ELSEVIER

Information Sciences 123 (2000) 281–293

INFORMATION SCIENCES
AN INTERNATIONAL JOURNAL

www.elsevier.com/locate/ins

Theory and application of a novel fuzzy PID controller using a simplified Takagi–Sugeno rule scheme

Hao Ying [a,b,*]

[a] *Department of Physiology and Biophysics, The University of Texas Medical Branch, Galveston, TX 77555, USA*

[b] *Biomedical Engineering Center, The University of Texas Medical Branch, Galveston, TX 77555, USA*

Received 20 November 1998; received in revised form 28 May 1999; accepted 29 October 1999

Abstract

We first present a novel general-purpose nonlinear PID controller realized via fuzzy PID control that uses our newly-introduced simplified Takagi–Sugeno (TS) rule scheme. Analytical structure of the fuzzy PID controller is derived and its structure is analyzed in relation to the linear PID controller. The unique features of the fuzzy controller are as follows. First, the proportional, integral and derivative gains constantly vary with the output of the system under control. The gain variation leads to a shorter rise-time, a less overshoot and a smaller settling-time as compared to a comparable linear PID controller. Second, the characteristics of the gain variation are determined by the fuzzy rules, and can intuitively be designed. We have also investigated the local stability of the fuzzy PID control systems. As an application demonstration, we have developed a fuzzy PID control system to regulate, in computer simulation, blood pressure in postsurgical patients. We have chosen this particular control problem because the studies in the literature have established that, in order to achieve satisfactory control results, using a nonlinear controller with variable gains is necessary. The simulation results show that the fuzzy PID controller significantly outperforms its linear counterpart, and is safer and more robust over a wide range of patient condition. © 2000 Published by Elsevier Science Inc. All rights reserved.

* Fax: +1-409-772-0751.
E-mail address: hying@utmb.edu (H. Ying).

PII: S0020-0255(99)00133-4

Keywords: Fuzzy control; PID control; Takagi–Sugeno fuzzy rule; Mean arterial pressure; Sodium nitropresside

1. Introduction

Fuzzy controllers are inherently nonlinear controllers, and hence fuzzy control technology can be viewed as a new, cost effective and practical way of developing nonlinear controllers [21]. The major advantage of this technology over the traditional control technology is its capability of capturing and utilizing qualitative human experience and knowledge in a quantitative manner through the use of fuzzy sets, fuzzy rules and fuzzy logic. However, carrying out analytical analysis and design of fuzzy control systems is difficult not only because the explicit structure of fuzzy controllers is generally unknown, but also due to their inherent nonlinear and time-varying nature.

There exist two different types of fuzzy controllers: the Mamdani type [6,10] and the Takagi–Sugeno (TS, for short) [15] type. They mainly differ in the fuzzy rule consequent: a Mamdani fuzzy controller utilizes fuzzy sets as the consequent whereas a TS fuzzy controller employs linear functions of input variables. Significant effort has been made to analytically study Mamdani fuzzy controllers (e.g., [2,4,5,7–9,12,17,22,24]). In contrast, analytical results of TS fuzzy controllers are still rather limited [16,18,20]. We recently derived analytical structures of some classes of TS fuzzy controllers, including the PI and PD types, and revealed their relationship with the classical controllers [25,27,28]. We also developed a new, more efficient TS rule scheme, called simplified TS fuzzy rule scheme, which greatly reduces the number of adjustable parameters in the rule consequent [27,28].

In the present paper, we will expand our studies from the fuzzy PI and PD controllers to a fuzzy PID controller that uses the simplified linear TS rule scheme. We revealed already, in a previous paper [27], its general structure to be a nonlinear PID controller with variable gains. However, the explicit expressions of the gains were not derived. We will first provide the configuration of the fuzzy controller and then derive its structure. We will then analyze the peculiar characteristics of the gain variation and point out the performance advantages that they bring to the fuzzy controller over the linear PID controller. Finally, as an application example, we will show, through computer simulation, how the fuzzy controller fits naturally and nicely to closed-loop control of blood pressure in postsurgical patients, a challenging control problem that has long been known for its need of a nonlinear controller.

