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Vertical integration and incentives to innovate

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Abstract

In this paper, two upstream innovators invest to improve process innovations used by two downstream producers. At the beginning of the game, each innovator licenses its technology to one producer and they can agree to integrate vertically. Then, investment takes place and successful innovators choose their licensees. When technologies are not costlessly substitutable, the prices of licenses rise with the size of the switching costs. This affects ex-ante incentives to invest, and efficient technologies with low switching costs may disappear. As a result, ex-ante vertical integration is privately beneficial.
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1. Introduction

Research and development (R&D) is a basic component of modern economies but market incentives and existing regulatory mechanisms are insufficient to induce firms to carry out their research projects efficiently. Although some large firms are monopolists in some R&D areas and have incentives to make substantial ex-ante investments, most innovations are developed in situations of oligopolistic

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R&D rivalry. Then, innovators are involved in patent races, in which losers do not receive patents and get small payoffs. Firms have incentives to protect themselves from such outcomes. For instance, they may integrate vertically with producers or develop technologies with switching costs. This last alternative increases the costs that producers and/or consumers incur if they decide to break the relationship with the innovator. Moreover, innovations become non-substitutable, which allows innovators to avoid patent races.

From a practical perspective, many industries have high equilibrium concentration levels and are characterized by the emergence of large firms able to carry out fundamental research, develop their innovations and market final products. Moreover, firms differentiate their innovations, which leads to different standards such as PC vs. Macintosh, Windows vs. Unix, PAL vs. SECAM, or (historically) VHS vs. Betamax. In so doing, innovators protect their monopoly power and can extract the surplus generated by their innovations, which makes them willing to embark on research projects in the first place.

Those strategies are associated with two important issues. First, there is an obvious relationship between non-substitutability and the emergence of standards. In that respect, it is sometimes argued that some inefficient standards are precisely those that survive. Given the impact of innovations on economic performance, the possibility that inefficient standards might be adopted is of primary concern. It is important to understand why this might occur and to find out how it could be avoided. Second, there is a link between substitutability and incentives to integrate vertically. Indeed, the degree of substitutability between innovations affects the prices of licenses and the extent to which innovators can capture the returns from their innovations, which can motivate vertical integration. Since the decision of two firms to integrate vertically is generally affected by the decisions of other firms, it is unclear whether the socially efficient industrial structure emerges. In our view, those two issues are closely related and the aim of this paper is to investigate them simultaneously. In particular, given that vertical integration is always a possible strategy for firms, there is a priori no reason why efficient technologies should disappear.

The literature on vertical integration focuses on mergers between producers of conventional inputs and outputs. The analyses determine when vertical mergers take place, identify the conditions under which market foreclosure is a consequence or a purpose of integration, and characterize the situations in which integration is beneficial.[1] By contrast, vertical integration in the specific case of

[1] See Hart and Tirole (1990), Ordover et al. (1990), Bolton and Whinston (1991, 1993) and Rey and Tirole (forthcoming). See also Chemla (forthcoming) for an analysis of the impact of downstream competition on the incentives to integrate vertically.

innovators and producers has received little attention. In the present analysis, the environment in which integration takes place between innovators and producers is different from that investigated in the tradition of Grossman and Hart (1986). Indeed vertical integration between producers of conventional inputs and outputs affects only competition on upstream and downstream markets. In the case of an input-innovation however, vertical integration may also modify incentives for R&D. In particular, the selection of high R&D investments and the discovery of high quality innovations are more likely in situations in which innovators can capture large benefits, which occurs with a higher probability if firms have good prospects to be monopolists on the product market. This suggests that the willingness of antitrust authorities to favor competition on the downstream market may be at variance with the interests of R&D policy makers, whose aim is to provide incentives for research efforts.[2]

We consider a model in which two upstream innovators undertake costly and uncertain R&D activities, which consist in improving existing technologies. Innovations differ in efficiency levels and can be licensed to two producers that compete in the downstream market for the sale of a product. We assume that each producer uses the technology developed by one innovator before any investment is made. Technologies are not costlessly substitutable, and each innovator can take out a patent for its innovation in case of success, independently of the outcome of its competitor's research activity. Once innovations are realized, a producer can adopt a new technology but it incurs a switching cost. We show that the prices of licenses vary with the size of switching costs. Easily substitutable technologies can command only a low price, while innovators can benefit from a lock-in effect for technologies with high switching costs. These price effects affect ex-ante private incentives to invest in R&D and, in particular, efficient technologies with low switching costs may disappear. In this framework, innovators and producers may find it profitable to integrate vertically before investing. Lastly, we characterize the potential effects of vertical integration on welfare. The analysis suggests that integration is desirable only if the likelihood that the most efficient standards survive increases.

The paper proceeds as follows. Section 2 presents the model. Section 3 investigates the private and social benefits of vertical integration. Section 4 addresses some concluding remarks.

[2] The interaction between patent protection and competition policy has received some attention in the literature. For instance, Martin (forthcoming) considers a model where two industries differ in their ability to improve the quality of technologies and analyzes the impact of antitrust policy on R&D incentives. Assuming that the competition authority selects a policy at each period and has limited resources to investigate realized prices, it is shown that tougher ex-ante competition policy increases the difference between pre-innovation and post-innovation payoffs, which fosters investment.

2. A model of vertical integration in R&D

We consider two upstream research laboratories U_i, $i=\{a, b\}$ and two downstream producers D_j, $j=\{1, 2\}$. At the beginning of the game, there are two possible technologies to produce the final good, one developed by U_a and another developed by U_b. Each producer is matched with one innovator. More precisely, producer D_1 uses the technology U_a develops, and D_2 uses the technology U_b develops. A technology is characterized by a marginal cost of producing the final good. We assume that both technologies are equally efficient and are represented by the marginal cost β_o. In the rest of the paper, we will call it technology *o*. Producers sell perfectly substitutable products, and their profits are normalized to 0. The industry evolves as follows:

First stage. Upstream innovator U_a (resp. U_b) and downstream producer D_1 (resp. D_2) decide whether to integrate vertically. We assume that vertical integration occurs whenever there is a positive surplus to share between the two parties.[3] If they do so, they pay a fixed cost of integration *V*, and integration is irreversible. Then the upstream laboratory becomes the *research division* and the downstream firm becomes the *production division* of the integrated firm. We denote by I_a (resp. I_b) the integrated firm if U_a and D_1 (resp. U_b and D_2) integrate vertically. There are four industry structures to consider:

– *no integration* ($\mathbf{v_n}$): none of the firms integrate vertically;
– *partial integration* ($\mathbf{v_a}$): U_a and D_1 integrate vertically while U_b and D_2 remain independent;
– *partial integration* ($\mathbf{v_b}$): U_b and D_2 integrate vertically while U_a and D_1 remain independent;
– *full integration* ($\mathbf{v_f}$): U_a and D_1 as well as U_b and D_2 integrate vertically.

Second stage. Each innovator embarks on R&D and chooses an investment e_i. It obtains an innovation of fixed value $\beta_i < \beta_o$ with probability $\pi(e_i)$, which is increasing and concave in e_i. We assume for simplicity that efficiency levels of innovations β_a and β_b are common knowledge. We also assume that $\beta_a < \beta_b$. This reflects the fact that U_a has better prospects than U_b (this assumption will be discussed later on). At the end of the research stage, all agents observe whether innovators are successful or not.

Third stage. Each innovating researcher patents its innovation and is allowed to sell licenses to producers. An innovator or an integrated firm can offer a licensing contract to any isolated producer. However, an integrated firm is not allowed to

[3] In particular, we do not discuss the nature of the offer to integrate, and do not make any assumption on how the surplus is split ex-post.

purchase the technology of another innovator when its research division fails.[4] Innovators compete for the sale of their innovations and producers decide which technology they adopt. For future reference, U_a's (resp. U_b's) upgrade will be called technology a (resp. technology b). Moreover, we denote a scenario in which producer D_1 uses technology i and producer D_2 uses technology i', by $[\mathbf{i}, \mathbf{i}']$ where i and i' are in $\{a, b, o\}$.

Adoption of technology: when a producer adopts a new standard, then it has to pay a switching cost. Since producer D_1 uses the technology developed by U_a in the status quo situation, it has to incur a cost $c_{1b} \geqslant 0$ to switch to technology b. Similarly, D_2 incurs $c_{2a} \geqslant 0$ to adopt technology a. Naturally, if a producer does not adopt any upgrade or adopts the upgrade of the technology he uses in the status-quo, it does not incur any switching cost. Formally $c_{1o} = c_{2o} = c_{1a} = c_{2b} = 0$. We assume for simplicity that new innovations are drastic. When a producer buys either β_a or β_b, its competitor makes losses if it uses β_o: it exits the market and gets a payoff normalized to zero.

Licensing contracts: innovator i specifies the license fees producer(s) have to pay in exchange of the technology for each market structure in which technology i can be used. Specifically, a license fee is a transfer $t_{ji}^{i'}$ that has to be paid by producer j to innovator i when the other producer adopts technology i'. Licensing contracts are offered simultaneously. We assume that transfers are non-negative and, in particular, that an innovator cannot bribe a downstream firm not to produce.

Fourth stage. Downstream firms or production divisions compete à la Cournot on the product market. They sell quantities q_1 and q_2 at price $p(q_1, q_2)$. The inverse demand for the final good is $p(q_1, q_2) = \gamma - \rho(q_1 + q_2)$ where $\gamma > 0$ and $\rho > 0$.[5] Lastly, license fees are paid.

3. The effects of vertical integration

We solve the game by backward induction. In Section 3.1, we analyze the competition between producers in the last stage, conditional on the technologies adopted. In Section 3.2, we determine the ex-post licensing contracts offered by

[4] Implicitly we assume that each research laboratory dedicates itself to the production of innovations in a given standard and is not able to produce other kinds of innovations. In other words, we assume that the cost of changing the research line is such that the research laboratory prefers preventing the production division from using another standard when integration occurs. A detailed analysis of the contracts signed between two firms that decide to integrate vertically would be of interest but beyond the scope of this paper.

[5] Our results hold for any demand function $p(q_1, q_2)$ differentiable, decreasing and concave with respect to each argument and with a zero cross-derivative. The cross-derivative condition is a sufficient condition to have a unique equilibrium in the fourth stage.

innovators. Section 3.3 characterizes the equilibrium levels of investment as well as the structure of the industry that emerges in the first stage. Lastly, in Section 3.4, we address some welfare considerations.

3.1. Ex-post competition

Let us first introduce the following assumption.

Assumption 1. $\beta_a < \beta_b < \beta_o = \gamma$ and $\beta_a \in (2\beta_b - \gamma,\ \beta_b)$.

This assumption guarantees that both producers have incentives to produce and compete when they adopt new technologies, even when one innovator has a technological advantage (uses technology a while the rival uses b). It also guarantees that profits are 0 in the status quo situation and an innovator does not produce when it does not use a new technology while its competitor does.[6] In terms of our notations, [**i**, **o**] with $i \neq o$ means that producer 1 becomes a monopolist and [**o**, **i**′] with $i' \neq o$ means that producer 2 becomes a monopolist.

If a producer uses technology $i \neq o$ and is a monopolist (this occurs when its competitor can only use technology β_o and exits), it produces q_i^m and its profit is B_i^m, where:

$$q_i^m = \frac{\gamma - \beta_i}{2\rho} \quad \text{and} \quad B_i^m = \frac{(\gamma - \beta_i)^2}{4\rho}. \tag{1}$$

If a producer uses technology $i \neq o$ and its competitor uses technology $i' \neq o$, it produces $q_{ii'}^d$ and gets $B_{ii'}^d$,where:

$$q_{ii'}^d = \frac{\gamma - 2\beta_i + \beta_{i'}}{3\rho} \quad \text{and} \quad B_{ii'}^d = \frac{(\gamma - 2\beta_i + \beta_{i'})^2}{9\rho} \tag{2}$$

and its competitor produces $q_{i'i}^d$ and gets a profit $B_{i'i}^d$, where:

$$q_{i'i}^d = \frac{\gamma - 2\beta_{i'} + \beta_i}{3\rho} \quad \text{and} \quad B_{i'i}^d = \frac{(\gamma - 2\beta_{i'} + \beta_i)^2}{9\rho}. \tag{3}$$

It is easy to check that equilibrium quantities and profits have the following properties:

$$\frac{\partial q_{ii'}^d}{\partial \beta_i} < 0\ , \quad \frac{\partial B_{ii'}^d}{\partial \beta_i} < 0\ , \quad \frac{\partial q_{i'i}^d}{\partial \beta_i} > 0\ , \quad \frac{\partial B_{i'i}^d}{\partial \beta_i} > 0\ , \tag{4}$$

[6] We could assume alternatively that $\gamma > \beta_o > \beta_b > \beta_a$ and normalize profits obtained in the status-quo situation to zero.

$$\frac{\partial q_i^m}{\partial \beta_i} < 0 \quad \text{and} \quad \frac{\partial B_i^m}{\partial \beta_i} < 0. \tag{5}$$

Let us recall some standard results that will be used extensively in the rest of the paper. First, the firm with a cost advantage will produce more than its competitor and make a higher profit at the Cournot equilibrium ($B_{ab}^d > B_{ba}^d$). Second, it is more profitable for a producer to be a monopolist rather than a duopolist ($B_i^m > B_{ii'}^d$). Third, the monopoly price decreases with the quality of the innovation ($p(q_b^m) > p(q_a^m)$) and competition reduces the equilibrium price ($p(q_i^m) > p(q_{ii'}^d, q_{i'i}^d)$). Fourth, when agents use the same technologies, the aggregate profit is smaller than the monopoly profit ($2B_{ii}^d < B_i^m$). Moreover, the monopoly profit of a firm that uses innovation a is greater than the sum of duopolistic profits ($B_a^m - B_{ab}^d - B_{ba}^d > 0$).[7] Lastly, when β_a and β_b are relatively close, the profit of a monopolist using technology b is higher than the sum of duopolistic profits under asymmetric competition ($B_b^m - B_{ab}^d - B_{ba}^d > 0$). By contrast, when the difference between β_a and β_b is substantial, the aggregate profit under competition is greater than the monopolistic profit derived with technology b ($B_b^m - B_{ab}^d - B_{ba}^d < 0$).[8] For future reference, let us introduce the following definition:

Definition 1. Technologies are called *lowly differentiated* when $B_b^m - B_{ab}^d - B_{ba}^d > 0$, and *highly differentiated* when $B_b^m - B_{ab}^d - B_{ba}^d < 0$.

