Published for SISSA by Springer

Received: September 21, 2015
Revised: February 10, 2016
Accepted: February 21, 2016
Published: March 11, 2016

Soft gluon resummation for associated $t\bar{t}H$ production at the LHC

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Abstract: We perform resummation of soft gluon corrections to the total cross section for the process $pp \to t\bar{t}H$. The resummation is carried out at next-to-leading-logarithmic (NLL) accuracy using the Mellin space technique, extending its application to the class of $2 \to 3$ processes. We present an analytical result for the soft anomalous dimension for a hadronic production of two coloured massive particles in association with a colour singlet. We discuss the impact of resummation on the numerical prediction for the associated Higgs boson production with top quarks at the LHC.

Keywords: Resummation, Higgs Physics, QCD

ArXiv ePrint: 1509.02780

doi:10.1007/JHEP03(2016)065

Contents

1 Introduction

Establishing the properties of the Higgs boson discovered at the LHC in 2012 [1, 2], in particular its couplings to the Standard Model (SM) particles, is one of the main tasks of the current LHC run. Since the SM Higgs boson couples to fermions proportionally to their masses, the top-Higgs Yukawa coupling is expected to be especially sensitive to the underlying physics. A direct way to probe the strength of the coupling without making any assumptions regarding its nature is provided by the measurement of Higgs production rates in the $pp \to t\bar{t}H$ process. Although the production cross section is low and the collision energy and the luminosity available so far have not been sufficient enough to measure a Higgs signal in Run 1 [3–7], such a measurement in Run 2 is eagerly awaited. Correspondingly, precision predictions for the $pp \to t\bar{t}H$ production process are of great importance and a lot of effort has been invested in the recent years to improve the theoretical accuracy.

The next-to-leading-order (NLO) QCD, i.e. $\mathcal{O}(\alpha_s^3\alpha)$ predictions are already known for some time [8–13] and have been newly recalculated and matched to parton showers in [14–17]. As of late, the mixed QCD-weak corrections [18] and QCD-EW corrections [19, 20] of $\mathcal{O}(\alpha_s^2\alpha^2)$ are also available. Furthermore, the NLO QCD corrections to the hadronic $t\bar{t}H$ production with top and antitop quarks decaying into bottom quarks and leptons have been recently obtained [21]. Concurrently, new methods for a better measurement of the process have been proposed e.g. in [22] or in [23]. In general, for the LHC collision energies of Run 2, the NLO QCD corrections are $\sim 20\%$, whereas the size of the (electro)weak correction is more than ten times smaller. The scale uncertainty of the NLO QCD corrections is estimated to be $\sim 10\%$ [8–13, 24]. While matching fixed-order predictions to parton showers pursued recently by many groups in such frameworks as aMC@NLO [14, 15, 25], POWHEG BOX [16, 17, 26] or SHERPA [27] allows for a more accurate description of final state characteristics, it does not change the predictions for the overall production rates. An improvement in the accuracy with which these rates are known can only be achieved by calculating higher order corrections. However, calculations of the next-to-next-to-leading-order corrections are currently technically out of reach. It is nevertheless interesting to ask the question what is the size and the effect of certain classes of corrections of higher than NLO accuracy. In particular, we focus here on taking into account contributions from soft gluon emission to all orders in perturbation theory. The traditional (Mellin-space) resummation formalism which is applied in this type of calculations has been very well developed and copiously employed for description of the $2 \to 2$ type processes at the Born level. The universality of resummation concepts warrants their applications to scattering processes with many partons in the final state, as shown in a general analytical treatment developed for arbitrary number of partons [28–30]. Recently, the soft gluon resummation technique in the soft collinear effective theory (SCET) framework was applied to $pp \to t\bar{t}W^{\pm}$ [31]. So far, however, no calculations in the traditional resummation framework for processes involving $2 \to 3$ scattering at the Born level have been performed.

In this paper we take the first step in this direction by developing the Mellin-space threshold resummation formalism at the next-to-leading-logarithmic (NLL) accuracy for the case of $2 \to 3$ processes with two coloured massive particles in the final state. We then apply this formalism in order to estimate the impact of soft gluon corrections on the predictions for the total $t\bar{t}H$ production rate. In this particular case, the threshold region is reached when the square of the partonic center-of-mass (c.o.m.) energy, $\sqrt{\hat{s}}$, approaches $M = 2m_t + m_H$, where m_t is the top quark mass and m_H is the Higgs boson mass. In the threshold region, the cross section receives enhancement in the form of logarithmic corrections in $\beta = \sqrt{1 - M^2/\hat{s}}$. The quantity β measures the distance from absolute production threshold and can be related to the maximal velocity of the $t\bar{t}$ system. Additionally, in the threshold region the virtual QCD corrections are also enhanced due to Coulomb-type interactions between the two final state top quarks which become large when the top quark velocity in the $t\bar{t}$ c.o.m. frame $\beta_{kl} \to 0$ with $\beta_{kl} = \sqrt{1 - 4m_t^2/\hat{s}_{kl}}$ and $\hat{s}_{kl} = (p_t + p_{\bar{t}})^2$. However, the contributions to the total cross section from the threshold region are strongly suppressed by the β^4 factor originating from the massive three particle phase space. Nevertheless, one expects that the threshold corrections can still have a non-negligible impact on the predictions.

