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Asymptotic Behaviors of the Eigenvalues and Solution of a Fourth Order Boundary Value Problem

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Abstract In this paper, we consider the spectral problem of the form: $y^{(4)} + p_3(x)y' + p_4(x)y = \lambda^4\rho(x)y, 0 \le x \le a$
$y^{(j)}(0) = 0, j = 0,1$ Where λ is a spectral parameter in which $\lambda = \sigma + i\delta$, where $\sum_{k=1}^{4}(iw_j\lambda)^{k-1}y^{(4-k)}(a,\lambda) = 0, j = 2,3$
$i = \sqrt{-1}$; $p_3(x), p_4(x)$ and $\rho(x)$ are real valued functions and we assume that $\rho(x) > 0, p_4(x) \in C[0,a]$, $p_3(x) \in C^2[0,a]$ and $\rho(x) \in C^4[0,a]$. Asymptotic formulas for eigenvalues and solutions of the consider boundary value problem are established.

Keywords: *spectral problem, eigenvalues, eigenfunctions, asymptotic formulas*

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1. Introduction

The history of spectral theory is the history of a beautiful and important area of mathematics with close links to physics and with a strong influence on the development of functional analysis. Its roots lies in three areas :1) discrete systems described by matrices (or quadratics forms) and continuous systems described by 2) differential equations or 3) integral equations.

The spectral theory of linear operators has as its basic origins, on the one hand, linear algebra-more precise theorems on the reduction of quadratic forms to sums of squares-and on the other hand, problems in the theory oscillations(vibration strings, membranes, etc).

Many authors studied the asymptotic formulas for eigenvalues and corresponding eigen functions to different types of spectral problems [1,2,3] and [5,6,7].

In this paper, we study the behavior of the solutions and asymptotic behaviors of eigenvalues of the fourth order boundary value problem of the form:

$$l(y) = y^{(4)} + p_3(x)y' + p_4(x)y = \lambda^4\rho(x)y, 0 \le x \le a$$
$$U_j(y) = y^{(j)}(0) = 0, j = 0,1 \qquad (1)$$
$$U_j(y) = \sum_{k=1}^{4}(iw_j\lambda)^{k-1}y^{(4-k)}(a,\lambda) = 0, j = 2,3$$

Where $p_3(x), p_4(x)$ and $\rho(x)$ are real-valued functions and $\rho(x) > 0$ and λ is a spectral parameter in which $\lambda = \sigma + i\tau, \sigma, \tau \in R, i = \sqrt{-1}$.

Here we assume that $p_4(x) \in C[0,a]$, $p_3(x) \in C^2[0,a]$, and $\rho(x) \in C^4[0,a]$. We have introduced the sectors T_k and their conjugates $\overline{T_k}$ (relative to the x-axis). Let λ be located in some fixed sector T_k or $\overline{T_k}$, and let w_j's for $j = 0,1,2,3$ be different roots of unity of degree 4, and ordered so that for all $\lambda \in T_k (or \overline{T_k})$ satisfied the inequality:

$$\mathrm{Re}(iw_k\lambda) \le \mathrm{Re}(iw_{k+1}\lambda), for(k = \overline{0,2}) \qquad (2)$$

Numbering depends on the selected sector. Entire complex plane of $\lambda = \sigma + i\tau$, is divided into 8 sector T_k and $\overline{T_k}$ (in the plane λ which is determined by the inequalities $\frac{k\pi}{2} \le \arg \le \frac{k\pi}{2} + \frac{\pi}{4}, k = 0,1,2,3$ and we assume that $w_k = \sqrt[4]{1}$ and $\varphi_k = iw_k\sqrt[4]{\rho(x)}$.

