Chapter 6

The Special Theory of Relativity

6.1 Introduction

The puzzling properties of light and the ether remained through the turn of the century and up to 1904: the speed of light (as described by the equations of electromagnetism) did not depend on the motion of the observer and, stranger still, the medium in which light propagates could not be described consistently.

A final effort was made in order to understand in a "fundamental" way the negative result of the Michelson-Morley experiment. It was postulated (independently) by Fitz-Gerald and by Lorentz that matter moving through the ether is compressed, the degree of compression being just so that there is a negative result in the M&M experiment. The claim was that the ether wind does slow down and speed up light, but it also contracts all objects and these two effects conspire to give no effect in all experiments.

A calculation shows that an object of length ℓ moving with velocity v with respect to the ether should be contracted to length ℓ' given by

$$\ell' = \ell \sqrt{1 - \frac{v^2}{c^2}}$$

(where c is the speed of light) in order to get the null result required.

So in order to understand the gamut of experimental results the ether had to be a very tenuous medium that could not be felt or tasted, nonetheless the strongest materials would be squashed by it by an amount which makes it impossible to see the ether's effects. The amount a material would be squashed, though admittedly very small, would always be there and is independent of the composition of the object going through the ether (see Fig. 6.1). This is a situation like the one I used in the " little green men on the moon" example (see Sect. **??**): the ether has was awarded the property that no experiment could determine its presence; the ether hypothesis is not falsifiable.

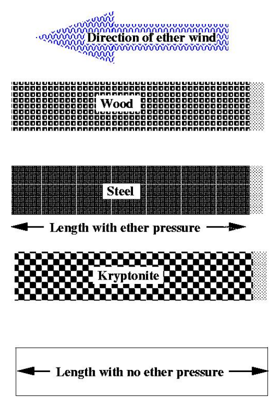


Figure 6.1: The idea behind the Lorentz–Fitz-Gerald contraction.

6.2 Enter Einstein

In 1905 Einstein published three papers. The first (dealing with the so-called "photoelectric effect") gave a very strong impulse to quantum theory, and got him the Nobel prize in 1921. The second dealt with the movement of small particles in a fluid (Brownian motion).

The third paper (Fig. 6.3) of 1905 was called *On the electrodynamics of moving bodies,* it changed the face of physics and the way we understand nature.

This paper starts with a very simple (and well known) example: if a magnet is moved inside a coil a current is generated, if the magnet is kept fixed and the coil is moved again the same current is produced (Fig. 6.4). This, together with the difficulties in detecting the motion with respect to the ether, led Einstein to postulate that

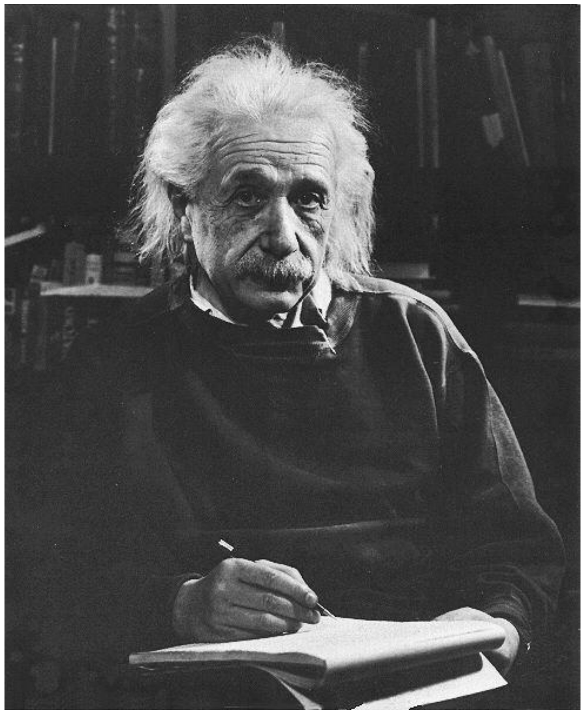


Figure 6.2: Albert Einstein (in his later years)

the same laws of electrodynamics and optics will be valid for all frames of reference for which the laws of mechanics hold good

which is known as the *Principle of Relativity*.

In order to understand the implications of the Principle of Relativity we need (again) the concept of an inertial observer (see Sec. **??**). This is a person which, when observing an object on which no forces act, finds that it moves with constant speed in a straight line, or else is at rest. In terms of inertial observers we can restate the Principle of Relativity:

all the laws of physics are the same for all inertial observers.

All the laws of physics are the same for all inertial observers

Galileo made a very similar statement but he referred only to the laws of mechanics, Einstein's achievement was not only to provide a generalization, but to derive a host of strange, surprising, unexpected and wonderful consequences from it.

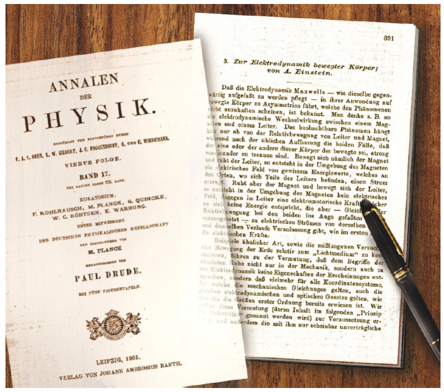


Figure 6.3: The 1905 paper on Special Relativity

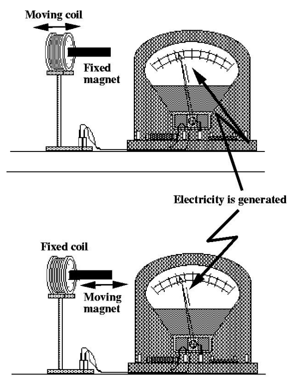


Figure 6.4: Illustration of one of the experimental facts that lead Einstein to the Principle of Relativity.

6.2.1 The first prediction: the speed of light and the demise of Newton's mechanics

Now that we have stated the Principle of Relativity we can examine its implications, and almost immediately we find reason to worry.

The speed of light c is the same for all inertial observers

Maxwell's equations (the equations of electromagnetism, see page **??**) contain a quantity we called c, the speed of light, which is given without reference to any inertial observer. So, if we accept the Principle of Relativity *and* trust Maxwell's equations, we must conclude that c is the same for all inertial observers. So if Jack measures the speed of a beam of light while sitting at the top of the hill, and Jill also measures the speed of the same beam of light while running up the hill, they should get exactly the same answer, no matter how fast Jill runs. It is often said that Einstein "proved that everything is relative" but, in fact, his first conclusion was that the speed of light is *absolute*.

This property of light is very different from, say, the properties of peas as described by the mechanics of Newton: if a person rides on a scooter and shoots peas, these move faster than the peas shot by a person standing by (see Sect. **??**). In contrast if the person on the scooter turns on a laser and the person standing by does the same when they coincide on the street, these two laser beams will reach Pluto at the same time (Fig. 6.5); this happens even if the scooter moves at 99% of the speed of light.