The associated production of a Higgs boson with a $t\bar{t}$ pair involves four coloured partons at the Born level and as such is characterized by a non-trivial colour flow. The colour structure influences the contributions from wide-angle soft gluon emissions which have to be included at the NLL accuracy. The evolution of the colour exchange at NLL is governed by the one-loop soft anomalous dimension [28, 32–36]. Starting from four coloured partons in the process, the soft anomalous dimension is a matrix and is known for heavy-quark [32, 33, 37], dijet [34–36] and supersymmetric particle production [38–40, 43], as well as for the general case of $2 \to n$ QCD processes [28–30]. Here we adopt the calculations of the soft anomalous dimension for the case of $2 \to 3$ processes with two coloured massive particles in the final state.

2 Resummation for $2 \to 3$ processes with two massive colored particles in the final state

The resummation of soft gluon corrections to the total cross section $\sigma_{pp \to t\bar{t}H}$ is performed in Mellin space, where the Mellin moments are taken w.r.t. the variable $\rho = M^2/S$. At the partonic level, the Mellin moments for the process $ij \to klB$, where i, j denote massless coloured partons, k, l two massive quarks and B a massive colour-singlet particle, is given by

$$\hat{\sigma}_{ij\to klB,N}(m_k, m_l, m_B, \mu_F^2, \mu_R^2) = \int_0^1 d\hat{\rho}\, \hat{\rho}^{N-1} \hat{\sigma}_{ij\to klB}(\hat{\rho}, m_k, m_l, m_B, \mu_F^2, \mu_R^2) \tag{2.1}$$

with $\hat{\rho} = 1 - \beta^2$.

At LO, the $t\bar{t}H$ production receives contributions from the $q\bar{q}$ and gg channels. We analyze the colour structure of the underlying processes in the s-channel color bases, $\{c_I^q\}$ and $\{c_I^g\}$, with $c_{\mathbf{1}}^q = \delta^{\alpha_i \alpha_j}\delta^{\alpha_k \alpha_l}$, $c_{\mathbf{8}}^q = T^a_{\alpha_i \alpha_j} T^a_{\alpha_k \alpha_l}$, $c_{\mathbf{1}}^g = \delta^{a_i a_j}\, \delta^{\alpha_k \alpha_l}$, $c_{\mathbf{8S}}^g = T^b_{\alpha_l \alpha_k} d^{b a_i a_j}$, $c_{\mathbf{8A}}^g = i T^b_{\alpha_l \alpha_k} f^{b a_i a_j}$. In this basis the soft anomalous dimension matrix becomes diagonal in the production threshold limit [32, 33] and the NLL resummed cross section in the N-space has the form [32, 33, 37]

$$\hat{\sigma}^{(\mathrm{res})}_{ij\to klB,N} = \sum_I \hat{\sigma}^{(0)}_{ij\to klB,I,N}\, C_{ij\to klB,I}\, \Delta^i_{N+1} \Delta^j_{N+1} \Delta^{(\mathrm{int})}_{ij\to klB,I,N+1}, \tag{2.2}$$

where we suppress explicit dependence on the scales. The index I in eq. (2.2) distinguishes between contributions from different colour channels. The colour-channel-dependent contributions to the LO partonic cross sections in Mellin-moment space are denoted by $\hat{\sigma}^{(0)}_{ij\to klB,I,N}$. The radiative factors Δ^i_N describe the effect of the soft gluon radiation collinear to the initial state partons and are universal. Large-angle soft gluon emission is accounted for by the factors $\Delta^{(\mathrm{int})}_{ij\to klB,I,N}$ which depend on the partonic process under consideration and the colour configuration of the participating particles. The expressions for the radiative factors in the $\overline{\mathrm{MS}}$ factorisation scheme read (see e.g. [37])

$$\begin{aligned} \ln\Delta^i_N &= \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \int_{\mu_F^2}^{M^2(1-z)^2} \frac{dq^2}{q^2} A_i(\alpha_\mathrm{s}(q^2))\,, \\ \ln\Delta^{(\mathrm{int})}_{ij\to klB,I,N} &= \int_0^1 dz \frac{z^{N-1}-1}{1-z} D_{ij\to klB,I}(\alpha_\mathrm{s}(M^2(1-z)^2)). \end{aligned} \tag{2.3}$$