2. Configuration of the TS fuzzy PID controller

The TS fuzzy PID controller under this investigation uses the same input variables as the linear PID controller does:

$$\begin{aligned} x_1(n) &= \mathrm{SP}(n) - y(n), \\ x_2(n) &= x_1(n) - x_1(n-1), \\ x_3(n) &= x_2(n) - x_2(n-1), \end{aligned} \tag{1}$$

where $\mathrm{SP}(n)$ is the setpoint/reference signal of system output, and $y(n)$ is the system output at sampling time n. Variables $x_1(n), x_2(n)$ and $x_3(n)$ represent the position, velocity and acceleration of the system output. Each variable is fuzzified by two input fuzzy sets, namely "Positive" and "Negative" and their mathematical definitions are identical for the input variables:

$$\begin{aligned} \mu_\mathrm{P}(x_i) &= \begin{cases} 0, & x_i < -L \\ \frac{x_i+L}{2L}, & -L \leqslant x_i \leqslant L \\ 1, & x_i > L \end{cases} \\ \mu_\mathrm{N}(x_i) &= \begin{cases} 1, & x_i < -L \\ \frac{-x_i+L}{2L}, & -L \leqslant x_i \leqslant L \\ 0, & x_i > L \end{cases} \end{aligned} \tag{2}$$

where the subscripts P and N mean "Positive" and "Negative," respectively. L is a design parameter whose value affects the control performance and should be carefully chosen during design. Fig. 1 gives a graphical illustration of their mathematical definitions.

A total eight different combinations of the input fuzzy sets exist, and hence we need eight fuzzy rules to cover them. Instead of using the original TS fuzzy rule scheme [15], we use our simplified linear TS rules scheme:

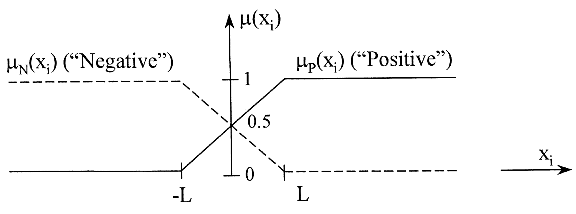


Fig. 1. The graphical definition of the input fuzzy sets given in Eq. (2). Input variables, specified as x_i where $i = 1, 2$ or 3, are fuzzified by the identical fuzzy sets.

$$
\begin{aligned}
R_1 &: \text{IF } x_1(n) \text{ is } A_{11} \text{ AND } x_2(n) \text{ is } A_{21} \text{ AND } x_3(n) \text{ is } A_{31} \\
&\quad \text{THEN } v_1(n) = k_1(a_1x_1(n) + a_2x_2(n) + a_3x_3(n)) \\
R_2 &: \text{IF } x_1(n) \text{ is } A_{12} \text{ AND } x_2(n) \text{ is } A_{22} \text{ AND } x_3(n) \text{ is } A_{32} \\
&\quad \text{THEN } v_2(n) = k_2v_1(n) \\
&\vdots \\
&\vdots \\
R_8 &: \text{IF } x_1(n) \text{ is } A_{18} \text{ AND } x_2(n) \text{ is } A_{28} \text{ AND } x_3(n) \text{ is } A_{38} \\
&\quad \text{THEN } v_8(n) = k_8v_1(n),
\end{aligned}
\tag{3}
$$

where $v_j(n)$ denotes the contribution of the jth rule to the controller output. Without loss of generality, we assume that $k_1 = 1$. In the rule consequent, $a_1, a_2, a_3, k_2, \ldots, k_8$ are ten constant parameters. Compared with the original TS rule scheme, parameter reduction is significant: 10 vs. 24. The rule consequent is proportional and k_j's represent the proportionality. Although the proportionality is fixed, all the consequent, being linear functions of input variables, constantly change with input variables. This will be shown later. Our simplified rule scheme is not restrictive as it can produce fuzzy controllers and systems that are universal approximators [26]. We use the product fuzzy logic AND operator to combine the three membership values in each of the rule antecedents to generate a combined membership, denoted as μ_j, for $v_j(n)$:

$$\mu_j = \mu_\beta(x_1)\mu_\beta(x_2)\mu_\beta(x_3),$$

where the subscript β is either P (representing "Positive") or N (signifying "Negative").

Finally, we use the popular centroid defuzzifier for defuzzification. The result can be either incremental output of the fuzzy controller, $\Delta u(n)$, if $v_j(n)$ represents incremental output, or output of the fuzzy controller, $u(n)$, if $v_j(n)$ means controller output:

$$\Delta u(n) = \frac{\sum_{j=1}^{8} \mu_j \; v_j(n)}{\sum_{j=1}^{8} \mu_j} \quad \text{or} \quad u(n) = \frac{\sum_{j=1}^{8} \mu_j \; v_j(n)}{\sum_{j=1}^{8} \mu_j}. \tag{4}$$

Note that there is no difference on the right sides of the two equations in form: the differences are on the left side as well as in the meaning of $v_j(n)$.

