

Effect of Pretwist on Free Vibration Characteristics Determination of Metallic Curved Blade Replaced with Composite Material

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Abstract This paper addresses the effects of a pretwist on the behavior of curved blade. The blade is considered to be clamped at one end on the axis of rotation. The vibration analysis of pre-twisted blade is performed by using finite element method based solver Ansys. The effects of pretwist on the natural frequencies and mode shapes of metallic curved blade replaced with composite are investigated. The methodology used to determine the Eigen frequency characteristics are determined and the results emphasizing the influence played by the ply angle, pretwist and stacking sequence on the free vibration characteristics are presented and pertinent conclusions are outlined.

Keywords: composite, finite elements, hexahedral mesh, nodes, pre twist

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1. Introduction

Rotating components have the shape of pre twisted blades find its applications commonly in several types of engineering & technology fields such as turbines, marine propulsion, wind mill blades, aircraft wings, turbo machines and aircraft rotary wings. The design of such rotating components includes determination of its modal characteristics which are to be determined accurately. Since the normal modes of motion and associated natural frequencies of vibration of blades depends on the magnitude of pretwist angle. The main objective in this paper is to study the effects of pretwist on the natural frequencies of curved blade. The pretwist angles ranging between 0 & $\pi/2$ are considered. A modeling method for vibration analysis of rotating pre twisted blade was presented by [1]. The paper studies the effects of the dimensionless parameters on the modal characteristics of rotating pre twisted blades and particularly eigen value loci of veering phenomena and associated mode shapes are observed. To obtain more accurate results of natural frequencies of a pre twisted cantilever blading allowing for torsion bending, rotary inertia and deflections due to shear are derived by [2] applying the standard variation Ritz method the equation of motion are deduced. Donholter [3] considered the static displacements and natural frequencies of pre twisted beams. The Effects of breadth taper & depth taper on the vibrational characteristics of pre twisted cantilever blading are discussed by [4]. An analytical method to determine the natural frequencies of rotating beams was presented by [5].

The equation of motion and associated mode shapes for a beam tapered in width and tapered in both width and depth are derived by [6]. A theoretical study to determine the vibrations of a pre twisted cantilever blading was done by [7]. The blading is pre twisted linearly about the centroidal axis of its cross section up to an angle of $\pi/2c$ and is considered to be mounted en cased at the root. White [8] employed greens functions to derive the conditions of orthogonally for a uniform pre twisted blade executing bending-bending vibrations. Diprima and Handle [9] solved the equation of motion of a pre twisted cantilever blade by the Rayleigh Ritz principle. Martin [10] used the mathematical theory of naturally bent and twisted rods as given in Loves theory of elasticity and applied beam problem. Rao [11] used Galerkin method to determine the first five natural frequencies of a pre twisted cantilever blade.

1.1. Composite as Replacement to Metallic

Fiber reinforce plastic finds its applications in various fields of engineering and technology, because of its specific properties being high strength, light weight, corrosion resistance. The deformation, stiffness of the composite can be varied by arranging the orientation of the fibers. The applicability of replacing metallic blade with the composite material is that composite being lighter and corrosion resistant. Another advantage of replacing metallic with composite is that the deformation of the composite can be controlled to improve its performance. [12] Presented a paper using the combined classical momentum and blade element theory to design the wind turbine blade aerodynamic shape. [13] Conducted

mechanical property test and vibration characteristics test of carbon fiber reinforced plastics specimens that can be adopted as new, marine propeller materials [14] determined the vibration characteristics of pre twisted metal matrix composite blade using beam and plate theories. The stability analysis of an angle ply laminated composite twisted panels using finite element method was done [15] where an eight noded iso parametric quad shell element is used to develop finite element procedure. J.G Russell [16] developed a method for blade construction employing CFRP in a basic load carrying spar with a GFRP outer shell having aero foil form. Bhumbla et al. [17] studied the natural frequencies and mode shapes of spinning laminated composite plates. Pandit et al [18] used a nine noded iso parametric plate bending element for the analysis of free un damped vibration of isotropic and fiber oriented laminated composite plates. Ishan kucukrendeci and omer K.Morgil [19] investigated the effects of elastic boundary conditions on the linear un damped free vibrations of a five layer symmetrical rectangular plate. Khoa and Thinh [20] developed a rectangular non-conforming element based on reddy's higher order shear deformation plate theory to analyze the laminated composite plates. Patel et al [21] used the finite element method for analyzing the free vibration of shell structures. A simple two noded shear flexible axisymmetric Shell element based on field consistency approach is employed. The free vibrations of thick cross ply laminated composite cylindrical shells spinning with the axis based on the first order shear deformation theory was investigated by Lam & Qiam [22]. Kumar & Palaninatham [23] studied the finite element analysis of laminated shells by using the degenerated shell element. Reddy et al [24] used the finite element method and artificial neural networks to optimize the stacking sequence of laminated composite plates using distance based optimal design in the design of experiments.