The coefficients A_i, $D_{ij\to klB,I}$ are power series in the coupling constant α_s,

$$A_i = \left(\frac{\alpha_\mathrm{s}}{\pi}\right) A_i^{(1)} + \left(\frac{\alpha_\mathrm{s}}{\pi}\right)^2 A_i^{(2)} + \ldots, \qquad D_{ij\to klB,I} = \left(\frac{\alpha_\mathrm{s}}{\pi}\right) D^{(1)}_{ij\to klB,I} + \ldots \tag{2.4}$$

The universal LL and NLL coefficients $A_i^{(1)}$, $A_i^{(2)}$ are well known [44, 45] and given by $A_i^{(1)} = C_i$, $A_i^{(2)} = \frac{1}{2}\, C_i \left(\left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_\mathrm{A} - \frac{5}{9} n_f\right)$ with $C_g = C_\mathrm{A} = 3$, and $C_q = C_\mathrm{F} = 4/3$.

The NLL coefficients $D_{ij\to klB,I}$ are obtained by taking the threshold limit $\hat{s} \to M^2 = (m_k + m_l + m_B)^2$ of the gauge-invariant soft anomalous dimension matrices $\Gamma^{ij\to klB}$. In this limit $\Gamma^{ij\to klB} = \frac{\alpha_\mathrm{s}}{\pi}\mathrm{diag}(\gamma_1^{ij}, \ldots)$ and $D_{ij\to klB,I} = 2\mathrm{Re}(\gamma_I^{ij})$. The calculations of $\Gamma^{ij\to klB}$ apply the methods developed in the heavy quark pair-production [32, 33] to the process at hand, taking into account $2 \to 3$ kinematics, and yield

$$\Gamma^{q\bar{q}\to klB} = \frac{\alpha_s}{\pi} \begin{bmatrix} -C_{\mathrm{F}}(L_{\beta,kl}+1) & \frac{C_{\mathrm{F}}}{C_{\mathrm{A}}}\Omega_3 \\ 2\Omega_3 & \frac{1}{2}[(C_{\mathrm{A}}-2C_{\mathrm{F}})(L_{\beta,kl}+1)+C_{\mathrm{A}}\Lambda_3+(8C_{\mathrm{F}}-3C_{\mathrm{A}})\Omega_3] \end{bmatrix}, \tag{2.5}$$

$$\Gamma^{gg\to klB} = \frac{\alpha_s}{\pi} \begin{bmatrix} \Gamma^{gg}_{11} & 0 & \Omega_3 \\ 0 & \Gamma^{gg}_{22} & \frac{N_c}{2}\Omega_3 \\ 2\Omega_3 & \frac{N_c^2-4}{2N_c}\Omega_3 & \Gamma^{gg}_{33} \end{bmatrix}, \tag{2.6}$$

with

$$\begin{aligned} \Gamma^{gg}_{11} &= -C_{\mathrm{F}}(L_{\beta,kl}+1), \\ \Gamma^{gg}_{22} &= \Gamma^{gg}_{33} = \frac{1}{2}((C_{\mathrm{A}}-2C_{\mathrm{F}})(L_{\beta,kl}+1)+C_{\mathrm{A}}\Lambda_3), \end{aligned}$$

where

$$\begin{aligned} \Lambda_3 &= (T_1(m_k)+T_2(m_l)+U_1(m_l)+U_2(m_k))/2, \\ \Omega_3 &= (T_1(m_k)+T_2(m_l)-U_1(m_l)-U_2(m_k))/2, \end{aligned}$$

and

$$L_{\beta,kl} = \frac{\kappa^2+\beta_{kl}^2}{2\kappa\beta_{kl}}\left(\log\left(\frac{\kappa-\beta_{kl}}{\kappa+\beta_{kl}}\right)+i\pi\right), \tag{2.7}$$

$$T_i(m) = \frac{1}{2}\left(\ln((m^2-t_i)^2/(m^2\hat{s}))-1+i\pi\right), \tag{2.8}$$

$$U_i(m) = \frac{1}{2}\left(\ln((m^2-u_i)^2/(m^2\hat{s}))-1+i\pi\right), \tag{2.9}$$

$$\kappa = \sqrt{1-(m_k-m_l)^2/s_{kl}}, \qquad s_{kl} = (p_k+p_l)^2, \tag{2.10}$$

$$t_1 = (p_i-p_k)^2, \qquad t_2 = (p_j-p_l)^2, \qquad u_1 = (p_i-p_l)^2, \qquad u_2 = (p_j-p_k)^2. \tag{2.11}$$

Eqs. (2.5), (2.6) reproduce the known results for heavy quark-antiquark (squark-antisquark) pair- production soft anomalous dimension [32, 33, 38–40] in the limit $p_B \to 0$. Also, our result for $\Gamma^{q\bar{q}\to klB}$ agrees with the result obtained in the SCET framework in [41, 42]. It can be also explicitly seen that in the limit $\hat{s} \to (2m_t+m_H)^2$ the non-diagonal elements vanish and the diagonal elements give $D_{q\bar{q}\to klB,I} = \{0, -N_c\}$, $D_{gg\to klB,I} = \{0, -N_c, -N_c\}$, which are the same coefficients as for the heavy-quark pair production $D_{ij\to kl}$. This confirms a simple physical intuition that the properties of the soft emission in the absolute threshold limit are only driven by the colour structure of the subprocesses and do not depend on the their kinematics.

