LABORATORY EXERCISE IN REACTOR PHYSICS (I)

PARAMETERS FOR NEUTRON TRANSPORT

Instructor:
Jitka Žáková, jitka.zakova@neutron.kth.se

INTRODUCTION

Neutron transport refers to the phenomena (motions and interactions) that take place during a neutron's journey from the point where it is created to the point where it is absorbed or leaks out of the system. The neutron transport in a medium can be described by macroscopic parameters such as **diffusion length**, **slowing-down length** and **relaxation length**. In the current exercise, these parameters will be measured for water, using Californium-252 as a neutron source.

Neutrons can be created in nuclear fission, which can be either spontaneous (a decay mode of some heavy nuclei) or neutron-induced (as in a nuclear reactor). In a fission event, one or a few neutrons are released. These neutrons are emitted with a high average kinetic energy. In subsequent collisions with the surrounding medium, the neutrons keep losing their energy (are being slowed down) until they reach the thermal energy of the medium- they become **thermalized**. Once the neutrons become thermalized, they keep moving randomly as they are constantly undergoing collisions with surrounding nuclei. This random motion is called **diffusion**. During the diffusion, the average energy of the neutrons does not change (energy gains are as probable as energy losses), because the neutrons are in thermal equilibrium with the surroundings. The process of diffusion is finished when the neutrons are finally absorbed.

Because of the similarity between the behaviour of the neutrons and gas atoms, the mathematical model developed for the kinetic theory of ideal gases may be used to describe the neutron motion. In this exercise, the basics of this theory are going to be employed together with experimental equipment to determine the following parameters:

- **Diffusion length L**, proportional to the average straight distance thermal neutrons travel before they are absorbed.
- **Slowing-down length L_S**, proportional to the average straight distance fast neutrons travel from the point where they are born to the point where they become thermalized.
- **Relaxation length λ,** is the attenuation of uncollided (fast) neutrons in their "first flight." At each relaxation length, the uncollided flux falls by a factor of *e*.

EXPERIMENTAL SET-UP

Figure 1 shows the experimental setup. The setup consists of a water-filled stainless steel tank, which contains both the neutron source (marked by a red dot in the centre of the figure) and the detector (marked by a blue box close to the side of the tank). The distance of the detector and the source (*r*) can be varied with help of a precise ruler mechanism, which is mounted on the top of the tank. Figure 1 also shows an optional hollow cadmium sphere, which can be mounted around the source in order to "filter out" thermal neutrons. The details of this procedure are given below.

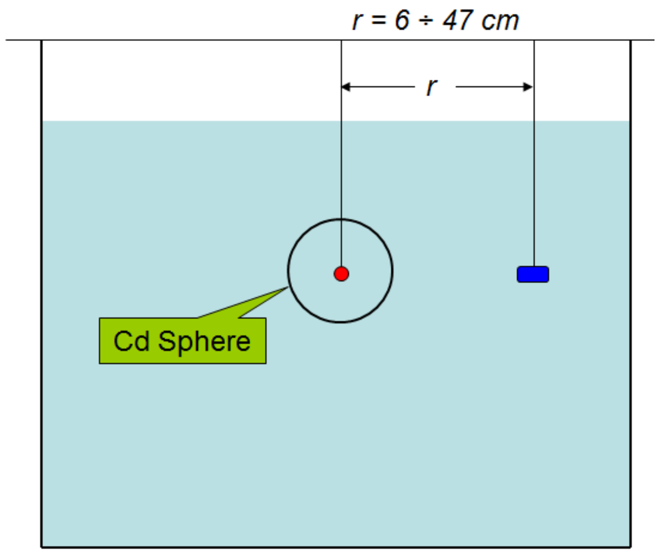

Figure 1: The water tank with the neutron source

The neutron source employed in this lab is a pellet, which contains 50 μg of Californium-252 encapsulated in steel. ^{252}Cf is unstable and decays mainly by emission of α-particles, with a half-life of 2.73 years. About 3% of the decays occur by spontaneous fission, where neutrons with an average kinetic energy of about 2 MeV are emitted.