3. The fuzzy PID controller as a nonlinear PID controller with variable gains

We now derive the explicit structure of the fuzzy PID controller and relate the resulting structure to the linear PID controller. We will focus on the incremental type of the fuzzy controller but the expressions are exactly the same for the other type.

Due to the nature of the chosen membership functions of the input fuzzy sets (i.e., $\mu_P(x_i) + \mu_N(x_i) = 1$), it can easily be proven that the denominator of (4) is always equal to 1. Hence, (4) reduces to

$$\begin{aligned}\Delta u(n) &= \sum_{j=1}^{8} \mu_j v_j(n) = v_1(n) \times \sum_{j=1}^{8} \mu_j k_j \\ &= G(x_1, x_2, x_3)(a_1 x_1(n) + a_2 x_2(n) + a_3 x_3(n)),\end{aligned} \tag{5}$$

where

$$\begin{aligned}G(x_1, x_2, x_3) = \sum_{j=1}^{8} \mu_j\ k_j &= k_1 \mu_P(x_1)\mu_P(x_2)\mu_P(x_3) + k_2 \mu_P(x_1)\mu_P(x_2)\mu_N(x_3) \\ &+ k_3 \mu_P(x_1)\mu_N(x_2)\mu_P(x_3) + k_4 \mu_P(x_1)\mu_N(x_2)\mu_N(x_3) \\ &+ k_5 \mu_N(x_1)\mu_P(x_2)\mu_P(x_3) + k_6 \mu_N(x_1)\mu_P(x_2)\mu_N(x_3) \\ &+ k_7 \mu_N(x_1)\mu_N(x_2)\mu_P(x_3) + k_8 \mu_N(x_1)\mu_N(x_2)\mu_N(x_3).\end{aligned} \tag{6}$$

Recall that a discrete-time linear PID controller in incremental form is:

$$\Delta u_{PID}(n) = \bar{K}_i x_1(n) + \bar{K}_p x_2(n) + \bar{K}_d x_3(n), \tag{7}$$

where $\bar{K}_p, \bar{K}_i$ and $\bar{K}_d$ are proportional-gain, integral-gain and derivative-gain, respectively. In comparison, the fuzzy PID controller is a nonlinear PID controller with variable proportional-gain, integral-gain and derivative-gain being $a_2 \times G(x_1, x_2, x_3), a_1 \times G(x_1, x_2, x_3)$ and $a_3 \times G(x_1, x_2, x_3)$, respectively. The same can be said to the fuzzy PID controller using $u(n)$ as output: it is a position-form nonlinear PID controller with variables gains.

Since the gains are proportional to $G(x_1, x_2, x_3)$, we only need to study $G(x_1, x_2, x_3)$ in order to understand the characteristics of the variable gains. $G(x_1, x_2, x_3)$ is defined over the input space of $(-\infty, \infty) \times (-\infty, \infty) \times (-\infty, \infty)$. Inside the cube $[-L, L] \times [-L, L] \times [-L, L]$, it is explicitly derived as

$$\begin{aligned}G(x_1, x_2, x_3) = \frac{k_1}{8L^3}\big(&c_1 L^3 + c_2 L^2 x_1 + c_3 L^2 x_2 + c_4 L^2 x_3 + c_5 L x_1 x_2 \\ &+ c_6 L x_1 x_3 + c_7 L x_2 x_3 + c_8 x_1 x_2 x_3\big),\end{aligned} \tag{8}$$

where

$$
\begin{aligned}
c_1 &= 1 + k_2 + k_3 + k_4 + k_5 + k_6 + k_7 + k_8,\\
c_2 &= 1 + k_2 + k_3 + k_4 - k_5 - k_6 - k_7 - k_8,\\
c_3 &= 1 + k_2 - k_3 - k_4 + k_5 + k_6 - k_7 - k_8,\\
c_4 &= 1 - k_2 + k_3 - k_4 + k_5 - k_6 + k_7 - k_8,\\
c_5 &= 1 + k_2 - k_3 - k_4 - k_5 - k_6 + k_7 + k_8,\\
c_6 &= 1 - k_2 + k_3 - k_4 - k_5 + k_6 - k_7 + k_8,\\
c_7 &= 1 - k_2 - k_3 + k_4 + k_5 - k_6 - k_7 + k_8,\\
c_8 &= 1 - k_2 - k_3 + k_4 - k_5 + k_6 + k_7 - k_8.
\end{aligned} \tag{9}
$$

$G(x_1, x_2, x_3)$ outside the cube has also been derived in a similar fashion, but the result is omitted here for brevity. $G(x_1, x_2, x_3)$ given in (8) is of most importance because it is more nonlinear than anywhere else in the input space [27,28]. Hence, we will discuss this region only below.