The quantities produced in the third stage are represented by the vector $q = (q_1, q_2)$. Let $G(q)$ denote gross social welfare, where $G'(q) = p(q)$. The revenue of firm j is $R_j(q_j) = q_j p(q)$ and the net surplus of consumers is:

$$J(q) = G(q) - R_1(q_1) - R_2(q_2) \tag{6}$$

Ex-post welfare (i.e. after industry structure and investments into innovations have been made) is gross social welfare less production costs. Therefore,

$$\begin{aligned} W(q) &= G(q) - \beta_{i(1)} q_1 - \beta_{i(2)} q_2 \\ &= \gamma(q_1 + q_2) - \frac{1}{2}\rho(q_1 + q_2)^2 - \beta_{i(1)} q_1 - \beta_{i(2)} q_2 \end{aligned} \tag{7}$$

where $i(j)$ represents the technology i adopted by firm j. Note that welfare is increasing in the quantities up to a point. Conditional on the emergence of a monopolistic situation, a social planner prefers that a be used rather than b.

[7] Indeed, $\partial[B_{ab}^d + B_{ba}^d]/\partial\beta_a \equiv -2\gamma + 10\beta_a - 8\beta_b < 0$ for all β_a that satisfy Assumption 1. Since $\lim_{\beta_a \to 2\beta_b - \gamma} B_{ab}^d + B_{ba}^d = B_a^m$, then $B_a^m - B_{ab}^d - B_{ba}^d > 0$.

[8] We know that $B_{ab}^d - B_{ba}^d$ is decreasing from B_a^m to $2B_{bb}^d$. Then $\lim_{\beta_a \to 2\beta_b - \gamma} B_b^m - B_{ab}^d - B_{ba}^d = B_b^m - B_a^m < 0$ and $\lim_{\beta_a \to \beta_b} B_b^m - B_{ab}^d - B_{ba}^d = B_b^m - 2B_{bb}^d > 0$. Therefore, there exists $\hat{\beta}_a$ such that $B_b^m - B_{ab}^d - B_{ba}^d$ is negative if $\beta_a < \hat{\beta}_a$ and positive if $\beta_a > \hat{\beta}_a$.

Naturally, welfare is higher when the marginal cost of production is smaller, i.e. $W(q^d_{aa}, q^d_{aa}) > W(q^d_{bb}, q^d_{bb})$. Lastly, competition increases total welfare, i.e. $W(q^d_{ab}, q^d_{ba}) > W(q^m_a) > W(q^m_b)$.

3.2. Licensing contracts

Once the outcome of the research stage is observed, contracts are offered to producers. Given these contracts, each producer accepts to adopt a technology at a given price if and only if it is better off by doing so rather than choosing the other technology or not producing at all. Naturally, this choice depends on which technology is transferred to the rival.

It is important to note that vertical integration reduces the set of possible scenarios. When both innovators succeed and if they have not merged in the first stage, one situation among the following will emerge. On the one hand, competition may be duopolistic. More precisely, (i) each producer keeps his usual technology, (ii) each producer switches to the other technology, (iii) both producers use technology a or (iv) both producers use technology b. Naturally, when a branch integrates vertically, the innovator in the other branch (whether it is integrated or not) cannot attract the production division. Then (ii) never occurs, (iii) (resp. (iv)) cannot emerge under partial integration ($\mathbf{v_b}$) (resp. ($\mathbf{v_a}$)) and full integration ($\mathbf{v_f}$). On the other hand, one producer may become a monopolist on the product market. Again, vertical integration reduces the set of scenarios since the production division of an integrated branch never produces under the technology of the other branch. Lastly, when only one innovator succeeds under no integration ($\mathbf{v_n}$), it can sell licenses to either one or both producers. For the same reason as before, fewer scenarios can emerge under vertical integration.

Producer D_j accepts a contract from U_i if both his participation constraint (IR) and his incentive compatibility constraint (IC) are satisfied, i.e. if conditional on the rival adopting technology i',

$$B_{ii'} - t^{i'}_{ji} - c_{ji} \geqslant 0 \tag{IR}$$

$$B_{ii'} - t^{i'}_{ji} - c_{ji} \geqslant B_{ki'} - t^{i'}_{jk} - c_{jk} \tag{IC}$$

where $k = \{a, b, o\} - \{i\}$ and B_{xy} represents the profits obtained in the last stage. We make the following assumption to avoid mixed strategies.

Assumption 2.

(i) When D_1 (resp. D_2) is indifferent between keeping technology a (resp. b) or switching, then it selects the first option.
(ii) If D_1 and D_2 can either buy the upgrade of their usual partners U_a and U_b, or

both switch, they both switch if and only if both are better-off under that option. Otherwise, they remain matched to their usual partner.

Point (i) specifies the choice of a producer when it is indifferent between the two technologies (i.e. when its incentive compatibility constraint is binding). Point (ii) avoids coordination problems.[9]

Before stating our first result, we have to introduce several definitions. First, the equilibrium will depend on the ability of each innovator to sell licenses and to avoid scenarios in which it gets zero profit. In particular, if U_b makes an offer to D_2 such that no offer from U_a is incentive compatible (IC), then U_b can obtain a payment from D_2. Naturally, the same argument applies to U_a. However, even when U_a can make a competitive offer to D_2, that offer is made in equilibrium if and only if it is profitable. In other words, U_b may specify a license fee to D_2 such that U_a does not find it profitable to make an offer to attract D_2.

Definition 2. An offer from U_i to D_j such that no offer from U_i's rival is acceptable by D_j is called an (*IC*)*-preventive offer*. An offer from U_i to D_j such that U_i's rival does not find it profitable to make also an offer to D_j is called a *profit-preventive transfer*.

Second, the presence of switching costs affects the possibility of making acceptable offers when both innovators succeed. In particular, innovators may benefit from *lock-in effects*. The aim of each innovator is two-fold. First, it wants to avoid situations in which it gets a zero payoff. Second, it maximizes profits, which might be achieved by attracting the producer in the other branch. The size of switching costs affects the likelihood of these outcomes.

Definition 3. Given the presence of switching costs,

(i) U_a (resp. U_b) benefits from a *strong lock-in effect* when $c_{1b} > B_b^m$ (resp. $c_{2a} > B_a^m$);
(ii) U_a (resp. U_b) benefits from a *weak lock-in effect* when $c_{1b} \in [B_{ba}^b, B_b^m]$ (resp. $c_{2a} \in [B_{ab}^b, B_a^m]$);
(iii) U_a (resp. U_b) has *no lock-in effect* when $c_{1b} < B_{ba}^d$ (resp. $c_{2a} < B_{ab}^d$).

When U_a benefits from a strong lock-in effect, then U_b cannot attract D_1 and [**b**, **o**], [**b**, **b**] are prevented at no cost. When U_a benefits from a weak lock-in effect, then D_1 cannot adopt technology b if D_2 adopts technology a. In that case, [**b**, **o**], [**b**, **b**] might be feasible but c_{1b} is sufficiently high to discourage U_b from

[9] Point (ii) allows us to select one equilibrium when multiplicity is at stake. This occurs only in one situation (see Appendix B) and we restrict attention to the Pareto dominant equilibrium.

attracting D_1. Naturally, the same applies to U_b. When innovators do not benefit from lock-in effect, both innovators can adopt a new technology. Overall, whenever an innovator benefits from lock-in effects, it has more freedom to attract the producer in the other branch.

Our first result characterizes the possible scenarios when both innovators are successful under no integration ($\mathbf{v_n}$). In particular, we determine the cases where zero-payoff scenarios can be prevented.

Lemma 1. *Under no integration* ($\mathbf{v_n}$) *and if both innovators are successful,*

(i) [**a**, **o**] *and* [**o**, **b**] *cannot occur*;
(ii) [**o**, **a**] (*resp.* [**b**, **o**]) *can occur if only* U_a (*resp.* U_b) *benefits from lock-in effects*;
(iii) U_b *can avoid* [**a**, **a**] *by making a preventive offer to* D_2 *when* c_{2a} *is relatively high compared to the potential gain of switching.*
(iv) *given its technological advantage,* U_a *can always prevent* [**b**, **b**];
(v) [**b**, **a**] *might occur only in the absence of lock-in effects.*

Proof. See Appendix A.

First, U_a can always prevent D_2 from becoming monopolist if it chooses technology b. The reason is simply that U_a can decide to transfer the technology to D_1 when D_2 adopts b in exchange of any payment $t^b_{1a} \leqslant B^d_{ab}$. Then, D_1 adopts technology a whenever D_2 adopts b and [**o**, **b**] cannot occur. Similarly, U_b can prevent [**a**, **o**] from occurring.

Second, when one innovator benefits from relatively stronger lock-in effects than its competitor, it can exploit its advantage to make the competing technology exit the market. For instance, if U_a benefits from strong lock-in effects while technology b can be easily adopted, U_a can specify a high license fee to D_1 (who has no other choice than accepting it), but its comparative advantage in terms of switching costs offers him two possible interesting alternatives. On the one hand, it may decide to sell only one license to D_2. On the other hand, it may want to attract both producers. However, as long as U_a develops a technology that is relatively costly for an adopting producer (c_{2a} is sufficiently high), U_b is able to make a preventive offer to D_2 and to avoid zero-payoff scenarios. Interestingly, if U_a benefits only from weak lock-in effects, U_b can prevent the monopolistic scenario in which only D_2 buys technology a. Indeed, U_b can make offers to attract both producers in order to force U_a to make a preventive offer to D_1.

Third, when both innovators benefit from lock-in effects, each of them has few possibilities to make incentive-compatible offers to the producer in the other branch. Then, license fees do not need to be low to prevent zero-payoff scenarios. Whenever (IC)-preventive or profit-preventive offers are necessary, royalties do not need to be small. In particular, when both innovators benefit from strong lock-in effects, their usual respective licensees cannot adopt new technologies since their profits would be dissipated by switching costs. In that case, U_a and U_b have all the bargaining power and any offer to their usual licensee are accepted. Then, they charge the highest license fees.

Lastly, in the absence of lock-in effects, [**b**, **a**] is a feasible scenario. However, given that innovators incur switching costs when they adopt a new technology, the payoffs that innovators can obtain are reduced relative to scenario [**a**, **b**].[10] Moreover, monopolistic situations are automatically avoided, but [**a**, **a**] and [**b**, **b**] are feasible. However, it is important to note that the result is affected quantitatively by the relative size of the innovations: U_a is more likely to attract D_2 than U_b is likely to attract D_1 since any firm that uses a makes higher profits than if it uses b. In particular, if c_{2a} is equal to zero and U_b would give its upgrade for free, D_2 would be likely to make a positive payment to acquire the superior technology a since $B^d_{ba} < B^d_{aa} < B^m_a$. Similar arguments apply to U_a. However, its technological advantage allows him to avoid zero-payoff scenarios more often. In particular, since $B^d_{ab} > B^d_{bb}$, offering a free upgrade of technology a to D_1 prevents U_b from attracting both producers, even when $c_{1b}$is equal to zero. Moreover, competition between innovators becomes tighter when switching costs decrease. In particular, given its technological disadvantage, U_b has high incentives to attract both producers ($2B^d_{bb} > B^d_{ba}$) and to respond aggressively to any strategy of its competitor.

We know from Lemma 1 which scenarios are possible depending on the size of the switching costs. Our next step is to determine the Nash equilibrium in each case. The main result of this section characterizes the equilibrium outcome of the game.

Proposition 1. *Under no integration* ($\mathbf{v_n}$), *the equilibrium licensing contracts are such that*:

(i) *If* U_a (*resp*. U_b) *is the unique successful innovator, then* D_1 (*resp*. D_2) *buys technology* a (*resp*. b), *becomes a monopolist and pays* B^m_a (*resp*. B^m_b);

(ii) *If both* U_a *and* U_b *are successful we have*:

(a) *If only* U_b *benefits from lock-in effects*, [**b**, **o**] *emerges when* $c_{1b} < B^m_b - B^d_{ab} - B^d_{ba}$;

(b) *If only* U_a *benefits from lock-in effects*, [**o**, **a**] *emerges when* $c_{2a} < B^m_a - B^d_{ab} - B^d_{ba}$, *and* [**a**, **a**] *emerges when* $c_{2a} < k(c_{1b})$ *where* $k(c_{1b}) = \max\{0; \min\{B^d_{aa} - B^d_{ba};\ 2B^d_{aa} + B^d_{bb} - B^d_{ba} - B^d_{ab} - c_{1b}\}\}$.

(c) *If* U_a *and* U_b *do not benefit from lock-in effects*, [**a**, **a**] *emerges when* $c_{2a} < k'$ *where* $k' = \max\{0; \min\{B^d_{aa} - B^d_{ba};\ -B^d_{ab} + 2B^d_{aa} + B^d_{bb} - 2B^d_{ba}\}\}$;

(d) *In all the other case*, [**a**, **b**] *is the unique equilibrium.*

[10] [**b**, **a**] is a wasteful outcome and will not emerge in equilibrium (see Proposition 1). Its feasibility helps innovators to make out-of-equilibrium threats to prevent zero-payoff scenarios.

