Dynamics of Firms and Trade in General Equilibrium*

(preliminary)

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Abstract

This paper develops a dynamic general equilibrium model that attempts to reconcile the observation that aggregate movements of exports and imports are disconnected from real exchange rate movements, while firm-level exports co-move significantly with the real exchange rate. Firms are heterogenous, facing recurrent aggregate and firm-product specific productivity shocks, choose which goods to export, and decide to enter and exit the business endogenously. We calibrate and estimate the model with both aggregate and firm level data from Japan.

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1 Introduction

Figure 1a displays the aggregate real values of exports and imports together with the real exchange rate in Japan during the period of 1973-2011 in logarithmic scale. The real exchange rate is defined as the relative price between Japan's trading partners and Japan.[1] As the trading partners' goods become relatively more expensive, we expect that Japanese exports would increase and imports would decrease through substitution effect. However, such a relationship between trade and the real exchange rate is not evident in Figure 1a. As the Japanese real exchange rate depreciates, exports do not necessarily increase. During the entire sample period, the correlation coefficient between exports and the real exchange rate is -0.02, and that for imports is -0.18 after detrending all the annual data by the log-linear trends. This lack of correlation, or correlation contrary to what we expect is an example of the so called "exchange rate disconnect puzzle," one of the six major puzzles in international macroeconomics according to Obstfeld and Rogoff (2000). This weak or opposite correlation between aggregate exports, imports and the exchange rate is observed in many other countries as well (see Hooper, Johnson, and Marquez (2000), and Dekle, Jeong, and Ryoo (2007)).[2,3]

Interestingly, the exchange rate disconnect is sensitive to the method of detrending in Japan. If we use Hodrick and Prescott filter of a smooth coefficient of 100 as in Figure 1b, then aggregate exports moved in the same direction with the real exchange rate with the correlation coefficient of 0.51. But aggregate imports also moved in the same direction with a correlation coefficient of 0.18, and the co-movement between the real exchange rate and aggregate import became stronger since 1990. (The correlation coefficient for 1990-2011 is 0.54 in the Hodrick-Prescott filtered data and 0.25 for the log-linear detrended data.) These co-movements during this

[1] The real exchange rate is measured as the ratio of the weighted average of the prices of Japan's major trading partners in yen term to Japanese prices, where the weights are the time-varying trading shares from the Bank of Japan. Aggregate real value of exports and imports are measured in billions of year 1998 yen using deflator of export and import in National Income and Product Account. (Source: Cabinet Office of Japan.)

[2] The list of other countries showing such weak correlation is Canada, France, Germany, Italy, the U.K., and the U.S. This empirical puzzle was first documented by Orcutt (1950).

[3] Note that this "exchange rate disconnect puzzle" is different from the so called "J-curve effect." The exchange rate disconnect puzzle is about the lack of association between the movements of exchange rates and *gross export quantities* while the J-curve effect is about the sluggish and J-shaped adjustment of *net export* in response to an improvement in the terms of trade. See Backus, Kehoe, and Kydland (1994) for the J-curve effect.

period suggest that a general equilibrium linkage may be important in order to understand the dynamics of trade and exchange rates in Japan, where intermediate goods trade is increasingly more important in imports and exports.

In contrast to the results using aggregate data, recent empirical studies using firm-level data have found a more robust relationship between exports and the exchange rate. Among other studies, Verhoogen (2008) finds that following the 1994 peso devaluation, Mexican firms increased their exports. Fitzgerald and Haller (2008), Dekle and Ryoo (2007), and Tybout and Roberts (1997) find a positive association between exports and an exchange rate depreciation for Irish, Japanese and Colombian firms, respectively.

The column 1 of Table 1 reports panel regression using our panel data of Japanese firms listed on the stock exchanges of Japan.[4] The dependent variable is firm level real export values (export value divided by GDP deflator) from 1985 to 1999, and the regressors are the aggregate real exchange rate, the weighted average of real GDP of Japanese trading partners, Japanese aggregate TFP, firm level TFP and firm fixed effect.[5] The regression coefficient of export values on the real exchange rate is significant and equal to 0.37 (i.e., 1% devaluation is associated with an increase of export value by 0.37%). The regression coefficients of export values on foreign GDP and aggregate TFP are both positive (0.40 and 0.38) and significant. In addition, the regression coefficient of measured firm-level TFP is equal to 2.1 and significant.

In this paper, we develop a dynamic general equilibrium model with heterogeneous firms that attempts to reconcile the different responses of exports and imports to exchange rates at both the aggregate and at the firm levels. Our model is a real business cycle model of a small open economy with a rich production structure. Firms are heterogeneous, facing re-

[4]The raw data used here and in our paper cover mostly firms which are publicly traded in stock exchanges in the Tokyo Stock Exchange and partially in the Mothers (comparable to NASDAQ). The particular data set that we use were compiled by the Development Bank of Japan (or "Kaigin," in Japanese prior to the 2008 re-organization of government-owned enterprises, when the name of the bank was changed). Kaigin data cover a respectable portion (60 percent in 2000) of the entire Japanese economy in terms of total sales. However, the number of employees in Kaigin data are only 40 percent of all employees (Fukao, et. al., 2008). The criteria for listed firms are based on market capitalization, profit and the other measures in the past, and the criteria has evolved over time. See http://www.tse.or.jp/english/rules/listcriteria/index.html for the detail.

[5]The weight of average real GDP of Japanese trading partner is time varying share of aggregate Japanese export. Japanese aggregate TFP is obtained from Hayashi and Prescott (2002).

current aggregate and firm-product specific productivity shocks, and decide to enter and exit endogenously.

We make a few choices to model heterogeneous firms to reflect our panel data of Japanese firms listed on the stock exchanges of Japan. In a well-known paper, Eaton and Kortum (2002) and Melitz (2003) showed that, when firms are heterogeneous in its total factor productivity and need to cover costs for exports, only highly productive and large firms export. Das, Roberts, and Tybout (2007) provide an empirical study showing that the difference in total factor productivity among producers explains whether they export or not, which is the so-called extensive margin of trade. In our Japanese panel data, we also find a strong relationship between firm size and exporting status. The average total sales of the incumbent exporting firm is about twice as much as the non-exporting firms. When firms are different only because their total factor productivity are different, however, the share of exports in total sales (export share) should be strongly correlated with firm size among the exporting firms (in addition to whether or not the firm exports at all). Our Japanese firm level data do not support this prediction. The correlation between the export share and total sales is weak. The average correlation coefficient is only 0.08 among all firms. Among exporting firms, the correlation coefficient becomes even lower at 0.05. This weak correlation remains robust even after controlling for industry and year effects.

Another interesting observation from Japanese firm level data is that a significant number of firms stay in business even if their profits are negative. About 8 percent of Japanese firms in our sample report negative profits in a given year. This fraction becomes even bigger at 11 percent among the firms who always export. Despite such negative profits, Japanese listed firms do not easily exit from the business, although entry into and exit from the export market are more frequent.[6]

[6]Strictly speaking, in our sample of Japanese listed firms, firms that drop out of the sample are "delisted." Of the 2386 firms in our sample that we examine between 1985 and 1999, 104 firms became "delisted." We examined the circumstances surrounding the de-listing of all of these 104 firms and the vast majority were delisted because of bankruptcy or "ceasing to do business." A small number disappeared as independent firms because of mergers with stronger firms. Thus, we are on reasonably firm ground when we equate a firm that has been "delisted" as essentially "exiting" from production.