For completness we display the explicit NLL expressions for the resummed factors in the Mellin space, which were used in our numerical implementation:

$$\ln \Delta_N^i \stackrel{\text{NLL}}{=} g_i^{(1)}\left(b_0\,\alpha_s(\mu_R^2)\ln N\right)\,\ln N \;+\; g_i^{(2)}\left(b_0\,\alpha_s(\mu_R^2)\ln N, M^2, \mu_R^2, \mu_F^2\right)\,, \tag{2.12}$$

$$\ln \Delta^{(\text{int})}_{ij\to kl\,B,I,N} \stackrel{\text{NLL}}{=} h^{(2)}_{ij\to kl\,B,I}\left(b_0\,\alpha_s(\mu_R^2)\ln N\right) \tag{2.13}$$

with

$$g_i^{(1)}(\lambda) = \frac{A_i^{(1)}}{2\pi b_0 \lambda}\,\left[2\lambda + (1-2\lambda)\,\ln(1-2\lambda)\right]\,, \tag{2.14}$$

$$\begin{aligned} g_i^{(2)}(\lambda, M^2, \mu_R^2, \mu_F^2) = &-\frac{A_i^{(1)}\gamma_E}{\pi b_0}\,\ln(1-2\lambda) \\ &+\frac{A_i^{(1)} b_1}{2\pi b_0^3}\left[2\lambda + \ln(1-2\lambda) + \frac{1}{2}\ln^2(1-2\lambda)\right] \\ &-\frac{A_i^{(2)}}{2\pi^2 b_0^2}\,\left[\,2\lambda + \ln(1-2\lambda)\,\right] \\ &-\frac{A_i^{(1)}}{2\pi b_0}\,\left[\,2\lambda + \ln(1-2\lambda)\,\right]\ln\left(\frac{\mu_R^2}{M^2}\right) \\ &+\frac{A_i^{(1)}}{2\pi b_0}\,2\lambda\,\ln\left(\frac{\mu_F^2}{M^2}\right)\,, \end{aligned} \tag{2.15}$$

$$h^{(2)}_{ij\to kl\,B,I}(\lambda) = \frac{\ln(1-2\lambda)}{2\pi b_0}\,D_{ij\to kl\,B,I}\,, \tag{2.16}$$

where b_0 and b_1 are the first two coefficients of the QCD β-function,

$$b_0 = \frac{11 C_A - 4 T_R n_f}{12\pi}\,, \qquad b_1 = \frac{17 C_A^2 - 10 C_A T_R n_f - 6 C_F T_R n_f}{24\pi^2}\,. \tag{2.17}$$

The coefficients

$$C_{ij\to klB,I} = 1 + \frac{\alpha_s}{\pi} C^{(1)}_{ij\to klB,I} + \ldots$$

contain all non-logarithmic contributions to the NLO cross section taken in the threshold limit. More specifically, these consist of Coulomb corrections, N-independent hard contributions from virtual corrections and N-independent non-logarithmic contributions from soft emissions. Although formally the coefficients $C_{ij\to klB,I}$ begin to contribute at NNLL accuracy, in our numerical studies of the $pp \to t\bar{t}H$ process we consider both the case of $C_{ij\to klB,I} = 1$, i.e. with the first-order corrections to the coefficients neglected, as well as the case with these corrections included. In the latter case we treat the Coulomb corrections and the hard contributions additively, i.e.

$$C^{(1)}_{ij\to klB,I} = C^{(1,\text{hard})}_{ij\to klB,I} + C^{(1,\text{Coul})}_{ij\to klB,I}.$$

For k, l denoting massive quarks the Coulomb corrections are $C^{(1,\text{Coul})}_{ij\to klB,\mathbf{1}} = C_\text{F}\pi^2/(2\beta_{kl})$ and $C^{(1,\text{Coul})}_{ij\to klB,\mathbf{8}} = (C_\text{F} - C_\text{A}/2)\pi^2/(2\beta_{kl})$. The additive treatment is consistent with NLL resummation and matching to NLO. We note that in general Coulomb corrections can also be resummed [46–50]. A combined resummation of Coulomb and soft corrections is, however, beyond the scope of this paper.