If only one innovator is successful under partial integration ($\mathbf{v_a}$) *and* ($\mathbf{v_b}$) *or full integration* ($\mathbf{v_f}$), *the result is the same as under no integration* ($\mathbf{v_n}$). *When both innovators succeed under* ($\mathbf{v_a}$) (*resp.* ($\mathbf{v_b}$)), I_a (*resp.* I_b) *does not produce and sells a license to* D_2 (*resp.* D_1) *when* $c_{2a} < B_a^m - B_{ab}^d - B_{ba}^d$ (*resp.* $c_{1b} < B_b^m - B_{ab}^d - B_{ba}^d$). *Otherwise, the unique equilibrium is* [**a**, **b**].

Proof. See Appendix B.

When U_a (respectively U_b) is the unique successful innovator under no integration ($v_{\mathbf{n}}$), it can extract all the surplus generated by its innovation. Since $B_a^m > B_a^m - c_{2a} > 2B_{aa}^d - c_{2a}$, it is always optimal to sell a license only to D_1 (respectively D_2) that becomes monopolist and pays B_a^m (resp. B_b^m) for it. Naturally, under vertical integration, this result is unchanged.

Suppose that both innovators are successful and that none of them has integrated vertically in the first stage. We have three possible situations:

First, when both innovators benefit from strong lock-in effects, producers cannot switch. Then the equilibrium is [**a**, **b**] and producers are left with no rent.

Second, when only one innovator benefits from (strong or weak) lock-in effects, it can threaten its competitor with a zero-payoff scenario. The region of switching costs in which technology b disappears corresponds simply to the region in which neither (IC)-preventive offers nor profit-preventive offers exist. More precisely, technology b disappears as a result of D_2 adopting a as a monopolist when D_2 and U_a have a surplus to share, even when U_b gives the upgrade of technology b for free to D_2. In particular, if the maximal payment that U_a can extract from D_1, B_{ab}^d, is smaller than the surplus D_2 gets by adopting technology a as a monopolist, $B_a^m - c_{2a} - B_{ba}^d$, then U_a and D_2 have a surplus to share. In other words, [**o**, **a**] emerges when $c_{2a} < B_a^m - B_{ab}^d - B_{ba}^d$.

Moreover, when U_a benefits only from weak lock-in effects, U_b can set preventive transfers that automatically prevent D_2 from adopting technology a as a monopolist. Still, U_a can attract both producers and this occurs when D_1, D_2 and U_a have a surplus to share, even when U_b gives the upgrade of technology b for free to any producer. More precisely, conditional on D_2 adopting technology a, D_1 accepts an offer from U_a if and only if its surplus is positive, i.e. $B_{aa}^d - t_{1a}^a \geqslant 0$.[11] Similarly, conditional on D_1 adopting technology a, D_2 accepts an offer from U_a if $B_{aa}^d - c_{2a} - t_{2a}^a \geqslant B_{ba}^d$. U_a is willing to make these offers if $t_{1a}^a + t_{2a}^a$ is greater than the benefit it obtains from transferring the technology only to D_1. However, D_1 is

[11] Remember this is the case because U_a benefits from a weak lock-in effect that allows him to attract D_1 at no cost if D_2 adopts a.

willing to adopt a and pay t_{1a}^{b} if $B_{ab}^{d} - t_{1a}^{b} \geqslant B_{bb}^{d} - c_{1b}$, which implies that the maximal payment U_a can extract is $t_{1a}^{b} = B_{ab}^{d} - B_{bb}^{d} + c_{1b}^{d}$. Overall D_1 and D_2 adopt technology a if $c_{2a} < k(c_{1b})$. The reasoning is similar for technology a except that U_b cannot attract both producers at the same time. Then a disappears only when the maximal payment that U_b can extract from D_2 is smaller than the surplus D_1 gets by adopting technology b as a monopolist, i.e. if $c_{1b} < B_{b}^{m} - B_{ba}^{d} - B_{ab}^{d}$. It is important to note that when technologies are highly differentiated, U_b and D_2 can never reach an agreement. As a consequence, a can disappear only when technologies are lowly differentiated.

Third, when there is no lock-in effect, each innovator competes to attract both producers and to prevent its competitor from attracting them. This competition results in a decrease in license fees. Moreover, U_a's technological advantage may result in the adoption of technology a by both producers. Again, this is the case when D_1, D_2 and U_a have a surplus to share even if technology b is free. As in the previous point, both producers adopt a if U_a's benefit $t_{1a}^{a} + t_{2a}^{a}$ is greater than the benefit it obtains from keeping only its usual licensee (which is again $B_{ab}^{d} + B_{bb}^{d} - c_{1b}$), and if each producer makes a positive surplus in accepting the offer. More precisely, conditional on its competitor accepting an offer from U_a, D_1 (resp. D_2) adopts technology a if $B_{aa}^{d} - t_{1a}^{a} \geqslant B_{ba}^{d} - c_{1b}$ (resp. $B_{aa}^{d} - c_{2a} - t_{2a}^{a} \geqslant B_{ba}^{d}$). Again, combining the three previous requirements, both producers adopt technology a if $c_{2a} < k'$.

The effect of integration is as follows. If U_a and D_1 integrate vertically in the first stage and if both innovators are successful, U_b cannot attract D_1. Then D_1 never adopts technology b. Moreover, the internalization of the switching cost allows the integrated firm to behave as if it benefited from strong lock-in effects. In particular, I_a can want to attract D_2 as a monopolist. Since U_b no longer has the option of attracting D_1 to force U_a to make an offer to D_1, it can only make attractive offers to D_2. When c_{2a} is sufficiently large, D_2 cannot accept an offer from the integrated branch and the equilibrium is necessarily [**a**, **b**]. When c_{2a} takes intermediate values, U_b can avoid the monopolistic scenario by setting preventive transfers. When c_{2a} is sufficiently low however, D_2 buys technology a, I_a's production division does not produce and technology b disappears. Again, this occurs when I_a and D_2 have a surplus to share. Obviously, the result is the same in the case of integration between U_b and D_2. Naturally, when full integration occurs, all the possibilities to attract the producer in the other branch disappear and the equilibrium is [**a**, **b**] in all the cases.

In the following paragraphs, we discuss in more detail how the license fees paid by producers are affected by the switching costs. When both innovations are successful, the payoff of innovator U_i is denoted by $P_i^h(c_{1b}, c_{2a}; a, b)$ and the payoff of producer j is $P_j^h(c_{1b}, c_{2a}; a, b)$ for all $h \in \{v_n, v_b, v_a, v_f\}$. When only innovator i is successful, only the producer that is ex ante matched with U_i produces and is left with no rent. Then, i's payoff is B_i^m. Naturally, all other players get no payoff.

Proposition 2. *Suppose that both innovators are successful. Under no integration* ($\mathbf{v_n}$), *the payoff of* U_a *is non-decreasing in* c_{1b} *and the payoff of* U_b *is non-decreasing in* c_{2a}. *In the absence of lock-in effects, the payoff of* U_b *can be positively affected by an increase in* c_{1b}. *Otherwise, the payoff of* U_a (*resp.* U_b) *is non-increasing in* c_{2a} (*resp.* c_{1b}). *Lastly, under partial integration* ($\mathbf{v_a}$) (*resp.* ($\mathbf{v_b}$), *the payoff of* I_a (*resp.* I_b) *is decreasing in* c_{2a} (*resp.* c_{1b}), *while the payoff of* U_b (*resp.* U_a) *is increasing in* c_{2a} (*resp.* c_{1b}).

Proof. See Appendix B.

First, note that the license fee that an innovator charges to a given producer depends on the switching cost of that producer. When U_a (resp. U_b) sells a license to D_2 (resp. D_1), the higher c_{2a} (resp. c_{1b}), the smaller the payment U_a (resp. U_b) can receive from D_2 (resp. D_1). By contrast, the higher c_{1b} (resp. c_{2a}), the smaller the chances D_1 (resp. D_2) switches and the higher the revenue U_a (resp. U_b) extracts from D_1 (resp. D_1).

Second, in the absence of lock-in effects, both producers can be easily attracted. Recall that when c_{1b} decreases, D_1 has higher incentives to switch to technology b, but U_a can secure some payments by attracting D_2. However, conditional on attracting D_1, U_b wants to attract also D_2. Then, U_a has no other choice than decreasing its offer to D_1 to prevent U_b from attracting D_1 in the first place. In other words, U_b's payoff is potentially affected negatively when c_{1b} decreases. This results from the tight competition between innovators not only to attract producers but also to prevent them both from adopting the same (competing) technology.

Lastly, when U_a and D_1 integrate vertically, c_{1b} is internalized and cannot affect payoffs. Either I_a attracts D_2 as a monopolist, in which case its payoff is as small as c_{2a} is high, or U_b sets a preventive transfer to avoid the monopolistic scenario. Then, U_b's payoff depends on c_{2a} and is as high as that switching cost is high. Naturally, the same argument applies when U_b and D_2 integrate vertically.

Before analyzing the incentives to invest and to integrate vertically, let us make two important remarks that follow directly from Propositions 1 and 2.

The equilibrium is affected by the difference between β_a and β_b. When technologies are highly differentiated, then U_a is always able to prevent U_b from attracting D_1 as a monopolist. In other words, the low-cost standard never disappears. However, the high-cost standard may not survive: if c_{2a} is small, the equilibrium is **[a, a]** when c_{1b} is also small, and [**o**, **a**] when c_{1b} is large. By contrast, when technologies are lowly differentiated, both monopolistic scenarios are possible but the region in which U_a attracts both producers is as small as the difference between the qualities of the two technologies decrease. Besides, when the innovations have the same quality, U_a and U_b can prevent [**b**, **b**] and [**a**, **a**] respectively. Overall, the difference in cost levels affects the survival of standards and it is not necessarily the case that the low-cost standard always survives. This can be summarized by the following corollary.

Corollary 1. *When technologies β_a and β_b are highly differentiated, only the least efficient one may disappear. When technologies are lowly differentiated, even the most efficient one may disappear.*[12]

It is also important to note that the survival of a standard is affected by the *presence* and the *relative size* of switching costs. In particular, if U_b does not benefit from lock-in effects, a will never disappear (see Proposition 1). Moreover, as switching costs go to zero, the competition becomes so tight that U_b may have no other choice than giving the license for free. In doing so, D_1, D_2 and U_a have a surplus to share, and as a result b disappears. This occurs when $k' > 0$ in Proposition 1.[13] As a consequence, the presence of switching costs is a reason why a high-cost standard can survive and a low-cost standard disappear. In their absence, producers find no reason to adopt an inefficient technology as long as a more efficient technology is available at a competitive price. To sum up,

Corollary 2. *The most efficient technology can disappear only when its switching cost c_{2a} is sufficiently higher than the switching cost of the lesser technology c_{1b}. Further, in the absence of switching costs ($c_{1b} = 0$ and $c_{2a} = 0$), only technology b may disappear.*

3.3. The incentives to integrate

In Section 3.2, we have characterized the technologies that will be present in the industry, given the market structure. In this subsection, we determine the optimal decisions of firms in stage 1, that is, we analyze their incentives to integrate.

Innovators or research divisions determine their provisions of efforts by maximizing their respective expected utility:

$$u_a^h(e_a, e_b) = \pi(e_a)\pi(e_b)P_a^h(c_{1b}, c_{2a}; a, b) + \pi(e_a)[1 - \pi(e_b)]B_a^m - e_a \tag{8}$$

$$u_b^h(e_a, e_b) = \pi(e_a)\pi(e_b)P_b^h(c_{1b}, c_{2a}; a, b) + \pi(e_b)[1 - \pi(e_a)]B_b^m - e_b \tag{9}$$

with $h \in \{v_n, v_a, v_b, v_f\}$.

Assumption 3. $\pi''(e)/\pi'(e) \geqslant \pi'(0)[B_a^m - B_{ab}^d]/B_a^m$ for all $\beta_a < \beta_b$.

[12] This result comes from a suggestion of an anonymous referee to whom I am grateful. Technically, it follows from Proposition 1.

[13] In that case, each producer pays $B_{aa}^d - B_{ba}^d$. Note that for any greater payment, each producer prefers buying b conditional on its competitor adopting a.

Assumption 3 is technical and allows us to have a unique solution in Lemma 2 (see Appendix C for details).[14]

Lemma 2. *The equilibrium efforts are as follows*:

(i) *Under no integration* ($\mathbf{v_n}$) *and in the absence of lock-in effects, U_i's optimal effort may not be monotonic in c_{1b}. With lock-in effects, U_a's effort is decreasing in c_{2a} and increasing in c_{1b}, and U_b's effort is increasing in c_{2a} and decreasing in c_{1b}.*
(ii) *Under partial integration* ($\mathbf{v_a}$) *or* ($\mathbf{v_b}$), *the optimal effort of the integrated firm decreases with the cost of adopting its technology, while the effort of the non-integrated innovator increases with that cost. In equilibrium, the integrated firm puts forth more effort than under no integration* ($\mathbf{v_n}$), *whereas the non integrated innovator diminishes its effort.*
(iii) *Under full integration* ($\mathbf{v_f}$), *I_a* (*resp. I_b*) *invests more than under partial integration* ($\mathbf{v_b}$) (*resp.* ($\mathbf{v_a}$)) *and less than under partial integration* ($\mathbf{v_a}$) (*resp.* ($\mathbf{v_b}$)).

Proof. See Appendix C.

