon in the 19th century, the location of the *Bureau du cadastre*. At the end of the 18th century, the Roman portico had not yet been added. (Source: Wikipedia)

1.4.1 Work organization

In his celebrated work *The Wealth of Nations*, Adam Smith[72] considered the example of the division of labor in a pin-factory. According to Smith, "a workman not educated to this business, nor acquainted with the use of the machinery employed in it, could scarce, perhaps, with his utmost industry, make one pin in a day." But if the work is split and specialized, "ten persons, therefore, could make among them upwards of forty-eight thousand pins in a day."[73]

Inspired by Smith, Prony decided to use manufacturing processes to compute the logarithms.[74] Many tasks were similar and could be parallelized.

[71] [Grattan-Guinness (1990b), p. 177]

[72] [Smith (1776), Peaucelle (2006), Peaucelle (2007)]

[73] Smith's conclusion should of course be relativized, in particular because it is set in an "ideal" factory where people are merely one-operation machines, and would slow down the work tremendously by applying themselves to tasks for which they have no training. But employees learn, and the discrepancy between Smith's specialization and the 'one-person-does-it-all' version is not as extreme as Smith thought. Moreover, even if the workman who did everything were qualified in his multiple tasks, he would still usually do less than several workmen qualified in only one task.

[74] [Anonymous (1820 or 1821), p. 7], Smith is not mentioned at all in the 1801 notice.

Prony's division was however not an exact copy of the pin-factory, because there was mainly one computing task, which was divided in about twenty computers, each doing a similar work. In the pin-factory, each of Smith's ten workers were specialized, and doing a specific task. There was no such specialization in Prony's scheme, except for the task of providing blank sheets with initial values, and checking the values. Most of the computations were only of one type.

So, with this inspiration, Prony organized the logarithm-factory in three groups:

- In the first group, there were five or six mathematicians of "very high merit," but only Adrien-Marie Legendre (1752–1833) is named explicitely by Prony.[75] Their role was to elaborate formulæ and to compute fundamental values, such as coefficients, number of digits, etc. Prony must certainly be included in this group. Jean-Baptiste Joseph Delambre (1749–1822) is known to have been close to the computations, and probably also Charles de Borda (1733–1799), but they were perhaps not meant by Prony. In his course of the *École Polytechnique* [Riche de Prony (1796b), p. 555], Prony only mentions José María de Lanz (1764–1839) and Charles Haros who worked on Mouton's interpolation problem. There was probably some overlap between the first and second groups, some members working both on the analytical part and on the application of the formulæ. Obviously, the first

Prony only mentions applying the methods of division of labour [Riche de Prony (1801), p. 2]. In 1819, Lacroix observed that the 1801 report is above all interesting because it shows the utility of the division of labour to the execution of the most long and difficult calculations [Lacroix (1819), p. 19]. In 1820 and 1824, Prony wrote that he accidentally found Smith's book in an antiquarian bookstore, opened it randomly on the chapter on the division of labour (which happens to be chapter one), and conceived the plan to construct logarithms like one constructs pins. Then Prony wrote that he was prepared to this conception by certain classes he was then teaching at the *École Polytechnique* [Riche de Prony (1824), pp. 35–36]. But at that time, there was not yet an *École Polytechnique* and not even its predecessor. As others have remarked, some of Prony's writings are inconsistent. Prony finally wrote that after having conceived his plan, he went to the countryside and established the foundations of the new factory. How much of this story is really true is not known. Prony's example also inspired Babbage, and it has later become a favorite example either for economists studying the division of labor, or for cognitive scientists exploring the metaphor of the mind as a computer. Some recent articles exploring these ideas are [Gigerenzer and Goldstein (1996), Green (2001), Langlois (2003), Boden (2006), Bullock (2008), Langlois and Garzarelli (2008)]. On the general question of human computing before computers, see the very interesting book by David Grier [Grier (2005)].

[75] [Riche de Prony (1801), p. 4] Delambre wrote that Legendre presided the analytical part for some time [Delambre (1810)].

group had only a temporary existence.[76]

- In the second group, there were computers acquainted with calculus, sometimes called "*calculateurs*." They computed the values of the initial logarithms and of the initial differences Δ, using the formulæ provided by the first group. They then gave calculation sheets to the members of the third group. They were also in charge of checking the results which came back from the third group. Members of this group must also have computed the first 10000 logarithms to 19 places. The members of this group may have varied, and may not be reflected accurately in the salary summaries. For instance, at the end of 1795, there is a group of *mathématiciens* comprising Langlet père, Antoine Joseph Reboul (1738–1816),[77] Jacques Joseph Grou, Theveneau,[78] and Charles Haros,[79] but at that time, the work on the tables was mostly finished, so that these *mathématiciens* may actually be the *calculateurs* mentioned

[76] According to Grattan-Guinness, the first group comprised also Carnot and Prieur [Grattan-Guinness (1990b), p. 179], but I do not know the source of this information. Grier also wrote that Legendre and Carnot were part of that group, referring to Babbage who mentions none of them [Grier (2005), p. 37 and note 40].

[77] Born in Montpellier, 1738–1816, Reboul was a benedictine of the Congregation of St. Maur and professor of mathematics and physics at the Sorèze military school.(A.N. $F^{1bI}44$). He apparently published tables of Venus in 1811.

[78] Probably Charles-Marie-Simon Théveneau (1759–1821) [Nielsen (1929), p. 229]. Théveneau edited Clairaut's algebra in 1801 and was also a poet. He is mentioned by Callet as having compared the centesimal tables in Callet's tables with the *Tables du cadastre* [Callet (1795), p. VI].

[79] We know that Haros worked on the development of formulæ for the computation of logarithms. After 1795, Haros was one of the computers of the *Connaissance des tems*. Among Haros' scientific works, there is an *Instruction abrégée sur les nouvelles mesures qui doivent être introduites dans toute la République, au 1er vendémiaire an 10 : avec des tables de rapports et de réductions* (1801, also with editions in at least 1802 and 1810), *Comptes faits à la manière de Barême sur les nouveaux Poids et Mesures, avec les prix proportionnels à l'usage des commerçans etc.* (1802) and also an article anticipating the Farey sequence ("Tables pour évaluer une fraction ordinaire avec autant de décimales qu'on voudra ; et pour trouver la fraction ordinaire la plus simple, et qui approche sensiblement d'une fraction décimale," *Journal de l'Ecole Polytechnique* 4(11) (Messidor X), 364–368). See also Roger Mansuy, *Les calculs du citoyen Haros — L'apprentissage du calcul décimal*, 2008, 3 pages, and [Guthery (2010)]. On 2 July 1809, Haros' widow wrote to Prony about tables of logarithms started by her late husband, and that she wanted to be examined by Prony, being now needy. According to her, Haros had wanted to compute the logarithms up to a million, and he thought that it would earn him a lot of money. (PC: Ms. 1745) One might want to correlate this information with the manuscript 8-place tables located in the *Ponts et chaussées* archives, but these tables are probably unrelated, first because the 8-place tables do not bear the name of Haros, nor any note alluding to such an origin, and second because two identical copies of Haros' tables would probably not have ended up in Prony's hands.

by Prony in 1801 [Riche de Prony (1801)]. According to Garnier, Nicolas Maurice Chompré (1750–1825) was also involved,[80] probably in this section.[81] We also know that Nicolas-Antoine Guillard (ca. 1760–1820), a French mathematician employed at the cadastre in 1794, was working on the analytical part of the computation of the tables.[82] Jean-Guillaume Garnier (1766–1840),[83] Charles Plauzoles,[84] José María de Lanz (1764–1839),[85] Nicolas Halma (1755–1828),[86] Étienne-Marie Barruel (1749–1818),[87] Marc-Antoine Parseval (1755–1836),[88] or Jean Baptiste Plessis,[89] may have been among the members of this group at one time or another.[90]

- The third group was the largest and was in charge of the interpolation; probable members of this group were Jean Baptiste Letellier, Jean Désiré Guyétant, Bridanne, Pierre Antoine Jannin, Alexandre, Ange Christophe Gabaille (born ca. 1771),[91] Thomas Robert Philippe Louis Gineste (born ca. 1768), René Bulton, Pierre Mamet (born ca. 1774),

[80]Chompré wrote several books on mathematics and physics, and translated English and Italian works into French. He probably started to work at the Cadastre in Vendémiaire III.(A.N. $F^{1bI}44$)

[81][Garnier (1826), p. 118]

[82][Michaud (1839), pp. 260–261] Guillard was professor of mathematics and published in particular a *Traité élémentaire d'Arithmétique décimale* in 1802, as well as a new edition of Bezout's *Cours de mathématiques*.

[83]Garnier published a number of books and was professor at the *École Polytechnique*. He was chief of the geometrical section of the cadastre until 1797. Mascart wrote that Garnier worked at the Cadastre until 1794, but it is not correct, as testified by the payment summaries [Mascart (1919), p. 562].

[84]Plauzoles published a table of logarithms in 1809 [de Plauzoles (1809)].

[85]Together with Agustín de Betancourt, Lanz developed Hachette's classification of mechanisms. In 1808, they published the *Essai sur la composition des machines*.

[86]Among his many activities, Halma published the first French translation of Ptolemy's *Almagest*, based on the original Greek text.

[87]Barruel wrote several books on physics and was among the first professors at the *École Polytechnique*.

[88]Parseval is most famous for what became known as "Parseval's theorem," first published (but not proven) in 1799.

[89]Plessis was *ingénieur-géographe* and later author of cartographic tables. He is mentioned in Puissant's *Traité de topographie*, 1807, and presumably gave his name to the Plessis ellipsoid, which was the standard ellipsoid used in France in 1817.

[90][Bret (1991), p. 123], [Gillispie (2004), p. 483] In 1822, another anonymous author named Garnier (then professor at the university of Gand), Legendre, Chompré, Plessis (then capitaine in the *corps des ingénieurs géographes*), Haros, Théveneau, Plauzoles (who died as deputy-chief of the new cadastre), "Langlais" (former professor at the *École royale militaire* in Paris, and since employed at the *Bureau des longitudes*), as those who may all have been part of this second group [Anonymous (1822)].

[91]Some of the ages are given in a letter by Prony dated 2 Nivôse IV (23 December

> Hervet, Saget (père), La Bussierre, Jean Baptiste André Vibert, Humaird, Étienne Antoine François Baudouin (born ca. 1768), Louis Saget (fils), Mazerat, Marc Antoine Parisot, Henry, Leprestre, Pierre Simon Pigeou,[92] and a few others.[93] These computers had only to perform additions and subtractions, and put the results on pages submitted to them by the second group. Some of the names appear in the tables.[94]

There is some uncertainty regarding the actual number of computers. For instance, in 1801, Prony wrote that there were about sixty or eighty computers,[95] but in 1832, he wrote that there had been between 150 and

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1795) in which he requests that a number of employees of the *Bureau du cadastre* not be requisitioned for the army.(A.N. $F^{1bI}44$)

[92]Probably Pierre-Simon Pigeou (1765 Reims–1812 Trèves), *ingénieur-géographe*, who worked with Jean Joseph Tranchot in Corsica and then on the left bank of the Rhine (see [Berthaut (1902), p. 315] and Michel Desbrière, "Les travaux dirigés par Jean Joseph Tranchot sur la rive gauche du Rhin, 1801–1814," *Bulletin du Comité Français de Cartographie*, number 191, March 2007, pp. 13–24).

[93]According to Dupin who seems to have been the first to mention it in 1824, and who was a friend of Prony, some members of the third group were former hairdressers who had been made jobless during the Revolution as a consequence of the change of fashion [Dupin (1825), p. 173], [Grattan-Guinness (1990b), p. 179]. Walckenaer, on the other hand, puts this assertion in doubt [Walckenaer (1940)], but then cites another anecdote by Dupin whom he considers trustworthy. Walckenaer's doubts can therefore safely be ignored. It should also be observed that in the same article, Walckenaer wrote that the manuscript of the tables at the *Observatoire* was the original one and the one at the *Institut* its copy, when the truth is that these manuscripts are at the same level. None is a copy of the other. For another critical appraisal of Walckenaer's text, see p. 35 of Arthur Birembaut, "Les deux déterminations de l'unité de masse du système métrique," *Revue d'histoire des sciences et de leurs applications*, 1959, volume 12, issue 1, pp. 25–54. Prony seems never to have mentioned hairdressers, but he wrote in 1824 that several of the computers sought and found a kind of safe haven, one that *political circumstances made them necessary* [Riche de Prony (1824), pp. 36–37]. Prony read this notice on 7 June 1824 at the Academy of Sciences, of which Dupin was a member, and Dupin's words are from a lecture given in November of the same year. It is tempting to link the two. The "hairdressers" were popularized by Grattan-Guinness' article [Grattan-Guinness (1990a)], but in our opinion Grattan-Guinness gave too much importance to a detail, as if all computers were from that trade. Perhaps there were only two or three of them. It is unfortunate that other authors have amplified this idea.

[94]Detailed lists of employees of the Cadastre starting in Vendémiaire III (September-October 1794) are available at the *Archives Nationales*. These lists include the salary, and, for some of them, the section of the Cadastre to which they were belonging.(A.N. $F^{1bI}44$) Concerning the rate of calculations, it is interesting to mention a letter by Louis Saget (fils) to Prony, dated 17 Fructidor III (3 September 1795), who asked for a raise, claiming to compute 200 logarithms per day, and to be one of the best computers (A.N. $F^{14}2146$). He wrote that he earns 2600 *francs*, whereas the other computers earn 3400 *francs*. Prony decided to give him 3000 *francs*.

[95][Riche de Prony (1801), p. 5]

200.[96,97,98,99,100]

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[96][Riche de Prony (1832), pp. 67–68] These figures seem widely exaggerated, and at any one time, there were probably a lot less computers. The number of computers of the tables was probably never greater than 20 or 25, and Lalande even put them at 15 [de Lalande (1795)]. In Vendémiaire III (September-October 1794), for instance, when the table of sines was finished, and presumably work going on with the other tables, the Cadastre was comprised of 44 employees, namely Prony (1st class), Garnier, Plauzoles, and Antoine de Chézy (director of the *École nationale des ponts et chaussées* in 1797–1798, see [Bradley (1994), p. 235], [Brunot and Coquand (1982), p. 30]) (all 2nd class), Lanz, Jean Baptiste Plessis, Antoine Joseph Reboul, Barruel, Langlet, Nicolas Antoine Guillard, Portail, Lecuit, Blanchet, Dujardin, Denayer (probably Jean Isidore, born ca. 1768), Guignet, Bosio (probably the painter Jean-François Bosio (1764–1827)), Jean Jacques Le Queu (1757–1826, famous for his architectural drawings), Ducamp, Gelée, Duprat, Charles Haros, Rousseaux, Jean Baptiste Letellier, Jean Désiré Guyétant, François Hubert Tinet, Gabaille, Pierre Antoine Jannin, Bridanne, Kitzinger (3rd class), Marie, Bouquet, Pounnery, Berny, Balzac, Humaird, Berthier, Bertrand (4th class), Saget, Bruyant (5th class), Butel, François (6th class), Naslot, Leurson (7th class). Prony had a salary of 6000 *livres* a year, the salary of the 2nd class was ranging from 5000 to 4500 *livres* a year, the 3rd class from 4000 to 3200, the 4th class from 2760 to 2500, the 5th class from 2200 to 2000, the 6th class from 1800 to 1500, and the 7th class had 1400 *livres* per year.(A.N. $F^{1bI}44$)

[97]A summary of Frimaire IV (November-December 1795) shows that there were a total of 63 employees (including Prony) and that they were grouped in two divisions, the first of geographers headed by Renard and Chézy, the second of computers, headed by Garnier (chef) and Plauzoles (sous-chef). Employees of the first division were in turn grouped in three "*brigades*": 1st (Éloi Lafeuillade and Langlet fils), 2nd (J. F. L. L'Evesque, François Benazet, Antoine Charles Boucher, Louis-Marie Charpentier, Pigeou, Henry, Pierre Eustache LeDuc (ca. 1772–1799 Cairo), LePrestre), and 3rd (Bruno Plagniol (b. ca. 1773), Jean-Pierre Faurie, Jean Junie, Boullée, Louis-Jacques Bourgeois, P. Cadillion, Ferat). A number of these first geographers are also given by Bret, in the list of geographers hired in the year II [Bret (2009), p. 145–147]. In this first division, there were also "*Géographes dessinateurs*" (geographers drawers) (Blanchet, Dujardin, Benayev, Bouquet) and two employees responsible for making and computing tables (Charles Michel Gelée, Jean Baptiste Bertrand, the latter perhaps the Bertrand from [Baudouin-Matuszek (1997)]). The second division was divided into sections. The first section were the "*mathématiciens*." There were nine of them, the first four working on the *Connaissance des tems* (Lanz, Jean Baptiste Marion, Nicolas Antoine Guillard and Dufort), and the others on the *Tables du cadastre* (Langlet père, Reboul, Jacques Joseph Grou, Theveneau, Charles Haros). Finally, this division had a second section made of 18 computers and verifiers: Jean Baptiste Letellier, Jean Désiré Guyétant, Bridanne, Pierre Antoine Jannin, Alexandre, Gabaille, Gineste, René Bulton, Pierre Mamet, Hervet, Saget (père), La Bussierre, Jean Baptiste André Vibert, Humaird, Antoine Baudouin, Louis Saget (fils), Mazerat and Marc Antoine Parisot. As can be observed by the names appearing in the tables themselves, some of the computers of the tables were part of the first division, at least at that time. In Nivôse V (December 1796-January 1797), 23 employees were explicitely assigned to the *Tables du cadastre*.(A.N. $F^{1bI}44$)

[98]One may question whether others have been working on the tables, not registered on these lists, but this seems unlikely. First, all the names appearing in the tables have also

1.4.2 Computing (1793–1796)

After Prony received the commission to build the tables, and came up with a division of the calculators in three sections, it seems that things went very quickly. It is very likely that the tables were computed in order, and not all at the same time, which would have been possible, but would not have brought any advantages. The table of sines was computed first, starting in 1793. It was probably completed in Fructidor II (August-September 1794).[101]

been found in the salary summaries. Second, one of the employees (Louis Saget) wrote about his work as a computer of logarithms, and this rules out that the employees listed were not the computers. The figures given by Prony seem therefore wrong, although we don't have a good explanation why this is so.

[99]The exaggerated number of computers, as well as the great size and number of manuscript volumes, and other difficulties, seem to have led to superlative descriptions. For instance, Grattan-Guinness wrote of Prony directing "an enormous team" and also spoke of the "gigantic" tables [Grattan-Guinness (1993)]. Prony, however, is probably the first to blame for these exaggerations.

[100]A later account was also given by the novelist Maria Edgeworth (1767–1849). She was visiting France in 1820 and met Prony. In a letter dated 4 June 1820, she wrote the following account: "During Buonaparte's Spanish War he employed Prony to make logarithm, astronomical, and nautical tables on a magnificent scale. Prony found that to execute what was required would take him and all the philosophers of France a hundred and fifty years. He was very unhappy, having to do with a despot who *would* have his will executed, when the first volume of Smith's *Wealth of Nations* fell into his hands. He opened on the division of Labour, our favourite pin-making: 'Ha, ha ! voilà mon affaire ; je ferai mes calcules (sic) comme on fait des épingles !' And he divided the labour among two hundred men, who knew no more than the simple rules of arithmetic, whom he assembled in one large building, and these men-machines worked on, and the tables are now complete." [Edgeworth (1894), vol. 1, p. 291] This account is interesting, because it contains some errors. For instance, although there was a French-Spanish war between 1793 and 1795 (the so-called "War of the Pyrenees"), Bonaparte was not involved in it. Perhaps Prony mentioned a war with Spain, and Edgeworth made a confusion with the Peninsula War, opposing France and Spain in 1808. Bonaparte also had nothing to do with the *Tables du cadastre*. In view of these errors, one has to guess that they are both the results of Edgeworth's confusion, and probably of exaggerations by Prony. Perhaps Edgeworth was subjugated by Prony of whom she wrote that he "is enough without any other person to keep the most active mind in conversation of all sorts, scientific, literary, humorous." [Edgeworth (1894), vol. 1, p. 289] Two weeks before, on May 20, she had written "Prony, with his hair nearly in my plate, was telling me most entertaining anecdotes of Buonaparte."

[101]Report dated Fructidor II, A.N. $F^{14}2146$. We can easily obtain some idea on the efficiency of the computations. According to Lalande, 600 logarithms were computed daily, and we also know that a good computer could compute 200 logarithms in a day. If we assume an average of 100 correct logarithms per computer in a day, then computing 400000 logarithms or sines, twice, represents about $\frac{800000}{100} = 8000$ man-days of work. If these computations are done in 500 days, about $\frac{8000}{500} = 16$ computers are necessary. It is of course difficult to compare the real efficiency with other historical computations, because

In order to speed up the work, forms had been used for the sines, and the report of Fructidor II writes that the computations would then turn to the logarithms of numbers, and that 1100 sheets with forms should be made according to an annexed model.[102] But based on the making of the abridged table in 1795, we think that it is more likely that the logarithms of sines and tangents were then started, as well as possibly the beginning of the logarithms of numbers, since the logarithms of the numbers 1 to 5000 were needed for the first logarithms of sines and tangents.[103]

Composition must have started immediately, after completion of the sines, for Lalande writes that the printing of the table of sines had started in 1794 with 22 decimals, and with differences up to the fifth order.

Gillispie estimated that each calculator made 900 to 1000 additions or subtractions in the day's work, which is consistent with about 200 logarithm values.[104]

All the work was done twice, but this obviously mainly applied to the computations. The first group was probably not made of two sections, although in some (but perhaps not all) cases different formulæ were designed, and the fundamental values may have been computed twice. The second and third groups were certainly divided in two sections which were in charge of a similar and independent work.

In order to speed up the making of the cadastre, the decree of the *Comité de Salut Public* (Committee of Public Safety)[105] of 22 Floréal II (11 May 1794) ordered that eight computers be added to the geometric division of the cadastre.[106] A few months later, and after the printing of the first tables, on 4 Pluviôse III (23 January 1795), it was decided to set up a *Bureau des correcteurs*[107] in order to check for errors in the printed tables and therefore speed up the making of the tables. Eight persons were hired. These correctors were apparently first assigned to the computation of the reduced tables, then

we often do not exactly know how many computers were involved, and what methods and shortcuts were used.

[102]Given that 190000 logarithms had to be computed, and that these logarithms would fill 3800 pages, we find that 950 sheets were necessary. A slightly larger number of sheets were probably printed in case of anticipated errors. A similar amount of pages was needed for the logarithms of sines and tangents.

[103]However, it remains to be seeen whether the logarithms of numbers used in these sections are those of the *Tables du cadastre*, or those of Briggs or another source.

[104][Gillispie (2004), p. 484]

[105]The *Comité de Salut Public* was the executive government in France during the Reign of Terror (27 June 1793—27 July 1794).

[106]A.N. $F^{17}1238$

[107]On the planned organization of this *Bureau*, see a report from 12 Nivôse III (1 January 1795).(A.N. $F^{17}1238$)

to the work on the main tables.[108]

A long report of 2 Thermidor II (20 July 1794) gave a detailed description of the projected tables.[109] The sines would be computed to 25 places, and printed to 22 places with five columns of differences, every 10000th of the quadrant. The logarithms of sines and tangents would be computed to 15 places and published to 12 places, every 100000th of the quadrant. The logarithms of numbers would be computed to 12 places from 1 to 200000. At that time, only the sines had been completed. The report also sketched the layout of the tables. The table of sines would have 100 pages, the logarithms of sines and tangents would have 500 pages (together), and the logarithms of numbers 400 pages. Although the report does not state it explicitely, this suggests that there would have been four columns of 100 logarithms of sines or tangents per page, and five columns of 100 logarithms of numbers per page, probably with first differences. We call this project, project 1.

The contract with Didot was based on this report[110] and stated that Didot would make a first printing of 500 copies, which had to be delivered 18 months later.[111]

Lalande was one of the first to describe the project in 1795. He wrote that Prony had fifteen computers trained by him, and that they were doing all computations twice. 600 results were obtained daily.[112] He wrote that

[108]Some of the correctors were hired at the beginning of 1795. These correctors were Bazin, Blondel, Pedon, Labussierre, Petit, Place, Sinquin, and Vernier. A letter from 29 Pluviôse III (17 February 1795) also suggested to replace Barruel, who left one of the positions to be professor of physics at the newly founded *École Polytechnique*, by Theveneau.(A.N. F^{14}2146)

[109]A.N. F^{17}1238

[110]See also Prony's report from 1 Ventôse IV (20 February 1796) summarizing the financial difficulties since the beginning of the contract.(A.N. F^{17}1238) The initial contract was for 270000 *livres*, of which 50000 had to be payed right away, and the remaining part 15000 *livres* every month. Didot had to print 1000 pages in 500 copies.

[111]In 1825, a report by Bouvard, Prony and Arago stated that the initial aim was to reduce the tables to 1200 pages, of which 500 had been composed.(A.N. F^{17}13571) This, however, is probably a misunderstanding going back to Prony, and this mistake has been repeated numerous times since. In 1801, Prony indeed wrote that the contract with Didot would have resulted in 1200 pages [Riche de Prony (1801)], but the contract does not explicitely mention this amount of plates.(A.N. F^{17}1238) Instead, the 500 composed pages very likely correspond to project 3 which would have totalled about 2000 pages, whereas the 1200 pages seem to be an extrapolation of the initial project of 1000 pages, plus some introduction. When project 4 was set up in 1819, things became even more confuse, because the 1200 pages then meant only part of the initial project. (Didot to the Interior minister, 13 September 1822, A.N. F^{17}13571) In 1819, Prony wrote erroneously that the initial contract was for 1200 plates, each of which would have had 100 lines. (note dated 2 March 1819, Archives of the *Académie des sciences*, Prony file, also in PC: Ms. 1183)

[112][de Lalande (1795)] This was then quoted by Bode [Bode (1795), p. 215] and again

the logarithms of sines and tangents would be published with 12 decimals and two columns of differences, the logarithms of the numbers up to 200000 with 12 decimals and two columns of differences, the logarithms of the first 10000 numbers with 25 decimals, as well as the logarithms of the ratios of sines and tangents to their arcs for the first 5000 one hundred thousands of the quadrant, with 12 decimals and two columns of differences.[113]

An undated description of the projected tables at the *Archives Nationales* almost totally agrees with Lalande's description, except that the initial target was to compute the first 10000 logarithms to 28 places.[114] We call this project, project 2. This project probably followed the report of 2 Thermidor II, and evolved into the actual computations, which we call project 3. It was probably towards the end of 1794 that the accuracy was reduced to 19 places. We have reconstructed project 2, as this is the one which is best specified.[115]

According to the description of the tables found in Callet's tables of logarithms,[116] it seems that a table of tangents with 22 exact decimals and all necessary differences for each centesimal degree was also planned, although the interpolation itself would not be carried out. This is consistent with the introductory volume of the tables, which has a section explaining how tangents could be computed.[117]

Moreover, on 6 Ventôse III (24 February 1795), Prony was asked to collaborate with Lagrange and Laplace to make reduced tables of logarithms of sines and tangents for the students of the *École Normale*,[118] and that these

by Hobert and Ideler, when the latter compared their 300 daily results with those of Prony. [Hobert and Ideler (1799), p. XXXIII]

[113][de Lalande (1803), p. 743]

[114]A.N. F[17]1238

[115]Interestingly, Edward Sang computed the logarithms of all numbers up to 20000 to 28 places. Sang's aim was to compute a table of nine-place logarithms from 100000 to one million, an endeavour of which only a by-product and fragments were published [Sang (1871), Sang (1872a)]. See Craik [Craik (2003), p. 55] and Fletcher [Fletcher et al. (1962), p. 159].

[116][Callet (1795), p. VI]

[117]The actual pivots are however nowhere to be found and have probably not been computed.

[118]The *École Normale de l'an III* (*École Normale* "of year III") was created in 1794 and had only a brief existence. The more than 1000 students of the *École Normale* were delegates from the various regions of France, and the purpose was to have them later in charge of organizing the education in the provinces. The professors of mathematics were Lagrange and Laplace, and Monge was professor of descriptive geometry (see [de Laplace et al. (1992)] for details on their lessons). The first course was given on 1 Pluviôse III (20 January 1795) and the last on 30 Floréal III (19 May 1795). The school failed because of the heterogeneity of the students. On the *École Normale de l'an III*, see [Gillispie (2004), pp. 494–520] and [Dupuy (1895)]. On education reforms during the

tables would be printed and distributed to the students at the cost of the Nation.[119] The archives contain no partial or total printing of these tables, and it is likely that they were never printed after the end of the *École Normale*.

Prony had thought first of using the Great Tables in order to extract the small tables from them, which suggests that the tables of logarithms of sines and tangents had already been computed by March 1795. Eventually, however, these tables were obtained by new interpolations, although the pivots were certainly copied from the Great Tables (see sections 2.8 and 4.12).[120] Prony writes that this table was completed independently in nine days, and not extracted from the main tables.[121]

The core of the tables must have been completed around mid-1796 and they filled 17 large in folio volumes, each in two copies.

When the tables were completed, some of the calculators were transferred to the newly created *Bureau des longitudes* to work on astronomical tables.[122]

A 1796 review of Callet's table is also informative and presumes that Prony's "vast and laborious undertaking is probably now finished." According to this same review, Prony's tables also contained a table of tangents true to 22 places, as well as a collection of astronomical tables.[123]

In 1798–1799, Thomas Bugge, the Danish Astronomer Royal, visited France as a member of the International commission on the metric system, and in his travel account, he described the *Bureau des longitudes* as well as the *Bureau du cadastre*, "under the superintendance of the excellent Prony."[124] He briefly described the work on tables, writing that most of the logarithms are already calculated.[125] This, however, does not imply that the core of the Great Tables were not complete, as the *Bureau du cadastre* was certainly busy with auxiliary tables, with which it probably moved at a slower pace. Bugge also described the soon to be published tables by Borda,

Revolution, see [Boulad-Ayoub (1996)]. It was only in 1808 that Napoleon created a new school which eventually became the elite *École Normale Supérieure*. In his second lesson at the *École Normale* on 9 Pluviôse III (28 January 1795), Laplace spoke of logarithms, but did not mention the *Tables du cadastre*. Lagrange drew the history of logarithms a week later, also without mentioning Prony's work [de Laplace et al. (1992)].

[119] See *Rapport au Comité des travaux publics*, 19 Ventôse III (9 March 1795) (A.N. F^{14}2146)

[120] The 1820 note on the joint British-French publication of the tables seems also to imply that the corresponding part of the Great Tables was finished by the time Prony was asked to make small ones [Anonymous (1820 or 1821), p. 8].

[121] [Riche de Prony (1824), p. 39] However, as we will detail it later, there is the possibility that only part of this abridged table was recomputed.

[122] [Bigourdan (1928), pp. A.25–A.28]

[123] [Anonymous (1796a), pp. 573–574]

[124] [Crosland (1969), p. 124]

[125] [Crosland (1969), p. 125]

and the difficulties of the latter to secure paper for the printing.[126]

In 1800, Lalande mentioned Hobert and Ideler's decimal tables and wrote that they "will facilitate astronomical calculations, until the more extensive tables, which C. Proney [sic] caused to be calculated at the *Bureau du Cadastre*, and which began to be printed some years ago, are finished."[127] But a few years later, in his *Bibliographie*, he wrote that although work was in full activity, the printing was discontinued.[128]

1.4.3 Printing

A decree of the *Comité de Salut Public* (Committee of Public Safety) of 22 Floréal II (11 May 1794) ordered that 10000 copies of the tables be printed at the expense of the Republic.[129] Consequently, the *Commission des Travaux Publics* entered a contract with Firmin Didot on 2 Thermidor II (20 July 1794) for printing the tables.[130]

In Didot's claim of 9 Nivôse IV (30 December 1795),[131] he wrote that 527 pages (100 pages of natural sines, 17 pages of logarithms of sine ratios, 17 pages of logarithms of tangent ratios, 200 pages of logarithms of sines, and 193 pages of logarithms of tangents) had been composed, but that only the natural sines had been soldered. In addition, enough digits were ready for about 400 more pages.[132] This seems to indicate that the logarithms of numbers were computed last, but in fact they may have been computed at the same time as the logarithms of sines and tangents, a view supported by the use of the same forms. If the logarithms of sines and tangents had been computed last, special forms might have been printed for these tables, which was not the case.

In 1819, a note on the printing of the tables[133] considered that the logarithms of sine and tangent ratios could be printed with 300 values per page, hence 17 pages for each ratios. It is possible that this layout was already

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[126][Crosland (1969), p. 126]

[127][de Lalande (1800), p. 40], [de Lalande (1803), pp. 812–813]

[128][de Lalande (1803), p. 744]

[129][Gillispie (2004), p. 484], A.N. $F^{17}1238$

[130][Gillispie (2004), p. 484]

[131]A.N. $F^{17}1238$

[132]Most interestingly, Didot gives the detail of the amount of each digit: 372000 '1's, 414000 '2's, 300000 '3's, 357000 '4's, 345000 '5's, 333000 '6's, 207000 '7's, 402000 '8's, 267000 '9's, 300000 '0's, all in packets, 344500 dots and commas, as well as 90000 loose digits. A report by Bouvard, Prony and Arago, dated 26 January 1825, stated that the total weight of these 500 plates was about 7.5 tons, hence about 15 kg per plate.(A.N. $F^{17}13571$)

[133]PC: Ms. 1181.

envisioned in project 3. But this same note also considered that a packed printing of the logarithms of sines and tangents would cover 500 pages each, so that the trigonometric part of the tables, with some introduction, would be about 1200 pages.

Printing of course also required corrections. In a much later letter written on the 18th September 1819, Ambroise Didot considered that one sheet (four pages) had to be reread four times, in addition to the proofreading made in the printer's shop, and one can guess that it would have been the same in the 1790s.[134]

Although the printing was never completed, Firmin Didot kept the plates until the beginning of the 1830s, when they were probably recycled. This explains why there was still hope for printing over a period of almost 40 years.

1.4.4 Delays

The collapse of paper money, the so-called *Assignat* (figure 1.4),[135] during the Directory eventually led the printer to desist.[136] The cost of printing was becoming more and more important and could no longer be afforded.[137]

On 1 Ventôse IV (20 February 1796), Prony wrote to the Interior Minister, explaining Didot's financial difficulties.[138] According to Prony, of the 1000 pages initially planned, 527 were set, and the material for 400 more pages

[134]PC: Ms. 1181.

[135]See [Levasseur (1894), Hawtrey (1918)]. According to Lewis, if one held 3000 *livres* of *assignats* in 1790, it would have been worth only one *livre* by 1796 [Lewis (1999), p. 62].

[136]By 9 Nivôse IV, Didot had received the following payments (*livres* before 18 Germinal III, and *francs* afterwards): 50000 (2 Fructidor II), 15000 (29 Vendémiaire III, 9 Frimaire III, 24 Nivôse III, 9 Pluviôse III, 22 Ventôse III, 20 Germinal III, 12 Floréal III, 23 Prairial III, 12 Messidor III, 24 Thermidor III, 16 Fructidor III, 10 Vendémiaire IV, and 18 Brumaire IV).(A.N. F^{17}1238)

[137]A note of 23 Thermidor II (10 August 1794) by Didot mentions his needs for lead, antimony (being used for making lead used in type metal harder), wood, candles, tallow, oil, and coal. On 18 Vendémiaire III (9 October 1794), Didot gives a precise list of the materials he needs: *cinq milliers de régule d'antimoine* (a *millier* was 489.506 kg, so this is about 2.5 tons of antimony), *cinquante livres de cuivre en lingot* (50 pounds of copper in ingots), *cinquante voies de bois pelard neuf* (about 96 steres of wood), *cinq voies de charbon de terre* (about 5 to 10 tons of coal), *vingt cinq voies de charbon de bois* (about 25 to 50 cubic meters of charcoal), *trois cent livres de chandelles* (300 pounds of candles), *trois cent livres de suif* (300 pounds of tallow), *cinquante limes et rapes* (50 files), *trois cent rames de papier grand raisin* (300 hundred reams of 500 sheets of *grand raisin* paper). Other needs expressed later concern red copper, tin, oil, ink, and sodium carbonate or potassium hydroxide for cleaning the type metal. Costs are also given.(A.N. F^{17}1238) See also § 5.2 on Didot's 1797 patent for details on the composition of type metal.

[138]A.N. F^{17}1238

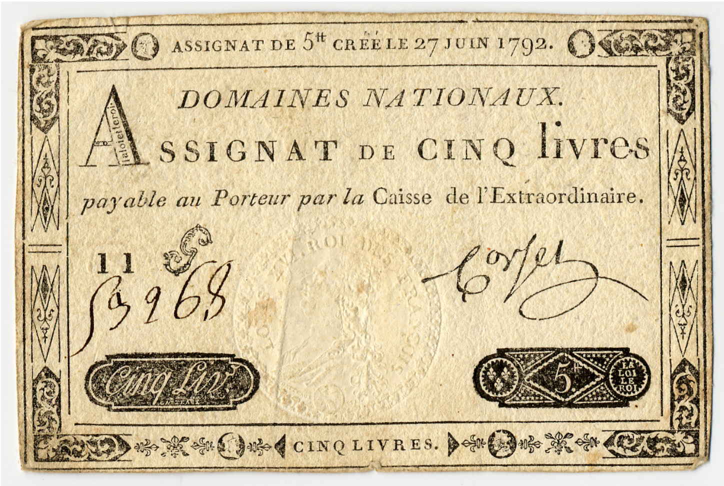

Figure 1.4: Assignat of 5 *livres*. (Source: Wikipedia)

was ready. Prony wrote that "the entire completion of the work is only a matter of work-force and composition of the plates," implying that the goal was to compose 1000 pages.[139] Prony explained that the gold value of the 270000 *livres* was 129600 *francs*. The 245000 *francs* which had been payed in *assignats* actually reduced to 45000 *francs* in gold, so that 84600 *francs* were still due to Didot. Prony stressed that although Didot had done the three fourths of the work, he was only payed a third of the metallic value of the contract. Now, since Didot has had to increase the amount of metal in the plates, Didot was actually demanding an additional 5000 *francs*, that is, a total of 89500 *francs* to complete the work, and wanted first 60000 *francs* to cover the money he advanced. It seems that this money was never given to Didot.

Moreover, the whole administration of the cadastre fell into pieces. The ambitious *École des Géographes* created in 1795, actually only started in 1797. It was open to the graduates of the *École Polytechnique*, but it attracted very few of them.[140] No surveyors were sent to the countryside, and

[139]If this is correct, we have a discrepancy with the existing fragment of the logarithms of tangents, since it is not compatible with this amount of pages. This fragment may be a page composed later, perhaps around 1824.

[140][Berthaut (1902), p. 149], [Bret (1991), p. 124], [Bret (2009)], [Gillispie (2004), p. 486], [Grattan-Guinness (1990b), p. 111] Detailed lists of professors and students with their salaries are kept at the *Archives Nationales*.(A.N. F^{1bI}44) On the content of the classes, see also PC: Ms. 2148.

in 1801 Prony eventually resigned from the Cadastre,[141] which didn't have any real activity anymore. It was officially abolished on 3 Germinal X (24 March 1802),[142] and Bonaparte closed the *École des Géographes* in 1803.[143] Meanwhile, in 1801, Prony became member of the *Bureau des longitudes*.[144]

In 1861, summarizing the activities of the *Bureau du cadastre*, Noizet wrote:

> "Ce bureau ne s'occupa que de travaux de pure théorie, au point de vue scientifique, pour préparer et déterminer les procédés par lesquels l'opération devait être exécutée sur le terrain. Ces travaux, qui n'ont consisté que dans des dispositions préliminaires sans réalisation, et même sans qu'un système complet ait été arrêté, ont disparu sans laisser aucune trace."[145]

On 4 Nivôse X (25 December 1801), the cadastre having been terminated, the *Bureau des longitudes*[146] suggested to ask that its computers be put under the responsibility of Prony who would be attached to the *Bureau des longitudes*. On 20 Floréal X (10 May 1802), it was announced that the first consul (Napoleon Bonaparte) had named Prony to the *Bureau des longitudes*.[147]

In 1804, Garnier, echoing Delambre's 1801 report on the tables, expressed his hopes for the tables to be printed once peace was reinstalled:

> "(...) espérons que dans des temps de paix et de bonheur, un Gouvernement ami des arts, ordonnera l'achèvement d'un ouvrage qui doit être désiré de tous ceux qui cultivent les sciences mathématiques : un tel vœu, émis par les premiers géomètres, est pour moi une raison de plus de me féliciter d'avoir coopéré à ce grand œuvre, sous le *citoyen Prony*, alors directeur du cadastre."[148]

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[141] [Gillispie (2004), p. 486]

[142] [Noizet (1861), p. 18]. Berthaud writes that the equipment went to the *Dépôt de la Guerre* [Berthaut (1902), p. 244], [Grattan-Guinness (1990b), p. 179].

[143] [Bret (1991), p. 125] A new school was opened soon afterwards.

[144] [Grattan-Guinness (1990b), p. 111]

[145] [Noizet (1861), p. 18]

[146] The *Bureau des longitudes* was created by the *Convention nationale*, by the law of 7 Messidor III (25 June 1795). Among its attributions, the *Bureau des longitudes* was overseeing the activities of the observatories, in particular the *Observatoire de Paris*. One of the tasks of the *Bureau des longitudes* was to compute and publish the *Connaissance des tems* [Denisart et al. (1807), pp. 165–166].

[147] Minutes of the *Bureau des longitudes* meetings [Feurtet (2005)].

[148] [Garnier (1804), p. 248] The first sentence is copied from Delambre's *Rapport sur les grandes tables trigonométriques décimales du cadastre* [Riche de Prony (1801)].

At the end of the 1790s or the beginning of the 1800s, there were a number of mentions about the completion of Prony's tables, or about their publication. At the time of the publication of Hobert and Ideler's decimal table in 1799, Lalande wrote that the printing of the cadastre tables had been started several years before.[149] In 1800, Lacroix wrote in his *Traité des différences et des séries*:

> "Prony, qui a dirigé ce beau travail, le plus étendu qu'on ait encore exécuté dans ce genre, ne manquera pas sans doute de faire connoître en détail les méthodes dont on s'est servi pour en simplifier le calcul. Les tables des sinus sont déjà stéréotypées, et il est bien à désirer qu'on en fasse bientôt jouir l'Europe savante ; il ne reste plus qu'à imprimer les logarithmes des sinus et des tangentes qui ont été calculées avec seize décimales."[150]

When Borda's tables were about to be published by Delambre, Lalande wrote that Prony's tables have a much wider scope, but that the "difficulty of printing may seriously delay the advantage that is expected from them."[151]

In 1806, describing the main tables of logarithms, John Bonnycastle made the following comment:

> "Besides these, several other tables, of a different kind, have been lately published by the French; in which the quadrant is divided, according to their new system of measures, into 100 degrees, the degree into 100 minutes, and the minute into 100 seconds; the principal of which are the second edition of the Tables Portatives of Callet, beautifully printed in stereotype, at Paris, by Didot, 8vo., 1795, with great additions and improvements; the Trigonometrical Tables of Borda, in 4to. an. ix., revised and enriched with various new precepts and formulæ by Delambre; and the tables lately published at Berlin, by Hobert and Ideler, which are also adapted to the decimal division of the circle, and are highly praised for their accuracy by the French computers."[152]

In 1807, Gergonne mentioned the great tables which are "currently" being computed at the *Bureau du cadastre*, but this is very anachronistic and "currently" seems to refer to events 10 years past.[153]

[149] [de Lalande (1803), pp. 812–813]

[150] [Lacroix (1800), p. 53] Lacroix writes incorrectly that the logarithms of sines and tangents were computed with 16 decimals, and he omits the logarithms of the numbers.

[151] [de Lalande (1803), p. 831]

[152] [Bonnycastle (1806), p. XXI]

[153] [Gergonne (1807)]

1.4.5 Revival under the Consulate

An attempt was made to revive the project in 1801 under the Consulate.[154] Laplace, Lagrange and Delambre wrote a report dated 11 Germinal IX (1 April 1801) on the publication of the tables, but nothing came out of it.[155]

In a letter dated 8 Nivôse XI (29 December 1802), the Interior minister Jean-Antoine Chaptal wrote to Didot in such terms that he admitted that the Government owed 45000 *francs* to Didot.[156] Didot claimed for years that he was due this money, but the following Governments always postponed payment.

Delambre pointed out the importance of having these accurate tables so that they could serve as the model for future tables.[157] This already happened at that time. According to Prony, Hobert and Ideler checked their decimal tables on Prony's tables before publishing them in 1799.[158] And Borda's tables were themselves checked on Prony's tables by Delambre.[159]

In 1802, Delamétherie announced that the tables had been completed.[160]

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[154] [Gillispie (2004), p. 484]

[155] [Lagrange et al. (1801)]

[156] A.N. F^{17}13571.

[157] This is expressed in the 1801 report [Lagrange et al. (1801)]. See also [Gillispie (2004), p. 485]

[158] [Anonymous (1820 or 1821), pp. 5–6], [Riche de Prony (1824), p. 40], [Gillispie (2004), p. 485]. I did not, however, find a direct reference to such a verification in Hobert and Ideler's tables.

[159] [Gillispie (2004), p. 485] During the years III-VI, and before he became member of the *Bureau des longitudes*, Prony attended several of its meetings, in particular so that Borda's tables could be checked on the *Tables du cadastre*. During the meeting of 12 Thermidor III, a memoir about decimal sine tables was discussed and it was decided that Borda's tables would be printed. On the 27 Fructidor III, Borda showed proofs of the tables of decimal sines. The next month, on 7 Vendémiaire IV, it was decided to ask Prony to bring his tables in order to check those that were going to be printed. Prony came to the next meeting on 12 Vendémiaire IV and offered to communicate his tables of decimal sines. Then, the following year, Lalande suggested to write to Prony in order to obtain a copy of the 100000 logarithms that were computed under his direction (19 Thermidor V) and on 14 Pluviôse VI, Prony came and announced that already half of the 100000 logarithms had been copied for the Bureau. A few years later, on 11 Ventôse XII, Prony discussed an 8-place table that was being computed under his direction for every second. It is not totally clear which table was meant. This is perhaps the 8-place table in the *Ponts et chaussées* archives (Ms. 243 and Ms. Fol.2773). See section 1.5 of this study. (Minutes of the *Bureau des longitudes* meetings [Feurtet (2005)])

[160] [Delamétherie (1802)] Delamétherie's description of the *Tables du cadastre* is slightly incorrect. He wrote that the logarithms from 1 to 100000 (instead of 10000) have been computed with 19 decimals, and that those from 100000 (instead of 10000) to 200000 were computed with 24 decimals (instead of 14). This description may have been copied from an earlier incorrect description, perhaps the description published in

Delamétherie appeared wishful that the tables can be printed.

That same year, Legendre mentionned the tables in the 4th edition of his *Éléments de géométrie* (1802),[161] describing them as one of the "most beautiful monument erected for the sciences." The description is omitted in the 6th (1806) and later editions.

In 1808, Firmin Didot's catalogue gave the *Tables du cadastre* "in press"[162] and also announced reduced tables by Prony which seem never to have been printed (see below).

Edward Sang, who would eventually compute even larger tables, recounted the impact of the Cadastre Tables on him in 1815:

> "About 1815, in our school, the boys were exercised in computing short tables of logarithms and of sines and tangents, in order to gain the right to use Hutton's seven-place tables; and well do I recollect the almost awe with which we listened to descriptions of the extent and value of the renowned Cadastre Tables."[163]

In 1816, Legendre published an excerpt of the tables of the logarithms of numbers.[164]

The same year, the article on logarithms in the Encyclopædia Perthensis wrote that "[t]his immense work, which was begun to be printed at the expense of the French government, was suspended at the fall of the assignats, and was not resumed in 1801; since which period, we have not heard of its farther progress."[165]

There has also been some unfair criticism, or perhaps chauvinism. In his history of the French cadastre published in 1818, Benzenberg wrote for instance that Prony had as many as 13 computers and still did not manage to do as much work in five years as Hobert and Ideler did in two years.[166]

1801 [Anonymous (1801)]. This error also shows up in Delambre [Delambre (1810)] and in a later work by Peirce [Peirce (1873), p. 24]. The initial error is probably a mere typographical error.

[161] [Legendre (1802), p. 359]

[162] See for instance at the end of the 7th edition of Legendre's *Éléments de géométrie*, published by Firmin Didot in 1808 [Legendre (1808)].

[163] [Anonymous (1907–1908), p. 185]

[164] [Legendre (1816)]

[165] [Anonymous (1816), p. 324]

[166] [Benzenberg (1818)], cited through [Anonymous (1819b)]. Benzenberg was of course defending the work of the Germans, but the scope of both works is very different.

1.4.6 Involvement of the British government (1819–1824)

The project of printing the tables was again revived in 1819 through Davies Gilbert (1767–1839), member of the British Parliament and of the Royal Society, and the scientist Charles Blagden (1748–1820).[167] It seems that Gilbert somehow heard of Prony's tables, perhaps through Babbage, and wrote to the *Bureau des longitudes* in February 1819. Blagden, who was living in France at the time, then proposed to Gilbert the idea to publish the tables jointly by the French and British government.[168,169]

On the 27th of May 1819, the House of Commons, on the motion of Davies Gilbert, resolved to present an address to the Prince Regent, "praying that he would direct His Majesty's Minister at the Court of France to take such measures as may be deemed expedient for procuring the large Manuscript

[167]Some documents related to the project of joint publication of logarithms are located in the library of the Royal Society (Papers of Sir Charles Blagden, CB/4/7/5, one folder dated 1819). As far as we could see, only four pieces of this folder are related to the publication of the tables. In piece 48, Blagden makes some observations, probably on Prony's introduction to the table: "Calculations of tangents not begun," "some attention necessary to get 12 decimals quite exact," etc. Piece 1 are instructions by Davies Gilbert to Blagden about various ideas for printing the logarithms (12 July 1819). A translation by Delambre of this document is found in the *Ponts et chaussées*, Ms. 1181. Piece 3 is a copy of a letter from Marquis Dessolles, then Prime Minister of France, to Charles Stuart, ambassador to France, in answer of a letter written by Stuart to Dessolles on 28 June about the choice of Blagden (15 July 1819). Piece 2 is a copy of a letter by Charles Stuart to Castlereagh (19 July 1819).

[168]Gillispie wrote that it was Blagden who proposed to Gilbert the idea of a joint publication [Gillispie (2004), p. 485]. The project was discussed in the meetings of the *Bureau des longitudes* at least on the following dates between 24 February 1819 and 16 October 1833 (minutes before and after these dates have not been consulted, but according to Jean-Marie Feurtet, this list is complete): 24 February 1819, 3 March 1819, 17 March 1819, 24 March 1819, 16 June 1819, 23 June 1819, 28 July 1819, 27 August 1819, 8 September 1819, 15 December 1819, 20 December 1820, 7 March 1821, 13 June 1821, 20 June 1821, 10 October 1821, 4 November 1823, 18 November 1823, 5 April 1824, 16 June 1824, 23 June 1824, 21 July 1824, 19 January 1825, 9 March 1825, 12 April 1826, 3 May 1826, 2 August 1826, 16 August 1826, 30 August 1826, 6 September 1826, 18 October 1826, 9 October 1833, and 16 October 1833. In addition, during the meeting of 14 June 1826, Prony described new tables of logarithms printed in London, and these were presumably Babbage's published in 1827 [Babbage (1827)]. Jean-Marie Feurtet, who transcribed all the minutes from 1795 to 1854, plans to put these transcriptions online in 2010 on the *Bureau des longitudes*' site.

[169]A note dated 1 December 1820 on this planned publication was printed by Firmin Didot and summarized the discussions up to that moment [Anonymous (1820 or 1821)]. This note contains some errors, for instance about Didot's initial contract. It seems to imply that the 1794 contract was for printing 1200 pages for a sum of 144000 *francs*, but these are the figures from the 1819 project. This may explain why Gillispie wrongly used these figures in the revolutionary frame [Gillispie (2004), p. 484].

