HIGH PERFORMANCE PARALLEL IMPLEMENTATION OF ADAPTIVE BEAMFORMING USING SINUSOIDAL DITHERS

Peter G. Vouras and Trac D. Tran

The Johns Hopkins University, Baltimore, MD 21218

ABSTRACT

This paper describes the implementation on parallel processors of a novel new technique for performing adaptive array processing whereby the gradient of the cost function to be minimized is estimated in near real-time through the use of sinusoidal dithers. Results will be shown for an optimized implementation of the algorithm on the Cray-XD1 computer utilizing both the available Opteron general processors and the Virtex-II Field Programmable Gate Array (FPGA) coprocessors.

Index Terms— dither, adaptive beamforming, Cray XD1

1. INTRODUCTION

Recall the Generalized Sidelobe Canceler (GSC) as described in [1]. In this case, we wish to minimize the output power of an array, $y(n) = \boldsymbol{w}^{\mathbf{H}}\boldsymbol{u}(n)$, subject to a set of linear constraints as in,

$$\underset{\boldsymbol{w}}{\text{minimize}}\ J(n) = E\left[\left|y(n)\right|^2\right]$$

$$\text{such that } \boldsymbol{C}^H \boldsymbol{w} = \boldsymbol{f}.$$

Here $\boldsymbol{w}$ represents an N-by-1 vector of complex weights and $\boldsymbol{u}(n)$ is an N-by-1 vector of spatial samples at time instant n taken from the N array elements. Assuming there are L constraints, $\boldsymbol{C}$ is the N-by-L constraint matrix and $\boldsymbol{f}$ is the L-by-1 gain vector.

The problem can be recast as an unconstrained optimization problem by decomposing the weight vector $\boldsymbol{w}$ into the sum of a quiescent component, $\boldsymbol{w}_{\mathbf{q}}$, and an adaptive ($N$-$L$)-by-1 component, $\boldsymbol{w}_{\mathbf{a}}$,

$$\boldsymbol{w} = \boldsymbol{w}_q - \boldsymbol{B}\boldsymbol{w}_a .$$

The columns of the N-by-(N-L) matrix $\boldsymbol{B}$ form a basis for the orthogonal complement of the range space of $\boldsymbol{C}$. Now the array output can be written as,

$$y(n) = \boldsymbol{w}_q^H \boldsymbol{u}(n) - \boldsymbol{w}_a^H \boldsymbol{B}^H \boldsymbol{u}(n).$$

In this case, the objective function to minimize becomes,

$$\underset{\boldsymbol{w}_a}{\text{minimize}}\ J(n).$$

A steepest descent algorithm may be used to converge to the optimal solution by updating the adaptive weight vector at each time instant as in,

$$\boldsymbol{w}_a(n+1) = \boldsymbol{w}_a(n) - \mu\nabla_{\boldsymbol{w}_a} J(n)$$

where μ is a small positive real constant.

2. DITHERING TECHNIQUE

It is possible to apply sinusoidal dithers to the real and imaginary parts of each complex weight to estimate the gradient vector in near real-time [2]. Consider any objective function $J(\boldsymbol{u})$ where $\boldsymbol{u}$ is an N-by-1 vector. For each component of $\boldsymbol{u}$, superimpose a sinusoidal dither of different frequency, as in

$$\boldsymbol{u}' = \boldsymbol{u} + \boldsymbol{\theta} = \boldsymbol{u} + \alpha\left[\cos(\omega_1 t),\quad \cos(\omega_2 t), \ldots, \cos(\omega_N t)\right]^T$$

where α is a small scalar. Using the Taylor series expansion of $J(\boldsymbol{u})$ yields,

$$\begin{aligned} J(\boldsymbol{u}') &= J(\boldsymbol{u}+\boldsymbol{\theta}) = J(\boldsymbol{u}) + \nabla J(\boldsymbol{u})^T \boldsymbol{\theta} + \tfrac{1}{2}\boldsymbol{\theta}^T \nabla^2 J(\boldsymbol{u})\boldsymbol{\theta} + \ldots \\ &= J(\boldsymbol{u}) + \alpha \sum_{i=1}^{N} \left.\frac{\partial J}{\partial u_i}\right|_{\boldsymbol{u}} \cos(\omega_i t) + \ldots \end{aligned}$$

The components of the gradient vector can be determined exactly once we multiply $J(\boldsymbol{u}')$ with $\cos(\omega_j t)$, for j = 1, 2, …, N. The result after using trigonometric identities is,

$$\begin{aligned} J(\boldsymbol{u}')\cos(\omega_j t) &= J(\boldsymbol{u})\cos(\omega_j t) + \frac{\alpha}{2}\left.\frac{\partial J}{\partial u_j}\right|_{\boldsymbol{u}} \\ &+ \frac{\alpha}{2}\left.\frac{\partial J}{\partial u_j}\right|_{\boldsymbol{u}} \cos(2\omega_j t) + \frac{\alpha}{2}\sum_{i \neq j}\left.\frac{\partial J}{\partial u_i}\right|_{\boldsymbol{u}} \cos([\omega_i - \omega_j]t) \\ &+ \frac{\alpha}{2}\sum_{i \neq j}\left.\frac{\partial J}{\partial u_i}\right|_{\boldsymbol{u}} \cos([\omega_i + \omega_j]t) + H.O.T. \end{aligned}$$

Note that the only constant term on the right hand side is the jth component of the gradient vector scaled by the factor $\alpha/2$, and can therefore be recovered exactly by low-pass filtering $J(\boldsymbol{u}')\cos(\omega_j t)$. Once the entire gradient vector is reconstructed, it can be used in the steepest descent algorithm to compute the next iteration of the weight vector.