2. The Behavior of the Solution of Fourth Order Boundary Value Problem

The aim of this section is to estimate the behavior of the solutions to the given fourth order boundary value problem and finding their coefficients $A_i(x), i = 0,1,2,3,4$ from the following theorem:

Theorem 1:

All coefficients $A_i(x), i = 0,1,2,3,4$ in the linear independent solutions

$$y_k^{(j)}(x,\lambda) = (\lambda\varphi_k)^j e^{\lambda\int_0^x \varphi_k(t)dt}\left(A_0 + \frac{A_{1j}}{\lambda} + \frac{A_{2j}}{\lambda^2} + \frac{A_{3j}}{\lambda^3} + \frac{A_{4j}}{\lambda^4} + O(\frac{1}{\lambda^5})\right)$$

k=0,1,2,3 of equation(1)
with sufficient large $|\lambda|$ are :

$$A_0(x) = \rho^{-\frac{3}{8}}(x), A_{1j} = A_1 + \frac{A_0'}{\varphi_k}C_j^1 + \frac{\varphi_k'}{\varphi_k^2}A_0C_j^2$$

$$A_{2j} = A_2 + \frac{A_1'}{\varphi_k}C_j^1 + \frac{\varphi_k'}{\varphi_k^2}A_1C_j^2 + \frac{A_0''}{\varphi_k^2}C_j^2 + \frac{\varphi_k'}{\varphi_k^3}A_0'2C_j^2 + \frac{(\varphi_k')^2}{\varphi_k^4}A_03C_j^4 + \frac{\varphi_k''}{\varphi_k^3}A_0C_j^3$$

$$A_{3j} = A_3 + \frac{A_2'}{\varphi_k}C_j^1 + \frac{A_1''}{\varphi_k^2}C_j^2 + \frac{\varphi_k'}{\varphi_k^2}A_2C_j^2 + \frac{A_0''}{\varphi_k^3}C_j^1 + \frac{\varphi_k''}{\varphi_k^3}A_1C_j^3 + \frac{(\varphi_k')^2}{\varphi_k^4}A_13C_j^4 + \frac{\varphi_k'}{\varphi_k^3}A_1'2C_j^2 + \frac{\varphi_k'}{\varphi_k^4}A_0'6C_j^4 + \frac{\varphi_k''}{\varphi_k^4}A_0'4C_j^4 + \frac{\varphi_k''}{\varphi_k^4}A_0C_j^4,$$

$$A_{4j} = A_4 + \frac{A_3'}{\varphi_k}C_j^1 + \frac{A_2''}{\varphi_k^2}C_j^2 + \frac{\varphi_k'}{\varphi_k^2}A_3C_j^2 + \frac{A_1'''}{\varphi_k^3}C_j^1 + \frac{\varphi_k''}{\varphi_k^3}A_2C_j^3 + \frac{(\varphi_k')^2}{\varphi_k^4}A_23C_j^4 + \frac{\varphi_k'}{\varphi_k^3}A_2'2C_j^2 + \frac{\varphi_k'}{\varphi_k^4}A_1''6C_j^4 + \frac{\varphi_k''}{\varphi_k^4}A_1'4C_j^4 + \frac{\varphi_k'''}{\varphi_k^4}A_1C_j^4 + \frac{A_0^{(4)}}{\varphi_k^3}C_j^4,$$

$$A_3(x) = -A_0(x)\int_0^x [\frac{3}{2}\frac{A_2''}{\varphi_k} + 3\frac{\varphi_k'}{\varphi_k^2}A_2' + (\frac{3}{4}\frac{(\varphi_k')^2}{\varphi_k^3} + \frac{\varphi_k''}{\varphi_k^2})A_2 + \frac{A_1^{(3)}}{\varphi_k^2} + \frac{3}{2}\frac{\varphi_k'}{\varphi_k^3}A_1'' + \frac{\varphi_k''}{\varphi_k^3}A_1' + (\frac{\varphi_k^{(4)}}{4\varphi_k^3} + \frac{p_3}{4\varphi_k^2})A_1 + (\frac{A_0^{(4)}}{4\varphi_k^3} + \frac{p_3A_0'}{4\varphi_k^3} + \frac{p_4A_0}{4\varphi_k^3}]\frac{dt}{A_0}$$