$$^{252}_{98}Cf \rightarrow \begin{cases} 97\% \quad \alpha + {}^{248}_{96}Cm + 6\,MeV \\ 3\% \quad fission\ products + 3.8\,n\,/\,fission + 185\,MeV \end{cases} \qquad Half-life = 2.645\ years$$

Figure 2 shows the data acquisition system. The signal is generated in a Li-6 scintillator detector. Since neutrons cannot be detected directly, the detector employs a nuclear reaction ^{6}Li(n,α)^{3}H, which yields detectable alpha particle and triton. The detector incorporates a matrix of a lithium compound enriched to 95% in ^{6}Li dispersed in a transparent phosphor powder. ^{6}Li features a high capture cross-section (σ_c=940 b) for thermal neutrons. This means that if a thermal neutron passes through the Li matrix, it is most likely to be captured in ^{6}Li . Furthermore, there is Europium is added to the crystal to serve as scintillator. The resulting alpha particle and triton raise the europium atoms to excited states. These excited states rapidly emit visible light. The crystal is optically coupled to a photomultiplier (PM) tube, from where the pulses are guided towards a multi-channel analyzer.

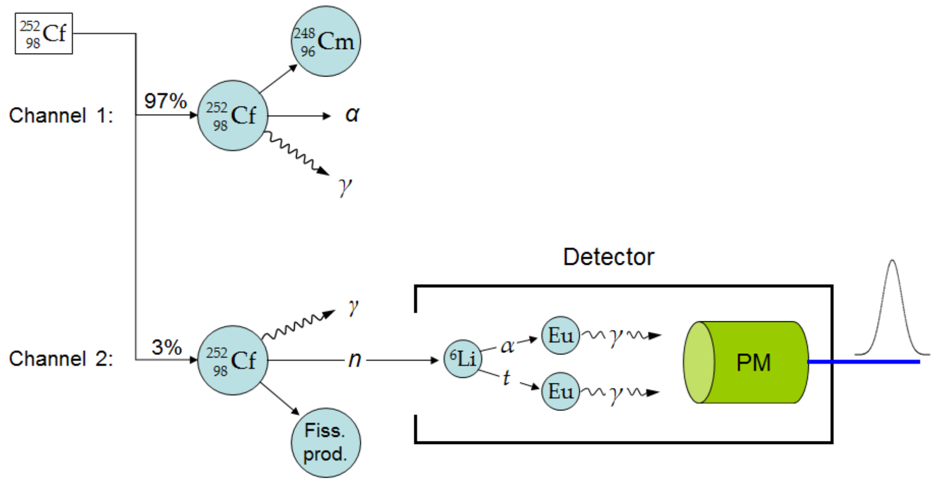

Figure 2: The detection system

A part of this laboratory exercise is dedicated to determining a parameter called diffusion length. Remember, that the process of diffusion takes place when the neutrons are in the equilibrium with their surroundings - they are thermalized. It is therefore reasonable to assume that to obtain the diffusion length a thermal neutron flux is going to be needed. However, there is nothing like a thermal neutron source in the nature, so we are going to use so called **Differential Measurement** to create a **virtual thermal neutron source** instead.

In the differential measurement technique, two separate measurements are performed, one with a bare neutron source and one with the source covered with a 1 mm thick, hollow cadmium sphere, 9 cm in diameter. The principle of this technique is outlined in figure 3. The detector is sensitive only to thermal neutrons. These detected neutrons can either be born thermal (yes, the source emits neutrons of all energies, the 2 MeV mentioned above is just the average!) or they become thermalized somewhere on the way between the source and the detector.

If the source is covered by something that efficiently captures thermal neutrons (for example Cd- see the cross-section in figure 4), only the portion of the neutrons that got thermalized outside the sphere (ϕ_{out}) gets detected. All the neutrons that were born thermal or got thermalized within the sphere (ϕ_{in}) are effectively captured in Cd and they do not reach the detector. The result of the flux measurement with the Cd sphere around the source is $\phi^{(2)}$ in figure 3 and it is equal to ϕ_{out}. If the source is not covered by the Cd sphere, both the neutrons that were thermal from the very beginning (ϕ_{in}) and the neutrons that got thermal on their way to the detector (ϕ_{out}) contribute to the measured count rate. In figure 3, this total flux is denoted by $\phi^{(1)}$. The difference between these measurements ($\phi_{th} = \phi_{in} = \phi^{(1)} - \phi^{(2)}$) represents the fraction of the source neutrons that have been born as thermal or slowed down to thermal energy within the cadmium sphere. This "virtual" source of thermal neutrons can be approximated by a point source.