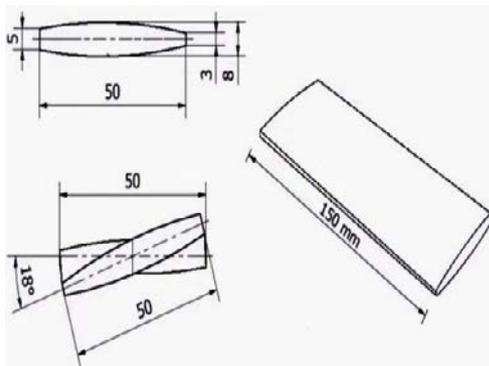


Figure 1. Dimensions of Curved Blade

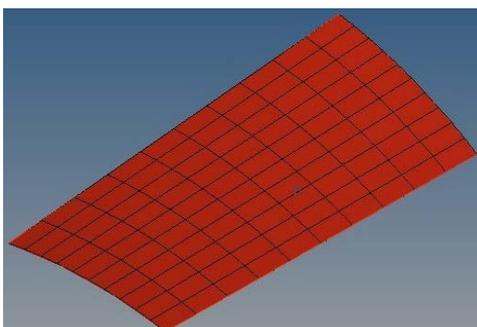


Figure 2. Extracted mid surface of Curved Blade

Table 1. Material properties and dimensions of blade

Blade Dimensions	Material	BC and loading
3 & 5 mm thick (at sides)	Titanium Ti-6Al-4	Fixed : Left face
8 mm thick (at the middle)	Modulus =113.8 Gpa	
50 mm wide(base and top)	Poisson's ratio =0.342	
150 mm long	Yield stress = 950 Mpa	
18° tilt angle(base s tip)	Density =4430 kg/m ³	

2. Material & Methods

Table 2. Material properties of composite for curved blade [12].

Material	E-glass fabric/epoxy
Ex	51.19[Gpa]
Ey	10.18[Gpa]
Ez	51.19[Gpa]
NUXY	0.278
NUYZ	0.05
NUZX	0.278
Gxy	2.433[Gpa]
Gyz	1.698[Gpa]
Gzx	2.4333[Gpa]
density	2 g/cc

2.1. Finite Element Method

The finite element methods are the techniques employed for approximating differential equations to continuous algebraic equations by a finite number of variables [7]. This method is one of the most practical ways of analyzing structures with a large number of degrees of freedom. To achieve more accurate results from Finite element method it is applicable to use fem based software in order to carry out the numerical computation part. In addition to saving time is also another factor which motivates specialists to use software instead of problems solving manually [17]. various Fem based software's are available such ads Ansys, Radiosis, Nastran, Abacus etc. are utilized for to sole engineering problem. In this study Ansys is used as baseline solver for determination of free