Tables of Logarithms of Numbers and of Parts of the Circle calculated in France, to be printed at the joint expense of the two Governments."[170]

In a letter written by Gilbert to Blagden on 15 July 1819, Gilbert gave precise specifications of the tables, but these specifications would have required the whole work to be redone, besides producing absolutely enormous volumes: logarithms of numbers from one to one million, and a division of the quadrant in a million parts. Blagden replied that with these conditions, there was no point to use Prony's tables. On 18 July 1819, Blagden wrote that

> "(...) le principal objet des tables de Mr. de Prony était d'établir le système décimal et de faciliter ce nouveau calcul, plutôt que de fournir des tables qui allassent plus loin que les tables existantes."[171]

A letter of 12 August 1819 from the Interior Minister to the *Bureau des longitudes* stated that the cost of printing would be 144000 F, to split between the two nations.[172] These 144000 F corresponded to the printing of the trigonometric part only, not including the logarithms of numbers.

On 28 August 1819, Delambre wrote a letter to the Interior Minister describing the advantages of the project:

> "Ainsi véritablement il y a un avantage marqué pour les nouvelles tables, ces tables non plus que celles de Briggs, ne serviront pas aux calculs usuels, mais dans des cas extraordinaires, comme celles de Briggs elles seront la source où viendront puiser tous ceux qui imprimeront des tables usuelles avec plus ou moins d'étendue, elles serviront de point de comparaison pour tout ce qui a été fait ou se fera.
>
> (...)
>
> Il n'y a pas de nécessité bien démontrée, mais un avantage réel à rendre impérissable un travail si neuf et si considérable. Une circonstance unique se présente et il faut en profiter. Le projet est annoncé, la demande officielle est faite, les journaux en ont parlé, il n'y a plus à délibérer."[173]

[170] [Sang (1872a)] This was cited by Edward Sang, who used the failure of these negotiations and the want for more extensive tables as a support for the publication of his own tables [Roegel (2010a)].

[171] PC: Ms. 1181. Blagden's assertion contradicts what Prony wrote a little later, namely that there was an explicit demand for a high accuracy [Riche de Prony (1824), p. 35].

[172] PC: Ms. 1181.

[173] PC: Ms. 1181. The first paragraph borrows heavily from the 1801 report [Lagrange et al. (1801)].

And indeed, perhaps as a consequence of Delambre's letter, the *Moniteur universel* dated 29 August 1819 was very enthusiastic:

> "Au milieu des discussions politiques qui agitent le Monde, et des intérêts divergens de la diplomatie, on doit voir avec plaisir ce concours, cette réunion des hommes instruits de deux nations grandes et éclairées, pour la publication d'un beau travail, propre à hâter les progrès des plus hautes connaissances, et à faciliter les calculs qui servent de base aux recherches et aux découvertes dans toutes les parties des sciences mathématiques et physiques."[174]

Castlereagh, Foreign Secretary, was persuaded of the merits of the project and informed the British ambassador in Paris, Sir Charles Stuart, that Blagden was to serve as British representative on the commission that would explore the matter. Although Blagden died in March 1820, negociations continued for a few years.[175]

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[174] [Anonymous (1819a)]

[175] At the *Bureau des longitudes*, discussions seem to have started on the 24 February 1819, when Arago read Gilbert's letter. On the next meeting, 3 March 1819, Laplace, Delambre, Arago, Biot and Burckhardt were given the task of writing a report on the proposal. On 17 March 1819, it was decided that Delambre's report would be adapted and given to Blagden, who would then transmit it to Gilbert. On 24 March 1819, a letter by Gilbert was read, and on 16 June 1819, it was announced that the British Parliament had approved of the project. On 23 June 1819, mention was made of a letter from the Interior Minister asking about the presumed cost. On 28 July 1819, the Interior Minister suggested to wait for the reply of the British Government. During the 27 August 1819 meeting, there were talks about the project which would be sent to the Minister. Then, on 8 September 1819, the commissionners wrote to F. Didot in order to find out about the current state of the printing. On 15 December 1819, the Minister sent to Delambre the documents concerning the old contract with Didot. The next mention of the tables occured one year later, probably as a consequence of Blagden's death. So, on 20 December 1820, it was announced that Davy would present a memoir of Prony to the Royal Society. This is possibly [Anonymous (1820 or 1821)], which is dated 1st December 1820. On 7 March 1821, it was announced that Gilbert Davies planned to come to Paris. Eventually, he attended the 20 June 1821 meeting. During that meeting, it was announced that the main objection to printing was the centesimal division of the circle. It was nevertheless agreed that the printing would be done in France. On 10 October 1821, Prony made a proposal for a partial printing. On 4 November 1823, it was announced that Davies Gilbert and Wollaston were named commissionners for the British Government. The Minister also announced that Didot was requesting payment for the composition and printing that he had already made. The Minister wanted the *Bureau des longitudes* to participate in this payment, but the *Bureau des longitudes* answered that it could not. On 18 November 1823, Arago communicated what he had learned about the printing of the tables, and it was decided to write to the Minister and to Gilbert and Wollaston in order to announce them that the *Bureau des longitudes* had nothing to do with the request which was made to

In September 1819, Delambre wrote to Blagden about various propositions, in particular that of printing also the logarithms of numbers, which would have added 97800 F to the 144000 F for the initial amount.[176]

This increase of cost, Didot's claim for the money that the Government owed them since 1796, the additional delays caused by Blagden's death as well as the frequent changes of governments, made the French Government unwilling to pursue this matter.

In 1822, in a section on the best tables in his book *Nouvelle méthode de nivellement trigonométrique* (1822), Prony wrote that "when the great tables computed using my methods and under my direction, and which the French and British government will print, will be available, we will have ressources much superior to the ones provided by the books I have just mentioned."[177]

Hutton also mentionned the tables in 1822[178] with a correct description of their contents.[179]

In 1822, in a letter to Humphry Davy, Charles Babbage used the *Tables du cadastre* as an example for supporting the mechanical calculation of tables,[180] but he probably only consulted the tables at the *Observatoire* a few years later for comparison with his own table of logarithms published in 1827.[181]

On 7 June 1824, Prony read a memoir on the tables at the *Académie des Sciences*,[182] in which he made a plea for the printing of the tables. Prony recalled that the tables were

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the British Government. On 5 April 1824, the Minister asked Arago if he had heard from London that the project had to be abandoned, but Arago didn't seem to be aware of it. However, on 16 June 1824, Davies Gilbert announced (through a letter dated June 5) that the British Board of Longitude had unanimously decided not to undertake the joint project. On 21 July 1824, the Minister also abandoned the project, but wanted a commission set up to assess the amount of the indemnity to be given to Didot. The commission was made of Prony, Bouvard and Arago. (Archives of the *Bureau des longitudes*) Additional details on the negociations with the British Government and on the discussions of this period can be found in the Archives of the *Ponts et chaussées*, Ms. 1181 and at the *Archives Nationales* (A.N. F^{17}13571). It is also possible that the Archives of the British Board of Longitude contain some relevant material, in particular on their final decision, but we have not consulted these documents. These archives are located at Cambridge University Library (EAD/GBR/0180/RGO 14) and possible places to check are boxes 7 and 8.

[176] PC: Ms. 1181.

[177] [Riche de Prony (1822), p. 32] This excerpt is criticized in the same year in a review of that book appearing in the *Annales belgiques des sciences, arts et littérature* [Anonymous (1822)]. The author of the review accuses Prony of taking all the credit of the work on the tables for himself, although his reaction seems somewhat excessive.

[178] [Hutton (1822), pp. 41 and 179]

[179] This description is not included in the previous edition, published in 1811.

[180] [Babbage (1822a)]

[181] [Babbage (1827), Campbell-Kelly (1988)]

[182] [Riche de Prony (1824)]

> "le monument de calcul logarithmique et trigonométrique le plus vaste et le plus complet qui ait jamais existé."[183]

and his notice ended with:

> "(...) l'Europe savante attend avec impatience l'issue de ces négociations ; elle ne voit pas sans inquiétude un monument, le plus grand de son genre, et dont la perte ne serait probablement jamais réparée, n'exister qu'en manuscrit, et se trouver ainsi sujet à des chances de destruction qui peuvent causer des regrets éternels aux amis des sciences."[184]

Nine days later, however, it was announced that the British Board of Longitude had voted against the project. According to the minutes of the *Bureau des longitudes*, the London Board of Longitude decided to abandon the project of publishing the *Tables du cadastre*, probably in May or early June 1824. No reason was given in the minutes of the *Bureau des longitudes*, but the most likely reasons are the increase of cost and the fact that the tables were based on the decimal division of the quadrant and that the British did not want to use that division. Edward Sang suggested a different reason, and he mentioned the rumour that the English Commissioners were dissatisfied of the soundness of the calculation,[185] but nothing in the French minutes seems to allude to such an observation.[186]

The French Interior Minister accepted the decision by the Board of Longitude, and decided not to follow the matter.[187] The Board of Longitude itself was abolished by act of Parliament in 1828.

A few years later, Augustus De Morgan reflected on the failed effort to publish the tables:

> "[In] 1820 a distinguished member of the Board of Longitude, London, was instructed by our government to propose to the Board of Longitude of Paris, to print an abridgment of these tables at

183[Riche de Prony (1824), p. 33]

184[Riche de Prony (1824), pp. 41–42]

185[Anonymous (1907–1908), p. 185]

186In 1875, Sang wrote that the involvement of the British Government was somehow artificially obtained: "Though sorely needed and urgently demanded, the new tables did not appear; and when expectation had been stretched to the utmost, the English Government, in 1819, at the instance of Mr Davies Gilbert, proposed to defray one-half of the expense." (...) "shall we accept (...) the refusal to print the tables as the measure of their value?"[Sang (1875a), pp. 435–436]

187*Bureau des longitudes*, minutes, 21 July 1824.

> the joint expense of the two countries: 5000*l* was named as the sum which our government was willing to advance for this purpose, but the proposal was declined, and the great 'Tables du Cadastre' are still confined, in manuscript, to the library of the Paris Observatory."[188]

1.4.7 The waning of the project (1824–1833)

Once the joint project had been rejected by the British Government, there still remained the problem of Didot's payment. Moreover, a reduced project came to light, namely that of printing only the sines and the logarithms of sines and tangents already composed, although apparently nothing came out of it.[189] A letter by Didot to the *Bureau des longitudes* on 20 January 1825 was requesting money to recompose the pages that had fallen or those which might be missing, and in a letter to the Interior minister in June 1825, Didot

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188 [De Morgan (1841)] In this article, De Morgan also wrote incorrectly that the logarithms of numbers from 1 to 100000 (instead of 10000) were given to 19 decimal places.

189 At the 19 January 1825 meeting of the *Bureau des longitudes*, there were discussions about a report from Prony for the Minister. On 9 March 1825, some documents about the money owed to Didot were sent to the Minister. On 12 April 1826, Prony announced that Didot was going to resume the printing of the tables of logarithms of sines and tangents. On 3 May 1826, there were discussions about which tables should be lent to Didot and Prony was considering lending some of his own papers (perhaps his own copy of the tables). On 2 August 1826, Arago presented in the name of Didot three copies of already set parts of the tables. Discussions were under way on the best means to correct the proofs. On 16 August 1826, Didot announced that he would send 100 new printed pages and asked if these pages, as well as the 100 previous ones, could be corrected. Didot wanted to have the *Observatoire* manuscript and claimed that it should have been given to him as his property when the printing was started. On 30 August 1826, Didot again requested the manuscript of the *Observatoire*, and gave assurance that it would be handled with great care. Didot needed it in order to resume the printing. On 6 September 1826, Prony explained that the conditions for printing had not been met in the 1790s, and that the tables could therefore not be considered Didot's property. The new project was only applying to the 500 pages formerly composed, which still had to be corrected and printed. Prony asked that the *Bureau des longitudes* lend Didot the manuscript so that he could correct the proofs of the 500 composed pages. The *Bureau des longitudes* decided to lend one volume of the *Observatoire* at a time to Prony, and to have the proofs corrected in a room at the *Ponts et chaussées*. On 18 October 1826, the *Bureau des longitudes* decided to lend the volumes directly to Didot. The next mention of the tables in the minutes of the *Bureau des longitudes* was on 9 October 1833, when Bouvard gave an account of his efforts to obtain from Didot that he return the volume of tables which he then had. On 16 October 1833, it was announced that Didot had returned the manuscript which had been lent to him, without further details. (Archives of the *Bureau des longitudes*) Additional details on the negociations with Didot can be found in the Archives of the *Ponts et chaussées*, Ms. 1182.

announced that they had not hesitated to recompose the 500 first pages.[190] However, one may question the utility of printing only partial tables, as the logarithms of sines and tangents certainly only covered about half of the quadrant.

In 1826, Garnier wrote that it was likely that the printing of the tables will never be completed.[191]

Prony also wrote an elementary textbook on how to use logarithms.[192] The *Tables du cadastre* are briefly mentioned, but somewhat incorrectly. Prony wrote that the tables at the *Observatoire* give the logarithms of numbers from 1 to 10000 with 25 places, and from 10000 to 200000 with 16 places, which is wrong in both cases.[193]

In 1836, the *Dictionnaire des sciences mathématiques pures et appliquées*,[194] echoing somewhat Prony,[195] wrote

> "On trouve dans l'avertissement placé en tête des *tables de Callet* la nomenclature des différentes parties de cette belle opération, qui n'est point encore publiée malgré l'offre faite, il y a quelques années, par le gouvernement anglais au gouvernement français d'imprimer ces tables aux frais communs de la France et de l'Angleterre. De tels monumens assurent cependant à la nation chez laquelle ils sont créés, un des genres de glore qu'elle doit le plus ambitionner ; il est infiniment à regretter qu'on laisser enfouie, en manuscrit, une production jugée sans égale par les Lagrange, les Laplace, et qu'on s'obstine ainsi à courir les chances de son irréparable perte, qui peut être occasionnée par un de ces accidens dont on a malheureusement tant d'exemples."

1.4.8 Legalization of the decimal system

The 1840s and 1850s were relatively quiet and by that time Prony's tables were almost forgotten. They were only mentioned once in a while, in particular in the context of the legalization of the decimal system.

In the 1830s, decimal tables had actually lost their interest, especially since 1812, when Napoleon abolished the requirement to have only decimal divisions. The decimal system became truly enforced only in 1840,[196] and as

[190] A.N. F^{17}13571.
[191] [Garnier (1826), p. 118]
[192] [Riche de Prony (1834)]
[193] [Riche de Prony (1834), p. 37]
[194] [de Montferrier (1836), p. 519]
[195] [Riche de Prony (1824), pp. 41–42]
[196] [Débarbat and Dumont (2006)]

soon as 1841, the *Popular encyclopedia* wrote that "it is high time that the French government should give [the tables] to the world."[197]

Various events were opportunities to remind the public of the great tables, although they were mostly seen as a monument that one would visit. For instance, in 1837, Arago, when reading a biography of Carnot, spoke of the "great, the incomparable tables of the cadastre."[198] In 1839, when Prony died, it was again Arago's turn to make a summary of his life, and he wrote that 99% of the tables were produced by laborers who knew only to add and subtract, and that "the 1% left was deduced from analytical formulæ by scientists to whom Prony was thus offering a refuge against the tempest."[199]

There was also often some confusion about the state of the tables, given that there were many of them, and that almost every table was copied or abridged from another one. So, it is not totally a surprise to read statements such as Airy's in his *Treatise on trigonometry*, in which he presented Borda's table (which he hadn't seen) as an abridgement of the *Tables du cadastre*:

> "An abridged form of the *Tables du Cadastre*, revised by Delambre, has, we believe, been edited by Borda; and must form a useful collection for the decimal division."[200]

1.4.9 The analysis of the tables (1858)

Up to 1858, there had been no serious analysis of the *Tables du cadastre*. This changed when Pierre Alexandre Francisque Lefort (1809–1878), a graduate of the *École Polytechnique* and the *École des Ponts et Chaussées* became interested in the tables.[201] He examined them for several months[202] and obtained that the manuscript still owned by Prony's heirs be given to the library of the *Institut*.[203] This somewhat revived the interest in the tables, Lefort suggested that a greater priority would be to use the *Tables du cadastre* to print 8-place tables,[204] and this may have led to the tables published in 1891 by the *Service géographique de l'armée*.[205]

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[197] [Anonymous (1841)]

[198] [Barral (1854), p. 561]

[199] [Barral (1855), p. 589–590]

[200] [Airy (1855), p. 94]

[201] Lefort was at the *École Polytechnique* when Prony was graduation examiner and at the *Ponts et chaussées* when he was director.

[202] [Lefort (1858a), p. 994]

[203] [Lefort (1858a), Lefort (1858b)]

[204] [Lefort (1858b), p. 146] See also Templeton's article [Templeton (1865)], answering Lefort's.

[205] [Service géographique de l'Armée (1891)]

In 1862, Jules Hoüel, writing of the advantages of the decimal division of the angles, made the wish that seven, six, five, or four-place tables be extracted from the *Tables du cadastre*, as the lack of such tables was felt by him as the main hindrance to the acceptance of that division [Hoüel (1867), p. 71].

When the next major independent calculation—Edward Sang's seven-place table of logarithms—was published in 1871,[206] Glaisher also gave a (slightly incorrect) description of the *Tables du cadastre*, claiming that the logarithms of numbers were given with "15 places of decimals."[207] Glaisher had apparently not seen the tables, and had based his account on Lefort's analysis,[208] although Lefort wrote correctly that 14 decimals were given.

By 1872, according to Glaisher, only Babbage and Lefort had used the *Tables du cadastre*, either for new tables, or for establishing erratas in Briggs' and Vlacq's tables, or merely for analyzing the tables. This, however, was not totally true, as Borda, Callet, Hobert and Ideler, the *Service géographique de l'armée*, Mendizábal-Tamborrel, and probably others, have at times used these tables.

When Edward Sang's project of a nine-place table became known,[209] and when Sang's article on his discovery of errors in Vlacq's table was published with this project,[210] an article in *Nature*[211] appeared very critical of Sang's claims and asserted that, contrary to Sang's writings, the *Tables du cadastre* had been used to check Vlacq's table, and that the errors found by Sang had mostly already been found by Lefort.

The article in *Nature* led Sang to publish a more detailed article on the *Tables du cadastre*, and on the need for new tables,[212] and an exchange with Lefort followed,[213] since Sang had actually not seen the *Tables du cadastre*, and only seen one of Lefort's articles, not his analysis published in the *Annales de l'Observatoire*.[214]

Sang was in particular very critical on the interpolation in the *Tables du cadastre*, as it represented too many steps, and only at the end would the computer know if his calculations were correct or not.[215] As a consequence,

[206][Sang (1871)] Sang's project was to publish a nine-place table of logarithms, of which the seven-place table is only a by-product.
[207][Glaisher (1872), p. 79]
[208][Lefort (1858b)]
[209][Sang (1872a)]
[210][Sang (1875c)]
[211][Anonymous (1874)]
[212][Sang (1875a)]
[213][Lefort (1875), Sang (1875b)]
[214][Lefort (1858b)]
[215]Sang was not alone to criticize the *Tables du cadastre*, but perhaps he was the most

Sang became convinced that the computers ended up exchanging and correcting their calculating sheets, contrary to the scheme set up by Prony, and this would cast a doubt on the accuracy and independence of the interpolations.[216] Sang wrote that

> "(...) the whole operation was conducted with a laxity of discipline which detracts enormously from its value."

Sang, however, had not seen the *Tables du cadastre*, and his critique seems exaggerated, although he may be right in isolated cases.[217] One would presumably imagine that the computers didn't want to risk their position, of which they showed pride, by cheating. There were errors, but errors were corrected. Presumably, the computations were done on separate sheets, and they were copied on the final sheets once the interpolation was done. This precluded copying erroneous computations, but it didn't prevent some minor internal errors which were within the range of the acceptable errors. As we will see, there is a great agreement between all the sheets in both sets, and this agreement can only be the result of a good verification structure, something which would not be the case if exchanges between some of the computers occurred. Moreover, a computer would have to show his calculation sheet only once it is complete, and it is doubtful whether exchanges could have been left unnoticed by the members of the second group.

Anyway, for Sang, the *Tables du cadastre* were useless:

> "these existent and unpublished tables barred the way [to progress]; for no private person would think of undertaking of new a work which had been already so well accomplished."[218]

And by writing *so well*, Sang meant quite the opposite.

Sang closed his article with

> "I call upon the whole body of cultivators of exact science to shake off this incubus, to hold these tables as non-existent, and to face

analytical one. Another critique is cited by Lefort in 1861, when reviewing a new edition of Callet's tables. The preface of this new edition attributed Callet's initial errors to the *formules expéditives mises en vogue par les auteurs des grandes Tables du Cadastre*, although these methods work correctly if properly used [Lefort (1861), p. 70]. Lefort also accused Callet of false erudition.

[216][Sang (1875a), p. 432]

[217]One such example may be the one related to the error in $0^{\mathrm{q}}.00243$ in the logarithms of the ratios arcs to sines, where obviously at least one person decided to conceal an error, see § 4.6.6.

[218][Sang (1875a), p. 435]

> manfully the problem of computing decimal Trigonometric Tables of extent and precision sufficient for their pioneers, and therefore capable of supplying all the shorter and less precise tables needed for their more ordinary pursuits."[219]

Sang's critique can also be read as a critique towards mechanical computing, as Sang was not supportive of Babbage's efforts to build a mechanical computer.[220]

Until the publication of the abridged tables in 1891, the *Tables du cadastre* were still mentioned once in a while. In 1873, Govi, for instance, in the report written in answer of Sang's specimen pages, still hoped that the tables would be published some day, but at the same time he realized that it would probably not happen any time soon and supported instead the publication of tables with 8 or 9 places such as those planned by Edward Sang.[221]

At about the same time as Sang published his remarks, a short note[222] in the *Comptes rendus hebdomadaires des séances de l'Académie des sciences* was echoing Govi's article[223] on Sang's project:

> S'il fallait émettre un vœu, ce serait celui que les gouvernements, intéressés à la détermination de l'arc du méridien et à l'unification du système des poids et des mesures, se missent d'accord pour publier enfin les *Grandes Tables*, calculées sous la direction de Prony, etc.

This was misunderstood as implying that the *Tables du cadastre* were about to be printed.

The reason for not printing the tables was sometimes misunderstood, as by the mathematician Joseph Bertrand who claimed that the tables had not yet been published, and probably never would because they make interpolation too difficult, and not for economical reasons.[224]

[219] [Sang (1875a), p. 436]

[220] [Daston (1994), pp. 201–202] See Sang's comments on the use of machines to aid calculations [Sang (1872b)].

[221] [Govi (1873), p. 167]

[222] [Anonymous (1875)]

[223] [Govi (1873)]

[224] [Bertrand (1870), Govi (1873)] Bertrand's objection was about the practicability of the step, and not, like Sang, on Prony's construction methods. In his article, he compared the application of Thoman's variant of the radix method to the interpolation in the *Tables du cadastre* using Newton's formula. But Bertrand forgot that the *Tables du cadastre* were not meant for a daily usage. Instead, they were meant as a standard from which other tables could be derived.

In 1873, Tennant was supporting the use of a decimalization for new tables, and wrote that "[i]t would be easy to use the MS. French Tables to compare with any published on this system."[225]

Andoyer, who published new tables in 1911, seemed familiar with Prony's tables. His tables also gave the logarithms for every centesimal part of the quadrant.

Numerous authors, among them Maurice d'Ocagne, did cite Prony's work, but usually without any further details.[226]

Some authors published partial tables, or tables with which it was possible to compute logarithms with a large number of decimals, but not giving them directly. Andoyer's 1922 tables, for instance, allows for the computation of logarithms with 13 places, by reducing the calculation to the use of logarithms of numbers from 100 to 1000 and from 100000 to 101000.[227]

1.4.10 Going beyond the *Tables du cadastre*

Only a few endeavours went beyond the *Tables du cadastre* before the advent of electronic computers. Many of these endeavours were left unfinished. Edward Sang and his daughters computed for instance more extensive tables, but they were never published. These tables were to serve as the basis of a table of nine-place logarithms,[228] of which Sang's 1871 table can be considered an abridgement.[229]

More extensive tables of the logarithms of numbers were published by Thompson in 1952.[230] Andoyer's 1911 table of logarithms of sines, cosines, tangents, and cotangents, are given to 14 places, but are sexagesimal.[231]

Tables of sines as extensive as those of Prony and in the decimal division do not seem to have been published. For instance, Andoyer's trigonometric tables give only 15 decimals.[232]

In 1910, when he published his trigonometric tables, Andoyer wrote that the "Tables trigonométriques n'ont donc bénéficié que de progrès insignifiants depuis l'invention des logarithmes, et l'œuvre même des fondateurs, Briggs et Vlacq, non surpassée, demeure entachée des nombreuses erreurs qui la déparent, tandis que les Tables du Cadastre restent inutiles à l'état de

[225] [Tennant (1873), p. 565]

[226] [d'Ocagne (1928)]

[227] [Andoyer (1922), Roegel (2010b)] This is a variant of the "radix method" [Glaisher (1915)].

[228] We have reconstructed this table in 2010 [Roegel (2010a)].

[229] See [Sang (1871), Roegel (2010c), Craik (2003)].

[230] [Thompson (1952)]

[231] [Andoyer (1911), Roegel (2010d)]

[232] [Andoyer (1915–1918), Roegel (2010e)]

manuscrit." He added that "leur étendue a été jusqu'à ce jour un obstacle insurmontable à leur publication et le Service géographique de l'Armée en a donné seulement une édition réduite à huit décimales en 1891, en même temps que M. de Mendizabal Tamborrel publiait des Tables analogues."[233] In 1911, he wrote that "(...) elles ont le grave tort d'être restées manuscrites, et de se prêter mal à l'impression."[234]

Andoyer seemed partly to attribute to the extent of the cadastre tables the fact that they were not printed:

> "J'ai encore été détourné de la division centésimale par les raisons suivantes : avec cette division, le seul intervalle convenable à adopter était celui des tables du Cadastre, et je me serais par suite heurté aux mêmes difficultés de publication ;"[235]

The *Tables du cadastre* have only been used in rare circumstances, for instance when computing the tables for the international ellipsoid reference adopted in 1924.[236]

1.5 Reduced tables

Several sets of reduced tables are related to the *Tables du cadastre*:

- Abridged tables by Prony (part of the *Tables du cadastre*). These tables give the logarithms of sines and tangents with 8 or 9 decimals (depending on the range), to be printed with 7 decimals, for every 10000th of the quadrant. The pivots were probably copied from the *Tables du cadastre*, but the interpolations were obtained by new calculations.

- Abridged table of sines.[237] This is a table giving the sines and cosines to 7 places, every 6″ (six sexagesimal seconds). Such a table was made at the *Bureau du cadastre* in 1795, but it is a sexagesimal table, and could not be made from the decimal sine table without some effort. It is likely that it was produced from a different source.

- Tables of logarithms to 8 places,[238] from 100000 to 200000. As no 8-place tables over that range were known by Prony's time, Callet[239]

233[Andoyer (1910)]
234[Andoyer (1911), p. VII]
235[Andoyer (1911), p. VII]
236[Perrier (1928)]
237PC: Fol. 305 and A.N. F^{17}1244B.
238PC: Ms. 243 and Ms. Fol.2773.
239[Callet (1795)]

covering only the range 100000–107999, and Newton[240] covering only the range 1–100000, this table is likely an extension of Callet's table based on the *Tables du cadastre*. The layout copies the one used by Callet. As mentioned earlier, Charles Haros had apparently computed a table of logarithms, but we believe it is unlikely that this is Haros' table. This table is certainly also unrelated to the table published in 1891 by the *Service géographique de l'armée*.

- Tables of antilogarithms, also to 8 places.[241] This seems likewise to be a new table, derived from the *Tables du cadastre*. In 1742, Dodson[242] went much beyond the present tables, publishing 300 pages of tables giving the antilogarithms from 0.00000 to 1.00000, to 11 places, but the current tables might have been used to check Dodson's tables. In 1844 and 1849, Shortrede published a 7-place table of antilogarithms,[243] as did Filipowski[244] in 1849.

- Another set of reduced tables were published in 1809. These tables, supervised by Charles Plauzoles,[245] were checked by former computers of the great cadastre tables. They contained the logarithms of numbers from 1 to 21750, the logarithms of sines, cosines, tangents and cotangents every $1'$ from $0°$ to $45°$, every $0^{q}.00001$ from $0^{q}.00000$ to $0^{q}.03000$, and every $0^{q}.0001$ from $0^{q}.0300$ to $0^{q}.5000$, all to six decimal places.[246]

- In his foreword to Plauzole's tables,[247] the printer Firmin Didot wrote that he was then preparing a new set of tables, computed by former computers of the cadastre tables, using Prony's methods, but this time payed by Didot. These tables are not the abridged volume which is part of the *Tables du cadastre*. These tables had been announced as being *sous presse* (getting printed) in Legendre's *Éléments de géométrie* (1808)[248] and they were also announced in a 1809 *Prospectus*.[249] They were supposed to be composed as follows in a quarto volume: the logarithms of numbers from 1 to 10000, the logarithms of numbers from

[240] [Newton (1658)]
[241] PC: Ms. Fol.2774.
[242] [Dodson (1742)]
[243] [Shortrede (1844)]
[244] [Filipowski (1849)]
[245] [Didot (1809a), de Plauzoles (1809)]
[246] The structure of Plauzoles' table was checked on the original edition and on the 4th printing published in 1830.
[247] [de Plauzoles (1809)]
[248] [Legendre (1808)]
[249] [Didot (1809b)]

10000 to 20000, by steps of 0.1, the logarithms of the whole numbers from 20000 to 100000, the logarithms of sines and cosines, for every second of the quadrant, all with seven or eight decimals. Didot wrote that they might be published in 1810, if there was a sufficient number of subscribers, but they do not seem to have been printed.

- An excerpt of the tables of the *cadastre* was published by the *Service géographique de l'armée* in 1891.[250] The tables gave the logarithms of the numbers 1 to 120000 and the logarithms of the sines, cosines, tangents and cotangents every 10 centesimal seconds, all to 8 places. The first part of the table does not seem to have been copied from the 8-place table mentioned above.

- Another set of tables was published in 1891 by Joaquín de Mendizábal-Tamborrel, and was compared with the *Tables du cadastre*.[251] These tables gave the logarithms of numbers 1–125000 to 8 places and the logarithms of trigonometric functions to 7 or 8 places.

Smaller tables using the decimal division were also issued, although not directly influenced by the *Tables du cadastre*. A very popular and widespread set of tables were those of Bouvart and Ratinet,[252] in use in France from the beginning of the 20th century to the 1970s. These tables gave the logarithms of the numbers 1–11000 and of the trigonometric functions to five places every decimal minute.

In 1935, Sarton wrote that "[m]any efforts have been made, and are still being made, in France to promote the decimal division of the quadrant."[253] He continues: "The simultaneous employment of two kinds of degrees would be confusing but for the fact that the French have two different names for them 90 degrés = 100 grades. (...) The prospects of the diffusion of the decimal division of the quadrant are not brilliant to day, and more's the pity, for our present system is disgraceful."[254] In 1938, mention was also made of projects of introducing the decimal division of the quadrant in Germany.[255]

[250][Derrécagaix (1891), Service géographique de l'Armée (1891), Radau (1891a)] See our reconstruction [Roegel (2010f)].
[251][de Mendizábal-Tamborrel (1891), Jacoby (1892a)] See our reconstruction [Roegel (2010g)].
[252][Bouvart and Ratinet (1957)]
[253][Sarton (1935), p. 201]
[254][Sarton (1935), p. 202]
[255][Sadler (1938)]

1.6 The manuscripts

The first detailed description of the manuscripts was published by Lefort in 1858.[256] The core is made of 17 large in-folio volumes, each set of four pages being obviously the work of one calculator. Lefort examined the set located at the Paris *Observatoire*[257] and this set still has exactly the same composition now. Lefort also located the second set which was in the hands of Prony's heirs[258] and had them transfer it to the library of the *Institut*.[259] In 1858, this second set had an introductory volume which was still incomplete. The missing parts were copied in 1862, so that the two introductory volumes now have the same contents. Both sets comprise 19 volumes, but the two sets are not totally identical. The *Institut* is alone to have a volume of the multiples of sines and cosines (volume 18), but this volume is obviously not really part of the set.[260] The *Observatoire*'s volume of abridged tables is missing at the *Institut*, but the second copy of the tables is actually located at the library of the *Ponts et chaussées*,[261] so that Prony's own set was the most complete, except for the introduction.[262]

The volumes which are in two copies, except the introduction, are nearly identical, the main changes being the slightly different layout (and values) of the logarithms of the numbers from 1 to 10000, and the different binding of the logarithms of tangents. In addition, the set at the *Observatoire* has a binding error in the tables of logarithms of sines.

The volumes at the *Observatoire* are numbered from 1 to 8 for the logarithms of the numbers, and from 1 to 4 for the logarithms of sines and the logarithms of tangents each. The three other volumes are not numbered. The volumes at the *Institut* are numbered from 1 to 18, except for the introductory volume which is not numbered. The numbers are only on the spines, and not within the volumes.

A detailed summary of the volumes follows:[263]

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[256] [Lefort (1858b)]

[257] [Riche de Prony (ca. 1793–1796a)] This set will sometimes be referred as "copy O" in this document.

[258] For a list of documents bequeathed by Mme de Corancez, Mme de Prony's niece, see [Bradley (1998), pp. 325–335].

[259] [Riche de Prony (ca. 1793–1796b)] This set will sometimes be referred as "copy I" in this document.

[260] A second, unbound copy, of this volume is located in the Archives of the *Ponts et chaussées*, Ms. Fol. 1890.

[261] PC: Ms. Fol. 242. This volume will sometimes be referred as "copy P" in this document.

[262] The Archives of the *Ponts et chaussées* do however contain drafts of the introductory volume, see Ms. 1745.

[263] An earlier description of the volumes was given by Grattan-

- *Observatoire*, 19 volumes (B6 1–19):[264]
  - introduction (one volume),
  - log. numbers (eight volumes),
  - log. sin (four volumes), 1) $0^q.00000$–$0^q.05000$ and $0^q.05000$–$0^q.25000$, 2) $0^q.25000$–$0^q.50000$, 3) $0^q.75000$–$1^q.00000$, 4) $0^q.50000$–$0^q.75000$
  - log. tan (four volumes), 1) $0^q.00000$–$0^q.05000$, $0^q.95000$–1.00000, and $0^q.05000$–$0^q.20000$, 2) $0^q.20000$–$0^q.45000$, 3) $0^q.45000$–$0^q.70000$, 4) $0^q.70000$–$0^q.95000$
  - logarithms of the ratios arcs to sines (included in the first volume of logarithms of sines),
  - logarithms of the ratios arcs to tangents (included in the first volume of logarithms of tangents),
  - sines (one volume)
  - abridged table (one volume);
- *Institut*, 19 volumes (Ms 1496–Ms 1514):
  - introduction (one volume, Ms 1514),
  - log. numbers (eight volumes, Ms 1496–Ms 1503),
  - log. sin (four volumes, Ms 1505–Ms 1508), 1) $0^q.00000$–$0^q.05000$ and $0^q.05000$–$0^q.25000$, 2) $0^q.25000$–$0^q.50000$, 3) $0^q.50000$–$0^q.75000$, 4) $0^q.75000$–$1^q.00000$
  - log. tan (four volumes, Ms 1509–Ms 1512), 1) $0^q.00000$–$0^q.05000$, $0^q.95000$–1.00000, and $0^q.05000$–$0^q.25000$, 2) $0^q.25000$–$0^q.50000$, 3) $0^q.50000$–$0^q.75000$, 4) $0^q.75000$–$0^q.95000$
  - logarithms of the ratios arcs to sines (included in the first volume of logarithms of sines),
  - logarithms of the ratios arcs to tangents (included in the first volume of logarithms of tangents),
  - sines (one volume, Ms 1504),
  - multiples of sines and cosines (one volume, Ms 1513);
- *Ponts et chaussées*:
  - abridged table (one volume, Ms. Fol. 242),

Guinness [Grattan-Guinness (1990a), p. 181], but was slightly incorrect.

[264]The exact call numbers of each volume are not given here, as they are barely legible.

– multiples of sines and cosines (one unbound volume, Ms. Fol. 1890)
– drafts of the introduction (Ms. 1745)

1.7 Going further

No systematical analysis of the construction and accuracy of the famed *Tables du cadastre* has ever been carried out. Lefort, the author of the first detailed analysis, and perhaps the only man since 1850 who was able to compare the two sets side by side, offered only a biased account, based to a great extent on Prony's own introduction which does not always describe accurately the content of the tables.

The manuscripts contain many idiosyncrasies, but Lefort,[265] and Grattan-Guinness in his recent accounts,[266] were almost mute on them.

The absence of any deeper analysis of the tables can be explained by the devaluation of the tables, by the fact that they remained in manuscript form, and by the difficulty of checking them.

It has often been stated that apart from Prony and Sang, basically almost every extensive table of logarithms printed between the 1630s and the beginning of the 20th century was based on Briggs' and Vlacq's tables.[267] The task of recomputing logarithms is a mighty one, and only few people have made that endeavour. It is therefore all the more understandable and natural that Lefort could only compare Prony's tables with other tables. And this is what he did, but he could do so only for the logarithms of numbers. Prony's trigonometric tables were decimal, which was not the case of the earlier extensive tables, and only some of the values could be checked easily.[268] Lefort was not able to compare the differences Δ^2, Δ^3, ..., which were never given.[269,270] Lefort may have recomputed a few pivot values, but certainly

[265] [Lefort (1858a)]

[266] [Grattan-Guinness (1990a), Grattan-Guinness (2003)]

[267] There are of course some other noteworthy tables, such as Thomson's table of logarithms from 1 to 120000 to 12 places [Glaisher (1874)], but these tables were never printed, not even partially, and were virtually unknown at the time of their computation.

[268] Moreover, the existing decimal tables, such as Borda's, were of a more restricted scope.

[269] One exception is Pitiscus' *Thesaurus mathematicus*, which gives the differences up to Δ^3, but these differences were tabulated, not computed independently [Pitiscus (1613)], and they were given to a different step as the one needed to check Prony's tables. Incidentally, there is a copy of Pitiscus' book in the Prony archives at the *École nationale des ponts et chaussées* (Fol. 423). It even contains a short table giving differences up to Δ^5.

[270] The second and fourth differences for the logarithms of numbers were however given later by Thompson in his *Logarithmetica Britannica* (1952) [Thompson (1952)], but the values are shifted. For instance, for $n = 10002$, Thompson gives $\delta_2 = 424120818382$, corresponding to Prony's $\Delta^2 = 4241208184$ for $n = 10001$. And Thompson gives $\delta_4 =$

not many.

26037, corresponding to Prony's $\Delta^4 = 2603683$ for $n = 10000$. Thompson needs only even differences, because he uses Everett's interpolation formula.

Chapter 2

Computational methods and tables

The first technical details on Prony's general methods were given in a report written in 1801 by Lagrange, Laplace and Delambre, but this report remained very vague.[271] Then in 1858, Lefort published a much more extensive description based on Prony's own description, and on a long examination of the tables themselves.[272] And almost a century and a half later, Grattan-Guinness gave additional details on the making of the tables.[273]

In this section, we describe in detail the methods used to compute the original tables, as well as some of the departures from the general rules claimed to have been used. We also describe how all the values were recomputed in the companion volumes.[274]

2.1 Interpolation

2.1.1 The method of differences

At the newly founded *École Polytechnique*, Prony gave a course on the method of differences. Prony's course was published in 1796 and was the first treatise in France that had used the concept of function throughout as

[271] [Lagrange et al. (1801)] Some details on specific parts were published by Lacroix a year before [Lacroix (1800), pp. 51–53]. Other details had been published in Prony's lessons at the *École Polytechnique* [Riche de Prony (1796c)].

[272] [Lefort (1858b)]

[273] [Grattan-Guinness (1990a), Grattan-Guinness (2003)]

[274] A list of the companion volumes is given in section 5.5. All the computations were done using the GNU `mpfr` library [Fousse et al. (2007)].

a basis.[275]

The general idea of the computation of logarithms or trigonometric values by Prony was to make exact computations for a number of pivots, and to perform an interpolation in between.[276] The pivots were regularly spaced by the constant interval Δx and for each pivot, a number of forward differences Δ^n were given. If x is a pivot, these differences are defined as follows:

$$\begin{aligned}
\Delta \quad f(x) &= f(x+\Delta x) - f(x) \\
\Delta^2 \quad f(x) &= \Delta^1 f(x+\Delta x) - \Delta^1 f(x) \\
\Delta^3 \quad f(x) &= \Delta^2 f(x+\Delta x) - \Delta^2 f(x) \\
&\dots \\
\Delta^{i+1} f(x) &= \Delta^i f(x+\Delta x) - \Delta^i f(x) \\
&\dots
\end{aligned}$$

Prony chose the number of differences Δ^i and the number of their digits in order to ensure that a certain number of digits were correct in the result. The logarithms of numbers, for instance, had to have 12 exact digits, or more exactly, the error had to be smaller than half a unit of the 13th place.

If there is an N such that $\Delta^n f(x_0)$ can be neglected for $n > N$, then the values of $\Delta^n f(x_0)$ can be used to express the values of $f(x)$ as follows, using Newton's forward difference formula:[277]

$$u_p = u_0 + p\Delta u_0 + p\frac{p-1}{2}\Delta^2 u_0 + p\frac{p-1}{2}\frac{p-2}{3}\Delta^3 u_0 + \cdots$$

u_0, u_1, u_2, ..., are the values of a given sequence, and they are obtained from the values of u_0 and $\Delta^n u_0$.

With this formula, $f(x)$ can in particular be computed for values between $f(x_0)$ and $f(x_0 + \Delta x)$.

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[275] [Riche de Prony (1796c)] See also [Schubring (2005), p. 287]

[276] Prony mainly refers to Mouton's interpolation method, described in detail by Maurice [Maurice (1844)], and this method, which was popularized by Lalande, Lagrange, and then Prony, is actually equivalent to Briggs' interpolation. Both are computing the differences of a subsequence using the differences of a sequence, and the subsequences are used to subtabulate the original function.

For the history of interpolation or the method of (finite) differences, see [de Lalande (1761a)], [de Lalande (1761b)], [Lagrange (1774)], [Lagrange (1780)], [Lagrange (1798)], [Delambre (1793)], [Lacroix (1800)], [Legendre (1815)], [Lacroix (1819)], [Maurice (1844)], [Radau (1891b)], [Markov (1896)], [Seliwanoff (1904)], [Selivanov et al. (1906)], [Gibb (1915)], [Meijering (2002)].

[277] It should be noted that special cases of this formula had been implicitely used by Briggs [Briggs (1624)], before Newton.

Prony did not make use of Newton's formula, but computed the differences, and then obtained the values of the interpolated function one by one, by mere additions or subtractions (if the differences are negative). Prony's method slightly differs from the strict use of Newton's formula, because of rounding.

Prony's subtabulation method has now long been considered obsolete, and has been replaced by more modern methods, such as those of Bessel and Everett. It is therefore instructive to read Sang's critique,[278] although Sang did not suggest replacements with the same convenience.[279]

2.1.2 Accuracy of the interpolation

If rounding is ignored, Newton's forward difference formula also gives the maximum error, in case u_0, Δu_0, ..., $\Delta^n u_0$ are not computed correctly. If $\mathcal{E}_0$ is the error on u_0, $\mathcal{E}_1$ the error on Δu_0, ..., $\mathcal{E}_n$ the error on $\Delta^n u_0$, then the total error is

$$\mathcal{E} = \mathcal{E}_0 + p\mathcal{E}_1 + p\frac{p-1}{2}\mathcal{E}_2 + \cdots + p\frac{p-1}{2}\frac{p-2}{3}\cdots\frac{p-(n-1)}{n}\mathcal{E}_n + \cdots \quad (2.1)$$

For the logarithms of numbers, we have $p = 200$, and therefore the final error is

$$\begin{aligned}\mathcal{E} = \mathcal{E}_0 + 200\mathcal{E}_1 + 19900\mathcal{E}_2 + 1313400\mathcal{E}_3 + 64684950\mathcal{E}_4 + 2535650040\mathcal{E}_5 \\ + 82408626300\mathcal{E}_6 + 2283896214600\mathcal{E}_7 + \cdots \quad (2.2)\end{aligned}$$

If we assume that all pivots are computed exactly, then each $\mathcal{E}_i$ is at most half a unit of its position, and if the variation of the last difference is bounded, this formula can be used to bound the total error, assuming no rounding in the interpolation. However, because of rounding the final error could in fact be larger, but the rounding errors too can be bound.

The factors of $\mathcal{E}_i$ determine the positions of the differences Δ^i.

[278] See [Sang (1875a)]. Comrie wrote of Sang's "masterly condemnation" of Prony's method [Comrie (1936), p. 227].

[279] For instance, Sang's own methods of computing the logarithms of primes require much more thought than mere additions and subtractions, and halving the interpolation intervals cancels the possibility to use one pivot to check the end of the previous interpolation. We believe that Prony's method was perfectly suited to computers who knew only to add and subtract, but evidently, the organization was not as perfect as it should have been, and the results would have been more accurate, had more care been taken, especially in the computation of some of the pivots.

2.1.3 The influence on Babbage

Prony's methods for the computation of a table had a direct influence on Charles Babbage's ideas, or at least, they added nicely to Babbage's plans. Babbage had wanted to secure the accuracy of tables, and he imagined a machine which would grind out successive values of a table, and even print them out. This machine was the "difference engine." Much has been written on Babbage's machines and successive designs and reconstructions, but at the beginning of his work,[280] Babbage explicitely quoted Prony's organization in a letter sent in 1822 to Humphry Davy.[281] Babbage may have learned from the *Tables du cadastre* through the discussions involving the French and British Governments for the joint printing of the tables, or he may have met Prony during the trip he made with John Herschel to Paris, probably in 1819.[282] One is tempted to imagine that he saw the tables at that time, but since he apparently did not mention them before 1822, it is probably unlikely. In any case, he examined them a few years later when preparing his own table of logarithms.

Babbage's difference engines do exactly embody the principles used by Prony for interpolating between pivots, except that Babbage would only use additions. If differences had to be subtracted, Babbage would in fact add their complement on the *word size*. For instance, if computations are done on 10 digits, subtracting 17 is like adding 9999999983. Babbage's machine would have replaced Prony's computers of the third section, without making any error. Besides the *finiteness* of the computations, the main other difference between Babbage's interpolation and that of Prony is that Babbage had all the differences at the same level, and therefore didn't have to take rounding into account.

Babbage's difference machines were not completed during his lifetime, but the first electronic computers were applied to the differencing of tables, either to compute differences from the table values, or to reconstruct table values from the differences.[283]

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[280] [Roegel (2009)]

[281] [Babbage (1822a)] Babbage gave more details on Prony's organization in the chapter devoted to the division of mental labor in his book on the economy of machinery and manufactures published in 1832 [Babbage (1832), pp. 153–157]. Babbage's description was based on the very rare note from 1820 [Anonymous (1820 or 1821)]. On the different interpretations, by Prony and Babbage, of "manufacturing," see especially [Daston (1994), pp. 196–198].

[282] [Hyman (1982), pp. 40–44]

[283] An early article on electronic computers and the method of differences is [Laderman and Abramowitz (1946)]. Mechanical difference engines have only been used to compute tables in isolated cases and semi-automatically.

2.2 Lagrange's formula for $\Delta^n f(x)$

In 1772, Lagrange published an article on the formal manipulation of series,[284] in which he showed that if u is a function of x and $\Delta u = u(x+\xi)-u(x)$, we have

$$\Delta^\lambda u = \left(e^{\frac{du}{dx}\xi} - 1\right)^\lambda,$$

provided that we identify $\left(\frac{du}{dx}\right)^k$ with $\frac{d^k u}{dx^k}$.

Lagrange derived this relation from the analogy between Newton's binomial formula and the n-th derivative of a product, an analogy discovered by Leibniz in 1695.[285]

Using Lagrange's result, Prony set up his method of interpolation making use of the values of $f(x)$, $\Delta f(x)$, $\Delta^2 f(x)$, etc. at certain pivot points. For a function f such as those under consideration, $\Delta^n f(x)$ can be expressed as

$$\begin{aligned}\Delta^n f(x) &= f^{(n)}(x)(\Delta x)^n + f^{(n+1)}(x)\frac{(\Delta x)^{n+1}}{(n+1)!} \times F(n, n+1) \\ &\qquad + f^{(n+2)}(x)\frac{(\Delta x)^{n+2}}{(n+2)!} \times F(n, n+2) + \cdots \\ &= \sum_{i=n}^{\infty} f^{(i)}(x)\frac{(\Delta x)^i}{i!} \times F(n, i)\end{aligned} \tag{2.3}$$

where $F(i, j)$ is defined for positive integers i, j by:

$$F(1, k) = 1 \text{ for } k \geq 1 \tag{2.4}$$
$$F(i, j) = 0 \text{ for } i > j \tag{2.5}$$
$$F(i, j) = n\big(F(i, j-1) + F(i-1, j-1)\big) \text{ in all other cases} \tag{2.6}$$

An expression equivalent to the above one is found in Prony's lessons at the *École Polytechnique*.[286]

The following table shows the first positive values of F:[287]

[284][Lagrange (1774)] For elementary treatises on this topic, see in particular Boole [Boole (1860)] and the recent surveys by Ferraro [Ferraro (2007), Ferraro (2008)].

[285][Ferraro (2007), p. 71]

[286][Riche de Prony (1796b), p. 554] On page 555, Prony mentions the work of Lanz and Haros on the problem of finding adequate formulæ.

[287]A table of the same function also appears in [Riche de Prony (1796b), p. 526].

	1	2	3	4	5	6	7	8
1	1	1	1	1	1	1	1	1
2		2	6	14	30	62	126	254
3			6	36	150	540	1806	5796
4				24	240	1560	8400	40824
5					120	1800	16800	126000
6						720	15120	191520
7							5040	141120
8								40320

These coefficients will be used several times in the sequel.

Now, assuming that we have computed u_0, Δu_0, $\Delta^2 u_0$, etc., we can construct the following table:

$$\begin{array}{lllllll}
u_0 & \Delta u_0 & \Delta^2 u_0 & \Delta^3 u_0 & \Delta^4 u_0 & \Delta^5 u_0 & \Delta^6 u_0 \\
u_1 & \Delta u_1 & \Delta^2 u_1 & \Delta^3 u_1 & \Delta^4 u_1 & \Delta^5 u_1 & \Delta^6 u_1 \\
u_2 & \Delta u_2 & \Delta^2 u_2 & \Delta^3 u_2 & \Delta^4 u_2 & \Delta^5 u_2 & \Delta^6 u_2 \\
u_3 & \Delta u_3 & \Delta^2 u_3 & \Delta^3 u_3 & \Delta^4 u_3 & \Delta^5 u_3 & \Delta^6 u_3 \\
\ldots
\end{array}$$

and compute the succeeding rows with

$$\begin{aligned}
u_p &= u_{p-1} + \Delta u_{p-1} \\
\Delta^{m-1} u_p &= \Delta^{m-1} u_{p-1} + \Delta^m u_{p-1}
\end{aligned}$$

2.3 Logarithms of the numbers

These logarithms span eight volumes (381 or 374 pages for the first volume, and 500 pages for each of the seven other volumes). Each volume covers 25000 numbers, and the whole set covers the numbers from 1 to 200000.

The logarithms are given with 19 decimals from 1 to 10000 and with 14 decimals from 10001 to 200000. The initial project seems to have been to compute the first 10000 logarithms to 28 decimals.[288] If Prony had indeed wanted to print the logarithms to 28 places (as is suggested by proofs at the *Archives Nationales*), he would actually have needed to compute even more places, in order to guarantee all these 28 places.

[288] At the time of Prony's calculations, there were already tables giving the logarithms with a greater number of decimals, but only for a small range. In 1706, Abraham Sharp had for instance published the logarithms of the integers from 1 to 100, and for the prime numbers up to 200, to 61 decimal places.