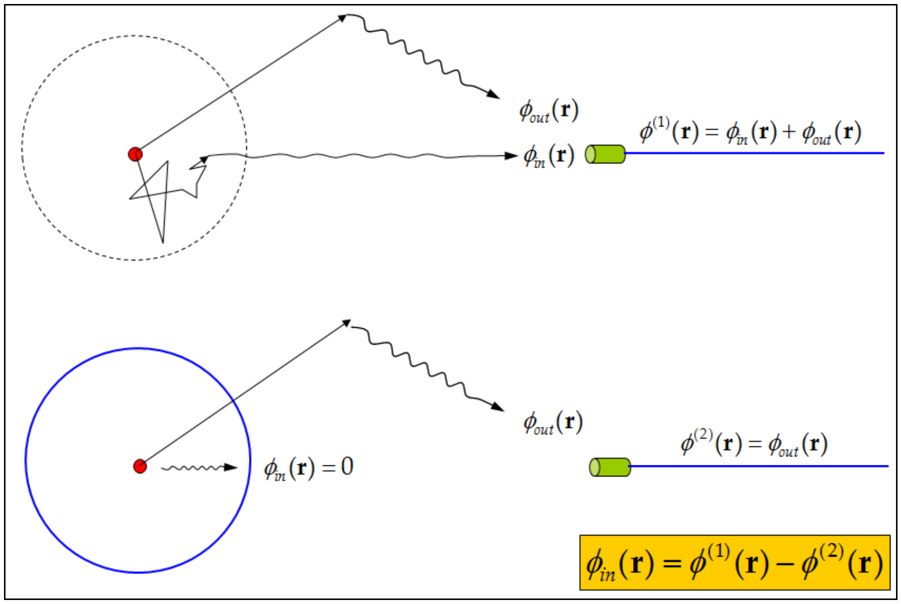

Figure 3: The differential measurement

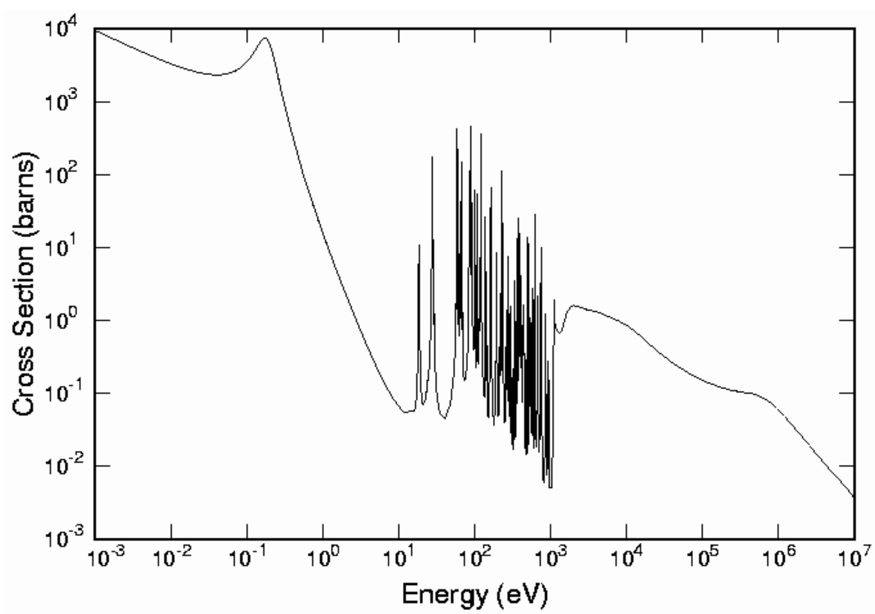

Figure 4: Capture cross-section for neutrons in cadmium.

