Finite Buffer Bulk Service Queue with Multiple Vacations

N. Sujatha, D. V. L. Prasanna

Abstract: In this paper, it is studied about finite buffer bulk service queue $M/G^{(a,b)}/1$ with multiple vacations and some analytical results, relations among queue length distributions, computations of state probabilities at various epochs are included.

Keywords: Bulk service queue, finite buffer, multiple vacations, probability generating function, probability density function.

I. INTRODUCTION

Queueing systems with vacations have wide applications in computer data communication systems, telecommunication engineering and manufacturing systems. In real life applications, queues with finite buffer are more realistic than queues with infinite buffer. In telecommunication networks messages are stored in the system if a server is busy. In such queues if an arrival finds the buffer space full then he is blocked from entering into the system and considered to be lost. The finite buffer queues are in general more difficult to analyze than the corresponding infinite buffer queues. The finite buffer M/G/1 queue with vacations has been studied previously by Chaudhry and Gupta[1] Fuhrmann [2], Gupta and Sikdar[3][4], Levy and Yechiali [5] , Sujatha, Srikanth and Srinivasu [6]. Gupta and Sikdar[3] analyzed a finite buffer M/G/1 queue with bulk service and single vacation. Further Gupta and Sikdar[4] analyzed a finite buffer bulk service queue $M/G^{(1,b)}/1$ with multiple vacations. In this paper, we have generalized the queueing model analyzed by Gupta and Sikdar[4]. Consider a single server queue where customers arrive according to Poisson process and are served by a single server in batches of maximum size b with a minimum threshold value a for specific values of a . When the server finishes serving a batch and finds less than a number of customers in the queue he goes away for a length of time called a vacation. However, on return from the vacation if he finds n ($a \leq n \leq b$) customers then he takes all of them for service. Otherwise, if he finds more than b customers waiting, he takes maximum b customers for service and continue to do so until the queue length reaches a. However, on return from a vacation, if he finds less than a number of customers waiting he immediately proceeds for another vacation. We assume that the system has finite buffer of size N(>b) i.e. the maximum number of customers allowed in the system at any time is N+b. If a customer on arrival finds N customers in the queue then he goes away without being served and assumed to be lost. Specifically, we consider a bulk service queue $M/G^{(a,b)}/1$ with finite buffer and multiple vacations.

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We analyze this queue using a combination of supplementary variable (with remaining service time of a batch in service and remaining vacation time of the server as supplementary variables) and embedded Markov chain techniques. The queue length distributions at service completion, vacation termination, departure and arbitrary epochs have been obtained. Finally, some important performance measures such as probability of blocking, average queue lengths etc. along with numerical results are presented.

II. PRELIMINARIES

Bulk Service queue: A queueing phenomenon in which customers are served in batches are called bulk service queues. For example, this type of queues occur frequently in loading and unloading of cargoes at a sea port, in traffic signal systems and in mass transportation system etc.

Finite buffer: If the maximum number of customers that can be allowed into the system is finite then the system is said to have finite buffer queue.

Multiple vacation: In multiple vacation when the server finishes serving a customer and the queue is empty, he goes for a vacation. On return, if he finds one or more customers waiting he serves the customers until the system empties then he takes another vacation. However, on return from a vacation if he finds no customer waiting, he immediately proceeds for another vacation and continues in this manner until atleast one customer is waiting in the queue.

Probability generating function: Probability generating function of a sequence $\{a_n\}$ is denoted by A(z) and is given by $A(z)=\sum_{n=0}^{\infty} a_n z^n$

Steady state: If the system is independent of time then the system is said to be in steady state.

III. BASIC NOTATIONS

$\boldsymbol{f^{*(j)}(\lambda)}$: j^{th} derivative of $\boldsymbol{f^{*(\theta)}}$ at $\theta=\lambda$ where $\boldsymbol{f^{*(\theta)} = \int_0^\infty e^{-\theta x} f(x)dx}$.

$\mathbf{g_j}$ **:** Probability that j customers arrive during a service time S of a batch. It is given by

$$g_j = \int_0^\infty \frac{(\lambda x)^j}{j!} e^{-\lambda x} dS(x) = \frac{(-\lambda)^j}{j!} S^{*(j)}(\lambda)$$

$\mathbf{h_j}$**:** Probability that j customers arrive during a vacation time V of a batch. It is given by

$$h_j = \int_0^\infty \frac{(\lambda x)^j}{j!} e^{-\lambda x} dV(x) = \frac{(-\lambda)^j}{j!} V^{*(j)}(\lambda)$$

$\boldsymbol{\phi_j}$**:** Probability that j customers arrive during an idle period. It is given by

$$\emptyset_j = \frac{h_j}{1-h_0} \quad , 1 \leq j \leq N\text{-}1 \text{ and}$$

$$\emptyset_N = \frac{h_N{}^c}{1-h_0} \text{ where } h_N{}^c = \sum_{i=N}^{\infty} h_i$$

$\mathbf{K_{i,j}}$**:** Probability of having j customers at a departure epoch (t+1) of a batch if there are i customers at a departure epoch

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t of a batch.