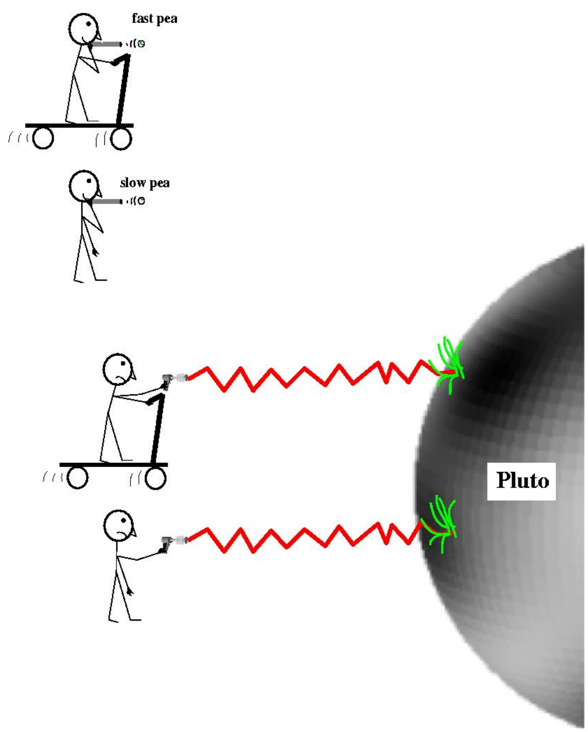


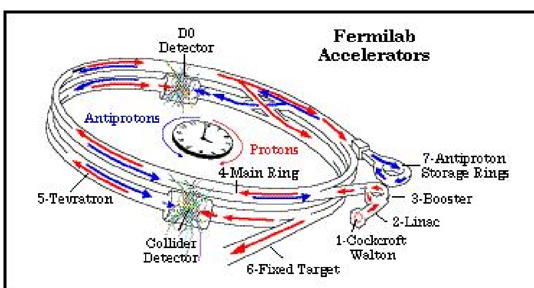
Figure 6.5: The pea shot from the scooter moves faster, yet both laser beams get to Pluto (it is really a photograph of Pluto) at the same time.

Newton would be horrified by this behavior of light beams: according to his mechanics velocities add, so that the laser beam from the scooter should reach Pluto sooner.

Thus, once Einstein adopted his Principle of Relativity, he was faced with a choice: either dismiss Newtonian mechanics or dismiss Maxwell's equations. It was impossible for them both to be right. Newton's mechanics had survived for about 250 years, it was universally accepted in the physics community, and its predictions agreed with all experiments (done up to 1905). Maxwell's equations, in contrast, were rather new, were not tested as thoroughly as Newton's, and were not universally accepted. Nonetheless Einstein took the daring path of siding with Maxwell and so challenged the whole edifice of the Newtonian theory. He was right.

Having chosen sides, Einstein assumed that Newton's mechanics were not a good description of Nature under *all* circumstances: it must then be only a good approximation. Einstein's work was then cut out for him: he needed to find a generalization of Newton's mechanics which is consistent with the Principle of Relativity, *and* which agrees with experiment as well as (or better than) Newton's theory. He was successful.

Significant discrepancies between Newton's and Einstein's mechanics become noticeable only at speeds close to c which explains why no problems were detected with Newton's theory before 1905: all experiments were done at speed very small compared to c. In this century a wealth of experimental evidence has been gathered which supports Einstein's mechanics in favor of Newton's. The best examples appear in experiments done since the 1950's using subatomic particles which are relatively easily accelerated to speeds approaching c. The behavior of such experiments completely vindicates Einstein's approach while being inexplicable from the Newtonian viewpoint.



gh energy accelerators. Most of the studies in subatomic ysics are done in enormous machines commonly called olliders" where electrically charged particles such as elecons and protons are accelerated to speeds very close to at of light and then forced to crash into each other. The sulting debris provides important clues as to the fundaental structure of matter. A popular design for a collider nsists of one or more concentric rings in which the colliding particles are piped and accelerated using electric and magnetic fields. Given the enormous speeds of the particles the design must be extremely accurate, even a very small error can send all the particles crashing into the walls of the ring. All calculations are done using Einstein's mechanics, and the behavior of the particles perfectly matches the predictions of the theory; a design of a collider using Newtonian mechanics would lead to a useless machine.

Concerning the addition of velocities, Newton's formula is, strictly speaking, not correct even for slow moving obejcts. The corrections are, however, very samll when the speeds are small compared to that of light. For example for the case of the passenger in a train in Fig. **??** if the speed of the ball is u and that of the train is v the speed measured from the platform is not $u+v$ as Newton would claim, but

$$(u+v) \times c^2/(c^2+uv)$$

that is, there is a small correction factor $c^2/(c^2+uv)$ which, for ordinary velocities is very small indeed, for example for the example $u = 1\text{m/s}$, $v = 2\text{m/s}$, this factor is 0.9999999998 (Newton would have predicted 1 instead). On the other hand, if both u and v are half the speed of light, the speed seen from the platform would be 80% of the speed of light (and not c as Newton would have expected). For the extreme case where either u or v (or both) are equal to c, the speed seen from the platform would again be c.

In conclusion: the Principle of Relativity together with Maxwell's equations imply that there is a universal speed whose value is the same to all inertial observers. This fact required several fundamental changes in the manner we understand the world.

6.2.2 The second prediction: Simultaneity is relative

One concept which is radically modified by the Principle of Relativity is that of simultaneity. Every-day experience indicates that the statement "two events happened at the same time" (*i.e.* they are simultaneous) is universal, and would be verified by any one looking into the matter. Thus I can say, "I got home at the same time you got to work" and nobody (usually) wonders about the consistency of such statement.

The surprising result is that two FBI agents looking into the matter but moving with respect to each other (and having very accurate clocks) would get conflicting answers. In order to illustrate this result we will consider two murder mysteries, one set in Victorian England which is analyzed using Newton's ideas, the other is set in outer space and is studied following Einstein's guidance.

The first murder mystery (*ca.* 1890)

Sherlock Holmes is called to investigate a murder: a man was found shot in a train car, with two bullets in his head. After much investigation Sherlock finds a hobo who was at a station as the train wheezed by. This man saw two men come in from opposite sides of a wagon and simultaneously fire their revolvers at a chap sitting right in the middle of the train car. Being a Newton acolyte, Holmes is a firm believer that simultaneity is a universal concept, and concludes that both men fired at the same time as an absolute fact. Inspector Lestrade (from Scotland Yard) manages to find both men, who are found guilty of the crime and die in the gallows.