Numbers from 1 to 10000

The first pages of the first volume give the logarithms of numbers 1 to 10000 to 19 decimal places (81 pages at the *Observatoire*, 74 pages at the *Institut*). The reason for this accuracy is certainly to anticipate the loss of accuracy when combining the logarithms of primes, and to be able to guarantee 12 exact decimals.[289] This table was obtained as follows. First, since

$$\ln \sqrt{\frac{1+y}{1-y}} = y + \frac{y^3}{3} + \frac{y^5}{5} + \cdots \tag{2.7}$$

setting $y = \frac{1}{2x^2-1}$, we have $\sqrt{\frac{1+y}{1-y}} = \frac{x}{\sqrt{x^2-1}}$ and therefore

$$\ln \frac{x}{\sqrt{x^2-1}} = \frac{1}{2x^2-1} + \frac{1}{3}\left(\frac{1}{2x^2-1}\right)^3 + \frac{1}{5}\left(\frac{1}{2x^2-1}\right)^5 + \cdots \tag{2.8}$$

Hence, the logarithms of the prime numbers from 3 to 10000 were computed with the formula

$$\begin{aligned}\log x = \frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1)& \\ + M\left[\frac{1}{2x^2-1} + \frac{1}{3}\left(\frac{1}{2x^2-1}\right)^3 + \frac{1}{5}\left(\frac{1}{2x^2-1}\right)^5 + \cdots\right]&\end{aligned} \tag{2.9}$$

where log is the decimal logarithm and $M = \frac{1}{\ln 10}$. For instance, $\log 103$ is computed from $\log 104$ and $\log 102$, but since the arguments are even, previously computed values of $\log x$ could be used: $\log 102 = \log 2 + \log 3 + \log 17$ and $\log 104 = 3\log 2 + \log 13$.

Only a few terms need to be computed for each value of x. For instance, for $x = 3$, the sum can be computed until $\frac{1}{15}\left(\frac{1}{18-1}\right)^{15}$. But for $x = 101$, three terms are already enough and for $x = 9973$, the last prime number before 10000, it is sufficent to keep only one term.[290]

[289] Nevertheless, the logarithms seem to be exact to 17 or 18 decimals. A note by Charles Blagden dated 1 August 1819 about the plans of printing the tables considered that the logarithms from 1 to 10000 could be printed to 17 places, and this seems to take into account the real accuracy of this part of the tables. (PC: Ms. 1181)

[290] It is possible that the logarithms of the numbers were computed using different formulæ for each manuscript, but Prony only gives the above formula. This may explain the discrepancies in the calculations, since the terms neglected may then be different. Another formula which may have been used is Borda's formula [de Borda and Delambre (1801),

The values of M and of $\log 2 = M \ln 2$ must have been taken from an earlier source[291] (for instance by 1748 Euler[292] had given 25 decimals for M and $\ln 2$, hence enough decimals for the first table) or computed anew, for instance using the formula

$$\ln 2 = \sum_{k \geq 1} \frac{1}{k 2^k} \tag{2.10}$$

Once all these logarithms had been computed, all the other values between 1 and 10000 were obtained by decomposition. As a consequence, the logarithms of numbers with a decomposition into many primes are less accurate than the logarithms of primes.

Legendre took an excerpt of the table contained in the *Observatoire* set comprising the logarithms of the odd numbers from 1163 to 1501 and the logarithms of all prime numbers from 1501 to 10000, plus 10007[293] and published them[294] in 1816 and again in 1826.

Legendre's tables contain a number of errors and since these errors are identical with those of the *Observatoire* set, but not with those of the *Institut*

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p. 6]:

$$\ln(x-2) - 2\ln(x-1) + 2\ln(x+1) - \ln(x+2) = -2\sum_{n=0}^{\infty} \frac{1}{2n+1}\left(\frac{2}{x^3-3x}\right)^{2n+1}$$

With this formula, $\ln(x+2)$ can be obtained from three preceding logarithms and a series converging very quickly. For instance, for $x = 95$, only two terms of the series are needed to obtain $\ln 97$ to 19 decimal places (assuming the previous values are computed accurately). With greater values of x, only one term is needed, but to secure the desired accuracy, one has of course to be careful with the propagation of errors. Borda's formula needs of course only be used when $x+2$ is prime. See [Warmus (1954), p. 12] for other useful formulæ.

Haros is in particular the author of several formulæ for computing logarithms, see [de Borda and Delambre (1801), p. 75], [Lacroix (1804)], [Garnier (1804)], [Bonnycastle (1813)], [Garnier (1814)], and [Guthery (2010), pp. 61–64]. Some of these formulæ were later extended by Lavernède [Gergonne (1807), Lavernède (1808), Lavernède (1810–1811a), Lavernède (1810–1811b)].

291 Prony's introductory volume writes "*Sa valeur est, comme on scait*, m = 0,43429 44819 03251 8."

292 [Euler (1748), pp. 91–92]

293 Although Legendre does explicitely refer to the *Tables du cadastre* as his source, log 10007 is not given to 19 places in the *Tables du cadastre*, and he must have taken the value from elsewhere, or recomputed it.

294 [Legendre (1816), table V] and [Legendre (1826), table V, pages 260–267]

set, there is no doubt about Legendre's source.[295,296]

Numbers from 10000 to 200000

Using the previous table, all logarithms from 10000 to 199800 were obtained by steps of 200:

$$\log(10000 + 200k) = \log 100 + \log(100 + 2k) = 2 + \log(100 + 2k) \quad (2.11)$$

with $0 \leq k < 950$.

Then, the six first differences $\Delta^i \log n$ were computed for $n = 10000 + 200k$. These can be computed using Lagrange's formula,[297] or directly as follows:

$$\Delta \log n = \log(n+1) - \log n = \log\left(1 + \frac{1}{n}\right) = \log(1+x) \quad \text{with } x = \frac{1}{n} \quad (2.12)$$

$$= M \times \ln(1+x) = M\left[x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots\right] \quad (2.13)$$

$$\Delta^2 \log n = \Delta \log(n+1) - \Delta \log n = M\left[\ln\left(1 + \frac{1}{n+1}\right) - \ln\left(1 + \frac{1}{n}\right)\right] \quad (2.14)$$

and so on. Eventually, we obtain[298]:

[295] These errors had already been noted by Sang who wrote that to make a list of the errors would be to make a list of all the primes [Sang (1875b)], [Fletcher et al. (1962), p. 872]. On the other hand, there seems to be a recurrent confusion as to the number of places of *computation* and the number of places of *accuracy*. Prony had the logarithms computed to 19 places from 1 to 10000 and to 14 places from 10000 to 200000, but he never claimed that all these decimals were exact. Prony's purpose was to have 12 exact decimals. Sang's 28 decimals were also not exact, but were chosen so as to guarantee 15 decimals in his million table.

[296] In view of the difference between calculated digits and accurate digits, Legendre should therefore have known better, and should not have taken all the decimals for his table.

[297] As an illustration of Lagrange's formula, we compute the first difference. Taking $u(x) = \log x$ and $\xi = 1$, we have $\Delta u = e^{\frac{du}{dx}} - 1 = \frac{du}{dx} + \frac{1}{2!}\frac{d^2u}{dx^2} + \frac{1}{3!}\frac{d^3u}{dx^3} + \cdots = M\left[\frac{1}{x} - \frac{1}{2x^2} + \frac{1}{3x^3} - \cdots\right]$.

[298] These formulæ were given by Lefort to the 6th order in 1858 [Lefort (1858b), p. 131], but with several typographical errors.

TABLE V.

Logarithmes à 19 décimales pour tous les nombres impairs de 1163 à 1501, et pour tous les nombres premiers de 1501 à 10000.

Nota. Cette Table fait suite aux logarithmes à 20 décimales des Tables de Gardiner, édit. d'Avignon. Elle est extraite des grandes Tables du Cadastre, déposées au Bureau des Longitudes, et dont la notice se trouve dans le tome V des Mémoires de l'Institut.

Nomb.	Logarithmes.	Nomb.	Logarithmes.	Nomb.	Logarithmes.
1163	06557 97147 28448 4114	1243	09447 11286 41644 7635	1323	12155 98441 87500 9733
1165	06632 59253 62037 7769	1245	09516 93514 31755 1459	1325	12221 58782 72826 6552
1167	06707 08560 45370 1735	1247	09586 64534 78542 6137	1327	12287 09228 64435 5119
1169	06781 45111 61840 1107	1249	09656 24383 74135 5120	1329	12352 49809 42731 9975
1171	06855 68950 72363 1299	1251	09725 73096 93419 9551	1331	12417 80554 74675 1223
1173	06929 80121 15529 2447	1253	09795 10709 94149 9998	1333	12483 01494 13859 2061
1175	07003 78666 07755 0740	1255	09864 37258 17056 9441	1335	12548 12657 00594 0268
1177	07077 64628 43434 6816	1257	09933 52776 85957 7472	1337	12613 14072 61984 3683
1179	07151 38050 95089 1354	1259	10002 57301 07862 5975	1339	12678 05770 12008 9744
1181	07224 98976 13514 7991	1261	10071 50865 73081 6210	1341	12742 87778 51598 9129
1183	07298 47446 27930 3691	1263	10140 33505 55330 7447	1343	12807 60126 68715 3565
1185	07371 83503 46122 6701	1265	10209 05255 11836 7244	1345	12872 22843 38426 7849
1187	07445 07189 54591 2204	1267	10277 66148 83441 3410	1347	12936 75957 22985 6122
1189	07518 18546 18691 5818	1269	10346 16220 94704 7763	1349	13001 19496 71904 2476
1191	07591 17614 82777 5032	1271	10414 55505 54008 1742	1351	13065 53490 22030 5913
1193	07664 04436 70341 8728	1273	10482 84036 53655 3957	1353	13129 77965 97622 9726
1195	07736 79052 84156 4898	1275	10551 01847 69973 9754	1355	13193 92952 10424 5343
1197	07809 41504 06410 6668	1277	10619 08972 63415 2866	1357	13257 98476 59737 0691
1199	07881 91830 98848 6760	1279	10687 05444 78653 9226	1359	13321 94567 32494 3114
1201	07954 30074 02906 0489	1281	10754 91297 44686 3019	1361	13385 81252 03334 6909
1203	08026 56273 39844 7438	1283	10822 66563 74928 5036	1363	13449 58558 34673 5517
1205	08098 70469 10887 1889	1285	10890 31276 67313 3420	1365	13513 26513 76774 8420
1207	08170 72700 97349 2146	1287	10957 85469 04386 6846	1367	13576 85145 67822 2790
1209	08242 63008 60771 8862	1289	11025 29173 53403 0241	1369	13640 34481 33989 9936
1211	08314 41431 43052 2453	1291	11092 62422 66420 3088	1371	13703 74547 89512 6597
1213	08386 08008 66572 9742	1293	11159 85248 80394 0381	1373	13767 05372 36755 1114
1215	08457 62779 34330 9913	1295	11226 97684 17270 6323	1375	13830 26981 66281 4550
1217	08529 05782 30064 9888	1297	11293 99760 84080 0814	1377	13893 39402 56923 6777
1219	08600 37056 18381 9245	1299	11360 91510 73027 8800	1379	13956 42661 75849 7581
1221	08671 56639 44882 4749	1301	11427 72965 61586 2544	1381	14019 36785 78631 2844
1223	08742 64570 36285 4633	1303	11494 44157 12584 6916	1383	14082 21801 09310 5824
1225	08813 60887 00551 2710	1305	11561 05116 74299 7667	1385	14144 97734 00467 3586
1227	08884 45627 27004 2409	1307	11627 55875 80544 2978	1387	14207 64610 73284 8627
1229	08955 18828 86454 0856	1309	11693 96465 50755 8000	1389	14270 22457 37615 5730
1231	09025 80529 31316 3078	1311	11760 26916 90084 2777	1391	14332 71299 92046 4100
1233	09096 30765 95731 6432	1313	11826 47260 89479 3435	1393	14395 11164 23963 4808
1235	09166 69575 95684 5355	1315	11892 57528 25776 6738	1395	14457 42076 09616 3591
1237	09236 96996 29120 6536	1317	11958 57749 61783 8079	1397	14519 64061 14181 9050
1239	09307 13063 76063 4583	1319	12024 47955 46365 2965	1399	14581 77144 91827 6288
1241	09377 17814 98729 8296	1321	12090 28176 14527 2041	1401	14643 81352 85774 6000

Table 2.1: An excerpt of the logarithms published by Legendre [Legendre (1826), table V, page 260]. There are more than 20 errors in this table, mostly on the last digit. The most important errors are those for 1253 (5 units of error), 1303 (10 units, but possibly a typo), and 1401 (4 units of error). The values (and errors) are identical to those found in the *Observatoire* manuscript. Compare the values with those in table 2.2. Moreover, there are 114 differences (out of 120) between the two manuscripts in this section.

	Log.					Log.					Log.			
N	Obs	Ins	exact		N	Obs	Ins	exact		N	Obs	Ins	exact	
1163	4114	4110	4114		1243	7635	7630	7635		1323	9733	9728	9733	
1165	7769	7766	7770		1245	1459	1456	1459		1325	6552	6548	6552	
1167	1735	1732	1735		1247	6137	6131	6137		1327	5119	5115	5119	
1169	1107	1103	1107		1249	5120	5118	5120		1329	9975	9970	9975	
1171	1299	1296	1299		1251	9551	9548	9550		1331	1223	1218	1223	
1173	2447	2443	2447		1253	9998	9998	0003	×	1333	2061	2055	2061	
1175	0740	0738	0740		1255	9441	9433	9441		1335	0268	0265	0268	
1177	6816	6809	6816		1257	7472	7468	7471		1337	3683	3680	3683	
1179	1354	1350	1354		1259	5975	5969	5976		1339	9744	9740	9744	
1181	7991	7987	7991		1261	6210	6207	6210		1341	9129	9123	9128	
1183	3691	3687	3691		1263	7447	7444	7447		1343	3565	3561	3565	
1185	6701	6697	6701		1265	7244	7241	7244		1345	7849	7844	7849	
1187	2204	2201	2205		1267	3410	3405	3410		1347	6122	6119	6122	
1189	5818	5817	5818		1269	7763	7757	7763		1349	2476	2471	2476	
1191	5032	5032	5032	×	1271	1742	1741	1742		1351	5913	5909	5913	
1193	8728	8723	8728		1273	3957	3951	3957		1353	9726	9725	9726	
1195	4898	4895	4898		1275	9754	9750	9754		1355	5343	5339	5343	
1197	6668	6663	6668		1277	2866	2860	2866		1357	0691	0688	0691	
1199	6760	6754	6760		1279	9226	9222	9226		1359	3114	3115	3115	
1201	0489	0487	0489		1281	3019	3016	3019		1361	6909	6904	6910	
1203	7438	7435	7438		1283	5036	5030	5037		1363	5517	5513	5517	
1205	1889	1887	1889		1285	3420	3416	3419		1365	8420	8416	8420	
1207	2146	2141	2146		1287	6846	6841	6845		1367	2790	2784	2790	
1209	8862	8859	8862		1289	0241	0241	0242	×	1369	9936	9928	9936	
1211	2453	2447	2453		1291	3088	3082	3088		1371	6597	6592	6596	
1213	9742	9738	9742		1293	0381	0378	0380		1373	1114	1109	1114	
1215	9913	9913	9913	×	1295	6323	6318	6323		1375	4550	4550	4551	×
1217	9888	9884	9889		1297	0814	0813	0815		1377	6777	6769	6777	
1219	9245	9239	9245		1299	8800	8798	8800		1379	7581	7576	7581	
1221	4749	4743	4749		1301	2544	2541	2544		1381	2844	2824	2844	
1223	4633	4628	4633		1303	6916	6902	6906		1383	5824	5820	5824	
1225	2710	2709	2710		1305	7667	7661	7667		1385	3586	3583	3586	
1227	2409	2407	2409		1307	2978	2969	2979		1387	8627	8619	8626	
1229	0856	0853	0856		1309	8000	7995	8000		1389	5730	5726	5730	
1231	3078	3076	3078		1311	2777	2773	2777		1391	4100	4094	4100	
1233	6432	6426	6432		1313	3435	3432	3435		1393	4808	4804	4808	
1235	5355	5352	5355		1315	6738	6734	6738		1395	3591	3587	3590	
1237	6536	6532	6536		1317	8079	8075	8079		1397	9050	9045	9050	
1239	4583	4580	4583		1319	2965	2956	2965		1399	6288	6283	6287	
1241	8296	8289	8296		1321	2041	2031	2041		1401	6000	6000	6004	×

Table 2.2: A comparison of the last four digits of the numbers in table 2.1 in the two manuscripts with the exact values. On this interval, most of the values at the *Observatoire* are correct, whereas almost every value at the *Institut* is wrong. Crosses indicate the six places (out of 120) where the last four digits are identical in the two manuscripts.

$$\Delta\ \log n = \quad M\left[\frac{1}{n} - \frac{1}{2n^2} + \frac{1}{3n^3} - \frac{1}{4n^4} + \frac{1}{5n^5} - \frac{1}{6n^6} + \frac{1}{7n^7} - \cdots\right] \tag{2.15}$$

$$\Delta^2 \log n = -M\left[\frac{1}{n^2} - \frac{2}{n^3} + \frac{7}{2n^4} - \frac{6}{n^5} + \frac{31}{3n^6} - \frac{18}{n^7} + \cdots\right] \tag{2.16}$$

$$\Delta^3 \log n = \quad M\left[\frac{2}{n^3} - \frac{9}{n^4} + \frac{30}{n^5} - \frac{90}{n^6} + \frac{258}{n^7} - \cdots\right] \tag{2.17}$$

$$\Delta^4 \log n = -M\left[\frac{6}{n^4} - \frac{48}{n^5} + \frac{260}{n^6} - \frac{1200}{n^7} + \cdots\right] \tag{2.18}$$

$$\Delta^5 \log n = \quad M\left[\frac{24}{n^5} - \frac{300}{n^6} + \frac{2400}{n^7} - \cdots\right] \tag{2.19}$$

$$\Delta^6 \log n = -M\left[\frac{120}{n^6} - \frac{2160}{n^7} + \cdots\right] \tag{2.20}$$

In his introductory volume,[299] Prony uses these formulæ to obtain the values of the differences for $n = 10400$.

For the auxiliary tables, we have used the same formulæ, but only with the terms of degree lower or equal to 6, our purpose being to equate Prony's results as much as possible.

The original volumes contain 51 logarithms per page, and every fourth page begins (ideally) with an exact value (n, $\log n$, $\Delta \log n$, etc.), whereas all other values are interpolated. The (tabulated) value of $\Delta^i \log n$ is an integer whose unit is at a fixed position on the whole range 10000–200000, except for Δ^5, whose position changes after 40000, for no clear reason.

The formula given above for the interpolation error determines the positions of the differences. Prony wanted the error not to exceed half a unit of the 13th place and he computed the logarithms to 14 places, Δ^1 to 16 places, Δ^2 to 18 places, etc. Then, the maximum error on the final result (assuming at most one unit of error in the initial pivot for any difference, assuming that no rounding takes place, and that Δ^6 is constant) is

$$\mathcal{E} = 10^{-14} + 200 \cdot 10^{-16} + 19900 \cdot 10^{-18} + \cdots \approx 10^{-13}$$

but this does of course assume that all errors accumulate in the same direction, which is not the case. Similar reasonings were used for determining the positions of the differences in the other tables.

The previous value of $\mathcal{E}$ does of course not guarantee that the value of the interpolated logarithm is correct to 12 places, but only that the error on that

[299] Copy *O*, introductory volume, p. 20.

logarithm is smaller than about 10^{-13}. Prony's objective was perhaps only that one, and not the one where 12 correctly rounded places are provided, although both aims are met almost always.

It is also interesting to look at the initial errors on the pivots. The following table shows the approximation resulting for $\Delta^i \log 10000$ from ignoring orders beyond the 6th order, together with the positions of the units at the beginning of the 10000–200000 interval.

Level	Unit	Neglected amount
$\Delta \ \log n$	-16	$6.2 \cdot 10^{-30}$
$\Delta^2 \log n$	-18	$7.8 \cdot 10^{-28}$
$\Delta^3 \log n$	-20	$1.1 \cdot 10^{-26}$
$\Delta^4 \log n$	-22	$5.2 \cdot 10^{-26}$
$\Delta^5 \log n$	-23	$1.0 \cdot 10^{-25}$
$\Delta^6 \log n$	-25	$9.4 \cdot 10^{-26}$

In particular, using these approximations, the value of Δ^6 is systematically wrong by about one unit at the beginning of the 10000–200000 range, but this error will quickly decrease. In the original tables, it would certainly have been desirable to compute $\Delta^6 \log n$ up to $\frac{1}{n^7}$ at least on the 10000–20000 range. In that case, the neglected amount would have been at most about $\frac{23940}{n^8}$ which is approximately $2.4 \cdot 10^{-28}$ for $n = 10000$.

2.4 Sines

The table of sines comprises one large volume of 400 pages and gives the sines and the differences every 10000ths of a quadrant, that is, for $\alpha = k\Delta x$ ($0 \leq k \leq 10000$), with $\Delta x = \frac{\pi}{20000}$ rd. The aim was to give the sines to 22 places. In the tables, the centesimal division of the quadrant is used, the quadrant being taken as the unit, and the argument goes therefore from $0^{\mathrm{q}}.0001$ to $1^{\mathrm{q}}.0000$, with 51 values spanning two pages, one value being common between one page and the next one.

This is one of the tables which were printed, the others being the tables of logarithms of sines and tangents. Lefort reports having seen six partial copies of the table of sines, but only two almost complete copies have been located in the *Ponts et chaussées* library[300] as well as fragments at the *Archives*

[300]PC: Fol. 294. In addition, it was reported in 1858 that the *École nationale des ponts et chaussées* had several copies of Prony's table of sines and that the *Institut* didn't have this table [Avril (1858)]. This may look as a contradiction, but what has been meant by Avril was certainly that the *Institut* didn't have a printed copy, which is correct.

Nationales.[301] The printed fragments give the sines to 22 places and five orders of differences.

Values of the sines at the pivot points

Since Prony wanted to give the sines to 22 places, he had to compute the pivots more accurately.[302]

For the table of sines, the introductory volume states that the pivot points are all degrees of the quadrant.[303] First, the sines were computed every 10 (centesimal) degrees using

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \tag{2.21}$$

with $x = \frac{k}{10} \cdot \frac{\pi}{2}$ $(1 \leq k \leq 9)$ using the then (1794) most accurate known value of π computed by Thomas Fantet De Lagny in 1719 to 112 correct places.[304]

The value taken was

$$\frac{\pi}{2} \approx 1.57079\ 63267\ 94896\ 61923\ 13216\ 92 \tag{2.22}$$

the last digit being rounded, and its 26 first powers were computed in an auxiliary table, with 28 digits.[305] The last digits of that table were slightly wrong, for instance $\left(\frac{\pi}{2}\right)^{26}$ was given as 125636.78163 10555 79582 50193 85 instead of 125636.78163 10555 79582 50193 77.

Since $\frac{\left(\frac{\pi}{2}\right)^{26}}{26!} \approx 3.1 \cdot 10^{-22}$, and since the sines were computed with 25 exact digits, it was actually necessary to compute some powers after x^{26} to ensure this accuracy.

The same procedure was used to compute $\sin x$ for $x = \frac{k}{100} \cdot \frac{\pi}{2}$ rd, with $1 \leq k \leq 9$.

Finally, all other sines from $0^q.11$ to $0^q.99$ were computed with the formula

$$\sin(a + b) = 2 \cos a \sin b + \sin(a - b) \tag{2.23}$$

[301] A.N. $F^{17}1238$ and A.N. $F^{17}13571$.

[302] After Prony, and before the advent of electronic computers, it seems that only Edward Sang computed working tables of sines in centesimal argument with a greater number of decimals, namely 33 [Fletcher et al. (1962), p. 175], [Craik (2003)]. Some authors have computed sines with more decimals, but larger steps [Fletcher et al. (1962), p. 175].

[303] A detailed analysis shows that it was in fact slightly different: 0^q was taken as a pivot, then the pivots were all degrees from $0^q.04$ to $0^q.99$. It is not clear why the pivots in $0^q.01$, $0^q.02$, and $0^q.03$ were not used.

[304] [de Lagny (1719)]

[305] This is table 1 in the introductory volume. On the computation of powers of π, see Glaisher's article [Glaisher (1876)]. Glaisher was aware of Prony's table of powers of π, but had not seen it. A more comprehensive list of computations of π^n is given by Fletcher [Fletcher et al. (1962), p. 122].

For instance,

$$\begin{aligned}\sin 0^{\mathrm{q}}.11 &= 2\cos 0^{\mathrm{q}}.10 \sin 0^{\mathrm{q}}.01 + \sin 0^{\mathrm{q}}.09\\ &= 2\sin 0^{\mathrm{q}}.90 \sin 0^{\mathrm{q}}.01 + \sin 0^{\mathrm{q}}.09\end{aligned}$$

and all latter quantities are known.

The values of the sines were checked using the following formula from Euler:

$$\sin x + \sin(0^{\mathrm{q}}.4 - x) + \sin(0^{\mathrm{q}}.8 + x) = \sin(0^{\mathrm{q}}.4 + x) + \sin(0^{\mathrm{q}}.8 - x) \quad (2.24)$$

This formula is an immediate consequence of $\cos 0^{\mathrm{q}}.4 = \frac{1}{4}\left(1+\sqrt{5}\right)$ and $\cos 0^{\mathrm{q}}.8 = \frac{1}{4}\left(-1+\sqrt{5}\right)$.

The 100 values following each pivot point (for instance from $0^{\mathrm{q}}.6101$ to $0^{\mathrm{q}}.6200$) were computed by interpolation and the following pivot point was used to check the interpolation.[306]

Values of the differences at the pivot points

Legendre computed the values of $\Delta^n \sin 0^{\mathrm{q}}$ and $\Delta^n \sin 1^{\mathrm{q}}$ as follows.[307] Setting $p = 2\sin\frac{\Delta x}{2}$, we have

$$p^2 = 2(1 - \cos\Delta x) \quad (2.25)$$

and therefore

$$\begin{aligned}\Delta \sin x &= \sin(\Delta x + x) - \sin x && (2.26)\\ &= \sin\Delta x \cos x + \cos\Delta x \sin x - \sin x && (2.27)\\ &= \sin\Delta x\cos x - p^2\cdot\frac{1}{2}\sin x && (2.28)\end{aligned}$$

[306]The *Archives Nationales* hold a file containing the verification of a number of sines at intervals of $0^{\mathrm{q}}.001$ to 22 places. $\sin 0^{\mathrm{q}}.011$, for instance, was computed as $2\cos 0^{\mathrm{q}}.01 \times \sin 0^{\mathrm{q}}.001 + \sin 0^{\mathrm{q}}.009$.(A.N. F^{17}1244B, *dossier* 6)

[307]This method is also detailed by Lacroix and attributed to Legendre, but I do not know if there is a published source before 1800 [Lacroix (1800), pp. 51–53].

Moreover[308]

$$\Delta^2 \sin x = -p^2(\Delta \sin x + \sin x) \tag{2.29}$$

and in general

$$\Delta^n \sin x = -p^2(\Delta^{n-1} \sin x + \Delta^{n-2} \sin x) \tag{2.30}$$

So, if we know $\sin \Delta x$ and p^2, we can easily compute all the differences. These quantities are obtained as follows:

$$p^2 = 2(1 - \cos \Delta x) = 2\left[\frac{(\Delta x)^2}{2!} - \frac{(\Delta x)^4}{4!} + \frac{(\Delta x)^6}{6!} - \cdots\right] \tag{2.31}$$

and

$$\sin \Delta x = \Delta x - \frac{(\Delta x)^3}{3!} + \frac{(\Delta x)^5}{5!} - \cdots \tag{2.32}$$

In his introduction, Prony shows the computation of $\Delta^n \sin 0^{\mathrm{q}}.20$ for $n \leq 8$.

Another method[309] is to compute the differences of the sines in x using the differences in 0^{q} and 1^{q}:

$$\Delta^n \sin x = \cos x \cdot \Delta^n \sin 0^{\mathrm{q}} + \sin x \cdot \Delta^n \sin 1^{\mathrm{q}} \tag{2.33}$$

In our recomputed tables (auxiliary volume 9a), we have recomputed all the differences directly, using the formulæ above, and not solely in 0^{q} and 1^{q}.

Since Prony wanted the sines with 22 exact places using the computation of differences, he concluded that he needed Δ^1 to $22 + 3$ places, Δ^2 with $22 + 4$ places, etc., the number of added digits beeing the number of digits of n, $n \cdot \frac{n-1}{2}$, $n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3}$, etc., for $n = 100$.

Prony concluded that the accuracies should be the following:[310]

Level	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6	Δ^7	Δ^8
Unit	-25	-26	-28	-29	-30	-32	-33	-34

[308]In [de Borda and Delambre (1801), p. 48], Delambre observes that $\Delta^2 \sin(x - \Delta x) = -p^2 \sin x$. This formula is actually given in Prony's introduction, as $\Delta^2 \sin x = -p^2 \sin(x + \Delta x)$ (Copy *O*, p. 3). Delambre notes that such a linear relationship may have been used to construct Hindu sine tables with differences [Delambre (1807)]. See also [van Brummelen (2009), p. 115]. Briggs later used this relationship in the *Trigonometria britannica* [Briggs and Gellibrand (1633)]. In [Lagrange et al. (1801)], Delambre is mentioned as having found very simple formulæ for all orders, when Legendre obtained even more convenient formulæ, although they could have been deduced from those of Delambre.

[309]Copy *O*, introductory volume, p. 6.

[310]Copy *O*, introductory volume, p. 4.

The accuracy of the main range of the actual table is slightly different, in that Δ^7 is at position -34, and this is consistent with the table given in the introductory volume, containing the values of the sines and the differences for all the pivots with the following accuracy:

Level	$\sin x$	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6	Δ^7	Δ^8
Unit	-25	-25	-26	-28	-29	-30	-32	-34	-34

The same positions were given for the entire interval. These positions correspond to the main subrange of the tables.

2.5 Tangents

There are no tables of tangents in the *Tables du cadastre*, but Prony's introduction explains how they could be computed in order to obtain 21 exact decimals. By "exact decimals," Prony means that the 21st decimal is correctly rounded.

Prony divided the quadrant in three parts: from 0^q to $0^q.5000$, from $0^q.5000$ to $0^q.9400$, and from $0^q.9400$ to $1^q.0000$.

2.5.1 Computation of tangents on 0^q–$0^q.5000$

In this interval, the tangents would have been computed using differences. The pivots were from 100 to 100, that is $0^q.0000$, $0^q.0100$, $0^q.0200$, ..., $0^q.4900$. Eleven differences would be used at the beginning and twelve towards the end of the interval. These tangents would have 22 exact decimals.

At the pivots, the tangents would have been computed by dividing the sine by the cosine. Both values would be taken from the table of sines.

Computation of the differences

The differences $\Delta^n \tan x$ for $n \leq 13$ would be computed using Lagrange's formula. Prony gave most of the coefficients in terms of $f(x) = \tan x$, $f'(x)$, $f''(x)$, etc., as well as the values for the particular case $x = 0$. The numerical values $\Delta^n \tan 0$ were given for $n \leq 11$.

Prony considered the number of decimals necessary for the computation of $(\Delta x)^n$, of $\tan^n x$ (which is used in the computation of $f^{(n)}(x)$, of the coefficients in the developments of $\Delta^n \tan x$ in Lagrange's formula, and of the $\Delta^n \tan x$ themselves. He concluded that $\Delta^1 \tan x$ should be computed with 25 decimals, $\Delta^2 \tan x$ with 27 decimals, then 29, 30, 31, 33, 34, 35, 36, 37, 38, 39 and 40 decimals for $\Delta^{13} \tan x$.

Prony then computed $\Delta^n \tan 0^{\mathrm{q}}.50$.

2.5.2 Computation of tangents on $0^{\mathrm{q}}.5000$–$0^{\mathrm{q}}.9400$

In this interval, the tangents would have 21 exact decimals and would be computed with

$$\tan(0^{\mathrm{q}}.5000 + a) = 2\tan 2a + \tan(0^{\mathrm{q}}.5000 - a) \tag{2.34}$$

Obviously, the tangents between $0^{\mathrm{q}}.7500$ and $0^{\mathrm{q}}.9400$ would be computed using earlier values in the interval $0^{\mathrm{q}}.5000$–$0^{\mathrm{q}}.9400$ and this would cause a loss of accuracy. For instance, the last value of the interval is in fact

$$\begin{aligned}
\tan 0^{\mathrm{q}}.94 &= 2\tan 0^{\mathrm{q}}.88 + \tan 0^{\mathrm{q}}.06 \\
&= 2(2\tan 0^{\mathrm{q}}.76 + \tan 0^{\mathrm{q}}.12) + \tan 0^{\mathrm{q}}.06 \\
&= 2(2(2\tan 0^{\mathrm{q}}.52 + \tan 0^{\mathrm{q}}.24) + \tan 0^{\mathrm{q}}.12) + \tan 0^{\mathrm{q}}.06 \\
&= 8\tan 0^{\mathrm{q}}.52 + 4\tan 0^{\mathrm{q}}.24 + 2\tan 0^{\mathrm{q}}.12 + \tan 0^{\mathrm{q}}.06 \\
&= 8(2\tan 0^{\mathrm{q}}.04 + \tan 0^{\mathrm{q}}.48) + 4\tan 0^{\mathrm{q}}.24 + 2\tan 0^{\mathrm{q}}.12 + \tan 0^{\mathrm{q}}.06 \\
&= 16\tan 0^{\mathrm{q}}.04 + 8\tan 0^{\mathrm{q}}.48 + 4\tan 0^{\mathrm{q}}.24 + 2\tan 0^{\mathrm{q}}.12 + \tan 0^{\mathrm{q}}.06
\end{aligned}$$

2.5.3 Computation of tangents on $0^{\mathrm{q}}.9400$–$1^{\mathrm{q}}.0000$

In this interval, the tangents would have 22 exact decimals and would be computed with $\tan x = \cot(1^{\mathrm{q}} - x)$ and (x being in radians)

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{3^2\cdot 5} - \frac{2x^5}{3^3\cdot 5\cdot 7} - \frac{x^7}{3^3\cdot 5^2\cdot 7} - \frac{2x^9}{3^5\cdot 5\cdot 7\cdot 11} - \frac{1382x^{11}}{3^6\cdot 5^3\cdot 7^2\cdot 11\cdot 13} - \cdots \tag{2.35}$$

In each pivot, $\frac{1}{x}$ is computed by division and the remaining part of the series by differences.

The formula tabulated would actually be $Z = \frac{R}{x} - \cot\frac{x}{R}$ with $R = \frac{1}{\Delta x} = \frac{20000}{\pi}$ and $x = 0$ up to 600. We have:

$$Z = \frac{x}{3R} + \frac{x^3}{3^2\cdot 5\cdot R^3} + \frac{2x^5}{3^3\cdot 5\cdot 7\cdot R^5} + \cdots \tag{2.36}$$

If the expansion of $\cot x$ is taken until x^{13}, the last neglected term is at most about $9\cdot 10^{-24}$.

Z would be obtained with 22 exact decimals for $x = 600$.

Then, $\cot\frac{x}{R}$ would be computed by subtracting the values of Z from the corresponding values $\frac{1}{\Delta x}$, $\frac{1}{2\Delta x}$, $\frac{1}{3\Delta x}$, etc.

Other computation methods

Prony did not use the formula (2.34), because, as shown above, the last values would accumulate errors and only contain 17 or 18 exact decimals. He gave the following example:

$$
\begin{aligned}
\tan 0^{\mathrm{q}}.9999 &= 2\tan 0^{\mathrm{q}}.9998 + \tan 0^{\mathrm{q}}.0001 \\
&= \ldots \\
&= 4096\tan(1^{\mathrm{q}} - 0^{\mathrm{q}}.4096) + 2048\tan 0^{\mathrm{q}}.2048 + 1024\tan 0^{\mathrm{q}}.1024 + \cdots \\
&\qquad + 4\tan 0^{\mathrm{q}}.0004 + 2\tan 0^{\mathrm{q}}.0002 + \tan 0^{\mathrm{q}}.0001 \\
&= 8192\tan 0^{\mathrm{q}}.1808 + 4096\tan 0^{\mathrm{q}}.4096 + 2048\tan 0^{\mathrm{q}}.2048 + \cdots + \tan 0^{\mathrm{q}}.0001
\end{aligned}
$$

Therefore, if the error in the first part is such that 22 decimals are exact, then the error can be multiplied by as much as 16383, and there could be a loss of 4 to 5 decimals. This of course assumes that the errors are in each case the largest ones and with the same sign, which is unlikely. The practical accuracy would actually have been better, but compounded with a constant uncertainty.

Prony also rejected the computation by division of the sine through the cosine which leads to a similar error.[311] Indeed, the worst case error for the division is that of $\tan 0^{\mathrm{q}}.9999$ when $\cos 0^{\mathrm{q}}.9999$ is in default by $\delta = 5 \cdot 10^{-23}$, assuming $\cos 0^{\mathrm{q}}.9999$ to be correct with 22 decimals.

$$
\begin{aligned}
\frac{\sin 0^{\mathrm{q}}.9999 + \delta}{\cos 0^{\mathrm{q}}.9999 - \delta} - \frac{\sin 0^{\mathrm{q}}.9999}{\cos 0^{\mathrm{q}}.9999} &\approx \frac{\sin 0^{\mathrm{q}}.9999}{\cos 0^{\mathrm{q}}.9999}\left(\frac{1 + \frac{\delta}{\sin 0^{\mathrm{q}}.9999}}{1 - \frac{\delta}{\cos 0^{\mathrm{q}}.9999}} - 1\right) \\
&\approx 6366.2\frac{\delta}{\cos 0^{\mathrm{q}}.9999} \\
&\approx 2 \cdot 10^{-15}
\end{aligned}
$$

Hence, in this case, only 14 decimals would be correct.

The limit $0^{\mathrm{q}}.9400$ was chosen because it is about at this position that one digit is lost. Indeed, from $0^{\mathrm{q}}.5000$ to $0^{\mathrm{q}}.9375$, at most four tangents from the first part are added, with coefficients totalling at most 15. The 21st decimal would therefore not be wrong by more than a unit. Prony rounded this limit

[311] Perhaps the first who quantified the errors arising from the division was Adrianus Romanus (1561–1615), at the time of the publication of Rheticus' *Opus palatinum* [Roegel (2010h)]. He gave precise rules for the number of extra decimals required to obtain a result with a certain accuracy [Bockstaele (1992)].

to $0^q.9400$ for practical reasons. Then, it was only necessary to compute 600 tangents with formula (2.35).

In a footnote, Prony stated that this method was tested by computing the first ten pivots, the tangents for each $x = \frac{k}{100}$ for $0 \le k \le 100$ and the data necessary for the computation of the last 600 tangents.

2.6 Logarithms of the sines

The tables of the logarithms of the sines comprise four volumes giving the sine of every angle $k\frac{\pi}{200000}$, for $0 \le k \le 100000$. Prony wanted to give the logarithms of the sines exact to 12 places.[312] The first volume contains the values of $\log \frac{x}{\sin x}$ for arcs from 0 to $0^q.05$ (5000 values on 100 pages), the logarithms of the sines for these 5000 values (50 pages), and the logarithms of the sines of the arcs from $0^q.05$ to $0^q.25$ with seven orders of differences (400 pages). The three remaining volumes contain the logarithms of the sines from $0^q.25$ to $0^q.50$ (500 pages), from $0^q.50$ to $0^q.75$ (500 pages), and from $0^q.75$ to $1^q.00$ (500 pages). The structure is the same in both sets of tables, except that two volumes of the *Observatoire* set have been swapped when they were bound and their spines should be exchanged.

Logarithms of the arc to sine ratios

Setting $a = \frac{\pi}{200000}$, the following function was tabulated

$$\mathcal{A}(x) = \log x - \log\sin(ax) \tag{2.37}$$

$$= \log\frac{1}{a} + \frac{Ma^2}{2\cdot 3}x^2 + \frac{Ma^4}{2^2\cdot 3^2\cdot 5}x^4 + \frac{Ma^6}{3^4\cdot 5\cdot 7}x^6 + \frac{Ma^8}{2^3\cdot 3^3\cdot 5^2\cdot 7}x^8 + \cdots \tag{2.38}$$

According to the introductory volume, the exact value of $\mathcal{A}$ was only computed for $x = 0$. All other values were obtained by interpolation, because $\mathcal{A}$ is almost constant over the interval $0^q.00$–$0^q.05$. But the actual computations display three interpolations on this interval, hence three pivots.

The differences $\Delta^n\mathcal{A}$ were obtained as follows, using Lagrange's formula:

[312] After Prony, and before the advent of electronic computers, it seems that only Edward Sang computed working tables of logarithms of sines in centesimal argument with a greater number of decimals, namely 15 [Fletcher et al. (1962), p. 199], [Craik (2003)]. Some authors have computed logarithms of sines with more decimals, but larger steps. The logarithms of the ratios arcs to sines do not seem to have been recomputed (before electronic computers) to a greater accuracy than Prony's with a similar step and range [Fletcher et al. (1962), p. 203].

$$\Delta\ \mathcal{A}(x) = \mathcal{A}^{(1)}(x)\Delta x + \mathcal{A}^{(2)}(x)(\Delta x)^2 \times \frac{F(1,2)}{2!} + \mathcal{A}^{(3)}(x)(\Delta x)^3 \times \frac{F(1,3)}{3!} + \cdots \tag{2.39}$$

$$\Delta^2\mathcal{A}(x) = \mathcal{A}^{(2)}(x)(\Delta x)^2 + \mathcal{A}^{(3)}(x)(\Delta x)^3 \times \frac{F(2,3)}{3!} + \mathcal{A}^{(4)}(x)(\Delta x)^4 \times \frac{F(2,4)}{4!} + \cdots \tag{2.40}$$

$$\cdots$$

$$\Delta^6\mathcal{A}(x) = \mathcal{A}^{(6)}(x)(\Delta x)^6 + \cdots \tag{2.41}$$

$$\cdots$$

In our case, $\Delta x = 1$, and therefore we have the simpler formulæ:

$$\Delta\ \mathcal{A}(x) = \mathcal{A}^{(1)}(x) + \mathcal{A}^{(2)}(x) \times \frac{F(1,2)}{2!} + \mathcal{A}^{(3)}(x) \times \frac{F(1,3)}{3!} + \cdots \tag{2.42}$$

$$\Delta^2\mathcal{A}(x) = \mathcal{A}^{(2)}(x) + \mathcal{A}^{(3)}(x) \times \frac{F(2,3)}{3!} + \mathcal{A}^{(4)}(x) \times \frac{F(2,4)}{4!} + \cdots \tag{2.43}$$

$$\cdots$$

$$\Delta^6\mathcal{A}(x) = \mathcal{A}^{(6)}(x) + \cdots \tag{2.44}$$

$$\cdots$$

In the original tables, these $\Delta^n\mathcal{A}(x)$ have only been computed for $x = 0$ and two other pivots, and the other values were obtained by interpolation. Only the terms up to x^6 have been used in $\mathcal{A}(x)$, and we have followed this limit in our reconstructions.

In the recomputed tables of the exact values, we have used the previous formulæ for all values $x \leq 5000$.

These tables span 100 pages with six orders of differences.

The accuracy of these tables is indicated by the following table, where the positions of the units at the beginning of the intervals are given:

Level	Unit	First neglected term for $x = 5000$
$\Delta\ \mathcal{A}(x)$	-18	$\mathcal{A}^{(7)}(x) \times \frac{F(1,7)}{7!} = Ma^8 \cdot \frac{200}{189} \cdot F(1,7) \approx 1.7 \cdot 10^{-39}$
$\Delta^2\mathcal{A}(x)$	-21	$\mathcal{A}^{(7)}(x) \times \frac{F(2,7)}{7!} = Ma^8 \cdot \frac{200}{189} \cdot F(2,7) \approx 2.1 \cdot 10^{-37}$
$\Delta^3\mathcal{A}(x)$	-24	$\mathcal{A}^{(7)}(x) \times \frac{F(3,7)}{7!} = Ma^8 \cdot \frac{200}{189} \cdot F(3,7) \approx 3.1 \cdot 10^{-36}$
$\Delta^4\mathcal{A}(x)$	-26	$\mathcal{A}^{(7)}(x) \times \frac{F(4,7)}{7!} = Ma^8 \cdot \frac{200}{189} \cdot F(4,7) \approx 1.4 \cdot 10^{-35}$
$\Delta^5\mathcal{A}(x)$	-31	$\mathcal{A}^{(7)}(x) \times \frac{F(5,7)}{7!} = Ma^8 \cdot \frac{200}{189} \cdot F(5,7) \approx 2.9 \cdot 10^{-35}$
$\Delta^6\mathcal{A}(x)$	-31	$\mathcal{A}^{(7)}(x) \times \frac{F(6,7)}{7!} = Ma^8 \cdot \frac{200}{189} \cdot F(6,7) \approx 2.6 \cdot 10^{-35}$

Logarithms of the sines over $0^{\mathrm{q}}.0$–$0^{\mathrm{q}}.05$

The previous table was then used to compute the logarithms of the sines over the same interval, since

$$\log\sin(ax) = \log x - \mathcal{A}(x) \tag{2.45}$$

For instance,[313]

$$\begin{aligned}
\log\sin 0^{\mathrm{q}}.01234 &= \log 1234 - \mathcal{A}(1234) &(2.46)\\
&= 3.09131\ 51596\ 972 - 4.80390\ 73191\ 9160 &(2.47)\\
&= 8.28740\ 78405\ 056 - 10 &(2.48)\\
&= \overline{2}.28740\ 78405\ 056 &(2.49)
\end{aligned}$$

and this value $\overline{2}.28740\ 78405\ 056$ is given in the second table.

This second table spans over 50 pages and gives on every page the values of x, $\log x$ and $\log\sin(ax)$ for 100 values of x.

In our recomputed tables, the values of the logarithms of the sines were computed directly.

Logarithms of the sines over $0^{\mathrm{q}}.05$–$1^{\mathrm{q}}.00$

Prony introduced pivot points by steps of $0^{\mathrm{q}}.002$ from $0^{\mathrm{q}}.05$ to $0^{\mathrm{q}}.5$ and by steps of $0^{\mathrm{q}}.01$ from $0^{\mathrm{q}}.5$ to $1^{\mathrm{q}}.0$. There are therefore 276 pivot points from $0^{\mathrm{q}}.05$ to $1^{\mathrm{q}}.0$.[314]

In each of these pivot points, the logarithms of the sines were computed by taking a 15 digits approximation of the sines, extracted from the table of sines. The 15 digits number N was decomposed as a sum of a fraction

[313] Like Prony, we use the notation $\overline{a}.b$ for $-a + 0.b$, which should not be confused with $-a.b$. The decimal part is consequently always positive.

[314] These pivots are given in the introductory volume (table 6), distinguishing the two ranges $0^{\mathrm{q}}.05$–$0^{\mathrm{q}}.50$ and $0^{\mathrm{q}}.50$–$1^{\mathrm{q}}.00$.

$\frac{p}{q}$ where $\log p$ and $\log q$ were known, and where $x = N - \frac{p}{q} \ll N$. x can be positive or negative. Then, using equation 2.7 with $y = \frac{x}{2\frac{p}{q}+x}$, we have $\sqrt{\frac{1+y}{1-y}} = \sqrt{\frac{Nq}{p}}$, therefore

$$\log\sqrt{\frac{Nq}{p}} = M\left[\frac{x}{2\frac{p}{q}+x} + \frac{1}{3}\left(\frac{x}{2\frac{p}{q}+x}\right)^3 + \cdots\right] \tag{2.50}$$

and hence $\log N$ was computed with

$$\log N = \log p - \log q + 2M\left[\frac{x}{2\frac{p}{q}+x} + \frac{1}{3}\left(\frac{x}{2\frac{p}{q}+x}\right)^3 + \cdots\right] \tag{2.51}$$

The values of $\Delta^n \log\sin x$ were computed for each of the pivot points, and values in between were interpolated. Lagrange's formulæ were used for $\Delta^n \log\sin x$. Setting $f(x) = \log\sin x$ and $q = \cot x$, we find easily

$$\begin{aligned}
f^{(1)}(x) &= \quad Mq \\
f^{(2)}(x) &= -M\,(1+q^2) \\
f^{(3)}(x) &= \quad 2M\,(q+q^3) \\
f^{(4)}(x) &= -2M\,(1+4q^2+3q^4) \\
f^{(5)}(x) &= \quad 2M\,(8q+20q^3+12q^5) \\
f^{(6)}(x) &= -2M\,(8+68q^2+120q^4+60q^6) \\
f^{(7)}(x) &= \quad 2M\,(136q+616q^3+840q^5+360q^7) \\
f^{(8)}(x) &= -2M\,(136+1984q^2+6048q^4+6720q^6+2520q^8)
\end{aligned} \tag{2.52}$$

with which the values of $\Delta^n \log\sin x$ can be computed. As an illustration, Prony showed the computation of $\Delta^n \log\sin 0^{\mathrm{q}}.052$ for $n \le 7$.[315]

As a consequence of the choice of pivots, we have interpolated intervals of 200 (4 pages) and 1000 values (20 pages).

In the recomputed exact values, the above formulæ were used for all values of x from $0^{\mathrm{q}}.05$ to $1^{\mathrm{q}}.00$ (1900 pages). The computations were done using $f^{(i)}$ with $i < 8$, in order to be as faithful as possible to the original computations.[316]

The accuracy of these tables is indicated by the following table, where $\Delta x = \frac{\pi}{200000}$, and where the third column gives the absolute value of the first neglected term:

[315] Copy *O*, introductory volume, p. 12.

[316] It is possible that $f^{(8)}$ was used in $\Delta^7 \log\sin x$, but a further investigation is required to ascertain it.

Level	Unit	First neglected term for $x = 5000\Delta x$
$\Delta\ \log\sin x$	-16	$f^{(8)}(x)\times(\Delta x)^8\times\frac{F(1,8)}{8!}\approx 1.4\cdot 10^{-31}$
$\Delta^2\log\sin x$	-18	$f^{(8)}(x)\times(\Delta x)^8\times\frac{F(2,8)}{8!}\approx 3.5\cdot 10^{-29}$
$\Delta^3\log\sin x$	-20	$f^{(8)}(x)\times(\Delta x)^8\times\frac{F(3,8)}{8!}\approx 8.1\cdot 10^{-28}$
$\Delta^4\log\sin x$	-22	$f^{(8)}(x)\times(\Delta x)^8\times\frac{F(4,8)}{8!}\approx 5.7\cdot 10^{-27}$
$\Delta^5\log\sin x$	-23	$f^{(8)}(x)\times(\Delta x)^8\times\frac{F(5,8)}{8!}\approx 1.8\cdot 10^{-26}$
$\Delta^6\log\sin x$	-25	$f^{(8)}(x)\times(\Delta x)^8\times\frac{F(6,8)}{8!}\approx 2.7\cdot 10^{-26}$
$\Delta^7\log\sin x$	-25	$f^{(8)}(x)\times(\Delta x)^8\times\frac{F(7,8)}{8!}\approx 2.0\cdot 10^{-26}$

2.7 Logarithms of the tangents

The tables of the logarithms of the tangents also comprise four volumes giving the tangent of every angle $k\frac{\pi}{200000}$, for $0 \le k \le 100000$. Prony wanted to give the logarithms of the tangents exact to 12 places.[317] The first volume of the *Observatoire* set contains the values of $\log\frac{x}{\tan x}$ for arcs from 0 to $0^{\mathrm{q}}.05$ (5000 values on 100 pages), the logarithms of the tangents and cotangents for these 5000 arcs (100 pages) and the logarithms of the tangents of the arcs from $0^{\mathrm{q}}.05$ to $0^{\mathrm{q}}.2$ with seven orders of differences (300 pages). The three remaining volumes contain the logarithms of the tangents from $0^{\mathrm{q}}.20$ to $0^{\mathrm{q}}.45$ (500 pages), from $0^{\mathrm{q}}.45$ to $0^{\mathrm{q}}.70$ (500 pages), and from $0^{\mathrm{q}}.70$ to $0^{\mathrm{q}}.95$ (500 pages).[318] The logarithms of the tangents from $0^{\mathrm{q}}.95$ to $1^{\mathrm{q}}.00$ are included in the first volume, since they are opposite to the logarithms of the cotangents of the complementary angles. The set at the *Institut* is similar, but with volume 14 covering the arcs $0^{\mathrm{q}}.05$ to $0^{\mathrm{q}}.25$ (400 pages), volume 15 covering the arcs $0^{\mathrm{q}}.25$ to $0^{\mathrm{q}}.50$ (500 pages), volume 16 covering the arcs $0^{\mathrm{q}}.50$ to $0^{\mathrm{q}}.75$ (500 pages), and volume 17 covering the arcs $0^{\mathrm{q}}.75$ to $0^{\mathrm{q}}.95$ (400 pages).[319]

[317] After Prony, and before the advent of electronic computers, it seems that only Edward Sang computed working tables of logarithms of tangents in centesimal argument with a greater number of decimals, namely 15 [Fletcher et al. (1962), p. 199], [Craik (2003)]. Some authors have computed logarithms of tangents with more decimals, but larger steps. The logarithms of the ratios arcs to tangents do not seem to have been recomputed (before electronic computers) to a greater accuracy than Prony's with a similar step and range [Fletcher et al. (1962), p. 203].