When an innovator increases its effort, it also increases its probability of innovating, which decreases the likelihood that its competitor will be the unique provider of an innovation. Moreover the benefit an innovator obtains when both compete in the licensing stage is smaller than the payoff it gets when it is the only successful innovator. As a consequence, if an innovator invests more, its competitor anticipates smaller gains and has incentives to decrease its own investment. Lastly, since switching costs affect the payoffs obtained when both innovators succeed, the level of investment varies with switching costs. Under no integration ($\mathbf{v_n}$), U_a (resp. U_b) is more likely to invest when c_{1b} (resp. c_{2a}) increases. Moreover, the presence of lock-in effects is beneficial for innovators. In particular, as long as D_1 cannot easily switch, a decrease in the difficulty of adopting technology a (i.e. a decrease in c_{2a}) increases U_a's likelihood of attracting D_2, which in turn increases its payoff as well as its incentives to invest. Naturally, the same argument applies for U_b. By contrast, in the absence of lock-in effects, a decrease in c_{1b} makes U_b more aggressive. In that case, U_a reacts by charging low fees, which forces U_b to also charge low fees (see Proposition 2). As a consequence, U_b's ex-ante incentives to invest are negatively affected by a decrease in c_{1b}. Since U_a tends to exert less effort when c_{1b} decreases, the overall effect is ambiguous.

Given that integration results in the internalization of all the benefits generated

[14] In terms of the primitives of the model, the condition is $-\pi''(e)/\pi'(e) \geqslant \pi'(0)\left[1-4(\gamma-2\beta_a+\beta_b)^2/9(\gamma-\beta_a)^2\right]$.

by the innovation, the effort of a research division is higher than the effort it exerts when it remains independent. By contrast, since a research laboratory increases its effort when it integrates vertically, the probability that its competitor will be the unique successful innovator decreases when integration occurs. Then, the expected benefit of the latter is smaller and it exerts a smaller effort compared to the no integration case ($\mathbf{v_n}$). The same argument applies if an independent innovator integrates when its competitor is already integrated. Moreover, when U_a and D_1 integrate vertically, the license fee charged to a producer can only depend on c_{2a}. Naturally, conditional on attracting D_2, I_a's payoff decreases with c_{2a}, as well as its incentives to invest. The argument is symmetric for I_b and c_{1b}.

In the remainder of the section, let $S_a^{v_a}(c_{1b}, c_{2a})$ (resp. $S_a^{v_f}(c_{1b}, c_{2a})$) denote I_a's surplus when U_b and D_2 remain independent (resp. U_b and D_2 also integrate). $S_b^{v_b}(c_{1b}, c_{2a})$ and $S_b^{v_f}(c_{1b}, c_{2a})$ are similarly defined.

Proposition 3. *Vertical integration generates a non-negative surplus. Moreover, $S_b^{v_f}(c_{1b}, c_{2a})$ and $S_b^{v_b}(c_{1b}, c_{2a})$ (resp. $S_a^{v_f}(c_{1b}, c_{2a})$ and $S_a^{v_a}(c_{1b}, c_{2a})$) are decreasing in c_{2a} (resp. c_{1b}).*

(i) *When U_a and U_b benefit from strong lock-in effects, the equilibrium is no integration ($\mathbf{v_n}$) for all $V>0$;*
(ii) *When U_b (resp. U_a) benefits from (strong or weak) lock-in effects, partial integration ($\mathbf{v_b}$) (resp. ($\mathbf{v_a}$)) never occurs;*
(iii) *Otherwise, both innovators have incentives to integrate vertically and the four industry structures may emerge depending on the cost V.*

Proof. See Appendix D.

The decision to integrate vertically is affected by the size of switching costs. When both innovators benefit from strong lock-in effects, they extract all the surplus from producers and they cannot gain by integrating vertically. Then, as long as integration is costly, the equilibrium in the first stage is no integration ($\mathbf{v_n}$). By contrast, when switching costs are such that an innovator cannot extract all the rents from its usual partner, integration generates a positive surplus. Indeed, it increases the expected revenue of the research laboratory, which gives it incentives to exert a higher effort. In addition, it decreases the incentives of its competitor to exert effort. Overall, partial integration versus no integration and full integration versus integration of competitors are privately beneficial. This implies also that the decision of each branch to integrate vertically is affected by the decision of the other branch. When only one innovator benefits from lock-in effects, it has relatively less incentive to integrate vertically than its competitor. As a consequence, if the cost of integration is very small, both branches decide to integrate vertically. When this cost is substantial however, only the developer of the technology that is most difficult to adopt may find vertical integration profitable.

Lastly, in the absence of lock-in effects, both branches have relatively high incentives to integrate. Nevertheless, the issue of the first stage depends not only on the relative size of the switching costs but also on the type of competition that emerges under no integration ($\mathbf{v_n}$) when both innovators succeed. In particular, the technological advantage of U_a makes it likely to attract both innovators when c_{2a} is small, although that option is never available to U_b. Moreover, U_b is relatively more aggressive than U_a, and the latter has to leave relatively more rents to D_1 than U_b has to leave to D_2. The combination of those different effects implies that all kinds of structures may emerge in equilibrium.

Remark. In the whole analysis, we could have assumed stochastic dominance instead of a deterministic advantage. Suppose for instance that innovators can make innovations $\underline{\beta}$ or $\bar{\beta}$. Moreover, assume that i's probability to get the most efficient technology $\underline{\beta}$ is $\nu_i(\underline{\beta}|e)$ when it exerts effort e with $\nu_a(\underline{\beta}|e) > \nu_b(\underline{\beta}|e)$. In that model, once the outcome of the R&D stage is observed the licensing contracts are the same as in Proposition 1. Then the innovator that has better prospects makes relatively more effort, as in our case. U_a's incentives to integrate are mostly affected by the switching costs, so the results of Proposition 3 still hold.[15]

3.4. Social costs and benefits of integration

Let $W^{h,\{a,b\}}$ (resp. $W^{h,\{i\}}$) represent ex-post welfare when the equilibrium in the first stage is $h \in \{v_n, v_a, v_b, v_f\}$ and when both innovators are successful (resp. only U_i is successful). Expected welfare is then:

$$E[W^h(e_a, e_b)] = \pi(e_a)\pi(e_b)W^{h,\{a,b\}} + \pi(e_a)(1 - \pi(e_b))w^{h,\{a\}} + \pi(e_b)(1 - \pi(e_a))W^{h,\{b\}} - e_a - e_b. \tag{10}$$

When only one innovator is successful, $W^{h,\{i\}} = W(q_i^m, 0)$ for all h. If both are successful, $W^{v_n,\{a,b\}}$ can take the values $W(q_{ab}^d, q_{ba}^d)$, $W(q_{aa}^d, q_{ba}^d)$, $W(q_a^m)$ or $W(q_b^m)$. Under partial integration ($\mathbf{v_a}$), the welfare is either $W(q_{ab}^d, q_{ba}^d)$ or $W(q_a^m)$. Similarly, under partial integration ($\mathbf{v_b}$), it is $W(q_{ab}^d, q_{ba}^d)$ or $W(q_b^m)$. When both branches integrate vertically, then $W^{v_f,\{a,b\}} = W(q_{ab}^d, q_{ba}^d)$.

The desirability of vertical integration builds on three important points. First, welfare is affected by ex-post competition. Other things equal, duopolistic competition is desirable. However, $W(q_{ab}^d, q_{ba}^d) \simeq W(q_a^m)$ and $W(q_a^m) > W(q_b^m)$. Then, vertical integration can be beneficial if it restores competition between adopters of lowly differentiated technologies, but also if the most efficient standard disappears under no integration while the least efficient standard disappears under

[15] If innovators have the same prospects ex-ante, the payoffs in the licensing stage are still affected by the presence of the switching costs. Then, the decision to make effort and to integrate vertically depend on c_{2a} and c_{1b}. The effect of integration is again to increase the overall intensity of research.

vertical integration. Second, welfare is affected by the intensity of research. Naturally, there exist socially optimal efforts e_a^* and e_b^*. Since innovators do not internalize the surplus of consumers, the privately selected efforts are not socially optimal.[16] However, since vertical integration affects ex-ante investment, it may increase or decrease the distance between privately chosen efforts and socially optimal efforts. Lastly, innovators take private decisions that depend on switching costs. The latter affect the way rents are split between innovators and producers but have no direct effect on welfare. However, they determine the kind of ex-post competition that will emerge on the product market, so they also influence (indirectly) the welfare of the economy.

When innovations are lowly differentiated, any kind of integration that makes ex-post duopolistic competition more likely is desirable. In other words, partial integration ($\mathbf{v_a}$) (resp. ($\mathbf{v_b}$)) is welfare improving when U_b (resp. U_a) benefits from relatively stronger lock-in effects and captures relatively more rents in the licensing stage. Given the results obtained in Proposition 3, private and social interests are aligned in that case. When innovations are highly differentiated however, welfare is higher when U_a is the unique successful innovator than when U_b innovates alone. Moreover, duopolistic ex post welfare is close to $W(q_a^m)$ when both innovate. As a consequence, a benevolent regulator would like to avoid partial integration ($\mathbf{v_b}$). Nevertheless, if U_b does not benefit from lock-in effects, it finds it profitable to integrate vertically with D_2. In that case, private and public objectives are not aligned.

4. Concluding remarks

Innovators often develop technologies that are not costlessly substitutable by adopting firms. We have shown that when licensees must incur switching costs to adopt different technologies, the prices of licenses vary with the size of the switching costs. Easily substitutable technologies can command only a low price, while innovators can benefit from a lock-in effect for technologies with high switching costs. This affects ex-ante private incentives to invest in R&D and, in particular, efficient technologies with low switching costs may disappear. In this framework, innovators and producers may find it profitable to integrate vertically, which is socially desirable if the likelihood that the most efficient standards survive increases. Overall, the presence of switching costs affects both incentives to embark in R&D and welfare.

We would like to conclude by pointing out two directions for future research. First, it could be of potential interest to analyze the incentives of firms to choose different lines of research leading to different standards. In other words, the

[16] They can be either smaller or larger than optimal efforts, depending on the parameters of the model.

question would be to know if there is a rationale for a new entrant to provide non easily substitutable innovations. Its benefit is clearly to avoid patent races, but its cost is the generation of switching costs. Naturally, the best strategy for the entrant is to select a technology such that firms that adopt it cannot switch to the current standard and firms that use the current standard can easily switch to the new one. Given that this may not be feasible, there is necessarily a trade-off between choosing a line of research generating low switching costs or a drastically different standard. Second, it would be interesting to determine the optimal contract between two integrated firms and whether one party should impose restrictions on future adoption of technologies developed by outside innovators.

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Appendix A. Proof of Lemma 1

(a) If only U_a (resp. U_b) is successful, U_a (resp. U_b) prefers to attract only one producer and extract all the surplus. Then D_1 (resp. D_2) produces alone and pays a transfer B_a^m (resp. B_b^m).

(b) Suppose both innovators are successful.

(b-1) Consider duopolistic situations.

– Participation and incentive compatibility imply:

[**a**, **a**] occurs if $t_{2a}^a \leqslant \min\{B_{aa}^d - c_{2a},\ B_{aa}^d - B_{ba}^d + t_{2b}^a - c_{2a}\}$ and $t_{1a}^a \leqslant \min\{B_{aa}^d,\ B_{aa}^d - B_{ba}^d + t_{1b}^a + c_{1b}\}$.

[**b**, **b**] occurs if $t_{1b}^b \leqslant \min\{B_{bb}^d - c_{1b},\ B_{bb}^d - B_{ab}^d + t_{1a}^b - c_{1b}\}$ and $t_{2b}^b \leqslant \min\{B_{bb}^d,\ B_{bb}^d - B_{ab}^d + t_{2a}^b + c_{2a}\}$.

[**b**, **a**] occurs if $t_{2a}^b \leqslant \min\{B_{ab}^d - c_{2a},\ B_{ab}^d - B_{bb}^d + t_{2b}^b - c_{2a}\}$ and $t_{1b}^a \leqslant \min\{B_{ba}^d - c_{1b}, B_{ba}^d - B_{aa}^d + t_{1a}^a - c_{1b}\}$.

(i) If $c_{2a} > B_{ab}^d$, D_2's dominant strategy is to adopt b: [**a**, **a**] and [**b**, **a**] cannot occur.

(ii) If $c_{1b} > B_{bb}^d$, D_1's dominant strategy is to adopt a: [**b**, **b**] and [**b**, **a**] cannot occur.

(iii) if $c_{1b} > B_{ba}^d$, D_1 adopts a if D_2 adopts b. Besides, [**b**, **a**] cannot occur.

– (IC)-preventive transfers.

(i) If $c_{2a} > B^d_{aa} - B^d_{ba}$, [**a**, **a**] can be prevented by setting $t^a_{2b} \in [0, \min\{B^d_{ba} - B^d_{aa} + c_{2a}, B^d_{ba}\}]$.

(ii) [**b**, **b**] can always be prevented by setting $t^b_{1a} \in [0, \min\{B^d_{ab} - B^d_{bb} + c_{1b}, B^d_{ab}\}]$.

– Profit-preventive transfers.

(i) If $c_{1b} > B^d_{ba}$ and $c_{2a} < B^d_{ab}$, the highest payoffs U_a can get from attracting both producers is $t^a_{1a} = B^d_{aa}$ (since D_1 does not switch to b when D_2 switches to a) and $t^a_{2a} = B^d_{aa} - B^d_{ba} - c_{2a} + t^a_{2b}$. U_a finds it profitable if $2B^d_{aa} - B^d_{ba} - c_{2a} + t^a_{2b} > t^b_{1a}$. If U_b sets $t^a_{2b} \leqslant t^a_{1a} - 2B^d_{aa} + B^d_{ba} + c_{2a}$ and if it is positive, U_a prefers not to attract both producers and [**a**, **a**] is prevented. Similarly, if U_a sets $t^b_{1a} \leqslant t^b_{2b} - 2B^d_{bb} + B^d_{ab} + c_{1b}$ (provided it is positive), [**b**, **b**] is prevented.