3. RESULTS

Fig. 1 shows the configuration of an adaptive array with an independent sinusoidal dither applied to the real and imaginary parts of each complex array element weight. The dither frequencies are spaced 100 Hz apart, the sampling frequency is 3200 Hz, and the scalar α was set equal to 0.01. The low pass filter (LPF) was implemented as a sliding window average of the data using a mask of 128 samples normalized to have unity DC gain. For a linear array with 5 elements, Fig. 2 shows the location of the dithers and the response of the LPF in the frequency domain. Fig. 3 illustrates the performance of the dithering algorithm when used to place a null in the direction of a jammer at 44º azimuth. Also shown for comparison in the figure is the beam pattern generated using the conventional GSC algorithm and the unadapted beam pattern. Fig. 4 illustrates the convergence of the Signal-to-Interference-Plus-Noise-Ratio (SINR) at the output of the dithering beamformer to that attained using a GSC.

Results in the presentation show measured latency for a parallel implementation of the dithering algorithm on the Cray XD1 supercomputer. The Cray XD1 offers both Opteron general purpose processors and Virtex-II FPGAs on the same processing board.

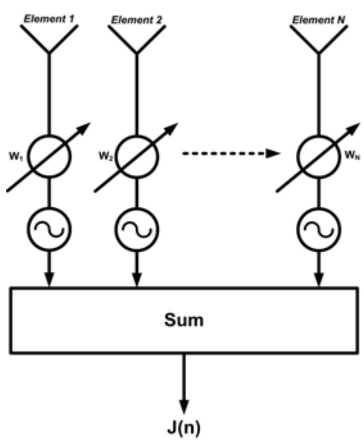

Fig. 1. Adaptive Array Configuration With Dithers

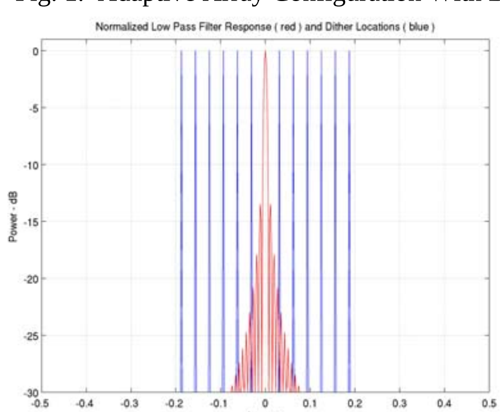

Fig. 2. Dither Locations and LPF Response

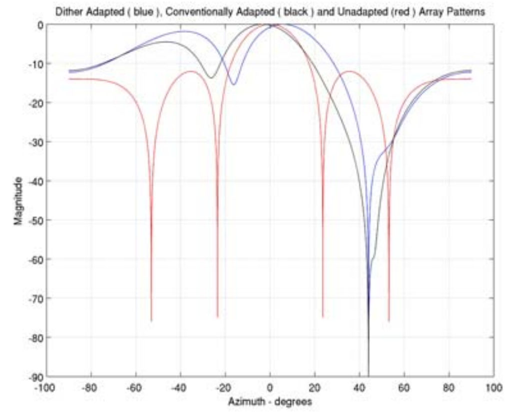

Fig. 3. Adapted and Unadapted Array Patterns

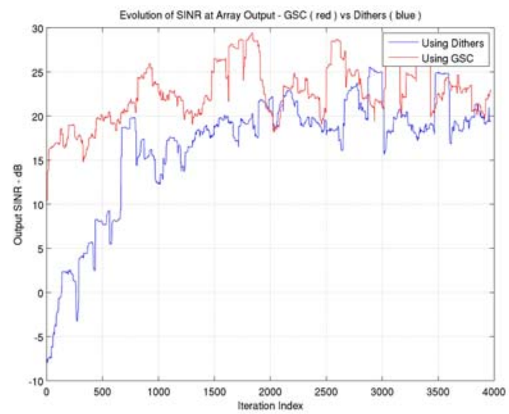

Fig. 4. Output SINR

4. REFERENCES

[1] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, Upper Saddle River, NJ, 1996.

[2] D. N. Loizos, P. P. Sotiriadis, and G. Cauwenberghs, "A Robust Continuous-Time Multi-Dithering Technique for Laser Communications Using Adaptive Optics," *Proceedings IEEE International Symposium on Circuits and Systems*, Kos, Greece, May 2006.