3 Theoretical predictions for the $pp \to t\bar{t}H$ process at NLO+NLL accuracy

The resummation-improved NLO+NLL cross sections for the $pp \to t\bar{t}H$ process are obtained through matching the NLL resummed expressions with the full NLO cross sections

$$\begin{aligned}\hat{\sigma}^{(\text{NLO+NLL})}_{h_1h_2\to kl}(\rho,\mu_F^2,\mu_R^2) &= \hat{\sigma}^{(\text{NLO})}_{h_1h_2\to klB}(\rho,\mu_F^2,\mu_R^2) + \hat{\sigma}^{(\text{res-exp})}_{h_1h_2\to klB}(\rho,\mu_F^2,\mu_R^2)\\ \text{with}\quad \hat{\sigma}^{(\text{res-exp})}_{h_1h_2\to klB} &= \sum_{i,j}\int_{\mathcal{C}}\frac{dN}{2\pi i}\,\rho^{-N} f^{(N+1)}_{i/h_1}(\mu_F^2)\, f^{(N+1)}_{j/h_2}(\mu_F^2)\\ &\quad\times\left[\hat{\sigma}^{(\text{res})}_{ij\to klB,N}(\mu_F^2,\mu_R^2) - \hat{\sigma}^{(\text{res})}_{ij\to klB,N}(\mu_F^2,\mu_R^2)\Big|_{(\text{NLO})}\right], \end{aligned} \tag{3.1}$$

where $\hat{\sigma}^{(\text{res})}_{ij\to klB,N}$ is given in eq. (2.2) and $\hat{\sigma}^{(\text{res})}_{ij\to klB,N}|_{(\text{NLO})}$ represents its perturbative expansion truncated at NLO. The moments of the parton distribution functions (pdf) $f_{i/h}(x,\mu_F^2)$ are defined in the standard way $f^{(N)}_{i/h}(\mu_F^2) \equiv \int_0^1 dx\, x^{N-1} f_{i/h}(x,\mu_F^2)$. The inverse Mellin transform (3.1) is evaluated numerically using a contour $\mathcal{C}$ in the complex-N space according to the "Minimal Prescription" method developed in ref. [51].

As mentioned in the previous section, the calculation of first-order contributions to the coefficients $C_{ij\to t\bar{t}H,I}$ requires knowledge of the NLO real corrections in the threshold limit as well as virtual corrections. In our calculations we follow the methodology of [52, 53], where the case of two massive coloured particle in the final state was considered. We have explicitly checked that adding a massive colour singlet particle in the final state does not introduce any extra terms dependent on the mass of the added particle. Thus the N-space results for the pair-production process of two massive coloured particles are also applicable in our $2 \to 3$ case. This way, the problem of calculating the $C^{(1)}_{ij\to t\bar{t}H,I}$ coefficients reduces to calculation of virtual corrections to the process. We extract them numerically using the publicly available POWHEG implementation of the $t\bar{t}H$ process [17], based on the calculations developed in [10–13]. The results were then cross-checked using the standalone MadLoop implementation in aMC@NLO [14]. Since the $q\bar{q}$ channel receives only colour-octet contributions, the extracted value contributing to $C^{(1,\text{hard})}_{q\bar{q}\to t\bar{t}H,\mathbf{8}}$ is exact. In the gg channel, however, both the singlet and octet production modes contribute. The implementation of the virtual corrections to $gg \to t\bar{t}H$ in POWHEG and in aMC@NLO does not allow for their separate extraction in each colour channel. Instead, we extract the value which contributes to the coefficient $\bar{C}^{(1,\text{hard})}_{gg\to t\bar{t}H}$ averaged over colour channels and use the same value to further calculate $C^{(1,\text{hard})}_{gg\to t\bar{t}H,\mathbf{1}}$ and $C^{(1,\text{hard})}_{gg\to t\bar{t}H,\mathbf{8}}$. In order to measure the size of the error introduced by this procedure, we then rescale this value by the ratios of the corresponding colour-channel dependent and colour averaged coefficients found for $gg \to t\bar{t}$ in [54]. The scale dependence of the $C^{(1)}_{ij\to klB,I}$ can be fully deduced from renormalization