1. DETERMINING THE DIFFUSION LENGTH - DIFFUSION THEORY

Even though most of the neutrons start their life with a higher energy than the energy of the surrounding media, subsequently they undergo slowing down until they reach a kinetic energy comparable to that of the surrounding nuclei and then they diffuse, we start our exercise directly in the diffusion phase, by measuring the diffusion length. The other two parameters, which are related to higher-energy neutrons we look into later.

Let us assume that we have successfully created a virtual source of thermal neutrons according to the description in the previous section. Now we have a set of data consisting of discrete values of distances and thermal fluxes at these distances. As said above, the thermal neutrons undergo collisions with the scattering nuclei in which the individual neutrons may gain or loose energy, but their average energy remains constant. The energy distribution of the thermal neutrons corresponds, like in the case of kinetic gas theory, to the **Maxwell-Boltzmann distribution**. The steady-state neutron distribution of the thermal neutrons in space is described by **diffusion equation**. In general, the energy group called "thermal neutrons" can be divided into several energy sub-groups. The number of the energy groups a user decides to employ depends on the problem he or she is trying to tackle and also, on the available computational power. For the purpose of this exercise, a **one-group diffusion equation** is going to be sufficient. The diffusion equation formally expressed in the following way:

(1)
$$\boxed{-D\nabla^2\phi(r)+\Sigma_a\phi(r)=\Sigma_f\phi(r)0+S(r)}$$

where

D = Diffusion coefficient $[cm]$
$\phi(r)$ = Neutron flux $[n/cm^2 s]$
Σ_f = Average macroscopic fission cross - section $[cm^{-1}]$
Σ_a = Average macroscopic absorption cross - section $[cm^{-1}]$
$S(r)$ = Source intensity $[n/s]$

The constants in equation 1 are one-group constants, which means, that they are weighted average values over the whole thermal energy range. In our particular experiment, the fission term $\Sigma_f\phi(r)$ is zero, since there is no fissile material present. The source term, S(r), is also zero everywhere in the medium, except at the particular point where the neutron source is located (*r=0*). In the one-group approximation for thermal neutrons, the point neutron source is assumed to isotropically emit monoenergetic (thermal) neutrons of intensity S_{th}. With the definition of the diffusion length $L=\sqrt{D/\Sigma_a}$, equation 1 is reduced to:

(2)
$$\boxed{\nabla^2\phi_{th}(r)-\frac{1}{L^2}\phi_{th}(r)=0} \qquad r\neq 0$$

The general solution to this equation in an "infinite medium" is:

(3) $$\phi_{th}(r) = A\frac{e^{-r/L}}{r} + B\frac{e^{r/L}}{r}$$

We have assumed a spherical symmetry and *r* is the distance enough from the source. The source is located at the centre of a spherical coordinate system. Mathematically, the source is specified as an appropriate boundary condition. The flux is assumed to be zero at the outer boundary of the system, which for simplicity is extended to infinity. That requires that B, in equation 3, is zero and the second term can be omitted (for a finite medium, B≠0 and the second term is obviously needed to satisfy a zero flux boundary condition at a finite radius). With the source intensity S_{th}, the constant A can then be determined to $A=S_{th}/4\pi D$. The solution involves the limit of the neutron current expressed by Fick's law (diffusion approximation). Fick's law is an approximation in that it assumes that neutron motion develops out of the free back-and-forth diffusion of neutrons. There is a net transfer of neutrons from regions of high neutron population to regions of low population simply because the neutron concentration is different. The final solution to equation 2 is:

(4) $$\boxed{\phi_{th}(r) = \frac{S_{th}}{4\pi D}\cdot\frac{e^{-r/L}}{r}} \qquad r \neq 0$$