Columns 2 to 7 of Table 1 present suggestive evidence that heterogeneity in profitability rather than total sales is important for explaining the heterogeneous reaction of firm exports to the real exchange rate. In columns 2 and 3 of Table 1, we split the firms into a high-profitability group (25%) and a low-profitability group (75%) based on the average profit-sales ratio in sample.[7] When we do the panel regression separately with firm fixed effects, we find that the regression coefficient of firm export values on the real exchange rate is marginally significantly larger for the low profitable firms than for the highly profitable firms. A 1% devaluation is associated with an increase of export values by 0.39% for low-profitable firms and 0.28% for highly profitable firms. When we split samples between big employers (35%) and small employers (65%, which are still not so small in Kaigin data), the regression coefficient of firm exports on the real exchange rate is larger for small employers than for large employers. If we divide samples between large firms (30%) and small firms (70%) in terms of average total sales in columns 6 and 7, however, the regression coefficients of firm export values on the real exchange rate are almost identical and small firms are less sensitive to foreign GDP and aggregate TFP than the large firms. This suggests that the export of marginally profitable firms rather than small sales firms are sensitive to changes in the real exchange rate and other aggregate conditions.

Previous empirical work has shown that product churning within existing firms is important. Bernard et. al. (2010) document that 94 percent of product additions by U.S. manufacturing firms occur within their existing production facilities and that 68 percent of firms change their product mix in a 5-year period between census. They show that product creation by existing and new firms accounts for 47 percent and the lost value from product destruction by existing and exiting firms accounts for 44 percent of output change within a 5-year period. Kawakami and Miyagawa (2013) use the Japanese Census of Manufacturers to find that the addition of new products accounted for a large part of output growth. For example, from 1998 to 2000, aggregate manufacturing output expanded by 7.3 percent. Of this expansion, the

[7] Because Kaigin data is not balanced, the proportion of high profitability firms is not equal to the proportion of observations of high profitability firms,

addition of new products accounted for 53.8 per cent while the dropped products subtracted 37.8 per cent so that the net addition of products account for 15.9 percent gain. The net entry of new firms accounted for 9.9 percent, while the net decline in the production of existing products subtracted 18.5 percent from output. (Thus we have the total change of output to be $15.9 + 9.9 - 18.5 = 7.3\%$).[8]

Given these empirical observations, we choose firms to produce multiple products and are heterogeneous in terms of the number of the products as well as the productivity distribution. Firms choose which products to produce and which products to export. Thus Eaton-Kortum and Melitz style extensive margin adjustment is mainly at the product level. This firm and product level heterogeneity helps explain the weak relationships among size, the export share and profitability in our firm-level data. Our firms also face recurrent idiosyncratic productivity shocks to each product, and thus they may not stop producing existing products with temporary negative profits in order to enjoy the option value of continuing production. We calibrate and estimate our model with both firm-level panel data and aggregate time series data. We then carry out quantitative exercises regarding the impact of shocks to productivity and preferences on aggregate and firm-level exports and other variables of interest.

Some other papers have tried to reconcile these aggregate and firm level exchange rate elasticities, but mostly in a partial equilibrium or static framework. Dekle, Jeong, and Ryoo (2007) show that in the aggregate export equation derived by consistently aggregating the firm level export equations, where industry level productivity and the export share are controlled for, the disconnect puzzle disappears. Berman, Martin, and Mayer (2009) use a model with heterogeneous firms to show that high productivity firms (who are heavily involved in exports) will raise their markups and prices instead of increasing their export quantities in response to an exchange rate depreciation. The authors show that this selection effect of a small quantity response of high productivity firms can explain the weak impact of exchange rate movements in

[8]Berman, Martin, and Meyer (2009) and Chatterjee, Dix-Carneiro and Vichynand (2013) examine how exchange rate fluctuations change the product mix of exportable goods, using French and Brazilian firm- and product-level data, respectively. They find that an exchange rate depreciation raises the number of goods exported by each firm.

aggregate data. There are some other recent papers that have tried to reconcile this discrepancy in a general equilibrium framework. Imbs and Majean (2009) and Feenstra, Russ, and Obstfeld (2010) show that the aggregation of heterogeneous industrial sectors can result in an aggregation bias in the elasticity of exports and imports with respect to the real exchange rate. Both of these papers are static and examine only the steady-state.

While our paper's initial motivation is to reconcile the differences in the reactions of trade volumes to the real exchange rate between aggregate and firm levels, our model has a broader contribution to the recently growing literature of macroeconomic dynamics and trade. Ghironi and Melitz (2005) and Alessandria and Choi (2007) analyze the effect of the extensive margin of production and exports on the international transmission of shocks to productivity and policy across countries. Fattal Jaef and Lopez (2013) add capital accumulation and endogenous labor supply to the Ghironi and Melitz (2005) model. Bilbie, Ghironi, and Melitz (2012) studies the role of endogenous entry of new products in propagating business cycle fluctuations. While these papers captures some aspects of aggregate dynamics and adjustment of firms, we believe that our model is the first model which attempts to capture the cyclical fluctuations of aggregate variables and firm level variables in a consistent general equilibrium model with firm-product heterogeneity.[9]

2 Model

There is a continuum of home firms $h \in \mathcal{H}_t$. Home firm h produces possibly multiple I_{ht} number of differentiated products for home and export markets at date t. Firm h produces q^H_{hit} amount of the i th differentiated product for the home market using labor l^H_{hit} and imported intermediate input m^{*H}_{hit}, according to a constant returns to scale technology

$$q^H_{hit} = a_{hit} Z_t \left(\frac{l^H_{hit}}{\gamma_L}\right)^{\gamma_L} \left(\frac{m^{*H}_{hit}}{1-\gamma_L}\right)^{1-\gamma_L}, \text{ for } i = 1, 2, .., I_{ht}.$$

[9] The other related papers on aggregate dynamics and trade include Arkolakis (2009), Atkeson and Burnstein (2010), Bernard, Redding and Schott (2011) and Klette and Kortum (2004). In terms of modeling, Klette and Kortum is the closest to ours, except they only study the steady state.

A variable a_{hit} is the productivity of firm h to produce the i th differentiated product at date t, Z_t is the aggregate productivity shock, and $\gamma_L \in (0,1)$ is the labor share. We assume no two firms produce the same product and distinguish the differentiated product by (h,i) - the i th product of firm h. Producing a differentiated product for export market has the same marginal productivity with the production for home market, but requires a constant fixed cost ϕ in terms of input composite for each variety as

$$q^F_{hit} = a_{hit} Z_t \left[\left(\frac{l^F_{hit}}{\gamma_L} \right)^{\gamma_L} \left(\frac{m^{*F}_{hit}}{1-\gamma_L} \right)^{1-\gamma_L} - \phi \right], \text{ for } i = 1, 2, ..I_{ht}.$$

Home final goods for home market is produced from all the differentiated products of home market according to a constant returns to scale CES production function as

$$Q^H_t = \left[\int_{h \in \mathcal{H}_t} \left(\sum_{i=1}^{I_{ht}} q^{H\ \frac{\theta-1}{\theta}}_{hit} \right) dh \right]^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$ is the elasticity of substitution between products. Home final goods for export market is produced from the differentiated products of export market as

$$Q^F_t = \left[\int_{h \in \mathcal{H}_t} \left(\sum_{i=1}^{I_{ht}} q^{F\ \frac{\theta-1}{\theta}}_{hit} \right) dh \right]^{\frac{\theta}{\theta-1}}.$$

Any new entrant who pays a sunk cost κ_{Et} in terms of home final goods at date t draws an opportunity of producing a new product from date $t+1$ with probability λ_E. The productivity a_{hit+1} of a new product (h,i) is distributed according to a Pareto distribution with lower bound parameter 1 and the shape parameter α. That is

$$\begin{aligned} \text{Prob}(a_{hit+1} &\leq a) = F(a) = 1 - a^{-\alpha}, \text{ for } a \in [1, \infty), \text{ where} \\ \alpha &> 1 \text{ and } \alpha > \theta - 1. \end{aligned} \qquad \text{(Assumption 1)}$$

The density function of the Pareto distribution is $f(a) \equiv F'(a) = \alpha a^{-(\alpha+1)}$, for $a \in [1, \infty)$. (Assumption 1) says that the shape parameter α of the Pareto distribution is larger than one and $\theta - 1$, which later guarantees that CES aggregates of final goods is well defined.

