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Effect of Shear on Stress Distribution in Redundant Frames

Chidolue C.A., Aginam C.H.

Abstract - ***In this paper, shear-modified expressions for fixed end moments and reactions were obtained for various beam loading conditions using the shear modified stiffness coefficients of elastic beams derived by the authors. By taking the effect of shear on the behavior of beam elements into consideration, a set of modified homogeneous solution of the beam elastic curve equation was obtained and used to derive expressions for fixed end moments and shears for beams with various end conditions and loading. The shear-modified fixed end moment expressions were used to analyze redundant frames. The results of the analysis were then compared with those obtained using the traditional expressions for fixed end moments and shears.***

Keywords: ***Elastic curve, shear- modified stiffness coefficients, fixed end moments, redundant frames, stress distribution.***

I. INTRODUCTION

Beam stiffness coefficients which make up the elements of the stiffness matrix of elastic beams were derived on the assumption that the beam element is subjected to pure bending. Even in the absence of externally applied load every structural beam possesses some self weight which gives rise to distributed load on the structural member. It is well known that once there is distributed load on a beam element, shear inevitably accompanies the induced bending moment. Thus, in real life, the condition of pure bending is hardly attained but can only be a sub-state in the analysis of elastic beams if superposition principle is in use.

Early work by Karamanski et al, [1], included the effect of shear in deriving stiffness coefficients of elastic beams by considering a beam element as member in pure bending, i.e., shear was initially ignored and there after its contribution on deflection of elastic curve was added as a secondary effect.

Other studies which included shear effects on response of structural beams are those of Osadebe and Mama, [2], Chugh, [3], Brush and Mitchel, [4].

Osadebe and Chidolue [5], derived a shear-modified differential equation of the elastic curve of a uniform beam element by considering simultaneous action of bending and shear.

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Chidolue C.A Department of Civil Engineering, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria

C.H. Aginam Department of Civil Engineering, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria.

After integrating the fourth order differential equations obtained, they used initial value approach to express the four unknown constants of integration in terms of initial values of deflection, slope, bending moment and shear. Thus the results obtained took into consideration the simultaneous action of bending and shear in beams.

In this work, shear-modified expressions for fixed end moments and reactions were derived for various beam loading conditions using the shear modified stiffness coefficients of elastic beams derived by the authors[5]. The expressions obtained were used to analyze redundant frames. The results of the analysis were then compared with those obtained using the traditional expressions for fixed end moments and reactions.

Three portal frames of different beam-column assemblages and loading conditions were analyzed using (a) expressions for fixed end moments obtained in books [6], [7], and (b) shear-modified expressions derived in this work.

II. THE SHEAR MODIFIED STIFFNESS COEFFICIENTS

A set of initial value homogeneous solution of the elastic curve equation is given as follows [2]:

$$y(x) = y_0 + \theta_0 x - \frac{M_0}{2EI}x^2 - \frac{Q_0}{6EI}x^3 \qquad \text{(a)}$$

$$\theta(x) = \theta_0 - \frac{M_0}{EI}x - \frac{Q_0}{2EI}x^2 \qquad \text{(b)} \qquad (1)$$

$$M(x) = M_0 + Q_0 x \qquad \text{(c)}$$

$$Q(x) = Q_0 \qquad \text{(d)}$$

where, y_0, θ_0, M_0 and Q_0 are the initial values of deflection, slope, bending moment and shear force, respectively on the elastic beam. x = distance along the beam measured from a known co-ordinate.

By taking into consideration, the effect of shear on the behavior of beam elements the authors obtained the following modified homogeneous solution of the elastic curve equation:

$$y(x) = y_0 + \theta_0 x - \frac{M_0 x^2}{2EI} - \frac{Q_0}{6EI}\left(x^3 - 3\beta^2 x\right) \qquad \text{(a)}$$

$$\theta(x) = \theta_0 - \frac{M_0 x}{EI} - \frac{Q_0}{2EI}\left(x^2 - \beta^2\right) \qquad \text{(b)}$$

$$M(x) = M_0 + Q_0 x \qquad \text{(c)}$$

$$Q(x) = Q_0 \qquad \text{(d)} \qquad (2)$$

where, β is a parameter governing shear modification factor and is given by,

$$\beta = 2r\sqrt{K\left(1+\nu\right)} \qquad (3)$$

r = radius of gyration = $\sqrt{I / A}$
I = Moment of inertia,
A = Cross sectional area,
K = Shape factor
ν = Poisson ratio
E = Elastic modulus

Equations (2a) to (2d) constitute a set of shear modified homogeneous solution of the elastic curve equation which can be used to derive the expressions for fixed end moments and reactions of beam-column assemblages.