(ii) If $c_{1b} < B^d_{ba}$ and $c_{2a} < B^d_{ab}$, the highest payoffs U_a can get from attracting both producers is $t^a_{1a} = B^d_{aa} - B^d_{ba} + c_{1b} + t^a_{1b}$ (since D_1 can switch to b when D_2 switches to a) and $t^a_{2a} = B^d_{aa} - B^d_{ba} - c_{2a} + t^a_{2b}$. It finds it profitable if $2B^d_{aa} - 2B^d_{ba} - c_{2a} + c_{1b} + t^a_{2b} - t^a_{1b} > t^b_{1a}$. If U_b sets t^a_{2b} and t^a_{1b} such that $t^a_{2b} + t^a_{1b} \leqslant t^b_{1a} - 2B^d_{aa} + 2B^d_{ba} + c_{2a} - c_{1b}$, [**a**, **a**] is prevented.

(b-2) Consider monopolistic situations.

– Given the presence of switching costs:

(i) If $c_{1b} > B^d_{ba}$, D_1 cannot adopt b if D_2 adopts a then U_a can attract D_2 as a monopolist. Otherwise, [**o**, **a**] is not feasible.

(ii) If $c_{2a} > B^d_{ab}$, D_2 cannot adopt a if D_1 adopts b and U_b can attract D_1 as a monopolist. Otherwise, [**b**, **o**] is not feasible.

– To prevent U_a from attracting D_1 as a monopolist, U_b can set $t^a_{2b} \leqslant B^d_{ba}$. Similarly, to prevent U_b from attracting D_2 as a monopolist, U_a can set $t^b_{1a} \leqslant B^d_{ab}$.

– A necessary condition to get [**o**, **a**] in equilibrium is that D_1 cannot adopt b and is not transferred a. If D_2 adopts a, then D_1 cannot adopt b when $c_{1b} > B^d_{ba}$ and does adopt a if U_a sets t^a_{1a} such that $B^d_{aa} - t^a_{1a} < 0$. Similarly, [**b**, **o**] can occur if $c_{2a} > B^d_{ab}$ and if U_b sets $t^b_{2b} > B^d_{bb}$.

– (IC)-preventive transfers. Participation and incentive compatibility imply that [**o**, **a**] occurs if $B^m_a - t^o_{2a} - c_{2a} \geqslant \min\{0, B^m_b - t^o_{2b}\}$. However U_a always sets $t^b_{1a} \leqslant B^d_{ab}$ such that [**o**, **b**] cannot occur. As a consequence, the necessary condition for [**o**, **a**] to occur is $B^m_a - t^o_{2a} - c_{2a} \geqslant \min\{0, B^d_{ab} - t^a_{2b}\}$. The reasoning is similar for [**b**, **o**]. Overall, participation and incentive compatibility imply:

If $c_{1b} > B^d_{ba}$, [**o**, **a**] occurs if $t^o_{2a} \leqslant \min\{B^m_a - c_{2a}, B^m_a - B^d_{ba} + t^a_{2b} - c_{2a}\}$.

If $c_{2a} > B^d_{ab}$, [**b**, **o**] occurs if $t^o_{1b} \leqslant \min\{B^m_b - c_{1b}, B^m_b - B^d_{ab} + t^b_{1a} - c_{1b}\}$.

(i) If $c_{2a} > B^m_a - B^d_{ba}$, U_b can prevent U_a from attracting D_2 as a monopolist by setting a transfer $t^a_{2b} \in [0, \min\{B^d_{ba}, B^d_{ba} - B^m_a + c_{2a}\}]$. Otherwise, there exists t^o_{2a} positive satisfying both D_2's participation and incentive compatibility constraints;

(ii) If $c_{1b} > B^m_b - B^d_{ab}$, U_a can prevent U_b from attracting D_1 as a monopolist by setting a transfer $t^b_{1a} \in [0, \min\{B^d_{ab}, B^d_{ab} - B^m_b + c_{1b}\}]$. Otherwise, there exists t^o_{1b} positive satisfying both D_2's participation and incentive compatibility constraints.

– Profit-preventive transfers

(i) The highest payment U_a can obtain from D_2 in [**o**, **a**] (provided D_1 does not adopt b) is $t^o_{2a} = B^m_a - c_{2a} - B^d_{ba} + t^a_{2b}$. It finds it profitable if $t^o_{2a} \geqslant t^b_{1a}$. If U_b sets $t^a_{2b} = -B^m_a + c_{2a} + B^d_{ba} + t^b_{1a}$ (provided it is positive), then [**o**, **a**] cannot occur. Note that if $c_{2a} < B^m_a - B^d_{ba} - B^d_{ab}$, such transfer does not exist. Note also that the profit-preventive transfer is always higher than the (IC)-preventive transfer. Then, if $c_{2a} < B^m_a - B^d_{ba} - B^d_{ab}$, the only way for U_b to prevent [**o**, **a**] is to attract D_1 (this is possible only when $c_{1b} < B^d_{ba}$).

(ii) Similarly, when $c_{1b} > B^m_b - B^d_{ba} - B^d_{ab}$, U_a can always set a profit-preventive transfer such that $t^b_{1a} = -B^m_b + c_{1b} + B^d_{ab} + t^a_{2b}$ to avoid [**b**, **o**]. When $c_{1b} < B^m_b - B^d_{ba} - B^d_{ab}$, there exists no preventive transfer. □

Appendix B. Proof of Propositions 1 and 2

(a) Suppose that there is no vertical integration.

Case 1. $c_{1b} > B^d_{bb}$ and $c_{2a} > B^d_{ab}$. The only possible situations are [**a**, **b**], [**b**, **o**] and [**o**, **a**].

– If $c_{1b} > B^m_b - B^d_{ba}$ and $c_{2a} > B^m_a - B^d_{ab}$, innovators do not have incentives to attract producers as monopolists. Both firms prefer to play [**a**, **b**] and to fix $t^b_{1a} = B^d_{ab}$ and $t^a_{2b} = B^d_{ba}$.

– If $c_{1b} < B^m_b - B^d_{ba}$ and $c_{2a} > B^m_a - B^d_{ab}$, U_b has incentives to attract D_1 as a monopolist. The maximum license fee it can charge is $t^o_{1b} = B^m_b - c_{1b} - B^d_{ab} + t^b_{1a}$. If $c_{1b} \geqslant B^m_b - B^d_{ab} - B^d_{ba}$, U_a fixes $t^b_{1a} = -B^m_b + c_{1b} + B^d_{ab} + t^a_{2b} > 0$, then [**b**, **o**] is prevented. The best U_b can do is to fix $t^a_{2b} = B^d_{ba}$. The equilibrium is [**a**, **b**] with $t^b_{1a} = -B^m_b + c_{1b} + B^d_{ab} + B^d_{ba}$ and $t^a_{2b} = B^d_{ba}$. By contrast, if $c_{1b} \geqslant B^m_b - B^d_{ab} - B^d_{ba}$, the best U_a can do is fix $t^b_{1a} = 0$ but this does not prevent D_1 from adopting b. Since D_2 cannot adopt a, the equilibrium is [**b**, **o**] with $t^o_{1b} = B^m_b - c_{1b} - B^d_{ab}$ (naturally, U_b sets $t^b_{2b} > B^d_{bb}$ so that D_2 cannot produce). This second equilibrium disappears if technologies are highly differentiated.

– If $c_{1b} > B^m_b - B^d_{ba}$ and $c_{2a} < B^m_a - B^d_{ab}$, the equilibrium is [**a**, **b**] with $t^b_{1a} = B^d_{ab}$, and $t^a_{2b} = -B^m_a + c_{2a} + B^d_{ab} + B^d_{ba}$ as long as $c_{2a} \leqslant B^m_a - B^d_{ba} - B^d_{ab}$. Otherwise, the equilibrium is [**o**, **a**] and $t^o_{2a} = B^m_a - c_{2a} - B^d_{ba} > B^d_{ab}$.

– If $c_{1b} < B^m_b - B^d_{ba}$ and $c_{2a} < B^m_a - B^d_{ab}$, both innovators want to attract producers as monopolists. To prevent [**o**, **a**], U_b sets $t^a_{2b} = -B^m_a + c_{2a} + B^d_{ab} + B^d_{ba}$. To prevent [**b**, **o**], U_a sets $t^b_{1a} = -B^m_b + c_{1b} + B^d_{ab} + B^d_{ba}$. Given these transfers (provided they are positive), D_1 adopts a and D_2 adopts b.

Case 2. $c_{1b} \in [B^d_{ba}, B^d_{bb}]$ and $c_{2a} > B_{ab}$. The only possible situations are [**a**, **b**], [**b**, **o**], [**o**, **a**] and [**b**, **b**]. Since $B^m_b - c_{1b} > B^d_{ba}$ and $2B^d_{bb} - c_{1b} > B^d_{ba}$, U_b wants to attract D_1. To attract D_1 as a monopolist, it must fix $t^o_{1b} = B^m_b - c_{1b} - B^d_{ab} + t^b_{1a}$. To attract both producers, U_b fixes $t^b_{2b} = B^d_{bb}$ (D_2 can be left without rents since it cannot accept an offer from U_a conditional on D_1 accepting an offer from U_b) and $t^b_{1b} = B^d_{bb} - B^d_{ab} - c_{1b} + t^b_{1a}$. Since $t^o_{1b} > t^b_{1b} + t^b_{2b}$, preventing [**b**, **o**] is sufficient to prevent also [**b**, **b**].

– If $c_{2a} > B^m_a - B^d_{ab}$, the equilibrium is [**a**, **b**] with $t^b_{1a} = -B^m_b + c_{1b} + B^d_{ab} + B^d_{ba}$ and $t^a_{2b} = B^d_{ba}$ as long as $c_{1b} \geqslant B^m_b - B^d_{ab} - B^d_{ba}$. Otherwise, it is [**b**, **o**].

– If $c_{2a} < B^m_a - B^d_{ab}$, the equilibrium is [**a**, **b**] with $t^a_{2b} = -B^m_a + c_{2a} + B^d_{ab} + B^d_{ba}$ and $t^b_{1a} = -B^m_b + c_{1b} + B^d_{ab} + B^d_{ba}$ as long as the transfers are positive.

Case 3. $c_{1b} < B^d_{ba}$ and $c_{2a} > B^d_{ab}$. The only possible situations are [**a**, **b**], [**b**, **o**] and [**b**, **b**]. U_b has always incentives to attract D_1. Using the same reasoning as before, the equilibrium is [**a**, **b**] with $t^b_{1a} = -B^m_b + c_{1b} + B^d_{ab} + B^d_{ba}$ and $t^a_{2b} = B^d_{ba}$ as long as $c_{1b} \leqslant B^m_b - B^d_{ab} - B^d_{ba}$.

Case 4. $c_{1b} > B^d_{bb}$ and $c_{2a} < B^d_{ab}$. We have three possible scenarios: [**a**, **b**], [**a**, **a**] and [**o**, **a**].

– If $c_{2a} < B^m_a - B^d_{ab}$, the equilibrium is [**a**, **b**] with $t^a_{2b} = -B^m_a + c_{2a} + B^d_{ab} + B^d_{ba}$ and $t^b_{1a} = B^d_{ab}$ as long as $c_{1b} \leqslant B^m_b - B^d_{ab} - B^d_{ba}$. Otherwise, U_a attracts D_2 as a monopolist.

– If $c_{2a} > B^m_a - B^d_{ab}$, the equilibrium is [**a**, **b**] with $t^a_{2b} = B^d_{ba}$ and $t^b_{1a} = B^d_{ab}$.

Case 5. $c_{1b} \in [B^d_{ba}, B^d_{bb}]$ and $c_{2a} < B^d_{ab}$. The possible scenarios are [**a**, **b**], [**o**, **a**], [**b**, **b**] and [**a**, **a**]. U_b has incentives to attract both producers. He can charge $t^b_{2b} = B^d_{bb}$ (since D_2 cannot switch if D_1 adopts a) and $t^b_{1b} = B^d_{bb} - B^d_{ab} + t^b_{1a} - c_{1b}$. If U_a sets the (IC)-preventive transfer $t^b_{1a} = B^d_{ab} - B^d_{bb} + c_{1b}$, or the profit-preventive transfer $t^b_{1a} = B^d_{ab} - B^d_{bb} + c_{1b} + t^a_{2b} - B^d_{bb}$. Since the (IC)-preventive transfer is always higher, U_a sets $t^b_{1a} = B^d_{ab} - B^d_{bb} + c_{1b}$ to prevent [**b**, **b**] and D_1 adopts a.

If U_a attracts both producers, it can charge $t^a_{1a} = B^d_{aa}$ and $t^a_{2a} = B^d_{aa} - B^d_{ba} - c_{2a} + t^a_{2b}$. If not, it gets $t^b_{1a} = B^d_{ab} - B^d_{bb} + c_{1b}$. To prevent [**a**, **a**], U_b charges a preventive transfer (if any): $t^a_{2b} = \max(\min(B^d_{ba} - B^d_{aa} + c_{2a},\ B^d_{ba}),\ \min(B^d_{ab} - B^d_{bb} + c^d_{1b} - 2B^d_{aa} + B^d_{ba} + c_{2a},\ B^d_{ba}))$

– If $c_{2a} > B^d_{aa}$, then $t^a_{2b} = B^d_{ba}$.

– Suppose $c_{2a} \in [B^d_{aa} - B^d_{ba}, B^d_{aa}]$ and consider the function $f(c_{2a}) = -c_{2a} - B^d_{ab} + 2B^d_{aa} + B^d_{bb}$. If $c_{1b} > f(c_{2a})$, then $t^a_{2b} = B^d_{ba}$. If $c_{1b} < f(c_{2a})$ and $c_{1b} > B^d_{aa} + B^d_{bb} - B^d_{ab}$, then $t^a_{2b} = c_{2a} - 2B^d_{aa} + B^d_{ba} + c_{1b} - B^d_{bb} + B^d_{ab}$. Otherwise $t^a_{2b} = c_{2a} - B^d_{aa} + B^d_{ba}$.