The second murder mystery (*ca.* 2330)

A murdered man is found in the cargo bay of the starship Enterprise with two head wounds caused by laser beams. The tragedy was observed from three places: a space station, the cargo bay itself, and a Klingon ship (a "bird of prey"). At the time of the crime the Enterprise was moving at a speed $c/2$ with respect to the space station; the bird of prey was moving in the same direction as the Enterprise at a speed $3c/4$ with respect to the space station (and was ahead of the Enterprise). To simplify the language we will say that both ships as seen from the space station were moving to the right (see Fig. 6.6).

Everyone agrees that the dead man was hit on the head by two laser beams simultaneously. These beams were fired by a klingon at the back of the cargo bay, and by a human at the front. They shot while they stood at the same distance from the victim. Both life-forms are arrested and put to trial.

Captain Kirk, then at the space station, acts as the human's lawyer. Kirk points out that the klingon must have fired first. Indeed, at the time of the murder the klingon was placed in such a way that the Enterprise carried the victim away from his laser bolt; in contrast, the ship carried the victim towards the human's laser bolt (Fig. 6.7). Since both bolts hit at the same time, *and they travel at the same speed c for all observers*, the klingon must have fired first. "The klingon's guilt is the greater one!" Kirk shouted dramatically, and sat down.

The captain of the bird-of-prey, who is (of course) acting as the klingon's lawyer, disagrees. His ship was moving to the right of the space station, but much faster than the Enterprise, hence, with respect to this ship, the Enterprise was moving to the *left*. "I can then use my esteemed colleague's arguments and categorically state that it was the *human* that fired first (see Fig. 6.8), it is *her* guilt that is the greatest."

Dr. McCoy happened to be in the cargo bay at the time of the shooting and testifies that he saw both the human and the klingon fire at the same time: since the beams hit the victim at the same time, and they were at the same distance, they must have fired at the same time (Fig. 6.9).

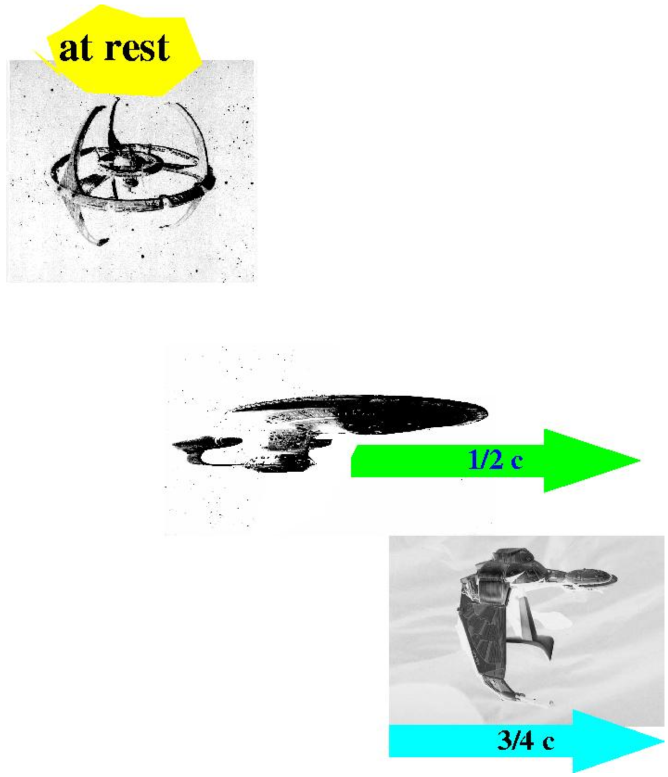


Figure 6.6: The setup for the second murder mystery. The velocities are measured with respect to the space station (labeled "at rest").

Now, the law (in this story) states that the guilty party is the one who fired first, but deciding who did fire first is impossible! This is so because events occurring at different places will not be simultaneous to all observers. The fact that c is the same for all observers implies that if two events separated by some distance (such as the firing of the lasers) are simultaneous to one observer (such as McCoy) they will *not be simultaneous* to observers moving relative to the first (such as Kirk and the Klingon captain). Even the ordering in time of these events is relative

Events occurring at different places will not be simultaneous to all observers

Simultaneity is relative for events separated by a non-zero distance. [1]

Let me use a short-hand and let **K** be the statement "the klingon shoots", while **H** denotes "the human shoots". Then

[1]This was explained by Spock to Kirk...at great length.

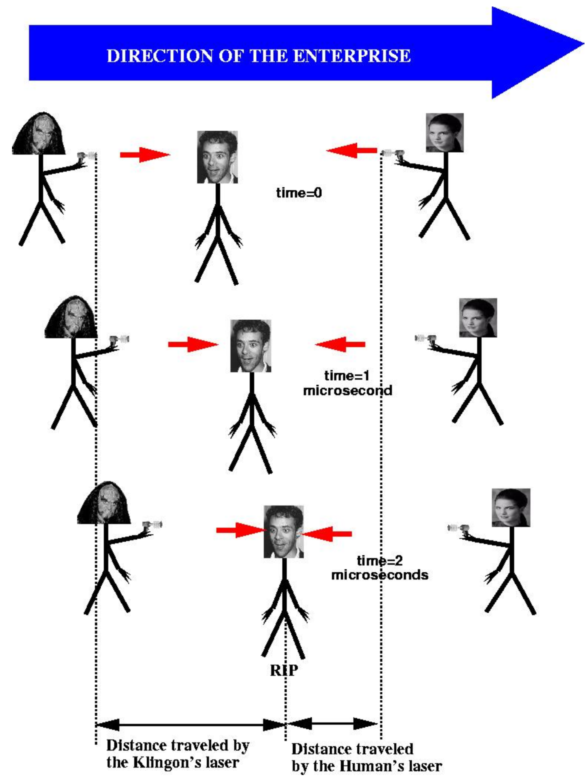


Figure 6.7: Illustration of Kirk's argument (the murder as seen from the space station)

Summary of the arguments				
K	happens before	**H**	as seen from the space station	(Kirk's argument)
H	happens before	**K**	as seen from space station	(Klingon capt.'s argument)
K	simultaneous with	**H**	as seen from Enterprise	(McCoy's argument)

So the Principle of Relativity forces us to conclude that in this situation the ordering of events in time is *relative*. But, this better not be true for *all* events: if the Principle of Relativity would predict that all time orderings are relative we could then imagine an observer who sees you, the reader, being born before your parents!

So there are events such as birth and death of a person which should occur in succession with the *same* ordering for any observer. And there are other events, such those in the shooting mystery, whose ordering in time is observer dependent. What is their difference?