Some important characteristics of the fuzzy PID controller can be observed from the expression of $G(x_1, x_2, x_3)$ in (8). First, $G(x_1, x_2, x_3)$ is a continuous function with respect to x_1, x_2 and x_3 over the entire input space. Accordingly, the gain variation is always continuous. Second, the characteristics of the gain variation are parameterized by the rule proportionality (i.e., k_j's). This is to say that *kj*'s determine the geometry of $G(x_1, x_2, x_3)$. For instance, at the equilibrium point (0, 0, 0), $G(0,0,0) = c_1/8$. The values of $G(x_1, x_2, x_3)$ at the eight vertexes of the cube are also directly related to k_j's (e.g., $G(L, -L, L) = k_3$). The values of $G(x_1, x_2, x_3)$ at these nine locations provide some rough ideas about the characteristics of the gain variation. Whether the gain variation is sensible in the context of control depends on the values of k_j's, which can be intuitively selected [28]. Interestingly, if all the values of k_j's are the same (i.e., 1), $G(x_1, x_2, x_3) = 1$ and the fuzzy controller actually becomes a conventional linear PID controller. In other words, the linear PID controller is just a special case of the fuzzy PID controller.

One of the best approaches to illustrating these characteristics is probably through graphical demonstration. Nevertheless, how $G(x_1, x_2, x_3)$ changes with the input variables is four-dimensional, and cannot be directly visualized. As an alternative, we will only discuss a simple case here, which demonstrates the same characteristics. We will exclude $x_3(n)$ in the analysis below and reduce the fuzzy PID controller to a fuzzy PI controller. This means that $a_3 = 0$, and $k_5 = k_6 = k_7 = k_8 = 0$ (Note that a fuzzy PI controller only needs four fuzzy rules). Under these conditions, we have (see also [27]):

$$G(x_1,x_2) = \frac{k_1}{4L^2}[(1+k_2+k_3+k_4)L^2 + (1+k_2-k_3-k_4)L\ x_1(n) + (1-k_2+k_3-k_4)L\ x_2(n) + (1-k_2-k_3+k_4)x_1(n)x_2(n)]. \tag{10}$$

Without losing generality, we now assume that $L = 1$, which is not restrictive as one can always rescale the input variables to fit any given intervals, and the scaled input variables can then be treated as $x_1(n)$ and $x_2(n)$. To further simplify the analysis, we let $k_2 = k_3 = 0$ and concentrate on the effect of k_4 on the characteristics of $G(x_1,x_2)$.

Fig. 2 shows three-dimensional plots of $G(x_1,x_2)$, when $k_4 = 1, 0.5$ and 0, respectively. The plots are with respect to $x_1(n)$ and $x_2(n)$ whose ranges are $[-2L, 2L]$. When $k_4 = 1$, $G(x_1,x_2)$ is a symmetrical function with respect to the lines $x_1(n) = x_2(n)$ and $x_1(n) = -x_2(n)$, which means that the proportional-gain and integral-gain vary symmetrically as well. The symmetry is achieved because $k_1 = k_4$ and $k_2 = k_3$. At (0, 0), $G(x_1,x_2) = 0.5$. $G(x_1,x_2)$ reaches its maximum, 1, at (L,L) and $(-L,-L)$, and achieves its minimum, 0, at $(L,-L)$ and $(-L,L)$. We define the proportional-gain and integral-gain at (0, 0) as steady-state proportional-gain and integral-gain, respectively. The gains at any other states are named dynamic proportional-gain and integral-gain. Based on the maximum and minimum of $G(x_1,x_2)$, one sees that the dynamic gains can be enlarged to as much as twice of the steady-state gains or can be reduced to as little as zero. In [27], we have provided a detailed analysis of the gain variation in the context of control and in comparison with the linear PI controller that uses the steady-state gains of the fuzzy controller. The analysis shows that the variable gains empower the fuzzy PI controller to outperform the linear PI controller.

The cases of $k_4 = 0.5$ and 0 in Fig. 2 are intended to show how the rule proportionality can be used to achieve different characteristics of the gain variation. The influence of the proportionality is evident as compared with the case of $k_4 = 1$. $G(x_1,x_2)$ is no longer a symmetrical function with respect to the lines $x_1(n) = -x_2(n)$ because $k_1 \neq k_4$. Moreover, the value of $G(x_1,x_2)$ around $[0,-2] \times [0,-2]$ decreases as the value of k_4 reduces. One can also adjust k_2 and k_3 to manipulate the gain variation characteristics. Indeed, an endless number of characteristics can be realized through these parameters. What kind of characteristics should be used depends on the particular control problem involved.