III. DETERMINATION OF SHEAR-MODIFIED FIXED END MOMENTS AND REACTIONS

A: Fixed- ended beam with point load

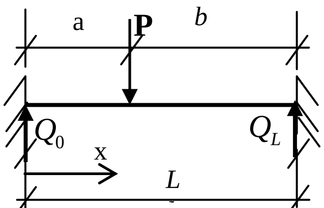

Fig. 1

A uniform beam of length L, subjected to a lateral point load P as shown in Fig. 1 was considered. Equations (2a) to (2d) constitute the set of homogeneous solution of the elastic beam curve, i.e., when the imposed lateral load is absent. In order to obtain a general solution we consider the additional effect of the imposed load on the beam.

Noting that the imposed load P has the similitude of shear, the particular integral for displacement y(x), slope θ(x), moment M(x), and shear Q(x), can be obtained by considering the imposed load P as the parameter Q_0 in Fig. 1. However, the origin is seemingly displaced to the point of application of load P, so that initial distance which measured x in the homogeneous solution will now measure (x-a) distance in the particular integral. Also, since the parameter Q_0 has opposite direction with P, we introduce P with negative sign and obtain the particular integrals as follows.

$$y_p = \frac{P}{6EI}\left[\left(x-a\right)^3 - 3\beta^2\left(x-a\right)\right] \qquad (a)$$

$$\theta_p = \frac{P}{2EI}\left[\left(x-a\right)^2 - \beta^2\right] \qquad (b)$$

$$M_p = -P\left(x-a\right) \qquad (c)$$

$$Q_p = -P \qquad (d) \qquad (4)$$

Addition of these particular integrals, (4a) to (4d), to the set of homogeneous solutions, (2a) to (2d), gives a set of general solution, (5a) to (5d). Thus:

$$y(x) = y_0 + \theta_0(x) - \frac{M_0}{2EI}x^2 - \frac{Q_0}{6EI}\left(x^3 - 3\beta^2 x\right) + \frac{P}{6EI}\left[\left(x-a\right)^3 - 3\beta^2\left(x-a\right)\right] \qquad (a)$$

$$\theta(x) = \theta_0 - \frac{M_0 x}{EI} - \frac{Q_0}{2EI}\left(x^2 - \beta^2\right) + \frac{P}{2EI}\left[\left(x-a\right)^2 - \beta^2\right]$$

(b)

$$M(x) = M_0 + Q_0 x - P\left(x-a\right) \qquad (c)$$

$$Q(x) = Q_0 - P \qquad (d) \qquad (5)$$

In order to obtain fixed end moments and shears under the action of the point load P, we note that the displacement and slope at both ends of the beam are zero, i.e., y(0) = 0, θ(0) = 0, y(L)= 0, $\theta(L) = 0$.

Applying these conditions to (5a) and (5b) we obtain that ,

$$3M_0 L^2 + Q_0\left(L^3 - 3L\beta^2\right) - P\left(b^3 - 3b\beta^2\right) = 0$$

and

$$2M_0 L + Q_0\left(L^2 - \beta^2\right) - P\left(b^2 - \beta^2\right) = 0$$

Solving and simplifying gives:

$$Q_0 = \frac{Pb^2}{L^3}\left[\frac{3L\left(1-\alpha\right) - 2b\left(1-3\alpha\right)}{\left(1+3\mu\right)}\right] \qquad (a)$$

$$M_0 = -\frac{Pb^2}{L^2}\left[\frac{L\eta_1 - b\eta_2}{1+3\mu}\right] \qquad (b)$$

$$Q_L = \frac{Pb^2}{L^3}\left[\frac{3L\left(1-\alpha\right) - 2b\left(1-3\alpha\right)}{\left(1+3\mu\right)} - \frac{L^3}{b^2}\right] \qquad (c)$$

$$M_L = -\frac{Pb^2}{L^2}\left[\frac{a^2 + \lambda_1 + \lambda_2}{b\left(1+3\mu\right)}\right] \qquad (d) \qquad (6)$$