At r=0, the flux becomes singular. This physically unrealistic singularity results from the mathematical artifice of a point source, i.e. a source with no extension in space. It is seen that the spatial distribution of neutrons emanating from a point source (emitting only thermal neutrons), in an infinite homogeneous medium decreases with the distance r from the source as 1/r times an exponential function. The exponential function describes the attenuation governed by neutron absorption and scattering. If the macroscopic absorption cross-section would be zero, $\Sigma_a=0$, then $L=\infty$ and the exponential function is unity; only geometrical attenuation remains, resulting in a diffusion theory flux proportional to 1/r. Note that diffusion results in a smaller decrease of the neutron flux than the decrease during dispersion in free space, where the mere geometrical spread of neutrons leads to a $1/r^2$ dependency of the flux. This can be explained by the fact that in a scattering medium, some neutrons may recoil "backwards", but in a vacuum all neutrons travel in the "forward" direction.

It should be recognized that since Fick's law is applied to the solution of equation 2, equation 4 is somewhat inaccurate for distances less than a few mean free paths away from the source (What is the mean free path in water?). In regions of strong absorption, or at boundaries between materials with very different neutron transport properties, or at a neutron source, the assumption of free diffusion does not apply.

Physical interpretation of the diffusion length

The flux solution derived in equation 4 can be employed to develop a physical interpretation of the diffusion length. The mean square distance, $\overline{r_{th}^{\,2}}$ travelled by neutrons from their point of thermalization to their final absorption can be expressed mathematically as the neutron normalized distribution function:

(5) $$\overline{r_{th}^2} = \frac{1}{N}\sum_{i=1}^{N} r_i^2 = \frac{\int_0^\infty r^2\Sigma_a\phi_{th}(r)4\pi r^2 dr}{\int_0^\infty \Sigma_a\phi_{th}(r)4\pi r^2 dr} = \frac{1}{S_{th}}\int_0^\infty \Sigma_a\phi_{th}(r)4\pi r^4 dr$$

The source intensity is equal to the total rate of absorption of neutrons in the entire infinite medium; $S_{th} = \int_0^\infty \Sigma_a \phi_{th}(r) 4\pi r^2 dr$. Substituting the relationship for the flux in equation 4, equation 5 becomes:

(6) $$\overline{r_{th}^2} = \frac{1}{L^2} \int_0^\infty r^3 e^{-\frac{r}{L}} dr = 6L^2$$

or

(7) $$L^2 = \frac{1}{6} \overline{r_{th}^2}$$

Therefore, the square of the diffusion length can also be interpreted as one-sixth of the mean square straight distance travelled by thermal neutrons from their point of thermalization to the location where they are absorbed. Inherent in this interpretation is that the diffusion occurs with constant neutron energy.

By rewriting equation 4 in the form of a straight line and fitting a straight line to the obtained values of $\phi_{th}(r)$, the diffusion length L can be determined.

Figure 5 visualizes the concept of the mean straight distances. $\overline{r_s}$ is the mean straight distance a neutron travels from the point where it is born to the point, where it becomes in the thermal equilibrium with the surroundings (slowing down – therefore the index "s"). $\overline{r_{th}}$ is the mean straight distance a neutron travels during diffusion (thermal neutron – therefore the index "th") and $\overline{r}$ denotes the total mean straight distance a neutron travels during its lifetime. Please, observe that the calculations in the following sections use the fact that $\overline{r_s}$ and $\overline{r_{th}}$ are orthogonal. Figure 6 explains this assumption.