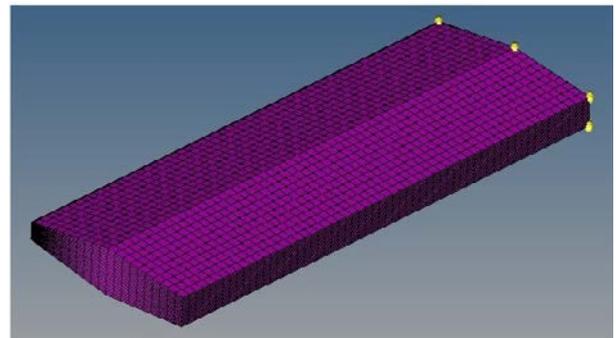


Figure 3. Metallic curved blade meshed model

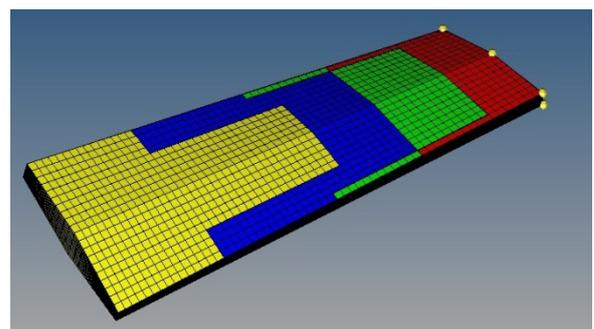


Figure 4. Composite curved blade meshed model

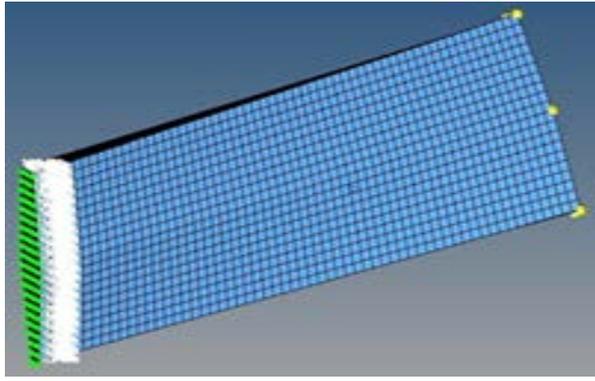


Figure 5. Metallic curved blade with constraints

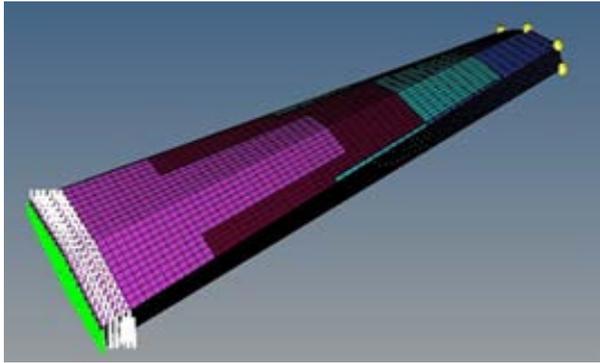


Figure 6. Composite curved blade with constraints

3. Modal Analysis

The following are the assumptions made in this study

1. The rate of pretwist along the longitudinal axis of the blade is uniform.

The equation of motion is derived based on the following assumptions.

The rate of pretwist along the longitudinal axis of the blade is uniform.

The shear and rotary inertia effects of the blade are negligible due to slender shape of the blade.

The neutral and centroidal axis of the blade coincides with each other in the cross section.

Gyroscopic coupling between the stretching and bending motions are negligible and

No external forces act on the blade.

Figure 7 shows the Configuration of a uniformly pre twisted blade where is the pretwist angle at the free end and b_1, b_2, b_3 represent the coordinate axis with respect to the fixed end a_1, a_2, a_3 . The generic point P_0 lying on the undeformed axis move to P when the blade is deformed.

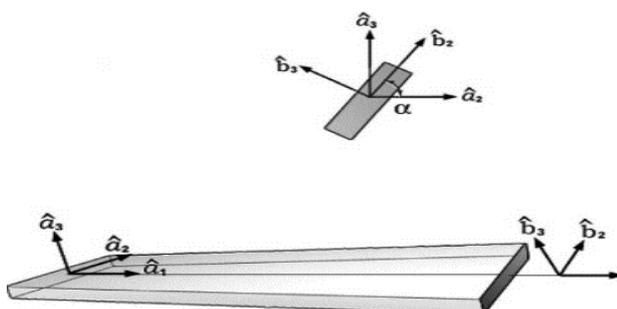
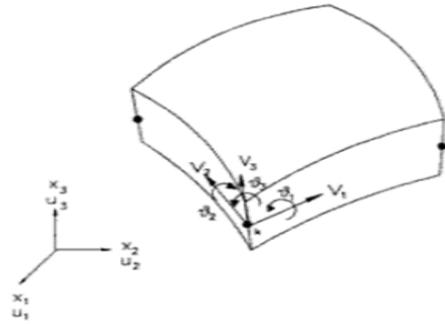