The one clue is the following: in the story the assassins came in from

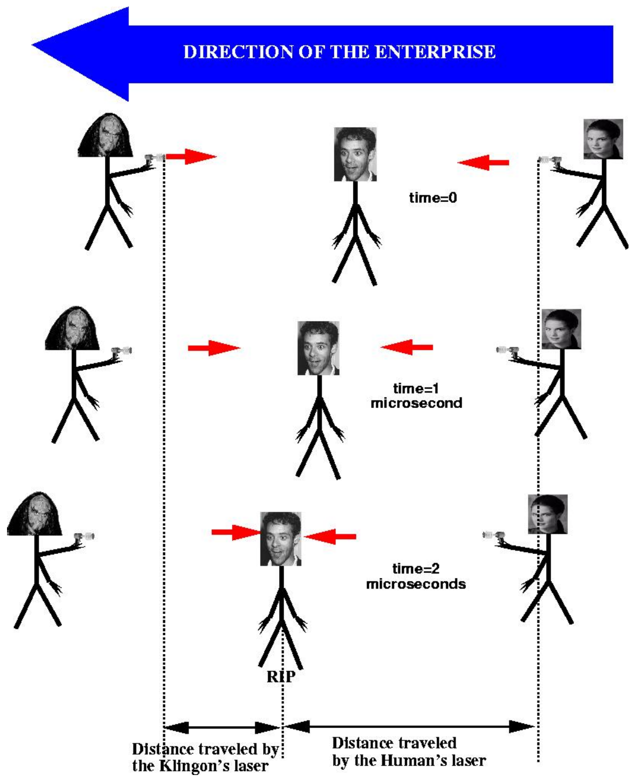


Figure 6.8: Illustration of Klingon captain's argument (the murder as seen from the bird of prey)

opposite sides of a cargo bay and shot the victim. Since lasers travel at the speed of light, the human will receive the image of the klingon shooting only after she herself has fired (in order to see anything we must receive light from some source); the same is true for the klingon. So, *when they fired they could not have been aware of each other's action.*

This is not the same as for birth and death: a cat is born and then the dog eats the cat. It is then possible for you to tell your dog, that is, to send him a signal, that the cat was born. This signal reaches the dog before he performs his grim action (Fig. 6.10)

So two events A (cat is born) and B (dog eats cat) are ordered in the same way in time for all observers if we can send a signal at the time one event occurs (A) which will reach an observer who will witness the second event (B). In this case everyone will agree that A occurs before B, no matter what the relative speed of the observer. An extreme case consists of those

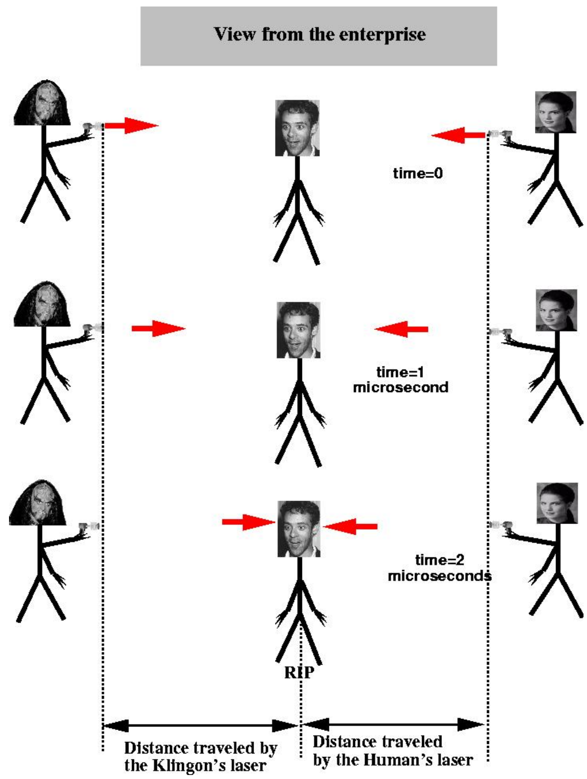


Figure 6.9: Illustration of McCoy's argument (the murder as seen from the Enterprise).

events occurring at the same time at the same place will be seen to occur at the same time by all observers (everyone agrees that the laser beams hit the victim at the same time).

In contrast if no signals sent at the time A occurs can reach an observer before B happens, then the ordering in time of A and B depends on the relative velocity of the observer.

So there is no hope of going back in time with the winning Loto number and becoming a millionaire. If you think about it, the number of paradoxes which would arise if all time orderings were relative would be enormous: if you could go back in time, there would be two of you: one a pauper and the other a millionaire...but which one *is* you? Fortunately the Special Theory of Relativity simply disallows such situations.

Why did all this happen? Because the speed of light is always c. Both laser bolts will be seen to travel at the same speed by all observers, and

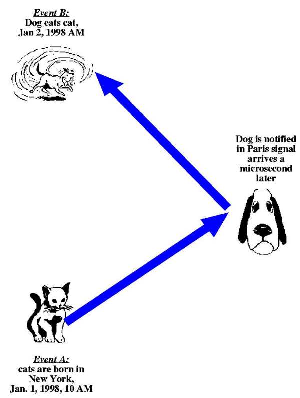


Figure 6.10: Illustration of events whose time ordering is *the same* for all observers.

because c is not infinite, the time it takes to reach a target depends on how the target is moving.

I will emphasize again the conclusions. Since the speed of light is the same for every observer in an inertial frame of reference, two things that are simultaneous to one observer will not be so according to other observers. The inescapable conclusion is that simultaneity is not an absolute concept: the statement "two events at different places occurred at the same time" is true only in a certain inertial reference frame and will be found to be incorrect in other frames.

Despite this there *are* events that everyone will agree are simultaneous: any two events happening at the same time and *at the same spot* will be seen to coincide by any observer. It is when the events are separated by a distance that simultaneity is relative. If events occurring at the same time and place for one observer were seen to occur at different times by another observer one can imagine going to a reference frame where the bullet that killed Lincoln went by his seat one hour before the president sat down. In this frame he was never assassinated!

One thing that Principle of Relativity does not permit is for some events which occur sequentially and such that the first affects the second to be inverted in order. For example it is impossible to go to a frame of reference in which the end of an exam occurs before it begins. It is only events that are mutually independent whose ordering in time can be inverted: two babies could be seen to be born one before the other or vice-versa, but only if they are not born at the same time at the same spot, so Jacob could not be the first born to Isaac (as opposed to Essau) in some frame of reference...the

Bible's story is, in this sense, frame independent.

6.2.3 The third prediction: The demise of Universal Time

Another peculiar and surprising consequence of the Principle of Relativity is that time intervals are no longer universal but depend on the frame of reference. Consider, for example, a clock consisting of a light source and detector. The source emits a light pulse, the pulse goes up and is reflected at a height h by a mirror. It is then detected and this determines one unit of time. See Fig 6.11.