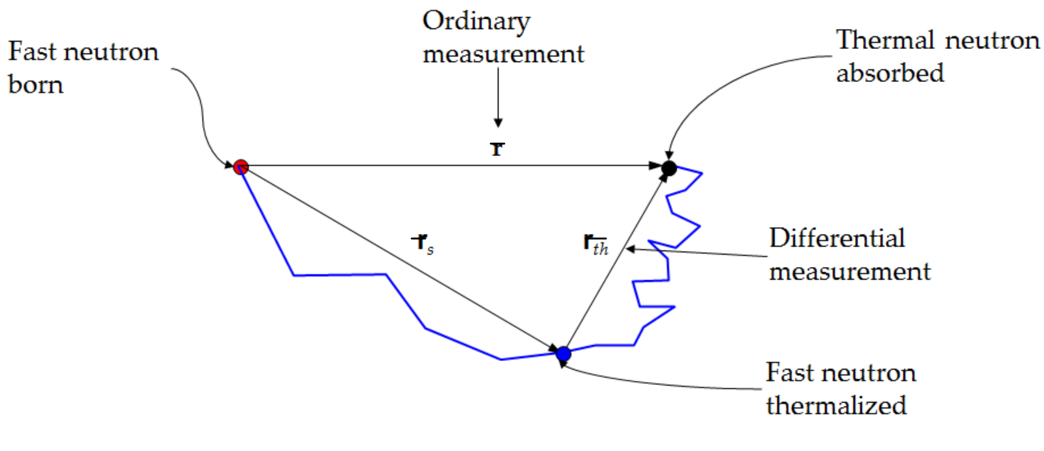

Figure 5: The mean straight distances a neutron travels during its lifetime and the related measurement types.

During the slowing down phase, a neutron travels in some direction, being scattered from this direction by an average angle θ = 0. Once the neutron reaches the thermal equilibrium with the surrounding media however, the direction of travel vanishes. All directions become equally probable, as the neutron is being scattered in collisions with surrounding nuclei. Therefore, the average angle between the straight track travelled during the slowing down and during diffusion is 90 degrees.

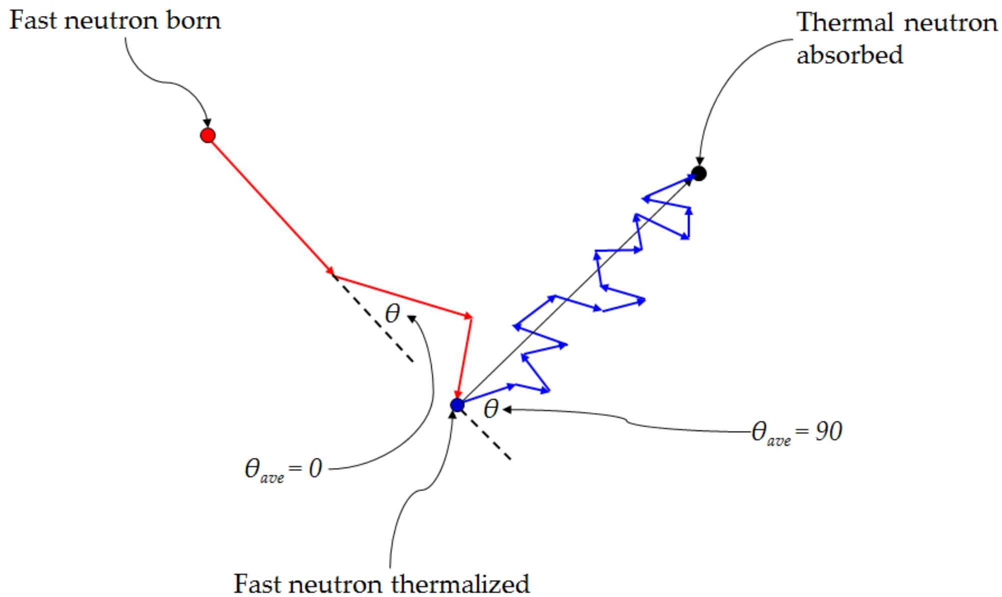

Figure 6: The orthogonality property.

DETERMINING THE SLOWING DOWN LENGTH L_S

Analogically to the diffusion length in equation 5, the slowing down length, L_s may be calculated as an average over the neutron population:

(8) $$\overline{r_s^2} = \frac{1}{N}\sum_{i=1}^{N} r_i^2 = \frac{\int_0^\infty 4\pi r^2 \phi(r) r^2 dr}{\int_0^\infty 4\pi r^2 \phi(r) dr} = 6L_s^2$$

The flux in equation 8 is the flux that is undergoing the slowing down. The detector detects the thermal neutrons that were either born thermal or got thermalized on the way, or- if the source is covered by the Cd sphere, the detector detects only the portion of flux that was slowed down between the sphere and the detector. Either way, the detected flux also contains the neutrons that had been undergoing diffusion. Instead of trying to separate the slowing-down flux, let us define another quantity, which we can easily obtain from out measurements. This will help us to calculate the desired L_s. This quantity is called the **migration area** or a square of the **migration length**. It is related to the mean straight distance a neutron travels during its lifetime ($\overline{r}$ in figure 5).