$$A_2(x) = -A_0(x)\int_0^x [\frac{3}{2}\frac{A_1''}{\varphi_k} + 3\frac{\varphi_k'}{\varphi_k^2}A_1' + (\frac{3}{4}\frac{(\varphi_k')^2}{\varphi_k^3} + \frac{\varphi_k''}{\varphi_k^2})A_1 + \frac{A_0^{(3)}}{\varphi_k^2} + \frac{3}{2}\frac{\varphi_k'}{\varphi_k^3}A_0'' + \frac{\varphi_k''}{\varphi_k^3}A_0' + (\frac{\varphi_k^{(3)}}{4\varphi_k^3} + \frac{p_3}{4\varphi_k^2})A_0]\frac{dt}{A_0}$$

$$A_4(x) = -A_0(x)\int_0^x [\frac{3}{2}\frac{A_3''}{\varphi_k} + 3\frac{\varphi_k'}{\varphi_k^2}A_3' + (\frac{3}{4}\frac{(\varphi_k')^2}{\varphi_k^3} + \frac{\varphi_k''}{\varphi_k^2})A_3 + \frac{A_2^{(3)}}{\varphi_k^2} + \frac{3}{2}\frac{\varphi_k'}{\varphi_k^3}A_2'' + \frac{\varphi_k''}{\varphi_k^3}A_2' + (\frac{\varphi_k^{(3)}}{4\varphi_k^3} + \frac{p_3}{4\varphi_k^2})A_2 + (\frac{A_1^{(4)}}{4\varphi_k^3} + \frac{p_3A_1'}{4\varphi_k^3} + \frac{p_4A_1}{4\varphi_k^3}]\frac{dt}{A_0}$$

Proof:

From [4], proved that the solutions of equation (1) for sufficient large $|\lambda|$ which can be written in the form

$$y_k(x,\lambda) = e^{\lambda\int_0^x \varphi_k(t)dt}(\sum_{i=0}^{4}\frac{A_i(x)}{\lambda^i} + O(\frac{1}{\lambda^5})) \qquad (3)$$

By differentiating (3) up to fourth order with respect to x, the following relations are obtained:

$$y_k'(x,\lambda) = (\lambda\varphi_k)e^{\lambda\int_0^x \varphi_k(t)dt}\left[A_0 + \sum_{i=0}^{3}\frac{1}{\lambda^{i+1}}(A_{i+1} + \frac{A_i'}{\varphi_k}) + O(\frac{1}{\lambda^5})\right] \qquad (4)$$

$$y_k''(x,\lambda) = (\lambda\varphi_k)^2 e^{\lambda\int_0^x \varphi_k(t)dt}[A_0 + \frac{1}{\lambda}\left(A_1 + 2\frac{A_0'}{\varphi_k} + \frac{\varphi_k'}{\varphi_k^2}A_0\right) + \sum_{i=0}^{2}\frac{1}{\lambda^{i+2}}(A_{i+2} + 2\frac{A_{i+1}'}{\varphi_k} + \frac{\varphi_k'}{\varphi_k^2}A_{i+1} + \frac{A_i''}{\varphi_k^2}) + O(\frac{1}{\lambda^5})]$$