Figure 7. Configuration of pre twisted blade

3.1. Geometry and Deformation of Degenerated Shell



A generic point of a shell may be described in terms of the position vectors of the nodes and shape functions. Two dimensional shape functions are used to describe a position in the element. N^k is the two dimensional shape function in $\xi-\eta$ plane. H^k is the one dimensional shape function along the ζ axis.

$$x_i(\xi, \eta, \zeta) = \sum_{k=1}^n N^k(\xi, \eta)x_i^k + \sum_{k=1}^n N^k(\xi, \eta)H^k(\zeta)V_{3i}^k \quad (1)$$

$(i = 1, 2, 3)$

Where x_i^k is the position vector of node k in the reference surface; V_{3i} is the unit vector at the node k ; and n is the number of nodes per element.

The unit vector is defines as

$$V_{3i}^k = \frac{(x_i^k)^{top} - (x_i^k)^{bottom}}{\left\| (x_i^k)^{top} - (x_i^k)^{bottom} \right\|} \quad (2)$$

Where top and bottom indicate the top and bottom surfaces of the shell and $\| \|$ denotes the euclidean norm.

The one dimensional shape function H^k is expressed as

$$H^k(\zeta) = \left[\frac{1}{4}(1+\zeta)(1-\zeta) - \frac{1}{4}(1-\zeta)(1+\zeta) \right] \frac{\left\| (x_i^k)^{top} - (x_i^k)^{bottom} \right\|}{\left\| (x_i^k)^{top} - (x_i^k)^{bottom} \right\|} \quad (3)$$

In which ζ indicates the location of the reference surface and varies from -1 to $+1$. $\zeta = 0$ represents the mid surface. The shape functions are obtained as follows

$$H_1(\xi, \eta) = \left[\frac{1}{4}(1-\xi)(1-\eta) \right] \quad (4)$$

$$H_2(\xi, \eta) = \left[\frac{1}{4}(1+\xi)(1-\eta) \right] \quad (5)$$

$$H_3(\xi, \eta) = \left[\frac{1}{4}(1+\xi)(1+\eta) \right] \quad (6)$$

$$H_4(\xi, \eta) = \left[\frac{1}{4}(1-\xi)(1+\eta) \right] \quad (7)$$

The displacement field in a shell can be written as

$$u_i(\xi, \eta, \zeta) = \sum_{k=1}^n N^k(\xi, \eta) u_i^k + \sum_{k=1}^n N^k(\xi, \eta) H^k(\zeta) (-V_{2i}^k \theta_1^k + V_{1i}^k \theta_2^k) \quad (8)$$

(i = 1, 2, 3)

In which u_i is the displacement along the x_i axis, u_i^k is the nodal displacement at the node k and unit vectors V_{1i}^k, V_{2i}^k lie along the reference surface. θ_1^k, θ_2^k are rotational degrees of freedom along the unit vectors.

3.2. Strain Displacement Relations

Six strain components are computed from above equation by taking the derivative with respect to the x_i axis. The matrix takes the form

$$\{\varepsilon\} = [B]\{d\} \quad (9)$$

and

$$\{\varepsilon\} = \{\varepsilon_{11} \ \varepsilon_{22} \ \varepsilon_{33} \ \nu_{12} \ \nu_{23} \ \nu_{13}\}^T \quad (10)$$