(9) $$M^2 \equiv \frac{1}{6}\overline{r^2} = L^2 + L_s^2$$

Containing information about both the slowing down and the diffusion phase, the flux from an uncovered neutron source can be used to calculate the migration area.

(10) $$M^2 \equiv \frac{1}{6}\overline{r^2} = \frac{1}{6}\frac{\int_0^\infty\int_0^\infty r^2\Sigma_a(r,E)\phi(r,E)dE4\pi r^2 dr}{\int_0^\infty\int_0^\infty \Sigma_a(r,E)\phi(r,E)dE4\pi r^2 dr} = \frac{1}{6}\frac{\int_0^\infty r^4\phi(r)dr}{\int_0^\infty r^2\phi(r)dr}$$

$\phi(r)$ is the intensity measured by the detector at position r. To arrive to the final shape of the equation 10, following approximation was employed for resolving the energy dependency of the cross-section (equation 11). The absorption cross-section of water is negligible for neutrons with higher then thermal energy. For the neutron energies lower than the thermal threshold, the absorption cross section is constant, can it be taken out of the integral and cancelled.

(11) $$\Sigma_a^{(H_2O)}(r,E) \approx \begin{cases} 0, & E > E_{th} \\ \Sigma_a \approx const, & E < E_{th} \end{cases}$$

A correction arises from the circumstance that the flux cannot be measured at large enough distances. It is necessary to make some theoretical or empirical estimate of how the flux behaves, and extrapolate from the measurement region to large r. One possibility is that the flux at large r goes as

(12) $$\phi(r) \propto \frac{1}{r^2}e^{-\frac{r}{\lambda}}$$

This corresponds to attenuation of the uncollided flux from a point source, with an average relaxation length, λ.

To evaluate the M from the corrected data, $r^2\phi(r)$ is plotted, the area under the extrapolated portion determined analytically, and the remaining area calculated by some numerical integration scheme, e.g. Simpson's rule. The $r^4\phi(r)$ may be determined from the smoothed or fitted $r^2\phi(r)$ curve, and integrated similarly. Then L_s is determined from equation 9.

The effect of the neutrons that have suffered no collisions is neglected in the simple age-diffusion theory. The "fast" neutrons are those produced in fissions that have not been moderated by collisions. Physically, this comes about because age theory refers to neutrons that experience a continuous average deceleration. The number of such fast neutrons will decrease with distance from the source as:

(13) $$\phi(r) \propto \frac{1}{r^2} e^{-\frac{r}{\lambda}}$$

That is, a simple exponential decrease combined with an inverse square characteristic of a point source in an infinite medium. Equation 13 describes the "first flight intensity" of fast neutrons. Fast neutrons travel, on the average, a certain mean free path λ before undergoing a collision. The mean free path appears in the exponential and for this reason it is sometimes called the "relaxation length," i.e., the exponential decreases by a factor *e* for every mean free path away from the source. For that reason, the scattering cross-section of these fast neutrons is referred to as the removal cross-section, $\sigma_{removal}$. The concept of the "removal cross section" is based on the assumption that there is exponential removal of fast neutrons.

If $\ln(\phi(r)\cdot r^2)$ is plotted against r, there will be a linear portion with slope $-\lambda$.

In this calculation, a flux measured without Cd sphere in distances $r \geq 15$cm is to be used.

1) Which general rules apply when handling radioactive objects?

2) Of what order of magnitude is the neutron mean-free path in water?

3) When ^{252}Cf- undergoes fission, neutrons are released. Why does nucleus fission? What energy do the neutrons have?

4) What is the energy distribution of the neutrons that pass through the cadmium sphere?

5) What would the relation $\phi_{th}(r) = \frac{S_0}{4\pi D} \cdot \frac{e^{-r/L}}{r}$ look like if it was written so that its plot became a straight line?