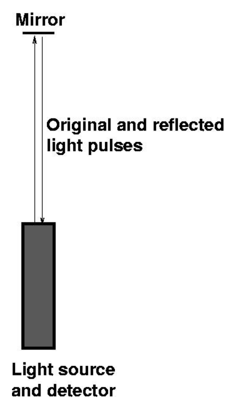


Figure 6.11: A clock at rest with respect to the observer

The time it takes the light pulse to come and go is $t_0 = 2h/c$. This is precisely the time it would be measured by any observer carrying any other clock as long as this observer is not moving with respect to the above timepiece.

Now let's consider what an observer moving with respect to this simple clock sees. This is shown in Fig. 6.12

It is clear that the distance traveled by the beam is larger than the up-down trip observed by the first person. But since the speed of the light beam is the same for both observers, the time measured by the second observer will be *larger*. If we have two such clocks one is at rest with respect to us and the other is moving, we find that the moving clock slows down, moreover, the faster it moves the slower it ticks. This is called *time dilation*: a moving clock ticks slower.

Time dilation: a moving clock ticks slower

This argument was based on the simple clock of Fig. 6.11, will it be true for *all* clocks? To examine this question let's assume we have another clock (a Rollex, for example) which gives ticks *same* way no matter how

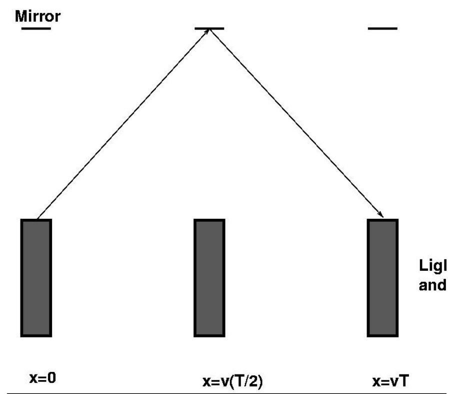
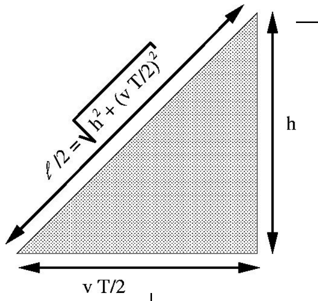


Figure 6.12: A clock moving with speed v to the right with respect to the observer

it moves. You go on a long trip to a near-by star taking the Rollex with you *and* also a clock like the one in Fig. 6.11. Your spaceship, you will notice, has no windows (they had to cut the budget *somewhere!*), but you go anyway. You experience the effects of lift-off but after a while you appear to be at a standstill: you are then moving at a constant speed with respect to Earth. But remember we assumed that the Rollex still ticks the same way as the clocks on Earth, and we have proved that your light-clock does not. So you will see a mismatch between the Rollex and the light-clock: this is an experiment which is done completely inside the spaceship and which determines whether you are moving. If there were such a Rollex the Principle of Relativity would be violated.

If we accept the Principle of Relativity we must conclude that time dilation will occur for *any* clocks, be it a Rollex, a biological clock or a Cartier. Note that this follows from the Principle of Relativity and the validity of Maxwells' equations, no additional assumptions are required.

If an observer at rest with respect to a clock, finds that she is pregnant and eventually delivers, the whole process taking precisely nine months, another observer moving with respect to her (and the simple clock) will find this claim to be wrong, he will state that she had a longer pregnancy (or a very long delivery) but that in any case the whole thing took longer than nine months.



Time dilation and Pythagoras' theorem. The distance the light has to travel in Fig. 6.12 can be determined by using Pythagoras' theorem.
In this reference frame light travels along the long sides of the triangles, each has a length which I call $\ell/2$; let's call T the time it takes to complete the trip, by Pythagoras' theorem $\ell/2 = \sqrt{h^2 + (vT/2)^2}$. On the other hand $\ell = cT$ since light moves at speed c for any observer and it takes a time T (according to the moving observer!) for it to get back to the detector. Solving for T we get

$$T = \frac{(2h/c)}{\sqrt{1-(v/c)^2}} = \frac{T_0}{\sqrt{1-(v/c)^2}}.$$

Thus the observer in motion with respect to the clock will measure a time T greater than T_0, the precise expression being given by the above formula.

So how come we do not see this in ordinary life? The reason is that the effect is very small in everyday occurrences. To be precise it an observer at rest with respect to the clock in Fig. 6.11 measures a time T_0 then the observer which sees the clock move at speed v (and sees the situation depicted in Fig. 6.12) will measure a time T, where $T = T_0/\sqrt{1-v^2/c^2}$ (see the box above). So the effect reduces to the appearance of the factor $1/\sqrt{1-v^2/c^2}$ which in usual circumstances is very close to one (so that T is almost equal to T_0). For example an ordinary man moving at, say 90miles/hr (trying to get his wife to the hospital before she delivers), $v/c = 0.0000001 = 10^{-7}$ (approximately) so that the above factor is essentially one (up to a few hundredths of a trillionth). This is typical of the magnitude of the new effects predicted by Einstein's theory for everyday situations: they are in general very small since the velocities of things are usually very small compared to c.

There are some instances, however, in which the effects are observable. There are subatomic particles which are unstable and decay (the process by which they decay is irrelevant) in a very small time interval when measured in the laboratory. It has also been found that high intensity radiation coming from space and hitting the upper atmosphere generates these same particles (again the process is immaterial). To the initial surprise of the experimenters, these particles survive the trip down to surface of the earth, which takes longer, *as measured on the Earth*, than the particle's lifetime!

The surprise evaporated when it was noted that the particles are moving very fast with respect to the Earth, almost at the speed of light, so that a time interval which is very short when measured at rest with respect to the particle will be much longer when measured in the laboratory.

So the rate of all clocks depends on their state of motion. In this sense

The rate of all clocks depends on their state of motion.

Time is relative.

And while the effect is small in many cases, it is spectacular in others. This is a surprising consequence of the Principle of Relativity and requires a complete divorce from Newton's concept of time (which he assumed to flow evenly under all circumstances, see Sect. **??**): time intervals depend on the motion of the observer, there is no "universal" time.

Time dilation is a *prediction* of the theory which must not be accepted as dogma but should be verified experimentally. All experiments do agree with this prediction. The fact that the theory of relativity makes predictions which can be tested experimentally, is what makes this an honest theory: it is falsifiable. It has been accpeted not because of its beauty, but because these predictions have been verified.

6.2.4 Length contraction

So time is relative, what about distance? In order to think about this note that when we say that the distance between two objects is ℓ we imagine measuring the position of these objects simultaneously...but simultaneity is relative, so we can expect distance to be a relative concept also.