– Suppose $c_{2a} < B^d_{aa} - B^d_{ba}$. If $c_{1b} > f(c_{2a})$, then $t^a_{2b} = B^d_{ba}$. By contrast, if $c_{1b} < f(c_{2a})$, then $t^a_{2b} = c_{2a} - 2B^d_{aa} + B^d_{ba} + c_{1b} - B^d_{bb} + B^d_{ab}$ as long as it is positive. Let $g(c_{2a}) = -c_{2a} - B^d_{ab} + 2B^d_{aa} + B^d_{bb} - B^d_{ba}$. If $c_{1b} > g(c_{2a})$, the transfer is positive and the equilibrium is [**a**, **b**]. If $c_{1b} < g(c_{2a})$, the equilibrium is [**a**, **a**] with $t^a_{1a} = B^d_{aa}$ and $t^a_{2a} = B^d_{aa} - B^d_{ba} - c_{2a}$ and naturally $t^a_{1a} + t^a_{2a} > c_{1b} + B^d_{ab} - B^d_{bb}$.

Case 6. $c_{1b} < B^d_{ba}$ and $c_{2a} < B^d_{ab}$. We have four possible scenarios: [**a**, **b**], [**a**, **a**], [**b**, **b**] and [**b**, **a**]. Moreover, U_b has incentives to attract both innovators.

* To get [**a**, **b**] in equilibrium, innovators must set transfers such that $B^d_{ab} - t^b_{1a} \geqslant B^d_{ba} - c_{1b} - t^a_{1b}$ and $B^d_{ba} - t^a_{2b} \geqslant B^d_{ab} - c_{2a} - t^b_{2a}$. Moreover, to prevent [**a**, **a**] and [**b**, **b**], innovators must set (IC) or profit-preventive transfers. Overall, transfers must be such that:

$$\begin{aligned} t^b_{1a} = \min(&B^d_{ab} - B^d_{ba} + c_{1b} \\ &+ t^a_{1b}, \max(B^d_{ab} - B^d_{bb} + c_{1b}, \min(t^a_{2b} - 2B^d_{bb} + 2B^d_{ab} + c_{1b} - c_{2a} \\ &- t^b_{2a}, B^d_{ab}))) \end{aligned}$$

$$\begin{aligned} t^a_{2b} = \min(&B^d_{ba} - B^d_{ab} + c_{2a} \\ &+ t^b_{2a}, \max(B^d_{ba} - B^d_{aa} + t^a_{2a} + c_{2a}, \min(t^b_{1a} - 2B^d_{aa} + 2B^d_{ba} + c_{2a} - c_{1b} \\ &- t^a_{1b}, B^d_{ba})) \end{aligned}$$

* To get [**b**, **a**] in equilibrium, the two constraints $B^d_{ab} - t^b_{1a} < B^d_{ba} - c_{1b} - t^a_{1b}$ and $B^d_{ba} - t^a_{2b} < B^d_{ab} - c_{2a} - t^b_{2a}$ must be satisfied. Moreover, innovators must set (IC)- or profit-preventive transfers. Overall,

$$\begin{aligned} t^b_{2a} = \min(&B^d_{ab} - B^d_{ba} - c_{2a} \\ &+ t^a_{2b}, \max(B^d_{ab} - B^d_{bb} + t^b_{2b} - c_{2a}, \min(t^a_{1b} - 2B^d_{bb} + 2B^d_{ab} + c_{1b} - c_{2a} \\ &- t^b_{1a}, B^d_{ab} - c_{2a})) \end{aligned}$$

$$\begin{aligned} t^a_{1b} = \min(&B^d_{ba} - B^d_{ab} - c_{1b} \\ &+ t^b_{1a}, \max(B^d_{ba} - B^d_{aa} + t^a_{1a} - c_{1b}, \min(t^b_{2a} - 2B^d_{aa} + 2B^d_{ba} + c_{2a} - c_{1b} \\ &- t^a_{2b}, B^d_{ba} - c_{1b})) \end{aligned}$$

– Suppose $c_{2a} \in [B^d_{aa} - B^d_{ba}, B^d_{ab}]$.

The equilibrium is [**a**, **b**], with $t^a_{1b} = 0$ and $t^b_{1a} = c_{1b} + B^d_{ab} - B^d_{bb}$. If $c_{2a} > B^d_{aa}$, then $t^a_{2b} = B^d_{ba}$ and $t^b_{2a} = B^d_{ab} - c_{2a}$. If $c_{2a} < B^d_{aa}$ and $B^d_{aa} + B^d_{bb} - B^d_{ab} - B^d_{ba} > 0$, then $t^a_{2b} = c_{2a} + B^d_{ba} - B^d_{aa}$ and $t^b_{2a} \in [B^d_{ab} - B^d_{aa}, B^d_{ab} - c_{2a}]$. Last, if $c_{2a} < B^d_{aa}$ and $B^d_{aa} + B^d_{bb} - B^d_{ab} - B^d_{ba} < 0$, then $t^a_{2b} = \min(B^d_{ba}, c_{2a} + 2B^d_{ba} - 2B^d_{aa} + B^d_{ab} - B^d_{bb}) >$

0. Both innovators get smaller payoffs in any equilibrium of type [**b**, **a**].[17]

– Suppose $c_{2a} < B^d_{aa} - B^d_{ba}$. This case is similar to the previous one except that now U_b cannot set a preventive transfer to prevent U_a from attracting both producers.

If $B^d_{aa} + B^d_{bb} - B^d_{ab} - B^d_{ba} > 0$, then $t^{ab}_{2b} = c_{2a} + B^d_{ba} - B^d_{aa} < 0$ and the equilibrium is [**a**, **a**]. If $B^d_{aa} + B^d_{bb} - B^d_{ab} - B^d_{ba} < 0$, then $t^{b}_{1a} = c_{1b} + B^d_{ab} - B^d_{ab}$ and $t^{ab}_{2b} = \min(B^d_{ba}, c_{2a} + 2B^d_{ba} - 2B^d_{aa} + B^d_{ab} - B^d_{bb}) > 0$ if $c_{2a} > 2B^d_{aa} - B^d_{ab} - 2B^d_{ba} + B^d_{bb}$, in which case the equilibrium is [**a**, **b**]. Again, both innovators get smaller payoffs in any equilibrium of type [**b**, **a**].

In the rest of the proof, we assume that $B^d_{ab} < B^m_a - B^d_{ab}$ and $B^d_{bb} \geqslant B^m_b - B^d_{ab} - B^d_{ba}$.[18] We can summarize the previous results as follows:

The equilibrium is [**a**, **b**] in Regions:

(**1**) $c_{1b} > B^m_b - B^d_{ba}$ and $c_{2a} > B^m_a - B^d_{ab}$: $P^{v_n}_a = B^d_{ab}$;
$P^{v_n}_1 = 0; \quad P^{v_n}_b = B^d_{ba}; \quad P^{v_n}_2 = 0.$

(**2**) $c_{1b} > B^m_b - B^d_{ba}$ and $c_{2a} \in [B^m_a - B^d_{ab} - B^d_{ba}, B^m_a - B^d_{ab}]$;

(**3**) $c_{1b} \in [B^d_{bb}, B^m_b - B^d_{ba}]$ and $c_{2a} \in [B^m_a - B^d_{ab} - B^d_{ba}, B^d_{ab}]$:
$P^{v_n}_a = B^d_{ab}; \quad P^{v_n}_1 = 0; \quad P^{v_n}_b = c_{2a} + B^d_{ba} + B^d_{ab} - B^m_a; \quad P^{v_n}_2 = B^m_a - B^d_{ab} - c_{2a}.$

(**4**) $c_{1b} \in [B^m_b - B^d_{ab} - B^d_{ba}, B^m_b - B^d_{ba}]$ and $c_{2a} > B^m_a - B^d_{ab}$:

(**5**) $c_{1b} \in [B^m_b - B^d_{ba} - B^d_{ab}, B^d_{bb}]$ and $c_{2a} \in [B^d_{ab}, B^m_a - B^d_{ab}]$:
$P^{v_n}_a = c_{1b} + B^d_{ba} + B^d_{ab} - B^m_b; \quad P^{v_n}_1 = B^m_b - B^d_{ba} \quad c_{1b}; \quad P^{v_n}_b = B^d_{ba}; \quad P^{v_n}_2 = 0.$

(**6**) $c_{1b} \in [B^d_{bb}, B^m_b - B^d_{ba}]$ and $c_{2a} \in [B^d_{ab}, B^m_a - B^d_{ab}]$:
$P^{v_n}_a = c_{1b} + B^d_{ba} + B^d_{ab} - B^m_b; \quad P^{v_n}_1 = B^m_b - B^d_{ba} - c_{1b};$
$P^{v_n}_b = c_{2a} + B^d_{ba} + B^d_{ab} - B^m_a; \quad P^{v_n}_2 = B^m_a - B^d_{ab} - c_{2a}.$

[17] Formally, if $B^d_{aa} + B^d_{bb} - B^d_{ab} - B^d_{ba} > 0$, then $t^{ba}_{2a} = B^d_{ab} - c_{2a} - 2B^d_{bb} + B^d_{ba}$ and $t^{ba}_{1b} = B_{ba} - c_{1b} - 2B^d_{bb} - 2B^d_{aa} + 2B^d_{ba} + B^d_{ab}$. If $B^d_{aa} + B^d_{bb} - B^d_{ab} - B^d_{ba} < 0$ and $c_{2a} > B^d_{ab} - B^d_{bb}$, then $t^{ba}_{2a} = B^d_{ab} - c_{2a} - 2B^d_{bb} + B^d_{ba}$ and $t^{ba}_{1b} = \min(B_{ba} - c_{1b} - 2B^d_{bb} - 2B^d_{aa} + 2B^d_{ba} + B^d_{ab}, B^d_{ba} - c_{1b})$. Last, if $B^d_{aa} + B^d_{bb} - B^d_{ab} - B^d_{ba} < 0$ and $c_{2a} < B^d_{ab} - B^d_{bb}$, we have $t^{ba}_{2a} = B^d_{ab} - c_{2a} - B^d_{bb}$ and $t^{ba}_{1b} = \min(B^d_{ab} - c_{1b} - B^d_{bb} - 2B^d_{aa} + 2B^d_{ba} - c_{1b}, B^d_{ba} - c_{1b})$.

[18] This allows us to characterize all the possible cases. Indeed, some of them disappear when either $B^d_{ab} > B^m_a - B^d_{ab}$ or $B^d_{bb} \leqslant B^m_b - B^d_{ab} - B^d_{ba}$.

(7) $c_{1b} \in [\max(B^d_{ba}, B^d_{bb} + B^d_{aa} - B^d_{ab}), B^d_{bb}]$ and $c_{1b} > f(c_{2a})$ and $c_{2a} \in [2B^d_{aa} - B^d_{ab}, B^d_{ab}]$, or

$c_{1b} < \max(B^d_{ba}, B^d_{bb} + B^d_{aa} - B^d_{ab})]$ and $c_{2a} \in [\min(2B^d_{aa} + B^d_{bb} - B^d_{ab} - B^d_{ba}, B^d_{aa}), B^d_{ab}]$:

$$P_a^{v_n} = c_{1b} + B_{ab} - B_{bb}; \quad P_1^{v_n} = B^d_{bb} - c_{1b}; \quad P_b^{v_n} = B^d_{ba}; \quad P_2^{v_n} = 0.$$

(8) $c_{1b} \in [\max(B^d_{ba}, B^d_{bb} + B^d_{aa} - B^d_{ab}), B^d_{bb}]$ and $c_{1b} \in [g(c_{2a}), f(c_{2a})]$ and $c_{2a} \in [2B^d_{aa} - B^d_{ab} - B^d_{ba}, \min(2B^d_{aa} + B^d_{bb} - B^d_{ab} - B^d_{ba}, B^d_{aa})]$:

$$P_a^{v_n} = c_{1b} + B_{ab} - B_{bb}; \quad P_1^{v_n} = B^d_{bb} - c_{1b};$$

$$P_b^{v_n} = c_{1b} + B_{ab} - B_{bb} - 2B^d_{aa} + c_{2a} + B^d_{ba}; \quad P_2^{v_n} = 0.$$

(9) $c_{1b} < \max(B^d_{ba}, B^d_{bb} + B^d_{aa} - B^d_{ab})$ and $c_{2a} \in [\min(2B^d_{aa} + B^d_{bb} - B^d_{ab} - 2B^d_{ba}, B^d_{aa} - B^d_{ba}), \min(2B^d_{aa} + B^d_{bb} - B^d_{ab} - B^d_{ba}, B^d_{aa})]$:

$$P_a^{v_n} = c_{1b} + B_{ab} - B_{bb}; \quad P_1^{v_n} = B^d_{bb} - c_{1b}$$

$$P_b^{v_n} = c_{2a} + B^d_{ba} - 2B^d_{aa} - B^d_{bb} + B^d_{ab} + g(\min(2B^d_{aa} + B^d_{bb} - B^d_{ab} - 2B^d_{ba}, B^d_{aa} - B^d_{ba})).$$

The equilibrium is [**o**, **a**] in Regions:

(10) $c_{1b} > B^m_b - B^d_{ba}$ and $c_{2a} < B^m_a - B^d_{ab} - B^d_{ba}$:

(11) $c_{1b} \in [B^d_{bb}, B^m_b - B^d_{ba}]$ and $c_{2a} < B^m_a - B^d_{ab} - B^d_{ba}$:

$$P_a^{v_n} = B^m_a - c_{2a} - B^d_{ba} \quad P_1^{v_n} = 0; \quad P_b^{v_n} = 0 \quad P_2^{v_n} = B^d_{ba}.$$

Note that in that case, U_a offers also $t^a_{1a} > B^d_{aa}$ to D_1 to prevent it from adopting a.