Where,

$$\eta_1 = \left(1 - \alpha - 3\mu + 3\alpha\mu\right)$$

$$\eta_2 = \left(1 - \mu - 3\alpha + 3\alpha\mu\right)$$

$$\lambda_1 = 2\alpha L - 3\left(\alpha b + \mu L - \alpha\mu L\right),$$

$$\lambda_2 = b\left(\mu - 3\alpha - 3\alpha\mu\right) + 3\mu L^2$$

$$\alpha = \beta^2 / b^2 \quad \text{and} \quad \mu = \beta^2 / L^2$$

Equations (6a) to (6d) are the shear modified fixed end moments and reactions for fixed ended beam with point load shown in Fig. 1. The expressions within the square brackets are the shear modification factors which tend to unity when shear effects are neglected.

B: Fixed ended beam with moment at arbitrary point along the beam

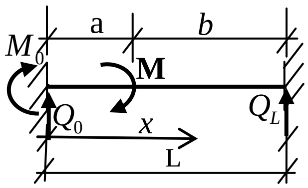

Fig. 2

In this case the parameter M has similitude with M_0 in the homogeneous solution and is of the same sign (direction). Therefore by replacing M_0 with M and changing x to (x-a) in the homogeneous solution (2a) to (2d), the particular integrals , (7a) to (7d) are obtained.

$$y_p = -\frac{M}{2EI}(x-a)^2 \qquad \text{(a)}$$

$$\theta_p = -\frac{M}{EI}(x-a) \qquad \text{(b)}$$

$$M_p = M \qquad \text{(c)}$$

$$Q_p = 0 \qquad \text{(d)} \qquad (7)$$

The general solution is obtained by adding (2) and (7). Thus:

$$y(x) = y_0 + \theta_0 x - \frac{M_0}{2EI}x^2 - \frac{M}{2EI}(x-a)^2 - \frac{Q_0}{6EI}\left(x^3 - 3\beta^2 x\right) \qquad \text{(a)}$$

$$\theta(x) = \theta_0 - \frac{M_0 x}{EI} - \frac{M}{EI}(x-a) - \frac{Q_0}{2EI}\left(x^2 - \beta^2\right) \qquad \text{(b)}$$

$$M(x) = M_0 + M + Q_0 x \qquad \text{(c)}$$

$$Q(x) = Q_0 \qquad \text{(d)} \qquad (8)$$

The deflection and slope at both ends of the beam are zero. Thus, y(0) = 0; $\theta(0) = 0$; y(L) = 0; $\theta(L) = 0$. Applying these conditions to (8a) and (8b) we obtain that:

$$3M_0 L^2 + 3Mb^2 + Q_0\left(L^3 - 3L\beta^2\right) = 0$$

$$2M_0 L + 2Mb + Q_0\left(L^2 - \beta^2\right) = 0$$

Solving and simplifying yields,

$$Q_0 = -\frac{6Mab}{L^3}\left[\frac{1}{1+3\mu}\right] \qquad \text{(a)}$$

$$M_0 = \frac{Mb}{L^2}\left[3a\left(\frac{1-\mu}{1+3\mu}\right) - L\right] \qquad \text{(b)}$$

$$Q_L = -\frac{6Mab}{L^3}\left[\frac{1}{1+3\mu}\right] \qquad \text{(c)}$$

$$M_L = \frac{Ma}{L^2}\left[L - 3b\left(\frac{\mu+1}{1+3\mu}\right)\right] \qquad \text{(d)} \qquad (9)$$

C: Fixed ended beam with uniformly distributed load (u.d.l)