[318] Our reconstruction follows the divisions of the set at the *Observatoire*, but it can easily be used to check the manuscripts at the *Institut*.

[319] The table of logarithms of tangents was at least partially printed, and the *Archives Nationales* hold the $0^{\mathrm{q}}.06400$—$0^{\mathrm{q}}.06600$ and $0^{\mathrm{q}}.14600$—$0^{\mathrm{q}}.14800$ excerpts.(A.N. F^{17}13571) But the printed excerpts contain many typographical errors and they are obviously only proofs.

Logarithms of the arc to tangent ratios

Like with the logarithms of the sines, and setting $a = \frac{\pi}{200000}$, the following function was tabulated

$$\mathcal{A}'(x) = \log x - \log\tan(ax) \tag{2.53}$$
$$= \log\frac{1}{a} - \frac{Ma^2}{3}x^2 - \frac{7Ma^4}{2\cdot 3^2\cdot 5}x^4 - \frac{62Ma^6}{3^4\cdot 5\cdot 7}x^6 - \frac{127Ma^8}{2^2\cdot 3^3\cdot 5^2\cdot 7}x^8 - \cdots \tag{2.54}$$

This function can be derived from previous calculations, since $\mathcal{A}' = \mathcal{A} + \log\cos(ax)$. Moreover, like for the sines, the exact value of $\mathcal{A}'$ was in principle only needed for $x = 0$, and in this case $\mathcal{A}' = \mathcal{A}$. But in fact, four different pivots were used, contrary to the statements in Prony's introductory volume. All other values were obtained by interpolation, because $\mathcal{A}'$ is almost constant over the interval $0^{\mathrm{q}}.00$–$0^{\mathrm{q}}.05$.

The differences $\Delta^n\mathcal{A}'$ could have been obtained using Lagrange's formula, but they can also be obtained from earlier calculations since

$$\Delta^n\mathcal{A}' = \Delta^n\mathcal{A} + \Delta^n\log\cos(ax) = \Delta^n\mathcal{A} + \Delta^n\log\sin\left(\frac{\pi}{2} - ax\right) \tag{2.55}$$

In the original tables, these $\Delta^n\mathcal{A}'(x)$ have been computed for $x = 0$ and three other values, and the other values were obtained by interpolation. Only the terms up to x^6 have been used in $\mathcal{A}'(x)$, and we have taken the same limit in our reconstructions.

In the recomputed tables of the exact values, we have used Lagrange's formulæ for all values of $x \le 5000$.

These tables span 100 pages with six orders of differences.

The accuracy of these tables is indicated by the following table:

Level	Unit	First neglected term for $x = 5000$
$\Delta\ \mathcal{A}'(x)$	-18	$\mathcal{A}'^{(7)}(x)\times\frac{F(1,7)}{7!} = Ma^8\cdot\frac{50800}{189}\cdot F(1,7)\approx 4.3\cdot 10^{-37}$
$\Delta^2\mathcal{A}'(x)$	-21	$\mathcal{A}'^{(7)}(x)\times\frac{F(2,7)}{7!} = Ma^8\cdot\frac{50800}{189}\cdot F(2,7)\approx 5.5\cdot 10^{-35}$
$\Delta^3\mathcal{A}'(x)$	-24	$\mathcal{A}'^{(7)}(x)\times\frac{F(3,7)}{7!} = Ma^8\cdot\frac{50800}{189}\cdot F(3,7)\approx 7.8\cdot 10^{-34}$
$\Delta^4\mathcal{A}'(x)$	-26	$\mathcal{A}'^{(7)}(x)\times\frac{F(4,7)}{7!} = Ma^8\cdot\frac{50800}{189}\cdot F(4,7)\approx 3.6\cdot 10^{-33}$
$\Delta^5\mathcal{A}'(x)$	-31	$\mathcal{A}'^{(7)}(x)\times\frac{F(5,7)}{7!} = Ma^8\cdot\frac{50800}{189}\cdot F(5,7)\approx 7.3\cdot 10^{-33}$
$\Delta^6\mathcal{A}'(x)$	-33	$\mathcal{A}'^{(7)}(x)\times\frac{F(6,7)}{7!} = Ma^8\cdot\frac{50800}{189}\cdot F(6,7)\approx 6.5\cdot 10^{-33}$

Logarithms of the tangents and cotangents over $0^{\mathrm{q}}.0$–$0^{\mathrm{q}}.05$

The previous table was then used to compute the logarithms of the tangents and cotangents over the same interval, since

$$\log\tan(ax) = \log x - \mathcal{A}'(x) \tag{2.56}$$
$$\log\cot(ax) = \mathcal{A}'(x) - \log x \tag{2.57}$$

For instance,

$$\begin{aligned}\log\tan 0^{\mathrm{q}}.01234 &= \log 1234 - \mathcal{A}'(1234) &(2.58)\\ &= 3.09131\ 51596\ 97222... - 4.80382\ 57264\ 38588... &(2.59)\\ &= 8.28748\ 94332\ 5863... - 10 &(2.60)\\ &= \overline{2}.28748\ 94332\ 5863... &(2.61)\end{aligned}$$

and the value $\overline{2}.28748\ 94332\ 5863$ is given in the second table.

Likewise,

$$\begin{aligned}\log\tan 0^{\mathrm{q}}.98766 &= \log\cot(1 - 0^{\mathrm{q}}.98766) = \log\cot 0^{\mathrm{q}}.01234 &(2.62)\\ &= \mathcal{A}'(1234) - \log 1234 &(2.63)\\ &= 4.80382\ 57264\ 38588... - 3.09131\ 51596\ 97222... &(2.64)\\ &= 1.71251\ 05667\ 41366... &(2.65)\end{aligned}$$

and the value 1.71251 05667 4137 is given in the second table.

This second table spans over 100 pages and gives on every page the values of x, $\log x$ and $\log\tan(ax)$ and $\log\cot(ax)$ for 51 values of x, one value being common between one page and the next one.

In the recomputed tables, the values of the logarithms of the tangents and cotangents were computed directly.

Logarithms of the tangents over $0^{\mathrm{q}}.05$–$0^{\mathrm{q}}.95$

For the logarithms of the sines, the pivot points were divided into two groups. In the case of the logarithms of the tangents, things are somewhat simpler and the pivot points were all values $k \times 0^{\mathrm{q}}.002$, for $25 \le k < 475$, that is 0.05, 0.052, 0.054, ..., 0.948. For the interpolation, one interval spanned 200 values, or four pages.

For each pivot point x in the interval $0^{\mathrm{q}}.05$–$0^{\mathrm{q}}.95$, the value of $\log\tan x$ was computed with

$$\log\tan x = \log\sin x - \log\cos x \tag{2.66}$$

As a consequence of the different set of pivot points, almost all of the values of $\log\tan x$ were computed using interpolated values of $\log\sin x$. For instance, $\log\tan 0^{\mathrm{q}}.52 = \log\sin 0^{\mathrm{q}}.52 - \log\sin 0^{\mathrm{q}}.48$ can be computed exactly, but $\log\tan 0^{\mathrm{q}}.502 = \log\sin 0^{\mathrm{q}}.502 - \log\sin 0^{\mathrm{q}}.498$ uses an interpolated value for $\log\sin 0^{\mathrm{q}}.502$.

The differences for the pivot points are easily computed with[320]

$$\Delta^n \log\tan x = \Delta^n \log\sin x - \Delta^n \log\cos x \tag{2.67}$$

In particular, the first difference is

$$\begin{aligned}
\Delta \log\tan x &= \log\sin(x+\Delta x) - \log\sin x - \big[\log\cos(x+\Delta x) - \log\cos x\big] && (2.68)\\
&= \Delta\log\sin x + \log\sin\left(\frac{\pi}{2} - x\right) - \log\sin\left(\frac{\pi}{2} - (x+\Delta x)\right) && (2.69)\\
&= \Delta\log\sin x + \Delta\log\sin\left(\frac{\pi}{2} - x - \Delta x\right) && (2.70)
\end{aligned}$$

The second difference is

$$\begin{aligned}
\Delta^2 \log\tan x &= \Delta\log\tan(x+\Delta x) - \Delta\log\tan x && (2.71)\\
&= \Delta\log\sin(x+\Delta x) + \Delta\log\sin\left(\frac{\pi}{2} - x - 2\Delta x\right)\\
&\quad - \left[\Delta\log\sin x + \Delta\log\sin\left(\frac{\pi}{2} - x - \Delta x\right)\right] && (2.72)\\
&= \Delta^2\log\sin x - \left[\Delta\log\sin\left(\frac{\pi}{2} - x - \Delta x\right) - \Delta\log\sin\left(\frac{\pi}{2} - x - 2\Delta x\right)\right] && (2.73)\\
&= \Delta^2\log\sin x - \Delta^2\log\sin\left(\frac{\pi}{2} - x - 2\Delta x\right) && (2.74)
\end{aligned}$$

And in general, we have

$$\Delta^n \log\tan x = \Delta^n\log\sin x + (-1)^{n+1}\Delta^n\log\sin\left(\frac{\pi}{2} - x - n\Delta x\right) \tag{2.75}$$

In addition, a very useful property is:

$$\Delta^n \log\tan(0^{\mathrm{q}}.5 + x) = (-1)^{n+1}\Delta^n\log\tan(0^{\mathrm{q}}.5 - n\Delta x - x) \tag{2.76}$$

[320] Copy *O*, introductory volume, p. 15.

and we have in particular the well known

$$\log\tan(0^{\mathrm{q}}.5 + x) = -\log\tan(0^{\mathrm{q}}.5 - x) \tag{2.77}$$

In the original tables, the interpolated values of the differences from the log sin table were therefore also used. As an illustration of the calculation, Prony gives the example of $\Delta^n \log\tan 0^{\mathrm{q}}.052$ computed using formula (2.75).[321]

This formula can be used for any value of x. It may be used to compute the pivot $0^{\mathrm{q}}.502$:

$$\begin{aligned}
\Delta^1 \log\tan 0^{\mathrm{q}}.502 &= \Delta^1 \log\sin 0^{\mathrm{q}}.502 + \Delta^1 \log\sin(0^{\mathrm{q}}.498 - 0^{\mathrm{q}}.00001)\\
&= \Delta^1 \log\sin 0^{\mathrm{q}}.502 + \Delta^1 \log\sin 0^{\mathrm{q}}.49799\\
\Delta^2 \log\tan 0^{\mathrm{q}}.502 &= \Delta^2 \log\sin 0^{\mathrm{q}}.502 - \Delta^2 \log\sin 0^{\mathrm{q}}.49798\\
\Delta^3 \log\tan 0^{\mathrm{q}}.502 &= \Delta^3 \log\sin 0^{\mathrm{q}}.502 + \Delta^3 \log\sin 0^{\mathrm{q}}.49797\\
\Delta^4 \log\tan 0^{\mathrm{q}}.502 &= \Delta^4 \log\sin 0^{\mathrm{q}}.502 - \Delta^4 \log\sin 0^{\mathrm{q}}.49796\\
\Delta^5 \log\tan 0^{\mathrm{q}}.502 &= \Delta^5 \log\sin 0^{\mathrm{q}}.502 + \Delta^5 \log\sin 0^{\mathrm{q}}.49795\\
\Delta^6 \log\tan 0^{\mathrm{q}}.502 &= \Delta^6 \log\sin 0^{\mathrm{q}}.502 - \Delta^6 \log\sin 0^{\mathrm{q}}.49794\\
\Delta^7 \log\tan 0^{\mathrm{q}}.502 &= \Delta^7 \log\sin 0^{\mathrm{q}}.502 + \Delta^7 \log\sin 0^{\mathrm{q}}.49793
\end{aligned}$$

We have however checked these equations for several angles, namely $x = 0^{\mathrm{q}}.2$, $0^{\mathrm{q}}.502$, $0^{\mathrm{q}}.7$, and equation (2.75) was *exactly* satisfied only for $x = 0^{\mathrm{q}}.2$. In the two other cases, there were slight differences, often only of a unit in the last decimal place. But if formula (2.75) was used, there should have been no differences at all.

It therefore seems, but it remains to be checked, that formula (2.75) was only used for $x \leq 0^{\mathrm{q}}.5$, that is, for 225 pivots.

For the remaining pivots, Prony very likely used formulæ (2.76) and (2.77), no addition or subtraction being then necessary. The use of these formulæ is not mentioned in Prony's introduction. Whether they have been used or not is very easy to check.

The differences for $\log\cos x$ could also have been computed directly using Lagrange's formula. In that case, with $f(x) = \log\cos x$ and $q = \tan x$, we

[321] Copy *O*, introductory volume, pp. 15–16.

have

$$\begin{aligned}
f^{(1)}(x) &= -Mq \\
f^{(2)}(x) &= -M(1+q^2) \\
f^{(3)}(x) &= -2M(q+q^3) \\
f^{(4)}(x) &= -2M(1+4q^2+3q^4) \\
f^{(5)}(x) &= -2M(8q+20q^3+12q^5) \\
f^{(6)}(x) &= -2M(8+68q^2+120q^4+60q^6) \\
f^{(7)}(x) &= -2M(136q+616q^3+840q^5+360q^7) \\
f^{(8)}(x) &= -2M(136+1984q^2+6048q^4+6720q^6+2520q^8)
\end{aligned} \tag{2.78}$$

with which the values of $\Delta^n \log\cos x$ can be computed.

In the recomputed tables, these formulæ were used for all values of x from $0^{\mathrm{q}}.05$ to $0^{\mathrm{q}}.95$ (1800 pages). The computations were done using $f^{(i)}$ with $i < 8$, in order to be as faithful as possible to the original computations.

The accuracy of these tables is indicated by the following table, where $\Delta x = \frac{\pi}{200000}$.[322]

Level	First neglected term for $x = 95000\Delta x$
$\Delta \log\cos x$	$f^{(8)}(x) \times (\Delta x)^8 \times \frac{F(1,8)}{8!} \approx 1.4 \cdot 10^{-31}$
$\Delta^2 \log\cos x$	$f^{(8)}(x) \times (\Delta x)^8 \times \frac{F(2,8)}{8!} \approx 3.5 \cdot 10^{-29}$
$\Delta^3 \log\cos x$	$f^{(8)}(x) \times (\Delta x)^8 \times \frac{F(3,8)}{8!} \approx 8.1 \cdot 10^{-28}$
$\Delta^4 \log\cos x$	$f^{(8)}(x) \times (\Delta x)^8 \times \frac{F(4,8)}{8!} \approx 5.7 \cdot 10^{-27}$
$\Delta^5 \log\cos x$	$f^{(8)}(x) \times (\Delta x)^8 \times \frac{F(5,8)}{8!} \approx 1.8 \cdot 10^{-26}$
$\Delta^6 \log\cos x$	$f^{(8)}(x) \times (\Delta x)^8 \times \frac{F(6,8)}{8!} \approx 2.7 \cdot 10^{-26}$
$\Delta^7 \log\cos x$	$f^{(8)}(x) \times (\Delta x)^8 \times \frac{F(7,8)}{8!} \approx 2.0 \cdot 10^{-26}$

The positions of the units for $\Delta^i \log\tan x$ are given by the following table:

Level	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6	Δ^7
Unit	−16	−18	−20	−22	−23	−25	−25

2.8 Abridged tables

In addition to the full tables of logarithms of numbers, of sines, of logarithms of sines and tangents, and of the logarithms of the ratios between the arcs

[322] The values in this table are identical to those given in the table for $\Delta^i \log\sin x$ with $x = 5000\Delta x$.

and the sines and the tangents, Prony also had a shorter table of logarithms of sines and tangents computed to eight or nine places, in view of printing them with seven places.[323] The title of the table was: *Tables des logarithmes sinus et tangentes de 10000^e en 10000^e du quart de cercle, calculées avec huit et neuf décimales (pour être imprimées avec sept décimales exactes) au Bureau du Cadastre sous la direction de M. De Prony et formant un abrégé des grandes tables calculées au même Bureau, qui contiennent les logarithmes sinus et tangentes, avec 14 décimales (pour être imprimées avec 12 décimales exactes) de 100000^e en 100000^e du quart de cercle*.

It seems that this table was made for the students of the *École Normale*, although this is not mentioned on the cover of the manuscript. There are no known printed versions, and I have found no documents regarding a contract with a printer, so that it is likely that the tables were never printed once the *École Normale* closed in May 1795. Although the school had only a brief existence, the table was computed in this interval. Prony writes that this table was completed independently in nine days, and not extracted from the main tables.[324] One copy of this volume of abreviated tables is located in the *Observatoire* library and the other copy is in the library of the *École nationale des ponts et chaussées*.[325] The latter should have been part of the set at the library of the *Institut* but was obviously missed during the transfer.

Prony does not give any details on the methods used to compute this table, except that it was not merely copied from the Great Tables. There are however different possibilities. The logarithms of sines are obtained from interpolations between pivots, and it is unlikely that these pivots were recomputed. They have certainly been taken from the Great Tables. Some—but not all—of the pivots of the abridged table are also pivots of the Great Tables.

For the logarithms of tangents, the same may have occured. The pivots may have been taken from the Great Tables, but it is also possible that they were computed from the abridged logarithms of sines.

If the pivots were obtained from the Great Tables (volumes 10–17), which certainly was the case for the logarithms of sines, only the values of the logarithms could have been taken directly. The differences could not have been copied directly, because the step of the tables is not the same. However, the differences of the pivots of the abridged table can easily be obtained from those of the Great Tables.

The differences for the pivots of the logarithms of tangents may have

[323]This table should not be confused with a smaller table of seven-place sines and cosines in the sexagesimal division (PC: Fol. 305), which may have been computed at the *Bureau du cadastre*, but independently of the *Tables du cadastre*.

[324][Riche de Prony (1824), p. 39]

[325]PC: Ms. Fol. 242.

been computed from the abridged logarithms of sines, but this would have introduced a delay, since that part would first have had to be computed. On the other hand, if these differences have been obtained from the Great Tables, there would have been more computation for each pivot, but without the requirement to wait for the completion of certain interpolations. A detailed analysis might answer these questions.

Volume 20 also contains the values of $\mathcal{A}$ and $\mathcal{A}'$ from $0^{\mathrm{q}}.0000$ to $0^{\mathrm{q}}.0500$ and the same remarks apply to them. They may or may not have been copied from the Great Tables, even if Prony seems to say that the whole table was computed anew, which is certainly not true.

We have recomputed these tables using the above formulæ (our volumes 20a and 20b).

2.9 Multiples of sines and cosines

The set at the *Institut* also contains a volume of multiple of sines, not mentioned by Prony,[326] with a very simple structure (figure 2.1). For each angle α from $0^{\mathrm{q}}.000$ to $0^{\mathrm{q}}.500$, the sines and cosines are given with five decimals as well as their multiples $\frac{n}{10}\cos\alpha$ and $\frac{n}{10}\sin\alpha$, with $1 \leq n \leq 100$. The first page of the volume states that at least the first bundle from $0^{\mathrm{q}}.000$ to $0^{\mathrm{q}}.020$ was begun on 11 Ventôse an 4 (1st March 1796).[327]

An unbound copy of this volume is located at the *Ponts et chaussées*[328] and drafts that may be related to the calculations of this volume are located at the *Archives Nationales*[329] and at the *Ponts et chaussées*.[330]

[326]This volume is in fact mentioned in a note published in 1820 in support of the joint publication of the *Tables du cadastre* by the French and British Governments [Anonymous (1820 or 1821), p. 4].

[327]Grinevald writes that of one of the first tasks completed by the *Bureau du cadastre* were the tables of multiples of sines and cosines, "printed in 2000 copies," and refers to PC: Ms. Fol. 242 [Grinevald (2008), p. 162]. However, this manuscript is the manuscript of the abridged tables (as Grinevald makes it clear in his footnote), and not of the multiples of sines and cosines. Grinevald does not know the source of the "2000 copies " (personal communication, 2010).

[328]PC: Ms.Fol.1890.

[329]A.N. F^{17}1244B, *dossier* 5.

[330]PC: Ms. 1745.

Tables du cadastre, volume 18a (D. Roegel, 2010)

Arcs

0,005 0,995

hypothénuse	0,00785 0,99997	hypothénuse	0,00785 0,99997	hypothénuse	0,00785 0,99997	hypothénuse	0,00785 0,99997	hypothénuse	0,00785 0,99997
0,0	0,00000 0,00000	2,0	0,01571 1,99994	4,0	0,03142 3,99988	6,0	0,04712 5,99981	8,0	0,06283 7,99975
0,1	0,00079 0,10000	2,1	0,01649 2,09994	4,1	0,03220 4,09987	6,1	0,04791 6,09981	8,1	0,06362 8,09975
0,2	0,00157 0,19999	2,2	0,01728 2,19993	4,2	0,03299 4,19987	6,2	0,04869 6,19981	8,2	0,06440 8,19975
0,3	0,00236 0,29999	2,3	0,01806 2,29993	4,3	0,03377 4,29987	6,3	0,04948 6,29981	8,3	0,06519 8,29974
0,4	0,00314 0,39999	2,4	0,01885 2,39993	4,4	0,03456 4,39986	6,4	0,05026 6,39980	8,4	0,06597 8,39974
0,5	0,00393 0,49998	2,5	0,01963 2,49992	4,5	0,03534 4,49986	6,5	0,05105 6,49980	8,5	0,06676 8,49974
0,6	0,00471 0,59998	2,6	0,02042 2,59992	4,6	0,03613 4,59986	6,6	0,05184 6,59980	8,6	0,06754 8,59973
0,7	0,00550 0,69998	2,7	0,02121 2,69992	4,7	0,03691 4,69986	6,7	0,05262 6,69979	8,7	0,06833 8,69973
0,8	0,00628 0,79998	2,8	0,02199 2,79991	4,8	0,03770 4,79985	6,8	0,05341 6,79979	8,8	0,06911 8,79973
0,9	0,00707 0,89997	2,9	0,02278 2,89991	4,9	0,03848 4,89985	6,9	0,05419 6,89979	8,9	0,06990 8,89973
1,0	0,00785 0,99997	3,0	0,02356 2,99991	5,0	0,03927 4,99985	7,0	0,05498 6,99978	9,0	0,07069 8,99972
1,1	0,00864 1,09997	3,1	0,02435 3,09990	5,1	0,04005 5,09984	7,1	0,05576 7,09978	9,1	0,07147 9,09972
1,2	0,00942 1,19996	3,2	0,02513 3,19990	5,2	0,04084 5,19984	7,2	0,05655 7,19978	9,2	0,07226 9,19972
1,3	0,01021 1,29996	3,3	0,02592 3,29990	5,3	0,04163 5,29984	7,3	0,05733 7,29977	9,3	0,07304 9,29971
1,4	0,01100 1,39996	3,4	0,02670 3,39990	5,4	0,04241 5,39983	7,4	0,05812 7,39977	9,4	0,07383 9,39971
1,5	0,01178 1,49995	3,5	0,02749 3,49989	5,5	0,04320 5,49983	7,5	0,05890 7,49977	9,5	0,07461 9,49971
1,6	0,01257 1,59995	3,6	0,02827 3,59989	5,6	0,04398 5,59983	7,6	0,05969 7,59977	9,6	0,07540 9,59970
1,7	0,01335 1,69995	3,7	0,02906 3,69989	5,7	0,04477 5,69982	7,7	0,06048 7,69976	9,7	0,07618 9,69970
1,8	0,01414 1,79994	3,8	0,02984 3,79988	5,8	0,04555 5,79982	7,8	0,06126 7,79976	9,8	0,07697 9,79970
1,9	0,01492 1,89994	3,9	0,03063 3,89988	5,9	0,04634 5,89982	7,9	0,06205 7,89976	9,9	0,07775 9,89969
2,0	0,01571 1,99994	4,0	0,03142 3,99988	6,0	0,04712 5,99981	8,0	0,06283 7,99975	10,0	0,07854 9,99969

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Figure 2.1: An excerpt of the reconstruction of the multiples of sines and cosines volume.

Chapter 3

Practical interpolation and accuracy

3.1 The computers

As mentioned previously, a number of computers were employed to compute interpolations, by repeated additions or subtractions.

The drafts of the computers were copied by them on handwritten or printed forms. The final bound sheets do not contain the original calculations. Doing the calculations on these sheets would have been very inconvenient, and very error prone.[331]

There are a few cases where the calculators have left their name, usually at the end of an interpolation. The table 3.1 gives a partial list of these authors for the logarithms of numbers. The handwritings are clearly identifiable, and it should be possible to group the sheets according to the writings.

In at least one case, the calculator has also added the date of the computation. This is the case for the interval 181800–182000 at the *Observatoire* which ends with "fini le 27 ventôse 4e année Rép. Ferat."[332]

At the bottom of each page of the logarithms of numbers at the *Institut*, there is also a pencil-marked number, usually 6 or 7, whose meaning is not clear. In some rare cases, the values are 81 or 82.

[331]Lefort seems to regret that the results were copied, and are not the real computations, but binding the real calculations would have been impossible, given the many unavoidable calculation errors [Lefort (1858b)], [Lefort (1858a), p. 998].

[332]"Completed the 27 Ventôse year 4 (17 March 1796) of the Republic, Ferat."

Observatoire		*Institut*	
33800– 34000	Vibert and Saget fils	23800– 24000	Henry
39800– 40000	Guyétant	56800– 57000	Vibert
48800– 49000	Gineste	79800– 80000	Gabaille
51800– 52000	Leprestre	80000– 80200	Alexandre
55800– 56000	Jannin	81800– 82000	Henry
58800– 59000	Pigeou	109000–109200	Alexandre
63800– 64000	Pigeou	110800–111000	Ferat
65800– 66000	Bridanne	117800–118000	Ferat
72800– 73000	Pigeou	119800–120000	Gabaille
77800– 78000	Jannin	120000–120800	Alexandre
78800– 79000	Ant. Baudouin	124800–125000	Henry
79000– 79200	Bridanne	151800–152000	Gineste
92000– 92200	Bulton	193800–194000	Guyétant
130800–131000	Labussiere and Bridanne	194800–195000	Saget fils
181800–182000	Ferat		
190800–191000	Labussierre		

Table 3.1: Some identified interpolations for the logarithms of numbers. This table should not be considered representative of the frequency of identifications. We have assumed that each name corresponds to a 4-page interval, but it may in fact correspond to longer intervals in some cases.

Since this table contains 15 different names and since there were 20 to 25 computers, it may be possible to identify exactly the authors of all the computed interpolations, provided more named sheets can be located.

3.2 Forms for the interpolation

There are slightly more than 9000 pages of tables in each set (including the abridged tables and the tables of multiples of sines and cosines, but excluding the introductory volume), and therefore a total of about 18000 pages of tables. Forms were used for a great part of these tables.

3.2.1 Main forms

The pages making up the main interpolations are actually preprinted 4-page forms with a header and lines, but there seems to have been mostly (or only) forms with the heading "*Nombres*" (numbers) (figure 3.1). For the sections of log. sines and log. tangents, this word was often striked out and replaced by "*Arcs*" (figures 3.6 and 3.7).

The forms represent a rectangle of width 26.1 cm and height 41.1 cm (including the header). The header is 1.55 cm tall. The widths of the columns are (from left to right) 1.8 cm, 4.6 cm, 4.2 cm, 3.9 cm, 3.4 cm, 3.1 cm, 2.85 cm and 2.25 cm. The area for the values is divided in ten horizontal strips, the height of nine of the strips being 3.9 cm and the first being taller to accomodate one more line. There were only horizontal lines every five values, and the lines in between were added with the pencil. Each vertical column is divided with dashed lines for the groups of digits. The last dashed line of a column is bolder than the others.

It is interesting to observe a slight engraving error in the plates: the horizontal line between the values 15 and 16 goes slightly too far beyond the frame on the left. This feature was reproduced in figures 3.1, 3.6, and 3.7.

3.2.2 Forms for the sines

Printed forms were also used for the table of sines, but only after $0^{\mathrm{q}}.0350$. The forms were presumably designed and printed during the first phase of the computations. There are actually three different forms. The first form (figure 3.2) was used for the left-hand (verso) pages from $0^{\mathrm{q}}.0350$ to $1^{\mathrm{q}}.0000$.

The second form (figure 3.3) was used to show differences from Δ^3 up to Δ^6 and was used in both manuscripts from $0^{\mathrm{q}}.0350$ to $0^{\mathrm{q}}.4950$, and also from $0^{\mathrm{q}}.9350$ to $0^{\mathrm{q}}.9400$, which must be considered an anomaly.

The third form (figure 3.4) was used to show differences from Δ^3 up to Δ^7 in both manuscripts from $0^{\mathrm{q}}.4950$ to $1^{\mathrm{q}}.0000$, except in the range from $0^{\mathrm{q}}.9350$ to $0^{\mathrm{q}}.9400$ where the second form was used.

In addition, the last column of the second form was sometimes divided by pencil lines in two columns for Δ^6 and Δ^7, with their associated dashed lines (figure 3.5). This was done by filling and extending an existing dashed line, and adding another dashed line in Δ^6.

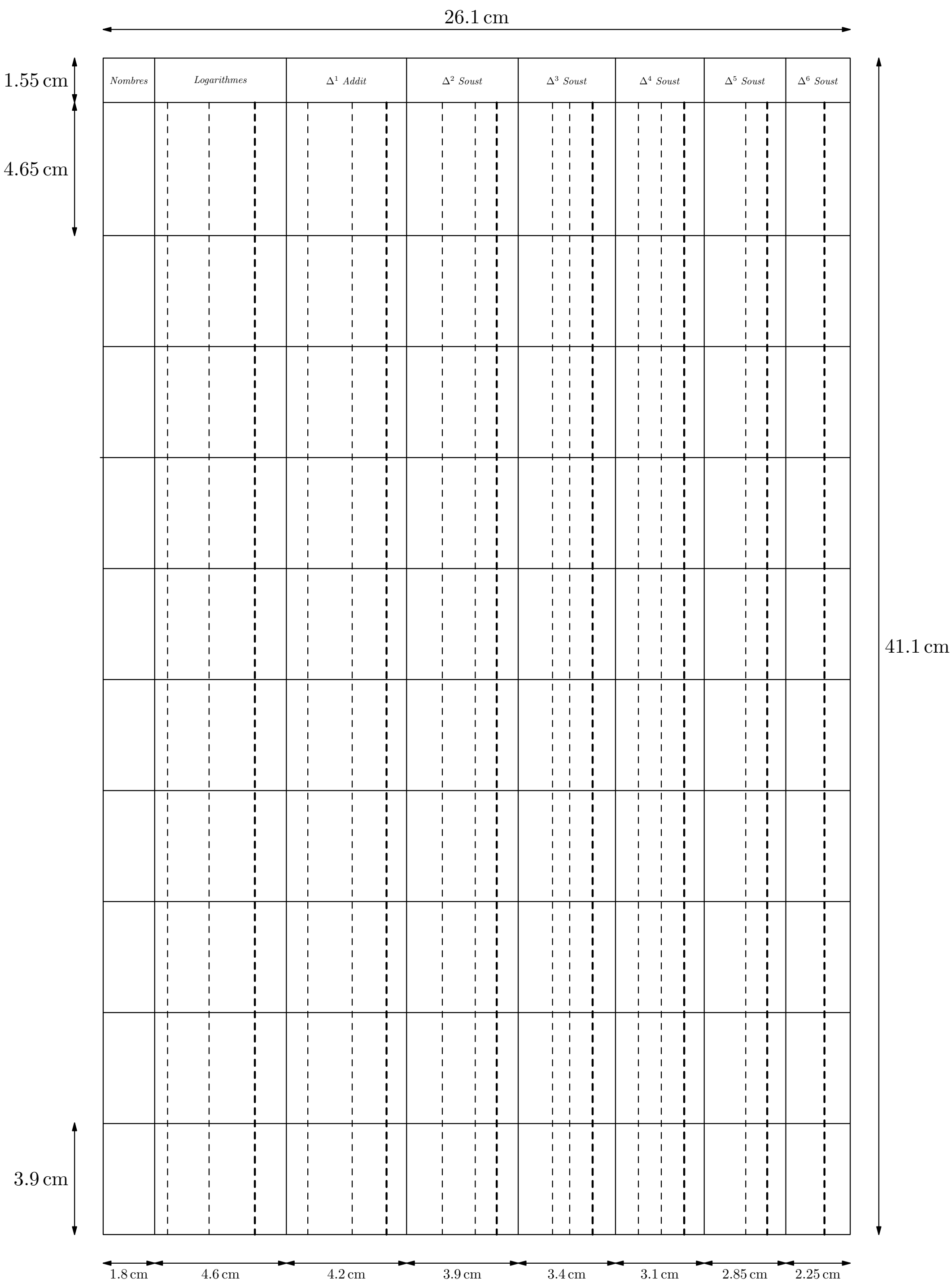

Figure 3.1: The dimensions of the forms used for the logarithms of numbers and the logarithms of sines and tangents. This sketch gives the correct relative dimensions, that is, each dimension is shown proportionally to the real one. We have in particular reproduced the printing error on the third horizontal dividing line.

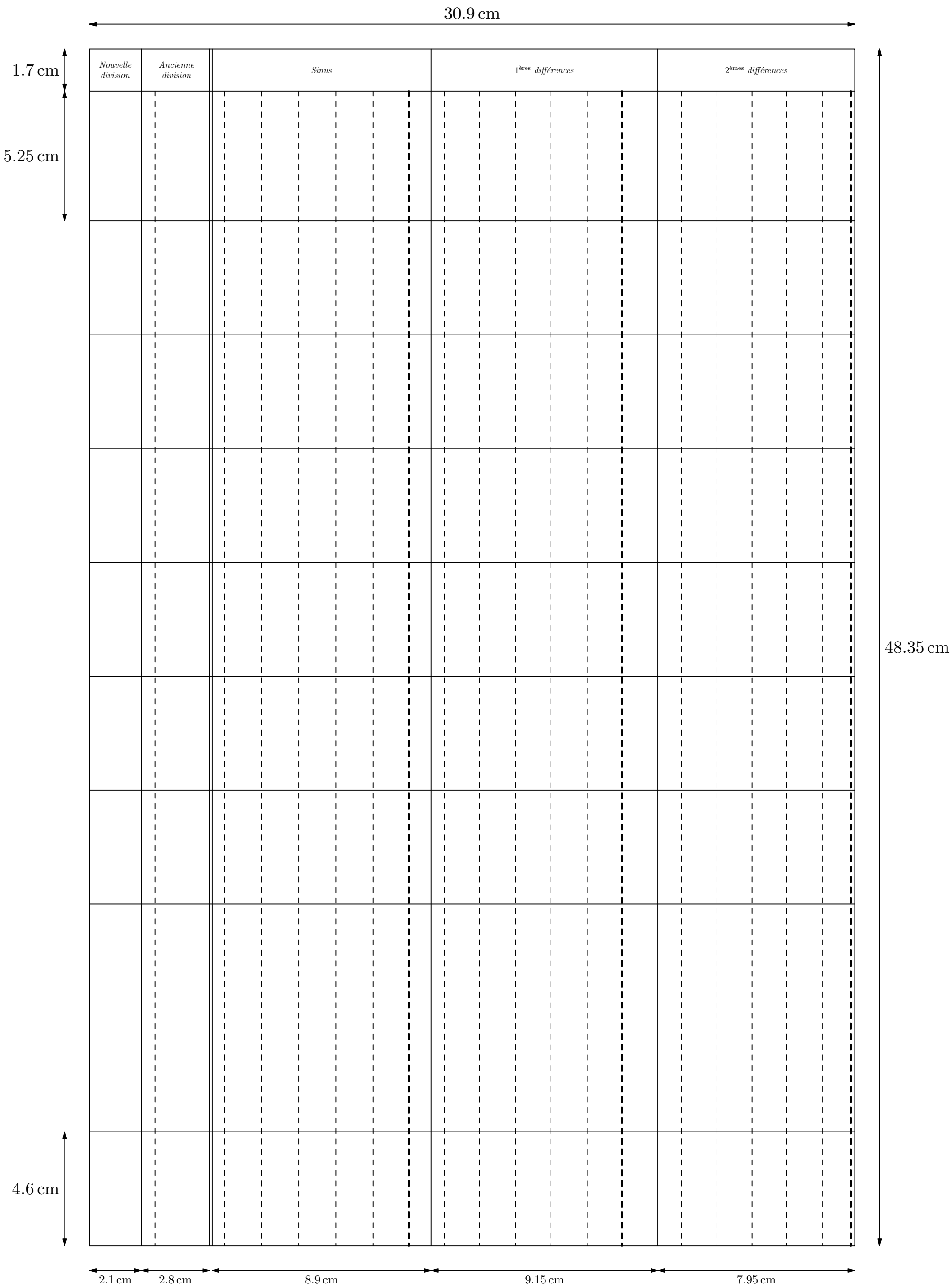

Figure 3.2: The dimensions of the forms used for the sines (verso pages) from $0^q.0350$ to $1^q.0000$.

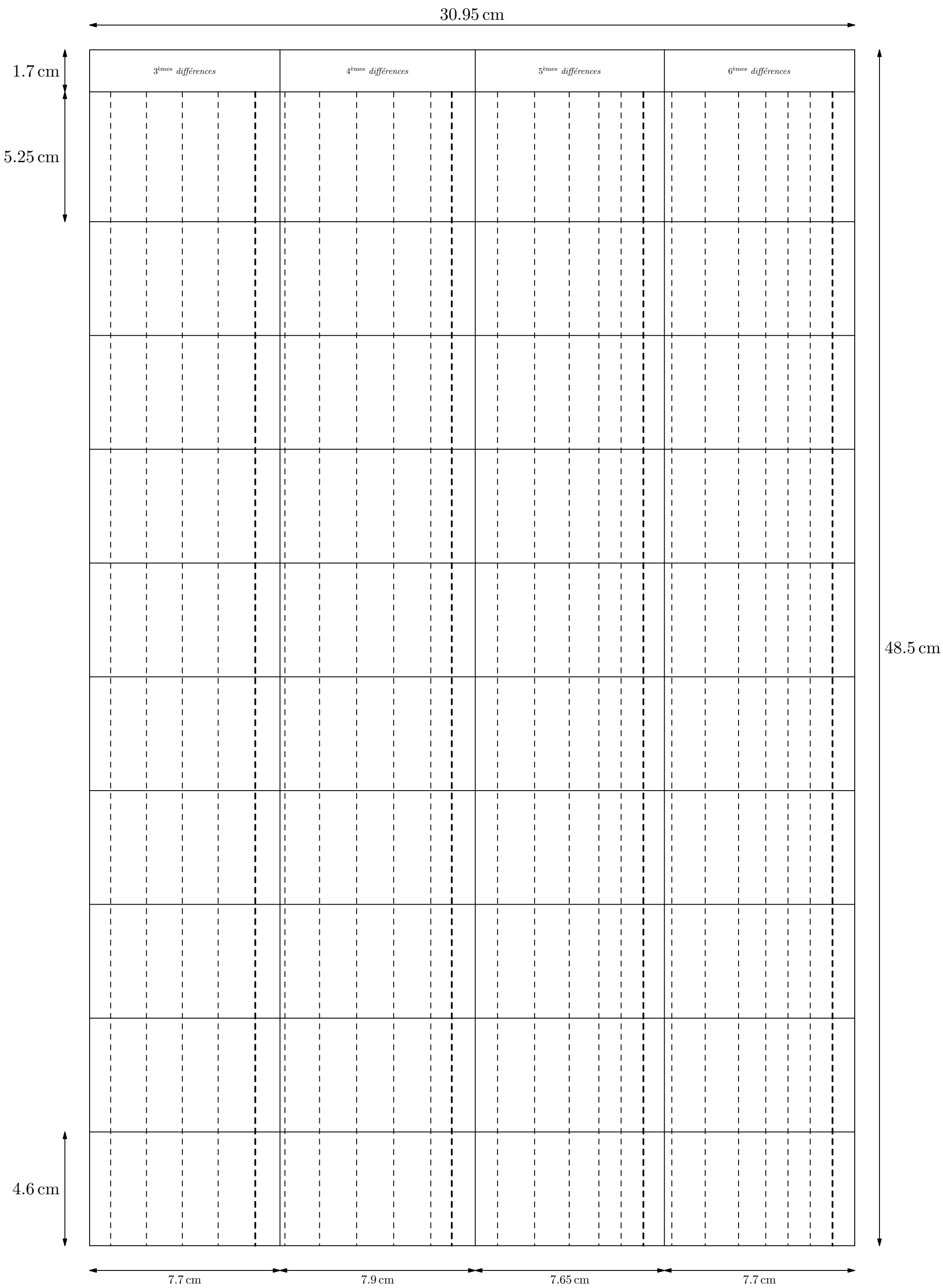

Figure 3.3: The dimensions of the forms used for the sines (recto pages, first type) mainly from $0^q.0350$ to $0^q.4950$.

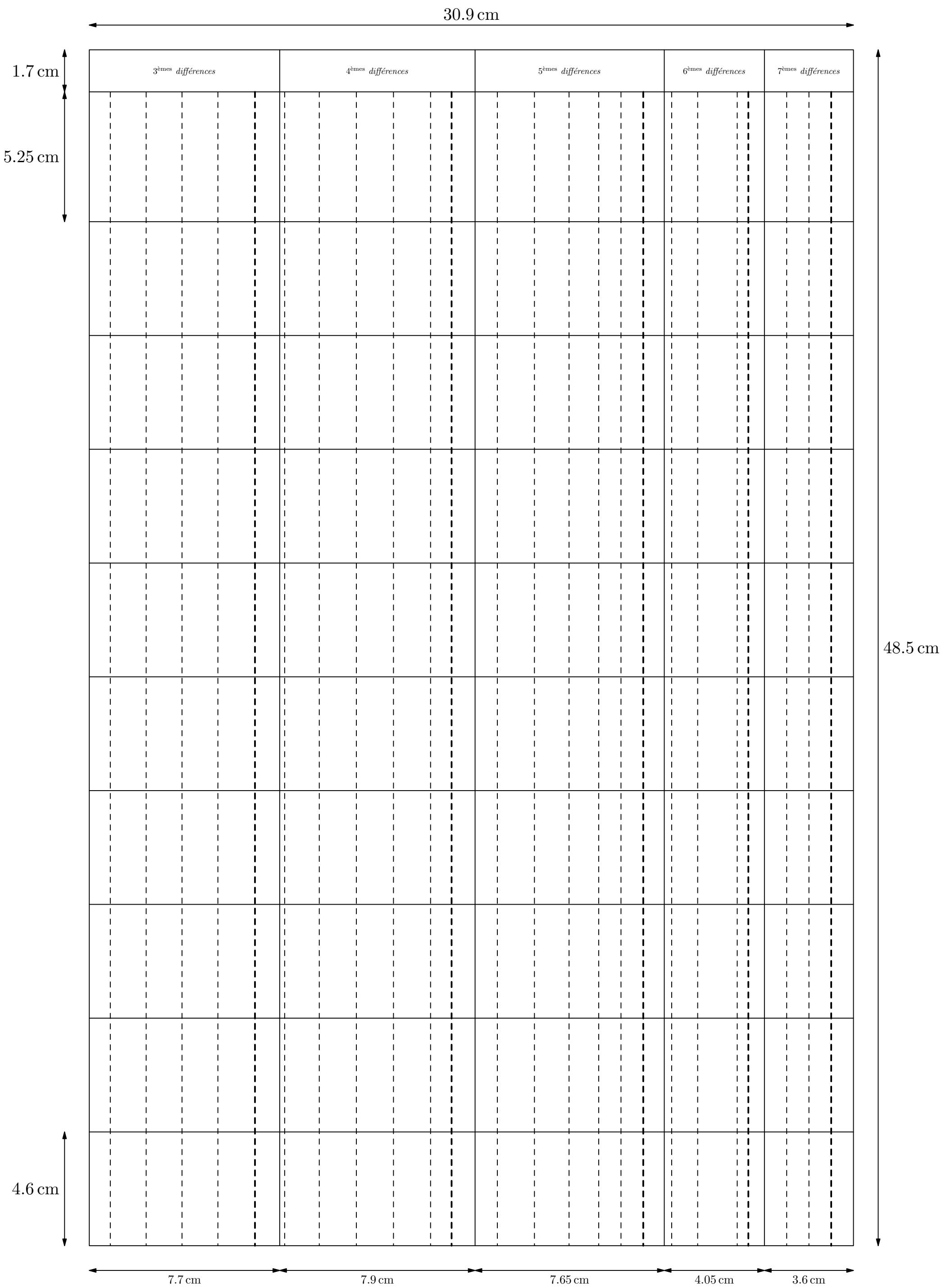

Figure 3.4: The dimensions of the forms used for the sines (recto pages, second type) mainly from $0^q.4950$ to $1^q.0000$.

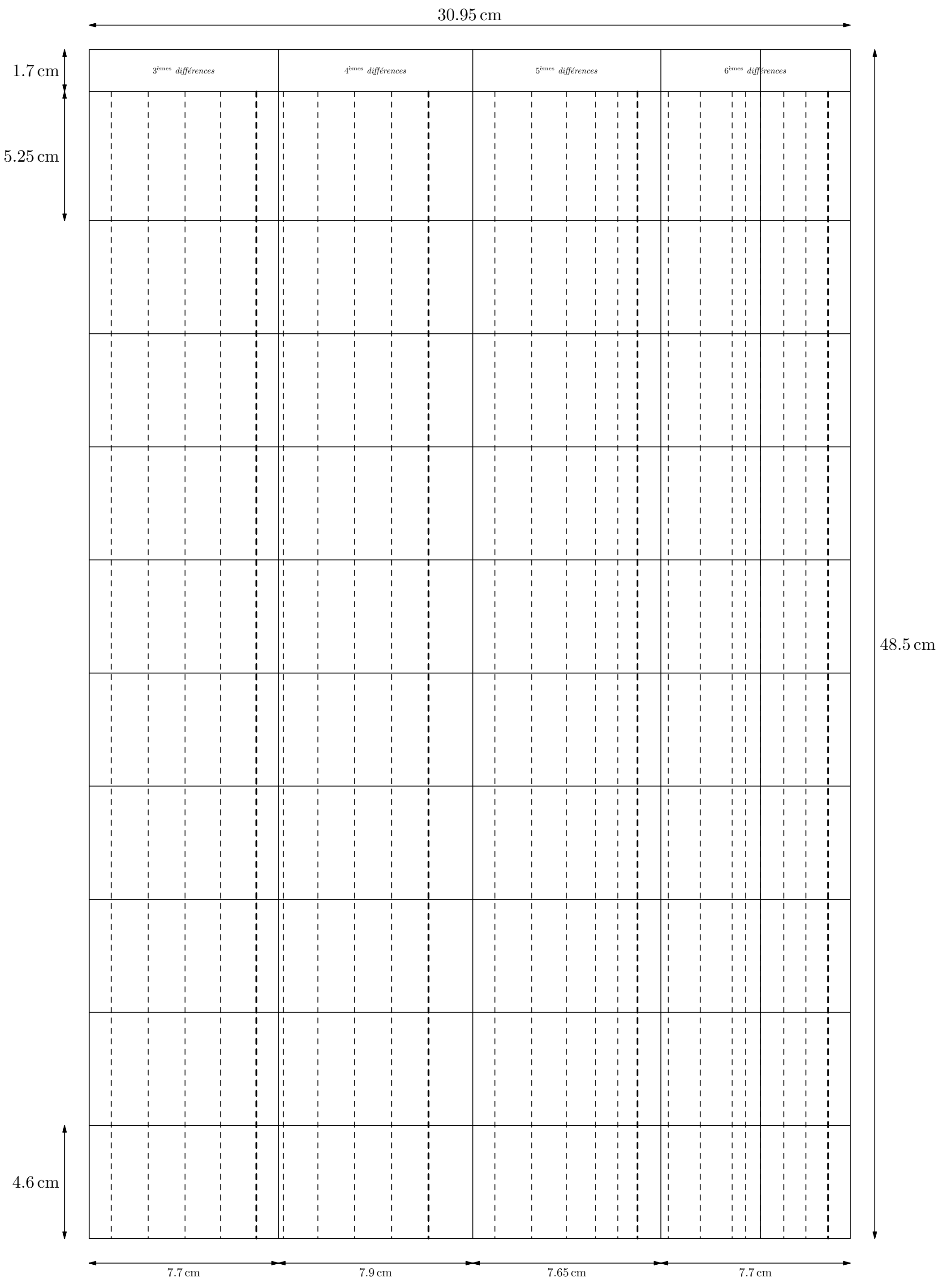

Figure 3.5: The first type of recto forms for the sines, where Δ^6 has been split in two areas by using an additional dashed line and filling and extending another line through the header. These forms have an additional header for Δ^7 marked with the pencil.

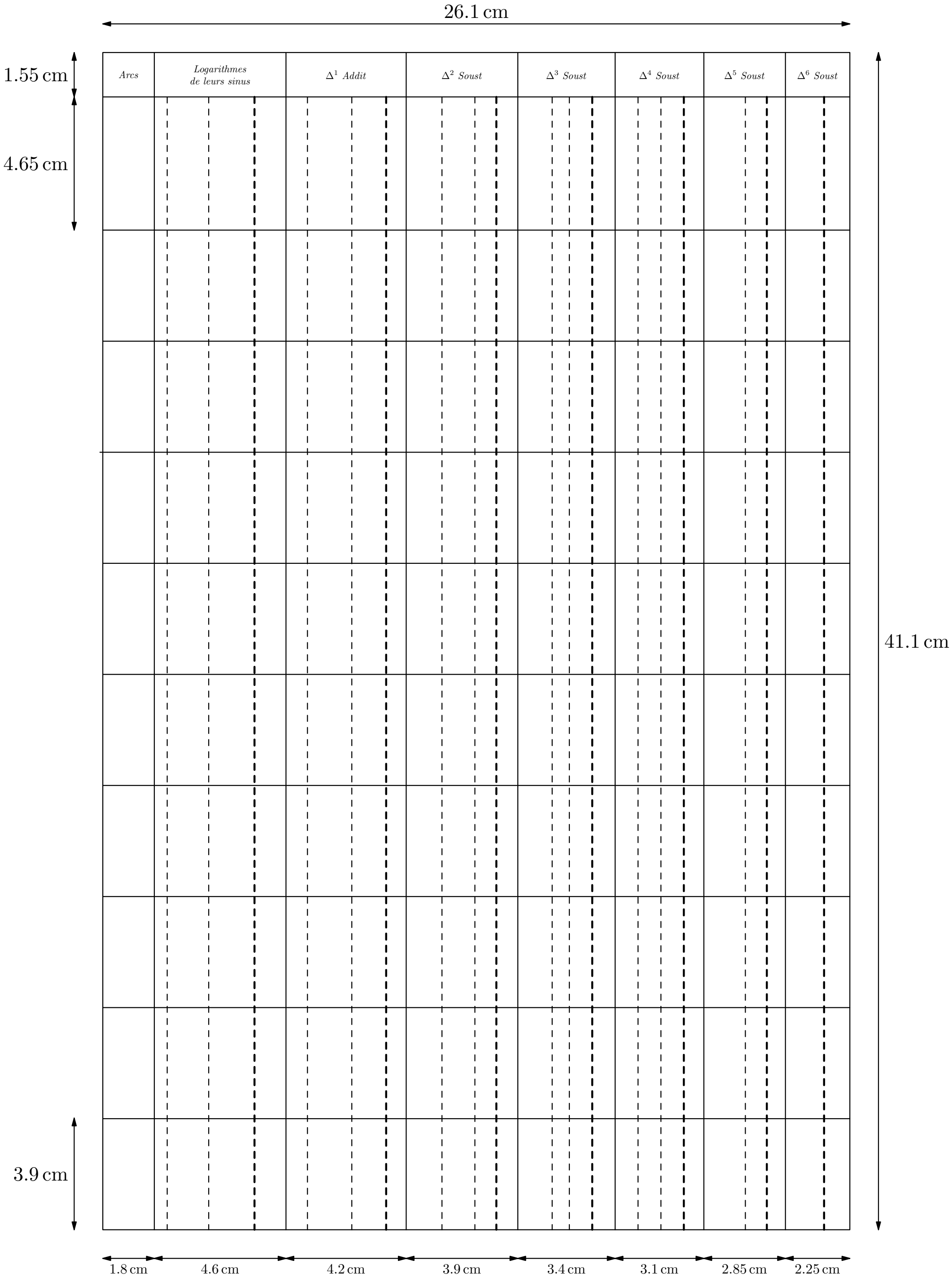

Figure 3.6: The form for the logarithms of numbers, adapted to the logarithms of sines.