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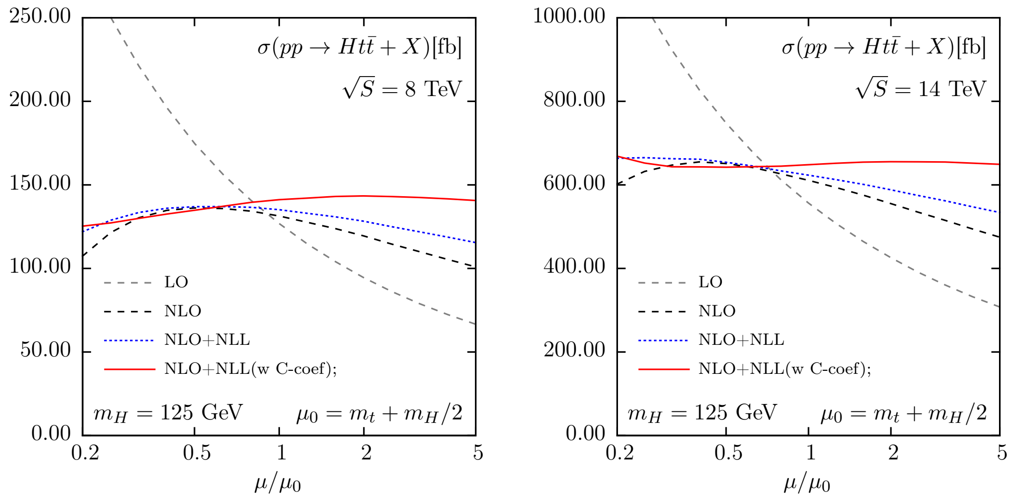

Figure 1. Scale dependence of the LO, NLO and NLO+NLL cross sections at $\sqrt{S} = 8$ and $\sqrt{S} = 14$ TeV LHC collision energy. The results are obtained while simultaneously varying μ_F and μ_R, $\mu = \mu_F = \mu_R$.

group arguments, in the same way as for the full NLO result. We have checked that numerical results obtained with the procedure which we use to extract the values of the coefficients at $\mu_0 = \mu_F = \mu_R$ show the same scale dependence as expected from exact analytical expressions.

In our phenomenological analysis we use $m_t = 173$ GeV, $m_H = 125$ GeV and choose the central scale $\mu_{F,0} = \mu_{R,0} = m_t + m_H/2$, in accordance with [24]. The NLO cross section is calculated using the aMC@NLO code [25]. In the implementation of the resummation formula, eq. (2.2), we numerically take a Mellin transform of the LO cross sections and the $C^{(1)}_{ij \to t\bar{t}H,I}$ coefficient terms which are both calculated in the x space. We perform the current analysis employing MMHT2014 [55] pdfs and use the corresponding values of α_s. Beside presenting the full result including non-zero $C^{(1)}_{ij \to t\bar{t}H,I}$ coefficients, we also show the results with $C_{ij \to t\bar{t}H,I} = 1$.

We begin our numerical study by analysing the scale dependence of the resummed total cross section for $pp \to t\bar{t}H$ at $\sqrt{S} = 8$ and 14 TeV, varying simultaneously the factorization and renormalization scales, μ_F and μ_R. As demonstrated in figure 1, adding the soft gluon corrections stabilizes the dependence on $\mu = \mu_F = \mu_R$ of the NLO+NLL predictions with respect to NLO. As an example, the central values and the scale error at $\sqrt{S} = 8$ TeV changes from $132^{+3.9\%}_{-9.3\%}$ fb at NLO to $141^{+1.4\%}_{-4.2\%}$ fb at NLO+NLL (with $C^{(1)}_{ij \to t\bar{t}H,I}$ coefficients included) and correspondingly, from $613^{+6.2\%}_{-9.4\%}$ fb to $650^{+0.8\%}_{-1.2\%}$ fb at $\sqrt{S} = 14$ TeV. It is also clear from figure 1 that the coefficients $C^{(1)}_{ij \to t\bar{t}H}$ strongly impact the predictions, especially at higher scales.

In order to understand these effects better, in figure 2 we analyse the dependence on the factorization and renormalization scale separately for the case study of $\sqrt{S} = 14$ TeV. We observe that the weak scale dependence present when the scales are varied simultaneously is a result of the cancellations between renormalization and factorization scale dependencies.