The equilibrium is [**b**, **o**] in Regions:

(12) $c_{1b} < B^m_b - B^d_{ba} - B^d_{ab}$ and $c_{2a} > B^m_a - B^d_{ab}$:

(13) $c_{1b} < B^m_b - B^d_{ba} - B^m_{ab}$ and $c_{2a} \in [B^d_{ab}, B^m_a - B^d_{ab}]$:

$$P_a^{v_n} = 0; \quad P_1^{v_n} = B^d_{ab}; \quad P_b^{v_n} = B^m_b - c_{1b} - B^d_{ab}; \quad P_2^{v_n} = 0.$$

Again, U_b offers also $t^b_{2b} > B^d_{bb}$ to D_2 to prevent it from adopting b.

The equilibrium is [**a**, **a**] in Region:

(**14**) $c_{1b} < \min(B^d_{bb}; g(c_{2a}))$ and $c_{2a} < \min(2B^d_{aa} + B^d_{bb} - B^d_{ab} - 2B^d_{ba}, B^d_{aa} - B^d_{ba})$:

$P^{v_n}_a = 2B^d_{aa} - B^d_{ba} - c_{2a} + \min(c_{1b} - B^d_{ba}, 0)$; $\quad P^{v_n}_1 = -\min(c_{1b} - B^d_{ba}, 0)$;

$P^{v_n}_b = 0$; $\quad P^{v_n}_2 = B^d_{ba}$.

(b) Suppose that U_a and D_1 integrate vertically.

Cases 1–3. The only possible scenarios are [**a**, **b**] and [**o**, **a**]. If $c_{2a} > B^m_a - B^d_{ab}$, I_a produces and gets B^d_{ab}. Then, U_b fixes $t^a_{2b} = B^d_{ba}$. If $c_{2a} < B^m_a - B^d_{ab}$, then I_a wants to attract D_2. To prevent it, U_b fixes $t^a_{2b} = -B^m_a + c_{2a} + B^d_{ab} + B^d_{ba}$ as long as it is positive.

Cases 4–6. The only possible scenarios are [**a**, **b**], [**o**, **a**] and [**a**, **a**]. If $c_{2a} < B^m_a - B^d_{ab}$, then I_a wants to attract D_2 but U_b fixes $t^a_{2b} = -B^m_a + c_{2a} + B^d_{ab} + B^d_{ba}$.

Overall, under ($\mathbf{v_a}$), the equilibrium is [**a**, **b**] when $c_{2a} > B^m_a - B^d_{ab} - B^d_{ba}$. In that case $P^{v_a}_a = B^d_{ab}$ and $P^{v_a}_b = \min(B^d_{ba}, -B^m_a + c_{2a} + B^d_{ab} + B^d_{ba})$. Note that $P^{v_a}_b = B^d_{ba}$ only in Regions (**1**), (**2**) and (**10**). Otherwise, the equilibrium is [**o**, **a**] with $P^{v_a}_a = B^m_a - c_{2a} - B^d_{ba}$ and $P^{v_a}_b = 0$.

(c) Suppose that U_b and D_2 integrate. The equilibrium is [**a**, **b**] when $c_{1b} > B^m_b - B^d_{ab} - B^d_{ba}$ with $t^a_{2b} = B^d_{ba}$ and $t^b_{1a} = \min(B^d_{ab}, -B^m_b + c_{1b} + B^d_{ab} + B^d_{ba})$. Then $t^a_{1b} = B^d_{ab}$ only in regions (**1**), (**4**) and (**12**). Otherwise, the equilibrium is [**b**, **o**] with $t^o_{1b} = B^m_b - c_{1b} - B^d_{ab}$. As under ($\mathbf{v_n}$), this last equilibrium disappears for highly differentiated technologies.

Overall, $P^h_a(c_{1b}, c_{2a}; a, b) \leqslant P^h_a(c_{1b}, c_{2a}; a) = B^m_a$ and $P^h_b(c_{1b}, c_{2a}; a, b) \leqslant P^h_b(c_{1b}, c_{2a}; b) = B^m_b$. Moreover, $\partial P^h_a(c_{1b}, c_{2a}; a, b)/\partial c_{1b} \geqslant 0$ for all $h \in \{v_n, v_b\}$ and $\partial P^h_a(c_{1b}, c_{2a}; a, b)/\partial c_{2a} \leqslant 0$ for all $h \in \{v_n, v_a\}$. Both derivatives are zero otherwise. Besides $\partial P^h_b(c_{1b}, c_{2a}; a, b)/\partial c_{2a} \geqslant 0$ for all $h \in \{v_n, v_a\}$ and $\partial P^h_b(c_{1b}, c_{2a}; a, b)/\partial c_{1b} \geqslant 0$ for $h = v_b$. When $h = v_n$, $\partial P^h_b(c_{1b}, c_{2a}; a, b)/\partial c_{1b} \geqslant 0$ in region (**8**). Otherwise, $\partial P^h_b(c_{1b}, c_{2a}; a, b)/\partial c_{1b} \leqslant 0$. Lastly, whenever a derivative is strictly negative, it is equal to -1 and whenever it is strictly positive, it is equal to 1. $\square$

Appendix C. Proof of Lemma 2

Note that $u^h_a(e_a, e_b)$ is concave in e_a and decreasing in e_b. The first order condition is:

$$\pi'(e_a)[\pi(e_b)P^h_a(c_{1b}, c_{2a}; a, b) + [1 - \pi(e_b)]B^m_a] = 1 \tag{C.1}$$

For all e_b selected by U_b, U_a's optimal effort is then a function of e_b, c_{1b} and c_{2a}.

We call it $\tilde{e}_a^h(e_b, c_{1b}, c_{2a})$. We will sometimes write it $\tilde{e}_a^h$ to simplify the notations. Differentiating (C.1) with respect to e_b, we get:

$$\pi''(\tilde{e}_a^h)[\pi(e_b)P_a^h(c_{1b}, c_{2a}; a, b) + [1 - \pi(e_b)]B_a^m] \times \frac{\partial \tilde{e}_a^h}{\partial e_b} + \pi'(\tilde{e}_a^h)\pi'(e_b)[P_a^h(c_{1b}, c_{2a}; a, b) - B_a^m] = 0.$$

Then $\tilde{e}_a^h(e_b, c_{1b}, c_{2a})$ is decreasing in e_b. Similarly, the optimal effort selected by innovator U_b is $\tilde{e}_b^h(e_a, c_{1b}, c_{2a})$ and is decreasing in e_a. The equilibrium is such that $\tilde{e}_b^h(e_a, c_{1b}, c_{2a}) = \tilde{e}_a^{h^{-1}}(e_a, c_{1b}, c_{2a})$ where $\tilde{e}_a^{h^{-1}}(e_a, c_{1b}, c_{2a})$ is the inverse function of $\tilde{e}_a^h(e_b, c_{1b}, c_{2a})$. A sufficient condition for uniqueness is $|\partial \tilde{e}_a^h / \partial e_b| < 1$, i.e.[19] $|(\partial / \partial e_a \partial e_b) u_a^h(e_a, e_b)| < |(\partial / \partial e_a \partial e_a) u_a^h(e_a, e_b)|$. Note that

$$\left| \frac{\partial}{\partial e_a\, \partial e_a} u_a^h(e_a, e_b) \right| = -\pi''(e_a)[\pi(e_b)\, P_a^h(c_{1b}, c_{2a}; a, b) + (1 - \pi(e_b))B_a^m] = A(e_b)$$

$$\left| \frac{\partial}{\partial e_a\, \partial e_b} u_a^h(e_a, e_b) \right| = \pi'(e_a)\pi'(e_b)\,[B_a^m - P_a^h(c_{1b}, c_{2a}; a, b)] = B(e_b)$$

$A(e_b)$ and $B(e_b)$ decrease in e_b and the equilibrium is unique if $B(0) < A(1)$, i.e. if $-\pi''(e_a)/\pi'(e_a) \geqslant \pi'(0)\big[B_a^m - P_a^h(c_{1b}, c_{2a}; a, b)/P_a^h(c_{1b}, c_{2a}; a, b)\big]$ which leads to Assumption 3.

Differentiating (C.1) with respect to c_{1b}, we get:

$$\pi''(\tilde{e}_a^h)[\pi(e_b)P_a^h(c_{1b}, c_{2a}; a, b) + [1 - \pi(e_b)]B_a^m] \times \frac{\partial \tilde{e}_a^h}{\partial c_{1b}} + \pi'(\tilde{e}_a^h)\, \pi(e_b) \frac{\partial P_a^h(c_{1b}, c_{2a}; a, b)}{\partial c_{1b}} = 0.$$

Then $\tilde{e}_a^h(e_b, c_{1b}, c_{2a})$ is non increasing in c_{1b}. Moreover $\tilde{e}_b^h(e_b, c_{1b}, c_{2a})$ is increasing in c_{2a}. Differentiating (C.1) with respect to c_{2a}, we have:

$$\pi''(\tilde{e}_a^h)[\pi(e_b)P_a^h(c_{1b}, c_{2a}; a, b) + [1 - \pi(e_b)]B_a^m] \times \frac{\partial \tilde{e}_a^h}{\partial c_{1b}} - \pi'(\tilde{e}_a^h)\pi(e_b) \frac{\partial P_a^h(c_{1b}, c_{2a}; a, b)}{\partial c_{2a}} = 0.$$

$\tilde{e}_a^h(e_b, c_{1b}, c_{2a})$ is decreasing in c_{2a} and $\tilde{e}_b^h(e_b, c_{1b}, c_{2a})$ is decreasing in c_{1b} except in Region (**8**) when $h = v_n$. Denote by $e_a^h(c_{1b}, c_{2a})$ (resp. $e_b^h(c_{1b}, c_{2a})$) U_a's (resp. U_b's) optimal effort under h. Note that $\tilde{e}_b^h(e_a^h(c_{1b}, c_{2a}), c_{1b}, c_{2a}) = e_b^h(c_{1b},$

[19] See Tirole (1988), chapter 5.

$c_{2a})$. Similarly $\tilde{e}_a^h(e_b^h(c_{1b}, c_{2a}), c_{1b}, c_{2a}) = e_a^h(c_{1b}, c_{2a})$. Differentiating those expressions with respect to c_{1b} and c_{2a} respectively:

$$\frac{\partial e_b^h}{\partial c_{1b}}\left[1-\frac{\partial \tilde{e}_b^h}{\partial e_a}\bigg|_{e_a^h}\frac{\partial \tilde{e}_a^h}{\partial e_b}\bigg|_{e_b^h}\right]=\frac{\partial \tilde{e}_b^h}{\partial e_a}\bigg|_{e_a^h}\frac{\partial \tilde{e}_a^h}{\partial c_{1b}}+\frac{\partial \tilde{e}_b^h}{\partial c_{1b}}$$

$$\frac{\partial e_a^h}{\partial c_{1b}}\left[1-\frac{\partial \tilde{e}_b^h}{\partial e_a}\bigg|_{e_a^h}\frac{\partial \tilde{e}_a^h}{\partial e_b}\bigg|_{e_b^h}\right]=\frac{\partial \tilde{e}_a^h}{\partial e_a}\bigg|_{e_a^h}\frac{\partial \tilde{e}_b^h}{\partial c_{1b}}+\frac{\partial \tilde{e}_a^h}{\partial c_{1b}}$$

$$\frac{\partial e_b^h}{\partial c_{2a}}\left[1-\frac{\partial \tilde{e}_b^h}{\partial e_a}\bigg|_{e_a^h}\frac{\partial \tilde{e}_a^h}{\partial e_b}\bigg|_{e_b^h}\right]=\frac{\partial \tilde{e}_b^h}{\partial e_a}\bigg|_{e_a^h}\frac{\partial \tilde{e}_a^h}{\partial c_{2a}}+\frac{\partial \tilde{e}_b^h}{\partial c_{2a}}$$

$$\frac{\partial e_a^h}{\partial c_{2a}}\left[1-\frac{\partial \tilde{e}_b^h}{\partial e_a}\bigg|_{e_a^h}\frac{\partial \tilde{e}_a^h}{\partial e_b}\bigg|_{e_b^h}\right]=\frac{\partial \tilde{e}_a^h}{\partial e_a}\bigg|_{e_a^h}\frac{\partial \tilde{e}_b^h}{\partial c_{2a}}+\frac{\partial \tilde{e}_a^h}{\partial c_{2a}}$$

(**1**): $e_a^{v_n} = e_a^{v_a} = e_a^{v_b} = e_a^{v_f}$; $e_b^{v_n} = e_b^{v_a} = e_b^{v_b} = e_b^{v_f}$

(**2**)–(**10**): $e_a^{v_n}$ is decreasing in c_{2a}, $e_b^{v_n}$ is increasing in c_{2a}:
$e_a^{v_n}(c_{2a}) = e_a^{v_a}(c_{2a}) > e_a^{v_b} = e_a^{v_f}$; $e_b^{v_n}(c_{2a}) = e_b^{v_a}(c_{2a}) < e_b^{v_b} = e_b^{v_f}$.

(**4**)–(**12**): $e_a^{v_n}$ is increasing in c_{1b}, $e_b^{v_n}$ is decreasing in c_{1b}:
$e_a^{v_n}(c_{1b}) = e_a^{v_b}(c_{1b}) < e_a^{v_a} = e_a^{v_f}$; $e_b^{v_n}(c_{1b}) = e_b^{v_b}(c_{1b}) > e_b^{v_a} = e_b^{v_f}$.