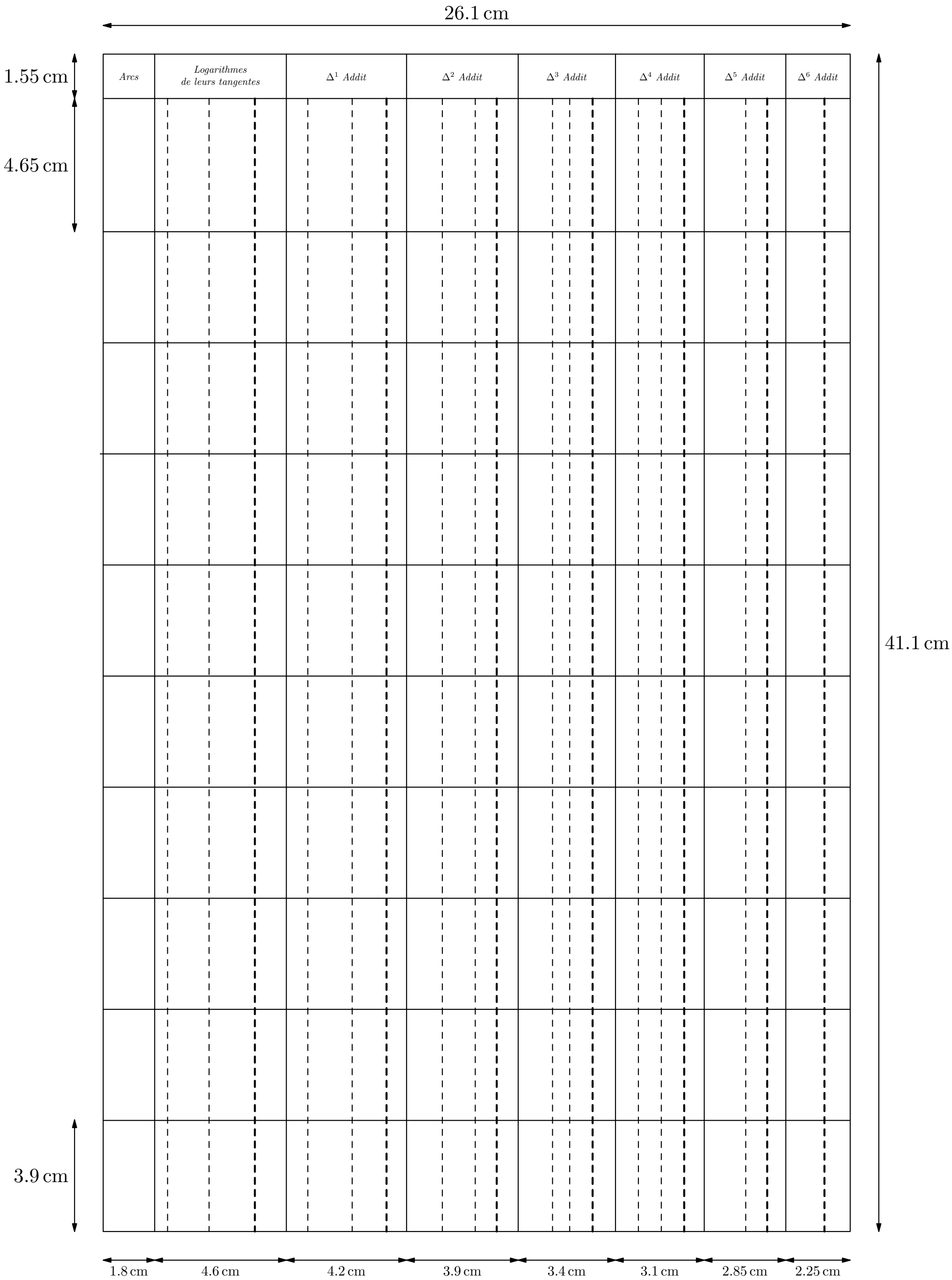

Figure 3.7: The form for the logarithms of numbers, adapted to the logarithms of tangents, for angles greater than $0^q.5$. The headings Δ^2 to Δ^6 are negative before $0^q.5$.

3.3 Interpolation methods

3.3.1 Forward and retrograde interpolations

The interpolation (in that case, subtabulation) was normally performed from one pivot to the next one. The first line contained (in principle) correct values and further lines were approximations of the real values. This step by step interpolation introduces errors, and the errors are greatest at the end of the intervals. For instance, the logarithms of numbers 10001 to 10200 could be computed from the pivot 10000, one after the other, and the error would likely be the greatest for 10200. We can call such an interpolation a "forward interpolation."

However, using the same pivots it is easy to devise a more accurate procedure. The error can be made a lot smaller by interpolating forwards and backwards.[333] For instance, 10200 might be a pivot, and it would be used to compute the logarithms from 10201 to 10300, but also backwards from 10199 to 10100. This would reduce the distance from a pivot, and therefore the maximum error on a logarithm.

The problem with this method is that the median values can not be checked easily. With the forward method alone, the end of the interpolation can be compared with the next pivot.

We have observed the occurences of retrograde interpolations in several cases. The most complete case occurs in the table of sines, and it may have been intentional.

There are many cases of short retrograde interpolations in the logarithms of numbers and perhaps for other logarithms as well. But these cases are obviously meant to cover errors. Retrograde interpolations seem almost always to be duplicate.[334] It can of course not be excluded that some retrograde interpolations contain errors and they should definitely be checked.

3.3.2 Choosing a method of interpolation

Given the initial values x, $\Delta f(x)$, $\Delta^2 f(x)$, $\ldots$, $\Delta^n f(x)$, the computation of the next values can proceed in various ways. The most common way to apply interpolation in the *Tables du cadastre* was to round $\Delta^n f(x)$ to the unit corresponding to $\Delta^{n-1} f(x)$ and to add these two values. Then, (the former value of) $\Delta^{n-1} f(x)$ was rounded similarly and added to $\Delta^{n-2} f(x)$,

[333] Interestingly, Hobert and Ideler applied this technique in their table, see their introduction [Hobert and Ideler (1799)].

[334] The only exception of which we are aware occurs in the table of the logarithms of tangents.

and so on. However, in some cases, rounding alters the value a lot, and a more accurate computation can be done by delaying the rounding. These are the two major variations in interpolation found in the manuscripts, and their use does not always follow clear rules. It is possible that certain pages mix these two kinds of interpolation. The calculators of the 2nd or 3rd sections may have taken some initiatives, and if these initiatives were producing more accurate results, the computations were kept. But the better interpolations were not systematically made.

In addition, with the method of interpolation used in these tables, the last difference is supposed to be constant on the interpolation interval. The basic rule is to keep Δ^n constant if Δ^{n+1} does not change the value of Δ^n, and if at the same time Δ^n changes the value of Δ^{n-1}. In other words, Δ^n represents a threshhold. Although this rule is followed most of the time, there are many irregularities. For instance, for the logarithms of numbers at the *Institut* (and presumably at the *Observatoire*), Δ^4 is constant in such intervals as 157400–158000, 159400–160000, 160400–161000, 161400–162000, 162200–162600, 163000–163200, 164600–164800 and other intervals before and after, but in fact Δ^4 should not be used in these places. It is Δ^3 which should be constant from 151600. But in order to have Δ^3 varying and Δ^4 constant, although after 151600 Δ^4 is normally rounded to 0, and therefore equivalent to have a constant value of Δ^3, which is contradictory, a change has to be applied. One solution is to keep Δ^4 unrounded and use all its digits. In 151600, for instance, we have the starting values

$$\begin{aligned}\Delta^3 &= 24929| \\ \Delta^4 &= |49\end{aligned}$$

the two digits of Δ^4 being located to the right of Δ^3. We can add two "0" to the first value of Δ^3. These digits are not the real digits of Δ^3, but the result obtained by the interpolation is still more accurate:

Δ^3	Δ^4
24929|00	49
24928|51	
24928|02	
...	

We can also add only one digit to Δ^3 and obtain

Δ^3	Δ^4
24929|0	49
24928|5	
24928|0	
...	

This principle is applied whenever a difference Δ^n which should be constant is made to vary, but also in a number of other cases. Lefort apparently thought that this procedure had not been used, but the truth is that it is actually common, especially in the interval 150000–200000. On the interval 175000–200000, Δ^3 is for instance constant only on a few pages, such as the interval 177000–177400.

3.3.3 Interpolation types

We can formally describe the two main types of interpolation used in the tables. Let L_i, Δ_i^1, Δ_i^2, …, Δ_i^n be the logarithm and differences for line i in the tables. We assume that all these values are represented by integers, and we have in particular $L_i = \mathrm{round}(10^{14} \log i)$ for the logarithms of numbers and the differences are integers for units at various positions. Let p_i be the position of column i. We have for instance $p_0 = -14$, $p_1 = -16$, etc. These positions may vary over the table. We set $r(i) = 10^{p_i - p_{i+1}}$. These are ratios used in the rounding procedure.

In the case of an interpolation, pivots are recomputed at regular intervals, typically every four pages.

First type (type B): "rounded interpolation"

In this interpolation, the values of Δ_{i+1}^j are merely computed from the values of Δ_i^j and Δ_i^{j+1} with the following formula:

$$\Delta_{i+1}^j = \Delta_i^j + \mathrm{round}\left(\frac{\Delta_i^{j+1}}{r(j)}\right).$$

In addition to an exact recomputation of the logarithms of numbers (volumes 1a to 8a), the sines (volume 9a), the logarithms of sines and tangents (volumes 10a to 17a), and of abridged logarithms of sines and tangents (volume 20a), these tables have also been recomputed using this type of interpolation, see volumes 1b, 2b, etc. We have used red digits to show how the wrong values spread, and we can see that the error slowly increases from the beginning to the end of an interpolation. Moreover, the changes in the structure of the tables are reflected in the accuracy. In the logarithms of numbers 15000 and 46200, there is a change in the differences which are kept constant, and this causes an great loss of accuracy at these points, which is only gradually reduced.

Second type (type C): "hidden interpolation"

In this interpolation, we maintain a "hidden" value H_i^j for row i and column j. The hidden values have at most one additional digit[335] compared to the "visible" value Δ_i^j. We set therefore $s(j) = \min(r(j), 10)$ for $j < n$ and $s(n) = 1$. At the beginning of the interpolation ($i = 0$), if Δ^n is the last difference considered, we set

$$H_0^j = \Delta_0^j \times s(j)$$

For example, for the interpolation on the logarithms of numbers starting at 25000, with $n = 6$, we have $\Delta^4 = 66686$, $\Delta^5 = 107$, and therefore $H^4 = 66686\underline{0}$ because $p_4 = -22$ and $p_5 = -23$, and $H^5 = 107\underline{0}$ because $p_5 = -23$ and $p_6 = -25$.

The '0's added to Δ^4 and Δ^5 are guard digits. They will not appear in the final results, but they make it possible to do more accurate interpolations.

The interpolation is then performed on the hidden values:

$$H_{i+1}^j = H_i^j + \text{round}\left(\frac{H_i^{j+1} \times s(j)}{s(j+1) \times r(j)}\right)$$

Finally, the tabulated values are obtained from the hidden values with

$$\Delta_{i+1}^j = \text{round}\left(\frac{H_{i+1}^j}{s(j)}\right).$$

In this interpolation, the values of Δ_{i+1}^j are not computed from the values of Δ_i^j and Δ_i^{j+1}, but from the values of H_{i+1}^j. The latter values are only computed using the pivot values and the constant values Δ_i^n, but no rounding occurs except the final rounding.

For the purpose of comparisons of parts of the actual tables, all eight volumes of logarithms of numbers have been recomputed using this type of interpolation, see volumes 1c, 2c, ..., 8c. This type of interpolation has not been applied to the other volumes. Like previously, the column of logarithms has its wrong digits marked in red.

For a comparison of the accuracy of the two interpolations, see table 3.2.

3.3.4 A note on rounding

In our reconstructions of exact values, the rounding of non integer values was done to the nearest integer, except for half integers which were rounded

[335] We could of course also consider the case of more than one additional digit, but the manuscripts mainly seem confined to this case.

	Last four digits		
Intervals	exact (A)	Int. 1 (B)	Int. 2 (C)
10000–10200	6192	6204	6186
...	9878	9838	9875
...	6477	6473	6473
...	8695	8652	8691
...	5823	5845	5817
...	7018	7019	7020
...	3647	3618	3648
...	2692	2718	2685
...	0613	0626	0613
11800–12000	4762	4750	4762
13000–13200	0585	0598	0582
...	6481	6487	6477
...	7022	7016	7021
...	0124	0113	0123
...	7824	7802	7821
...	8306	8280	8304
...	9525	9486	9527
...	8444	8406	8441
...	9496	9461	9500
14800–15000	5568	5545	5567
15000–15200	$\boxed{4477}$	$\boxed{4501}$	$\boxed{4512}$
...	3646	3663	3682
...	5446	5484	5478
...	5442	5477	5472
...	5592	5625	5619
...	4263	4281	4287
...	4770	4792	4793
...	4006	4043	4028
...	2586	2615	2607
...	7827	7848	7845
17000–17200	0755	0752	0771
39400–39600	2551	2547	2552
39600–39800	7369	7364	7371
39800–40000	2796	2794	2796
40000–40200	8447	8445	8445
40200–40400	1060	1070	1061
45800–46000	8157	8166	8156
46000–46200	5613	5619	5610
46200–46400	$\boxed{5488}$	$\boxed{5505}$	$\boxed{5478}$
...	9000	9004	8987
...	7412	7433	7397
...	3572	3579	3562
47000–47200	3409	3419	3394
49800–50000	3602	3617	3591
199400–199600	5135	5195	5132
199600–199800	8996	8977	8995
199800–200000	6398	6422	6397

Table 3.2: The last four digits of both (theoretical) interpolations compared with the exact values, at different positions of the logarithms of numbers. The boxed values show the threshholds at 15000 at 46200.

to the nearest *even* integer. This rule avoids the so-called rounding drift.[336] The value 2.5, for instance, is rounded to 2, and not to 3. The value 1.5 is also rounded to 2, and not to 1, because 2 is the nearest even integer.

In the reconstructed interpolations (volumes b and c), we have however used the more common rounding, where half integers are rounded away from zero. Although such a rounding introduces a drift, our purpose was not to avoid the drift, but to better approximate the original computations which were done at a time when the rounding drift was not known.

3.3.5 A classification of interpolation methods

We can now enumerate the main types of deviations from the standard interpolation scheme (the "rounded" interpolation).

C_5: in this case, there is an additional 0 for Δ^5; this occurs in both manuscripts from 10200 to 10400: we have

$$\begin{aligned}\Delta^5(10200)_e &= 9410\\ \Delta^5(10200)_b &= 9429\underline{0}\end{aligned}$$

such a deviation also occurs in the 1st volume of log. tan.

C_4: here, one digit is added to Δ^4; one such example is for the logarithms of numbers, when $n = 47000$, $\Delta^4 = 5339\underline{0}$ and $\Delta^5 = 5$; the added digit was underlined;

C_3: in this case, one digit is added to Δ^3; this happens for instance in the logarithms of numbers at 67600, when instead of $\Delta^3 = 28|11|55$ and $\Delta^4 = 12|48$, we have $\Delta^3 = 28|115|50$ and $\Delta^4 = 124|8$; this case is exceptionnal, and taking it into account has only a minor influence on the result;

C_{23}: in this case, there is one additional digit for Δ^2 and Δ^3, if $\Delta^4 < 50$; this case only occurs a few times, for instance in the logarithms of numbers at 162200;

it may also occur when $\Delta^4 \geq 50$, for instance between 149000 and 150000 ('0' has been added to the values of Δ^2 and Δ^3 for 149000, 149200, 149400, etc.); this should be compared with the interval 148800–149000, where $\Delta^4 = 53$ and the rounded value 1 is subtracted for each

[336] For more on rounding algorithms, see `http://en.wikipedia.org/wiki/Rounding`.

line to Δ^3; subtracting 1 seems excessive, and may have prompted this more accurate scheme; these changes are found in both manuscripts;

C_{12}**:** in this case, there are two additional digits for Δ^1 and Δ^2, and Δ^3 is constant; this happens for instance in 163200, when $\Delta^1 = \ldots 60\underline{00}$, $\Delta^2 = \ldots 71\underline{00}$, and $\Delta^3 = \ldots 19982$.

These are the types which were observed, but it is possible that other variations are used in some places of the tables.

3.4 Structure of the differences

3.4.1 Groups of numbers and dashed lines

The following table is an excerpt of the introduction to the 1891 reduced tables and gives a good idea of the structure of the interpolations in the *Tables du cadastre*.[337]

Arcs.	Logarithmes de leurs sinus.				Δ^1 Addit.				Δ^2 Soust.				Δ^3 Soust.				Δ^4 Soust.			Δ^5 Soust.		Δ^6 Soust.
0,35000	$\bar{1}$	71808	51017	9400	1	11321	0302	14	3	9249	27	12	2	01	20	00	1	74	08	2	0	0
1		71809	62338	9702	1	11317	1052	87	3	9247	25	92	2	01	18	26	1	74	06			
2		71810	73656	0755	1	11313	1805	61	3	9245	24	74	2	01	16	52	1	74	04			
3		71811	84969	2561	1	11309	2560	36	3	9243	23	57	2	01	14	78	1	74	02			
4		71812	96278	5121	1	11305	3317	12	3	9241	22	42	2	01	13	04	1	74	00			
5		71814	07583	8438	1	11301	4075	90	3	9239	21	29	2	01	11	30	1	73	98	2	0	
6		71815	18885	2514	1	11297	4836	69	3	9237	20	18	2	01	09	56	1	73	96			
7		71816	30182	7351	1	11293	5599	49	3	9235	19	08	2	01	07	82	1	73	94			
8		71817	41476	2950	1	11289	6364	30	3	9233	18	00	2	01	06	08	1	73	92			
9		71818	52765	9314	1	11285	7131	12	3	9231	16	94	2	01	04	34	1	73	90			
0,35010		71819	64051	6445	1	11281	7899	95	3	9229	15	90	2	01	02	60	1	73	88	2	0	

This table contains seven main columns, one for the value of log sin from $0^{\mathrm{q}}.35000$ to $0^{\mathrm{q}}.35010$ and six for the differences Δ^1 to Δ^6. The logarithms are given to 14 places and each Δ^i adds some decimals. The added figures are distinguished by a thicker dashed line. We can therefore see that Δ^1 is given to 16 places. The first two vertical divisions of Δ^1 correspond to the last two divisions of log sin. Δ^2, Δ^3, Δ^4 also add two digits each, but Δ^5 has only one additional digit, which is 0 here.

[337] [Service géographique de l'Armée (1891)]

3.4.2 Vertical position of the constant Δ^n

When a table is computed by interpolation, the constant value Δ^n is seldom written on every line of the table, but usually only every five lines. The value then appears either above or below the line dividing every group of five values.

3.5 Accuracy

3.5.1 General considerations

Prony chose to compute the logarithms of the pivots with 14 decimals, so that at least 12 decimals would be correct at the end of the interpolations, or, more exactly, so that the error was smaller than half a unit of the 12th decimal ($5 \cdot 10^{-13}$). This choice seems somewhat to contradict the initial aims of the project which were to compute "the most vast and imposing monument" ever made. Prony's tables aimed to guarantee 12 decimals (in the above sense), but Briggs' 1624 and 1633 tables were providing 14 decimals, although the 14th decimal was often in error.[338] However, although Briggs' tables may sometimes be more accurate (and it remains to be checked how often this is the case), the *Tables du cadastre* have undoubtedly been more thoroughly checked and give the values of the logarithms of trigonometric functions at a smaller interval.[339] They may therefore still be considered more accurate than Briggs' tables.[340]

[338][Briggs (1624), Briggs and Gellibrand (1633)]

[339]In Briggs' tables, the quadrant is divided in $90 \times 100 = 9000$ parts. In the *Tables du cadastre*, it is divided in 100000 parts (log sin and log tan) or 10000 parts (sines). In Vlacq's *Trigonometria artificialis* [Vlacq (1633)], the quadrant is divided in $90 \times 60 \times 6 = 32400$ parts, but the logarithms are only given to 10 places.

[340]The objection is sometimes raised as to the need of such a high accuracy, and Grattan-Guinness wrote for instance that Prony did not explain why the project required so "gigantic tables"[Grattan-Guinness (1990a), p. 183], [Grattan-Guinness (1993)]. But this is not totally true. First, Prony had the task to build tables which were superior to all the existing tables of similar scope, and he or Carnot decided to double most of the figures of the previous tables. This may explain why the sines (which were computed first) were computed to 29 places, because 29 is about twice the number of places given by Briggs in 1633. Another possibility is that Prony chose to obtain a final accuracy of 22 places for the sines because this was Briggs' accuracy for his fundamental sines [Briggs and Gellibrand (1633)]. Later, Prony had planned to compute the logarithms of the first 10000 integers to 28 places, which is twice the number of places of Briggs' 1624 table. The logarithms of numbers were going to be computed from 1 to 200000, which is twice Briggs' ideal interval, and twice Vlacq's interval. Of course, some of the initial decisions were later changed, and the final accuracies may no longer reflect the initial plans. Prony explained his choices

Although Edward Sang had not seen the *Tables du cadastre*, he was very critical towards their usefulness for checking Briggs' or Vlacq's tables.[341] Obviously, if the error on the unrounded logarithm is less than $5 \cdot 10^{-13}$, the 13th and 14th decimals necessarily uncertain and the values of the *Tables du cadastre* cannot be used reliably (except in certain cases) to check these decimals in other tables. There is also no absolute certainty on the 12th and lower decimals, the number of correct decimals being only superior to the number of common decimals in the values with the two extreme errors.

Even if the values in the *Tables du cadastre* are correct to 12 places, rounding them to 10 may give incorrect results. As an illustration, Sang gives seven examples where the values given by the *Tables du cadastre* may lead to an incorrect rounding. Sang noted that, using the *Tables du cadastre*, Lefort had concluded that Vlacq's value for $\log 26188$ should be 4.41810 23323, but in fact Vlacq's initial value (and also Vega's) 4.41810 23322 is the correct one, because $\log 26188 = 4.41810\ 23322\ 49959\ldots$[342] There are six other similar examples where the *Tables du cadastre* cannot establish the certainty of the 10th place.

These examples are the following ones:[343]

Number	Logarithm (20th place rounded)	*Tables du cadastre* (copy *O*)
26188	4.41810 23322 49959 00920	4.41810 23322 5014
29163	4.46483 21978 49968 31667	4.46483 21978 5005
30499	4.48428 55999 50010 73882	4.48428 55999 4997
31735	4.50153 85026 49975 27403	4.50153 85026 5005
34162	4.53354 32883 50038 92375	4.53354 32883 4997
34358	4.53602 78753 50011 99957	4.53602 78753 4998
60096	4.77884 55662 49998 09339	4.77884 55662 5001

Sang correctly gave the 20 first rounded decimals, using his 28-place table.

In each of these seven examples, the values given by the *Tables du cadastre* are such that the 12th place is correctly rounded, but the logarithm rounded to the 10th place may give an incorrect value.

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for the number of places of the differences, given his assumptions to interpolate over a number of values, and these assumptions were themselves dictated by the organization of the work-force, that is, the way interpolations were done. Finally, if such high accuracies were initially planned, it was of course to erect a definitive table standard, that other table makers could use in the future.

[341] [Sang (1890)]

[342] [Lefort (1858b)]

[343] Sang's article mistakenly indicates the number 34182 instead of 34162.

More recently, Thompson has voiced the opinion that the method of differences was used wastefully in the *Tables du cadastre*, echoing therefore Sang's remarks.[344] Thompson also observed that G. and E. Scheutz had followed a similar course in 1857, computing 400 logarithms in each direction.[345]

3.5.2 Log. 1–10000

The logarithms in the two manuscripts were probably computed independently and not sufficiently (or not at all) compared. If different formulæ were used, the choice of the neglected terms may have been inadequate in certain cases, and this would explain why certain ranges seem to be more accurate than others in this section. This contrasts with the effort put in the other parts of the tables.

On the other hand, these discrepancies may be perfectly normal, and not inconsistent with the purpose to provide 12 exact decimals which may have applied also to this section of the tables.

3.5.3 Identity of the manuscripts and corrections

From what we can tell, apart from the sections with the logarithms 1–10000, the two sets of manuscripts are nearly identical. When some values are not correct in one manuscript, the same error can almost certainly be found in the other manuscript. Other anomalies (for instance about which Δ^i should be constant) are also duplicated, and only seldom do they occur in one set only. This shows that the methods set up by Prony to ensure the identity of the two sets proved very effective, although this identity did not guarantee

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[344] [Thompson (1952), vol. 1, p. xxxv]

[345] [Scheutz and Scheutz (1857)]

the intrinsic correctness of the computations.[346,347]

Since some of the anomalies are unlikely to appear independently systematically, it is clear that when the independent computations were compared, they must have been found to differ, and one of the computations was then redone, in order to reach the identity of the two sets. Only a careful examination of two computations can reveal which one contains errors, and only then can that computation be redone. It is advisable to redo the computation, and not merely to copy the sheet which is presumed correct.

However, it should be observed that some pages have been corrected here and there by gluing new paper strips on the pages. It is not rare to see for instance entire columns that were replaced. It seems that these corrections are themselves duplicated in both manuscripts (at least in our samples), and this may indicate corrections made at a later stage. Somehow, two sheets were probably made identical, but it must have been later decided that the computation either was incorrect, or could be improved, and the improvement was made identically in both manuscripts.

Finally and only in rare cases, the digits of a given difference were put in the wrong column. In copy O, for instance, for the logarithms of numbers 163100–163150, Δ^3 is located in the column devoted to Δ^4 and Δ^4 is located

[346] A report to the *Comité de Salut Public* (Committee of Public Safety) from 12 Nivôse III (1 January 1795) gives details on the planned organization of the *Bureau de correction* whose task was in principle to check the printed proofs, and not the calculations themselves. The eight correctors were going to be divided in two groups of four persons. The printer would print four copies of each page, two for each group. In each group, two members would have the two corresponding manuscripts, and the other two would have the printed proof. It was therefore assumed that this group of four correctors would at the same time check for the identity of both manuscripts and check that the printed proofs are identical to the manuscripts. Each printed proof was supposed to be compared with one of the manuscripts. The same verification would be done the next day in the other group with the two other proofs. After this process, there would be four proofs, perhaps with corrections. Two cases would then be considered. If the four proof pages do not bear exactly the same corrections and if the original proof pages themselves are identical (which might not be the case, if characters fell and were not correctly put back in place), these proof pages would be locked away, the printer would provide new ones and the process would be repeated. On the other hand, if the four proof pages are in total agreement, one would be sent to the printer for the correction, a new proof page would be printed in four copies and checked again, but using a simpler process. The printer was supposed to follow the work and provide a steady flow of proof pages. How much of this procedure was really put in practice is not known.(A.N. F^{17}1238)

[347] In his article on Babbage's calculating engine, Lardner writes that "[w]e have reason to know, that M. Prony experienced it on many occasions in the management of the great French tables, when he found three, and even a greater number of computers, working separately and independently, to return him the same numerical result, and *that result wrong*" [Lardner (1834), p. 278].

in the column devoted to Δ^5.

3.5.4 Anomalies

It is important to realize that the original manuscripts exhibit many anomalies, and even a certain anarchy. The identity of both manuscripts does not entail their correctness. Some causes and manifestations of these anomalies are

- that the digits have been written by about twenty different persons, and are consequently not homogeneous, and sometimes confusing; this may have caused errors when digits were copied;
- that the interpolation may use inconsistent numbers of digits, and that the choice of which Δ^n should be constant may also be inconsistent (problem of specification and organization);
- that the pivots may be inconsistently distributed, some of them being full pivots, and others only partial pivots (that is, some, but not all values were recomputed).

All these reasons, compounded with computation errors, may account for various divergences between actual and theoretical interpolations.

3.6 Strategies for retrograde interpolation

In a number of cases, a certain part at the end of an interpolation interval has been covered by a blank sheet on which a retrograde interpolation was performed. The fact that an interpolation goes backwards is not obvious at first sight, but becomes clear by comparing the last line of the new (covered) part with the first line of the next page, and by observing an anomaly for the differences at the beginning of the newly added interval. The last difference, in particular, should be that of the next page.

It seems that what Lefort termed as "corrections de sentiment" were probably these retrograde interpolations. Lefort didn't identify these interpolations as being retrograde, and only observed that in some places the value of Δ from the next interval was used. He also seemed to believe that these corrections were arbitrary, hence the way he termed them.

An example of a retrograde interpolation, between $\log\tan 0^{\mathrm{q}}.72981$ and $\log\tan 0^{\mathrm{q}}.73000$ (copy O), is:

Arc	Log.	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5
$0^q.72980$	...1416	...1040	...7599	...6906	...7888	71
$0^q.72981$	...8997	...0093	...3362	...7398	...7929	74
⋮	⋮	⋮	⋮	⋮	⋮	⋮
$0^q.73000$	...4054	...9552	...3889	...6516	...8062	74

We can see a gap in the values of Δ^5, because from $0^q.72981$ to $0^q.73000$, the value of Δ^5 was the one corresponding to the next page. Moreover, there are also gaps for the other differences. Δ^4 should for instance increase by steps of 7, but it doesn't do so between $0^q.72980$ and $0^q.72981$.

However, the corrections were probably not arbitrary. They may have been triggered when the result of the interpolation was differing from the next pivot by more than an certain amount, although there appear to be many exceptions to this rule. The purpose of the retrograde interpolations was certainly to increase the accuracy and not to hide errors.

In order to find a possible threshhold, assuming it exists, one can consider the differences between the normal (forward) interpolated values at every pivot with the new pivot, and try to establish a correlation with the use of a retrograde interpolation. This is what was done in table 4.3 for the logarithms of numbers, although we have not reached a conclusive result.

As far as we could see, both manuscripts have the same corrections, except for one interpolation in the logarithms of tangents. Moreover, when there is a retrograde interpolation, the glued strip covers the entire width, except in rare cases where the last Δ is the same as on the next page, for instance.

3.7 Correction of errors

In general, when there is a computation error, the wrong parts are either scratched, or covered with a strip of varying size (depending on the extent of the error). Sometimes, the whole page is covered.

Chapter 4

Description of the manuscripts

Mais au premier rang des richesses bibliographiques de l'Observatoire, on doit placer les grandes Tables logarithmiques et trigonométriques manuscrites, en 17 volumes grand in-folio, calculées au cadastre sous la direction de M. de Prony (...)[348]

The manuscript volumes of the *Tables du cadastre* exist in two copies, one at the library of the Paris observatory, and the other at the library of the *Institut*. The latter was found by Lefort in 1858 among Prony's *Nachlass* and then given to the *Institut*, of which Prony was a member.

The location of the first set at the *Observatoire* may seem a little puzzling, but it seems that it is Prony's move to the *Bureau des longitudes* in 1801 which explains that the 19 volumes of tables initially kept at the Cadastre were transfered to the library of the *Observatoire*, which depends on the *Bureau des longitudes*. Moreover, the meetings of the *Bureau des longitudes* were taking place at the *Observatoire* after 1804. Prony had probably taken home the other copy once the computation was complete. This may also explain why the copy now at the *Institut* does not bear any "Cadastre" stamp, although these tables too have been computed at the *Bureau du cadastre*.

4.1 Paper and binding

In this section, we consider the material support of the tables, and some of its features. The main volumes are folios, that is they are made of sheets of

[348][Macarel and Boulatignier (1838), p. 635] The "17 volumes" are the main volumes, not including the introduction and the abridged tables.

paper folded once, and all these sheets appear in sequence. This is suitable for most interpolations which cover four pages (200 lines).

4.1.1 Paper

The paper used for the main volumes is called "raisin" (grapes, in French) as it supposedly contains the watermark of grapes. There are in fact different watermarks throughout the pages, and it would be interesting to study them in depth. Some pages of the *Observatoire* set of the logarithms of numbers with clear watermarks are the pages of the intervals 100950–101000 (perhaps showing a lyre), 121250–121300, or 121450–121500 (perhaps showing a coat of arms with a *fleur-de-lis*). There also seems to be some watermarked text, for instance on the interval 123050–123100.

The dimensions of the pages of the volumes of logarithms of numbers, sines, and tangents are all about 30 cm × 46.5 cm. The volume of sines is larger and the pages are about 35.5 cm × 53 cm. The volume of multiples of sines at the *Institut* uses pages of dimensions 28 cm × 43 cm, but some parts use larger dimensions.

According to Lalande,[349] the "grand-raisin" format has a width of "22 pouces 8 lignes," which is about $\left(22+\frac{8}{12}\right)\times 2.7\,\mathrm{cm}\approx 61\,\mathrm{cm}$ and a height of "17 pouces," which is about $17\times 2.7\,\mathrm{cm}\approx 46\,\mathrm{cm}$. When this is folded in two, it gives about 30.5 cm × 46 cm. We can therefore conclude more precisely that the main tables used the "grand-raisin" format.

The paper is usually in excellent condition, but it is not always clean. There are marks of watering, as well as ink stains in some places.[350]

4.1.2 Binding

The two sets of manuscripts have been bound by different binders. In the *Observatoire* set, the binder is given by his label: "Tessier, relieur et doreur de la trésorerie nationale et du Bureau de la Guerre, rue de la Harpe n°132." No binder is named in the *Institut* volumes.

The spines of the set at the *Institut* bear the words "*Grandes tables de Prony*," but since these tables were transferred to the *Institut* only in 1858, it seems likely that the labels (and numbering) go back to this period.

Inside each volume of the *Observatoire*, we find the following short description of the tables: "*Tables calculées au Bureau du Cadastre sous la direction et d'après les méthodes de Monsieur De Prony et sur lesquelles il*

[349] *L'art de faire le papier*, 1820 [de Lalande (1820)]
[350] For a nice ink stain, see the interval log 88000—log 88050 at the *Institut*.

a été fait un Rapport à la première classe de l'Institut National par Messieurs Delagrange, Delaplace, et Delambre publié dans le cinquième volume des Mémoires de cette classe." There is no such mention at the *Institut*.

The external dimensions of the main volumes (logarithms of numbers, sines and tangents) of both sets are 330 mm × 490 mm. As mentioned previously, the volume of sines is larger: 355 mm × 530 mm (paper) and 360 mm × 560 mm (binding).

Inside the volumes of the *Institut*, each page has a printed folio, but this number was only added after the sheets were bound. For instance, volume 1 has folios 1 to 189. The first part (log. 1–10000) covers folios 1–37 (74 pages), the second part (log. 10000–25000) covers folios 38–188 (151 pages, the two pages following 12050 being blank), and the last folio is also blank. The *Observatoire* set does not have such printed sheet numbers.

An interesting note was added at the beginning of the fifth volume of logarithms of numbers at the *Institut*. This note reads: *En secouant la poussière de ce cinquième volume une feuille s'en est détachée. L'ignorance de la personne qui l'a ramassée l'a empêchée de la remettre à la place qui lui convient, aussi l'a-t-elle fixée sur un fil au commencement du livre.* I am not sure which sheet is concerned, but perhaps a closer analysis can locate it.

4.1.3 Stamps

All the volumes bear some stamps, but they are not all the same:[351] the pages of the *Institut* copy have an oval stamp BIBLIOTHEQUE INSTITUT, but there are sometimes also older stamps "Bibliotheque de l'Institut national," for instance at the beginning of certain volumes such as the volume of multiples of sines. These stamps must have been added in 1858 or later.

The pages of the *Observatoire* copy have a stamp "Commission des travaux publics — Cad$^{\text{tre}}$" (for instance on the first page of the volume of log sin for the interval $0^{\text{q}}.50000$—$0^{\text{q}}.75000$), or merely "Cad$^{\text{tre}}$."

The introductory volume at the *Observatoire* is an exception to this and contains an "Observatoire de Paris" stamp.

[351] In 1858, Lefort wrote that both sets bear the mark of the Cadastre, but that does not seem true [Lefort (1858a), p. 995].

4.2 Introductory volume

The introductory volume describes the methods used to construct the tables. At the *Observatoire*, the cover of this thin volume bears the title

Prony
Grandes tables logarithmiques et trigonométriques
Introduction

The title inside the volume is "*Exposition des procédés employés pour la construction des Grandes tables logarithmiques et Trigonométriques calculées d'après les méthodes et sous la direction de M^r. de Prony*" written on a strip covering the same text as the one at the *Institut* (see below). This strip must therefore have been added later than 1862. This volume does not contain any "Cadastre" stamp, contrary to the other 18 volumes, but an "Observatoire de Paris" stamp.

The dimensions of this volume are 34 cm × 44.5 cm (binding) and 33 cm × 43.5 cm (paper). This volume has the following structure (some pages are left blank): contents (one page), *Des Sinus en parties du Rayon* (pages 1–6), *Calcul des Logarithmes sinus et des Log. Tangentes* (pages 7–18), *Calcul des Logarithmes des Nombres depuis l'unité jusqu'à 200 000* (pages 19–22), *Calcul des tangentes en parties du Rayon* (pages 23–28), *Table 1* (page 31), *Table 2* (pages 34–35), *Table 2* (cont'ed) (pages 38–39), *Table 3* (page 41), *Table 4* (pages 42–43), *De l'Interpolation* (pages 45–50), *Table 5* (page 51), *Table 6* (pages 53–57), and *Table 7* (pages 59–73 and 75–78).

At the *Institut*, the spine bears "*Exposition des méthodes*" and the title of the volume is "*Exposition des Méthodes employées pour la construction des grandes Tables Trigonométriques et Logarithmiques calculées au Bureau du Cadastre sous la direction et d'après les méthodes de M^r. de Prony*." This title is followed by the mention "*Collationné par les soussignés et certifié conforme à l'exemplaire déposé aux archives de l'Observatoire. Paris, le 8 Mai 1862. A Lanvin, G. Leveau.*[352]"

The dimensions of the volume at the *Institut* are 35.5 cm × 53 cm (binding) and 34 cm × 52.3 cm (paper). This volume has the following structure (some pages are left blank): there is no table of contents, then follows *Des Sinus en parties du Rayon* (pages 5–10), *Calcul des Logarithmes Sinus et des Log. Tangentes* (pages 10–19), *Calcul des Logarithmes des Nombres depuis l'unité jusqu'à 200 000* (pages 19–22), *Calcul des Tangentes en parties du Rayon* (pages 22–27), *Table 1* (page 28), *Table 2* (pages 29–30), *Table 3* (page 31),

[352] This is certainly Gustave Leveau (1841–1911), who was astronomer at the Paris observatory from 1857 until his death.

Table 4 (pages 32), *De l'Interpolation* (pages 33–38), *Table 5* (page 38), *Table 6* (pages 39–43), and *Table 7* (pages 44–59 and 61–63). The text of both volumes is assumed to be identical, but we have not checked it in detail.

The tables contained in this introductory volume are the following:

- table 1 is a table of the first 26 powers of $\frac{\pi}{2}$;
- table 2 gives the (theoretical) pivots of the sine table;
- tables 3 and 4 are tables used for the computation of the differences;
- table 5 is a somewhat unrelated table for an application of interpolation;
- table 6 gives the (theoretical) pivots of the logarithms of sines; and
- table 7 gives the (theoretical) pivots for the logarithms of numbers.

Lefort[353] heavily based his analysis on this introduction, and we also took it into account in our analysis, especially in chapter 2. We hope that this introductory volume will be published sometime in the future.

The Archives of the *École nationale des ponts et chaussées* hold a draft of this volume.[354]

[353] [Lefort (1858b)]
[354] PC: Ms. 1745.

4.3 Logarithms from 1 to 10000

The logarithms of the numbers from 1 to 10000 are laid out similarly in the two manuscripts, but this is the part where the manuscrits differ most. In copy *O*, the tables span 81 pages (twoside), with three columns of logarithms per page, usually with 120 to 130 values per page. Copy *I* is not identical to copy *O*, and there the tables span 74 pages, also with three columns per page.[355] The main reason for this difference is that contrary to most of the other tables, no preprinted sheets were used. Those who filled the tables tried to balance the columns, but there are great variations from one page to another and great variations from one manuscript to the other. The first page of *I*, for instance, has columns for the numbers 1–39, 40–78 and 79–117, whereas the first page of *O* has columns 1–38, 39–76 and 77–114. Some of the pages have columns of unequal sizes.

We can conclude that this part of the manuscript was not conceived as rigorously as the others. Perhaps this part of the tables was computed last, and eventually left in this state when the project stalled. The logarithms in this section were not compared, and if they were, the computations were not redone, probably by lack of time or work-force.[356] The pages containing these logarithms do very certainly not have the appearance they had meant to have. They are merely in an unfinished state, but were still bound with the other parts who had gone through a more thorough verification procedure.

[355] For the sake of completeness, we give here the last values of the pages of these two sets. This, alone, will show the great difference between the two sets for this section. *Institut*: 117, 234, 354, 474, 588, 696, 813, 927, 1044, 1161, 1260, 1365, 1470, 1575, 1680, 1785, 1890, 1991, 2150, 2309, 2468, 2627, 2754, 2880, 3006, 3132, 3270, 3396, 3522, 3654, 3850, 4048, 4246, 4444, 4642, 4840, 5038, 5236, 5416, 5605, 5806, 6002, 6163, 6322, 6484, 6646, 6790, 6937, 7081, 7231, 7354, 7468, 7579, 7690, 7807, 7925, 8042, 8160, 8280, 8400, 8520, 8640, 8760, 8880, 9000, 9120, 9240, 9360, 9480, 9600, 9690, 9780, 9882, and 10000. *Observatoire*: 114, 228, 342, 459, 576, 696, 813, 930, 1044, 1161, 1287, 1413, 1539, 1665, 1794, 1920, 2043, 2172, 2295, 2418, 2544, 2670, 2793, 2916, 3038, 3158, 3287, 3416, 3545, 3674, 3795, 3916, 4036, 4156, 4282, 4405, 4528, 4651, 4768, 4885, 5002, 5119, 5261, 5423, 5543, 5672, 5825, 5978, 6128, 6278, 6398, 6518, 6638, 6758, 6893, 7010, 7127, 7262, 7388, 7512, 7635, 7758, 7878, 7998, 8118, 8239, 8359, 8479, 8599, 8719, 8839, 8959, 9079, 9199, 9319, 9439, 9559, 9679, 9787, 9895, and 10000. We have reconstructed approximations of both versions in volume 1a.

[356] We must remember that this section was most certainly computed by the second group of the logarithm-factory. Now, perhaps this proves true Prony's assertion that those who knew the most were not the best computers [Riche de Prony (1801), p. 5]. It may also be that there was no comparable verification procedure for the second group as for the third group of computers. And of course, the formulæ used by both groups were perhaps not the same, and the results may differ if too large terms have been neglected, as the neglected terms may be different. On the other hand, as we have written earlier, if the purpose was to obtain 12 exact decimals, both tables are possibly correct.

However, in spite of these obvious differences, Prony seems to have had a great confidence in the computations.[357] But unbeknownst to him, this section contains many errors, and probably errors larger than he thought. An illustration of Prony's confidence is shown by the error on $\log 1082$, reported by Lefort. This error in the *Tables du cadastre* was found because Prony claimed that Briggs' value was incorrect.[358] Both manuscripts have $\log 1082 = 3.03422\ 7260\underline{8}\ 70550\ 6321$ but the underlined digit is wrong and should be 7. At the *Institut*, the wrong "8" was circled with the pencil and a "7" was added next to it, presumably by Lefort. This correction was not made at the *Observatoire*. This error is strange, because it should not occur in both copies, and if it does, it does not explain why there are so many other discrepancies between the two manuscripts.

Still considering $\log 1082$, we can check its value in the first volume of logarithms of sines. There, we have $\log 1082 = 3.03422\ 72607\ 7055$, in both sets. The error does not appear there, and it is not totally clear where that value comes from. And the first volume of logarithms of tangents has the value $\log 1082 = 3.03422\ 72607\ 706$ (both manuscripts).

The samples that were taken show that there is a great variability in the differences between the manuscripts. In some places, the values of the two manuscripts are often in agreement, and in others, almost all values differ. Contrary to the main table of logarithms of numbers, the two calculations were not carefully compared after having been computed independently by two groups of calculators.

In particular, we can see that Legendre's table of logarithms[359] is based on the *Observatoire* set, and that its many errors are in total agreement with it. The set at the *Institut* has different errors and cannot have been used by Legendre. It is particularly surprising that Legendre hasn't checked the two manuscripts, and this can only be explained by the confidence he had that the two manuscripts were identical. But then, this was 20 years after the computations, and Legendre may have forgotten about the discrepancies. In any case, this means that Legendre was certainly not much involved in the verification.

The errors can be put in two groups: one of differences between the manuscripts, and another of differences between the manuscripts and the

[357] This was also observed by Lefort who writes that "Prony and his aids had an almost unshakeable trust in the absolute perfection of the results they had obtained."[Lefort (1858a), p. 996].

[358] See Lefort [Lefort (1858a), p. 997] and section 4.4.2 in this document. Lefort writes that it did not occur to Prony that some of the errors he had found in Briggs' *Arithmetica logarithmica* could be errors in the *Tables du cadastre*.

[359] [Legendre (1816), table V] and [Legendre (1826), table V, page 260].

exact values. Among the first, we have for instance:

N	I	O	exact
5100	...3656	...3654	...3658
6000	...6313	...6323	...6325
6850	...5730	...5720	...5733

There are however many small absolute errors, common to both manuscripts, for instance:

N	I/O	exact
2300	...8787	...8789
7100	...2858	...2861
7400	...1916	...1920

The error for $N = 7400$ is the largest we have found, but there are possibly even larger errors.[360]

It seems that in certain ranges the error is always in the same direction. This may be due to the neglect of a term that should not have been neglected and hopefully a closer analysis will reveal the exact causes of the main discrepancies. It may be possible to find a simple formula expressing the error.

Comparing only the last four digits in three different ranges, we consider three intervals:

- Interval 1–114: in this interval, the two manuscripts are totally identical, and there is a total of eight absolute errors ($n = 33$, 48, 58, 67, 102, 104, 106, and 114); in each case, the last digits of the logarithms are in excess of 1 in the manuscript;

- Interval 1163–1401 (Legendre's section): there are many differences (see tables 2.1 and 2.2); the two manuscripts differ on 114 values (out of 120);

- Interval 9896–10000: there are also many differences: each manuscript has 102 errors (out of 105 values) and the two manuscripts differ in 21 cases; (table 4.1)

We can also correlate the values of the logarithms, knowing that some logarithms have been computed using other logarithms. We have for instance

[360]When we examined this part of the tables, we have mainly sampled the values $N = 50k$.

	Log.					Log.					Log.			
N	Obs	Ins	exact		*N*	Obs	Ins	exact		*N*	Obs	Ins	exact	
9896	2388	2388	2393		9931	7058	7057	7061	×	9966	1102	1102	1101	
9897	3167	3167	3171		9932	6969	6969	6973		9967	6468	6468	6471	
9898	4305	4306	4309	×	9933	8345	8347	8352	×	9968	3958	3958	3963	
9899	4353	4353	4358		9934	5373	5372	5379	×	9969	4706	4706	4709	
9900	9150	9150	9153		9935	9553	9553	9557		9970	7196	7196	7199	
9901	1815	1814	1819	×	9936	9710	9708	9716	×	9971	7284	7284	7287	
9902	2760	2759	2764	×	9937	2003	2003	2008		9972	8183	8183	8188	
9903	9683	9683	9687		9938	9908	9906	9913	×	9973	0476	0476	0482	
9904	7572	7572	7577		9939	4232	4232	4238		9974	2107	2107	2113	
9905	8706	8707	8711	×	9940	3116	3116	3120		9975	8388	8388	8391	
9906	2655	2655	2660		9941	2019	2019	2024		9976	1987	1987	1994	
9907	6282	6282	6287		9942	3741	3741	3746		9977	2964	2964	2966	
9908	3745	3745	3749		9943	8415	8412	8415	×	9978	8718	8718	8722	
9909	6495	6495	6499		9944	3486	3486	3492		9979	4041	4041	4046	
9910	3282	3282	3284		9945	3768	3766	3771	×	9980	1092	1092	1094	
9911	0143	0143	0149		9946	1378	1378	1383		9981	9390	9390	9393	
9912	0436	0436	0439		9947	5790	5790	5795		9982	5840	5841	5845	×
9913	4794	4794	4796		9948	3804	3804	3808		9983	4725	4725	4724	
9914	1158	1158	1164		9949	9562	9562	9566		9984	7680	7678	7682	×
9915	4784	4784	4787		9950	4546	4546	4549		9985	3741	3741	3746	
9916	8206	8206	8214		9951	7576	7576	7578		9986	9316	9316	9320	
9917	1291	1291	1295		9952	4813	4813	4816		9987	8184	8184	8188	
9918	1082	1082	1187		9953	9764	9764	9769		9988	1508	1508	1513	
9919	2346	2346	2351		9954	3282	3282	3285		9989	7832	7832	7837	
9920	6555	6555	6557		9955	3553	3555	3558	×	9990	3081	3081	3087	
9921	2827	2827	2882		9956	6123	6123	6128		9991	0570	0570	0570	
9922	7710	7712	7712	×	9957	3877	3877	3880		9992	0974	0974	0977	
9923	4740	4741	4744	×	9958	7044	7045	7048	×	9993	2379	2379	2385	
9924	4979	4979	4984		9959	3214	3214	3216		9994	0251	0253	0257	×
9925	6750	6750	6754		9960	7314	7312	7316	×	9995	7438	7438	7442	
9926	5683	5684	5688	×	9961	1626	1626	1632		9996	4172	4172	4177	
9927	4678	4678	4733		9962	5793	5795	5800	×	9997	8085	8085	8088	
9928	4145	4145	4153		9963	6807	6807	6810		9998	4189	4189	4192	
9929	1523	1523	1529		9964	8998	8998	9005		9999	4893	4893	4896	
9930	1756	1756	1759		9965	4080	4080	4083		10000	0000	0000	0000	

Table 4.1: The last four digits for the logarithms on the interval 9896–10000. Only three values are correct in each manuscript. The 21 "×"s indicate all places where the last four digits differ in the two manuscripts. In this interval, the differences never exceed three units.

$\log 2 = \ldots 63981\,1952$, $\log 1253 = \ldots 94149\,9998$ (in both manuscripts), and we have again $\log 2506 = \ldots 58131\,1950$, which is the sum of the two previous values. Although there is an error of five units in the last place of $\log 1253$, the equality $\log 2506 = \log 2 + \log 1253$ is still satisfied. This is not always true, though. Copy O gives for instance $\log 1303 = \ldots 12584\,6916$ (which may be a typographical error for the correct $\ldots 12584\,6906$), whereas copy I gives $\log 1303 = \ldots 12584\,6902$ which is off by 4 units in the last place. Still, copy I satisfies $\log 2606 = \log 2 + \log 1303$, whereas copy O does not, both manuscripts having $\log 2606 = \ldots 76565\,8854$. A more detailed investigation of the errors should be conducted and the errors should be correlated and classified.

4.4 Logarithms from 10000 to 200000

These logarithms span eight volumes, with 15000 values in the first volume and 25000 values in each of the remaining seven volumes. There are 51 values per page, one value being common between one page and the next one. From three to six differences are used at any time. The larger the values of n, the less higher differences are used, because these differences become smaller and smaller.

The units of each $\Delta^i \log n$ are located at certain positions and these positions do not change on the 10000–200000 interval, except for $\Delta^5 \log n$ after 40000. We have used these threshholds in all of our reconstructions.