$\mathbf{p_n^+}$: Steady state probability that n customers are left in the system at a departure epoch of a batch.

$\boldsymbol{\pi_n^+}$: Probability (n customers in the queue just prior to service completion epoch/≥0 customers in the queue just prior to service completion or vacation termination epoch)

$\boldsymbol{\omega_n^+}$: Probability (n customers in the queue just prior to vacation termination epoch/≥0 customers in the queue just prior to service completion or vacation termination epoch)

$\mathbf{p_n}$: Probability that there are n customers in the queue at an arbitrary epoch.

$\boldsymbol{\pi_n}$: Probability that there are n customers in the queue at service completion epoch.

$\boldsymbol{\omega_n}$: Probability that there are n customers in the queue at vacation termination epoch.

IV. SOME IMPORTANT LEMMA

Lemma 1 The probability that the server is busy is given by

$$\rho^1 = \frac{E(S)k}{E(S)k + E(V)s_0}, a = 1$$

$$\rho^1 = \frac{E(S)k^2}{E(S)k^2 + E(V)(ks_1 + h_1 s_0)}, a = 2$$

$$\rho^1 = \frac{E(S)k^3}{E(S)k^3 + E(V)(k^2 s_2 + kh_1 s_1 + s_0(h_2 k + {h_1}^2))}, a = 3$$

$$\rho^1 = \frac{E(S)k^4}{E(S)k^4 + E(V)\left(k^3 s_3 + k^2 h_1 s_2 + ks_1(h_2 k + {h_1}^2) + s_0 m\right)}, a = 4$$

where $k=1-h_0$, $s_i = \sum_{k=0}^{i} p_k^+$, $m = h_3 k^2 + 2h_1 h_2 k + {h_1}^3$, $E(S) = -S^{*(1)}(0)$, $E(V) = -V^{*(1)}(0)$.

Lemma 2 The expression of σ in terms of ρ^1 is given by

$$\sigma = \frac{\rho^1 E(V) + (1-\rho^1)E(S)}{E(S)E(V)}$$

Lemma 3 The relation between $\{\pi_n^+\}$ and $\{p_n^+\}$ is given by

$$\pi_n^+ = \frac{\rho^1}{\sigma E(S)} p_n^+, \; 0 \le n \le N.$$

Here p_n^+, $0 \le n \le N$ are obtained by solving the equations $p^+ P = p^+$ using GTH algorithm where $P = [p_{ij}]$ is one-step transition probability matrix which is given by

$$P = \begin{cases} k_{i,j}, & 0 \le i \le (a-1), 0 \le j \le (N-1) \\ g_j, & a \le i \le b, 0 \le j \le (N-1) \\ g_{j-(i-b)}, & j \ge (i-b), i \ge (b+1), 0 \le j \le (N-1) \\ {k_{i,j}}^c, & 0 \le i \le (a-1), j = N \\ {g_j}^c, & a \le i \le b, j = N \\ {g^c}_{j-(i-b)}, & b+1 \le i \le N, j = N \\ 0, & otherwise \end{cases}$$

$$k_{i,j} = \begin{cases} \sum_{n=a-i}^{b-i} \emptyset_n g_j, & 0 \le i \le (a-1), j = 0 \\ \sum_{n=a-i}^{b-i} \emptyset_n g_j + \sum_{n=b-i+1}^{b-i+j} \emptyset_n g_{j-(n-b+i)} + \emptyset_N g_{j-(N-b+i)}, & 0 \le i \le (a-1), 1 \le j \le (N-1), \\ & j-(n-b+i) \ge 0, j-(N-b+i) \ge 0 \end{cases}$$

and ${g_i}^c = \sum_{r=i}^{\infty} g_r, \; b \le i \le N, \; {\emptyset_N}^c = \frac{{h_N}^c}{1-h_0}$

Lemma 4 In steady state, we have

$$\omega_n^+ = \sum_{k=0}^{n} \frac{\pi_k^+}{1-h_0} h_{n-k} + \sum_{k=0}^{n-1} \frac{\omega_k^+}{1-h_0} h_{n-k}, \; 0 \le n \le (a-1),$$

$$\omega_n^+ = \sum_{k=0}^{a-1} (\pi_k^+ + \omega_k^+) h_{n-k}, \; a \le n \le (N-1)$$

$$\omega_N^+ = \sum_{k=0}^{a-1} (\pi_k^+ + \omega_k^+) {h_N}^c$$

with $\sum_{n=0}^{N} (\pi_n^+ + \omega_n^+) = 1$.