An incumbent firm who already has existing products must pay fixed maintenance cost κ (in terms of home final goods) for each product in order to produce and maintain its productivity.

That is, the firm that wants to maintain I_{ht} number of products must pay κI_{ht}. If the firm does not pay the fixed cost for an existing product, it loses the technology for this product for sure and forever. For the product which the firm pays the maintenance cost, the same productivity is maintained in the next period with probability $1-\delta$ and looses the productivity with probability δ:

$$a_{hit+1} = \begin{cases} a_{hit}, \text{ with probability } 1-\delta \\ 0, \text{ with probability } \delta \end{cases}.$$

In addition, independently from the success or failure of maintaining the existing product, each product that firm pays the maintenance cost yields an opportunity to produce another new product with probability $\lambda\delta$, and the productivity of the new product is distributed according to the same Pareto distribution of new products. We assume

$$\lambda < 1. \qquad \text{(Assumption 2)}$$

Thus, while the number of products each firm produces may increase or decrease depending on the success or failure of the maintenance as well as the draws of new products, the number of products tend to decline on average. This guarantees there are new entries in the neighborhood of the steady state. Because firms are heterogeneous in the number of products as well as in the productivity distribution, we can show that there are only weak relationships among size, the export share and profitability across firms - an important feature of our Japanese data.

Home final goods are either consumed by households and government, or used for the entry sunk costs of the new entrants, or for the maintenance costs of the existing products,

$$Q_t^H = C_t + G_t + \kappa_{Et} N_{Et} + \kappa N_t. \qquad (1)$$

Variables C_t and G_t are consumption of households and government, N_{Et} is the measure of entering firms, and N_t is the measure of existing differentiated products which incumbent firms try to maintain. We consider that the costs of drawing new technology and maintaining old technology include both intangible and tangible capital investment, and we abstract from the other tangible capital investment. Although each new entrant takes the sunk cost of entry as

exogenous, it is an increasing function of the number of entries in the aggregate as

$$\kappa_{Et} = \kappa_E \left(\frac{N_{Et}}{N_E} \right)^{\eta}, \tag{2}$$

where κ_E and η are positive parameters and N_E is the steady state measure of the new entrants.

A representative household supplies labor L_t to earn wage income, consumes final goods C_t, and holds home and foreign short-term real bonds D_t and D_t^* to maximize the expected utility,

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \psi_0 \frac{L_t^{1+1/\psi}}{1+1/\psi} + \xi_t^* \ln D_t^* \right),$$

subject to the budget constraint

$$C_t + D_t + \epsilon_t D_t^* = w_{Lt} L_t + \Pi_t + R_{t-1} D_{t-1} + \epsilon_t R_{t-1}^* D_{t-1}^* - T_t. \tag{3}$$

Variable ϵ_t is real exchange rate (the relative price of foreign and home final goods), w_{Lt} is real wage rate, Π_t is the sum of real net profits distribution of firms, R_{t-1} and R_{t-1}^* are home and foreign one-period real gross interest rates from date $t-1$ to t, and T_t is lump-sum tax. Note that, although both home and foreign bonds are used as means of saving, we assume that the holding of foreign bonds facilitates international transactions, hence is in the utility function. The utility from holding foreign bonds is subject to the "liquidity shock" ξ_t^*.[10]

We assume that all home imports are intermediate inputs to production, and that the imported input price is normalized to be one in terms of foreign final goods. We assume that foreign aggregate demand for home exports are given by

$$Q_t^F = \left(p_t^F \right)^{-\varphi} Y_t^*, \tag{4}$$

where Y_t^* is an exogenous foreign demand parameter and p_t^F is an endogenous export price in terms of foreign final goods. A parameter φ is the elasticity of demand for home export final goods, which we assume it to be relatively inelastic

$$0 < \varphi < 1. \qquad \text{(Assumption 3)}$$

[10] The idea is similar to money in the utility function. Section 5.3.8 of Obstsfeld and Rogoff (1996) presents a model with both home and foreign money in the utility function to analyze the phenomenon of dollarization. Alternatively, we can formulate that home households face an international borrowing constraint and that the utility from foreign bond holding is $\xi^* \ln(D_t^* + \zeta_t^*)$ where $\zeta_t^* > 0$ is the credit line of foreign lenders to the home representative household which is stochastic. We ignore the utility of home bonds for simplicity.

We assume that foreigners do not hold home bond. Then, foreign bond holdings D_t^* of the home household evolves along with exports and imports as

$$D_t^* = R_{t-1}^* D_{t-1}^* + p_t^F Q_t^F - M_t^*, \tag{5}$$

where $M_t^* = \int_{h\in\mathcal{H}_t} \left[\sum_{i=1}^{I_{ht}} (m_{hit}^{*H} + m_{hit}^{*F})\right] dh$ is the total imported input of the home country.

Because tax is lump sum and households are infinitely lived without financing constraint, Ricardian equivalence theorem holds. Thus without loss of generality, we consider the government has the balanced budget with zero net supply of bond

$$\begin{aligned} G_t &= T_t \\ D_t &= 0. \end{aligned} \tag{6}$$

Here, because the foreigners do not hold home bond, the home bond holding of the home representative household is equal to zero in equilibrium.

3 Competitive Equilibrium

3.1 Firm's Production

The market for final goods and factors of production are perfectly competitive, while the market for differentiated products are monopolistically competitive. From the usual feature of the CES production function of final goods from differentiated products, each firm faces a downward sloping demand curve for the product variety in home and foreign markets as a function of its prices p_{hit}^H and p_{hit}^F, such that

$$\begin{aligned} q_{hit}^H &= \left(\frac{p_{hit}^H}{p_t^H}\right)^{-\theta} Q_t^H, \\ q_{hit}^F &= \left(\frac{p_{hit}^F}{p_t^F}\right)^{-\theta} Q_t^F, \end{aligned}$$

where p_t^H and p_t^F are the aggregate price indices of final goods in home and export markets given by

$$\begin{aligned} p_t^H &= \left[\int_{h\in\mathcal{H}_t}\left(\sum_{i=1}^{I_{ht}}\left(p_{hit}^H\right)^{1-\theta}\right)dh\right]^{\frac{1}{1-\theta}} = 1, \qquad (7)\\ p_t^F &= \left[\int_{h\in\mathcal{H}_t}\left(\sum_{i=1}^{I_{ht}}\left(p_{hit}^F\right)^{1-\theta}\right)dh\right]^{\frac{1}{1-\theta}}. \end{aligned}$$

We use home final goods as the numeraire in the home market (i.e., $p_t^H = 1$), and foreign final goods as the numeraire in the foreign market.