	Positions in the manuscripts						
Intervals	$\log n$	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
10000– 40000	-14	-16	-18	-20	-22	-23	-25
40000–200000	"	"	"	"	"	-24	"

Table 7 in the introductory volume gives the pivots of these tables, following this division 10000–40000 and 40000–200000, except that most of the values of Δ^2 and Δ^3 are given with an additional digit in 149000 and afterwards. It remains to be seen whether this added digit is significative, or if it is always 0.

Excerpts[361] of the table are:

<table>
<tr><th>N</th><th>Log.
(−14)</th><th>Δ¹
(−16)</th><th>Δ²
(−19)</th><th>Δ³
(−21)</th><th>Δ⁴
(−22)</th><th>Δ⁵
(−24)</th><th>Δ⁶
(−25)</th></tr>
<tr><td>10000
10000_x</td><td>4.00000|00000|0000
"</td><td>4|34272|7686|27
"</td><td>43|4207|63|82
"</td><td>86|81|98|10
98|23</td><td>2|60|36|82
36|83</td><td>10|41|0
"</td><td>5|21</td></tr>
<tr><td>10001
10001_x</td><td>4.00004|34272|7686
"</td><td>4|34229|3478|63
"</td><td>43|4120|81|84
"</td><td>86|79|37|73
37|86</td><td>2|60|26|41
26|42</td><td>10|40|5
"</td><td>5|21</td></tr>
<tr><td>40000_e</td><td>4.60205|99913|2794</td><td>1|08572|2633|17</td><td>2|7142|04|95</td><td>1|35|70|09</td><td>1|01|83</td><td>1|0</td><td></td></tr>
<tr><td>40000_b
(exact)</td><td>4.60205|99913|2796
"</td><td>1|08572|2633|28
"</td><td>2|7142|04|80
"</td><td>1|35|70|18
"</td><td>1|01|77
"</td><td>1|02
"</td><td></td></tr>
<tr><td>46200_e</td><td>4.66464|19755|5619</td><td>94002|1172|97</td><td>2|0346|11|20</td><td>88|08|34</td><td>56|19</td><td>0|51</td><td></td></tr>
<tr><td>46200_b
46200_x</td><td>4.66464|19755|5613
"</td><td>94002|1172|76
"</td><td>2|0346|11|81
11|80</td><td>88|07|37
"</td><td>57|19
"</td><td>0|49
|50</td><td></td></tr>
<tr><td>199999
199999_x</td><td>5.30102|78241|8616
" 8614</td><td>21714|7783|77
" 83|82</td><td>1085|73|691
73|62</td><td>1|08|502
1|08|57</td><td>16
16</td><td></td><td></td></tr>
<tr><td>200000
200000_x</td><td>5.30102|99956|6400
" 6398</td><td>21714|6698|03
" 6698|09</td><td>1085|72|606
72|53</td><td>1|08|500
1|08|57</td><td>16
16</td><td></td><td></td></tr>
</table>

[361] In all excerpts, the positions given in the headers correspond to the rightmost digits in the columns. Therefore, in the present table, the rightmost digit for Δ^5 in 10000 is at position -23, because the rightmost digit *of the column* (for instance for 46200) is at position -24. Moreover, the vertical bars divide the digits in groups corresponding to the theoretical dashed lines in the forms (see section 3.4.1). In some examples, the vertical bars are not aligned, because the positions of the Δ^i may have changed within a column. The actual tables do not always agree with the theoretical positions of the bars. The suffixes x, b and e correspond to "exact," "begin," and "end" values of an interpolation.

In this excerpt, the values of Δ^2 and Δ^3 in 199999 and 200000 seem to be at positions -19 and -21, but this is only so because '0's were added at the beginning of the interpolation. In 199800, Δ^2 and Δ^3 were only computed to 18 and 20 places.

The characteristic of $\log n$ is sometimes given, most of the time only on the first and last lines, sometimes only on the first, and sometimes on each line. In our reconstructions, it was given on every line.

4.4.1 Truncation lines

The copy of the tables at the *Institut* contains red lines in the columns of the logarithm and of the first two differences, in order to indicate where the values would be truncated (and rounded) for printing. These three red lines are located after the 12th decimal. The three published columns would therefore have had the same unit. No such truncation lines appear in copy O.

This truncation corresponds to the second printing project, as reproduced in a companion volume (see section 5.1).

4.4.2 Comparison with Briggs' tables

Next to every $\log n$ for $n = 10000$ to 20000 and 90001 to 100000, copy O contains the 13th and 14th digits from Briggs' *Arithmetica logarithmica*.[362] In some rare instances, for instance for 99973 and 99974, the 12th digit is also given. Briggs covers the intervals 1–20000 and 90001–100000, but the section 1–10000 of the *Tables du cadastre* was not annotated. These digits are marked with the pencil, and do not appear in copy I. The annotations obviously serve the purpose of verification, and were certainly added by Jean Baptiste Letellier and Jean Désiré Guyétant, two calculators of the *Bureau du cadastre*.[363] Some of the interpolated intervals bear their names, for instance the intervals 52000–52050 ("fait Letellier") and 56800–57000 ("calculé par Guyétant"). Letellier and Guyétant seem to have mainly checked the last decimals, but although they seem to have found the error on $\log 1082$ (see § 4.3), they wrongly attributed it to Briggs. Since the two manuscripts had the same value of $\log 1082$, they must have been convinced that the *Tables du cadastre* were correct, and they did not perform the elementary verification on the values of the differences. In fact, since the accuracy of the *Tables du cadastre* is only to 12 places, the errors recorded by Letellier and Guyétant do not extend beyond the 11th place. Lefort, however, did

[362] [Briggs (1624)] We have also made a reconstruction of Briggs' tables in 2010 [Roegel (2010i)].

[363] [Lefort (1858b), p. 147]

regret that a check of the 14 first places was not conducted using the first 10000 logarithms of the *Tables du cadastre*, which are correct to 17 or 18 places [Lefort (1858b), p. 147].

These annotations appear in volume 1 (1–25000) and 4 (75000–100000) of the logarithms of numbers. In the first volume, they are put in a new column at the right side of the pages. In the fourth volume, they are added immediately at the right of the column of logarithms. An excerpt of these annotations is given below ("B" for Briggs):

n	B	n	B	n	B
		19950	77	99950	13
10001	87	19951	16	99951	77
10002	66	19952	47	99952	94
10003	22	19953	81	99953	64
10004	47	19954	30	99954	87
10005	23	19955	04	99955	63
10006	39	19956	14	99956	92
10007	82	19957	72	99957	74
10008	37	19958	87	99958	10
10009	92	19959	71	99959	99
10010	32	19960	35	99960	41
10011	46	19961	90	99961	37
10012	19	19962	46	99962	87
10013	38	19963	15	99963	91
10014	89	19964	08	99964	48
10015	60	19965	35	99965	59

These comparisons were the basis of a new errata to Briggs' *Arithmetica logarithmica*, which was appended by Prony to Briggs' volume at the *Bibliothèque Sainte-Geneviève*.[364] This volume was initially lacking several pages, including apparently the errata which was located before the introduction.[365] The missing pages have been added in handwritten form, perhaps at the same time as a new errata was added at the end of the volume by Prony. This errata begins with

Errata pour les tables logarithmiques de Briggs

[364] Bibliothèque Sainte-Geneviève, FOL V 64(2) INV 84 RES. Prony has apparently added the same errata to his own copy of the *Arithmetica logarithmica* [Lefort (1858a), p. 996], which he bought in Montpellier, and which is probably volume 4.411 in the *Ponts et chaussées* library. (minutes of the *Bureau des longitudes*, 8 Frimaire XI, where Prony announced his errata [Feurtet (2005)])

[365] The 1624 *Arithmetica logarithmica* is available on *google books* (id: `L88WAAAAQAAJ`).

(Exempl. du C^n Prony)

> *Cet errata est composé, 1^o de celui qui est en tête de l'introduction latine ; 2^o des fautes qu'ont trouvées les C^{ns} Letellier et Guyetant, calculateurs au Bureau du Cadastre, en collationnant la Table de Briggs sur les grandes tables du cadastre, ces dernières fautes sont indiquées par le signe *.*

According to a handwritten note in Briggs' volume, the errata provided by Prony was checked by Lefort in 1857 who found that a number of errors reported by Prony, and attributed to Briggs, were actually errors in the *Tables du cadastre* themselves. Prony's errata contains 158 errors originating from Briggs and 31 errors originating from Letellier and Guyétant. Prony states that several errors in Briggs' errata do not occur in that copy, but I believe he was wrong. These cases actually concern values which are duplicated at the bottom and the top of a column, one of which is incorrect (this concerns the entries for 11867, 12734 and a few others).

Among the 31 new errors, four have been erroneously attributed to Briggs:

- $\log 1082$: here Prony believes that his values are correct, and claims Briggs is in error;
- $\log 1154$: Prony corrects Briggs' value $3.06220\ 5808\underline{8}\ldots$ (which is correct) into $3.06220\ 5808\underline{7}\ldots$;
- $\log 1158$: Prony corrects Briggs' value $3.06370\ 8559\underline{3}\ldots$ (which is correct) into $3.06370\ 8559\underline{5}\ldots$;
- $\log 4219$: Prony corrects Briggs' logarithm $3.625\underline{3}0\ 95253\ 818\underline{8}$ into $3.625\underline{2}0\ 95253\ 818\underline{1}$; the first correction ($3 \rightarrow 2$) is indeed an error in Briggs' volume, but the second correction ($8 \rightarrow 1$) introduces a new error.

Lefort has also observed that beyond 10000, all the errors reported concern the first eleven decimals. Lefort concluded that Prony or Guyétant and Letellier were aware that the *Tables du cadastre* could not be used to check reliably the last two decimals of Briggs' *Arithmetica logarithmica*, and this is of course consistent with the accuracy with which the tables were constructed [Lefort (1858a), p. 997]. The fact that the first 10000 logarithms of the *Arithmetica logarithmica* could be checked with a greater accuracy is due to the computation of the corresponding logarithms to 19 places in the *Tables du cadastre*.

More comprehensive erratas for Briggs' *Arithmetica logarithmica* were given by Lefort[366] and eventually by Thompson.[367]

4.4.3 Corrections by the *Service géographique de l'armée*

A number of errors were also found in the manuscript at the *Observatoire* when the *Tables du cadastre* were used as a basis for the 8-place tables published in 1891 by the *Service géographique de l'armée*.[368] Some of the errors highlight differences between the two manuscripts, for instance:

- $\log 72587$ was given as $4.86085\ \underline{9}8475\ 0722$, but the underlined digit should have been 8; it was corrected with the pencil in 1887;
- $\log 78447$ was given as $4.89457\ 63\underline{4}97\ 8584$, but the underlined digit should have been 3.

None of these errors occur in the manuscript at the *Institut*.

4.4.4 The pivots and their accuracy

The values $n = 10000 + 200k$ are pivots, for $10000 \leq n \leq 199800$. These pivots are located at the top of the pages, every four pages. It is for these values that $\log n$, $\Delta \log n$, ..., $\Delta^6 \log n$ have been computed in advance by the members of the second section.

Interpolation then took place from one pivot to the next one, hence over four pages. This, at least, is the general scheme, and there are some variations mentioned below.

Our survey shows that the pivots were computed very accurately, and are almost always identical to the exact values, except for $\Delta^6 \log n$ right at the beginning of the range (10000–15000), as we did anticipate it (although perhaps not for the good reasons):

[366] [Lefort (1858b)]
[367] [Thompson (1952)]
[368] [Service géographique de l'Armée (1891), Roegel (2010f)]

	$\Delta^6 \log n$	
n	tables	exact
10200	460	463
10400	413	412
10800	339	328
11000	300	294
11200	261	264
12000	176	175
13600	83	82

The other pivot values of $\Delta^6 \log n$ over the interval 10000–15000 are all correct.

In addition, at the beginning of the tables, there appear to be large errors on Δ^3. This contrasts with the fact that the values of the logarithm, of Δ^1 and of Δ^2 are usually correct, and that those of Δ^4 and Δ^5 only have small errors (about one unit).

For instance, for 10000, the tables give $\Delta^3 = \ldots 10$ (instead of the exact $\ldots 23$), $\Delta^4 = \ldots 82$ (instead of the exact $\ldots 83$). For 10400, both manuscripts have $\Delta^1 = \ldots 2788$ (correct), $\Delta^2 = \ldots 4573$ (correct), $\Delta^3 = \ldots 38|31$ (exact: 3842), $\Delta^4 = \ldots 57|06$ (exact: 5708). For 10600, both manuscripts have $\Delta^1 = \ldots 0569$ (correct), $\Delta^2 = \ldots 6264$ (correct), and $\Delta^3 = \ldots 74|44$ (exact: 7454). For 10800, both manuscripts have $\Delta^3 = \ldots 2663$ (exact: 2671), $\Delta^4 = \ldots 3896$ (exact: 3898). For 11000, both manuscripts have $\Delta^3 = \ldots 1678$ (exact: 1686). For 46200, both manuscripts have $\Delta^4 = 57|19$ (correct) and $\Delta^5 = 0|49$. For 50200, the tables give $\Delta^4 = \ldots 4102$ (instead of the exact $\ldots 4103$). In 60400, both manuscripts have $\Delta^4 = 19|55$ (exact: 19|58) In 126600, $\Delta^4 = \ldots 102$ (before and after the pivot) but it should be $\ldots 101$. And for 199800, $\Delta^2 = \ldots 9001$, but should be $\ldots 9000$.

Most of the time, though, there are no errors. The error on 126600, for instance, is the only error on a pivot value between 125000 and 127000 inclusive, that is on 11 pivots.

4.4.5 Constant differences Δ^i

For each interpolation interval, there is some Δ^i which is considered constant. At the beginning of the range, it is Δ^6 which is constant, and later, lower orders become constant. In general, Δ^n is considered constant when Δ^{n+1} is rounded to 0.

Δ^6 is constant by interval, approximately from 10000 to 15000. Δ^5 is constant by interval, approximately from 15000 to 46200. Δ^4 is constant by interval, approximately from 46200 to 200000, but with many exceptions.

Constant difference Δ^6

At the beginning of the 10000–200000 interval, Δ^6 is constant over four pages. From 10000 to 10200, we have for instance $\Delta^6 = 521$.

Δ^6 is used for the interpolation until $n = 15000$, but there are irregularities. The limit of 15000 is explained by the fact that from the beginning until the pivot $n = 14800$, the rounded-truncated value of $\Delta^6 \log n$ added to Δ^5 is greater or equal to 1. But for the pivot $n = 15000$, we have $\frac{\Delta^6 \log n}{100} \approx 0.46$ and further values are all smaller than 0.5 and are rounded to 0.

Δ^6 is still used as a constant after 15000 in certain cases, but the interpolation is then obviously performed differently (for otherwise $\Delta^5 \log n$ would be constant, and $\Delta^6 \log n$ would not really be used).

Constant difference Δ^5

$\Delta^5 \log n$ is constant by interval from 15000 until 46200, but with some irregularities. In other words, $\Delta^5 \log n$ sometimes varies even beyond 15000. The limit 46200 approximately corresponds to $\Delta^5 \log n < 50$ (assuming two more places from Δ^4 to Δ^5), or when the value added to Δ^4 after rounding is 0.

The interpolation ending in 40000 has $\Delta^5 = 10$, but then after 40000, $\Delta^5 \log n$ is computed with one digit more than before. The position of Δ^5's unit becomes -24 instead of -23 before 40000. For instance, in 40000 the tables have $\Delta^5 \log n = 1|02$ (correct), in 43000 (both manuscripts) $\Delta^4 = 76|21$ (exact: 7620) and $\Delta^5 = 0|71$ and in 43400 $\Delta^5 \log n = 0|68$ (correct).

The digit added to Δ^5 has no effect from 40000 to 46200, that is, as long as Δ^4 varies and as long as the rounded interpolation is used. In every case, Δ^5 is rounded to 1 when it is subtracted from Δ^4.

One possible explanation for this change in 40000 is to keep two digits for $\Delta^5 \log n$, because the rounded version of $\Delta^5 \log n$ would have become smaller than 10 starting at pivot 40200.

Constant differences Δ^4 and Δ^3

After 46200, either Δ^4 or Δ^3 remains constant over four pages, but there are many irregularities. Δ^5 is still given at position -24, but is normally not used. Examples of exceptions are the intervals 49400–49600 and 51000–51200 where Δ^4 still varies, in both manuscripts.

Δ^4 should actually be used as long as it contributes to Δ^3, that is as long as $\Delta^4 \geq 50$, hence until 151600.

In practice, we observe that $\Delta^4 \log n$ is constant over four-pages intervals from 46200 to 151600, but with some irregularities. From 151600 to 175000, $\Delta^3 \log n$ is constant over four pages, but there are many anomalies.

From 175000 to 200000, usually $\Delta^4 \log n$ is again constant, but with some exceptions.

With the first (rounded) interpolation scheme, $\Delta^3 \log n$ should have been constant (over four pages) between 151600 and 200000. The fact that it often is not the case certainly corresponds to a change of policy in order to make the computations more accurate.

In our reconstructions, we have considered Δ^4 constant from 46200 to 200000 (by interval). We may provide reconstructions that mimick the original idiosyncrasies more faithfully in the future.

4.4.6 Accuracy of interpolated values

Depending on how the interpolation is done, the error can vary. We have already shown in section § 3.3.3 that an interpolation using hidden digits is more accurate than the "natural" interpolation by mere rounding, and that the threshholds of 15000 and 46200 have serious consequences on the accuracy of the values of the logarithms.

In both manuscripts, we have for instance the following case, which uses the first type of interpolation identified previously, and which is in volume 2b:

N	Δ^4	Δ^5
45600	60|26	0|53
	60|25	
	60|24	
	...	
45650	59|76	

In this case, Δ^4 has been decremented by 1 at each step, resulting in a final value which is far from the exact value (5999). The second type of interpolation considered above produces 6001, off by only two units. In this example, however, the value obtained for the logarithm of 45650 only differs by one or two units of the fourteenth place from the correct value. In other cases, and especially at the end of the interpolations, the differences are much larger, and the second type of interpolation is much more accurate than the first one.

Table 4.2 lists a number of discrepancies resulting from interpolation. Knowing that the pivots were computed correctly for 75000, 89800, 94800, 99800, 110000, 114800, 174800 and 199800 (except Δ^2), we can exploit that table and see if the errors observed are those that we expected. The simulated interpolations show that most of the entries of this table correspond to the rounded interpolation (volumes B). The discrepancies of 82000 (whose logarithm should have been ...8358 according to the rounded interpolation) are explained by a one-unit error on Δ^1 in 81800, which led to two incorrect roundings in the logarithms of 81911 and 81916.

The last entries of the table, however, are closer to the values obtained with the more exact interpolation type (volumes C), as alluded to previously.

It should however be observed that the roundings are not always done consistently, and some interpolations may have been done more or less accurately, resulting in discrepancies with our simulations.

The manuscripts contain observations on the accuracy of the interpolation in at least two cases. For instance, in copy O, at the end of the first interpolation, annotations give the difference with the following pivot:

Log	7	units	less	on the	14th	digit
Δ^1	9	"	more	"	16th	"
Δ^2	5	"	less	"	17th	"
Δ^3	11	"	more	"	19th	"
Δ^4	6	"	less	"	21st	"
Δ^5	19	"	less	"	23rd	"

In fact, for the first line, the manuscript writes 5 instead of 7, but this is most certainly a typo.

<table>
<tr><th>N</th><th>Log.
(−14)</th><th>Δ^1
(−16)</th><th>Δ^2
(−18)</th><th>Δ^3
(−20)</th><th>Δ^4
(−22)</th><th>Δ^5
(−24)</th><th>Δ^6
(−25)</th></tr>
<tr><td>10000_x</td><td>4.00000|00000|0000</td><td>4|34272|7686|27</td><td>43|4207|63|82</td><td>86|81|98|23</td><td>2|60|36|83</td><td>10|41|0</td><td>5|21</td></tr>
<tr><td>10000_b</td><td>4.00000|00000|0000</td><td>4|34272|7686|27</td><td>43|4207|63|82</td><td>86|81|98|10</td><td>2|60|36|82</td><td>10|41|0</td><td>5|21</td></tr>
<tr><td>⋮</td><td>⋮</td><td>⋮</td><td>⋮</td><td>⋮</td><td>⋮</td><td>⋮</td><td>⋮</td></tr>
<tr><td>10200_e
(error)</td><td>4.00860|01717|6185
(−7)</td><td>4|25758|0336|85
(+9)</td><td>41|7348|45|75
(−53)</td><td>81|81|30|79
(+111)</td><td>2|40|53|82
(−54)</td><td>9|41|0
(−19)</td><td>5|21</td></tr>
<tr><td>10200_b</td><td>4.00860|01717|6192</td><td>4|25758|0336|76</td><td>41|7348|46|28</td><td>81|81|29|68</td><td>2|40|54|36</td><td>9|42|90</td><td>4|60</td></tr>
<tr><td>10200_x</td><td>4.00860|01717|6192</td><td>4|25758|0336|76</td><td>41|7348|46|28</td><td>81|81|29|80</td><td>2|40|54|39</td><td>9|42|9</td><td>4|63</td></tr>
</table>

The "begin" values are the same as those in the introductory volume at the *Institut*.

A similar observation can be found in copy I at position 13200.

4.4.7 Retrograde interpolations

The logarithms of numbers contain many retrograde interpolations which were not reproduced in our reconstructions.[369] A non-exhaustive list is given

[369] They might be reproduced in the future, once a complete inventory is available.

	Last digits of						
n	$\log n$	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5	
25200	8157 (8154)	0807 (0785)	0908 (0966)	6785 (6692)	4486 (4594)	107 (103)	= vol. b
75050	7928 (7929)	7165 (7163)	3009 (3012)	5475 (5464)			= vol. b
75100	0418 (0417)	0817 (0811)	0369 (0381)	5075 (5054)			= vol. b
75150	2298 (2293)	5737 (5723)	7929 (7954)	4675 (4645)			= vol. b
75200	9179 (9164)	1831 (1800)	5689 (5731)	4275 (4237)			= vol. b
80000	9211 (9194)	0993 (0948)	6737 (6816)	9715 (9637)			= vol. b
82000	8360 (8372)	8729 (8755)	7174 (7134)	7483 (7525)			≈ vol. b
90000	3932 (3932)	4353 (4353)	5412 (5411)	9140 (9142)			= vol. b
95000	8899 (8885)	8044 (8017)	0214 (0259)	1346 (1303)			= vol. b
100000	9975 (0000)	0995 (1045)	8655 (8580)	6778 (6855)			= vol. b
110200	1571 (1577)	3772 (3796)	1337 (1295)	4856 (4901)			= vol. b
115000	5333 (5361)	3302 (3363)	8429 (8331)	7008 (7109)			= vol. b
125000	0823 (0806)	9594 (9579)	4387 (4402)	4484 (4470)			= vol. b
150000	5577 (5568)	8951 (8951)	17170 (1720)	57380 (5735)	51/52		≈ vol. c
175000	8634 (8629)	6631 (6632)	08866 (0882)	62020 (6206)	28 (28)		≈ vol. c
200000	6400 (6398)	9803 (9809)	72606 (7253)	08500 (0857)			≈ vol. c

Table 4.2: Some errors due to interpolation in the logarithms of numbers. The values shown are the interpolated values at position n. The correct values are given between parenthesis. All values are identical in both manuscripts, except Δ^4 for 150000 which is 51 at the *Observatoire*, and 52 at the *Institut*.

in table 4.3. There is apparently no retrograde interpolation occuring in only one of the sets, and we haven't found any in the volumes 175000–200000, assuming we haven't missed anything. Except in two cases, these interpolations are all characterized by a wide strip of paper covering the old (forward) interpolation. The two known exceptions occur in copy *I* and are the intervals 100191–100200 and 100576–100600. In these two cases, the retrograde interpolations are part of the normal page. There are possibly other such interpolations which are not as easy to detect as those with added strips.

It should be noticed that the retrograde interpolations almost always occur in the 10 to 30 last lines of a 200-interval.

These interpolations may have been triggered by the discrepancy between the end of a normal (forward) interpolation and the next pivot. Table 4.3 shows these differences in the case of the rounded interpolation, that is, the less accurate one (volumes 'B'). Except in a few cases, all the differences are equal to 29 or greater. However, there are also a number of interpolations where the difference is greater than 29, but which have not led to a retrograde interpolation. It is therefore not totally clear what exactly triggered these interpolations.

Using the second type of interpolation, the discrepancies are a lot smaller, usually at most one unit on the 14th place on the interval 100000–200000. The greatest errors occur after the threshholds at 15000 and 46200.

It should also be observed that some corrections may look like retrograde interpolations, but are not. For instance, in copy *O*, a strip was glued over the 107650–107700 interval, but it is actually a normal forward interpolation. The same is true in the same set for intervals 125150–125200, 130350–130400, 180450–180500, and probably others.

interval	last error (rnd. int.)	interval	last error (rnd. int.)
11391–11400	29	100191–100200	31
12586–12600	38	100576–100600	37
14791–14800	35	101381–101400	35
15791–15800	35	101586–101600	29
15986–16000	33	113586–113600	31
17791–17800	31	114776–114800	35
20586–20600	50	115186–115200	34
54191–54200	31	116981–117000	31
54581–54600	32	118986–119000	31
59781–59800	35	135381–135400	33
63191–63200	31	138171–138200	35
69181–69200	31	141376–141400	35
70986–71000	32	141986–142000	32
72391–72400	32	143586–143600	32
79791–79800	31	145571–145600	35
83181–83200	34	147181–147200	32
86591–86600	31	147391–147400	33
87986–88000	33	148371–148400	35
93386–93400	32	148981–149000	35
93586–93600	31	150761–150800	41
		151761–151800	14
		152956–153000	35
		156950–157000	33
		157181–157200	32
		164581–164600	32
		166176–166200	35

Table 4.3: Partial list of retrograde interpolations in the logarithms of numbers. These retrograde interpolation occur in both manuscripts. The column "last error" gives the absolute error on the logarithm at the end of each rounded interpolation.

4.5 Sines

The title of this table is "*Sinus en parties du rayon depuis 0 jusqu'à 10000*." In other words, the sines are given in the modern way, the radius being taken equal to 1. These sines are sometimes called *natural sines*, as opposed to the logarithms of sines.

The sines are given with 29 decimals at the beginning of the range, and with 25 decimals[370] after $0^q.0350$.

Printed forms with special headers are used after $0^q.0350$ (except in case of anomalies) (figures 3.2, 3.3, 3.4, and 3.5). These forms have columns for the first six or seven differences. When extra differences are given, their headers and columns appear handwritten.

A (tiny) excerpt of the table of sines is shown in an article by Grattan-Guinness.[371]

We have reconstructed two versions of this table, one with the exact values (volume 9a) and one with the rounded interpolation (volume 9b). Δ^{i+1} being large compared to Δ^i, there was no need to construct a more elaborate interpolation in which Δ^i would have been extended for a greater accuracy.

4.5.1 Structure

The interval $0^q.0000$–$1^q.0000$ is mainly divided into two subintervals: $0^q.0000$ to $0^q.0350$ and $0^q.0350$ to $1^q.0000$.

Sines from $0^q.0000$ to $0^q.0350$

In this interval, $0^q.0000$ is a pivot and there is a continuous interpolation from $0^q.0000$ to $0^q.0350$. Δ^6 was computed in advance for each of the 351 values of the angle and the other values are obtained by interpolation.

The positions of the units are as follows:

	Levels						
	Sines	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
Positions	-29	-29	-29	-32	-35	-38	-41

Excerpts of the table for this first part are:

[370] Bigourdan, in his inventory of the manuscripts at the *Observatoire*, writes incorrectly that the sines are given with 20 decimals [Bigourdan (1895), p. F.28].

[371] [Grattan-Guinness (2003), p. 113]

<table>
<tr><th>Arc</th><th>Sinus (−29)</th><th>Δ^1 (−29)</th><th>Δ^2 (−29)</th></tr>
<tr><td>$0^{\mathrm{q}}.0000$</td><td>0.00000|00000|00000|00000|00000|0000</td><td>15|70796|32033|52556|52138|1218</td><td>3875|78456|11296|8971</td></tr>
<tr><td>$0^{\mathrm{q}}.0000_x$</td><td>0.00000|00000|00000|00000|00000|0000</td><td>15|70796|32033|52556|52138|1218</td><td>3875|78456|11296|8971</td></tr>
<tr><td>$0^{\mathrm{q}}.0001$</td><td>0.00015|70796|32033|52556|52138|1218</td><td>15|70796|28157|74100|40841|2247</td><td>7751|56902|66282|2871</td></tr>
<tr><td>$0^{\mathrm{q}}.0001_x$</td><td>0.00015|70796|32033|52556|52138|1218</td><td>15|70796|28157|74100|40841|2247</td><td>7751|56902|66282|2871</td></tr>
<tr><td>$0^{\mathrm{q}}.0200$</td><td>0.03141|07590|78128|29383|91837|5272</td><td>15|70017|35202|63716|87351|4356</td><td>7|78903|27603|79122|0022</td></tr>
<tr><td>$0^{\mathrm{q}}.0200_x$</td><td>0.03141|07590|78128|29383|91836|7382</td><td>15|70017|35202|63716|87351|4192</td><td>7|78903|27603|79122|0023</td></tr>
<tr><td>$0^{\mathrm{q}}.0201$</td><td>0.03156|77608|13330|93100|79188|9628</td><td>15|70009|56299|36113|08229|4334</td><td>7|82777|11935|31154|6086</td></tr>
<tr><td>$0^{\mathrm{q}}.0201_x$</td><td>0.03156|77608|13330|93100|79188|1574</td><td>15|70009|56299|36113|08229|4169</td><td>7|82777|11935|31154|6087</td></tr>
<tr><td>$0^{\mathrm{q}}.0350_e$</td><td>0.05495|01799|12445|74736|33989|9663</td><td>15|68416|22096|34706|18646|6268</td><td>13|59711|25287|48128|3286</td></tr>
<tr><td>$0^{\mathrm{q}}.0350_x$</td><td>0.05495|01799|12445|74736|35945|0333</td><td>15|68416|22096|34706|18727|4306</td><td>13|59711|25287|48125|8537</td></tr>
</table>

and

<table>
<tr><th>Arc</th><th>Δ^3 (−32)</th><th>Δ^4 (−35)</th><th>Δ^5 (−38)</th><th>Δ^6 (−41)</th></tr>
<tr><td>$0^{\mathrm{q}}.0000$</td><td>3875|78446|54985|3899|821</td><td>19|12622|9905|812|746</td><td>9|56311|4363|005|364|064</td><td>707|881|194|694|215</td></tr>
<tr><td>$0^{\mathrm{q}}.0000_x$</td><td>3875|78446|54985|3899|821</td><td>19|12622|9905|812|746</td><td>9|56311|4363|005|364|064</td><td>707|881|194|694|215</td></tr>
<tr><td>$0^{\mathrm{q}}.0001$</td><td>3875|78427|42362|3994|008</td><td>28|68934|4268|818|110</td><td>9|56311|3655|124|169|370</td><td>943|841|565|755|869</td></tr>
<tr><td>$0^{\mathrm{q}}.0001_x$</td><td>3875|78427|42362|3994|008</td><td>28|68934|4268|818|111</td><td>9|56311|3655|124|169|369</td><td>943|841|565|755|869</td></tr>
<tr><td>$0^{\mathrm{q}}.0200$</td><td>3873|84331|52032|6063|882</td><td>1931|42512|1588|555|902</td><td>9|55827|7582|557|051|619</td><td>4|7891|845|648|777|840</td></tr>
<tr><td>$0^{\mathrm{q}}.0200_x$</td><td>3873|84331|52032|6063|883</td><td>1931|42512|1588|555|897</td><td>9|55827|7582|557|051|618</td><td>4|7891|845|648|777|839</td></tr>
<tr><td>$0^{\mathrm{q}}.0201$</td><td>3873|82400|09520|4475|326</td><td>1940|98339|9171|112|954</td><td>9|55822|9690|711|402|841</td><td>4|8127|685|512|848|084</td></tr>
<tr><td>$0^{\mathrm{q}}.0201_x$</td><td>3873|82400|09520|4475|327</td><td>1940|98339|9171|112|949</td><td>9|55822|9690|711|402|840</td><td>4|8127|685|512|848|083</td></tr>
<tr><td>$0^{\mathrm{q}}.0350_e$</td><td>3869|87835|18027|2500|749</td><td>3364|50157|6576|173|094</td><td>9|54845|9067|775|988|383</td><td>8|3251|347|532|162|715</td></tr>
<tr><td>$0^{\mathrm{q}}.0350_x$</td><td>3869|87835|18027|2000|747</td><td>3364|50157|6581|173|083</td><td>9|54845|9067|775|988|387</td><td>8|3251|347|532|162|715</td></tr>
</table>

These two excerpts are identical in both manuscripts. We have given the exact values ($\ldots_x$) after the table values. It should be noted that the tables in the introductory volumes give only the sines of the first values to 25 and not to 29 places.

The previous table displays a large difference between $\sin 0^{\mathrm{q}}.0350_e$ (end value) and $\sin 0^{\mathrm{q}}.0350_x$ (exact value), which is a consequence of an interpolation error which will be detailed later.

The four last digits of the first and last values of Δ^6 on that interval are (in both manuscripts):

Angle	Δ^6 table	Δ^6 exact	Angle	Δ^6 table	Δ^6 exact
$0^{\mathrm{q}}.0000$	...4215	...4215	$0^{\mathrm{q}}.0340$	...2860	...2861
$0^{\mathrm{q}}.0001$	...5869	...5869	$0^{\mathrm{q}}.0341$	...9187	...9188
$0^{\mathrm{q}}.0002$	...9166	...9166	$0^{\mathrm{q}}.0342$	...1163	...1164
$0^{\mathrm{q}}.0003$	...2018	...2018	$0^{\mathrm{q}}.0343$	...5221	...5222
$0^{\mathrm{q}}.0004$	...2337	...2336	$0^{\mathrm{q}}.0344$	...7844	...7845
$0^{\mathrm{q}}.0005$	...8036	...8036	$0^{\mathrm{q}}.0345$	...5563	...5564
$0^{\mathrm{q}}.0006$	...7030	...7030	$0^{\mathrm{q}}.0346$	...4960	...4960
$0^{\mathrm{q}}.0007$	...7235	...7235	$0^{\mathrm{q}}.0347$	...2667	...2667
$0^{\mathrm{q}}.0008$	...6567	...6567	$0^{\mathrm{q}}.0348$	...5365	...5365
$0^{\mathrm{q}}.0009$	...2945	...2944	$0^{\mathrm{q}}.0349$	...9787	...9787
$0^{\mathrm{q}}.0010$	...4288	...4287	$0^{\mathrm{q}}.0350$	...2715	...2715

Sines from $0^q.0350$ to $1^q.0000$

In this interval, the pivots are $0^q.0400$, $0^q.0500$, $0^q.0600$, …, $0^q.9900$. In addition, there are now differences Δ^7 and Δ^8. The difference Δ^8 remains constant between two pivots. Moreover, from $0^q.0350$ to $0^q.0400$, $\Delta^8 \sin x = 235$, which is the value for $0^q.0400$, and the interpolation is *retrograde*. In other words, the interpolation starts at $0^q.0400$ and goes backwards until $0^q.0350$. $\Delta^8 \sin x = 235$ from $0^q.0350$ to $0^q.0500$.

The positions of the units are as follows (most of the time) from $0^q.0350$ to $1^q.0000$:

	Unit levels								
Intervals	Sines	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6	Δ^7	Δ^8
$0^q.0350$–$0^q.0400$	−25	−25	−26	−28	−29	−30	−32	−34	−34
$0^q.0400$–$0^q.0600$	−25	−26	−27	−29	−30	−31	−33	−34	−34
$0^q.0600$–$0^q.0700$	−25	−26	−27	−29	−30	−31	−32	−34	−34
$0^q.0700$–$1^q.0000$	−25	−25	−26	−28	−29	−30	−32	−34	−34

The table of pivots in the introductory volume uses the positions of the interval $0^q.0700$–$1^q.0000$ for the whole range 0^q–1^q and there is no special treatment of the interval 0^q–$0^q.0350$.

The changes of accuracy are obvious on the following excerpts of the table for $0^q.0350$, $0^q.0400$, and $0^q.0700$.

<table>
<tr><th>Arc</th><th>Sinus (−25)</th><th>Δ^1 (−26)</th><th>Δ^2 (−27)</th></tr>
<tr><td>$0^q.0350_b$</td><td>0.05495|01799|12445|74736|35843</td><td>15|68416|22096|34706|18737</td><td>13|59711|25287|48126|4</td></tr>
<tr><td>$0^q.0350_x$</td><td>0.05495|01799|12445|74736|35945</td><td>15|68416|22096|34706|18727</td><td>13|59711|25287|48125|9</td></tr>
<tr><td>$0^q.0400_e$</td><td>0.06279|05195|29313|37607|61782</td><td>15|67688|96614|15297|18223</td><td>15|53162|08401|93774|3</td></tr>
<tr><td>$0^q.0400_b$</td><td>0.06279|05195|29313|37607|61782</td><td>15|67688|96614|15297|18223|2</td><td>15|53162|08401|93774|3|1</td></tr>
<tr><td>$0^q.0400_x$</td><td>0.06279|05195|29313|37607|61782</td><td>15|67688|96614|15297|18223|3</td><td>15|53162|08401|93774|3|2</td></tr>
<tr><td>$0^q.0700_e$</td><td>0.10973|43110|91045|26802|42629</td><td>15|61296|67345|79428|59267|8</td><td>27|11437|93879|14949|2|3</td></tr>
<tr><td>$0^q.0700_b$</td><td>0.10973|43110|91045|26802|44775</td><td>15|61296|67345|79428|59415</td><td>27|11437|93879|14938|6</td></tr>
<tr><td>$0^q.0700_x$</td><td>0.10973|43110|91045|26802|44775</td><td>15|61296|67345|79428|59416</td><td>27|11437|93879|14938|6</td></tr>
</table>

and

<table>
<tr><th>Arc</th><th>Δ^3
(−29)</th><th>Δ^4
(−30)</th><th>Δ^5
(−31)</th><th>Δ^6
(−33)</th><th>Δ^7
(−34)</th><th>Δ^8
(−34)</th></tr>
<tr><td>$0^q.0350_b$</td><td>3869|87835|18027|1|8 4</td><td>3364|50157|6|5 8|6</td><td>9|54845|9|0 6|6|3</td><td>8|3|2 5|1|2|2 0</td><td>2 3|5|6|0 4|1 1</td><td>2|3 5</td></tr>
<tr><td>$0^q.0350_x$</td><td>3869|87835|18027|2|0 0</td><td>3364|50157|6|5 8|1</td><td>9|54845|9|0 6|7|8</td><td>8|3|2 5|1|3|4 8</td><td>2 3|5|5|9 6|7 3</td><td>2|0 6</td></tr>
<tr><td>$0^q.0400_e$</td><td>3868|07914|92506|4|8 0</td><td>3841|81792|9|8 6|0</td><td>9|54400|7|9 3|6|1</td><td>9|5|0 2|8|5|4 7</td><td>2 3|5|4|8 6|6 1</td><td>2|3 5</td></tr>
<tr><td>$0^q.0400_b$</td><td>3868|07914|92506|4|7|98</td><td>3841|81792|9|8|6 0|1</td><td>9|54400|7|9|3 6|1|2</td><td>9|5|0|2 8|5|4|66</td><td>2|3 5|4|8|6 6|1</td><td>2 3|5</td></tr>
<tr><td>$0^q.0400_x$</td><td>3868|07914|92506|4|7|98</td><td>3841|81792|9|8|6 0|1</td><td>9|54400|7|9|3 6|1|2</td><td>9|5|0|2 8|5|4|66</td><td>2|3 5|4|8|6 6|1</td><td>2 3|5</td></tr>
<tr><td>$0^q.0700_e$</td><td>3852|27821|99712|8|0|73</td><td>6699|71005|4|7|8 7|3</td><td>9|50495|0|1|6 7|1|3</td><td>16|5|5 4|4|1|9 3</td><td>2|3 4|5|5|0|0 0</td><td>3|5 1</td></tr>
<tr><td>$0^q.0700_b$</td><td>3852|27821|99712|0|7 5</td><td>6699|71005|5|2 3|7</td><td>9|50495|0|1 9|0|2</td><td>16|5|5 4|3|2|4 4</td><td>2 3|4|5|2 1|1 6</td><td>4|0 9</td></tr>
<tr><td>$0^q.0700_x$</td><td>3852|27821|99712|0|7 5</td><td>6699|71005|5|2 3|7</td><td>9|50495|0|1 9|0|2</td><td>16|5|5 4|3|2|4 5</td><td>2 3|4|5|2 1|1 6</td><td>4|0 9</td></tr>
</table>

These excerpts are identical in both manuscripts.

The introductory volume at the *Institut* gives the same values for $0^{\mathrm{q}}.0700_b$ and all other values are given with the same accuracy.

The intervals $0^{\mathrm{q}}.0350$—$0^{\mathrm{q}}.0400$, $0^{\mathrm{q}}.0400$—$0^{\mathrm{q}}.0600$, $0^{\mathrm{q}}.0600$—$0^{\mathrm{q}}.0700$, and $0^{\mathrm{q}}.0700$—$1^{\mathrm{q}}.0000$ were taken into account in both reconstructed volumes.

4.5.2 Retrograde (or backward) interpolation

The retrograde interpolation from $0^{\mathrm{q}}.0400$ to $0^{\mathrm{q}}.0350$, with the pivot $0^{\mathrm{q}}.0400$, is the only such "anomaly" found in the table of sines, and we have reproduced it in our reconstruction of the interpolation. But this interpolation is certainly not an accident, and is found in both manuscripts. Many less systematic retrograde interpolations appear in the logarithms of numbers, of sines and tangents.

Contrary to other interpolations, this retrograde interpolation does not end with a pivot value, and therefore checking the accuracy of the computation is not as straightforward, as in the usual forward interpolation scheme. There are actually two interpolated lines for $0^{\mathrm{q}}.0350$ (the vertical bar is set after the 22nd decimal):

$$\begin{array}{lcll}
\sin 0^{\mathrm{q}}.0350 & = & 0.05495\ 01799\ 12445\ 74736\ 35|945\ 0333 & \text{(exact)} \\
 & = & 0.05495\ 01799\ 12445\ 74736\ 33|989\ 9663 & \text{(forward)} \\
 & = & 0.05495\ 01799\ 12445\ 74736\ 35|843 & \text{(backward)}
\end{array}$$

As will be explained later (§ 4.5.6), the value of the forward interpolation is corrupted by an error. The correct value in case of rounded interpolation should have been 0.05495 01799 12445 74736 35|950 5788 (see volume 9b).

A careful examination of copy I shows that the left part of the current interpolation from $0^{\mathrm{q}}.0400$ to $0^{\mathrm{q}}.0350$ covers an older forward interpolation where the sine was given to 29 places. The same can be observed on copy O, although it is difficult to see what lies under the current interpolation. Moreover, in copy O, there is also an older interpolation from $0^{\mathrm{q}}.0400$ to $0^{\mathrm{q}}.0450$ (left part) which is now covered.

4.5.3 Truncation lines

The two tables of sines appear to be quasi-identical, but the version of the *Institut* has red lines for truncation. Such lines are rare or inexistant in the *Observatoire* version. There are actually even two sets of red lines:

- a first set of red lines are those after the 10th decimal in the columns of the sines and of the first two differences; this may have been because of a planned excerpt of sines with 10 decimals;

- a second set of red lines are located after the 22nd decimal in the column of sines, and after the 20th decimal in the columns Δ^1 to Δ^5; the red lines for Δ^3 to Δ^5 are only given systematically until $0^q.0500$ and irregularly afterwards.

The rounded figures are added in red ink like on the table of logarithms of the numbers.

The second set of red lines is compatible with the known printed copies of the table of sines[372] which give the sines with 22 decimals, and all five differences with 20 decimals (see section 5.1).

4.5.4 The last values of the table

The last double page of the table of sines normally should have some negative values for Δ^1, Δ^3, Δ^5, and Δ^7. Both manuscripts actually show these values as positive values, but separate them from the previous ones in the same columns with a horizontal line.

The last values in both manuscripts are:

Arc	Sinus (−25)	Δ^1 (−25)	Δ^2 (−26)
$0^q.9996$	0.99999|98026|07918|47215|22577	863|59034|07031|55771	246|74008|21235|88656|7
$0^q.9996_x$	0.99999|98026|07918|47215|21444	863|59034|07031|55699	246|74008|21235|88660|1
$0^q.9997$	0.99999|98889|66952|54246|78348	616|85025|85795|67114	246|74009|73437|58692|5
$0^q.9997_x$	0.99999|98889|66952|54246|77143	616|85025|85795|67039	246|74009|73437|58696|0
$0^q.9998$	0.99999|99506|51978|40042|45462	370|11016|12358|08421	246|74010|64758|60864|2
$0^q.9998_x$	0.99999|99506|51978|40042|44182	370|11016|12358|08343	246|74010|64758|60867|8
$0^q.9999$	0.99999|99876|62994|52400|53883	123|37005|47599|47557	246|74010|95198|94946|4
$0^q.9999_x$	0.99999|99876|62994|52400|52525	123|37005|47599|47475	246|74010|95198|94950|1
$1^q.0000$	1.00000|00000|00000|00000|01440	123|37005|47599|47389	246|74010|64758|60864|0
$1^q.0000_x$	1.00000|00000|00000|00000|00000	−123|37005|47599|47475	246|74010|64758|60867|8

and

[372] *École nationale des ponts et chaussées*, Fol. 294. There are two almost complete printed copies of the table of sines (each of 100 pages), but a few pages have defects and a few others are blank. The headers are "Angles," "Sinus," "Différences 1^{eres} addit," etc. Our reconstruction is based on these copies. The *Archives Nationales* also hold two excerpts of these tables, one similarly laid out ($0^q.4300$–$0^q.4400$), and another with different headers ($0^q.0000$–$0^q.0100$).(A.N. F^{17}1238, A.N. F^{17}13571) These two different headers correspond perhaps to two different printings, one in the 1790s, the other in the 1820s. Or perhaps both are from the 1820s, but one before, and the other after Didot corrected some tables.

<table>
<tr><th>Arc</th><th>Δ^3
(−28)</th><th>Δ^4
(−29)</th><th>Δ^5
(−30)</th><th>Δ^6
(−32)</th><th>Δ^7
(−34)</th><th>Δ^8
(−34)</th></tr>
<tr><td>$0^q.9996$</td><td>1|52201|70035|8|17</td><td>60880|67864|1|57|5</td><td>225|3|25|6|2</td><td>150|2|17|0|5|97</td><td>18|83</td><td>37|06</td></tr>
<tr><td>$0^q.9996_x$</td><td>1|52201|70035|9|30</td><td>60880|67864|1|54|9</td><td>225|3|25|5|9</td><td>150|2|17|0|5|87</td><td>18|53</td><td>37|06</td></tr>
<tr><td>$0^q.9997$</td><td>91321|02171|6|59</td><td>60880|68089|4|83|1</td><td>75|1|08|5|6</td><td>150|2|17|0|6|16</td><td>18|23</td><td>37|06</td></tr>
<tr><td>$0^q.9997_x$</td><td>91321|02171|7|75</td><td>60880|68089|4|80|4</td><td>75|1|08|5|3</td><td>150|2|17|0|6|06</td><td>−18|53</td><td>37|06</td></tr>
<tr><td>$0^q.9998$</td><td>30440|34082|1|76</td><td>60880|68164|5|91|7</td><td>75|1|08|5|0</td><td>150|2|17|0|5|98</td><td>55|29</td><td>37|06</td></tr>
<tr><td>$0^q.9998_x$</td><td>30440|34082|2|94</td><td>60880|68164|5|89|0</td><td>−75|1|08|5|3</td><td>150|2|17|0|5|87</td><td>−55|60</td><td>37|06</td></tr>
<tr><td>$0^q.9999$</td><td>30440|34082|4|16</td><td>60880|68089|4|83|2</td><td>225|3|25|5|6</td><td>150|2|17|0|5|43</td><td>92|35</td><td>37|06</td></tr>
<tr><td>$0^q.9999_x$</td><td>−30440|34082|2|94</td><td>60880|68089|4|80|4</td><td>−225|3|25|5|9</td><td>150|2|17|0|5|32</td><td>−92|66</td><td>37|06</td></tr>
<tr><td>$1^q.0000$</td><td>91321|02171|8|99</td><td>60880|67864|1|57|6</td><td>375|5|42|6|1</td><td>150|2|17|0|4|51</td><td>1|29|41</td><td>37|06</td></tr>
<tr><td>$1^q.0000_x$</td><td>−91321|02171|7|75</td><td>60880|67864|1|54|9</td><td>−375|5|42|6|4</td><td>150|2|17|0|4|39</td><td>−1|29|73</td><td>37|06</td></tr>
</table>

The columns Δ^4 to Δ^8 and the lines $0^q.9996$ to $1^q.0000$ are covered by a strip in copy I, but in copy O it only covers the columns Δ^4 to Δ^7. In copy O, the former value of Δ^8, which is still 3706, is not covered.

4.5.5 Accuracy

On the interval $0^q.0000$–$0^q.0050$, the values of Δ^2 to Δ^6 are very close to the theoretical values (usually, there is not more than one unit of error), but Δ^1 appears less accurate.

A note inserted in copy I, bearing the title "*Observation sur les Log. des nombres, des sinus, &a*", says that the calculators in charge of the interpolation were only checking their computations every 5 results, or every 10 results, or even at larger intervals, and that there may therefore be errors in between that were cancelled out, one such case having been observed on the table of sines when the proofs were corrected. This note was meant for the printing and asks to compare the two manuscripts.

4.5.6 Errors

Uncorrected error

During the examination of the manuscripts, we found a relatively important error in $0^q.0250$: for the $0^q.0250$ line at the top of the page, Δ^4 was copied from the end of the previous page, but a '9' was mistakenly replaced by a '4.' This then spoiled all values of Δ^4 from $0^q.0250$ to $0^q.0300$, and therefore Δ^3, Δ^2, Δ^1, and the sines. The values of the sines are then not totally correct. For instance, the value of $\sin 0^q.0350$ computed by the forward interpolation has only 21 correct decimals after rounding (...3$\underline{4}$, 4 is incorrect), and not 22 as should have been the case by construction (...36). As a consequence of the error in $0^q.0250$, the value of $\sin 0^q.0350$ (and of previous sines) is incorrect by two units of the 22nd decimal.

On the contrary, the value of $\sin 0^q.0350$ computed by the retrograde interpolation is correct (after rounding) to 22 (and only 22) places.

This error is identical in both manuscripts.

The fact that the error appears in both manuscripts may be explained as follows: the correct line $0^q.0250$ must have been incorrectly copied on a new page for the first interpolation $0^q.0250$–$0^q.0300$, and it must have been copied from there to the second page for the interpolation. Otherwise, such an error seems very unlikely, except if there was an obvious readability problem with that digit.

It is however surprising that this error was not detected, given that the two interpolations differed by a greater amount than the one which was allowed, that is, one of the values had less than 22 exact decimals.

Other errors

Other errors occur. From $0^q.4400$ to $0^q.4500$, for instance, strips cover the ends of all columns from the sines to Δ^5. This occurs in both manuscripts. There must have been an error in $0^q.4400$, and it must have been found only after the volumes were bound.

4.6 Logarithms of the arc to sine ratios

The logarithms of the arc to sine ratios fill the first 100 pages of volume 10 at the *Institut*, and the corresponding volume at the *Observatoire*.

4.6.1 Forms

This section of the tables does not make use of any special form, and the forms which were used are merely those for the logarithms of numbers (figure 3.1), with handwritten adaptations. "Logarithmes" was sometimes overwritten by $\mathcal{A}$, "Δ^2 Soust" by Δ^2+, "Δ^3 Soust" by Δ^3+, "Δ^4 Soust" by Δ^4+, "Δ^5 Soust" by Δ^5+, and "Δ^6 Soust" by Δ^6+. "Δ' Addit" was normally not changed, since the sign of Δ^1 is the same as in the original forms.

4.6.2 Truncation lines

The tables in copy I contain red truncation lines, and there are none in copy O. These lines are located in the columns of $\mathcal{A}$ and of the first two differences, after the 12th decimal. The rounded values are only given for $\mathcal{A}$ and Δ^1. These truncation lines correspond to the ca. 1794 project 2 (see section 5.1).