$$y_k'''(x,\lambda) = (\lambda\varphi_k)^3 e^{\lambda\int_0^x \varphi_k(t)dt}[A_0 + \frac{1}{\lambda}(A_1 + 3\frac{A_0'}{\varphi_k} + 3\frac{\varphi_k'}{\varphi_k^2}A_0) + \frac{1}{\lambda^2}(A_2 + 3\frac{A_1'}{\varphi_k} + 3\frac{\varphi_k'}{\varphi_k^2}A_1 + 3\frac{A_0''}{\varphi_k^2} + 3\frac{\varphi_k'}{\varphi_k^3}A_0' + \frac{\varphi_k''}{\varphi_k^3}A_0) + \sum_{i=0}^{1}\frac{1}{\lambda^{i+3}}\left(A_{i+3} + 3\frac{A_{i+2}'}{\varphi_k} + 3\frac{\varphi_k'}{\varphi_k^2}A_{i+2} + 3\frac{A_{i+1}''}{\varphi_k^2} + 3\frac{\varphi_k'}{\varphi_k^3}A_{i+1}' + \frac{\varphi_k''}{\varphi_k^3}A_{i+1} + \frac{A_i'''}{\varphi_k^3}\right) + O(\frac{1}{\lambda^5})]$$

$$y_k^{(4)}(x,\lambda) = (\lambda\varphi_k)^4 e^{\lambda\int_0^x \varphi_k(t)dt}[A_0 + \frac{1}{\lambda}(A_1 + 4\frac{A_0'}{\varphi_k} + 6\frac{\varphi_k'}{\varphi_k^2}A_0)$$

$$+\frac{1}{\lambda^2}(A_2+4\frac{A_1^{'}}{\varphi_k}+6\frac{\varphi_k^{'}}{\varphi_k^2}A_1+6\frac{A_0^{''}}{\varphi_k^2}+12\frac{\varphi_k^{'}}{\varphi_k^3}A_0^{'}$$
$$+(3\frac{(\varphi_k^{'})^2}{\varphi_k^4}+4\frac{\varphi_k^{''}}{\varphi_k^3})A_0)+\frac{1}{\lambda^3}(A_3+4\frac{A_2^{'}}{\varphi_k}+6\frac{\varphi_k^{'}}{\varphi_k^2}A_2+6\frac{A_1^{''}}{\varphi_k^2}$$
$$+12\frac{\varphi_k^{'}}{\varphi_k^3}A_1^{'}+(3\frac{(\varphi_k^{'})^2}{\varphi_k^4}+4\frac{\varphi_k^{''}}{\varphi_k^3})A_1+4\frac{A_0^{'''}}{\varphi_k^3}+6\frac{\varphi_k^{'}}{\varphi_k^4}A_0^{''}$$
$$+4\frac{\varphi_k^{''}}{\varphi_k^4}A_0^{'}+\frac{\varphi_k^{'''}}{\varphi_k^4}A_0)+\frac{1}{\lambda^4}(A_4+4\frac{A_3^{'}}{\varphi_k}+6\frac{\varphi_k^{'}}{\varphi_k^2}A_3+6\frac{A_2^{''}}{\varphi_k^2}$$
$$+12\frac{\varphi_k^{'}}{\varphi_k^3}A_2^{'}+(3\frac{(\varphi_k^{'})^2}{\varphi_k^4}+4\frac{\varphi_k^{''}}{\varphi_k^3})A_2+$$
$$4\frac{A_1^{'''}}{\varphi_k^3}+6\frac{\varphi_k^{'}}{\varphi_k^4}A_1^{''}+4\frac{\varphi_k^{''}}{\varphi_k^4}A_1^{'}+\frac{\varphi_k^{'''}}{\varphi_k^4}A_1+\frac{A_0^{(4)}}{\varphi_k^4})+O(\frac{1}{\lambda^5})] \quad (5)$$

Now, we substitute the resulting equations (3),(4) and (5)

With equation in the equation (1), we obtain the results.