The fuzzy PID controller has one more input variable (i.e., $x_3(n)$) and five more parameters than the fuzzy PI controller (i.e., a_3 and k_5 to k_8). Hence, it is more difficult to analyze because of the higher dimensionality. In theory, its control performance is enhanced by the additional degrees of freedom introduced by the extra parameters. Such enhancement, however, is at the expense of difficult parameter tuning and complicated controller structure. One remedy is

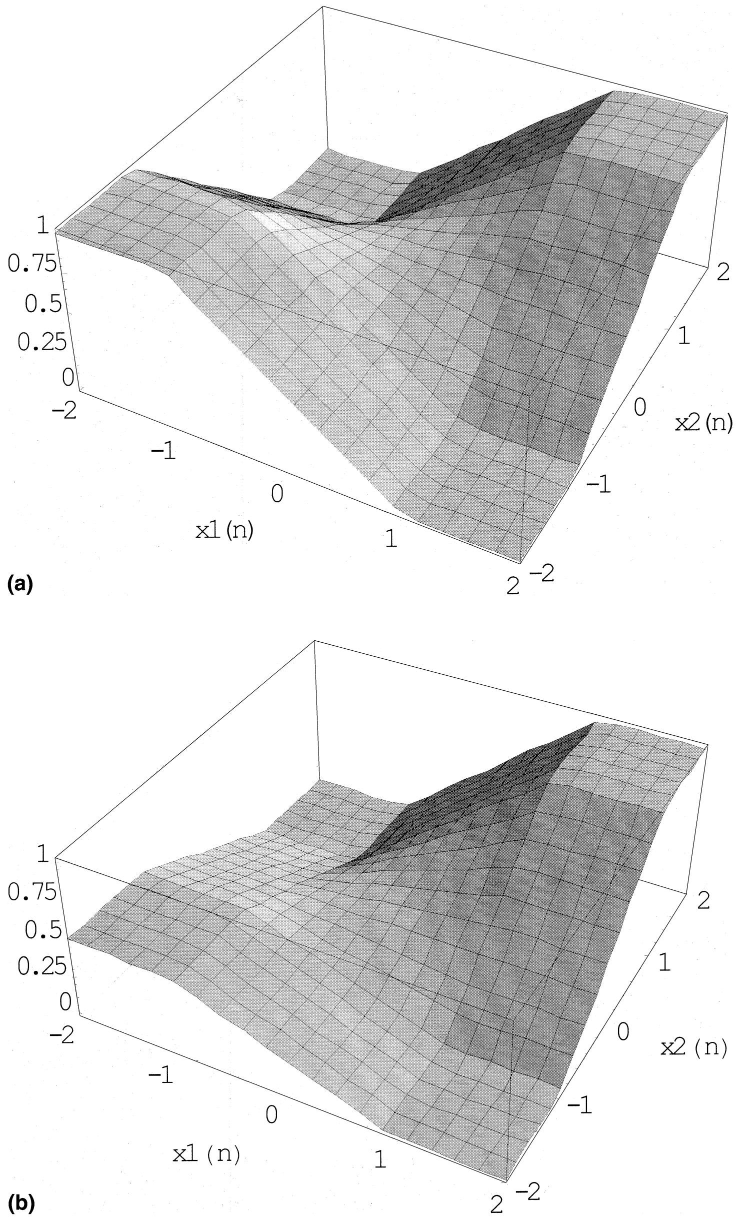
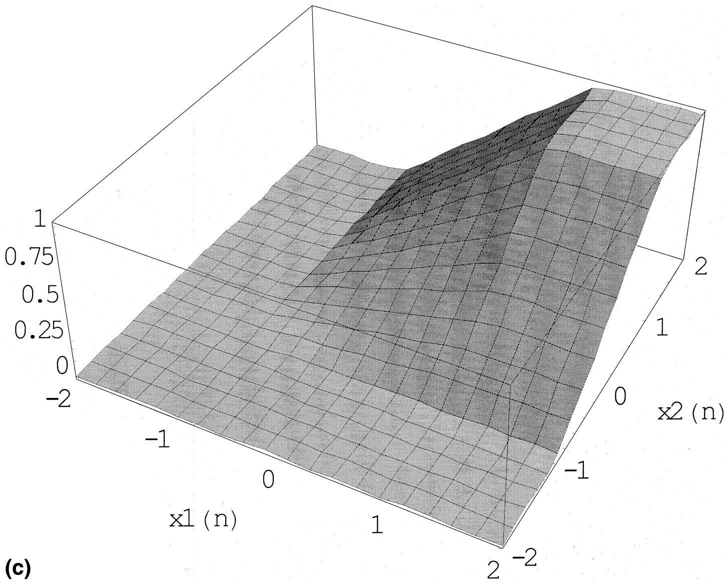


Fig. 2. Three-dimensional plots of $G(x_1, x_2)$ when (a) $k_4 = 1$; (b) $k_4 = 0.5$; and (c) $k_4 = 0$. Values of the other parameters are: $L = 1, k_1 = 1, k_2 = k_3 = 0$. The mathematical expression is given in (8) where k_5 to k_8 are 0.