Recall that the production function of all the differentiated products have a common component: Cobb-Douglas function of input composite of labor and imported intermediate input. Moreover, the ratios of labor to imported intermediate input are equal across firms when firms minimize the costs under perfectly competitive factor market. Let x_{hit}^H and x_{hit}^F be input composites used for producing differentiated products for the home and export markets. Then the production function can be simplified to

$$\begin{aligned} q_{hit}^H &= a_{hit}Z_t \cdot x_{hit}^H,\\ q_{hit}^F &= a_{hit}Z_t \cdot \left(x_{hit}^F - \phi\right). \end{aligned}$$

Then, the sum of input composite use is equal to the aggregate production of the input composite,

$$\int_{h\in\mathcal{H}_t}\left(\sum_{i=1}^{I_{ht}}(x_{hit}^H + x_{hit}^F)\right)dh \equiv X_t = \left(\frac{L_t}{\gamma_L}\right)^{\gamma_L}\left(\frac{M_t^*}{1-\gamma_L}\right)^{1-\gamma_L}.$$

Because the price of imported inputs at home is equal to the real exchange rate (due to our choice of numeraire), the cost minimization implies that the unit cost of the input composite w_t and the demands for labor and imported inputs are given by

$$\begin{aligned} w_t &= (w_{Lt})^{\gamma_L}\epsilon_t^{1-\gamma_L}, \qquad (8)\\ L_t &= \gamma_L\frac{w_t X_t}{w_{Lt}}, \qquad (9)\\ M_t^* &= (1-\gamma_L)\frac{w_t X_t}{\epsilon_t}. \qquad (10) \end{aligned}$$

Maximizing current profits, each firm sets the product prices p_{hit}^H and p_{hit}^F as mark-ups over their unit production cost such that

$$p_{hit}^H = \frac{\theta}{\theta-1}\frac{w_t}{a_{hit}Z_t} \equiv p_t^H(a_{hit}), \tag{11}$$

$$p_{hit}^F = \frac{\theta}{\theta-1}\frac{w_t/\epsilon_t}{a_{hit}Z_t} \equiv p_t^F(a_{hit}). \tag{12}$$

Then, the quantities q_{hit}^H and q_{hit}^F of each product for home and foreign market depend on its own productivity a_{hit} only (aside from aggregate variables) such that

$$q_{hit}^H = \left(\frac{p_t^H(a_{hit})}{p_t^H}\right)^{-\theta} Q_t^H \equiv q_t^H(a_{hit}), \tag{13}$$

$$q_{hit}^F = \left(\frac{p_t^F(a_{hit})}{p_t^F}\right)^{-\theta} Q_t^F \equiv q_t^F(a_{hit}). \tag{14}$$

That is, although each firm may produce multiple differentiated products, firm's choice on how much to produce and whether to continue to produce for each product is independent from the choices of other products, like the "amoeba management".[11]

We conjecture that in equilibrium, all firms choose to pay the fixed maintenance cost for the product with positive productivity, (which we will verify later). Then, the total measure of differentiated products evolves through maintenance and new entries as:

$$N_{t+1} = (1-\delta+\delta\lambda)\,N_t + \lambda_E N_{Et}. \tag{15}$$

The first term in the right hand side is the measure of maintained and spin-out products in which $1-\delta+\delta\lambda<1$ by Assumption 2. The second term is the introduction of new products by entrants. Let $N_t(a)$ be the measure of products with productivity a. Then, from the specific feature of our idiosyncratic productivity evolution, $N_t(a)$ is a proportional to N_t as:

$$N_t(a) = f(a)N_t.$$

Thus, from (7) and (11), the price index for home final goods for the home market becomes

$$1 = p_t^H = \left[\int_1^\infty p_t^H(a)^{1-\theta} N_t f(a) da\right]^{\frac{1}{1-\theta}} = \frac{\theta}{\theta-1}\frac{w_t}{A_t^H}.$$

[11] The founder of Kyocera (a Japanese technology company), Kazuo Inamori, proposes an "amoeba" management style, in which each production unit makes relatively independent production decisions, while the number of production units multiply and shrink like "amoebas." Our technology can be seen as a justification for the "amoeba" management style.

Variable A_t^H is the aggregate productivity of home firms in home market, given by

$$A_t^H \equiv \overline{a} N_t^{\frac{1}{\theta-1}} Z_t, \tag{16}$$

and $\overline{a}$ is the average productivity of products that are produced for home market,

$$\overline{a} \equiv \left[\int_1^\infty a^{\theta-1} f(a) da\right]^{\frac{1}{\theta-1}} = \left(\frac{\alpha}{\alpha+1-\theta}\right)^{\frac{1}{\theta-1}}.$$

Note that this implies that the unit cost of input composite is given by

$$w_t = \frac{\theta-1}{\theta} A_t^H. \tag{17}$$

Due to the presence of the fixed cost of exporting, we conjecture that there is a lower bound of productivity level $\underline{a}_t > 1$ at which the product makes zero profit for exporting such that

$$\pi_t^F(\underline{a}_t) = \epsilon_t p_t^F(\underline{a}_t) q_t^F(\underline{a}_t) - w_t\left[\frac{q_t^F(\underline{a}_t)}{\underline{a}_t Z_t} + \phi\right] = w_t\left[\frac{1}{\theta-1}\frac{q_t^F(\underline{a}_t)}{\underline{a}_t Z_t} - \phi\right] = 0, \tag{18}$$

Thus only a fraction $Prob(a \geq \underline{a}_t) = (\underline{a}_t)^{-\alpha} < 1$ of maintained products are exported.

In Appendix A, we show that the lower bound of productivity for export which clears the export market is given by

$$\underline{a}_t = \left[\frac{\alpha(\theta-1)\phi}{\alpha+1-\theta}\frac{A_t^H N_t}{\epsilon_t^\varphi Y_t^*}\right]^{\frac{\theta-1}{\alpha(\theta-1)+(\alpha+1-\theta)(1-\varphi)}}. \tag{19}$$

(The details of the competitive equilibrium are all in Appendix A.) We verify the conjecture that $\underline{a}_t > 1$ so that some products with low productivity are not exported, if and only if

$$\frac{\epsilon_t^\varphi Y_t^*}{A_t^H N_t} < \frac{\alpha(\theta-1)\phi}{\alpha+1-\theta}. \tag{Condition 1}$$

If this condition is not satisfied, all home products would be exported, which contradicts with the data. Thus, we restrict our attention to the case where Condition 1 is satisfied.

The export sales S_t^F in terms of home final good turns out to be

$$\begin{aligned} S_t^F &\equiv \epsilon_t p_t^F Q_t^F \\ &= (\underline{a}_t)^{\frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}} \epsilon_t^\varphi Y_t^*. \end{aligned} \tag{20}$$

Our price and quantity index take into account the effect of the varieties of products in the market. If the measured data do not fully take into account the changes in the varieties of products, then we have measurement errors. This problem is particularly serious for export, because the fraction of products exported can change quickly. Thus, instead of looking at the price and quantity of export separately, we examine the implication for the real export sales value in terms of home final goods or foreign final goods.

3.2 Market Clearing and Free Entry

From the utility maximization of the representative household, we have

$$1 = R_t E_t \left(\Lambda_{t,t+1}\right), \tag{21}$$

$$\xi_t^* \frac{C_t}{D_t^*} = \epsilon_t - R_t^* E_t \left(\Lambda_{t,t+1}\epsilon_{t+1}\right), \tag{22}$$

$$w_{Lt} = \psi_0 L_t^{\frac{1}{\psi}} C_t, \tag{23}$$

where $\Lambda_{t,t+1} = \beta C_t / C_{t+1}$. The first equation is a standard Euler equation for home bond holding. The second equation is an Euler equation for foreign bond holding, where the left hand side is the marginal rate of substitution between foreign bond service and consumption and the right hand side term is the opportunity cost of holding one unit of the foreign bond for one period. The third equation is the labor supply condition.