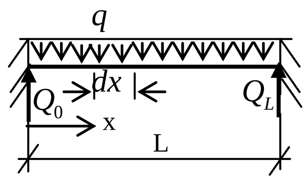

Fig. 3

In the case of fixed ended beam with uniformly distributed load, Q = qdx and the particular integrals, y_p, θ_p, M_P, and Q_p are obtained by integrating the appropriate expressions for deflection, slope, moment and shear, in (2), containing Q_0, and replacing Q_0 with qdx. Thus:

$$dy_p = \frac{qdx}{6EI}\left(x^3 - 3\beta^2 x\right)$$

$$\to y_p = \frac{q}{6EI}\int\left(x^3 - 3\beta^2 x\right)dx$$

$$y_p = \frac{q}{6EI}\left[\frac{x^4}{4} - \frac{3\beta^2 x^2}{2}\right] \qquad \text{(a)}$$

$$\theta_p = \int\frac{qdx}{2EI}\left(x^2 - 3\beta^2\right)dx = \frac{qx^3}{6EI} - \frac{q\beta^2 x}{2EI} \qquad \text{(b)}$$

$$M_p = -\int qxdx = -\frac{qx^2}{2} \qquad \text{(c)}$$

$$Q_p = -\int qdx = -qx \qquad \text{(d)} \qquad (9)$$

Hence, the general solution is obtained by adding (2) and (9). Thus:

$$y(x) = y_0 + \theta_0(x) - \frac{M_0}{2EI}x^2 - \frac{Q_0}{6EI}\left(x^3 - 3\beta^2 x\right) + \frac{qx^4}{24EI} - \frac{q\beta^2 x^2}{4EI} \qquad \text{(a)}$$

$$\theta(x) = \theta_0 - \frac{M_0 x}{EI} - \frac{Q_0}{2EI}\left(x^2 - \beta^2\right) + \frac{qx^3}{6EI} - \frac{q\beta^2 x}{2EI} \qquad \text{(b}$$

$$M(x) = M_0 + Q_0 x - \frac{qx^2}{2} \qquad \text{(c)}$$

$$Q(x) = Q_0 - qx \qquad \text{(d)} \qquad (10)$$

The displacement and slope at both ends of the beam are zero, i.e, y(0) = 0, $\theta(0) = 0$, y(L) = 0, $\theta(L) = 0$
From (10a) and (10b) we obtain that;

$$12M_0L^2 + 4Q_0\left(L^3 - 3\beta^2 L\right) = qL^4 - 6q\beta^2 L^2$$

and $12M_0L^2 + 6Q_0L\left(L^2 - \beta^2\right) = 2qL^4 - 6q\beta^2L^2$

Solving and simplifying gives :

$$Q_0 = \frac{qL}{2}\left[\frac{6\mu - 1}{3\mu - 1}\right] \quad \text{(a)}$$

$$M_0 = \frac{qL^2}{12}\left[\frac{1 - 9\mu - 36\mu^2}{3\mu - 1}\right] \quad \text{(b)}$$

$$Q_L = -\frac{qL}{2}\left[\frac{1}{3\mu - 1}\right] \quad \text{(c)}$$

$$M_L = \frac{qL^2}{12}\left[\frac{1 - 9\mu - 36\mu^2}{3\mu - 1}\right] \quad \text{(d)} \qquad (11)$$

D. Propped cantilever with point load

This case is similar to case *A* except for the boundary conditions:

y(0) = 0; y(L) = 0

$\theta(0) = \theta_0 = 0$, $M(L) = M_L = 0$

Applying these to the set of general solutions, eqns (4a) to (4d) we obtain that:

$$3M_0L^2 + Q_0\left(L^3 - 3L\beta^2\right) = P\left(b^3 - 3b\beta^2\right)$$

and

$$3M_0L^2 + 3Q_0L^3 = 3PL^2b$$

which solutions are;

$$Q_0 = -\frac{Pb}{L^3}\left[\frac{b^2 - 3\beta^2 - 3L^2}{2 + 3\mu}\right] \quad \text{(a)}$$

$$M_0 = \frac{Pb}{L^2}\left[\frac{b^2 - L^2 - 3\beta^2 + 3\mu L^2}{2 + 3\mu}\right] \quad \text{(b)}$$