Lemma 5 Relation between ω_n, ω_n^+, π_n^+ are given by

$$\omega_n = \frac{\sigma}{\lambda}\left(\sum_{i=0}^{n} \pi_i^+\right), 0 \le n \le (a-1),$$

$$\omega_n = \frac{\sigma}{\lambda}\left(\sum_{i=0}^{a-1} \pi_i^+ - \sum_{j=a}^{n} \omega_j^+\right), a \le n \le (N-1),$$

$$\omega_N = 1 - \rho^1 - \frac{\sigma}{\lambda}\left(\sum_{i=0}^{a-1} (N-i)\pi_i^+ - \sum_{j=a}^{N-1} (N-j)\omega_j^+\right)$$

Lemma 6 Relation between π_n, π_n^+, ω_n^+ is given by

$$\pi_0 = \frac{\sigma}{\lambda}\left(\sum_{i=a}^{b} (\pi_i^+ + \omega_i^+) - \pi_0^+\right),$$

$$\pi_n = \frac{\sigma}{\lambda}\left(\sum_{i=a}^{b+n} (\pi_i^+ + \omega_i^+) - \sum_{i=0}^{n} \pi_i^+\right), 1 \le n \le (N-b)$$

$$\pi_n = \frac{\sigma}{\lambda}\left(\sum_{i=a}^{N} (\pi_i^+ + \omega_i^+) - \sum_{i=0}^{n} \pi_i^+\right), (N-b+1) \le n \le (N-1)$$

$$\pi_N = \rho^1 - \sum_{n=0}^{N-1} \pi_n$$

Lemma 7 Relation between p_n, π_n, ω_n is given by

$$p_n = P(N_q = n) = \pi_n + \omega_n, 0 \le n \le N$$

V. PERFORMANCE MEASURES

We calculate various performance measures such as average queue length $L_q = \sum_{n=0}^{N} np_n$, average queue length when the server is busy $L_{q_1} = \sum_{n=0}^{N} n\pi_n$, average queue length when the server is on vacation $L_{q0} = \sum_{n=0}^{N} n\omega_n$ .The blocking probability of a customer is given by $p_{loss} = \pi_N + \omega_N$.

VI. NUMERICAL RESULTS

We have performed extensive numerical work of this model for a combination of various service and vacation time distributions viz. exponential (M), Erlang (E_2, each phase having a mean $\frac{1}{\mu}$) and deterministic (D). We are presenting a few of them. All calculations have been done in double precision but they are reported here in four decimal places. The notations used in the tables are same as those defined earlier in this chapter. The probability distributions of number of customers in the queue (pmf) and their cumulative (cum) probabilities at various epochs are presented in Tables 3.2.1 - 3.2.3. We have presented state probabilities only for few values of n (0 ≤ n ≤ N) in various columns of the tables. Performance measures are presented at the bottom of the tables.

In Table 3.2.1, results of $\mathbf{M/M^{(2,8)}/1/15}$ when vacation time follows deterministic distribution are given. Results of $\mathbf{M/M^{(3,15)}/1/30}$ when vacation time is Erlang (E_2) distributed are shown in Table 3.2.2. We have presented results of $\mathbf{M/E_2^{(3,15)}/1/25}$ when vacation time follows deterministic distribution are presented in Table 3.2.3. In Figure 1, effect of mean vacation time (E(V)) on the probability of loss (p_{loss}) has been shown. This is done for the case when

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service time follows Erlang (E_2) distribution and vacation time follows deterministic distribution. It is observed from the figure that for fixed ρ, as E(V) increases p_{loss} also increases. This is because the server being in vacation for longer duration of time greater accumulation of customers there by increasing the p_{loss}. In Figure 2, effect of N on blocking probability has been presented. This is done for the case when service time follows Erlang (E_2) distribution and exponential distribution with a fixed vacation time distribution (D) and when service time follows exponential distribution and vacation time follows Erlang (E_2) distribution. This shows that as N increases, blocking probability p_{loss} decreases.