(c)

Fig. 2. (Continued).

to let some parameters be zero, as did above to obtain a fuzzy PI controller. Useful and interesting nonlinear gain variation characteristics can still be generated to produce a superior controller even when the majority of k_j's are zero.

One may wonder about the stability of the control systems involving the fuzzy PID controller. We have established a simple yet powerful criterion: the fuzzy PID control system is locally stable around the equilibrium point if and only if the corresponding linear PID control system is locally stable [24]. This criterion is tight in that it is a necessary and sufficient criterion. Global stability criteria can also be established either through the Lyapunov's methods [16,18,19] or through the Small Gain Theorem [3].

This novel fuzzy PID controller is general-purpose and can be used for control problems in diverse fields. Most practical control problems are nonlinear in nature and require nonlinear control as the best solution. Because of the technical difficulties, a nonlinear control problem is usually approximated as a linear problem and solved as such using the relatively simpler linear control theory. Among all the linear controllers, the PID controller is the most popular and effective controller for its simplicity in structure and tuning [1]. Based on the above analysis of the fuzzy PID controller, we can say that if a linear PID controller works, the fuzzy PID controller will, too, and could perform even better for the same system.

Control problems in biomedicine are intrinsically nonlinear, time-varying and often associated with time-delay. They are the most challenging control problems in existence, and demand sophisticated nonlinear controllers with time-varying characteristics. Below, we will show how the above-developed fuzzy PID controller fits naturally and nicely to an important medical control problem.

4. A medical application of the fuzzy PID controller

We will first briefly describe the medical problem. After the open-heart surgery, the patient stays in a Cardiac Surgical Intensive Care Unit for recovery. Some patients exhibit hypertension, that is, mean arterial pressure (MAP) higher than normal, which should be treated to prevent potential complications. The blood pressure is lowered and maintained at a normal level (usually 80 mm Hg) through manually regulating infusion rate of a vasodilating drug, called sodium nitropresside (SNP), that is delivered to the patient intravenously. Increasing the infusion rate lowers the blood pressure whereas decreasing the rate creates an opposite effect on the blood pressure. A mathematical model has been established in the literature to describe the dynamic relationship between SNP infusion rate and change in MAP (i.e., ΔMAP) [13]

$$\frac{\Delta\mathrm{MAP}(s)}{\mathrm{SNP}(s)} = \frac{K\mathrm{e}^{-30s}(1 + 0.4\mathrm{e}^{-50s})}{1 + 40s}, \tag{11}$$

where K represents the sensitivity of patients to SNP. K is -0.72 for the typical patients, -0.18 for the insensitive patients and -2.88 for the over-sensitive patients. The ratio is 1:16 between the insensitive and the over-sensitive.

Because a lower blood pressure, say 50 mm Hg, is far more life-threatening than a higher one, say 110 mm Hg, biased change of the infusion rate must be implemented for patient's safety. The infusion rate must be reduced much more quickly when the blood pressure is quite below the normal level. The farther the blood pressure is below the normal level, the faster the rate reduction must be. The first automatic blood pressure controller implemented clinically in 1970s [13] indeed used this kind of biased control strategies. Specifically, the controller was a PI controller with a decision table that had seven rules for changing the proportional-gain and integral-gain according to the current state of the blood pressure. One of the rules doubled the controller gains whenever the blood pressure became 5 mm Hg below the normal level, trying to speed up the reduction of the infusion rate. In late 1980s, we developed a Mamdani fuzzy PI controller to clinically treat the patients with superior results to those of the linear PID controller [23].