(**6**): $e_a^{v_n}$ is increasing in c_{1b} and decreasing in c_{2a}, $e_b^{v_n}$ is decreasing in c_{1b} and-increasing in c_{2a}:

$$e_a^{v_n}(c_{1b}) = e_a^{v_b}(c_{1b}) < e_a^{v_a}(c_{2a}); \quad e_b^{v_n}(c_{1b}) = e_b^{v_b}(c_{1b}) > e_b^{v_a}(c_{2a}).$$

(**3**)–(**11**): $e_a^{v_n}$ is decreasing in c_{2a}, $e_b^{v_n}$ is increasing in c_{2a}:
$e_a^{v_n}(c_{2a}) = e_a^{v_a}(c_{2a}) > e_a^{v_a}(c_{1b})$; $e_b^{v_n}(c_{2a}) = e_b^{v_a}(c_{2a}) < e_b^{v_b}(c_{1b})$.

(**5**)–(**13**): $e_a^{v_n}$ is increasing in c_{1b}, $e_b^{v_n}$ is decreasing in c_{1b}:
$e_a^{v_n}(c_{1b}) = e_a^{v_b}(c_{1b}) < e_a^{v_a}(c_{2a})$; $e_b^{v_n}(c_{1b}) = e_b^{v_b}(c_{1b}) > e_b^{v_a}(c_{2a})$.

(**7**)–(**8**)–(**9**)–(**14**): $e_a^{v_a}(c_{2a})$ is decreasing in c_{2a}, $e_b^{v_a}(c_{2a})$ is increasing in c_{2a}.

Moreover, $e_a^{v_b}(c_{1b})$ is increasing in c_{1b} and $e_b^{v_b}(c_{1b})$ is decreasing in c_{1b}. Under no integration, we have: In (**7**), $e_a^{v_n}$ is increasing in c_{1b}, $e_b^{v_n}$ is decreasing in c_{1b}. In (**8**), $\partial\tilde{e}_a^{v_n}/\partial c_{1b} < \partial\tilde{e}_b^{v_n}/\partial c_{1b}$, then $e_b^{v_n}$ is increasing in c_{1b}. The variations of $e_a^{v_n}$ with c_{1b} are ambiguous. Moreover $e_a^{v_n}$ is decreasing in c_{2a} and $e_b^{v_n}$ is increasing in c_{2a}. In (**9**), $e_a^{v_n}$ is increasing in c_{1b} and decreasing in c_{2a}, $e_b^{v_n}$ is decreasing in c_{1b} and increasing in c_{2a}. Last in (**14**), $e_a^{v_n}$ is increasing in c_{1b} and decreasing in c_{2a}, $e_b^{v_n}$ is decreasing in c_{1b} and increasing in c_{2a}. □

Appendix D. Proof of Proposition 3

Denote by $\overline{u}_a^{v_n}(c_{1b}, c_{2a})$ and $\overline{u}_b^{v_n}(c_{1b}, c_{2a})$ the equilibrium expected utility of innovators under (**v_n**). Similarly $\overline{u}_a^{v_a}(c_{2a})$ is I_a's equilibrium expected utility and $\overline{u}_b^{v_b}(c_{1b})$ is I_b's equilibrium expected utility. Lastly, $\overline{w}_1^h(c_{1b}, c_{2a})$ (resp. $\overline{w}_2^h(c_{1b}, c_{2a})$) represents the equilibrium expected utility of producer D_1 when $h=\{v_n, v_b\}$ (resp. when $h=\{v_n, v_a\}$. More precisely,

$$\overline{w}_1^h(c_{1b}, c_{2a}) = \pi(e_a^h)\pi(e_b^h)P_1^h(c_{1b}, c_{2a}; a, b)$$
$$\overline{w}_2^h(c_{1b}, c_{2a}) = \pi(e_a^h)\pi(e_b^h)P_2^h(c_{1b}, c_{2a}; a, b)$$

From Lemma 2, $u_a^{v_a}(e_a^{v_a}, e_b^{v_a}) \geqslant u_a^{v_a}(e_a^{no}, e_b^{v_a}) \geqslant u_a^{v_a}(e_a^{v_n}, e_b^{v_n})$. Moreover, since $P_a^{v_a}(c_{2a}; a, b) \geqslant P_a^{v_n}(c_{1b}, c_{2a}; a, b)$, we also have $u_a^{v_a}(e_a^{v_n}, e_b^{v_n}) \geqslant u_a^{v_n}(e_a^{v_n}, e_b^{v_n})$. Then, $\overline{u}_a^{v_a}(c_{2a}) \geqslant \overline{u}_a^{no}(c_{1b}, c_{2a})$. Similarly, $u_a^{\overline{v}_f} \geqslant u_a^{\overline{v}_b}(c_{2a})$. Lemma 2 implies also that $u_a^{v_b}(e_a^{v_b}, e_b^{v_b}) \leqslant u_a^{v_b}(e_a^{v_b}, e_b^{v_n})$. Since $P_a^{v_b}(c_{1b}; a, b) \leqslant P_a^{v_n}(c_{1b}, c_{2a}; a, b)$, then $u_a^{v_b}(e_a^{v_b}, e_b^{v_n}) \leqslant u_a^{v_n}(e_a^{v_b}, e_b^{v_n}) \leqslant u_a^{v_n}(e_a^{v_n}, e_b^{v_n})$. Therefore, $\overline{u}_a^{v_b}(c_{1b}) \leqslant \overline{u}_a^{v_n}(c_{1b}, c_{2a})$. Similarly, $\overline{u}_a^{v_f} \leqslant \overline{u}_a^{v_a}(c_{2a})$. In addition, $u_a^{\overline{v}_n}(c_{1b}, c_{2a}) + w_1^{\overline{v}_n}(c_{1b}, c_{2a}) \leqslant u_a^{v_a}(e_a^{v_n}, e_b^{v_n}) < u_a^{v_a}(e_a^{v_a}, e_b^{v_a})$ and $\overline{u}_a^{v_b}(c_{1b}, c_{2a}) + \overline{w}_1^{v_b}(c_{1b}, c_{2a}) \leqslant u_a^{v_f}(e_a^{v_b}, e_b^{v_b}) < u_a^{v_f}(e_a^{v_f}, e_b^{v_f})$. Naturally, the same remarks hold for U_b and D_2. Vertical integration generates a surplus:

$$S_a^{v_a}(c_{2a}) = \overline{u}_a^{v_a}(c_{2a}) - [\overline{u}_a^{v_n}(c_{1b}, c_{2a}) + \overline{w}_1^{v_n}(c_{1b}, c_{2a})]$$
$$S_a^{v_f}(c_{1b}) = \overline{u}_a^{v_f} - [\overline{u}_a^{v_b}(c_{1b}) + \overline{w}_1^{v_b}(c_{1b})]$$
$$S_b^{v_b}(c_{1b}, c_{2a}) = \overline{u}_b^{v_b}(c_{1b}) - [\overline{u}_b^{v_n}(c_{1b}, c_{2a}) + \overline{w}_2^{v_n}(c_{1b}, c_{2a})]$$
$$S_b^{v_f}(c_{2a}) = \overline{u}_b^{v_f} - [\overline{u}_b^{v_a}(c_{2a}) + \overline{w}_2^{v_b}(c_{2a})]$$

Given our previous results, $S_a^{v_a}(c_{2a}) \geqslant 0$, $S_b^{v_b}(c_{1b}, c_{2a}) \geqslant 0$, $S_a^{v_f} \geqslant 0$ and $S_b^{v_f} \geqslant 0$. Moreover, $\partial S_a^{v_f}/\partial c_{2a} = \partial S_b^{v_f}/\partial c_{1b} = 0$.

The optimal first stage decision depends on the levels of the switching costs. Let $s_a(s_b)$ be the optimal strategy of branch a when branch b chooses s_b where s_a, $s_b \in \{I, NI\}$ with I = 'integration' and NI = 'no integration'. Denote by $s*_a$ and $s*_b$ the equilibrium strategies of branch a and b respectively. Note that (**v_b**) can occur if $S_b^{v_f} < S_b^{v_b}$ and $V \in [S_b^{v_f}, S_b^{v_b}]$. Similarly, (**v_a**) can occur if $S_a^{v_f} < S_b^{v_a}$ and $V \in [S_a^{v_f}, S_b^{v_a}]$. Using Appendix B and C we get

$$\frac{\partial S_a^{v_f}}{\partial c_{1b}} = -[\pi'(e_a^{v_b})[\pi(e_b^{v_b})[P_a^{v_b}(c_{1b}, c_{2a}; a, b) + P_1^{v_b}(c_{1b}, c_{2a}; a, b)] + (1-\pi(e_b^{v_b}))B_a^m] - 1]\frac{\partial e_a^{v_b}}{\partial c_{1b}} - \pi'(e_b^{v_b})\pi(e_a^{v_b})[P_a^{v_b}(c_{1b}, c_{2a}; a, b) + P_1^{v_b}(c_{1b}, c_{2a}; a, b) - B_a^m]\frac{\partial e_b^{v_b}}{\partial c_{1b}} \leqslant 0.$$

Similarly $\partial S_b^{v_f}/\partial c_{2a} \leqslant 0$.

$$\frac{\partial S_a^{v_a}}{\partial c_{1b}} = -[\pi'(e_a^{v_n})[\pi(e_b^{v_n})[P_a^{v_n}(c_{1b}, c_{2a}; a, b) + P_1^{v_n}(c_{1b}, c_{2a}; a, b)]$$
$$+ (1 - \pi(e_b^{v_n})B_a^m] - 1]\frac{\partial e_a^{v_n}}{\partial c_{1b}}$$
$$- \pi'(e_b^{v_n})\pi(e_a^{v_n})[P_a^{v_n}(c_{1b}, c_{2a}; a, b)$$
$$+ P_1^{v_n}(c_{1b}, c_{2a}; a, b) - B_a^m]\frac{\partial e_b^{v_n}}{\partial c_{1b}} \leqslant 0.$$

everywhere except in region (**8**) where it is positive. Similarly, $\partial S_b^{v_b}/\partial c_{2a} \leqslant 0$. Consider the different regions:

(**1**): $S_a^{v_a} = S_a^{v_f} = 0$, $S_b^{v_b} = S_b^{v_f} = 0$, then for all $V > 0$, $s*_a = NI$ and $s*_b = NI$.

(**2**)–(**10**): $S_a^{v_a} = 0$ and $S_a^{v_f} = 0$, then $s_a^* = NI$. Moreover, $S_b^{v_b} = S_b^{v_f} > 0$, then
–If $V < S_b^{v_b}$, $s_b^* = I$ and the equilibrium is ($\mathbf{v_b}$).
–If $V > S_b^{v_b}$, $s_b^* = NI$ and the equilibrium is ($\mathbf{v_n}$).

(**4**)–(**12**): $S_b^{v_b} = 0$ and $S_b^{v_f} = 0$, then $s_b^* = NI$. Moreover, $S_a^{v_a} = S_a^{v_f} > 0$.
–If $V < S_a^{v_a}$, $s_a^* = I$ and the equilibrium is ($\mathbf{v_a}$).
–If $V > S_a^{v_a}$, $s_a^* = NI$ and the equilibrium is ($\mathbf{v_n}$).

(**6**): $S_a^{v_a} > 0$ and $S_a^{v_f} > 0$.

Moreover $S_b^{v_b} > 0$ and $S_b^{v_b} > 0$. When $c_{1b} = B_b^m - B_{ba}^d$, then $S_a^{va} = S_a^{v_f} = 0$ and $S_b^{v_b} = S_b^{v_f}$. When c_{1b} decreases, $S_a^{v_a}$ and $S_a^{v_f}$ increase, while $S_b^{v_f}$ is constant. When $c_{1b} = B_{bb}^d$, then $S_b^{v_b} = 0$, $S_a^{v_a} > 0$ and $S_a^{v_f} > 0$. When c_{1b} is sufficiently large, then $S_a^{v_a}$ and $S_a^{v_f}$ are smaller then $S_b^{v_b}$ and $S_b^{v_f}$. In that case ($\mathbf{v_a}$) never occurs. When c_{1b} is sufficiently small, $S_b^{v_b} < S_b^{v_f}$, then ($\mathbf{v_b}$) never occurs.

(**3**)–(**11**):

$S_a^{v_a} = 0$, $S_b^{v_b} > 0$, $S_a^{v_f} > 0$ and $S_b^{v_f} > 0$.
–If $V < \min(S_b^{v_b}, S_a^{v_f}, S_b^{v_f})$, $s_a^* = I$ and $s_b^* = I$ and the equilibrium is ($\mathbf{v_f}$).
–If $V > \max(S_b^{v_b}, S_a^{v_f}, S_b^{v_f})$, $s_a^* = NI$ and $s_b^* = NI$ and the equilibrium is ($\mathbf{v_n}$).
–V takes intermediate values ($\mathbf{v_a}$) never occurs and ($\mathbf{v_b}$) is equilibrium if $V < S_b^{v_b}$.

(**5**)–(**13**):

$S_b^{v_b} = 0$, $S_b^{v_a} > 0$, $S_a^{v_f} > 0$ and $S_b^{v_f} > 0$.
–If $V < \min(S_a^{v_a}, S_a^{v_f}, S_b^{v_f})$, $s_a^* = I$ and $s_b^* = I$ and the equilibrium is ($\mathbf{v_f}$).
–If $V > \max(S_a^{v_a}, S_a^{v_f}, S_b^{v_f})$, $s_a^* = NI$ and $s_b^* = NI$ and the equilibrium is ($\mathbf{v_n}$).
–If V takes intermediate values ($\mathbf{v_b}$) never occurs and ($\mathbf{v_a}$) is equilibrium if $V < S_a^{v_a}$.

(**8**): $S_b^{v_b} < S_b^{v_f}$, then ($\mathbf{v_b}$) does not occur.

(**7**)–(**9**)–(**14**):

All industry structures can emerge depending on the size of V. □

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