Table 3.2.1: Queue length distributions at various epochs for **$M/M^{(2,8)}$/1/15** (vacation is deterministically distributed) with E(V)=0.565, λ=6.2, E(S)=0.90909 and ρ=0.704545.

n	p_n+ (pmf)	p_n+ (cum)	π_n+ (pmf)	π_n+ (cum)	w_n+ (pmf)	w_n+ (cum)
0	0.0978	0.0978	0.0811	0.0811	0.0025	0.0025
1	0.0908	0.1886	0.0753	0.1564	0.0114	0.0139
2	0.0835	0.2721	0.0693	0.2257	0.0246	0.0385
5	0.0627	0.4807	0.0520	0.3987	0.0274	0.1346
6	0.0566	0.5373	0.0469	0.4457	0.0179	0.1525
9	0.0491	0.7124	0.0407	0.5909	0.0020	0.1692
10	0.0417	0.7541	0.0346	0.6256	0.0007	0.1700
14	0.0217	0.8667	0.0180	0.7192	0.0001	0.1704
15	0.1330	1.0000	0.1103	0.8295	0.0000	0.1704

n	π_n (pmf)	π_n (cum)	w_n (pmf)	w_n (cum)	p_n (pmf)	p_n (cum)
0	0.0883	0.0883	0.0153	0.0153	0.1037	0.1037
1	0.0821	0.1705	0.0296	0.0450	0.1118	0.2156
2	0.0757	0.2463	0.0250	0.0700	0.1007	0.3163
5	0.0573	0.4362	0.0067	0.1074	0.0641	0.5436
6	0.0518	0.4881	0.0033	0.1108	0.0552	0.5989
9	0.0452	0.6484	0.0002	0.1131	0.0454	0.7615
10	0.0386	0.6870	0.0001	0.1132	0.0387	0.8002
14	0.0209	0.7938	0.0000	0.1132	0.0209	0.9070
15	0.0929	0.8867	0.0000	0.1132	0.0929	1.0000

ρ^1= 0.8867, Lq = 5.909, $\mathbf{Lq_1}$= 5.6530, $\mathbf{Lq_0}$ = 0.2559,P_{loss} = 0.0929.

Table 3.2.2: Queue length distributions at various epochs for **$M/M^{(3,15)}$/1/30** (vacation is Erlang (E_2) distributed) with E(V)=0.45,
λ=5.4, E(S)=1.84028 and ρ=0.6625.

n	p_n+ (pmf)	p_n+ (cum)	π_n+ (pmf)	π_n+ (cum)	w_n+ (pmf)	w_n+ (cum)
0	0.0520	0.0520	0.0418	0.0418	0.0107	0.0107
1	0.0502	0.1023	0.0403	0.0821	0.0250	0.0357
2	0.0483	0.1506	0.0387	0.1209	0.0404	0.0762
5	0.0421	0.2833	0.0338	0.2274	0.0198	0.1611
10	0.0322	0.4641	0.0259	0.3726	0.0020	0.1940
15	0.0350	0.6106	0.0281	0.4902	0.0001	0.1969
20	0.0217	0.7433	0.0174	0.5968	0.0000	0.1971
25	0.0134	0.8255	0.0107	0.6628	0.0000	0.1971
30	0.1318	1.0000	0.1058	0.8028	0.0000	0.1971

n	π_n (pmf)	π_n (cum)	w_n (pmf)	w_n (cum)	p_n (pmf)	p_n (cum)
0	0.0529	0.0529	0.0049	0.0049	0.0579	0.0579
1	0.0512	0.1042	0.0971	0.0146	0.0609	0.1189
2	0.0494	0.1536	0.0142	0.0289	0.0637	0.1826
5	0.0436	0.2905	0.0042	0.0497	0.0479	0.3402
10	0.0343	0.4804	0.0003	0.0561	0.0346	0.5365
15	0.0369	0.6379	0.0000	0.0566	0.0369	0.6945
20	0.0243	0.7825	0.0000	0.0566	0.0243	0.8391
25	0.0165	0.8794	0.0000	0.0566	0.0165	0.9360
30	0.0082	0.9433	0.0000	0.0566	0.0082	1.0000

ρ^1= 0.9433, Lq = 10.9813, $\mathbf{Lq_1}$= 10.8137, $\mathbf{Lq_0}$ = 0.1675,P_{loss} = 0.0082