$$[B] = [B^1 \ B^2 \ \dots \ B^n] \quad (11)$$

$$\{d\} = \{d^1 \ d^2 \ \dots \ d^n\}^T \quad (12)$$

The expression $[B^k]$ is given by

$$[B^k] = \begin{bmatrix} \frac{\partial N^k}{\partial x_1} & 0 & 0 & -g_1^k V_{21}^k & g_1^k V_{11}^k \\ 0 & \frac{\partial N^k}{\partial x_2} & 0 & -g_2^k V_{22}^k & g_2^k V_{12}^k \\ 0 & 0 & \frac{\partial N^k}{\partial x_3} & -g_3^k V_{33}^k & g_3^k V_{33}^k \\ \frac{\partial N^k}{\partial x_2} & \frac{\partial N^k}{\partial x_1} & 0 & -g_2^k V_{21}^k - g_1^k V_{22}^k & g_2^k V_{11}^k + g_1^k V_{12}^k \\ 0 & \frac{\partial N^k}{\partial x_3} & \frac{\partial N^k}{\partial x_2} & -g_3^k V_{22}^k - g_2^k V_{23}^k & -g_3^k V_{23}^k + g_2^k V_{13}^k \\ \frac{\partial N^k}{\partial x_3} & 0 & \frac{\partial N^k}{\partial x_1} & -g_3^k V_{21}^k - g_1^k V_{23}^k & -g_3^k V_{11}^k - g_1^k V_{13}^k \end{bmatrix} \quad (13)$$

In which

$$g_i^k = \frac{\partial N^k}{\partial x_i} H^k + N^k \frac{\partial H^k}{\partial x_i} \quad (14)$$

In addition the vector

$$\{d^k\} = \{u_1 \ u_2 \ u_3 \ \theta_1 \ \theta_2\} \quad (15)$$

Constitutive equations

Stresses and strains can be expressed as

$$\{\sigma'\} = [D']\{\varepsilon'\} \quad (16)$$

Where $\{\sigma'\}$ and $\{\varepsilon'\}$ are the stresses and strain components in the local axes along the reference plane. The constitutive matrix is given as

$$[D'] = \begin{bmatrix} \frac{E}{(1-\nu^2)} & \frac{E\nu}{(1-\nu^2)} & 0 & 0 & 0 & 0 \\ \frac{E\nu}{(1-\nu^2)} & \frac{E}{(1-\nu^2)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{E}{2(1+\nu)} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{kE}{2(1+\nu)} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{kE}{2(1+\nu)} \end{bmatrix} \quad (17)$$

In which E and ν are the elastic and poisons ratio respectively. The material property matrix

$$[D] = [T_\varepsilon]^T [D'] [T_\varepsilon] \quad (18)$$

Where the transformation matrix $[T_\varepsilon]$ is (19)

$$[T_\varepsilon] = \begin{bmatrix} l_{11}^2 & l_{12}^2 & l_{13}^2 & l_{11}l_{12} & l_{12}l_{13} & l_{13}l_{11} \\ l_{21}^2 & l_{22}^2 & l_{23}^2 & l_{21}l_{22} & l_{22}l_{23} & l_{23}l_{21} \\ l_{31}^2 & l_{32}^2 & l_{33}^2 & l_{31}l_{32} & l_{32}l_{33} & l_{33}l_{31} \\ 2l_{11}l_{21} & 2l_{12}l_{22} & 2l_{13}l_{23} & (l_{11}l_{22} + l_{21}l_{12}) & (l_{12}l_{23} + l_{22}l_{13}) & (l_{13}l_{21} + l_{23}l_{11}) \\ 2l_{21}l_{31} & 2l_{22}l_{32} & 2l_{23}l_{33} & (l_{21}l_{32} + l_{31}l_{22}) & (l_{22}l_{33} + l_{32}l_{23}) & (l_{23}l_{31} + l_{33}l_{21}) \\ 2l_{31}l_{11} & 2l_{32}l_{12} & 2l_{33}l_{13} & (l_{31}l_{12} + l_{11}l_{32}) & (l_{32}l_{13} + l_{12}l_{33}) & (l_{33}l_{11} + l_{13}l_{31}) \end{bmatrix}$$

Where l_{ij} are the directional cosines of the unit vector V_i with respect to the x_j axis

Element stiffness matrix

The element stiffness matrix is computed from

$$[K] = \int_{\Omega_e} [B]^T [D] [B] d\Omega \quad (20)$$

And the element mass matrix is given as

$$[M] = \int_{\Omega_e} [N]^T [\rho] [N] d\Omega \quad (21)$$

In general modal analysis consists of extraction of modal parameters i.e. Eigen frequencies; mode shapes of modal damping thesis

$$M\ddot{q} + C\dot{q} + kq = r \quad (22)$$

Where M, C, K mass, damping, stiffness matrix

\ddot{q}, \dot{q}, q acceleration, velocity, displacement

r external force vector

The equation (22) Compose the governing discrete motion, time invariant mathematical basis for modal analysis. However, the specific application of (22) differs in the type of analysis. In this work general modal analysis type in the form of free vibration are considered.