$$Q_L = -\frac{Pb}{L^3}\left[\frac{b^2 - 3\beta^2 - 3L^2}{2 + 3\mu} + \frac{L^3}{b}\right] \quad \text{(c)}$$

$$M_L = 0 \quad \text{(d)} \qquad (12)$$

E: Propped cantilever with moment at arbitrary point along the beam

This case is similar to case *B* except for the boundary conditions, and has (8a) to (8d) as general solutions. Applying the end conditions: y(0) = 0, $\theta_0 = 0$, y(L) = 0, M(L) = 0, to eqns (8a) and (8c) we obtain that;

$$3M_0L^2 + Q_0\left(L^3 - 3L\beta^2\right) = -3Mb^2$$

and

$$3M_0L^2 + 3Q_0L^3 = -3ML^2$$

Solving and simplifying gives; $Q_0 = -\frac{3M}{L}\left[\frac{L^2 - b^2}{3\beta^2 + 2L^2}\right]$

(a)

$$M_0 = M\left[\frac{3\left(L^2 - b^2\right) - \left(3\beta^2 + 2L^2\right)}{\left(3\beta^2 + 2L^2\right)}\right] \text{(b)}$$

$$Q_L = -\frac{3M}{L}\left[\frac{L^2 - b^2}{3\beta^2 + 2L^2}\right] \quad \text{(c)}$$

$$M_L = 0 \quad \text{(d)} \qquad (13)$$

F: Propped cantilever with u.d.l.

This case is similar to case *C*. The general solution is given by (10a) to (10d). The end conditions are; y(0) = 0, y(L) = 0, $\theta(0) = 0$, $M(L) = 0$

Applying the end conditions to (10a) and (10c) we obtain that ;

$$12M_0L^2 - 4Q_0\left(L^3 + 3\beta^2L\right) = qL^4 + 6qL^2\beta^2$$

And

$$2M_0 + 2Q_0L = qL^2$$

Hence,

$$Q_0 = \frac{qL}{4}\left[\frac{5 + 6\mu}{2 + 3\mu}\right] \quad \text{(a)}$$

$$M_0 = \frac{qL^2}{4}\left[\frac{1}{2 + 3\mu}\right] \quad \text{(b)}$$

$$Q_L = -\frac{3qL}{4}\left[\frac{1 + 2\mu}{2 + 3\mu}\right] \quad \text{(c)}$$

$$M_L = 0 \quad \text{(d)} \qquad (14)$$

The summary of these shear-modified fixed end moments and support reactions is on Table 2.

IV. APPLICATION OF SHEAR-MODIFIED FIXED END MOMENTS AND SUPPORT REACTIONS IN THE ANALYSIS OF REDUNDANT FRAMES

Figures 4, 5, and 6 show three redundant portal frames of different beam-column assemblages and loading. These portal frames are analyzed using (a) traditional expressions for fixed end moments and shears, (b) expressions for shear-modified fixed end moments and support reactions obtained in this work. Classical displacement method of analysis was used with transformed member rigidity.

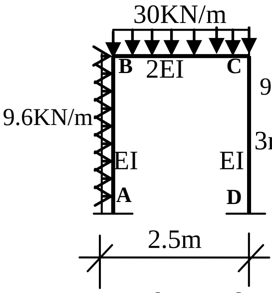

Fig. 4: Simple portal frame

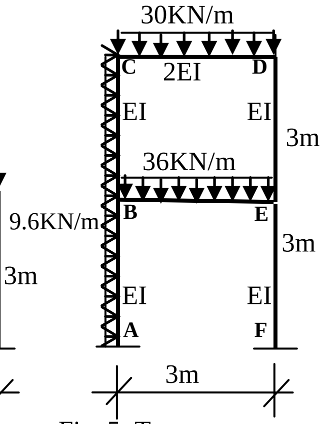

Fig. 5: Two storey portal frame

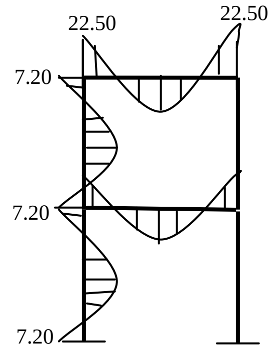

Fig. 7a: Fixed end moment diagram (shear ignored)

