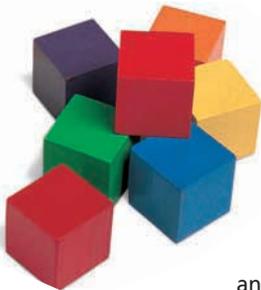


Research on the Benefits of Manipulatives

History of Manipulatives

Since ancient times, people of many different civilizations have used physical objects to help them solve everyday math problems. The ancient civilizations of Southwest Asia (the Middle East) used counting boards. These were wooden or clay trays covered with a thin layer of sand. The user would draw symbols in the sand to tally, for example, an account or take an inventory. The ancient Romans modified counting boards to create the world's first abacus. The Chinese abacus, which came into use centuries later, may have been an adaptation of the Roman abacus.



Similar devices were developed in the Americas. The Mayans and the Aztecs both had counting devices that featured corn kernels strung on string or wires that were stretched across a wooden frame. The Incas had their own unique counting tool—knotted strings called *quipu*.

The late 1800s saw the invention of the first true manipulatives—maneuverable objects that appeal to several different senses and are specifically designed for teaching mathematical concepts. Friedrich Froebel, a German educator who, in 1837, started the world's first kindergarten program, developed different types of objects to help his kindergartners recognize patterns and appreciate geometric forms found in nature. In the early 1900s, Italian-born educator Maria Montessori further advanced the idea that manipulatives are important in education. She designed many materials to help preschool and elementary school students discover and learn basic ideas in math and other subjects.

Since the early 1900s, manipulatives have come to be considered essential in teaching mathematics at the elementary-school level. In fact, for decades, the National Council of Teachers of Mathematics (NCTM) has recommended the use of manipulatives in teaching mathematical concepts at all grade levels.

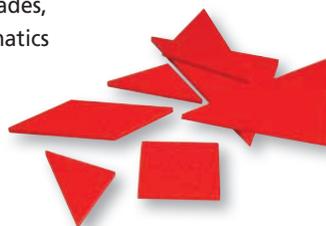
Manipulatives and Curriculum Standards

The NCTM calls for manipulatives to be used in teaching a wide variety of topics in mathematics.

- sorting—a pre-mathematical skill that aids in comprehension of patterns and functions
- ordering—a pre-mathematical skill that enhances number sense and other math-related abilities
- distinguishing patterns—the foundation for making mathematical generalizations
- recognizing geometric shapes and understanding relationships among them
- making measurements, using both nonstandard and standard units with application to both two- and three-dimensional objects
- understanding the base-ten system of numbers
- comprehending mathematical operations—addition, subtraction, multiplication, division
- recognizing relationships among mathematical operations
- exploring and describing spatial relationships
- identifying and describing different types of symmetry
- developing and utilizing spatial memory
- learning about and experimenting with transformations
- engaging in problem-solving
- representing mathematical ideas in a variety of ways
- connecting different concepts in mathematics
- communicating mathematical ideas effectively

Different states across the nation have also mandated the use of manipulatives for teaching math. These have included California, North Carolina, Texas, and Tennessee, among others. In addition, many local school districts mandate or strongly suggest manipulatives be used in teaching math especially for mathematics teaching at the elementary level.

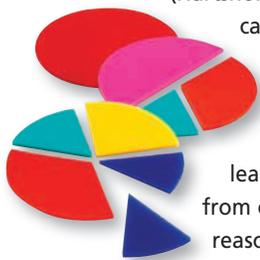
Manipulative use is recommended because it is supported by both learning theory and educational research in the classroom.



Concrete stage	Representational stage	Abstract stage
A mathematical concept is introduced with manipulatives; students explore the concept using the manipulatives in purposeful activity.	A mathematical concept is represented using pictures of some sort to stand for the concrete objects (the manipulatives) of the previous stage; students demonstrate how they can both visualize and communicate the concept at a pictorial level.	Mathematical symbols (numerals, operation signs, etc.) are used to express the concept in symbolic language; students demonstrate their understanding of the mathematical concept using the language of mathematics.

How Learning Theory Supports the Use of Manipulatives

The theory of experiential education revolves around the idea that learning is enhanced when students acquire knowledge through active processes that engage them (Hartshorn and Boren, 1990). Manipulatives



can be key in providing effective, active, engaging lessons in the teaching of mathematics.

Manipulatives help students learn by allowing them to move from concrete experiences to abstract reasoning (Heddens, 1986; Reisman, 1982; Ross and Kurtz, 1993). Experts in

education posit that this learning takes place in three stages.

The use of manipulatives helps students hone their mathematical thinking skills. According to Stein and Bovalino (2001), "Manipulatives can be important tools in helping students to think and reason in more meaningful ways. By giving students concrete ways to compare and operate on quantities, such manipulatives as pattern blocks, tiles, and cubes can contribute to the development of well-grounded, interconnected understandings of mathematical ideas."