3. Asymptotic Behaviors of Eigenvalues to the Problem (1)

The aim of this section is to study the asymptotic behaviors of eigenvalues in the following cases:

a. $\rho(a)=1 \& \rho^{'}(a)\neq 0$

b. $\rho(a)=1, \rho^{'}(a)=0 \& \rho^{''}(a)\neq 0$

c. $\rho(a)=1, \rho^{'}(a)=0, \rho^{''}(a)=0 \& \frac{5}{8}\rho^{'''}(a)+p_3(a)\neq 0$

d. $\rho(a)=1, \rho^{'}(a)=0, \rho^{''}(a)=0, \frac{5}{8}\rho^{'''}(a)+p_3(a)=0$ and $-\frac{1}{16}\rho^{(4)}(a)-\frac{3}{2}p_3^{'}(a)+p_4(a)\neq 0$

to the given spectral problem by the theorem :

Theorem 2:

Asymptotic behavior of eigenvalues for sufficiently large λ of the spectral boundary value problem in the irregular case and in the sector $C_0=\frac{\pm i}{4}C(p,q)$, $d=\int_0^a \sqrt[4]{\rho(t)}dt$ has the form:

$$\lambda_m=\frac{w_k}{d}(\pi m+2i\ln m+i\ln(\frac{\pi w_k}{d})-\frac{i}{2}\ln C_0)+o(1)$$

Where $\lambda_m\in T_k$.

and

$$\lambda_m=\frac{\overline{w_k}}{d}(\pi m-2i\ln m-i\ln(\frac{\pi w_k}{d})+\frac{i}{2}\ln C_0)+o(1)$$

Where $\lambda_m\in\overline{T_k}$

$m=N+1,N+2,...,N$ is natural number,

$$C_0=\frac{\pm i}{4}C(p,q), d=\int_0^a \sqrt[4]{\rho(t)}dt$$

$$a) C(p,q)=-\frac{3}{8}\rho^{'}(a)$$
$$b) C(p,q)=-\frac{5}{4}\rho^{''}(a)$$
$$c) C(p,q)=\frac{5}{8}\rho^{'''}(a)+p_3(a)$$
$$d) C(p,q)=-\frac{1}{16}\rho^{(4)}(a)-\frac{3}{2}p_3^{'}(a)+p_4(a)$$

Proof: Since from the given boundary conditions, we can write all as follows

$$U_0(y)=y(0), U_1(y)=y^{'}(0),$$
$$U_2(y)=y^{'''}(a,\lambda)+(iw_2\lambda)y^{''}(a,\lambda)$$
$$+(iw_2\lambda)^2 y^{'}(a,\lambda)+(iw_2\lambda)^3 y(a,\lambda)$$
$$U_3(y)=y^{'''}(a,\lambda)+(iw_3\lambda)y^{''}(a,\lambda)+$$
$$(iw_3\lambda)^2 y^{'}(a,\lambda)+(iw_3\lambda)^3 y(a,\lambda).$$

Thus,

$$U_0(y)=\frac{1}{\sqrt[8]{\rho^3(a)}}[1]$$
$$U_1(y)=[\lambda\varphi_k(0)A_0(0)+A_0^{'}(0)$$
$$+\frac{A_1^{'}(0)}{\lambda}+\frac{A_2^{'}(0)}{\lambda^2}+\frac{A_3^{'}(0)}{\lambda^3}+O(\frac{1}{\lambda^5})]$$

Where

$$A_0(0)=\frac{1}{\sqrt[8]{\rho^3(0)}}, A_0^{'}(0)=-\frac{3}{8}(\frac{\rho^{'}(0)}{\rho^{\frac{11}{8}}(0)}), \varphi_k(0)=iw_k\rho^{\frac{1}{4}}(0)$$

$$A_1^{'}(0)=-[\frac{3}{2}\frac{A_0^{''}(0)}{\varphi_k(0)}+3\frac{\varphi_k^{'}(0)}{\varphi_k^2(0)}A_0^{'}(0)$$
$$+(\frac{3}{4}\frac{(\varphi_k^{'}(0))^2}{\varphi_k^3(0)}+\frac{\varphi_k^{''}(0)}{\varphi_k^2(0)})A_0(0)]$$