We show in Appendix that the market clearing condition of labor and input composite implies

$$X_t = \frac{1}{\gamma_L(\psi_0 C_t)^{\psi}} \left[\frac{{w_t}^{1-\gamma_L+\psi}}{\epsilon_t^{(1-\gamma_L)(1+\psi)}}\right]^{\frac{1}{\gamma_L}} \tag{24}$$

$$= X_t^H + \phi \frac{\theta\alpha + 1 - \theta}{\alpha + 1 - \theta} \left(\underline{a}_t\right)^{-\alpha} N_t. \tag{25}$$

where X_t^H denote the aggregate composite input use for the home market. The home final goods market clearing implies

$$C_t + G_t + \kappa_E \left(\frac{N_{Et}}{N_E}\right)^{\eta} N_{Et} + \kappa N_t = A_t^H X_t^H. \tag{26}$$

From (5), (10) and (20), foreign bond holding evolves as

$$D_t^* = R_{t-1}^* D_{t-1}^* + (\underline{a}_t)^{\frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}} \epsilon_t^{\varphi-1} Y_t^* - (1-\gamma_L)\frac{w_t X_t}{\epsilon_t} \tag{27}$$

Let $V_t(a)$ be the value of the product with productivity a at the beginning of period (for which the fixed cost of maintenance is paid). The Bellman equation is

$$\begin{aligned} V_t(a) &= \pi_t^H(a) + \pi_t^F(a) - \kappa \\ &\quad + E_t \Lambda_{t,t+1} \left[(1-\delta) V_{t+1}(a) + \delta\lambda \int_1^\infty V_{t+1}(a') f(a') da' \right], \end{aligned}$$

where $\pi_t^H(a)$ and $\pi_t^F(a)$ are profit arising from selling a product with productivity $a_{hit} = a$ in the home and export markets. The free entry condition for a potential entrant is

$$\kappa_{Et} = \lambda_E E_t \left(\Lambda_{t,t+1} \overline{V}_{t+1} \right), \tag{28}$$

where $\overline{V}_t$ is the average value of the products produced as

$$\begin{aligned} \overline{V}_t &\equiv \int_1^\infty V_t(a) f(a) da \\ &= \overline{\pi}_t - \kappa + (1-\delta+\delta\lambda) E_t(\Lambda_{t,t+1} \overline{V}_{t+1}), \end{aligned} \tag{29}$$

and $\overline{\pi}_t$ is the average profit of the products with positive productivity $\overline{\pi}_t \equiv \int_1^\infty \left\{ \pi_t^H(a) + \pi_t^F(a) \right\} f(a) da$.

In Appendix, we show that the free entry condition can be written as

$$\kappa_{Et} - (1-\delta+\delta\lambda) E_t \left[\Lambda_{t,t+1} \kappa_{Et+1} \right] = \lambda_E E_t [\Lambda_{t,t+1} (\overline{\pi}_{t+1} - \kappa)]. \tag{30}$$

The left-hand side is the cost of increasing entry by one unit at present and reducing entry by $1-\delta+\delta\lambda$ in the next period. This increases the expected number of products by λ_E only in the next period. The right-hand side is the expected increase of the net profit in the next period. We can also show the average profit is

$$\overline{\pi}_t = \frac{w_t X_t}{(\theta-1) N_t} - \frac{\theta}{\theta-1} w_t \phi \cdot (\underline{a}_t)^{-\alpha}. \tag{31}$$

The first term in the right hand side is the average gross profit due to mark-up per product and the second term is the average fixed cost for export.

The necessary and sufficient condition that the firm strictly prefers to maintain a product with the lowest productivity by paying the fixed cost is $V_t(1) > 0$ for all t. A sufficient condition for this is

$$0 < \pi_t^H(1) - \kappa + \delta\lambda\frac{\kappa_{Et}}{\lambda_E}, \ \forall t. \qquad \text{(Condition 2)}$$

Notice that this condition is satisfied even if realized current net profits of each product is negative ($\pi_t^H(1) < \kappa$), because there is an option value for the low productivity product to spin-out a high productivity product. This helps explain why firms often record negative current profits. In addition, because some large firms may have a large number of low productivity products, there can be only a weak correlation between size and profitability across firms - another interesting aspect of Japanese firms.

3.3 Equilibrium Dynamics

The state of our economy is described by the set of variables $\mathcal{M}_t = (N_t, D_{t-1}^*, Z_t, G_t, Y_t^*, \xi_t^*, R_t^*)$ where the first two state variables are endogenous and the last five are exogenous. The equilibrium dynamics of our economy is described by the fourteen endogenous variables of $(w_t, \epsilon_t, R_t, A_t^H, \underline{a}_t, \kappa_{Et}, \overline{\pi}_t, X_t, X_t^H, C_t, T_t, N_{Et}, N_{t+1}, D_t^*)$ as functions of $\mathcal{M}_t$ which are determined by the fourteen equations: (2), (6), (15), (16), (17), (19), (21), (22), (24), (25), (26), (27), (30) and (31). The consumer budget constraint (3) is automatically satisfied once all the market clearing conditions are satisfied (by a variant of Walras' Law), noting that aggregate net profit distribution is equal to the average gross profit multiplied by the number of products produced net of intangible investment cost ($\Pi_t = \overline{\pi}_t N_t - \kappa N_t - \kappa_{Et} N_{Et}$). Notice that we do not have to keep track the distribution of productivity of firm-product pairs to describe the aggregate economy because production size and maintenance of each product is independent from those of the other products within each firm - "amoeba" feature of our production economy.

We can organize the equilibrium conditions. Aggregate productivity A_t^H and unit cost of input composite w_t are functions of only state variables. Given A_t^H and w_t, the variables $\left(\underline{a}_t, X_t, X_t^H, \overline{\pi}_t\right)$ can be arranged into functions of (C_t, ϵ_t) and the state variables. The interest rate R_t is a function of (C_t, C_{t+1}), and the lump-sum tax satisfies the balanced budget $T_t = G_t$.

Thus, the equilibrium dynamics are characterized by five variables $(C_t, \epsilon_t, N_{Et}, D_t^*, N_{t+1})$ as a function of $\mathcal{M}_t = (N_t, D_{t-1}^*, Z_t, G_t, Y_t^*, \xi_t^*, R_t^*)$ that satisfy the following five equations:

(i) Euler equation for foreign bond holding

$$\xi_t^* \frac{C_t}{D_t^*} = \epsilon_t - R_t^* E_t \left(\beta \frac{C_t}{C_{t+1}} \epsilon_{t+1} \right); \tag{32}$$

(ii) Dynamics of net foreign asset: (27);

(iii) Dynamics of measure of products: (15);

(iv) Free entry equation, obtained from combining equations (2), (30) and (31),

$$\begin{aligned} & \kappa_E \left\{ \left(\frac{N_{Et}}{N_E} \right)^{\eta} - (1 - \delta + \delta\lambda) E_t \left[\beta \frac{C_t}{C_{t+1}} \left(\frac{N_{Et+1}}{N_E} \right)^{\eta} \right] \right\} \\ = \; & \lambda E_t \left(\beta \frac{C_t}{C_{t+1}} \left\{ -\kappa + A_{t+1}^H \left[\frac{1}{\theta} \frac{X_{t+1}}{N_{t+1}} - \phi (\underline{a}_{t+1})^{-\alpha} \right] \right\} \right), \end{aligned} \tag{33}$$

where A_{t+1}^H, $\underline{a}_{t+1}$, and X_{t+1} are functions of N_{t+1}, ϵ_{t+1}, C_{t+1} and exogenous variables;

(v) Home final goods market clearing condition,

$$\begin{aligned} & C_t + G_t + \kappa_E \left(\frac{N_{Et}}{N_E} \right)^{\eta} N_{Et} + \kappa N_t \\ = \; & A_t^H \left[X_t - \frac{\alpha\theta + 1 - \theta}{\alpha + 1 - \theta} \phi (\underline{a}_t)^{-\alpha} \right]; \end{aligned} \tag{34}$$

After characterizing the equilibrium, we verify that conditions (Condition 1) and (Condition 2) are satisfied in equilibrium.