4.6.3 Positions of $\mathcal{A}$ and the Δ^n

The following are the positions of $\mathcal{A}$ and of its differences over the interval $0^q.00000$–$0^q.05000$:[373]

	Unit levels						
Position	$\mathcal{A}$	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
$0^q.000$–$0^q.007$	−13	−18	−21	−24	−26	−31	−31
$0^q.007$–$0^q.010$	−14	"	"	"	−26	"	"
$0^q.010$–$0^q.020$	−13	"	"	"	−26	"	"
$0^q.020$–$0^q.026$	"	"	"	"	−27	"	"
$0^q.026$–$0^q.050$	−14	"	"	"	−27	"	"

$\mathcal{A}$ is given with 13 decimals from the beginning until $0^q.026$, except between $0^q.007$ and $0^q.01$ where 14 decimals are given; after $0^q.026$, $\mathcal{A}$ is given with 14 decimals. The table in the next section (§ 4.7) gives the logarithms of the sines to 14 places after $0^q.007$ and it is therefore not consistent with the accuracy of the present table, since that table uses the values of the present

373 There are possibly other intervals with idiosyncrasies that escaped our attention.

table. Perhaps the interval $0^{q}.010$–$0^{q}.026$ was initially giving $\mathcal{A}$ to 14 places, and it was changed later.

In copy I, the last four digits of column $\mathcal{A}$ of $0^{q}.007$–$0^{q}.01$ are covered by a new strip. This is not the case in copy O, although $\mathcal{A}$ is also given with 14 decimals. In that case, possibly the whole page was replaced.

The following excerpt (identical in both manuscripts) illustrates the variations in accuracy:

Arc	$\mathcal{A}$ (−14)	Δ^1 (−18)	Δ^2 (−21)	Δ^3 (−24)	Δ^4 (−27)	Δ^5 (−31)	Δ^6 (−31)
$0^{q}.00$	4.80388|01229|698	178|5 9645	357|1 9289|427	5|288	3|525|35	41	17
$0^{q}.00_x$	4.80388|01229|698	178|5 9645	357|1 9289|418	5|288	3|525|35	41	17
$0^{q}.01_e$	4.80389|79827|6149	357|377 3|7742	357 2|1056|209	3|531|288	3|525|35	1 7041	17
$0^{q}.01_b$	4.80389|79827|614	357|377|3 7516	357|2 1055|690	3|530|918	3|526|18	1 6612	17
$0^{q}.01_x$	4.80389|79827|614	357|377|3 7516	357|2 1055|689	3|530|918	3|526|18	1 6610	17
$0^{q}.02_e$	4.80395|15638|989	714|611|4 2422	357|2 6346|931	7|055|818	3|526|18	3 3612	17
$0^{q}.02_b$	4.80395|15638|991	714|611|4 2974	357|2 6348|281	7|058|205	3|528|672	3|3192	17
$0^{q}.02_x$	4.80395|15638|991	714|611|4 2916	357|2 6348|281	7|058|207	3|528|673	3|3178	17
$0^{q}.026_e$	4.80400|08608|814	928|983|4 6713	357|3 1217|439	9|176|087	3|530|965	4|3392	17
$0^{q}.026_b$	4.80400|08608|8128	928|983 4|6713	357 3|1217|439	9|176|087	3|530|965	4|3392	17
$0^{q}.026_x$	4.80400|08608|8129	928|983 4|6661	357 3|1217|436	9|176|067	3|530|961	4|3120	17
$0^{q}.05_e$	4.80432|67059|2975	1786|878 1|6931	357 6|3417|321	17|666|821	3|546|255	8|4192	17
$0^{q}.05_x$	4.80432|67059|2958	1786|878 1|6777	357 6|3417|134	17|666|600	3|546|080	8|2884	17

4.6.4 Structure of the interpolation

The interpolation is in fact not as simple as alluded earlier. There are basically three distinct interpolations. The actual structure of the sine ratio table is as follows:

- (interpolation 1) from $0^{q}.00000$ to $0^{q}.01000$, we have an interpolation with $\Delta^4 = 3|525|35$; Δ^5 and Δ^6 are given (on the first and last lines), but not used; however, Δ^5 is interpolated from $\Delta^6 = 17$, hence $\Delta^5 = 17041 = 41 + 1000 \times 17$ for $0^{q}.01$; we have therefore a double interpolation; the same occurs in both manuscripts;

- all values were recomputed in $0^{q}.01$;

- (interpolation 2) from $0^{q}.01000$ to $0^{q}.02000$, we have an interpolation with $\Delta^4 = 3|526|18$, as well as a separate interpolation of Δ^5 from 16612 to $33612 = 16612 + 17 \times 1000$;

- all values were again recomputed in $0^{q}.02000$, but Δ^1 and Δ^5 were not computed very accurately; the error on Δ^1 at $0^{q}.02$ is the main cause of the discrepancy in $0^{q}.05$;

- (interpolation 3) from $0^q.02000$ to $0^q.05000$, we have an interpolation, with only $\Delta^6 = 17$ being constant, and Δ^4 and Δ^5 varying; moreover, in $0^q.02600$, when the number of decimals was increased, only $\mathcal{A}$ was recomputed $= \ldots 8128$ (both manuscripts); in this interpolation the values of Δ^5 are only given on the 1st and last line of each page, although they too are interpolated.

4.6.5 Pivots

The accuracy of the three pivots (0^q, $0^q.01$ and $0^q.02$) can be observed in the previous table. We can in particular note that the values of the differences for $0^q.00$ are exactly those given in the introductory volume.[374]

4.6.6 Errors

Error in $0^q.00243$ at the *Institut*

Copy *I* has an error in $0^q.00243$ where the sum $\Delta^3 = 858338 + 3525$ was initially mistakenly computed as 862863, instead of the correct value 861863. The error was corrected in the manuscript, and it is easy to spot, since there are many digit corrections from $0^q.00243$ until $0^q.00400$. The error did initially propagate over several pages, even beyond $0^q.00400$. The values of the logarithms, of Δ^1, Δ^2, and Δ^3 were incorrect and were corrected only on the $0^q.00243$–$0^q.004$ interval (with ink, not glued strips), but not from $0^q.004$ to $0^q.007$.

On the page ending with $0^q.004$, it is still possible to guess $\Delta^3 = 1416288$ for $0^q.004$, and see that it was corrected into 1415288. The latter value is also the one found in copy *O*.

Both manuscripts have therefore the same interpolated values at $0^q.00400_e$: 168, $\ldots$13164, $\ldots$72837, $\ldots$15288, $\ldots$352535, $\ldots$06841, 17. The values 13164 and 72837 are slightly wrong compared to our reconstruction (volume 10b), because of the error on Δ^2 at position $0^q.00000$.

Copy *O* does not have this error in $0^q.00243$.

From $0^q.004$ to $0^q.01$

On the following page of *both* manuscripts, starting with $0^q.004$, we have $\Delta^3 = 1416288$, which is the wrong value.

[374] Copy *O*, introductory volume, p. 14.

Both manuscripts give the new values at $0^q.00400_b$: 167, ... 13173, ... 72994, ... 1$\underline{6}$288, ... 352535, ... 06841, 17. These are the values from the previous wrong interpolation.

It therefore appears that although copy O has not had an error in $0^q.00243$, we still have the error on Δ^3 from copy I! This is actually very interesting, in that it shows that some interpolations went on before their earlier parts were checked. Moreover, although the error was noticed, twelve pages of computations (from $0^q.004$ to $0^q.01$) were left uncorrected. Either a corrector from the second section did not do his work properly, or more likely computers from the third section managed to smuggle in the false computation. However, the section section is necessarily also at fault, since it either did not check the computations well enough, or it intentionally let pass a mistake.

In copy I, we have the following last digits of $\mathcal{A}$:

Arc	$\mathcal{A}$
$0^q.00400$	... 167
1	223
	636
	407
	535
...	...
450	... 539
...	...
600	... 607
700	... 307

Then, from $0^q.007$ to $0^q.01$, the digits of $\mathcal{A}$ are corrected:

Arc	$\mathcal{A}$
$0^q.00700_b$	... 3100
$0^q.00800$	0264
$0^q.00900$	9847
$0^q.01000$	6149

For both manuscripts, at $0^q.007$, when passing to 14 decimals, $\mathcal{A}$ must therefore have been recomputed in ... 3100 (exact: 3102).

The error in $0^q.00243$ is stopped by the recomputation in $0^q.01$.

The value of Δ^3 in $0^q.01_e$ should have been 3530288 and the incorrect value goes back to the error in $0^q.00243$.

After $0^q.01$

There are at least two new errors after $0^q.01$. The first error is visible at the end of the $0^q.01$–$0^q.02$ interpolation. The value of Δ^3 in $0^q.02_e$ should have been 7056818 and the discrepancy is due to an error in $0^q.01124$ which has $\Delta^3 = 3967142$ instead of 3968142. Surprisingly, the error has been spotted, as witnessed by an asterisk and the correction of the wrong digit, but later values of Δ^3 were not corrected.

The second error is the one responsible for the error on $\mathcal{A}$ at the end of the last interpolation at $0^q.05_e$. It is caused by the error of Δ^1 at $0^q.02_b$ (62 units) which accumulates from $0^q.026_b$ (where $\mathcal{A}$ is recomputed) to $0^q.05_e$, totalling about $62 \times \frac{2400}{10000} \approx 15$.

4.7 Logarithms of sines from $0^q.00000$ to $0^q.05000$

The tables in this section cover 50 pages. The pages are only onesided (recto), the forms are handwritten with ink, not printed, and the pages have a smaller format than the other ones.

The values in this table are computed from the logarithms of the numbers and the values of $\mathcal{A}$ in the table of logarithms of the arc to sine ratios.

In our reconstruction (volume 10b), we give also the exact values, not the values copied from the interpolated tables.

There are four columns: (log, N, log sin, arc), and these four columns are duplicated:

Logarithmes	N	Log. sinus	Arcs	Logarithmes	N	Log. sinus	Arcs
...	...	...	...	...	...	...	...

The only change in the format of this table is that all the values of the logarithms are given with 13 decimals until $n = 700$, and with 14 decimals afterwards (starting also at $n = 700$, but on the next page). This is the case in both manuscripts. These features have been taken into account in the reconstructions.

This change of accuracy is however inconsistent with the previous table (section 4.6), as the current table uses the values of $\mathcal{A}$ from the table of logarithms of arc to sine ratios, but the changes of accuracies do not occur at the same places.

The question therefore arises whether the logarithms of numbers used in this table are those from the section 1–10000 of the logarithms of numbers, or those from Briggs, which are to 14 places. There is the possibility that the logarithms of numbers had not yet been computed when this part of the table was computed and that another source was used. It is of course also possible that the logarithms of sines on the interval $0^q.00000$ to $0^q.05000$ were only computed at the end, since they are neither needed for the computation of the logarithms of sines after $0^q.05000$, nor for the computation of the logarithms of tangents.

In order to answer this question, we have taken a small sample, namely the last four digits of the last values of logarithms of numbers of this table, and we have compared them to the corresponding values in Briggs' *Arithmetica logarithmica*:

	$\log n$	
n	copy O	Briggs
4990	2339	2339
4991	6612	6612
4992	7437	**7436**
4993	5511	5511
4994	1533	1533
4995	6200	6200
4996	0210	**0209**
4997	4259	**4258**
4998	9043	9043
4999	5259	5259
5000	3602	3602

It turns out that the last eleven logarithms of numbers used in this table have the correct values, which is not the case for the values in Briggs' *Arithmetica logarithmica*. It is therefore most likely that all the logarithms of numbers are those of the interval 1–10000 of Prony's tables, and were not extracted from another source, and certainly not from Briggs' table.

4.8 Logarithms of sines after $0^q.05000$

As with other sections of the tables, this part is also nearly identical in both manuscripts. There is however a binding error in copy O, in that the bindings of the volumes for $0^q.50000$–$0^q.75000$ and $0^q.75000$–$1^q.00000$ have mistakenly been exchanged. The spines do therefore incorrectly describe the contents of the volumes.

There are no specific forms for the logarithms of the sines and the general form for the logarithms of numbers was used. However, the printed column head *Nombres* is often striked out and replaced by *Arcs* and the printed column head *Logarithmes* is often supplemented by "*de leurs sinus*" (= "of their sines"). The "Δ Soust" headers are sometimes changed in "Δ Addit" or Δ^+.

4.8.1 Truncation lines

The tables of the *Institut* set have red truncation lines located in the columns of the logarithm and of the first difference, after the 12th decimal. The lines are not always clear, but the rounded digits are marked.

On some pages, there is also a red line in Δ^2 at -12.

These lines obviously correspond to the printing project (see section 5.1).

There are no such truncation lines at the *Observatoire*.

4.8.2 Positions of Δ^n

The differences are located at the following positions, with possible minor exceptions:

	Unit levels							
Intervals	$\log\sin x$	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6	Δ^7
$0^q.05000$–$0^q.50000$	−14	−16	−18	−20	−22	−23	−25	−25
$0^q.50000$–$0^q.99700$	−14	−17	−19	−22	−25	−27	−29	
$0^q.99700$–$1^q.00000$	−14	−16	−18	−22	−22			

The positions for the two main intervals are given in the introduction to the tables, except for Δ^7 which is given in -26.[375]

The introductory volume gives the (theoretical) pivots at the above positions but only with the two ranges $0^q.05000$–$0^q.50000$ and $0^q.50000$–$1^q.00000$.

The following table gives an excerpt (identical in both manuscripts) of the tables:

[375] Copy O, introductory volume, p. 11.

<table>
<tr><th>Arc</th><th>log sin (−14)</th><th>Δ^1 (−17)</th><th>Δ^2 (−19)</th><th>Δ^3 (−22)</th><th>Δ^4 (−25)</th><th>Δ^5 (−27)</th><th>Δ^6 (−29)</th><th>Δ^7 (−25)</th></tr>
<tr><td>$0^q.05000_b$</td><td>$\bar{2}$,89464|32984|0644</td><td>8|66715|2383|18</td><td>174|0059|64|1 2</td><td>694|24|4 4|3 7</td><td>41|62|5 6|71</td><td>3|32|70|4</td><td>33|2|33</td><td>40</td></tr>
<tr><td>$0^q.05000_x$</td><td>$\bar{2}$,89464|32984|0644</td><td>8|66715|2383|22</td><td>174|0059|64|1 3</td><td>694|24|4 4|3 7</td><td>41|62|5 6|70</td><td>3|32|70|6</td><td>33|2|34</td><td>40</td></tr>
<tr><td>$0^q.50000_e$</td><td>$\bar{1}$,84948|50021|6798</td><td>68217|7461|02</td><td>2|1430|90|5 3</td><td>67|3 1|5 6</td><td>4 2|97</td><td>3</td><td></td><td></td></tr>
<tr><td>$0^q.50000_b$</td><td>$\bar{1}$,84948|50021|6801</td><td>68217|7461|247</td><td>2|1430|90 0|38</td><td>67 3|2 2|928</td><td>4|2 29|758</td><td>332|18</td><td>3|33</td><td></td></tr>
<tr><td>$0^q.50000_x$</td><td>$\bar{1}$,84948|50021|6801</td><td>68217|7461|246</td><td>2|1430|90 0|38</td><td>67 3|2 2|929</td><td>4|2 29|759</td><td>332|17</td><td>3|34</td><td>0</td></tr>
<tr><td>$0^q.99998$</td><td>$\bar{1}$,99999|99997|8560</td><td>1|6073|58</td><td>1|0715|78|7 4</td><td>1</td><td>5 29</td><td></td><td></td><td></td></tr>
<tr><td>$0^q.99998_x$</td><td>$\bar{1}$,99999|99997|8568</td><td>1|6073|68</td><td>1|0715|78|6 8</td><td>264</td><td>5 29</td><td>0</td><td>10</td><td>0</td></tr>
<tr><td>$0^q.99999$</td><td>$\bar{1}$,99999|99999|4634</td><td>5357|79</td><td>1|0715|78|7 4</td><td>0</td><td>5 29</td><td></td><td></td><td></td></tr>
<tr><td>$0^q.99999_x$</td><td>$\bar{1}$,99999|99997|4642</td><td>5357|89</td><td>1|0715|78|6 8</td><td>−264</td><td>5 29</td><td>0</td><td>10</td><td>0</td></tr>
<tr><td>$1^q.00000$</td><td>$\bar{1}$,99999|99999|9992</td><td></td><td>1|0715|78|7 4</td><td>0</td><td>5 29</td><td></td><td></td><td></td></tr>
<tr><td>$1^q.00000_x$</td><td>0,00000|00000|0000</td><td>−5357|89</td><td>1|0715|78|6 8</td><td>−793</td><td>5 29</td><td>0</td><td>10</td><td>0</td></tr>
</table>

4.8.3 Constancy of Δ^n

For the reconstruction of the interpolations, we have considered the following threshholds:

Interval	Constant difference			
$0^q.05000$—$0^q.09400$				Δ^7
$0^q.09400$—$0^q.18200$			Δ^6	
$0^q.18200$—$0^q.47400$		Δ^5		
$0^q.47400$—$0^q.50000$	Δ^4			
$0^q.50000$—$0^q.70000$			Δ^6	
$0^q.70000$—$0^q.99700$		Δ^5		
$0^q.99700$—$1^q.00000$	Δ^4			

The fact that there are two distinct sequences for the constancies of Δ^n is consistent with the two main ranges for the positions of Δ^n, and must have been intended. There are however a number of variations.

In both manuscripts, Δ^7 is constant (over 4-page intervals) from $0^q.05000$ to $0^q.09400$. From $0^q.09400$ to $0^q.18200$, it is Δ^6 which is constant. From $0^q.18200$ to $0^q.47400$, Δ^5 is constant. From $0^q.47400$ to $0^q.50000$, Δ^4 is constant, but there are anomalies.

From $0^q.50000$ to $0^q.70000$, it is Δ^6 which is constant. It is normally constant on intervals of $0^q.01$ (20 pages), but there are anomalies. For instance, $\Delta^6 = 333$ on $0^q.50$–$0^q.51$, 296 on $0^q.51$–$0^q.52$, 263 on $0^q.52$–$0^q.529$ (in both manuscripts), 235 on $0^q.529$–$0^q.54$, 210 on $0^q.54$–$0^q.55$.

There are also some differences between the manuscripts. For instance, from $0^q.50800$ to $0^q.51000$, copy *I* has $\Delta^6 = 3$, but copy *O* has $\Delta^6 = 296$. The values at the beginning of $0^q.50800$ are: …6045, …535, …55843, …42512, …362, …30669, …296. Copy *I*'s only difference is $\Delta^6 = 3$. In addition, from $0^q.50950$ to $0^q.51000$, a strip covers Δ^6 in copy *I* with the value 333, which has no consequences.

From $0^q.70000$ to $0^q.99700$, Δ^5 is constant by interval, but with some anarchy: $\Delta^5 = 59|25$ from $0^q.70000$ to $0^q.70900$, 5489 (the value for $0^q.71000$)

from $0^q.70900$ to $0^q.71900$, 5087 (the value for $0^q.72000$) from $0^q.71900$ to $0^q.72900$, 4716 (the value for $0^q.73000$) from $0^q.72900$ to $0^q.74000$, 4373 from $0^q.74000$ to $0^q.74500$, 4374 from $0^q.74500$ to $0^q.74800$, 4054 (the value for $0^q.75000$) from $0^q.74800$ to $0^q.75000$. From $0^q.74500$ to $0^q.74800$, copy *O* has $\Delta^5 = 4373$, although this has no consequences.

From $0^q.98900$ to $0^q.99700$, $\Delta^5 = 105$ (approximate value for $0^q.99000$) and from $0^q.99700$ to $1^q.00000$, $\Delta^4 = 529$.

4.8.4 Pivots

According to Prony, the pivots are $0^q.050$, $0^q.052$, $0^q.054$, $0^q.056$, ..., $0^q.498$, $0^q.50$, $0^q.51$, $0^q.52$, $0^q.53$, ..., $0^q.99$.

The previous excerpt gives the values of the tables at the pivots $0^q.050$ and $0^q.50$ and the accuracy is very good, the worst case being for Δ^1 with four units of error.

In general, the computations seem to be quite accurate, and sometimes the table values agree totally with our computed values, for instance in the two following cases:

Angle	log sin	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5
$0^q.24800$	4758	8730	7199	4336	8904	111
$0^q.49800$	9391	3071	4009	1756	4297	

4.8.5 Discrepancy in $0^q.51000$

The interpolated value for $0^q.51000$ is (in both manuscripts) 6911, 5316, 6027, 3703, 3620, 0069, and 296 (copy *O*) or 333 (copy *I*). The value 333 is the value for the start at 51000. The difference between the two manuscripts has no consequences. The value of the computed pivot $0^q.51000$, instead, is 6910, 5308, 6028, 3703, 3621, 0069, 296, the exact values being 6910, 5308, 6028, 3704, 3622, 0069, and 297. As can be observed in our reconstruction (volume 12b), the interpolation should have resulted in a much greater discrepancy, and the interpolated value for $0^q.51000$ therefore looks very suspicious. Perhaps this is the left-over of a previous retrograde interpolation, and the apparently interpolated $0^q.51000$ would then be an earlier pivot computation.

4.8.6 Retrograde interpolations

Retrograde interpolations occur also in the logarithms of sines. Examples are the following intervals, in both manuscripts:

Intervals
$0^q.33986$–$0^q.34000$
$0^q.40370$–$0^q.40400$
$0^q.47986$–$0^q.48000$
$0^q.48991$–$0^q.49000$

The retrograde interpolation from $0^q.49786$ to $0^q.49800$ slightly differs from the other ones (in both manuscripts). There is a glued strip, but this strip does not cover Δ^5, because Δ^4 is constant. And it is the value of Δ^4 from the next page which was taken. We have $\Delta^5 = 0.4$ before $0^q.49800$ and 0.3 afterwards, but it is not taken into account.

4.8.7 Indication of degrees

In copy I, the number of equivalent sexagesimal degrees is sometimes given in the margin. For instance, at the left of $0^q.37100$, we have $33^\circ\!\!,39$. This does possibly also occur in some places in copy O.

4.8.8 Interpolation corrections

A whole new page is glued on the interval $0^q.38450$–$0^q.38500$ (copy I), but there is no retrograde interpolation. On copy O, many column strips are glued.

Sometimes, the fact that calculations were redone is clearly stated. For instance, at the beginning of $\log\sin 0^q.84000$ in copy I, we find the mention "*Log. Sinus de 0,84000 à 0,85000 Cahier recommencé.*"

4.8.9 Errors

Several errors have been corrected in copy O during the preparation of the 1891 tables:[376]

- $\log\sin 0^q.75283 = \overline{1}, 96641\ \underline{1}0023\ 6418$; the underlined digit was corrected to 0; (there was no error in copy I)
- $\log\sin 0^q.86843 = \overline{1}, 99065\ \underline{9}3430\ 9100$; the underlined digit was corrected in red into 0 by Ch. de Villedeuil, from the *Service géographique de l'armée*, sometime before 1891 (there was no error in copy I). Villedeuil is mentioned in the preface of the 1891 tables.[377]

Copy I certainly has many other errors which should be located.

[376] [Service géographique de l'Armée (1891)]
[377] [Service géographique de l'Armée (1891), Roegel (2010f)]

4.8.10 Fragments

The *École nationale des ponts et chaussées*[378] holds a separate sheet for the interpolation of $\log\sin$ from $0^q.57900$ to $0^q.58000$, which was probably discarded and should be compared with the actual tables, using our companion volumes.

[378] PC: Ms. 1745.

4.9 Logarithms of the arc to tangent ratios

The logarithms of the arc to tangent ratios fill the first 100 pages of volume 14 at the *Institut*, and the corresponding volume at the *Observatoire*.

4.9.1 Forms

These tables do not make use of any special form, and the forms which were used were those for the logarithms of numbers (figure 3.1), with handwritten adaptations. “Logarithmes” was sometimes overwritten by $\mathcal{A}'$, “Δ' Addit” by *Soust*, “Δ^2 Soust” by Δ^2+, “Δ^3 Soust” by Δ^3+, “Δ^4 Soust” by Δ^4+, “Δ^5 Soust” by Δ^5+, and “Δ^6 Soust” by Δ^6+.

4.9.2 Truncation lines

The truncation (and rounding) is shown in copy I at positions -12 for $\mathcal{A}'$ and Δ^1, but the lines are usually not drawn. The rounded figures are written in red.

These truncation lines correspond to the printing project (see section 5.1).

4.9.3 Positions of $\mathcal{A}'$ and the Δ^n

The following are the positions of $\mathcal{A}'$ and of its differences over the interval $0^q.00000$–$0^q.05000$:[379]

	Unit levels						
Intervals	$\mathcal{A}'$	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
$0^q.00000$–$0^q.00300$	−13	−18	−21	−24	−26	−31	−33
$0^q.00300$–$0^q.00500$	−14	−18	−21	−24	−26	−31	−33
$0^q.00500$–$0^q.01550$	−13	−18	−21	−24	−26	−31	−33
$0^q.01550$–$0^q.03000$	−14	−18	−21	−24	−26	−31	−33
$0^q.03000$–$0^q.03800$	−14	−17	−19	−22	−25	−27	−33
$0^q.03800$–$0^q.04000$	−14	−16	−18	−20	−22	/	/
$0^q.04000$–$0^q.05000$	−14	−17	−19	−22	−25	−27	−33

There are a number of changes in accuracy, and some idiosyncrasies may have been unrecorded. The following excerpt (identical in both manuscripts) illustrates these changes in accuracy:

[379] There are possibly other intervals with idiosyncrasies that escaped our attention.

Arc	$\mathcal{A}'$ (−14)	Δ^1 (−18)	Δ^2 (−21)	Δ^3 (−24)	Δ^4 (−26)	Δ^5 (−31)	Δ^6 (−33)
$0^q.00000$	4.80388|01229|698	357|1 9289	714|3 85 7 8|853	74|032	4 9|3 54|94	2569	1027|25
$0^q.00000_x$	4.80388|01229|698	357|1 9289	714|3 85 7 8|861	74|032	4 9|3 54|94	2568	1027|25
$0^q.00050_e$	4.80388|00336|715	36|076|4 9290	714|3 86 4 3|014	2|5 41|782	4 9|3 54|97	53919	1027|25
$0^q.00050_x$	4.80388|00336|716	36|076|4 9291	714|3 86 4 3|023	2|5 41|782	4 9|3 55|07	53930	1027|25
$0^q.01550_e$	4.80379|42955|053	1107|961|8 5735	714|9 79 6 7|695	7 6|6 38|055	4 9|4 78|62	1 5|94419	1027|25
$0^q.01550_b$	4.80379|42955|0536	1107|961 8|5735	714 9|79 6 7|695	7 6|6 38|055	4 9|4 78|62	1 5|94419	1027|25
$0^q.01550_x$	4.80379|42955|0517	1107|961 8|5743	714 9|79 6 7|747	7 6|6 38|124	4 9|4 78|65	1 5|94800	1027|25
$0^q.020_e$	4.80373|72128|9971	1429|787 6|0296	715 3|74 5 5|974	9 8|9 21|075	4 9|5 60|65	2 0|56569	1027|25
$0^q.020_x$	4.80373|72128|9976	1429|787 6|0335	715 3|74 5 6|063	9 8|9 21|179	4 9|5 60|80	2 0|57061	1027|25
$0^q.030_e$	4.80355|84826|8828	2145|738 7|2060	716 6|11 7 1|050	1 4 8|6 01|572	4 9|8 17|69	3 0|83569	1027|25
$0^q.030_b$	4.80355|84826|8805	2145|738 7|239	716 6|11 7|1 8	1 4|8 6|02 8	4 9 8|19 2	3|1 0	
$0^q.030_x$	4.80355|84826|8811	2145|738 7|226	716 6|11 7|1 3	1 4|8 6|01 8	4 9 8|17 8	3|0 8	
$0^q.038_e$	4.80336|39072|0190	2719|545 4|202	717 9|59 9|4 7	1 8|8 5|51 9	5 0 0|59 2	3|1 0	
$0^q.038_b$	4.80336|39072|0157	2719|545 4|35	717 9|60|0 5	1|8 8|5 9	5|0 2		
$0^q.038_x$	4.80336|39072|0165	2719|545 4|31	717 9|60|0 2	1|8 8|5 6	5|0 1		
$0^q.04_e$	4.80330|80873|0142	2863|175 6|32	718 3|47|1 5	1|9 8|5 9	5|0 2		
$0^q.04_b$	4.80330|80873|0142	2863|175 6|322	718 3|47 1|5 2	1 9|8 5|94 4	5 0 1|82 1	4|1 5	
$0^q.04_x$	4.80330|80873|0144	2863|175 6|185	718 3|47 1|2 6	1 9|8 5|90 7	5 0 1|77 6	4|1 1	
$0^q.05_e$	4.80298|58532|1635	3582|598 3|169	720 5|84 4|2 1	2 4|8 9|76 4	5 0 5|82 1	4|1 5	
$0^q.05_x$	4.80298|58532|1615	3582|598 2|625	720 5|84 3|9 6	2 4|8 9|90 7	5 0 6|40 0	5|1 4	

The values of the differences for $0^q.00$ are almost exactly those given in the introductory volume.[380]

4.9.4 Accuracy of $\mathcal{A}'$

- from $0^q.00000$ to $0^q.01550$, $\mathcal{A}'$ is given with 13 decimals; however, on the $0^q.003$–$0^q.005$ interval, $\mathcal{A}'$ is given with 14 decimals (there are glued strips in both manuscripts); a similar observation can be made here as for the ratios of arcs to sines, namely that the accuracy in the interval $0^q.003$–$0^q.005$ is not consistent with the table in the next section (§ 4.10), where $\mathcal{A}'$ is assumed to have 13 places before $0^q.01550$; the table in the next section may therefore have used values of $\mathcal{A}'$ from an earlier computation, now hidden under glued strips;

- from $0^q.01550$ to $0^q.05000$, $\mathcal{A}'$ is given with 14 decimals (without a glued strip); however, on the $0^q.1550$–$0^q.01600$ interval, there are 14 decimals, but with a glued strip (*I* and *O*); $\mathcal{A}'$ was probably recomputed in $0^q.01550$ and interpolated on these 50 values.

4.9.5 Pivots

In the reconstruction, we assume that $0^q.00$, $0^q.03$, $0^q.038$, and $0^q.04$ are the pivots.

[380]Copy *O*, introductory volume, p. 17.

Except for $0^q.00000$, these pivots are not computed very accurately. The initial values at $0^q.00000$ are almost all correct, the greatest error being for Δ^2. In turn, this error causes the discrepancy observed in the above table for Δ^2 in $0^q.00050$.

4.9.6 Structure of the interpolation

Like for the arc to sine ratios, the interpolation is more complex than in Prony's description. There are actually four distinct interpolations.

- (interpolation 1) from $0^q.00000$ to $0^q.03000$, we have one long interpolation with $\Delta^6 = 1027|25$; the errors seem to be mainly due to the initial error on Δ^2;
- $0^q.03000$ is a pivot, we have new values, and the number of decimals changes; the value of $\mathcal{A}'$ is not correct: both manuscripts have $\mathcal{A}' = \ldots 8805$ (correct: 8811), and the previous interpolation gave $\ldots 8828$;
- (interpolation 2) from $0^q.03000$ to $0^q.03800$, $\Delta^5 = 310$;
- the values of $0^q.03800$ are new and it is probably a pivot;
- (interpolation 3) from $0^q.03800$ to $0^q.04000$, $\Delta^4 = 502$; this is an anomaly, occuring in both manuscripts; it is Δ^5 that should be constant; in fact, the pages $0^q.03800$–$0^q.04000$ seem to have been replaced in both manuscripts;
- $0^q.04000$ is a pivot (new values) or the result of a cancelled interpolation (former pages $0^q.03800$–$0^q.04000$);
- (interpolation 4) from $0^q.04000$ to $0^q.05000$, $\Delta^5 = 415$.

Some of the interpolations may contain errors which have not been recorded here.

4.10 Logarithms of tangents from $0^q.00000$ to $0^q.05000$

The tables in this section cover 100 pages. The pages are only onesided (recto), the forms are handwritten with ink, not printed, and the pages have a smaller format (29 cm × 44.5 cm) than the other ones.

The tables give $\mathcal{A}' - \log x$, $\log x - \mathcal{A}'$ and $\log x$ for the arcs $0^q.00000$ to $0^q.05000$ and numbers 0 to 5000. The values of $\mathcal{A}'$ and $\log x$ were taken from the previous table and from the table of the logarithms of numbers.

$\log\cot x =$ $\mathcal{A}' - \log x$	Arc	$\log\tan x =$ $\log x - \mathcal{A}'$	N	$\log x$
...	$0^q.00000$		0	
...	...	...	...	...
...	$0^q.05000$		5000	

The only change in the format of this table is that all logarithm values are given with 13 decimals until $0^q.01550$ and with 14 decimals afterwards (starting with $0^q.01550$ on a new page). This occurs in both manuscripts.

This change of accuracy in $0^q.01550$ is not consistent with the changes in the tables of the previous section (§ 4.9), and the present tables possibly have used the results of earlier interpolations of the previous table.

These features were included in our reconstructions.

In addition, this section of the tables also has red lines, namely after the 12th decimal until $0^q.01550$, and after the 13th decimal after $0^q.01550$.

Finally, we can make the same observation as for the corresponding table of logarithms of sines, and it is most likely that the logarithms of numbers used in this table are those from the 1–10000 section of logarithms of numbers described previously.

4.11 Logarithms of tangents after $0^q.05000$

Like for the logarithms of sines, there are no specific forms for the logarithms of the tangents and the general form for the logarithms of numbers was used. However, the printed column head *Nombres* is often striked out and replaced by *Arcs* and the printed column head *Logarithmes* is often supplemented by "*de leurs tangentes*." The "Δ Soust" headers are sometimes changed in "Δ Addit" or Δ^+.

4.11.1 Truncation lines

Copy *I* has red lines for the logarithms, for Δ^1 and Δ^2. They are located at positions -12. There are no truncation lines in copy *O*.

These truncation lines obviously correspond to the printing project (see section 5.1). The *Archives Nationales* hold two printed fragments corresponding to these truncation lines. They cover the ranges $0^q.06400$ to $0^q.06600$ and $0^q.14600$—$0^q.14800$, and contain many errors, probably because the proof pages have never been checked.[381]

4.11.2 Pivots

For the reconstruction, we have assumed that the pivots are the angles $k \times 0^q.002$ for $25 \leq k < 475$ (450 pivots from $0^q.05$ to $0^q.948$), but in the manuscript tables the values were probably never computed from scratch. They are pivots based on other computed values. As mentioned earlier, it seems that the pivots before $0^q.5$ were computed using the logarithms of sines, but that the other pivots were computed using earlier tabulated values of the logarithms of tangents. Our volumes 14b–17b currently reconstruct interpolations using the exact pivots, but a future reconstruction of the interpolations should use the interpolated values from the logarithms of sines to compute the pivots of the logarithms of tangents. Nevertheless, the volumes 14b–17b show what accuracy could have been obtained if the pivots had been computed exactly, and it also allows for a comparison of the interpolated pivots with the exact ones.

The following table shows an excerpt of this table (identical in both manuscripts):

[381] A.N. $F^{17}13571$

Arc	log tan (−14)	Δ^1 (−16)	Δ^2 (−18)	Δ^3 (−20)	Δ^4 (−22)	Δ^5 (−23)
$0^q.20000_e$	$\overline{1}$,51177|60385|4112	2|32116|5513|45	10|0359|12|17	10|96|86|45	16|19|47	34|3
$0^q.20000_x$	$\overline{1}$,51177|60385|4103	2|32116|5513|37	10|0359|12|39	10|96|85|85	16|20|64	32|6
$0^q.20000_b$	$\overline{1}$,51177|60385|4103	2|32116|5513|37	10|0359|12|38	10|96|85|84	16|20|64	32|6
$0^q.21000_e$	$\overline{1}$,53450|39806|0808	2|22602|8755|05	9|0149|08|47	9|49|82|32	13|30|06	26|8
$0^q.21000_x$	$\overline{1}$,53450|39806|0797	2|22602|8754|71	9|0149|09|19	9|49|81|05	13|31|75	25|5
$0^q.21000_b$	$\overline{1}$,53450|39806|0797	2|22602|8754|70	9|0149|09|19	9|49|81|05	13|31|75	25|5
$0^q.22000_e$	$\overline{1}$,55632|92248|5400	2|14041|2771|30	8|1276|14|28	8|28|45|07	11|03|59	21|2
$0^q.22000_x$	$\overline{1}$,55632|92248|5406	2|14041|2771|31	8|1276|14|28	8|28|44|93	11|04|05	20|2
$0^q.22000_b$	$\overline{1}$,55632|92248|5406	2|14041|2771|32	8|1276|14|27	8|28|44|92	11|04|05	20|2
$0^q.70000_e$	0.29283|41192|1606	1|68648|1167|50	3|8496|92|37	3|42|21|13	3|09|81	4|2
$0^q.70000_x$	0.29283|41192|1601	1|68648|1167|56	3|8496|92|65	3|42|21|66	3|10|37	4|3
$0^q.70000_b$	0.29283|41192|1601	1|68648|1167|56	3|8496|92|64	3|42|21|65	3|10|36	4|4
$0^q.71000_e$	0.30989|70417|8509	1|72674|0901|46	4|2081|65|71	3|75|55|09	3|57|67	5|2
$0^q.71000_x$	0.30989|70417|8509	1|72674|0901|47	4|2081|65|71	3|75|55|07	3|57|67	5|1
$0^q.71000_b$	0.30989|70417|8509	1|72674|0901|46	4|2081|65|71	3|75|55|09	3|57|67	5|2
$0^q.72000_e$	0.32738|10415|9085	1|77076|0070|40	4|6024|73|67	4|14|03|76	4|13|73	6|2
$0^q.72000_x$	0.32738|10415|9083	1|77076|0070|52	4|6024|73|68	4|14|03|74	4|13|73	6|1
$0^q.72000_b$	0.32738|10415|9083	1|77076|0070|52	4|6024|73|67	4|14|03|76	4|13|73	6|2

In the previous excerpt, Δ^5 is constant by interval, and we therefore did not give Δ^6 nor Δ^7.

Note that the values in $0^q.71000_e$ and $0^q.71000_b$ are identical, because a retrograde interpolation takes place before $0^q.71000$.

4.11.3 Position of the Δ^n

The following table shows the positions of the differences in the tables, with possible minor exceptions:

	Positions							
Intervals	log tan	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6	Δ^7
$0^q.05000$—$0^q.84900$	−14	−16	−18	−20	−22	−23	−25	−25
$0^q.84900$—$0^q.95000$	−14	−16	−18	−20	−22	−24	−25	−25

In our reconstruction, because of the change in $0^q.84900$, we have recomputed the value of Δ^5, although $0^q.84900$ is not considered a pivot.

Δ^7 is only given from $0^q.05000$ to $0^q.09400$ and from $0^q.91400$ (where $\Delta^7 = 1$) to $0^q.95000$.

Before $0^q.84900$, Δ^5 is usually at −23, but sometimes the interpolation is done with an additional 0 at the beginning. It may also happen that there is a genuine computation to 24 places.

The following excerpt (identical in both manuscripts) illustrates the changes in the positions of the differences:

<table>
<tr><th>Arc</th><th>log tan
(−14)</th><th>Δ^1
(−16)</th><th>Δ^2
(−18)</th><th>Δ^3
(−20)</th><th>Δ^4
(−22)</th><th>Δ^5
(−24)</th><th>Δ^6
(−25)</th><th>Δ^7
(−25)</th></tr>
<tr><td>$0^{q}.05000$</td><td>$\overline{2}$,89598|41511|1987</td><td>8|72084|7148|16</td><td>172|9277|45|46</td><td>694|27|11|04</td><td>41|62|51|29</td><td>3|32|70|4</td><td>33|2|3 3</td><td>40</td></tr>
<tr><td>$0^{q}.05000_x$</td><td>$\overline{2}$,89598|41511|1987</td><td>8|72084|7148|20</td><td>172|9277|45|47</td><td>694|27|11|04</td><td>41|62|51|28</td><td>3|32|70|6</td><td>33|2|3 4</td><td>40</td></tr>
<tr><td>$0^{q}.05200_e$</td><td>$\overline{2}$,91309|05766|7896</td><td>8|38828|1658|91</td><td>159|8290|14|67</td><td>617|23|09|71</td><td>35|57|99|73</td><td>2|74|18|4</td><td>25|2|3 3</td><td></td></tr>
<tr><td>$0^{q}.05200_x$</td><td>$\overline{2}$,91309|05766|7871</td><td>8|38828|1657|75</td><td>159|8290|19|02</td><td>617|22|96|17</td><td>35|58|34|48</td><td>2|73|48|6</td><td>26|2|6 9</td><td>30</td></tr>
<tr><td>$0^{q}.05200_b$</td><td>$\overline{2}$,91309|05766|7868</td><td>8|38828|1657|87</td><td>159|8290|19|06</td><td>617|22|96|18</td><td>35|58|34|49</td><td>2|73|48|5</td><td>26|2|6 8</td><td>30</td></tr>
<tr><td>$0^{q}.06000_e$</td><td>$\overline{2}$,97555|96936|7425</td><td>7|28068|3355|50</td><td>119|8738|25|72</td><td>401|85|34|53</td><td>20|07|79|93</td><td>1|33|95|2</td><td>10|8|4 6</td><td></td></tr>
<tr><td>$0^{q}.06000_x$</td><td>$\overline{2}$,97555|96936|7410</td><td>7|28068|3355|07</td><td>119|8738|26|93</td><td>401|85|30|88</td><td>20|07|89|30</td><td>1|33|76|3</td><td>11|1|3 7</td><td>11</td></tr>
<tr><td>$0^{q}.06000_b$</td><td>$\overline{2}$,97555|96936|7410</td><td>7|28068|3355|08</td><td>119|8738|26|92</td><td>401|85|30|88</td><td>20|07|89|30</td><td>1|33|76|2</td><td>11|1|3 6</td><td>11</td></tr>
<tr><td>$0^{q}.09400_e$</td><td>$\overline{1}$,17242|00418|8905</td><td>4|68775|3837|38</td><td>48|4035|84|35</td><td>104|57|37|98</td><td>3|33|35|34</td><td>14|27|70</td><td>6|5 7</td><td></td></tr>
<tr><td>$0^{q}.09400_x$</td><td>$\overline{1}$,17242|00418|8879</td><td>4|68775|3836|90</td><td>48|4035|85|53</td><td>104|57|34|98</td><td>3|33|41|43</td><td>14|18|3</td><td>7|5 4</td><td></td></tr>
<tr><td>$0^{q}.09400_b$</td><td>$\overline{1}$,17242|00418|8879</td><td>4|68775|3836|89</td><td>48|4035|85|53</td><td>104|57|34|97</td><td>3|33|41|44</td><td>14|18|40</td><td>7|5 4</td><td></td></tr>
<tr><td>$0^{q}.50000_e$</td><td>0.00000|00000|0011</td><td>1|36437|6354|18</td><td>1|34|76</td><td>1|34|66|51</td><td>68</td><td>6</td><td>0</td><td>0</td></tr>
<tr><td>$0^{q}.50000_x$</td><td>0.00000|00000|0000</td><td>1|36437|6354|07</td><td>1|34|66</td><td>1|34|65|85</td><td>1</td><td>7</td><td>0</td><td>0</td></tr>
<tr><td>$0^{q}.50000_b$</td><td>0.00000|00000|0000</td><td>1|36437|6354|07</td><td>1|34|66</td><td>1|34|65|85</td><td>1</td><td>7</td><td>0</td><td>0</td></tr>
<tr><td>$0^{q}.91400_e$</td><td>0.86672|85660|5418</td><td>5|11219|0238|47</td><td>58|0009|71|71</td><td>136|67|31|56</td><td>4|76|66|40</td><td>22|03|91</td><td>1|1 2|6</td><td></td></tr>
<tr><td>$0^{q}.91400_x$</td><td>0.86672|85660|5432</td><td>5|11219|0238|96</td><td>58|0009|73|28</td><td>136|67|35|94</td><td>4|76|75|73</td><td>22|18|89</td><td>1|2 9|1</td><td>1</td></tr>
<tr><td>$0^{q}.91400_b$</td><td>0.86672|85660|5432</td><td>5|11219|0238|96</td><td>58|0009|73|28</td><td>136|67|35|94</td><td>4|76|75|73</td><td>22|18|85</td><td>1|2 9|1</td><td>1</td></tr>
<tr><td>$0^{q}.95000_e$</td><td>1.10401|58488|8014</td><td>8|72257|7119|08</td><td>173|0667|19|69</td><td>695|52|03|73</td><td>41|75|49|42</td><td>3|33|67|39</td><td>32|4 4|8</td><td>30</td></tr>
<tr><td>$0^{q}.95000_x$</td><td>1.10401|58488|8013</td><td>8|72257|7120|34</td><td>173|0667|24|69</td><td>695|52|18|57</td><td>41|75|85|44</td><td>3|34|37|34</td><td>33|4 7|4</td><td>40</td></tr>
</table>

4.11.4 Accuracy

The values of Δ^i at the pivots are often slightly wrong, but this is probably mainly due to the fact that these values were computed from interpolated logarithms of sines. The values of the differences for $0^{q}.052$ are almost exactly those given in the introductory volume.[382]

4.11.5 Interpolation adaptations

A '0' is often added to Δ^5 for the interpolation and this was not done in our reconstructions. For instance, at $0^{q}.09200_b$, the manuscripts give $\Delta^5 = 15|79|30$, but the 0 is added for the computation. For this pivot, $\Delta^6 = 85|7$ (instead of the correct $85|8$) and $\Delta^7 = 1$ which is correct. But for both manuscripts, we have $\Delta^5 = 15869$ at the interpolation ending in $0^{q}.09200$. So, Δ^5 seems to be moved from position -23 to -24, but it is only an impression. This adaptation is done once in a while.

4.11.6 Interpolated values

In $0^{q}.50$, copy I gives $\log\tan = 000\ldots33$. At this position, I gives $\Delta^4 = 68$ (exact: -1), essentially because the correct value $\Delta^4 = 132$ for $0^{q}.49800$ has

[382] Copy O, introductory volume, p. 16.

been decremented by 200 (Δ^5 being rounded to 1), and $132 - 200 = -68$ and the sign is not indicated. The change of sign is shown above by a cross, but the cross is misplaced because of these rounded calculations.

The final interpolated value for $0^q.95000$ is (in both manuscripts): $\log\tan = 1.1040\ldots8014$ (exact: 8013), $\Delta^1 = \ldots71|19|08$ (exact: 712034), $\Delta^2 = 173|06|67|19|69$ (exact: 17306672469), $\Delta^3 = 695|52|03|73$ (exact: 695521857), $\Delta^4 = 41|75|49|42$ (exact: 41758544), $\Delta^5 = 333|67|39$ (exact: 3343734), $\Delta^6 = 3244|8$ (exact: 33474), $\Delta^7 = 30$ (exact: 40). 30 is the value of Δ^7 for $0^q.94800$.

4.11.7 Constancy of Δ^n

For the reconstruction of the interpolations, we have considered the following threshholds:

Interval	Constant diff.		
$0^q.05000$—$0^q.09400$			Δ^7
$0^q.09400$—$0^q.18200$		Δ^6	
$0^q.18200$—$0^q.84900$	Δ^5		
$0^q.84900$—$0^q.91400$		Δ^6	
$0^q.91400$—$0^q.95000$			Δ^7

In this table, Δ^4 is never constant, except in case of anomalies.

4.11.8 Values of the logarithms

After $0^q.50$, logarithms are positive.

4.11.9 Retrograde interpolations

There are a number of retrograde interpolations, and the following ones (the list of which may be incomplete) are found in both manuscripts:

$0^q.24366$–$0^q.24400$	$0^q.70991$–$0^q.71000$
$0^q.53750$–$0^q.53800$	$0^q.72791$–$0^q.72800$
$0^q.53986$–$0^q.54000$	$0^q.72981$–$0^q.73000$
$0^q.56391$–$0^q.56400$	$0^q.75850$–$0^q.75900$
$0^q.56981$–$0^q.57000$	$0^q.84191$–$0^q.84200$
$0^q.58971$–$0^q.59000$	$0^q.84391$–$0^q.84400$
$0^q.59966$–$0^q.60000$	$0^q.89936$–$0^q.89950$
$0^q.63581$–$0^q.63600$	$0^q.89950$–$0^q.90000$
$0^q.65991$–$0^q.66000$	$0^q.94196$–$0^q.94200$

There are sometimes some minor differences between the manuscripts. For instance, for the interpolation $0^q.63581$–$0^q.63600$, copy I has $\Delta^5 = 17$ (instead of 18 on the next page). The reason why this is so is that the strip was not glued over Δ^5. Copy O instead has $\Delta^5 = 18$. This difference has no incidence, because in both cases Δ^5 is rounded to 2 when added to Δ^4.

All known retrograde interpolations occur in both manuscripts, except for the $0^q.74750$–$0^q.74800$ interpolation. This interpolation is a normal forward interpolation in copy O. In copy O, no strip is glued on the interval $0^q.74750$–$0^q.74800$, and $\Delta^5 = 99$. In copy I instead, a strip is glued on the four last digits of the logarithm and on the differences Δ^1–Δ^5, and we have a retrograde interpolation.

The values at the beginning of the $0^q.74750$–$0^q.74800$ interval are:

	log.	Δ^1	Δ^2	Δ^3	Δ^4	Δ^5
I	…7654	…7575	…2175	…2792	63230	103
O	…7684	…7588	…2186	…2805	63211	99
theor. int.	…7669	…7572	…2180	…2803	63210	99

4.11.10 Unidentified interpolations

There are at least two cases of interpolations which are not as clearly retrograde as the other ones which have been observed:

- first case:
  - $0^q.52941$–$0^q.52950$ (I, O): retrograde interpolation with respect to the following page;
  - $0^q.52950$–$0^q.53000$ (I, O): the whole page is glued, perhaps a retrograde interpolation, but it is not clear, because only Δ^1, Δ^4 and Δ^5 coincide for $0^q.53000$ (values for copy I):

	log tan	Δ^2	Δ^3
$0^q.53000_e$	…7425	…1989	…6786
$0^q.53000_b$	…7427	…1992	…6790

- A similar problem occurs on the $0^q.50950$—$0^q.51000$ interval, but without any previous inverse fragment. Neither $0^q.50950$, nor $0^q.51000$ shares its values with the previous or following intervals.

The most plausible explanation seems to be that we have retrograde interpolations, but that these interpolations rest on earlier computations of the pivots $0^q.51$ and $0^q.53$. When the pivots were recomputed, these discrepancies surfaced.

4.11.11 Indication of degrees

Like for the log sin tables, in some cases the equivalent angle in degrees is indicated in the margin. For instance, in copy O, for $0^q.36300$, we have $32^\circ\!.67$.

4.11.12 Interpolation corrections

Some of the interpolations have been corrected by gluing strips of paper with new interpolations. Some examples are:

- In copy O, the interval $0^q.84050$–$0^q.84100$ is totally covered by a new strip, but it is not a retrograde interpolation. In copy I, there is no new strip.
- In copy O, the values of the logarithms and of Δ^1 on the interval $0^q.84100$–$0^q.84150$ are covered by a strip, but it is not a retrograde interpolation. In copy I, there is no new strip.
- In copy O, the values of the logarithms and of Δ^1 on the interval $0^q.84150$–$0^q.84200$ are covered by a strip. Another strip was glued on top of it for the interval $0^q.84191$–$0^q.84200$ which is a retrograde interpolation. We have therefore two layers of corrections. In copy I, there is only one layer.
- In copy I, the values of the logarithms and of Δ^1 on the interval $0^q.65700$–$0^q.65800$ are covered by a strip. This is only partially the case in copy O.