3.3. Parametric Extraction of Free Response System

The numerical modal parameter will be extracted from finite element response analysis. For curved blade, neglecting the effect of damping in the finite element analysis, the free response analysis are described by the homogeneous equation

$$M\ddot{q} + Kq = 0 \quad (23)$$

The natural frequency are obtained using (23) and corresponding mode shapes are plotted by using the natural frequencies.

4. Results &Conclusions

In this present study the effect of pre twist, fiber orientations and layer lamination on the vibration response of a twisted blade are considered. To investigate the difference in vibration response for the blade shell 181 elements are used for finite element modeling and the results are varied in steps of twist angles from 0 to 90. The table shows the quantitative comparisons of the 10 natural frequencies of a non-rotating blade. The blade is meshed up with shell elements using Hyper mesh solver and the numerical results are obtained using ANSYS. Table 4 shows the non-dimensional frequency parameters of twisted composite blade model. The Table 3 shows the natural frequencies of a Titanium blade with various pre twist angles and the corresponding graphs represents the mode shapes as a function of natural frequencies and pre twist angles. The Table 4 shows the variation in natural frequencies of a pre twisted blade at an angle of 18°. Table 5 shows the results of composite blades with various pre twist angles.

Table 3. Natural frequencies of titanium curved blade for various pretwist angles

$\alpha=0$	$\alpha=15$	$\alpha=18$	$\alpha=30$	$\alpha=45$	$\alpha=60$
304.13	296.22	304.14	297.88	302.58	301.5
1519.0	1552.2	1518.3	1384.7	1237.4	1092.6
1709.6	1697.8	1709.6	1740.7	1803.4	1879.6
2048.0	1977.5	2048.5	2191.2	2420.6	2636.0
5034.1	4997.6	5034.0	4847.3	4635.7	4337.6
5297.4	5294.0	5297.6	5414.5	5610.6	5875.6
7807.3	7760.5	7806.3	7902.3	8112.4	8282.2
8478.8	8480.2	8478.4	8468.9	8540.4	8419.6
9382.5	9434.5	9383.2	9519.1	9434.2	9315.2
9807.7	9681.9	9807.6	9674.4	9906.3	10205.0

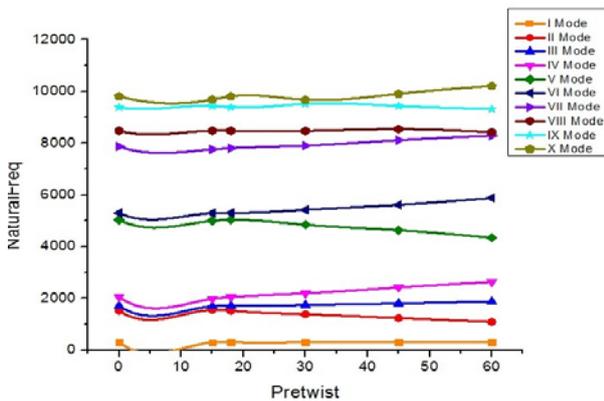


Figure 8. Natural freq vs. Pretwist

Table 4. Natural frequencies of composite curved blade for pretwist angle $\alpha=18^\circ$

6-layers	12-layers	20-layers	37-layers	50-layers
264.98	293.66	288.55	284.06	283.04
1378.9	1452.7	1420.5	1413.0	1418.5
1802.5	1754.5	1801.1	1834.0	1844.7
1911.5	1973.9	1932.0	1910.3	1910.7
4424.4	4851.3	4794.0	4733.6	4720.0
5554.6	5476.1	5585.3	5665.2	5688.0
7725.5	7686.4	7509.4	7492.4	7541.9
8040.4	8031.4	7789.9	7768.9	7832.5
8572.2	9204.8	9206.1	9133.0	9113.6
9739.7	9965.3	9968.6	10017.0	10040.0