Home real GDP is given as the sum of consumption, government purchase, intangible investment, net export value as

$$\begin{aligned} Y_t &= C_t + G_t + \kappa_{Et} N_{Et} + \kappa N_t + \epsilon_t p_t^F Q_t^F - \epsilon_t M_t^* \\ &= w_t \left[\frac{\theta}{\theta - 1} X_t - \phi \left(\underline{a}_t \right)^{-\alpha} N_t - (1 - \gamma_L) X_t \right] \\ &= \gamma_L w_t X_t + w_t \left[\frac{1}{\theta - 1} X_t - \phi \left(\underline{a}_t \right)^{-\alpha} N_t \right]. \end{aligned} \tag{35}$$

The first term of RHS of (35) is wage income, and the second term is profit, i.e., the return on capital.

4 Calibration

4.1 Parameter Choice and Moment Comparison

Appendix B describes the steady state equilibrium of our economy. Define $\widehat{X}_t$ as the proportional deviation of X_t from the steady state value X as

$$\widehat{X}_t = \ln X_t - \ln X \simeq \frac{X_t - X}{X}.$$

We assume the proportional deviation of the exogenous shocks $\widehat{\boldsymbol{Z}_t}' = (\widehat{Z}_t \ \widehat{Y_t^*} \ \widehat{G}_t \ \widehat{\xi}_t^* \ )'$ follow an independent AR(1) process

$$\widehat{\boldsymbol{Z}_t} = \begin{pmatrix} \rho_z & 0 & 0 & 0 \\ 0 & \rho_{Y^*} & 0 & 0 \\ 0 & 0 & \rho_G & 0 \\ 0 & 0 & 0 & \rho_{\xi^*} \end{pmatrix} \widehat{\boldsymbol{Z}}_{t-1} + \begin{pmatrix} \varepsilon_{Zt} \\ \varepsilon_{Y^*t} \\ \varepsilon_{Gt} \\ \varepsilon_{\xi^*t} \end{pmatrix}, \tag{36}$$

where the last terms in the right hand side are mutually independent exogenous shocks to aggregate TFP, foreign demand, government purchase, and liquidity service of foreign bond. In calibration, we decided to abstract from the shock to the foreign interest rate, because it has a similar effect with the shock to the liquidity service of foreign bond as both tend to increase the demand for foreign bond.[12] When we log linearize the endogenous and exogenous variables around the steady state, we can numerically derive the state space representation of the evolution of endogenous state variables as

$$\begin{pmatrix} \widehat{N}_{t+1} \\ \widehat{D_t^*} \end{pmatrix} = \begin{pmatrix} B_{NN} & B_{ND^*} \\ B_{D^*N} & B_{D^*D^*} \end{pmatrix} \begin{pmatrix} \widehat{N}_t \\ \widehat{D^*}_{t-1} \end{pmatrix} + \begin{pmatrix} B_{NZ} & B_{NY^*} & B_{NG} & B_{N\xi^*} \\ B_{D^*Z} & B_{D^*Y^*} & B_{D^*G} & B_{D^*\xi^*} \end{pmatrix} \widehat{\boldsymbol{Z}}_t. \tag{37}$$

We can also derive the state space representation of consumption, entry and real exchange rate as

$$\begin{pmatrix} \widehat{C}_t \\ \widehat{N_{Et}} \\ \widehat{\epsilon}_t \end{pmatrix} = \begin{pmatrix} B_{CN} & B_{CD^*} \\ B_{N_EN} & B_{N_ED^*} \\ B_{\epsilon N} & B_{\epsilon D^*} \end{pmatrix} \begin{pmatrix} \widehat{N}_t \\ \widehat{D^*}_{t-1} \end{pmatrix} + \begin{pmatrix} B_{CZ} & B_{CY^*} & B_{CG} & B_{C\xi^*} \\ B_{N_EZ} & B_{N_EY^*} & B_{N_EG} & B_{N_E\xi^*} \\ B_{\epsilon Z} & B_{\epsilon Y^*} & B_{\epsilon G} & B_{\epsilon\xi^*} \end{pmatrix} \widehat{\boldsymbol{Z}}_t, \tag{38}$$

where B_{ij}'s are constant coefficients which are functions of parameters. These three equations $(36, 37, 38)$ characterize the joint stochastic process of the endogenous and exogenous

[12] Neumeyer and Perri (2005) analyze the importance of foreign interest rate shock to the emerging economy.

state variables $\left(\widehat{N}_{t+1}, \widehat{D_t^*}, \widehat{\boldsymbol{Z}}_t\right)$ and endogenous control and jump variables $(\widehat{C}_t, \widehat{N_{Et}}, \widehat{\epsilon}_t)$. The other endogenous variables can also be solved as functions of endogenous and exogenous state variables.

Table 2a summarizes the choice of the parameters for calibration of our annual model. We follow convention for some parameters $\beta = 0.92$ and $R^* = 1.05$. The steady state value of $Z = 1$ is normalization. We choose the steady state value of government purchase to consumption ratio $G/C = 0.28$ and the share of imported material in total cost $1 - \gamma_L = 0.15$ to make them roughly comparable to the Japanese data. Concerning the elasticity of entry cost with respect to aggregate entry, we do not have good data to match and we fix $\eta = 0.1$. Then we choose the other parameters to make the aggregate and the steady state cross sectional moments listed in Table 2b comparable to the aggregate and Kaigin data. Because Kaigin data is only for relatively large firms, we decided to consider the largest firms in our simulated economy (1.56% out of 100,000 simulated firms) such that they generate 60% of total sales of the economy (which is comparable to the sales share of firms in Kaigin data). To avoid a sharp cutoff of firms in terms of sales, we introduce multiplicative noise in the firms' sales to determine whether they enter the subset of large firms. The variance of this noise, denoted σ in the Table 2a, was also calibrated to match the cross-sectional moments of the Kaigin data.

Table 2b compares the steady state moments of Japanese annual aggregate data (1981-2012), Kaigin data and Model. Concerning the first three aggregate moments, we use the average ratio of H-P filter trends. (For an example, C/Y is the average of H-P filter trends of consumption and GDP over 1981-2012 period.) Concerning the ratio of new entry to intangible capital, we do not have comparable number and we fix $N_E/N = 0.1$. The remaining 11 moments are from Kaigin data of 1999. The average of total sales is exactly matched because of the choice of unit. The other steady state moments of our model are broadly consistent with the data, except for a few moments: average profit rate in the model is too high (15% instead of 3%) and is correlated too closely with revenue (71% instead of 7%) relative to the data. One possible explanation is that many Japanese firms tend to distribute a significant fraction of their profit as bonus to their employees. Another possibility is that profit is under-reported for tax purpose.

Table 3a summarizes the choice of the standard deviations and the first order serial correlation coefficients of the exogenous process of aggregate productivity, foreign demand, government purchase, and liquidity shock to the foreign bond demand, $\left(\sigma_Z, \sigma_{Y^*}, \sigma_G, \sigma_{\xi^*}, \rho_Z, \rho_{Y^*}, \rho_G, \rho_{\xi^*}\right)$. The number is for annual data calibration. In order to obtain these eight parameters, we use the moments of the log deviation of GDP, government purchase, intangible investment, export value and real exchange rate $\left(\widehat{Y}_t, \widehat{G}_t, \widehat{I}_t, \widehat{E_{Xt}}, \widehat{\epsilon}_t\right)$ from the H-P filter trends where $I_t = \kappa_{Et} N_{Et} + \kappa N_t$ and $E_{Xt} = \epsilon_t p_t^F Q_t^F$. More specifically, we use five variances $(E(\widehat{Y}_t)^2, E(\widehat{G}_t)^2, E(\widehat{I}_t)^2, E(\widehat{E_{Xt}})^2, E(\widehat{\epsilon}_t)^2)$, five first order auto-coveriances of $\left(\widehat{Y}_t, \widehat{G}_t, \widehat{I}_t, \widehat{E_{Xt}}, \widehat{\epsilon}_t\right)$, and four covariances with GDP $(E(\widehat{G}_t\widehat{Y}_t), E(\widehat{I}_t\widehat{Y}_t), E(\widehat{E_{Xt}}\widehat{Y}_t), E(\widehat{\epsilon}_t\widehat{Y}_t))$. We use two methods to obtain the parameters. One is to choose the parameters to minimize the weighted sum of the fourteen squared differences between data and the simulated moments, using the inverse of the Newey-West heteroskedasticity and autocorrelation robust (HAC) estimator of variance as an efficient weight. Another is to choose the parameters to minimize the weighted sum of the squared differences, using the subjective weight to reflect our emphasis of GDP, export and real exchange rates: The weight used is $(250, 100, 1, 70, 10)$ for the five variances, $(120, 80, 1, 60, 1)$ for the five first order auto-covariances and $(50, 1, 150, 1)$ for the four covariances with GDP. We restrict the serial correlation coefficient of the exogenous shocks to be between 0 and 0.95. Table 3b reports the sample and simulated values of fourteen moments for both the efficient weight method and the subjective weight method. The main differences are that, by using the subjective weight, we can match the variances of export and the correlation between export and GDP better than the efficient weight, while the correlation between the real exchange rate and GDP is more badly matched.[13] We use the parameter values obtained by the efficient weights in the following because it is closer to the convention.