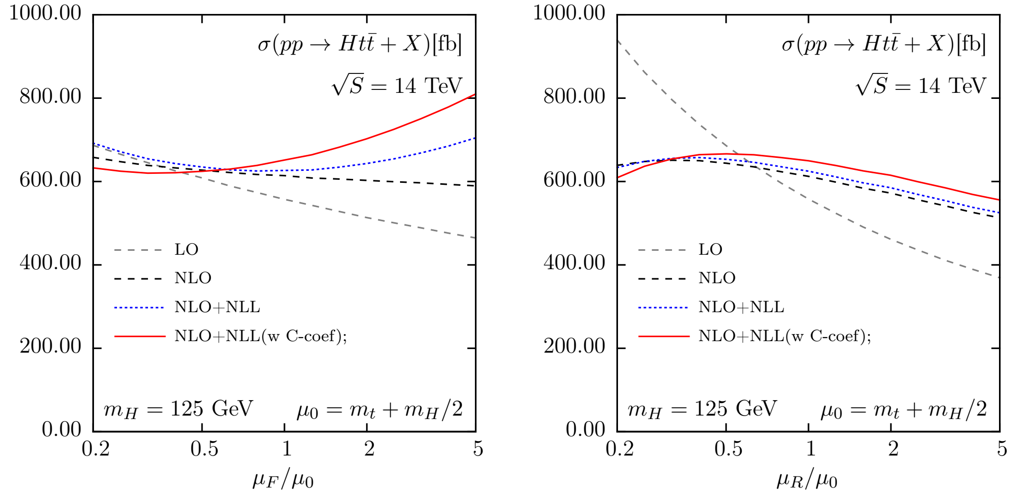

Figure 2. Factorization and renormalization scale dependence of the LO, NLO and NLO+NLL cross sections at $\sqrt{S} = 14\,\text{TeV}$ LHC collision energy. The results are obtained with $\mu_R = \mu_0$ for μ_F variation and and $\mu_F = \mu_0$ for μ_R variation.

A similar effect of the opposite behaviour of the total cross section under μ_F and μ_R variations was previously shown for the total cross section for the inclusive Higgs production in the gluon-fusion process [57]. The typical decrease of the cross section with increasing μ_R originates from running of α_s. The behaviour under variation of the factorization scale, on the other hand, is related to the effect of scaling violation of pdfs at probed values of x. In this context, it is interesting to observe that the NLO+NLL predictions in figure 2 show very little μ_F dependence around the central scale, in agreement with expectation of the factorization scale dependence in the resummed exponential and in the pdfs cancelling each other, here up to NLL. The relatively strong dependence on μ_F of the NLO+NLL predictions with non-zero $C^{(1)}_{ij\to t\bar{t}H,I}$ can be then easily understood: the resummed expression will take into account higher order scale dependent terms which involve both $C^{(1)}_{ij\to t\bar{t}H,I}$ and logarithms of N. These terms do not have their equivalent in the pdf evolution since the pdfs do not carry any process-specific information. Correspondingly, they are not cancelled and can lead to strong effects if the coefficients $C^{(1)}_{ij\to t\bar{t}H,I}$ are numerically substantial. As these terms can only provide a part of the full scale dependence at higher orders, it is to be expected that their impact will be significantly modified when NNLO corrections are known.

Given the arguments above, we choose to estimate the theoretical uncertainty due to scale variation using the 7-point method, where the minimum and maximum values obtained with $(\mu_F/\mu_0, \mu_R/\mu_0) = (0.5, 0.5), (0.5, 1), (1, 0.5), (1, 1), (1, 2), (2, 1), (2, 2)$ are considered. The effect of including NLL corrections is summarized in table 1 for the LHC collision energy of 8, 13 and 14 TeV. The NLO+NLL predictions show a significant reduction of the scale uncertainty, compared to NLO results. The reduction of the positive and negative scale errors amounts to around 20-30% of the NLO error for $\sqrt{S} = 13, 14\,\text{TeV}$ and to around 25–35% for $\sqrt{S} = 8\,\text{TeV}$. This general reduction trend is not sustained for the

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$\sqrt{S}$ [TeV]	NLO [fb]	NLO+NLL		NLO+NLL with C		pdf error
		Value [fb]	K-factor	Value [fb]	K-factor	
8	$132^{+3.9\%}_{-9.3\%}$	$135^{+3.0\%}_{-5.9\%}$	1.03	$141^{+7.7\%}_{-4.6\%}$	1.07	$^{+3.0\%}_{-2.7\%}$
13	$506^{+5.9\%}_{-9.4\%}$	$516^{+4.6\%}_{-6.5\%}$	1.02	$537^{+8.2\%}_{-5.5\%}$	1.06	$^{+2.3\%}_{-2.3\%}$
14	$613^{+6.2\%}_{-9.4\%}$	$625^{+4.6\%}_{-6.7\%}$	1.02	$650^{+7.9\%}_{-5.7\%}$	1.06	$^{+2.3\%}_{-2.2\%}$

Table 1. NLO+NLL and NLO total cross sections for $pp \to t\bar{t}H$ for various LHC collision energies. The error ranges given together with the NLO and NLO+NLL results indicate the scale uncertainty.

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positive error after including the $C^{(1)}_{ij\to t\bar{t}H,I}$ coefficients. More specifically, the negative error is further slightly reduced, while the positive error is increased. The origin of this increase can be traced back to the substantial dependence on μ_F of the resummed predictions with non-zero $C^{(1)}_{ij\to t\bar{t}H,I}$ coefficients, manifesting itself at larger scales. However, even after the redistribution of the error between the positive and negative parts, the overall size of the scale error, corresponding to the size of the error bar, is reduced after resummation by around 7% at 8 TeV and 10 (13)% at 13 (14) TeV with respect to the NLO uncertainties. The scale error of the predictions is still a few times larger than the pdf error, cf. table 1. For simplicity, the pdf error shown in table 1 is calculated for the NLO predictions, however adding the soft gluon correction can only minimally influence the value of the pdf error.[1]