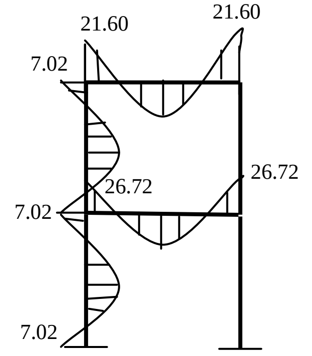

Fig. 7b: Fixed end moment diagram (shear included)

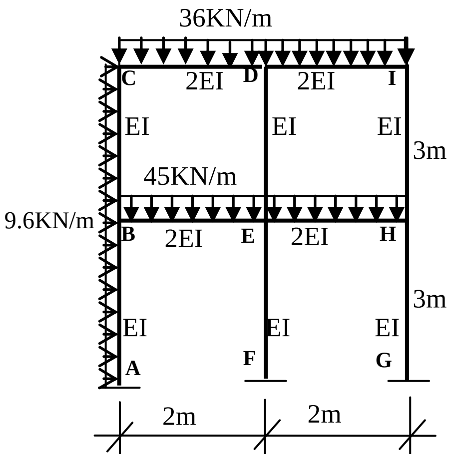

Fig. 6: Two storey multi- bay portal frame

Table 1: Values of shear-modification parameter $\beta = 2r\sqrt{K(1+\nu)}$		
Type of material	Shape of cross-section	β
Concrete beams and columns ($\nu = 0.35$	Rectangular (K = 1.5) Circular (K= 1.7)	$2.846\,r$ $3.03\,r$
Rolled steel sections ($\nu = 0.5$)	I-beams and other built up sections (K varies from 1.14 to 1.18)	Varies from $2.615\,r$ to $2.66\,r$

Considering a 330mm x 230mm concrete column and 300mm x 230mm beam for the frames in Figs.7, 8, and 9, we obtain the following parameters.

$A_{col} = 52900mm^2$, $I_{col} = 2.332x10^8\,mm^4$

$r_{col}^2 = 4408mm^2$, $\beta_{col}^2 = 35705mm^2$ $\quad \mu_{col} = 0.004$

$A_{beam} = 66700mm^2$, $r_{beam}^2 = 6998mm^2$

$I_{beam} = 4.675x10^8\,mm^4$, $\beta_{beam}^2 = 56684mm^2$,

$\mu_{beam} = 0.0063$

Therefore, shear-modification factors for fixed end moments are $\left(54\mu^2+15\mu-1\right)/\left(3\mu+1\right)=$ -0.975 for columns and -0.960 for beams. The joint moments obtained using traditional expressions for fixed end moment and shear modified expressions for fixed end moments, for the two storey portal frame, are shown in Figs. 7(a) and 7(b) respectively. Tables 3, 4, and 5 show the results of the analysis of the portal frames.