Table 3.2.3: Queue length distributions at various epochs for **$M/E_2^{(3,15)}$/1/25** (vacation time is deterministically distributed) with E(V)=0. 5, λ=6.4, E(S)=1.93549 and ρ=0.825807

n	π_n (pmf)	π_n (cum)	w_n (pmf)	w_n (cum)	p_n (pmf)	p_n (cum)
0	0.0451	0.0451	0.0008	0.0008	0.0460	0.0460
1	0.0465	0.0916	0.0024	0.0033	0.0489	0.0950
2	0.0471	0.1388	0.0045	0.0078	0.0517	0.1467
4	0.0468	0.2329	0.0025	0.0140	0.0493	0.2470
5	0.0459	0.2789	0.0015	0.0156	0.0475	0.2945
10	0.0501	0.4993	0.0000	0.0170	0.0501	0.5163
15	0.0333	0.6986	0.0000	0.0170	0.0333	0.7156
20	0.0204	0.8247	0.0000	0.0170	0.0204	0.8417
25	0.0948	0.9829	0.0000	0.0170	0.0948	1.0000

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n	p_n+ (pmf)	p_n+ (cum)	π_n+ (pmf)	π_n+ (cum)	w_n+ (pmf)	w_n+ (cum)
0	0.0110	0.0110	0.0103	0.0103	0.0004	0.0004
1	0.0198	0.0308	0.0185	0.0289	0.0022	0.0027
2	0.0265	0.0574	0.0248	0.0538	0.0062	0.0089
4	0.0354	0.1245	0.0332	0.1167	0.0130	0.0328
5	0.0380	0.1626	0.0356	0.1524	0.0118	0.0447
10	0.0432	0.3683	0.0405	0.3452	0.0004	0.0625
15	0.0394	0.5801	0.0369	0.5437	0.0000	0.0627
20	0.0276	0.7422	0.0258	0.6956	0.0000	0.0627
25	0.1690	1.0000	0.1584	0.9372	0.0000	0.0627

ρ^1= 0.982988, Lq = 11.1115, **Lq_1**= 11.0617, **Lq_0** = 0.049734, P_{loss} = 0.0948353

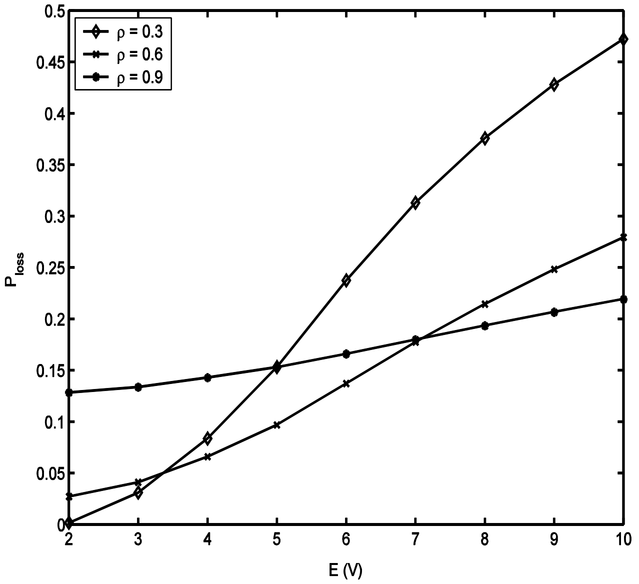

Figure 1: Effect of Mean Vacation Time on Blocking Probability (P_{loss})

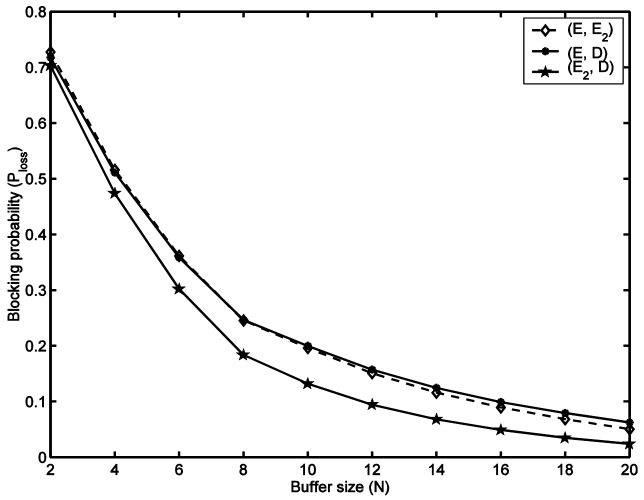

Figure 2: Effect of N on Blocking Probability (P_{loss}) for b=8 and ρ = 0.75

VII. CONCLUSIONS

We have analyzed a bulk service M/G/1 queue with finite buffer and multiple vacations when services are performed in batches of maximum size b with a minimum threshold value a. There is a scope for more generalization of this model i.e. it can be extended for higher values of a.

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