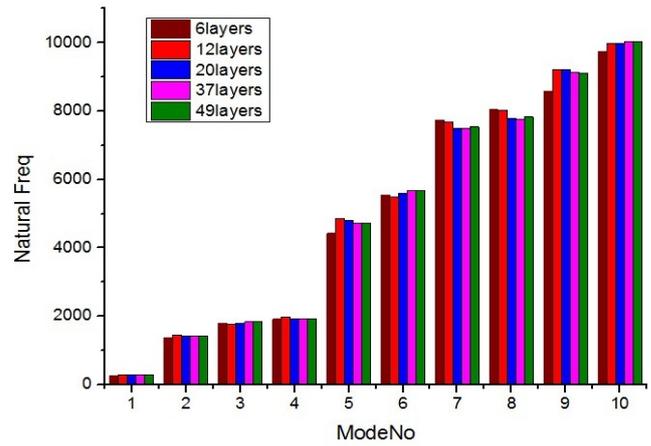


Figure 9. Natural freq vs. Mode No

Table 5. Natural frequencies of composite curved blade for various pretwist angles

$\alpha=0$	$\alpha=15$	$\alpha=18$	$\alpha=30$	$\alpha=45$	$\alpha=60$
283.2	276.57	283.04	313.44	319.76	284.1
1420.7	1442.9	1418.5	1472.3	1315.6	1021.3
1848.1	1802.4	1844.7	1912.0	1979.9	2000.6
1911.7	1869.3	1910.7	2347.0	2601.6	2479.7
4724.1	4662.9	4720.0	5132.3	4936.2	4109.8
5694.0	5629.1	5688.0	5930.7	6121.1	6210.6
7548.1	7521.3	7541.9	8564.0	8773.2	7650.2
7855.8	7817.3	7832.5	9094.1	9090.6	8104.5
9122.0	9000.2	9113.6	10136	10170	8936.4
10055	9877	10040	10516	10666	10674

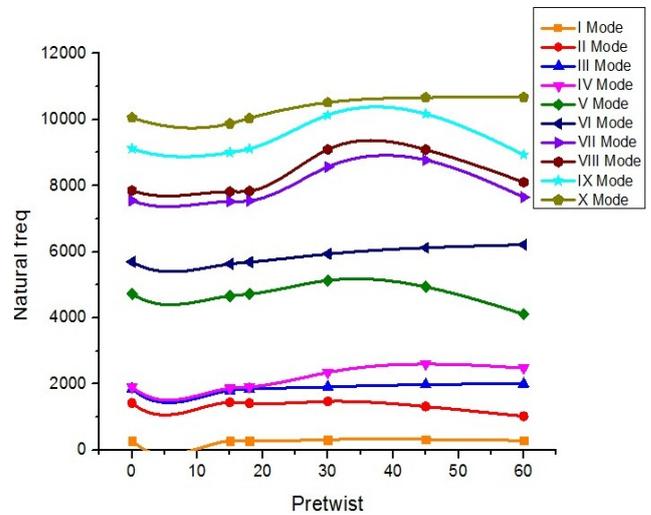


Figure 10. Natural freq vs. Pretwist

5. Conclusions

In this study vibration characteristics of a composite curved blade were analyzed using the finite element approach. The blade was modeled as a cantilever oriented arbitrarily with respect to the rotating axes. The effects of initial twist, fiber orientation, and composite laminates were investigated on the vibration characteristics of composite blade. Results of the analysis of the present study can be summarized as follows:

From Table 3, Table 5 for an initially twisted non rotating blade the bending frequency tend to decrease

whereas the twisting frequency tends to increase with the increase in the initial twist.

From Table 4 it can be observed that with increase in number of layers from 6 to 50 with symmetric ply sequence the operating frequencies are increased by 6%.

The different plyups changed the natural frequencies of the composite blades with this change the desired vibrations characteristics of the blade without modifying the shape is expected. Therefore initial twist and ply lays should be optimized when rotating blades are designed to obtain maximum fundamental natural frequencies

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