As expected on the basis of large phase-space suppression in the threshold regime, the predictions for total cross section at NLO+NLL are only moderately increased by 2–3% w.r.t. the full NLO result. Introducing the coefficients $C^{(1)}_{ij\to t\bar{t}H,I}$ leads to an increase in the K-factor of up to 6–7%, indicating the importance of constant terms in the threshold limit. Since the impact of soft corrections is bigger for processes taking place closer to threshold the K-factor gets slightly higher for smaller collider energies. We also check the impact of our approximated treatment of keeping parts of $C^{(1,\mathrm{hard})}_{gg\to t\bar{t}H,\mathbf{1}}$ and $C^{(1,\mathrm{hard})}_{gg\to t\bar{t}H,\mathbf{8}}$ coefficients coming from the virtual corrections equal to the colour channel averaged value, by rescaling at $\mu_F = \mu_R = \mu_0$ the averaged $\bar{C}^{(1,\mathrm{hard})}_{gg\to t\bar{t}H}$ coefficient with ratios $C^{(1,\mathrm{hard})}_{gg\to t\bar{t},I}/\bar{C}^{(1,\mathrm{hard})}_{gg\to t\bar{t}}$ taken from [54]. The procedure is motivated by obvious similarities between the colour structures of the $pp \to t\bar{t}$ and $pp \to t\bar{t}H$ cross sections considered at threshold. We find that such rescaling of the hadronic $t\bar{t}H$ cross section leads to a 3 per mille effect at 14 TeV, or a 5% effect on the correction itself. Therefore we do not expect that the exact knowledge of the $C^{(1)}_{gg\to t\bar{t}H,\mathbf{1}}$ and $C^{(1)}_{gg\to t\bar{t}H,\mathbf{8}}$ coefficients will have a significant impact on the hadronic NLO+NLL predictions. However, we stress that because of the large phase-space suppression in the threshold regime the resummed results, while systematically taking into account a well defined class of correction, should not be used to estimate the size of the NNLO total cross section, by e.g. methods of expansion of the resummed exponential.

[1] It should be however noted that the size of the pdf error presented here does not represent the uncertainty related to using fixed-order cross sections in the pdf fits, as opposed to using resummed cross sections. Recent results by the NNPDF collaboration [56] indicate that this effect would need to be investigated in the full phenomenological study of the $t\bar{t}H$ production and its theoretical uncertainty, which is a task beyond the scope of this paper.

In more detail, QCD corrections to the $t\bar{t}H$ cross section may be divided into logarithmically enhanced (up to the NLL accuracy) soft gluon corrections and the formally subleading pieces i.e. corrections that enter beyond the NLL accuracy at the absolute threshold limit. A direct numerical analysis of relative importance of the two classes of corrections in the NLO correction shows that the soft gluon logarithms do not dominate the exact NLO correction. In the window of renormalisation and factorisation scales $1/2\mu_0 < \mu_F = \mu_R < 2\mu_0$ the NLL result expanded to the NLO accuracy differs from the NLO cross-section by about 10%, which should be compared to the typical relative magnitude of the exact NLO correction in this scale window of up to 20%. We took into account some of these formally non-leading corrections via the $C^{(1)}$-coefficient determined in the absolute threshold limit. This approach, however, also does not provide a satisfactory approximation to the exact NLO correction. We conclude that a good approximation of the exact NLO correction requires inclusion of subleading pieces in the NLL expansion beyond the absolute threshold limit. Therefore our results should be viewed as an all-order improvement of a well defined sub-class of perturbative corrections to the $t\bar{t}H$ cross-section, which, however, omits other possibly important contributions in the full perturbative expansion.

4 Summary

We have investigated the impact of the soft gluon emission effects on the total cross section for the process $pp \to t\bar{t}H$ at the LHC. The resummation of soft gluon emission has been performed using the Mellin-moment resummation technique at the NLO+NLL accuracy. To the best of our knowledge, this is the first application of this method to a $2 \to 3$ process. Supplementing the NLO predictions with NLL corrections results in moderate modifications of the overall size of the total rates. The size of these modifications, as well as the size of the theoretical error due to scale variation is strongly influenced by the inclusion of the first-order hard matching coefficients into the resummation framework. The overall size of the theoretical scale error becomes smaller after resummation, albeit the reduction is relatively modest when the non-zero first-order hard matching coefficients are considered.

Note added. After the arXiv publication of this paper, ref. [58] appeared. The results of [58] seem to support our conclusion regarding the importance of the corrections from beyond the absolute threshold region.

Acknowledgments

This work has been supported in part by the DFG grant KU 3103/1. Support of the Polish National Science Centre grants no. DEC-2014/13/B/ST2/02486 is gratefully acknowledged. TS acknowledges support in the form of a scholarship of Marian Smoluchowski Research Consortium Matter Energy Future from KNOW funding.

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