To gain a deep understanding of mathematical ideas, students need to be able to integrate and connect a variety of concepts in many different ways. Clements (1999) calls this type of deep understanding "Integrated-Concrete" knowledge. The effective use of manipulatives can help students connect ideas and integrate their knowledge so that they gain a deep understanding of mathematical concepts.

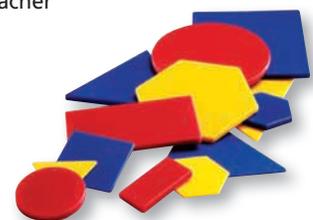
Teachers play a crucial role in helping students use manipulatives successfully, so that they move through the three stages of learning and arrive at a deep understanding of mathematical concepts.

How Research from the Classroom Supports the Use of Manipulatives

Over the past four decades, studies done at all different grade levels and in several different countries indicate that mathematics achievement increases when manipulatives are put to good use (Canny, 1984; Clements and Battista, 1990; Clements, 1999; Dienes, 1960; Driscoll, 1981; Fennema, 1972, 1973; Skemp, 1987; Sugiyama, 1987; Suydam, 1984). Additional research shows that use of manipulatives over the long-term provides more benefits than short-term use does (Sowell, 1989).

With long-term use of manipulatives in mathematics, educators have found that students make gains in the following general areas (Heddens; Picciotto, 1998; Sebesta and Martin, 2004):

- verbalizing mathematical thinking
- discussing mathematical ideas and concepts
- relating real-world situations to mathematical symbolism
- working collaboratively
- thinking divergently to find a variety of ways to solve problems
- expressing problems and solutions using a variety of mathematical symbols
- making presentations
- taking ownership of their learning experiences
- gaining confidence in their abilities to find solutions to mathematical problems using methods that they come up with themselves without relying on directions from the teacher





Studies have shown that students using manipulatives in specific mathematical subjects are more likely to achieve success than students who don't have the opportunity to work with manipulatives. Following are some specific areas in which research shows manipulatives are especially helpful:

Counting Some children need to use manipulatives to learn to count (Clements, 1999).

Place Value Using manipulatives increases students' understanding of place value (Phillips, 1989).

Computation Students learning computational skills tend to master and retain these skills more fully when manipulatives are used as part of their instruction (Carroll and Porter, 1997).

Problem Solving Using manipulatives has been shown to help students reduce errors and increase their scores on tests that require them to solve problems (Carroll and Porter, 1997; Clements, 1999; Krach, 1998).

Fractions Students who have appropriate manipulatives to help them learn fractions outperform students who rely only on textbooks when tested on these concepts (Jordan, Miller, and Mercer, 1998; Sebesta and Martin, 2004).

Ratios Students who have appropriate manipulatives to help them learn fractions also have significantly improved achievement when tested on ratios when compared to students who do not have exposure to these manipulatives (Jordan, Miller, and Mercer, 1998).

Algebraic Abilities Algebraic abilities include the ability to represent algebraic expressions, to interpret such expressions, to make connections between concepts when solving linear equations, and to communicate algebraic concepts. Research indicates that students who used manipulatives in their mathematics classes have higher algebraic abilities than those who did not use manipulatives (Chappell and Strutchens, 2001).

Manipulatives have also been shown to provide a strong foundation for students mastering the following mathematical concepts (The Access Center, October 1, 2004):

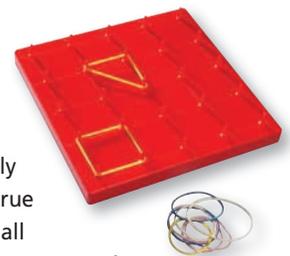
- number relations
- measurement
- decimals
- number bases
- percentages
- probability
- statistics

Well-known math educator Marilyn Burns considers manipulatives essential for teaching math to students of all levels. She finds that manipulatives help make math concepts accessible to almost all learners, while at the same time offering ample opportunities to challenge students who catch on quickly to the concepts being taught. Research indicates that using manipulatives is especially useful for teaching low achievers, students with learning disabilities, and English language learners (Marsh and Cooke, 1996; Ruzic and O'Connell, 2001).

Research also indicates that using manipulatives helps improve the environment in math classrooms. When students work with manipulatives and then are given a chance to reflect on their experiences, not only is mathematical learning enhanced, math anxiety is greatly reduced (Cain-Caston, 1996; Heuser, 2000). Exploring manipulatives, especially self-directed exploration, provides an exciting classroom environment and promotes in students a positive attitude toward learning (Heuser, 1999; Moch, 2001). Among the benefits several researchers found for using manipulatives was that they helped make learning fun (Moch, 2001; Smith et. al, 1999).

Summary

Research from both learning theory and classroom studies shows that using manipulatives to help teach math can positively affect student learning. This is true for students at all levels and of all abilities. It is also true for almost every topic covered in elementary school mathematics curricula. Papert (1980) calls manipulatives "objects to think with." Incorporating manipulatives into mathematics lessons in meaningful ways helps students grasp concepts with greater ease, making teaching most effective.



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