4.11.13 Errors

During the preparation of the 1891 tables, some errors have been corrected in copy O by the *Service géographique de l'armée*, in particular:

$0^q.80394$	(wrong)	0.4974$\underline{9}$ 85914 7138
	(exact)	0.49744 85914 7142
$0^q.80416$	(wrong)	0.4979$\underline{7}$ 84039 3024
	(exact)	0.49796 84039 3022
$0^q.80626$	(wrong)	0.50295 6$\underline{9}$268 7629
	(exact)	0.50295 60268 7628
$0^q.80722$	(wrong)	0.50525 1$\underline{7}$163 5466
	(exact)	0.50525 18163 5456
$0^q.93229$	(wrong)	0.97158 53$\underline{8}$06 2477
	(exact)	0.97158 53706 2481

In each case, there was a wrong digit in the value of log tan, and it was not the last digit.

Since errors do not propagate, it means that the results have been copied from other sheets.

Other errors corrected by the *Service géographique de l'armée* are:

- $\log\tan 0^{\mathrm{q}}.55981 = 0.08208\ 78\underline{2}19\ 9732$ (underlined digit should be 1, and actually the last two digits are false too, but this was not noted); the value is correct in copy *I* (that is, 0.08208 78119 9732)

- $\log\tan 0^{\mathrm{q}}.55982 = 0.08210\ 17\underline{1}02\ 4980$ (underlined digit should be 0, and actually the last two digits are false too, but this was not noted); the value seems correct in copy *I*

- $\log\tan 0^{\mathrm{q}}.55983 = 0.08211\ 55\underline{9}85\ 8525$ (underlined digit should be 8, and actually the last two digits are false too, but this was not noted); the value is correct in copy *I* (that is, 0.08211 55885 8525)

These three errors are consecutive and are not found in copy *I*.

The *Service géographique de l'armée* also found two errors within the retrograde interpolation $0^{\mathrm{q}}.63581$–$0^{\mathrm{q}}.63600$ in copy *O*:

- $\log\tan 0^{\mathrm{q}}.63581 = 0.19118\ 79\underline{7}82\ 2546$ (should be 8);

- $\log\tan 0^{\mathrm{q}}.63582 = 0.19120\ 29\underline{6}56\ 6298$ (should be 7).

The underlined digits are false. Copy *I* has the correct values.

4.12 Abridged tables of logarithms of sines and tangents

These tables contain the logarithms of sines and tangents to eight or nine places (depending on the interval), to be printed with seven places. There are two copies of the tables, one belonging to the set at the *Observatoire* (copy O), the other in the library of the *École nationale des ponts et chaussées* (copy P).[383] These two manuscripts are nearly identical, except for some minor details. They both carry a DE PRONY stamp.

The spines are not identical. Copy O has "*Tables des logarithmes de 10000 en 10000*" and copy P bears "*Tables manuscrites des logarithmes sinus et tangentes de 10000 en 10000*," with a binding from the *Ponts et chaussées*.

4.12.1 Truncation

The copy at the *Observatoire* contains red truncation lines after the 7th decimal for the logarithms and the first differences, but neither for Δ^2, nor Δ^3. The volume at the *Ponts et chaussées* has no truncation lines.

This truncation after seven places corresponds to the project of printing these tables at that accuracy. Since the other volumes containing truncation lines are those of the *Institut*, we can conclude that this volume should actually have been part of the *Institut* set, whereas the volume at the *Ponts et chaussées* should have been part of the *Observatoire* set.

4.12.2 Positions of the Δ^n

The following table gives the positions of the logarithms, of Δ^n, $\log n$, $\mathcal{A}$, and $\mathcal{A}'$ on the interval $0^{\mathrm{q}}.0000$—$1^{\mathrm{q}}.0000$. These positions were used in volumes 20a and 20b.

	Unit levels										
Intervals	$\log n$	$\mathcal{A}$	$\log\sin$	Δ_+	Δ^2_-	Δ^3_-	$\mathcal{A}'$	$\log\tan$	Δ_+	Δ^2_-	Δ^3_-
$0^{\mathrm{q}}.0000$—$0^{\mathrm{q}}.0500$	−8	−8	−8				−8	−8			
$0^{\mathrm{q}}.0500$—$0^{\mathrm{q}}.2500$			−8	−8				−8	−8		
$0^{\mathrm{q}}.2500$—$0^{\mathrm{q}}.5000$			−9	−10	−12	−12		−9	−10	−12	−12
$0^{\mathrm{q}}.5000$—$0^{\mathrm{q}}.7500$			−9	−10	−12	−12		−9			
$0^{\mathrm{q}}.7500$—$1^{\mathrm{q}}.0000$			−9	−10	−12	−12		−8			

[383]Ms. Fol. 242. The name Corancez also appears on the stamp on the title page of the volume.

It is in fact strange that the interpolations were not totally explicited, and this makes one wonder if all the interpolations are new, or if some parts have been directly lifted from the main tables.

4.12.3 Structure

The tables are structured in five parts: $0^q.0000$ to $0^q.0500$ (10 pages), $0^q.0500$ to $0^q.2500$ (40 pages), $0^q.2500$ to $0^q.5000$ (50 pages), $0^q.5000$ to $0^q.7500$ (50 pages), and $0^q.7500$ to $1^q.0000$ (50 pages). Each part is formated differently. Each page contains 51 values, the last one of a page being also the first value of the next page.

We have used interpolations only on the intervals on which the last given difference was constant by interval. We could have computed a hidden interpolation, but since the origin of some of the values is not yet totally clear, we have decided to postpone such an approach.

Moreover, contrary to the main volumes of the *Tables du cadastre* where the interpolations normally start at the top of a page and end at the bottom of a page, this table contains interpolations which end before the ends of the pages. In that case, there is no duplication between the last interpolated value, and the newly computed one. If however the interpolation ends at the bottom of a page, we are in the same configuration as in the main volumes.

From $0^q.0000$ to $0^q.0500$

In the first part of the volume (10 pages, starting recto), only the values of $\log\sin$, $\mathcal{A}$, $\log n$, $\log\tan$ and $\mathcal{A}'$ are given. They are all given with eight decimals. No differences are given. The value of $\log\sin$ is actually computed from $\log n$ and $\mathcal{A}$, and $\log\tan$ is computed from $\log n$ and $\mathcal{A}'$ (see eq. (2.45) and (2.56)).

The values of $\mathcal{A}$ and $\mathcal{A}'$ were computed by interpolation, or copied, but the interpolation (if any) is not detailed here.

Our reconstruction only gives the exact values.

From $0^q.0500$ to $0^q.2500$

In the second part of the table (40 pages, starting recto), the values of $\log\sin$, $\log\tan$ as well as the first differences, are given with eight decimals.

There was possibly an interpolation which is only summarized here. The values of Δ^2 are not given and they must have been used to obtain the Δ^1. The interpolation is either a new one, or the values were copied from the main tables.[384]

Our reconstruction only gives the exact values.

From $0^q.2500$ to $0^q.5000$

The third part of the volume (50 pages, starting recto) is computed by interpolation. The logarithms and the differences Δ^1, Δ^2 and Δ^3 are given both for the sines and the tangents.

In this interval, $\log\sin$, $\log\tan$, Δ^1, Δ^2, and Δ^3 are computed every $0^q.0040$: for $0^q.2500$, $0^q.2540$, $0^q.2580$, ..., $0^q.4980$. These values are also pivots in the great tables.[385]

We have reconstructed this interpolation for $\log\sin$ and $\log\tan$ (volume 20b).

From $0^q.5000$ to $0^q.7500$

The fourth part of the volume is 50 pages long and starts on a recto page. Differences are no longer given for $\log\tan$. Δ^1, Δ^2 and Δ^3 are only given for $\log\sin$. The fact that differences were not given for $\log\tan$ may mean that the values of the logarithms were copied from the Great Tables.

$\Delta^1\log\sin$ was recomputed in $0^q.5000$, but $\Delta^2\log\sin$ was not. Afterwards, $\log\sin$, $\Delta^1\log\sin$, $\Delta^2\log\sin$, and $\Delta^3\log\sin$ are computed every $0^q.0040$ from $0^q.5020$ to $0^q.7500$. In addition, $\Delta^3\log\sin$ is indicated on every first and last line of a page.[386]

We have reconstructed the interpolation for $\log\sin$ (volume 20b).

From $0^q.7500$ to $1^q.0000$

The last part of the volume is 50 pages long. Differences are no longer given for $\log\tan$. The fact that differences were not given for $\log\tan$ may mean that the values of the logarithms were copied from the Great Tables.

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[384] However, as mentioned earlier, the differences could not have been copied directly, as the step of the tables is not the same.

[385] Incidentally, one reason for recomputing the interpolations may have been to bypass computations which were not totally finished. In that case, it was indeed faster to start new interpolations than to wait for the main ones to be completed. The fact that the pivots of the abridged tables coincide with those of the main table for this interval may support this. On the other hand, not all pivots of the interval $0^q.5000$ to $1^q.0000$ in the abridged tables are pivots in the main table, so this again pleads for a completion of the main tables before starting the abridged ones.

[386] Not all of the pivots of the abridged table in this interval are pivots in the main tables.

In this interval, $\Delta^3 \log\sin$ is computed every $0^q.0040$ from $0^q.7500$ until the end.[387]

Normally, $\Delta^3 \log\sin$ becomes equal to 0 after $0^q.9080$ and the tables should reasonably use $\Delta^3 \log\sin = 0$ after $0^q.9100$. There are however some anomalies in the tables, and we have for instance $\Delta^3 = 1$ for $0^q.8900$, then $\Delta^3 = 0$ for $0^q.8940$ (and $\Delta^2 = 11000$ constant from $0^q.8940$ to $0^q.8979$), then again $\Delta^3 = 1$ for $0^q.8980$ (and $\Delta^2 = 11020$ which decreases until $\Delta^2 = 10981$ for $0^q.9019$), then $\Delta^3 = 0$ for $0^q.9020$ (and $\Delta^2 = 10970$, exact: 10973). There are several such variations until the end.

Both manuscripts do indeed have:

Arc	$\Delta^1 \log\sin$	$\Delta^2 \log\sin$	$\Delta^3 \log\sin$
$0^q.8979$	110300	11000	0
$0^q.8980$	110197	11020	1

The values of the logarithm, of $\Delta^1 = 110197$, and $\Delta^2 = 11020$ must be recomputations.

At the end, the value given for $\log\tan$ of $1^q.0000$ is "*infini positif*."

We have reconstructed the interpolation for $\log\sin$ (volume 20b).

4.12.4 Corrections

This volume also exhibits a number of corrections. A strip is for instance glued over the last four digits of $\log\sin$ from $0^q.7540$ to $0^q.7550$ (P) and from $0^q.7541$ to $0^q.7550$ (O). This strip appears as follows at the *Observatoire*:

$0^q.7541$	3356
$0^q.7542$	1094
$0^q.7543$	8819
$0^q.7544$	6531
$0^q.7545$	4231
$0^q.7546$	1919
$0^q.7547$	9594
$0^q.7548$	7257
$0^q.7549$	4907
$0^q.7550$	2544

[387] Like in the previous case, not all of the pivots of the abridged table in this interval are pivots in the main tables.

Another strip was glued on the intervals $0^q.8900$–$0^q.8940$ (P) and $0^q.8901$–$0^q.8939$ (O).

Some of the corrections were made as a consequence of the 1891 tables. For $0^q.0146$, for instance, log sin was given as $\overline{2}.36044\,467$ and it was corrected into $\overline{2}.36043\,467$ by Ch. de Villedeuil in 1888, as noted in the margin.

4.12.5 Accuracy

We have only sampled a few values to check the accuracy of this volume. The values for $0^q.0001$ are for instance all correct. This is also the case for $0^q.0500$ (first value) and $0^q.0550$.

For $0^q.2500$ (first value), the manuscripts have $\log\sin = \ldots 660$ (exact: 661), $\Delta\log\sin = \ldots 588$ (exact: 582), $\Delta^2\log\sin = \ldots 73150$ (exact: 73117), $\Delta^3\log\sin = 55$ (exact), $\log\tan = \ldots 314$ (exact: 315), $\Delta\log\tan = \ldots 223$ (exact: 217), $\Delta^2\log\tan = \ldots 589$ (exact: 561), $\Delta^3\log\tan = 57$ (exact).

For the interpolated value $0^q.5000$, log sin is correct, $\Delta\log\sin = 682082$ (copy P, 682085 in copy O, and exact value 682081), $\Delta^2\log\sin = 21420$ (exact: 21425), $\log\tan = \ldots 003$ (exact: 000), and $\Delta\log\tan = \ldots 4378$ (exact: 4376).

The last values of $\Delta^2\log\tan$ for the interpolation to $0^q.5000$ differ in both manuscripts. We have $\Delta^3\log\tan = 14$ in both cases, but the following values for $\Delta^2\log\tan$:

Arc	$\Delta^2\log\tan$ (O)	$\Delta^2\log\tan$ (P)
$0^q.4997$	20	18
$0^q.4998$	06	04
$0^q.4999$	/	−10
$0^q.5000$	/	/

For $0^q.7500$, both copies give the correct $\log\sin = \ldots 5346$, $\Delta\log\sin$ is correct too, $\Delta^2\log\sin = 12557$ (exact: 12553), and $\Delta^3\log\sin$ and log tan are correct.

4.13 Multiples of sines and cosines

These tables are actually unrelated to the tables of logarithms. The tables were probably computed between 1794 and 1796, but only when time permitted.

These tables comprise one volume and are only part of the set at the *Institut* (volume 18). The title of the volume is "*Multiples des sinus et cosinus des arcs depuis 0 jusqu'à 0,500*."

In an addition to a note dated March 19, 1819, Prony writes that this table is the only one which was not made in two copies.[388] However, the *Ponts et chaussées* contain a manuscript[389] which is exactly that volume, except that it is not bound. This unbound copy is made of 25 groups (bundles) of 20 pages of tables, except for the first group which has 21 pages. The unbound copy may be the original of which the *Institut* volume is a clean copy.

The tables give the values of the sines and cosines with 5 places every 1000th of the quadrant. Each page covers one angle value and gives 100 multiples of the corresponding sines and cosines. The copy at the *Institut* is also made of $21 + 24 \times 20$ pages of tables. The first group starts with the multiples of the sines and cosines of $0^q.000$ on a recto page and ends with the multiples of the sines and cosines of $0^q.020$ also on a recto page. The following page is left blank and the 480 remaining pages follow, ending with $0^q.500$ on a verso page.

It should be noted that the values are rounded, and that the rounded values—not the exact values—are used in the multiples. For instance, for $\alpha = 0^q.500$, the manuscripts give $\sin\alpha = \cos\alpha = 0.70711$ and $10\sin\alpha = 7.0711\underline{0}$, but the exact computation would have given $10\sin\alpha = 7.07107$.

At the end of the volume at the *Institut*, there are rounding marks to three places for every value. For instance, on the last page, for the factor 9.8, in addition to the value 6.92968 ($= 0.70711 \times 9.8$, rounded), we have the rounded value 6.930. Red truncation lines after the third decimal and the associated red rounded digits also appear in the unbound copy.

Some of the pages bear the names of the calculators. The first group at the *Institut* bears the inscription "*1er cahier commencé le 11 ventôse an 4*" (1st bundle started on 11 Ventôse an 4, = 1st March 1796) and at the end of the bundle it says "*Pigeou, fini le 5 Germinal*" (finished on 5 Germinal, = 25 March 1796). Other names appear in the *Institut* copy: Henry (2nd bundle, 1st page), Bulton (4th bundle, 1st page), Pigeou (5th bundle, last page), Letellier (6th bundle, 1st page), Pigeou (7th bundle, last page), Ferat

388 Archives de l'Académie des Sciences, Prony file.

389 PC: Ms.Fol.1890.

(15th bundle, last page), and Letellier (19th bundle, 1st page).

The bundles of the unbound copy at the *Ponts et chaussées* sometimes carry years on the cover, such as 1794 for the sines from $0^q.261$ to $0^q.280$ or 1796 for the sines from $0^q.381$ to $0^q.400$, and this seems to hint to it as being the original version of the *Institut* volume.

Chapter 5

Printing the tables

5.1 Planned structure

Although the *Tables du cadastre* have never been printed, there have been several projects for printing them, partial printings were made in the 1790s and attempts to revive the project took place in the 1820s.

Moreover, the plans seem to have evolved with time, and also with financial difficulties. The first detailed complete printing project seems to have been drafted in 1794, but there were subsequent variations and simplifications.

5.1.1 Project 1 (1794)

A long report of 2 Thermidor II (20 July 1794) gave a detailed description of the projected tables.[390] The sines were going to be computed to 25 places, and printed to 22 places with five columns of differences, every 10000th of the quadrant.[391] The logarithms of sines and tangents were going to be computed to 15 places and published to 12 places, every 100000th of the quadrant. The logarithms of numbers would be computed to 12 places from 1 to 200000. The report also sketches the layout of the tables. The table of sines would have 100 pages, the logarithms of sines and tangents would have 500 pages (together), and the logarithms of numbers 400 pages. Although the report does not state it explicitely, this suggests that there would have been four columns of 100 logarithms of sines or tangents per page, and five columns of 100 logarithms of numbers per page, probably with first differences. We call

[390] A.N. F^{17}1238

[391] By that time, the sines were almost complete. They were probably finished in August or September 1794.(A.N. F^{14}2146)

this project, project 1.

The total number of pages is 1000, which were envisioned in two volumes, one with the sines and the logarithms of numbers (500 pages), and the other with the logarithms of sines and tangents. The first volume would contain a 20 pages introduction.

5.1.2 Project 2 (ca. 1794)

The plans then seem to have evolved, perhaps after the printer found that the first project led to too crowded pages. In the new project,[392] the tables were to contain:

- sines with 22 decimals and five columns of differences (100 pages);
- logarithms of sines, with first and second differences, all to 12 decimal places (475 pages);
- logarithms of the ratios sines/arcs, with first and second differences, all to 12 decimal places (25 pages);
- logarithms of tangents, with first and second differences, all to 12 decimal places (450 pages);
- logarithms of the ratios tangents/arcs, with first and second differences, all to 12 decimal places, from $0^q.00000$ to $0^q.05000$ (25 pages) and logarithms of the ratios cotangents/arcs from $0^q.95000$ to $1^q.00000$ (25 pages);
- logarithms of the numbers from 1 to 10000 computed with 28 decimals[393] (50 pages);
- logarithms of the numbers from 10000 to 200000, with first and second differences, all to 12 decimal places (950 pages).

The total number of pages was to be 2100 pages. This project being the one with the best specifications, it is the one we chose to reproduce. But since the logarithms of the ratios of cotangents to arcs are merely obtained from the logarithms of the ratios tangents/arcs by changing the sign, they were

[392] A.N. $F^{17}1238$

[393] This section of the tables is the only one where the number of decimals kept for printing is possibly not smaller than the number of decimals used for the computation. It may actually have only been a tentative proposition from the printer, which was then quickly discarded.

not given in our reconstruction, which therefore only contains 2075 pages. Our reconstruction can be compared with two almost complete printings of the table of sines at the library of the *Ponts et chaussées*[394] and with the fragments of the logarithms of tangents. Moreover, the volumes at the *Institut* contain truncation lines after the 12th decimal, which obviously were aimed for printing the tables. The volumes at the *Observatoire* do not contain such truncation lines.

5.1.3 Project 3 (1794–1795)

The ambitious project 2 was never completed and was probably altered around 1794 or 1795. The most conspicuous change is that the logarithms of numbers from 1 to 10000 were only computed to 19 places, and not 28. For their printing, these logarithms would probably have been reduced even further to 12 places.

It seems however that the layout of the logarithms of sine and tangent ratios was also revised, and that these sections were eventually totally set. They were probably covering 17 pages each, so that it is likely that in this third project, there were 300 values per page in these sections. We have unfortunately not found any proof pages of these tables. Such a layout was then again suggested in a note from 1819.[395]

Finally, this note also considered the tables of logarithms of sines and tangents, and put them at 500 pages each, which is consistent with the second project. It seems very likely that the suggestions in the 1819 note were themselves consistent with the existing plates.

5.1.4 Project 4 (1819)

A new project was considered in 1819, when the British Government became involved in the joint publication of the tables.

A first idea was to print the sines (100 pages), the logarithms of sine and tangent ratios (17 pages + 17 pages), the logarithms of sines (500 pages), the logarithms of tangents (500 pages) and an introduction, for a total of 1200 pages.[396] The logarithms of numbers would not have been included, other

[394]PC: Fol. 294. Both copies are lacking pages $0^q.3400$–$0^q.3500$, $0^q.3600$–$0^q.3700$, $0^q.5400$–$0^q.5700$, and $0^q.9900$–$1^q.0000$ which are left blank, and one copy has some printing errors, such as missing figures, while the other has errors on the interval $0^q.0800$–$0^q.0900$. The paper on which the tables were printed has the dimensions 27 cm × 42.7 cm, which is a little smaller than the *grand-raisin* of the manuscripts. The tables themselves have the dimensions 21.5 cm × 36.6 cm.

[395]PC: Ms. 1181.

[396]PC: Ms. 1181.

tables being deemed sufficient.

Nevertheless, if the logarithms of numbers had been printed, they would have added 25 pages (logarithms from 1 to 10000, rounded to 17 places,[397] with 400 values per page and no differences), and 634 pages (logarithms from 10000 to 200000, with 300 values per page and the first difference). There would then have been a total of 1859 pages. Other variations were then considered, but none were put in practice.

5.1.5 Project 5 (1825)

Once the British Government withdrew from the project of joint printing of the tables, Didot first pursued the project of printing only the 500 already composed pages, but such a project was doomed, as the tables of logarithms of sines and tangents were incomplete. Didot may then have planned to compose the missing pages, but the project stalled again, and eventually was abandoned.

5.2 Stereotyping

Printers had been looking for means to print a book or other documents in many copies, without the burden of keeping all the printing plates. This was in particular needed for works which could be considered intemporal and which were likely to be often reprinted, for instance bibles or mathematical tables. A special application was the paper money, in particular the *assignats* of the 1790s. Attempts had actually been made since the early 18th century to replace plates with moveable type by solid plates made of only one piece. The advantages were multiple, the plates were easier to handle, they were not liable to drop parts, and they could be made a lot lighter than the original plates. Given that less metal was needed in the new plates, a plate could sometimes be six times lighter than before.[398]

The main idea was to build a plate with moveable type and to transform it into a solid plate, by impressing a matrice. Marie Gatteaux, who was involved in the printing of the *assignats*, made a number of experiments, in particular with the printing of an excerpt of Borda's table of logarithms.[399] Among the problems encountered, were those of producing a matrice by striking a plate of characters. If this was done with hot metal, it could lead to air-holes. In

[397] Charles Blagden gave this figure in 1819, probably accounting for the known inaccuracy of the 19 decimals computed. (PC: Ms. 1181)

[398] [Peignot (1802), p. 196]

[399] [Lambinet (1810), volume 2, p. 401]

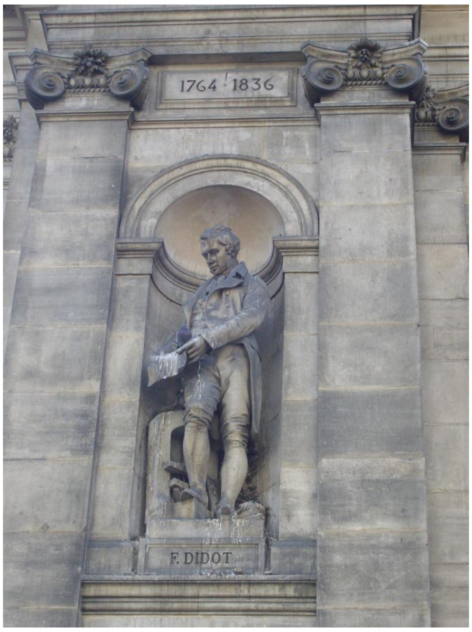

Figure 5.1: The statue of Firmin Didot on the *Hôtel de ville* in Paris. (Source: Wikipedia)

order to prevent these holes, the punches had to be in very hard metal, and the impression had to be done with a screw press without heating the lead plate.[400] A lot of effort was therefore put towards finding the good alloys for the characters. In his patent from 22 Frimaire VI (12 December 1797), Didot gives for instance the following composition: for 10 kilograms, there are 7 kilograms of lead, 2 kilograms of antimony and 1 kilogram of an alloy containing a tenth of copper, and nine tenths of tin.[401]

Firmin Didot (figure 5.1) was one of those who tried to develop the stereotyping technique in the 1790s, in particular in association with Louis-Étienne Herhan. All of them were involved in the fabrication of the *assignats*. The technique was called "stereotyping," for "solid type." At the turn of the century, there were about 60 stereotype volumes on Didot's catalog.

It should however be remarked that although one of the first "stereotype" volume by Didot was Callet's tables of logarithms, published in 1795, this work was actually using a different technique, namely that of Samuel Lucht-

[400][Lambinet (1810), volume 2, pp. 401–402] [Hodgson (1820), pp. 91–93]

[401][Boquillon (1837), p. 478] In his patent application, Didot writes that he had been using his composition for the previous two years.

mans at the beginning of the 18th century.[402] In his foreword to the tables, Firmin Didot summarily describes his new printing process and the advantages he expects from it.[403] The digits were soldered onto the plates, which could then be moved without risk. The main advantage was to ensure that the tables would become error-free. Didot did not claim that they were error-free then, but his plans were to make it possible for the errors to gradually vanish, by advertising the errors, and by ensuring that every correction is permanent and does not jeopardize other parts of the plates.

Many other experiments were made at that period. In a patent from year VI, Herhan describes for instance a technique in which the moveable characters in copper were sunk (*en creux*) and not in relief, then assembled, and from this assembly a plate for printing was immediately made with a different metal.[404]

Stereotyping was used by Charles Babbage and Scheutz in the 1820s and later for printing their tables of logarithms.

The *Encyclopædia Britannica* summarizes the advantages of stereotype printing:

> *"The expence of renewed composition in successive editions is thereby saved; and the additional capital expended in preparing the plates is, perhaps, more than compensated by the facility with which small editions of works can be printed without laying aside a stock of paper in a warehouse to meet the gradual sale. (...) but whatever may be the advantages in point of expence, its merit in point of accuracy is unquestionable. Dictionaries, classics, works on arithmetic and mathematics, once made accurate, may for ever be kept so with but little chance of error."* [405]

The main source on the history of the stereotyping process at the end of the 18th century is Camus' book.[406] Other summaries, borrowing heavily from Camus, are those of Peignot, Lambinet, Hodgson and Boquillon.[407]

402[Lambinet (1810), volume 2, pp. 415–416] [Timperley (1839), p. 585]
403[Callet (1795), p. III–V]
404[Encyclopædia Britannica (1824), p. 377]
405[Encyclopædia Britannica (1824), p. 379]
406[Camus (1802)]
407[Peignot (1802), Lambinet (1810), Hodgson (1820), Boquillon (1837)]

5.3 Truncating the computations

The volumes at the *Institute* contain red lines or marks corresponding to truncation.[408] For instance, the volumes with the logarithms of numbers contain a red separating line in the columns of the logarithm and of the first two differences. As a consequence of these lines, many digits had to be rounded, and the rounded figures are shown in red above the original values. In some volumes, only the corrected digits are given, and the red lines are not marked. Sometimes, several digits had to be changed. If for instance a value ends with 319|7, it was rounded to 320, and the 20 was written in red ink above 19. An example of a truncation changing many digits is that of $\log 21194 = 4.32621\ 29299\ 99|54$. The digits 30000 are marked over 29999 in red in copy I.

These red digits seem to have been wrongly interpreted by Grattan-Guinness as corrections of errors.[409]

In some cases (at least at the beginning of the table of sines which was the first to be computed), there are several red lines, perhaps because different truncations were considered at the beginning of the project.

The volumes at the *Observatoire*, on the other hand, have very few of these red lines. There may be truncation lines, but they are much more discrete and the rounded digits usually appear in black. It was of course not necessary to do the rounding twice. Instead it could be done once, and then checked, since the verification is straightforward.

5.4 The 1891 excerpt

In 1891, the French *Service géographique de l'armée* published an 8-place table of logarithms based on the *Tables du cadastre*.[410] The structure of these tables follows that of the original tables, and it is possible to see the *Tables du cadastre* through them.

We have produced a complete reconstruction of that table.[411]

[408] This also concerns the abridged volume at the *Observatoire*. It should also be mentioned that some red lines are not truncation lines for the publication, but truncation lines from one column to the other. These red lines were usually marked as thick interrupted lines and indicate the change of scale, or the number of added digits, from the previous column to the current one.

[409] [Grattan-Guinness (1990a), p. 183], [Grattan-Guinness (2003), p. 117]

[410] [Derrécagaix (1891), Service géographique de l'Armée (1891), Radau (1891a)]

[411] [Roegel (2010f)]

5.5 Description of the 47 auxiliary volumes

In order to ease the analysis of the original tables and to compare them, it was deemed useful to produce auxiliary tables. Those who have been working on the original tables have compared them with tables such as Briggs' *Arithmetica logarithmica*, but all these tables have different layouts, and the comparison with the *Tables du cadastre* is therefore extremely cumbersome and error prone. It is therefore much more useful to have new tables with the same layout as the original tables.

However, a unique set of tables would not be enough. Those wishing to verify the original tables need to check the accuracy of the pivot values, but also the accuracy of the interpolated values.[412] We have therefore produced a first set of volumes with exact computations (to the accuracy of the original pivot values) and volumes using interpolated values. We have however considered two different types of interpolation for the logarithms of numbers, because two main types of interpolation are used throughout the original manuscript.

1. volumes 1–8 (logarithms of numbers); these volumes give the logarithms of the numbers from 1 to 200000; the logarithms of the numbers 1–10000 are given in two different layouts, corresponding to the two manuscripts; only the exact values are given for them, and they appear in volume 1A only;
   (a) 1a–8a with "exact" computation for $\log n$, and $\Delta^i \log n$ using the terms up to $\frac{1}{n^6}$, for each value of n from 10000 to 200000. (since we stopped at $\frac{1}{n^6}$, there may be cases where the values are not exact but this is intentional)
   (b) 1b–8b with *rounded interpolation* using the approximate thresholds for the constancies of Δ^i; the pivots are computed like in volumes 1a–8a; these volumes show the erroneous digits of the logarithms in red;
   (c) 1c–8c with *hidden interpolation* using the approximate thresholds for the constancies of Δ^i; the pivots are computed like in volumes 1a–8a; these volumes show the erroneous digits of the logarithms in red;
2. volume 9 (sines)

[412]Although the interpolations provided in the auxiliary volumes are the ideal ones, using the "exact" values of the pivots, they can still be used to check interpolations where the pivots are not exactly the same, using the formula given in section 2.1.2.

   (a) 9a: exact computation; each value is computed exactly and not approximated as in volumes 1a–8a;
   (b) 9b: with rounded interpolation; the pivots are computed like in volume 9a; this volume shows the erroneous digits of the logarithms in red;

3. volumes 10–13 (logarithms of sines)
   (a) 10a–13a: "exact" computation; the values of $\mathcal{A}$ were intentionally truncated after x^6 and those of $\Delta^n \log \sin$ have been truncated after the 7th derivative;
   (b) 10b–13b: with rounded interpolation; only rounded interpolations were considered here; the pivots are computed like in volumes 10a–13a; these volumes show the erroneous digits of the logarithms in red;

4. volumes 14–17 (logarithms of tangents)
   (a) 14a–17a: "exact" computation; the values of $\mathcal{A}'$ were intentionally truncated after x^6 and those of $\Delta^n \log \tan$ have been truncated after the 7th derivative;
   (b) 14b–17b: with rounded interpolation; only rounded interpolations were considered here; the pivots are computed like in volumes 14a–17a; these volumes show the erroneous digits of the logarithms in red;

5. volume 18 (multiples of sines and cosines)
   (a) 18a: exact computation
   (b) 18b: with rounded computation

6. tables of volume 19 (introductory volume) with exact computation (not yet completed)

7. volume 20 (abridged tables)
   (a) 20a: "exact" computation, like in volumes 10a–17a;
   (b) 20b: with rounded interpolation; the pivots are computed like in volume 20a; this volume shows the erroneous digits of the logarithms in red;

8. complete set of printed tables (project 2): this is the closest approximation of what the tables were supposed to look like at the beginning (2100 pages); the values were computed as in the "exact" volumes 1–17.

Chapter 6

Conclusion and future research

In this study, we gave an overview of Prony's *Tables du cadastre* and we highlighted a number of features of the manuscripts, in particular their many idiosyncrasies. The tables do now no longer appear as *clean* as told by historical accounts, but this makes them perhaps the more interesting. The analysis of the tables should however not be considered complete. We have only taken various samples, and we did not go through the whole tables, page by page. Further work should be considered:

- The first 10000 logarithms differ significantly in both manuscripts, probably as a result of the formulæ used. The exact formulæ used should been identified, as well as the terms which have been neglected. Both tables should be compared with respect to the presumably aimed accuracy of 12 places.
- The whole Legendre excerpt should be compared with copy O (only the first page was checked).
- The correctness of our assumptions on the use of formulæ (2.75), (2.76) and (2.77) for the computation of the logarithms of tangents still remains to be checked.
- Without having to read all the values, it is actually feasible, and not too time-consuming, to go through all the pages and group them according to the type of interpolation they are using. There are possibly other types of interpolation than those we have given in this report. We need a fine-grained analysis of the interpolations, classifying them by interval, sometimes as small as 4-pages. All the deviations should be recorded. A comparison of the errors at the end of the interpolations with the expected results should make it possible to detect yet unnoticed errors. Standard forms should be used to facilitate the survey. In

addition, each interpolation could also be recomputed using the actual values, instead of the exact ones, as we have done.

- Another venture is to check the accuracy of all the pivot points. This, too, should not be too difficult, using the volumes of exact values that were prepared. For instance, there are 950 pivot points for the logarithms of numbers from 10000 to 200000.

- When errors take place in the pivot values, it should be possible, at least in some cases, to find the origin of the errors.

- Other retrograde interpolations should be identified. The fact that an interpolation goes backwards is not obvious at first sight, but becomes clear by comparing the last line of the new part with the first line of the next page, and by observing an anomaly for the differences at the beginning of the newly added interval. The last difference, in particular, should be that of the next page.

- In particular, it would be interesting to know if there are other cases of retrograde interpolations, such as that of the sine table between $0^q.0350$ and $0^q.0400$.

- Another example concerns the logarithms of numbers, where the interpolated values for 149000 are correct, and this is suspicious; did a retrograde interpolation from 149000 to 148800 take place, and, if so, why?

- The structure of the abridged table should be analyzed in depth and the source of the pivots located. For instance, have the differences for pivots of the logarithms of sines been computed anew or obtained from the Great Tables? Were the differences of the logarithms of tangents obtained from the abridged logarithms of sines or from the Great Tables?

- It would also be interesting to analyze the parts of the manuscripts which were covered by new strips (for instance new columns); were the original computations (which can sometimes be seen under the strips) incorrect, or merely not as accurate as they could be?

- All corrections related to the 1891 tables should be located.

- If something has been checked in only one manuscript, it should also be checked in the other. This applies to a few of our observations.

- All corrections should be noted, page by page, and all retrograde computations should be recomputed.
- The names of all the calculators should be copied, with their precise location.
- Research should be undertaken in the Archives of the London Board of Longitudes, in order to analyze the discussion that took place between 1819 and 1824 about the joint publication of the tables; in particular, the decision to abandon the publication was taken during a meeting that took place in May or June 1824 at the Board of Longitudes. The Archives of the Board of Longitudes are located at the Cambridge University Library (EAD/GBR/0180/RGO 14) and interesting pieces may be among boxes 7 and 8.
- Other archive files may contain useful material and should be examined: at the *Archives Nationales* (in particular A.N. $F^{20}283$), at the *Ponts et chaussées* (in particular Ms 2213), at archives of other administrations, archives of Didot, etc.
- The Archives of the *École Normale de l'an III* may also contain information on the demand that led to the construction of the abridged tables of logarithms of sines and tangents.

We hope that this list is not too daunting, but that it will help others to organize the further exploration of the great *Tables du cadastre*.

Chapter 7

Primary sources

A number of manuscripts and primary sources have been consulted (directly or indirectly) and used for this research. They are given below. A few items of possible interest, but which have not yet been consulted, have also been included in this list.

Paris observatory library

- B 6: *Tables du cadastre* (19 volumes)

Library of the *Institut* (Paris)

- Ms 1496–Ms 1514: *Tables du cadastre* (19 volumes)

Archives nationales (Paris)

- F^{1b}: *Personnel administratif*
  - $F^{1bI}44$
- F^{14}: *Travaux publics*
  - $F^{14}2146$
  - $F^{14}2153$
  - $F^{14}2304^2$
- F^{17}: *Instruction publique*
  - $F^{17}1237$
  - $F^{17}1238$

  - $F^{17}1244B$
  - $F^{17}1393$
  - $F^{17}13571$

- F^{20}: *Statistique*
  - $F^{20}283$: Various documents related to the cadastre of France, 1790–1807 (not consulted)

Bibliothèque nationale (Paris)

- ms. n.a. fr.15778 (not consulted)[413]

Library of the *École nationale des ponts et chaussées*

- 4.292/C16 (= [Riche de Prony (1801)]);
- 4.295/C16 contains the three documents
  - [Riche de Prony (1824)],
  - [Anonymous (1820 or 1821)], and
  - [Anonymous (ca. 1820)];
- 4.296/C16 (= [Riche de Prony (1824)])
- Ms. Fol.242: Abridged tables of logarithms of sines and tangents from the *Tables du cadastre*. These tables are in fact not really abridged, but were computed *de novo*.
- Ms. 243: Table of 8-place logarithms from 100000 to 200000, 167 pages. This table is certainly based on the *Tables du cadastre*. It is identical to Ms.Fol.2773.
- Fol. 294: Two almost complete printings of the sine table of the *Tables du cadastre*, 100 pages each.

[413] This file contains in particular a letter (fol.79) which is reproduced in [Bradley (1998), pp. 201–202], but the date given for the letter (25 Ventôse II) is incorrect; assuming only the year to be wrong, the correct date is 25 Ventôse XI, and this letter is then consistent with another letter sent by Chaptal to Didot on 8 Nivôse XI (A.N. $F^{17}13571$).

- Fol. 305: Table of sines and cosines to seven places, with their multiples $n \sin \alpha$, for $1 \leq n \leq 9$ and $0 \leq \alpha \leq 45°$ at intervals of one minute; the tables cover 135 pages, at a rate of $20'$ per page. These tables were printed in 1795, and perhaps computed at the *Bureau du cadastre*. They are however sexagesimal and cannot have been merely copied from the *Tables du cadastre*.[414]

- Fol. 423: Pitiscus: Thesaurus mathematicus, 1613 [Pitiscus (1613)]

- Ms. 1181: Letters and drafts related to the joint publication of the tables, 1819.

- Ms. 1182: Letters and various notes related to the money owed to Didot for the printing of the tables.

- Ms. 1183: Letters and various notes related to the money owed to Didot for the printing of the tables.

- Ms. 1745: Large file on the methods used to compute the *Tables du cadastre*.

- Ms. Fol.1890: This file contains several tables, in particular an unbound copy of the tables of multiples of sines and cosines for angles from $0^q.000$ to $0^q.500$.

- Ms. Fol.2773: Table of 8-place logarithms from 100000 to 200000, identical to Ms. 243.

- Ms. Fol.2774: Tables of antilogarithms and logarithms, also to eight places, presumably based on the *Tables du cadastre*.

Other files of possible interest and which were not consulted are Ms. 2147, Ms. 2148, Ms. 2149, Ms. 2150, Ms. 2199, Ms. 2213, Ms. 2402, Ms. 2485, Ms. 2713, and perhaps others.

Bureau des longitudes (Paris)

- minutes of the meetings from 24 February 1819 to 16 October 1833

[414]See A.N. F^{17}1244B

Archives of the *Académie des sciences* (Paris)

- Prony's biographical file[415]
- *Procès-verbaux des séances de l'Académie des Sciences*

Bibliothèque Sainte-Geneviève (Paris)

- Briggs' *Arithmetica logarithmica* (1624), with Prony's corrections [Briggs (1624)]

Library of the Royal Society (London)

- Charles Blagden Papers, file CB/4/7/5 (only some of the papers in this file are related to the project of joint publication of the tables)

[415] Gillispie mentions a manuscript dated 8 March 1819 in this file, but we were unable to locate it [Gillispie (2004), p. 485].

Acknowledgements

This work would not have been possible without the encouragement and help of many persons, first of all the curators of the libraries holding the primary sources used in this study. I would like, therefore, to first thank Laurence Bobis, head of the library of the Paris observatory and her staff, as well as Mireille Pastoureau, head of the library of the *Institut de France*, her staff and in particular Fabienne Queyroux. They all provided excellent working conditions during my visits.

Anybody who is working on Prony will end up at the library of the *École nationale des ponts et chaussées*, whose curator Catherine Masteau was most helpful. Other pieces of information were obtained from the Archives of the *Académie des sciences* and it is my pleasure to thank their curator Florence Greffe, and the documentalist Claudine Pouret. I owe a great debt to the staff of the *Archives Nationales* in Paris, which keep what is left from the Archives of the *Bureau du cadastre* and to André Lebeau, president of the Bureau des Longitudes, and its secretary Michel Tellier, for enabling access to the minutes of the Bureau's meetings. Jean-Marie Feurtet has also kindly provided excerpts of his soon to be published transcriptions of the minutes.

Joanna Corden was most helpful sending me copies of a file in Charles Blagden's papers at the Royal Society library, about the inception of the project of French-British joint publication of the *Tables du cadastre*.

I would also like to warmly thank my yeomen at the LORIA lab and at the University of Nancy libraries for locating many sources and having them copied or lent to me.

Charles Coulston Gillispie, Ivor Grattan-Guinness, Paul-Marie Grinevald, Patrice Bret, Scott B. Guthery, Margaret Bradley, and others, have answered questions or helped me to locate rare documents.

It was also a surprise to realize that Jean-Louis Peaucelle had studied the same topic at the same time, and wholly independently. He drew my attention to some documents I had not concentrated on, and led me to be clearer on certain topics.

Finally, it is my pleasure to thank Charles-Michel Marle, Paul Caspi, Gérard Berry, and Jean Dercourt, *secrétaire perpétuel* for the mathematical sciences at the *Académie des sciences*, for their interest and support.

References

J'aurais peut-être dû passer sous silence les livres faits par des imbéciles ou des fous ; mais encore était-il utile de mettre en garde mes lecteurs contre cette sorte d'écrivains.

Lalande [de Lalande (1803), p. viii]

The following list gives all the references that were used in this work, including the main tables, with the exception of the manuscripts found in various institutions which are detailed in chapter 7.

Not all of these references—in particular a number of works which make only passing references to Prony's work and which are impossible to list exhaustively—are explicitely mentioned in the text and are only included here for the sake of semi-completeness.

We also draw the attention of the reader to the fact that we have changed the case of most titles, these notices being not meant to be facsimiles of the original works.

[Airy (1855)] George Biddell Airy. *A treatise on trigonometry.* London: Richard Griffin and company, 1855.

[Alder (2002)] Ken Alder. *The measure of all things. The seven-year odyssey and hidden error that transformed the world.* New York: Free Press, 2002.

[Andoyer (1910)] Marie Henri Andoyer. Nouvelles Tables trigonométriques fondamentales. *Comptes rendus hebdomadaires des séances de l'Académie des sciences*, 150:591–592, janvier-juin 1910.

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[Andoyer (1915)] Marie Henri Andoyer. Fundamental trigonometrical and logarithmic tables. In [Knott (1915)], pages 243–260.

[Andoyer (1915–1918)] Marie Henri Andoyer. *Nouvelles tables trigonométriques fondamentales contenant les valeurs naturelles des lignes trigonométriques. . . .* Paris: Librairie A. Hermann et fils, 1915–1918. [3 volumes, reconstruction by D. Roegel in 2010 [Roegel (2010e)].]

[Andoyer (1922)] Marie Henri Andoyer. *Tables logarithmiques à treize décimales*. Paris: Librairie J. Hermann, 1922. [Reconstruction by D. Roegel in 2010 [Roegel (2010b)].]

[Anonymous (1796a)] Anonymous. Review of Callet's tables of logarithms. *The Monthly Review*, 21:570–574, 1796.

[Anonymous (1796b)] Anonymous. Literarische Nachrichten. *Intelligenzblatt der Allgemeine Literatur-Zeitung*, 3(169):1434, 1796. [Issue dated 14 December 1796. Contains a mention of the completion of the tables and that they are almost completely printed.]

[Anonymous (1796c)] Anonymous. Review of "Connaissance des Temps" for year 4 of the French Republic. *Allgemeine Literatur-Zeitung*, 2(152): 377–381, 1796. [Issue dated 18 May 1796. Column 381 mentions the expected tables by Prony.]

[Anonymous (1801)] Anonymous. Nachricht von den großen logarithmisch-trigonometrischen Tafeln die im *Bureau du Cadastre* unter Prony's Aussicht berechnet worden sind. *Magazin für den neuesten Zustand der Naturkunde mit Rücksicht auf die dazu gehörigen Hülfswissenschaften*, 3:616–619, 1801. [Translated from an article in the *Bulletin de la Société philomatique*.]

[Anonymous (1811)] Anonymous. Review of Dealtry's Principles of fluxions. *The Quarterly review*, 5:340–352, 1811.

[Anonymous (1816)] Anonymous. Logarithms. In *Encyclopædia Perthensis; or universal dictionary of the arts, sciences, literature, &c.*, volume 13. Edinburgh: John Brown, 1816.

[Anonymous (1819a)] Anonymous. Notice sur la publication des tables trigonométriques et logarithmiques par les gouvernements anglais et français. *Le moniteur universel*, 241:1145, 1819. [Issue dated "29 août 1819." The notice does not have any title and appears at the bottom of the first column of the page.]

[Anonymous (1819b)] Anonymous. Über das Cataster, von Benzenberg (review). *Jenaische Allgemeine Literatur-Zeitung*, 3(143):177–182, August 1819.

[Anonymous (1820 or 1821)] Anonymous. Note sur la publication, proposée par le gouvernement anglais, des grandes tables logarithmiques et trigonométriques de M. de Prony, 1820 or 1821. [This eight-page note is extremely rare. One copy is located in the ENPC Archives (4.295/C16), another was—and perhaps still is—located in the Archives of the *Académie des Sciences*.]

[Anonymous (ca. 1820)] Anonymous. Observations sur la proposition de transformer les grandes tables centésimales de M. de Prony en tables sexagésimales, ca. 1820. [8 pages, extremely rare, ENPC Archives (4.295/C16) and Archives Nationales]

[Anonymous (1822)] Anonymous. Nouvelle Méthode de nivellement trigonométrique, par M. de Prony (review). *Annales belgiques des sciences, arts et littérature*, 9(3):177–180, March 1822.

[Anonymous (1841)] Anonymous. Gaspard Clair François Marie Riche de Prony. In *The popular encyclopedia*, volume VII (supplement), pages 763–764. Glasgow: Blackie & Son, 1841.

[Anonymous (1842)] Anonymous. Mental division of labor — The French numerical tables. *The Penny magazine of the Society for the Diffusion of Useful Knowledge*, XI(626):2–4, 1842. [Issue dated 1 January 1842]

[Anonymous (1855)] Anonymous. Grandes tables logarithmiques de l'observatoire. *Nouvelles annales de mathématiques*, 14:14–18 (appendix), 1855.

[Anonymous (1858)] Anonymous. Compte rendu des académies, Bulletin de l'académie des sciences, Séance du lundi 16 mai 1858. *Le progrès, journal des sciences et de la profession médicales*, 1:585, 1858. [mention of the manuscript deposited at the *Institut*]

[Anonymous (1874)] Anonymous. Note about Edward Sang's project of computing a nine-figure table of logarithms. *Nature*, 10:471, 1874. [Issue of 8 October 1874. This note was reproduced in [Sang (1875a)].]

[Anonymous (1875)] Anonymous. Correspondance. *Comptes rendus hebdomadaires des séances de l'Académie des sciences*, 80(22): 1392–1393, janvier-juin 1875. [Minutes of the meeting of the 7 June 1875.]

[Anonymous (1898)] Anonymous. L'extension du système décimal aux mesures du temps et des angles. *Ciel et Terre*, 18:601–608, 1898.

[Anonymous (1901)] Anonymous. Chronique. *Nouvelles annales de mathématiques, 4e série*, 1 (supplément)(10):xxxiii–xxxvi, October 1901.

[Anonymous (1907–1908)] Anonymous. Dr Edward Sang's logarithmic, trigonometrical, and astronomical tables. *Proceedings of the Royal Society of Edinburgh*, pages 183–196, 1907–1908. [possibly by Cargill Gilston Knott]

[Archibald (1943a)] Raymond Clare Archibald. Tables of trigonometric functions in non-sexagesimal arguments. *Mathematical Tables and other Aids to Computation*, 1(2):33–44, April 1943.

[Archibald (1943b)] Raymond Clare Archibald. Review of "Coast and geodetic survey, *Table of S and T Values*, 1934". *Mathematical Tables and other Aids to Computation*, 1(3):83–85, 1943.

[Ashurst (1983)] F. Gareth Ashurst. *Pioneers of computing*. London: Frederick Muller Limited, 1983. [contains a brief mention of the *Tables du cadastre* p. 62 in the chapter devoted to Babbage]

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[Babbage (1827)] Charles Babbage. *Tables of logarithms of the natural numbers from 1 to 108,000*. London: Mawman, 1827. [Some excerpts are reproduced in volume 2 of [Campbell-Kelly (1989)]. A complete reconstruction of the tables was made by D. Roegel in 2010 [Roegel (2010j)].]

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[Bertrand (1870)] Joseph Bertrand. Table de logarithmes à 27 décimales pour les calculs de précision, par Fédor Thoman (review). *Journal des savants*, pages 750–760, December 1870.

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[Biot et al. (1858)] Jean-Baptiste Biot, Léonce Élie de Beaumont, and Urbain Le Verrier. Communication faite par M. Biot. *Comptes rendus hebdomadaires des séances de l'Académie des sciences*, 46: 911–912, 1858. [séance du 17 mai 1858]

[Bloch (1929)] Marc Bloch. Les plans parcellaires. *Annales d'histoire économique et sociale*, 1(3):392–398, 1929. [see pp. 392–393 on the creation of the French cadastre]

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[Boden (2006)] Margaret A. Boden. *Mind as machine: a history of cognitive science.* Oxford: Oxford University Press, 2006. [see p. 138 on Prony]

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[Bret (2002)] Patrice Bret. *L'État, l'armée, la science : l'invention de la recherche publique en France (1763–1830)*. Rennes: Presses universitaires de Rennes, 2002. [The *Tables du cadastre* are briefly mentioned pp. 191–192.]

[Bret (2009)] Patrice Bret. Du concours de l'an II à la suppression de l'Ecole des Géographes : la quête identitaire des ingénieurs géographes du Cadastre de Prony, 1794–1802. In Ana Cardoso de Matos, Maria Paula Diogo, Irina Gouzévitch, and André Grelon, editors, *Jogos de identidade profissional : os engenheiros, a formação e a acção – Les enjeux identitaires des ingénieurs : entre la formation et l'action – The quest for a professional identity: engineers between training and action*, pages 121–154. Lisbonne: Edições Colibri, 2009. [Proceedings of the conference held in Evora, 8–11 October 2003.]

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[Brunot and Coquand (1982)] André Brunot and Roger Coquand. *Le corps des ponts et chaussées.* Paris: Éditions du Centre national de la recherche scientifique, 1982.

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[Bullock (2008)] Seth Bullock. Charles Babbage and the emergence of automated reason. In Philip Husbands, Owen Holland, and Michael Wheeler, editors, *The Mechanical Mind in History*, pages 19–39. Cambridge, Ma.: MIT Press, 2008. [see p. 30 for Prony]

[C. (1811)] L. C. Notice sur les grandes tables logarithmiques et trigonométriques, calculées au bureau du cadastre, sous la direction du C. PRONY, Membre de l'Institut national, et Directeur de l'École des ponts et chaussées et du cadastre ; rapport fait sur ces tables par les CC. LAGRANGE, LAPLACE et DELAMBRE. *Bulletin des sciences, par la société philomatique de Paris*, 3:30–31, 1811. [I assume that L.C. = Lazare Carnot.]

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[Callet (1783)] Jean-François Callet. *Tables portatives de logarithmes, publiées à Londres par Gardiner, augmentées, et perfectionnées dans leur disposition par M. Callet, etc.* Paris: Firmin Ambroise Didot, 1783. [This edition does not yet contain tables for the decimal division.]

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