Figure 2a and 2b compare the model simulation and the data for the distribution of domestic sales and export sales. The distribution is roughly comparable, except that the model has a little too disperse distribution for domestic sales than Kaigin data. Figure 3a and 3b present

[13] We also tried to minimize the equally weighted sum of the squared differences, with the results somewhat in between the efficient and the subjective weight methods.

the densities of total sales conditional on firms being exporters or non-exporters for the data and the model. As is well-known, the exporters tends to have a larger total sales than the non-exporters with the average size twice as large in Kaigin data. Our model generates such a qualitative feature, but quantitatively the exporters in our model tend to have too large totals sales compared to the non-exporters. Even though we avoid the complete split of a standard Melitz (2003) model by allowing firms to produce multiple products, we do not fully capture heterogeneity among exporters and non-exporters (such as heterogeneity in transportation costs and the taste of foreigners across different products).

Table 4 compares the time series regression of aggregate export value on the real exchange rate, foreign GDP and aggregate TFP for the annual data from 1980 to 2010 in Japan. The regression coefficient of Japanese aggregate real export value on the real exchange rate is equal to 0.24 for the data deflated by export price index in column (1) and and equal to 0.83 for the data deflated by consumer price index in column (2), both data are detrended data by the log-linear trend. When we use the sixteen years of data generated by the model in column (3), the regression coefficient of the real export value on the real exchange rate is equal to 0.60 and is marginally significant. When we control for the foreign demand in column (4), the regression coefficient of export value on the foreign demand is equal to 0.99 and significant, which is consistent with data. The regression coefficient of aggregate export value on real exchange rate is now equal to 0.74 and very significant, which is larger than the coefficient of aggregate data. This suggests that the loose association of aggregate export and real exchange rate in our model is partly driven by the significant role of foreign demand shock, because an increase in foreign demand tends to increase export value and appreciate real exchange rate as we will see in the impulse response function in the following. The result of aggregate regression with simulated data does not change much when we control the aggregate TFP in column (5).

4.2 Impulse Responses of Aggregate Variables

Figure 4 presents the impulse responses of the aggregate variables to one standard deviation shock to the aggregate productivity. As in a standard open economy real business cycle model,

with a 0.9% positive aggregate productivity shock, output increases by 1% and consumption increases by 0.9%. Labor initial increases slightly before decreasing. As the home export becomes cheaper with higher productivity, the real exchange rate depreciates by 0.9%. The real export value in terms of home final goods increases by 0.7%, and the export value in terms foreign final goods decreases 0.2% perhaps because foreign demand for home export is relatively inelastic and the fraction of goods exported decreases by 0.4% in 2 to 5 years. The real import value increases by 0.9%. Because import increases more than export, net foreign asset decreases 0.2% in 2 to 5 years. As a measure of intangible capital (N_t) accumulates with vigorous intangible investment by 0.4% in 3 to 6 years, the expansionary effect persists beyond the persistence of the TFP shock itself.

Figure 5 presents the impulse responses to shocks to foreign demand for home export. In order to explain the volatility of export, our foreign demand shock is relatively large with standard deviation of 1.4% and persistent with the serial correlation coefficient of 0.94. With the increase in foreign demand by 1.4%, GDP increases by 0.2% and consumption increase very persistently by 0.15%, and labor increases slightly less persistently. Export value increases by 0.8% with the fraction of goods exported increases by 0.8%, while import increases by 0.2%. Then current account improves, and real exchange rate appreciates by 0.8% with the anticipation of net foreign asset accumulation (increase by about 0.15% in 8 to 20 years). Intangible capital increases slowly by nearly 0.15% in 7 to 20 years. In this way, the increase in foreign demand leads to an export-driven expansion of the home economy. Notice that the export increases despite of the real exchange rate appreciation.

Figure 6 presents the impulse responses to shocks to government purchase with serial correlation coefficient of 0.95. With an exogenous increase in government purchase by 0.8%, consumption decreases slightly and labor increases by 0.15% with a decline of household wealth. Output and import of intermediated goods increase by 0.15%. With the current account worsening, the net foreign asset decumulates and the real exchange rate depreciates by 0.15%, (partly because we do not have non-traded goods).

Figure 7 presents the impulse responses to shocks to liquidity service of foreign bond. In

order to explain the volatile and persistent real exchange rate movement, our shock to liquidity of foreign bond has a very large standard deviation of 22% in log scale with the serial correlation coefficient of 0.95. With one standard deviation increase in the liquidity service of foreign bond, the real exchange rate depreciates by 3%, leading to a decrease in import by 0.2% and an increase of export value and the fraction of goods exported by 2%. Net foreign asset increases by about 8% from 7 to 20 years. GDP falls by 0.2% and consumption falls by 0.6%, with decrease in intangible capital by 0.3% in 4 to 9 years. This is similar to "sudden stop," a financial shock induced current account reversal and recession.

4.3 Exchange Rate Disconnect at the Aggregate and Connect at the Firm Levels

From the above calibration, we learn that the real exchange rate tends to depreciate with positive shocks to aggregate TFP, government purchase and liquidity service of foreign bond. In contrast, the real exchange rate tends to appreciate with a positive shock to foreign demand for home export. In terms of the state space representation, we learn that, for a broad set of reasonable parameters, $B_{\epsilon Z}$, $B_{\epsilon G}$ and $B_{\epsilon \xi^*}$ are all positive, and $B_{\epsilon Y^*}$ is negative in (38). Also we show that the intangible capital stock evolves slowly even though the net foreign asset evolves faster.