To see this consider the above subatomic particles. As mentioned they are moving very fast but we can still imagine Superman (an unbiased observer if there is one) riding along with them. So we have two pictures: from the observer on earth Superman's clocks (accompanying the particle) are very slow, and so he/she can understand why it takes so long for the particle to decay. But for Superman the particle is at rest and so it must decay in its usual short time...the fact remains, however, that the particle does reach the earth. How can this be? Only if the distance which the particle traveled as measured in the frame of reference in which it is at rest is very short. This is the only way the observation that the particle reaches the earth's surface can be explained: for the observer on the earth this is because of time dilation, for the observer riding along with the particle, this is because of length contraction, see Fig. 6.13.

But we do not require peculiar subatomic particles in order to demonstrate length contraction (though the Principle of Relativity requires that if

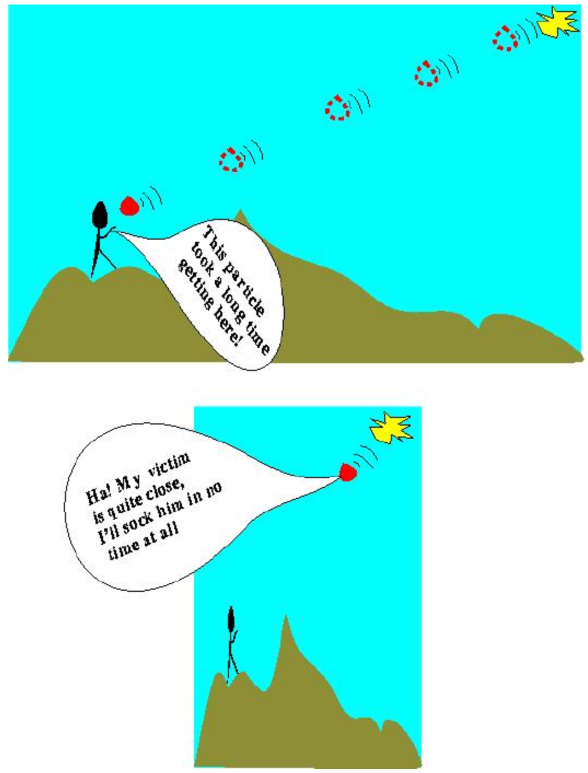


Figure 6.13: An observer measures a long life-time for the particles due to time dilation. The particles measures a short distance between itself and the observer due to length contraction.

it occurs for the example above it should occur in all systems, otherwise we could determine by comparison which system has an absolute motion). So consider the previous experiment with the moving clock (Fig 6.12).

- The observer watching the clock move with velocity v notes that in a time T the clock moves a distance $\ell = vT$.
- The observer riding with the clock notes that the same distance is covered in a time T_0; therefore the length measured by him/her is $\ell_0 = vT_0$ (He also sees the other observer receding with speed v.)
- Therefore we have $\ell = vT = vT_0/\sqrt{1-(v/c)^2} = \ell_0/\sqrt{1-(v/c)^2}$. Thus, the observer moving with the clock will measure a shorter length compared to the one measured by the other observer.

It is important to note that these expressions are *not* to be interpreted as "illusions", the an observer in motion with respect to a ruler will, when measuring its length, find a result smaller than the result of an observer at rest with respect to the ruler. An observer in motion with respect to a clock will measure a time larger than the ones measured by an observer at rest with the clock.

The question, "what is 'really' the length of a ruler?" has no answer for this length depends on the relative velocity of the ruler to the measuring device [2]. The same as with velocity, specifying lengths requires the framework provided by a frame of reference,

Length is relative. Length is relative

Note that this peculiar effect occurs only for lengths measured along the direction of motion and will *not* occur for lengths perpendicular to it. To see this imagine two identical trees, we sit at base of one and we observe the other move at constant speed with respect to us, its direction of motion is perpendicular to the trunk. In this setup as the roots of both trees coincide also will their tops, and so in *both* frames of reference we can *simultaneously* determine whether they have the same height; and they do.

This implies that a moving object will be seen thinner (due to length contraction) but not shorter. Thin fellows will look positively gaunt at speeds close to that of light.

These conclusions require we also abandon Newton's description of space: distances are observer-dependent, no longer notches in absolute space.

6.2.5 Paradoxes.

The above conclusions can be very confusing so it might be worthwhile to discuss the a bit.

Take for example length contraction: the Principle of Relativity implies that if we measure the length some rod while at rest with respect to it, and then we measure it when it is moving along its length, the second measurement yields a smaller value. The crucial point to keep in mind is the condition that the first measurement is made at rest with respect to the rod.

[2] One can, of course, say that *the* length of a ruler is the one measured while at rest with respect to it...but this is only a convention. Once the result of any length measurement is known (for any relative speed between ruler and measuring device), special relativity determines unambiguously what any other observer would measure.

Similarly suppose we have two clocks labeled 1 and 2. which are in perfect agreement when they are at rest with respect to each other. Suppose now these clocks are endowed with a relative velocity. Then when we look at clock 2 in the frame of reference in which clock 1 is at rest, clock 2 will be measured to tick slower compared to clock 1. Similarly, in the frame of reference in which clock 2 is stationary, clock 1 will run slower compared to clock 2.

These results can be traced back to the fact that simultaneous events are not preserved when we go from one reference frame to another.

There are many "paradoxes" which appear to imply that the Principle of Relativity is wrong. The do not, of course, but it is interesting to see how the Principle of Relativity defends itself.

1. Consider a man running with a ladder of length ℓ (measured at rest) and a barn also of length ℓ (again, when measured at rest). The barn has two doors and there are two persons standing at each of them; the door nearer to the ladder is open the farthest is closed. Now the man with the ladder runs fast towards the barn while the door persons have agreed to close the first door and open the second door as soon as the rear of the ladder goes through the first door.
   This is a paradox for the following reason. The ladder guy is in a frame of reference in which the ladder is at rest but the barn is moving toward him, hence he will find the length of the barn shortened (shorter than his ladder), and will conclude that the front of the ladder will hit the second door before the first door is closed.
   The barn people in contrast find the ladder shortened and will conclude that it will fit comfortably. There will even be a short lapse between the closing of the first door and the opening of the second, there will be no crash and the ladder guy will sail through.
   So who is right?
   The answer can be found by remembering that an even simultaneous for the barn people (the closing and opening of the doors) will not be simultaneous for the ladder guy. So, while for the door person the opening of the rear door and closing of the front occur at the same time, the ladder guy will see the person at the second door open it *before* the person at his rear closes that door and so he will sail through but only because, he would argue, the door guards were not synchronized.

2. There is an astronaut whose length is 6 ft and he sees a big slab of metal with which he/she is going to crash. This piece of metal has

a square hole of length 6 ft. (measured at rest with respect to the slab). From the point of view of the astronaut the hole is shrunk and so he will be hit...and die! From the point of view of an observer on the shuttle the plate is falling toward earth and the astronaut moving at right angles toward it, hence this observer would measure a short astronaut (5 ft) [3] and conclude that he/she will not be harmed (see Fig. 6.14). What does really happen?