With the fuzzy PID controller, these biased control strategies are realized in a more smooth and natural fashion. The key is to select the proper values of the design parameters. After experimenting some parameter values with the patient model (11), we found the following values adequate: $k_1 = 1, k_2 = 0.5$, $k_3 = k_4 = k_5 = k_6 = 0$, $k_7 = 0.1, k_8 = 0.85, L = 40, a_1 = -0.024, a_2 = -1.6$ and $a_3 = -25$. The sampling period was 10 s. Fig. 3 shows the simulated control performance comparisons between the fuzzy PID controller and the linear PID

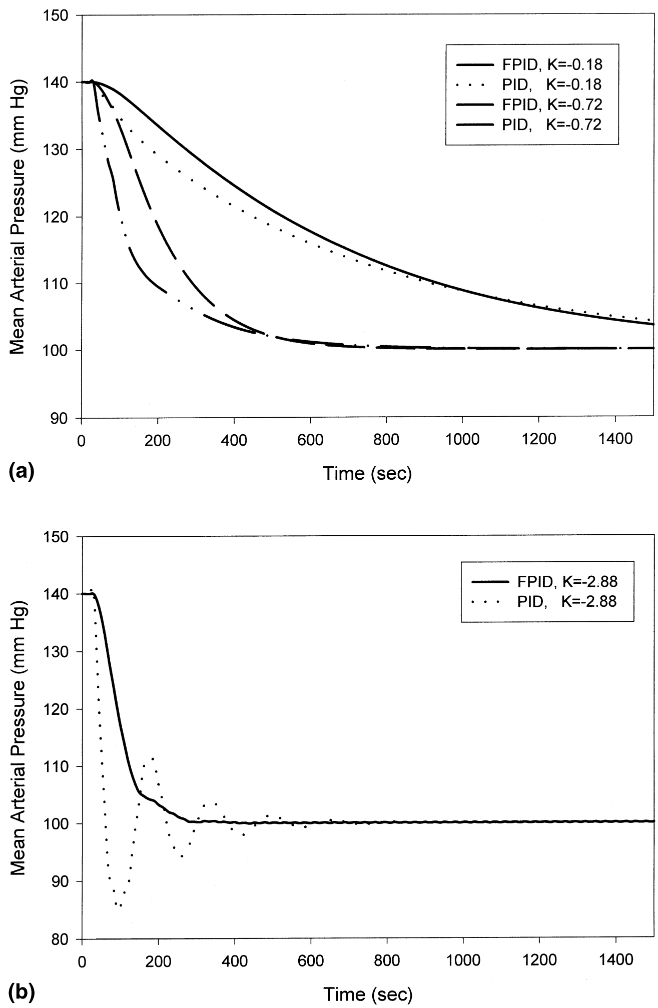


Fig. 3. Simulated performance comparisons of mean arterial pressure for patients with different sensitivities under the fuzzy PID control (FPID) and the linear PID control (PID) that uses the steady-state gains of the fuzzy controller: (a) typical patients, (b) insensitive patients, and (c) over-sensitive patients. The patient model (11) is used. The parameter values are: $k_1 = 1, k_2 = 0.5$, $k_3 = k_4 = k_5 = k_6 = 0$, $k_7 = 0.1, k_8 = 0.85, L = 40, a_1 = -0.024, a_2 = -1.6, a_3 = -25$, and the sampling period is 10 s.

controller that uses the steady-state gains of the fuzzy controller. All the typical, insensitive and over-sensitive patient cases are included. The parameters are fixed at the same values for both controllers in all the comparisons. For typical and insensitive patients, one sees from Fig. 3(a) that the rise-time of both control systems is about the same, although the response of the linear controller is slower during the initial time period (i.e., 0–300 s for the typical patients, and 0–600 s for the insensitive patients). No MAP overshoot exists for both systems. For the over-sensitive patients, however, the fuzzy control system performs much better: little overshoot and much more stable MAP response vs. dangerous 15 mm Hg overshoot and oscillatory MAP trajectory of the system controlled by the linear PI controller. The settling-time of the linear system is also significantly longer. The superiority of the fuzzy controller is especially convincing because it results in both smaller overshoot and smaller rise-time. According to the linear control theory, achieving these two seemingly contradictory control objectives simultaneously is virtually impossible [11]. One can only achieve one objective at the expense of the other. The fuzzy PID control system can do both because it is nonlinear [14].

5. Conclusions

We have developed a novel fuzzy PID controller, which is realized via our newly-introduced simplified TS rule scheme. Analytical analysis reveals that the fuzzy controller is a nonlinear PID controller with time-dependent variable gains. We have investigated the characteristics of the gains and shown the fuzzy controller to be superior to its linear counterpart due to the variable gains. To demonstrate the usefulness and effectiveness of the variable gains, we have developed a fuzzy PID controller for closed-loop control of mean arterial pressure in postsurgical patients through drug regulation. The simulation results show that the fuzzy PID control system indeed significantly outperforms the linear PID control system, especially with enhanced safety and robustness for the over-sensitive patients.