$$A_2^{'}(0)=-[\frac{3}{2}\frac{A_1^{''}(0)}{\varphi_k(0)}+3\frac{\varphi_k^{'}(0)}{\varphi_k^2(0)}A_1^{'}(0)$$
$$+(\frac{3}{4}\frac{(\varphi_k^{'}(0))^2}{\varphi_k^3(0)}+\frac{\varphi_k^{''}(0)}{\varphi_k^2(0)})A_1(0)+\frac{A_0^{'''}(0)}{\varphi_k^2(0)}+\frac{3}{2}\frac{\varphi_k^{'}(0)}{\varphi_k^3(0)}A_0^{''}(0)$$
$$+\frac{\varphi_k^{''}(0)}{\varphi_k^3(0)}A_0^{'}(0)+(\frac{\varphi_k^{'''}(0)}{4\varphi_k^3(0)}+\frac{p_3(0)}{4\varphi_k^2(0)})A_0(0)]$$

$$A_3^{'}(0)=-[\frac{3}{2}\frac{A_2^{''}(0)}{\varphi_k(0)}+3\frac{\varphi_k^{'}(0)}{\varphi_k^2(0)}A_2^{'}(0)$$
$$+(\frac{3}{4}\frac{(\varphi_k^{'}(0))^2}{\varphi_k^3(0)}+\frac{\varphi_k^{''}(0)}{\varphi_k^2(0)})A_2(0)+\frac{A_1^{'''}(0)}{\varphi_k^2(0)}+\frac{3}{2}\frac{\varphi_k^{'}(0)}{\varphi_k^3(0)}A_1^{''}(0)$$
$$+\frac{\varphi_k^{''}(0)}{\varphi_k^3(0)}A_1^{'}(0)+(\frac{\varphi_k^{'''}(0)}{4\varphi_k^3(0)}+\frac{p_3(0)}{4\varphi_k^2(0)})A_1(0)$$
$$+\frac{A_0^{(4)}(0)}{4\varphi_k^3(0)}+\frac{p_3(0)}{4\varphi_k^3(0)}A_0^{'}(0)+\frac{p_4(0)}{4\varphi_k^3(0)}A_0(0)]$$

$$U_j(y)=\sum_{k=1}^{4}(iw_j\lambda)^{k-1}y^{(4-k)}(a,\lambda)=0,\ j=2,3$$

$$\Delta(\lambda)=\det|U_i(y_i)|=0, i,\ j=0,1,2,3$$

Proceed to finding the zeros of determinant $\Delta(\lambda)$ for $|\lambda|\to\infty$ in this irregular case $\rho(a)=1$.

Let $f(\lambda)=\rho^{-\frac{1}{2}}(0).(i\lambda)^7$.

Therefore, by [5], we have

$$\lambda^4.e^{2iw_k\lambda d}=C_0[1]$$

Where $C_0=\frac{\pm i}{4}C(p,q)$, where the $<<+>>$ in front of the imaginary unit is taken as $\lambda\in T_k$, $<<->>$ for sector T_k

Let $\lambda\in T_k$

$$e^{2i\overline{w_k}\lambda d}=\lambda^{-4}C_0[1]$$

Solving this equation by the same way in [1], we obtain:

$$\lambda_m=\frac{\pi m w_k}{d}+\frac{2iw_k}{d}\ln\lambda_m-\frac{iw_k\ln C_0}{2d})+o(1)$$

Taking the initial approximation $\lambda_0=\frac{\pi m w_k}{d}$, the method of successive approximations we obtain:

$$\lambda_m=\frac{w_k}{d}(\pi m+2i\ln m+i\ln(\frac{\pi w_k}{d})-\frac{i}{2}\ln C_0)+o(1)$$

and

$$\lambda_m=\frac{\overline{w_k}}{d}(\pi m-2i\ln m-i\ln(\frac{\pi w_k}{d})+\frac{i}{2}\ln C_0)+o(1)$$

Where $\lambda_m\in\overline{T_k}$, $m=N+1,N+2,...,N$ is natural number.

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