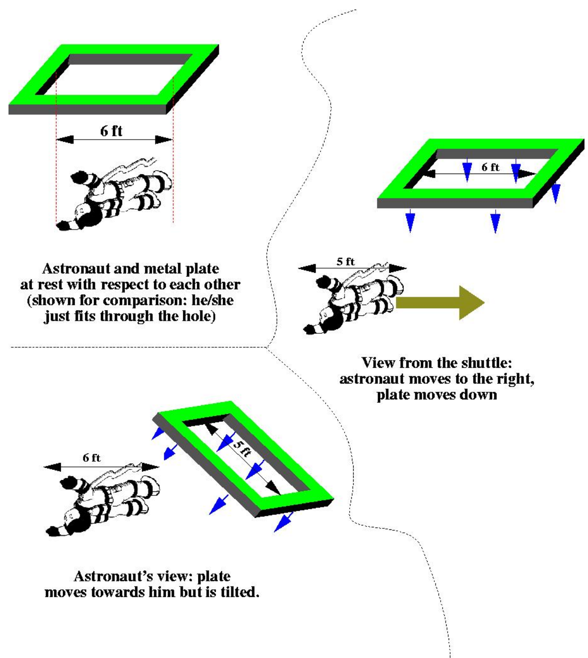


Figure 6.14: An astronaut's close encounter with a metal plate

The problem is solved in the same way as above. For the astronaut to be hit a simultaneous coincidence of his head and legs with the two extremes of the slab's hole should occur. In fact he is not hit. What is more peculiar is what he sees: he will see the slab tilt in such a way that he goes through the hole with no problem!

This story illustrates the peculiar look which big objects acquire at very large speeds. For example, a kettle moving close to the speed of light with respect to, say, the Mad Hatter will be observed to twist in

[3] This corresponds to an astronaut moving at about half the speed of light toward the plate.

a very curious way indeed, see Fig. 6.15.

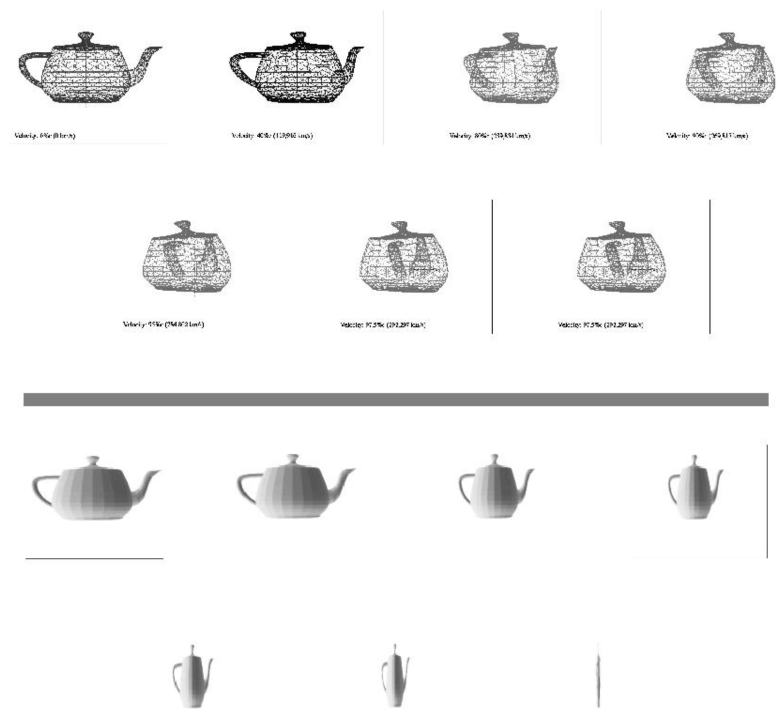


Figure 6.15: A relativistic kettle. The top view shows how the three dimensional view is distorted due to relativistic effects. The bottom view shows the corresponding behavior of a flat kettle which exhibits only length contraction.

Just as for the case of length contraction and time dilation, the effect on the kettle is *not* an optical illusion, but any unbiased observer (such as a photographic camera) would detect the above images precisely as shown. If the relative velocity between the observer and the kettle is known, one can use the formulas of special relativity to determine the shape of the object when at rest with respect to it...and we would obtain the first of the images: a nice kettle

3. Consider two identical twins. One goes to space on a round trip to Alpha-Centauri (the star nearest to the Sun) traveling at speeds very

close to c. The round trip takes 10 years as clocked on Earth [4]. As seen by the twin remaining on Earth all clocks on the ship slow down, including the biological clocks. Therefore he expects his traveling twin to age less than 10 years (about 4.5 years for these speeds; the difference is large since the speed is close to c).
On the other hand the twin in the spacecraft sees his brother (a together with the rest of the solar system) traveling backwards also at speeds close to c and he argues that Einstein requires the twin on Earth to age less than 10 years. Thus each one states that the other will be younger when they meet again!
The solution lies in the fact that the traveling twin is not always in an inertial frame of reference: he must decelerate as he reaches Alpha-Centauri and then accelerate back. Because of this the expressions for time dilation as measured by the traveling twin will not coincide with the ones given above (which are true only for observers in different inertial frames). It is the traveling twin that will be younger.

6.2.6 Space and Time

All events we witness are labeled by a series of numbers, three to tell us where it happened, and one to determine *when* it happened. All in all four numbers are needed. These numbers are determined by some measuring devices such as measuring rods and clocks.

According to Newton (see Sect. **??**) the properties of measuring rods and clocks can be made completely independent of the system which they measure (if it does not look like that, you can buy a higher quality device which will satisfy this criterion). But Einstein showed this is *not* the case: even Cartier watches slow down when compared to Seiko watches when they move with respect to each other. Even high density steel beams will be measured to be shorter than wimpy papers when their relative velocity is non-zero.

The measurements obtained by two observers in motion relative to each other are not identical, but they *are* related. For example, the times measured by two clocks are related by the time-dilation formula given earlier. Suppose observer A measures the location and time at which an event occurs: spider-man ran the 100 yard dash in 3 seconds flat. Now observer B, moving with respect to A, wants a description of this feat in his own coordinates. In order to find how many yards spider man ran *as measured by B*

[4]This corresponds to a speed of 90% that of light

this observer needs to know his velocity with respect to A, the distance spidy ran as measured by A (100 yds) and how long did he take as measured by A's clock (3 sec); it is *not* enough to know the distance and relative velocity, the *time* it took is also needed.