References

[1] P.B. Deshpande, Improve Quality Control on-line with PID Controllers, Chemical Engineering Progress, 1991, pp. 71–76.
[2] F. Bouslama, A. Ichikawa, Fuzzy control rules and their natural control laws, Fuzzy Sets and Systems 48 (1992) 65–86.
[3] G. Chen, H. Ying, BIBO stability of nonlinear fuzzy PI control systems, Journal of Intelligent and Fuzzy Systems 5 (1997) 245–256.
[4] A.E. Hajjaji, A. Rachid, Explicit formulas for fuzzy controller, Fuzzy Sets and Systems 62 (1994) 135–141.

[5] R. Langari, A nonlinear formulation of a class of fuzzy linguistic control algorithms, in: Proceedings of the 1992 American Control Conference, Chicago, IL, 1992.

[6] C.C. Lee, Fuzzy logic in control system: fuzzy logic controller (Parts I and II), IEEE Transactions on Systems Man and Cybernetics 20 (1990) 408–435.

[7] F.L. Lewis, K. Liu, Towards a paradigm for fuzzy logic control, Automatica 32 (1996) 167–181.

[8] H.A. Malki, H.D. Li, G. Chen, New design and stability analysis of fuzzy PD controllers, IEEE Transactions on Fuzzy Systems 2 (1994) 245–254.

[9] F. Matia, A. Jimenez, R. Galan, R. Sanz, Fuzzy controllers: Lifting the linear-nonlinear frontier, Fuzzy Sets and Systems 52 (1992) 113–129.

[10] J.M. Mendel, Fuzzy logic systems for engineering: a tutorial, Proceedings of the IEEE 83 (1995) 345–377.

[11] K. Ogata, System Dynamics, third ed., Prentice-Hall, New York, 1996.

[12] Y.M. Pok, J.X. Xu, An analysis of fuzzy control systems using vector space, in: Proceeding of the Second IEEE International Conference on Fuzzy Systems, San Francisco, California, 1993, pp. 363–368.

[13] L.C. Sheppard, Computer control of the infusion of vasoactive drug, Annuals of Biomedical Engineering 8 (1980) 431–444.

[14] J-J.E. Slotine, W. Li, Applied nonlinear control, Prentice-Hall, New York, 1991.

[15] T. Takagi, M. Sugeno, Fuzzy identification of systems and its applications to modeling and control, IEEE Transactions on Systems Man and Cybernetics 15 (1985) 116–132.

[16] K Tanaka, T. Ikeda, H.O. Wang, Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: Quadratic stabilizability H^{∞} control theory and linear matrix inequalities, IEEE Transactions on Fuzzy Systems 4 (1996) 1–13.

[17] K.L. Tang, R.J. Mulholland, Comparing fuzzy logic with classical controller designs, IEEE Transactions on Systems Man and Cybernetics 17 (1987) 1085–1087.

[18] H.O. Wang, K. Tanaka, M.F. Griffin, An approach to fuzzy control of nonlinear systems: Stability and design issue, IEEE Transactions on Fuzzy Systems 4 (1996) 14–23.

[19] L.X. Wang, Adaptive Fuzzy Systems and Control, Prentice-Hall, New York, 1994.

[20] R.R. Yager, D.P. Filev, Essentials of Fuzzy Modeling and Control, Wiley, New York, 1994.

[21] J. Yen, R. Langari, L.A. Zadeh, Industrial Applications of Fuzzy Control and Intelligent Systems, IEEE Press, New York, 1995.

[22] H. Ying, W. Siler, J.J. Buckley, Fuzzy control theory: a nonlinear case, Automatica 26 (1990) 513–520.

[23] H. Ying, M. McEachern, D. Eddleman, L.C. Sheppard, Fuzzy control of mean arterial pressure in postsurgical patients with sodium nitroprusside infusion, IEEE Transactions on Biomedical Engineering 39 (1992) 1060–1070.

[24] H. Ying, The simplest fuzzy controllers using different inference methods are different nonlinear proportional-integral controllers with variable gains, Automatica 29 (1993) 1579–1589.

[25] H. Ying, An analytical study on structure stability and design of general Takagi–Sugeno fuzzy control systems, Automatica 34 (1998).

[26] H. Ying, General Takagi–Sugeno fuzzy systems with simplified linear rule consequent are universal controllers models and filters, Information Sciences 108 (1998) 91–107.

[27] H. Ying, The Takagi–Sugeno fuzzy controllers using the simplified linear control rules are nonlinear variable gain controllers, Automatica 34 (1998) 157–167.

[28] H. Ying, Constructing nonlinear variable gain controllers via the Takagi–Sugeno fuzzy control, IEEE Transactions on Fuzzy Systems 6 (1998) 226–234.