Table 2: Summary of shear-modified fixed end moments and support reactions

Fixing and loading condition of beams	Traditional fixed end moments and support reactions	Shear modified fixed end moments and Support Reactions
A: Fixed- ended beam with point load	$M_0 = -\frac{Pb^2 a}{L^2}$ $M_L = \frac{Pba^2}{L^2}$ $Q_0 = \frac{Pb^2}{L^3}\left(L+2a\right)$ $Q_L = \frac{Pab^2}{L^2}$	$M_0 = -\frac{Pb^2}{L^2}\left[\frac{L\eta_1 - b\eta_2}{\left(1+3\mu\right)}\right]$ $M_L = -\frac{Pb^2}{L^2}\left[\frac{a^2+\lambda_1+\lambda_2}{b\left(1+3\mu\right)}\right]$ $Q_0 = \frac{Pb^2}{L^3}\left[\frac{3L\left(1-\alpha\right)-2b\left(1-3\alpha\right)}{\left(1+3\mu\right)}\right]$ $Q_L = \frac{Pb^2}{L^3}\left[\frac{3L\left(1-\alpha\right)-2b\left(1-3\alpha\right)}{\left(1+3\mu\right)} - \frac{L^3}{b^2}\right]$
B: Fixed ended beam with moment at arbitrary point along the beam	$M_0 = \frac{Mb}{L^2}(2a-b)$ $Q_0 = \frac{-6Mab}{L^3}$ $M_L = \frac{Ma}{L^2}\left(2b-a\right)$ $Q_L = -\frac{6Mab}{L^3}$	$M_0 = \frac{Mb}{L^2}\left[3a\left(\frac{1-\mu}{1+3\mu}\right)-L\right]$ $Q_0 = -\frac{6Mab}{L^3}\left[\frac{1}{1+3\mu}\right]$ $M_L = \frac{Ma}{L^2}\left[L-3b\left(\frac{\mu+1}{1+3\mu}\right)\right]$ $Q_L = -\frac{6Mab}{L^3}\left[\frac{1}{1+3\mu}\right]$
C: Fixed ended beam with uniformly distributed load (u.d.l)	$M_0 = \frac{qL^2}{12}$ $Q_0 = -\frac{qL}{2}$ $M_L = \frac{qL^2}{12}$ $Q_L = -\frac{qL}{2}$	$M_0 = \frac{qL^2}{12}\left[\frac{1-9\mu-36\mu^2}{3\mu-1}\right]$ $Q_0 = -\frac{qL}{2}\left[\frac{6\mu-1}{3\mu-1}\right]$ $M_L = \frac{qL^2}{12}\left[\frac{1-9\mu-36\mu^2}{3\mu-1}\right]$ $Q_L = -\frac{qL}{2}\left[\frac{1}{3\mu-1}\right]$

D. Propped cantilever with point load	$M_0 = -\frac{Pb}{L^2}\left[\frac{b^2 - L^2}{2}\right]$ $Q_0 = \frac{Pb}{L^3}\left[\frac{3L^2 - b^2}{2}\right]$ $M_L = 0$ $Q_L = \frac{Pb}{L^3}\left[\left(\frac{3L^2 - b^2}{2}\right) - \frac{L^3}{b}\right]$	$M_0 = \frac{Pb}{L^2}\left[\frac{b^2 - L^2 - 3\beta^2 + 3\mu L^2}{(2+3\mu)}\right]$ $Q_0 = \frac{Pb}{L^3}\left[\frac{b^2 - 3\beta^2 - 3L^2}{(2+3\mu)}\right]$ $M_L = 0$ $Q_L = -\frac{Pb}{L^3}\left[\frac{b^2 - 3\beta^2 - 3L^2}{(2+3\mu)} + \frac{L^3}{b}\right]$
E: Propped cantilever with moment at arbitrary point along the beam	$M_0 = \frac{M}{2L^2}\left[3\left(L^2 - b^2\right) - 2L^2\right]$ $Q_0 = \frac{-3M}{2L^3}\left(L^2 - b^2\right)$ $M_L = 0$ $Q_L = -\frac{3M}{2L^3}\left(L^2 - b^2\right)$	$M_0 = \frac{Mb}{L^2}\left[3a\left(\frac{1-\mu}{1+3\mu}\right) - L\right]$ $Q_0 = -\frac{3M}{L}\left[\frac{L^2 - b^2}{3\beta^2 + 2L^2}\right]$ $M_L = 0$ $Q_L = -\frac{3M}{L}\left[\frac{L^2 - b^2}{3\beta^2 + 2L^2}\right]$
F: Propped cantilever with u.d.l.	$M_0 = -\frac{qL^2}{8}$ $Q_0 = \frac{5}{8}qL$ $M_L = 0$ $Q_L = -\frac{3}{8}qL$	$M_0 = \frac{qL^2}{4}\left[\frac{1}{2+3\mu}\right]$ $Q_0 = \frac{qL}{4}\left[\frac{5+6\mu}{2+3\mu}\right]$ $M_L = 0$ $Q_L = -\frac{3qL}{4}\left[\frac{2\mu+1}{3\mu+2}\right]$

Table 3: Results for simple portal frame

Joint designation	Joint moment (shear neglected)	Joint moment (shear included)	Deference %
AB	7.284	7.409	+1.7
BA	22.545	21.119	-6.4
BC	-22.545	-22.117	-6.4
CB	-4.401	-4.896	+11.2
CD	4.401	4.897	+11.2
DC	9.012	9.328	+3.5