6) How does a scintillation detector with a photo-multiplier work? Why is ^{6}Li used?

7) Using the point source solution (equation 14) for q(r,τ) to derive the relation $\overline{r_s^2} = 6\tau$.

$$q(r,\tau) = \frac{S}{(4\pi\tau)^{3/2}} e^{-\frac{r^2}{4\tau}}, \qquad (14)$$

8) Can gamma radiation disturb our measurement? What are the possible sources of gamma radiation in the lab?

ABOUT THE REPORT – SPECIFIC TO THIS LAB

This section is a guideline specific to the Neutron Transport lab report. It is complementary to the general report guidelines presented on the website; both guidelines are to be followed. The division into the sections presented below serves merely as a suggestion, however the information content is mandatory. A report that lacks any of the information below will not be corrected.

- Abstract – the abstract should summarize your results and methods. It should be short (maximum seven sentences) but precise.
- Introduction – give a short introduction (maximum one page) to the experiment making it possible for a fellow student who has not performed this laboratory exercise to understand the rest of the report and to relate the result to what he or she already knew.
- Methods and tools – describe briefly, what equipment and theoretical framework you used.
- Results
    - Present the measured data from the experiment in a tidy table(s). The number of counts will be listed in form of value ± error.
    - Show the calculated count rates (counts per second) and the thermal flux, also accompanied by the error.
    - Present in sufficient detail all calculations and intermediate steps that lead to your results. Sufficient detail means that a reader understands quickly how you proceeded from one step to another.
    - Plot your data and show the fitting as you performed it when applicable. Show the values of the integrals you calculated.
    - Accompany all your results with appropriate unit. Optional: accompany your results with an error.
- Conclusions and discussion – Summarize and discuss your results and compare them to the literature. Use references. Suggest a possible explanation for the discrepancies.
- Preparatory exercises – Present the solutions of your preparatory exercises, preferably as an appendix to your report.

Least Square Method:

An approximation according to the least square method should be performed when a straight line (or a curve) is to be fitted to a number of measured values. Assume that (x_1, y_1), (x_2, y_2), ... (x_n, y_n) represent the values obtained in the measurements and that you are looking for a linear relation on the form $y = kx + l$. k and l should now be determined so that the quadratic sum

$$S = [y_1 - (kx_1 + l)]^2 + [y_2 - (kx_2 + l)]^2 + ... + [y_n - (kx_n + l)]^2 \quad (15)$$

becomes as small as possible. This is done according to:

$$k = \frac{\sum y_i (x_i - \bar{x})}{\sum (x_i - \bar{x})^2} \quad (16)$$

$$l = \bar{y} - k\bar{x} \quad (17)$$

The sum S is related to the standard deviation for the measurement and it is a measure of the discrepancies between the measured values and the fitted straight line.

Flux or count rate?

While the theory uses the neutron flux- ϕ , the detector shows a count rate. The flux can be calculated from the count rate if the efficiency of the detection system is known. That is not our case. However, we can assume that the efficiency is constant through all the measurements and as you may verify yourself, such a constant efficiency cancels out through the mathematical operations. Therefore you may refer to the measured values as a count rate or flux. However, try not to mix the terminology, once you decided.

How long does a measurement take?

In order to be able to subtract the values of the flux measured with and without the Cadmium sphere, a sufficient measurement precision is needed. The standard deviation associated with a measurement that is performed on a Poison-distributed process is $\sqrt{N}$, where N is the number of detected events. Assuming, that the distance and the time are measured with 100% accuracy, the relative error in the measurement is:

$$\varepsilon_{rel} = \frac{\sqrt{N}}{N} \frac{1}{\sqrt{N}} \quad (18)$$

In this experiment, the relative error should be ≤ 0.3%. The number of counts needed to achieve this level of relative error is going to be rather fast-achieved in the positions close to the source. However, for larger *r*, long measurements time can be expected.

Deadline

The deadline for the report is one week from the day of the laboratory exercise. Send your report to your instructor via email in pdf format.

Deadline

The report is to be prepared in a team of two people. In case there are an odd number of people in the group, three students may produce one report.