The fact that in order to compare results from different observers both position and time are required is completely foreign to Newtonian mechanics. Yet this is the way the universe is organized. Far from being independent, space (that is, position) and time are interlinked. In fact, the mathematical description of the Special Theory of Relativity is most naturally expressed by combining space and time into one object: *space-time*. A point in space-time determines the position *and* time of occurrence of an event.

Within Special Relativity space-time is unaltered by whatever is in it. There are rules that state how the measurements of two observers are related, but these rules are unaltered by the objects (and beings) that populate space-time, they are the same whether we look at a pea, an elephant or a star millions of times more massive than the Sun. Space and time are still the arena where Nature unfolds.

We will see when we describe the General Theory of Relativity (Chap. **??**) that space-time is far from being this imperturbable object where things just happen, it is in fact a dynamical system which affects and is affected by the matter in it. The development of our ideas of space and time from being independent of each other and imperturbable, to being meshed into space-time system, to being a dynamical object is one of the most profound developments derived from the general and special theories of relativity.

6.2.7 The top speed.

In all the above discussion all the effects would go away is the speed of light were infinite. If there is a top speed, which by definition has and absolute value (the same for all observers), then all the above effects return. It is because the equations found by Maxwell involve an absolute speed, and because they agree with experiment, and because nothing has been found to travel faster than light in vacuum, and because all the consequences of the Principle of Relativity are verified again and again with the top speed equal to c, that we believe this top speed to be precisely c.

Imagine, as Einstein did when a teenager, what would happen is you could move at the speed of light. As you go by a village (for example) you'd move at the same speed as all the light coming form that village. So, if you look around, you would see the same things all the time, nothing would ever change since you are riding along with a single image: the one carried by the light from the village at the time you passed it. In your frame of reference time would stand still! (we will see, however, that it is impossible for something having mass –such as you– to move at the speed of light. You can reach speeds very close to c but never reach the speed of light itself).

Imagine now what would happen if, for example, a rat manages to travel at a speed greater than c. Let's imagine that as the rat travels by you, you send a short laser light pulse after it. According to you the rat will gain on the light pulse steadily. Since the distance between the rat's tail and the front edge of the pulse increases the rat would think that the pulse is moving in the opposite direction. So you and the rat would disagree even on the direction along which the light is traveling. This, of course, contradicts the Principle of Relativity and/or Maxwell's equations and it shows that the Principle of Relativity together with Maxwells' equations imply that nothing can move faster than light.

This is a good feature: if a faster-than-light-rat could be found, the vermin farme of reference would have time flowing backwards. To see this imagine the rat going by the same village mentioned above. Since the rat moves faster than light it will steadily gain on the light beams than come from the village. As it looks around the rat will see the church clock strike 12, and, as it gains on some earlier images, the rat would wee the clock strike 11, etc. So events whose time orderings wer aboslute would no longer occur in the correct order in this frame.

6.2.8 Mass and energy.

How could it be that we cannot accelerate something to go faster than light? Surely we could kick a ball again and again and again until it travels faster right? No! and the reason is quite interesting.

As something is moving with respect to another object we say that the moving thing has a certain amount of energy by virtue of its motion. Energy is the ability to do some work, and, indeed, a moving thing can be lassoed and made to do some work, like pulling a car (of course in so doing it looses energy and slows down).

Now, when we have the above object moving, it will have a certain amount of energy. Einstein argued, the only way we can insure that it cannot be accelerated indefinitely, is if there is a universal equivalence between mass and energy. The more energy an object has, the heavier it will be. When we speed it up a little bit it becomes a bit heavier, and so it also becomes a bit harder to speed it up further. In fact, the closer we are to the speed of light, the larger the force is needed to accelerate the object; an infinite force is needed to speed up a material object to the speed of light: it never happens!

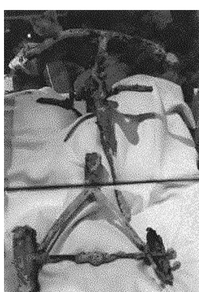
But there is more to the equivalence of mass and energy, for it also implies that an object of mass m has energy, just by virtue of its existence; the specific relationship is

$$E = mc^2.$$

This formula plays a basic role in nuclear reactions (and in atom bombs, for that matter): in these processes an atomic nucleus of initial mass M is transformed (either because the environment is tailored to insure this or because is is unstable and disintegrates spontaneously) into another object of smaller mass m. The difference in mass is released as energy in the amount $(M - m)c^2$.

To give an idea of how powerful this is, suppose we initially have a sheet of paper weighing 6gr, and that at the end we have something weighing half this amount. The energy released is then so big as to turn on a light bulb of 100W for about 86,000 years, or run a hair-drier for about 4000 years.

The energy released through the transformation of mass is also capable of destroying a whole planet (or at 'least' all life on it). Einstein was not aware of this application until much later in his life.



Shin's tricycle. Shin-ichi was a three year old boy who loved his tricycle. When the bomb was dropped, he was playing with his best friend, Kimiko. They died. They were buried in the garden of Shin-ichi's house together. In July 1985, 40 years later, their parents decided to move them to a proper grave.
From the story of *"Shin's Tricycle"* (Translation by Kazuko Hokumen-Jones and Jacky Copson):
Early in the morning, I began to dig open the grave with Kimi's mother, who had come to help. After digging for a while a rusty pipe began to show. "Oh! It's the tricycle!" Before I realized it I had started to sob. To tell you the truth, I'd forgotten all about the tricycle.
"Look! There's something white," someone cried. I felt like ice. Carefully we uncovered the bones using chop-sticks and brushes. There were a number of tiny bones.
"Shin-ichi, Shin-ichi." "Kimiko." Everyone's eyes were glued to the little white hands of the two children. They were still holding hands....

The principle $E = mc^2$ was used during the Second World War to develop what is now known as atomic weapons (Fig. 6.16). Shortly thereafter it was used to develop the hydrogen bomb. Atomic bombs were used during the Second World War in two Japanese cities, Hiroshima and Nagasaki. Hundreds of thousands of people died. The creation of nuclear weapons was one of the watersheds of the 20-th century, and it marks one of the most dramatic instances in which physics has affected the social structure of the planet. Yet the very same formulas also suggest the possibility of obtaining vast amounts of energy which can be used for constructive purposes. It is a burden of post-second World War physicists to deal with this issue, and to strive for decent and environmentally safe applications of nuclear power.

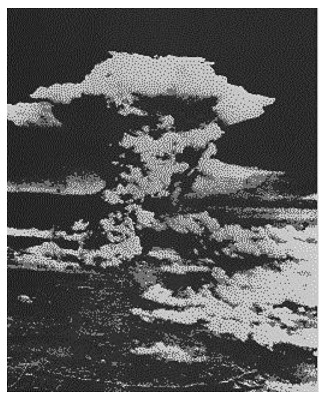


Figure 6.16: An atomic explosion.