Table 4: Results for two storey portal frame

Joint designation	Joint moment (shear neglected)	Joint moment (shear included)	Deference %
AB	32.20	32.38	+0.6
BA	41.11	43.74	+6.4
BC	-7.19	-7.13-36.61	-0.8
BE	-36.92	30.67	-0.8
CB	31.19	-30.67-1.81	-1.7
CD	-31.19	1.81	-1.7
DC	-1.20	-2.00	+50.8
DE	1.19	-19.82	+50.8
ED	12.59	21.82	-22.8
EB	-18.82	31.95	+5.3
EF	21.40		+2.0
FE	31.65		+9.5

Table 5: Results for two storey multi bay frame

Joint designation	Joint moment (shear neglected)	Joint moment (shear included)	Deference %
AB	24.76	25.05	+1.2
BA	43.50	43.07	-1.0
BC	-0.71	-0.70	-1.4
BE	-42.79	-42.37	-1.0
CB	23.72	23.22	2.1
CD	-23.71	-23.22	-2.1
DC	5.04	3.51	-30.4
DI	-22.65	21.25	-6.2
DE	17.61	17.74	7.4
EB	-6.84	-8.81	28.8
ED	16.09	16.23	8.7
EH	-45.47	43.80	-3.7
EF	36.22	36.38	0.4
FE	38.00	38.15	3.9
GH	33.13	33.40	0.8
HG	26.50	26.87	1.4
HE	-30.04	-30.90	2.9
HI	3.54	4.03	13.8
IH	7.10	7.60	7.0
ID	-7.10	-7.60	7.0

V. DISCUSSIONS AND CONCLUSION

Table 1 shows the values of shear modification factors for different materials and cross sections.

The summary of the derived expressions for shear modified fixed end moments and support reactions for various beam fixing and loading conditions are given in Table 2. The expressions within the square brackets are the shear modification parameters which, if shear is neglected [$\beta = 0$, $\alpha = 0$, $\mu = 0$], revert the expressions to the traditional fixed end moments and support reactions.

The fixed end moment diagrams for the two storey portal frame are shown in Figs. 7(a) and 7(b), from where it can be seen that shear in redundant frames decreases the fixed end moments.

The joint moments for the simple portal frame of Fig. 4 shown in Table 3 indicate that shear can increase or lower the bending moment distribution in a redundant frame. Hence, there was no defined pattern for stress variation in the simple portal frame. Similar observations were also made for two storey portal frame of Fig. 5, Table 4, and two storey multi-bay portal frame of Fig. 6, Table 5, where neither increase in load nor variation in frame configuration gave definite pattern for stress variation. For example, the maximum hogging moment for the simple portal frame **reduced** by 6.4% when shear effects were considered while those of two storey portal frame and two storey multi-bay frames **increased** by 6.4% and **reduced** by 3.7% respectively, when shear is taken into consideration.

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VI. CONCLUSION

Shear reduces the fixed end moments of redundant frames. The variation of stress distribution in redundant frames has no defined pattern . Some joint moments were numerically lower when shear was considered than when it was neglected while others were higher when shear was considered than when it was ignored. Increase / decrease in maximum joint moment was less than 7%. Consequently we conclude that the effect of shear on the stress distribution in redundant frames is not substantial and can be ignored since the error involved can be taken care of by the use of appropriate factor of safety for loads.

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Chinenye Alfred Chidolue obtained B. Eng (Civil) degree from the University of Zambia , M. Eng degree (Civil and Structural) from University of Sheffield and Ph. D. degree in Structural Engineering is being awaited from University of Nigeria, Nsukka. He is currently a Senior lecturer in Civil Engineering Department, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria

Chukwura Henry Aginam obtained B. Eng (Civil) degree from the University of Nigeria Nsukka , M. Sc. Degree (Structures) from University of Lagos, Nigeria and Ph. D. degree in Structural Engineering from University of Nigeria, Nsukka. He is currently a Senior lecturer in Civil Engineering Department, Nnamdi Azikiwe University, Awka, Anambra State, Nigeria