Recall that the lower bound of the productivity for export and aggregate export value in terms of home final goods are

$$\underline{a}_t = \left[\frac{\alpha\left(\theta-1\right)\phi}{\alpha+1-\theta} \frac{\overline{a} Z_t N_t^{\frac{\theta}{\theta-1}}}{\epsilon_t^{\varphi} Y_t^*} \right]^{\frac{\theta-1}{\Delta}}, \text{ and } S_t^F = \left(\underline{a}_t\right)^{\frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}} \epsilon_t^{\varphi} Y_t^*,$$

where $\Delta = \alpha(\theta-1) + (\alpha+1-\theta)(1-\varphi)$. Thus we have the aggregate export value as

$$\begin{aligned}
\widehat{S_t^F} &= \frac{\varphi\alpha(\theta-1)}{\Delta}\widehat{\epsilon}_t + \frac{\frac{\theta}{\theta-1}(\alpha+1-\theta)(1-\varphi)}{\Delta}\widehat{N}_t + \frac{(\alpha+1-\theta)(1-\varphi)}{\Delta}\widehat{Z}_t + \frac{\alpha(\theta-1)}{\Delta}\widehat{Y_t^*} \\
&= \frac{\varphi\alpha(\theta-1)B_{\epsilon Z} + (\alpha\text{+}1\text{-}\theta)(1\text{-}\varphi)}{\Delta}\widehat{Z}_t + \frac{\varphi\alpha(\theta-1)B_{\epsilon N} + \frac{\theta}{\theta-1}(\alpha+1-\theta)(1-\varphi)}{\Delta}\widehat{N}_t \\
&\quad + \frac{(\varphi B_{\epsilon Y^*}+1)\alpha(\theta-1)}{\Delta}\widehat{Y_t^*} + \frac{\varphi\alpha(\theta-1)}{\Delta}\left(B_{\epsilon D^*}\widehat{D^*}_{t-1} + B_{\epsilon G}\widehat{G}_t + B_{\epsilon\xi^*}\widehat{\xi_t^*}\right).
\end{aligned}$$

From here, we learn the aggregate export value and the real exchange rate tend to move in the same direction, i.e., real exchange rate depreciation and increase in export value are associated, if shocks to TFP, government purchase and liquidity demand of foreign asset are important. If shock to foreign demand is dominant, then real exchange appreciation and increase in export value are associated. Thus there are generally two ways to explain the disconnect between the real exchange rate and the aggregate export value: The first is that the shock to foreign demand is dominant. The second is that the partial effect of the real exchange rate on the aggregate export $\frac{\varphi\alpha(\theta-1)}{\Delta}$ is small, which is true only if the price elasticity of foreign demand for home exports φ is small.

On the other hand, for the export value of the individual product with productivity a, we have

$$\begin{aligned} s_t^F(a) &\equiv \epsilon_t p_t^F(a) q_t^F(a) = (\theta-1)\phi A_t^H \left(\frac{a}{\underline{a}_t}\right)^{\theta-1} I(a-\underline{a}_t) \\ &= (\theta-1)\phi \bar{a} a^{\theta-1} I(a-\underline{a}_t) Z_t N_t^{\frac{1}{\theta-1}} (\underline{a}_t)^{1-\theta}. \end{aligned}$$

where $I(a-\underline{a}_t)$ is an indicator function such that $I(a-\underline{a}_t)=1$ if $a-\underline{a}_t \geq 0$, and $=0$ otherwise. Thus we get

$$\begin{aligned} \widehat{s_t^F(a)} &= \widehat{I(a-\underline{a}_t)} + \frac{\varphi(\theta-1)^2}{\Delta}\widehat{\epsilon}_t + \frac{\frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1} - (\theta\text{-}1)^2}{\Delta}\widehat{N}_t + \frac{(\alpha\text{+}1\text{-}\theta)(1\text{-}\varphi)}{\Delta}\widehat{Z}_t + \frac{(\theta\text{-}1)^2}{\Delta}\widehat{Y_t^*} \\ &= \widehat{I(a-\underline{a}_t)} + + \frac{\varphi(\theta-1)^2 B_{\epsilon Z} + (\alpha\text{+}1\text{-}\theta)(1\text{-}\varphi)}{\Delta}\widehat{Z}_t + \frac{(\varphi B_{\epsilon Y^*}\text{+}1)(\theta\text{-}1)^2}{\Delta}\widehat{Y_t^*} \\ &\quad + \frac{\varphi(\theta\text{-}1)^2 B_{\epsilon N} + \frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1} - (\theta\text{-}1)^2}{\Delta}\widehat{N}_t + \frac{\varphi(\theta\text{-}1)^2}{\Delta}\left(B_{\epsilon D^*}\widehat{D^*}_{t-1} + B_{\epsilon G}\widehat{G}_t + B_{\epsilon\xi^*}\widehat{\xi_t^*}\right), \end{aligned}$$

Because $\theta - 1 < \alpha$ by Assumption 1, the export value of the individual product is less sensitive to the shocks unless there is the change in the extensive margin $\widehat{I(a-\underline{a}_t)}$. The response of the export value of an individual product to the real exchange rate depends upon whether there is an adjustment of the extensive margin. If a product has a very high productivity and is always exported (as always $a > \underline{a}_t$), then the export value of such product is not very responsive to the real exchange rate. Figure 8a describes the relationship between the export value of a high productive product and the real exchange rate. If a product has a productivity

in the neighborhood of the lower bound for the export, then the response of the export value is large because both intensive and extensive margins adjust to the real exchange rate. Figure 8b describes the response of the export value of a marginal product. When the real exchange rate appreciates (ϵ_t falls) due to financial shocks $\widehat{\xi}_t^*$, the lower bound of productivity for export increases. At some threshold $\underline{\epsilon}_t$, the productivity of this product becomes lower than the boundary, and the export value drops to zero. As in Green (2009), the exports of the low productivity products drop like "flies" when there is an adverse shock such as a real exchange rate appreciation.

Our Japanese firm-level data (Kaigin data) are mostly of relatively large firms, which typically produce multiple products. If a majority of products of some firm is close to the lower bound for export, then the export of this firm is sensitive to the real exchange rate shifts as in Figure 8b. Because such firms are common under Assumption 1, the firm-level export tends to react significantly to the real exchange rate. In contrast, the products with considerably higher productivity than the lower bound is not very sensitive to the real exchange rate shifts as in Figure 8a, and their share in the aggregate export is large. Thus the aggregate exports are less sensitive contemporaneously to the real exchange rate shift as in Figure 8c. This heterogeneous reaction of exports to the real exchange rate shift across different products helps explain why firm level exports co-move significantly with the real exchange rate, while aggregate exports appear to be "disconnected" from the real exchange rate.[14]

Table 5 presents the panel regression of Kaigin data to present some evidence to support this "drop like a flies" hypothesis. Table 5 conducts firm level real export value on real exchange rate, foreign GDP, aggregate TFP with the interaction terms with profit rate (profit-sales ratio), in addition to firm TFP. In the first column, the regression coefficient of real export value on the product of real exchange rate and profit rate is negative and significant at 10% level. The regression coefficient on the product of foreign GDP and profit rate is also negative and

[14]Our explanation of the extensive margin adjustment at product level is consistent with Dekle, Jeong and Ryoo (2007), which find that the apparent lack of relationship between the exchange rate and aggregate exports occur through the intensive margin of export sales within firms, rather than through the extensive margin of entry and exit of firms in the export market.

significant at 5 % level. Thus export of firms with higher profit rate tend to be less sensitive to the change of real exchange rate or foreign income. In columns 2, we find the regression coefficient of export value on real exchange rate is not significantly affected by sales. The regression coefficient of export value on foreign income is significantly affected by firm sales but the effect tend to be larger (rather than smaller) for larger firms.[15]

In order to see whether our model generates such heterogeneous reactions of firm export value to the aggregate variables, we present the result of panel regression of simulated data of our model in Table 6. Appendix C explains the detail of the panel calibration for sixteen years (which is the similar length as our Kaigin data). In the first column (column 0), we reproduce the first column of Table 5 for the comparison. In column 1, we present the regression of firm export value on the real exchange rate, foreign demand and aggregate TFP using our panel calibration data for sixteen years. The regression coefficients of export value on real exchange rate and foreign demand in simulated data are consistent with those of the Kaigin data, even though they are not significant. In column 2, we add the interaction term of real exchange rate and profit rate, and found that the regression coefficient of the interaction term is significantly negative - the firms with higher profit rate tend to increase their export less with the real exchange rate depreciation. In addition, the regression coefficient on real exchange rate is now positive and significant at 5% level. In column 3, we include full interaction terms to find that the firms with higher profitability are marginally less sensitive to the change of real exchange rate and the foreign demand. In the last column, we add the firm TFP, and found that the firm export significantly increases with firm level TFP. These panel regressions of simulated data from the model are broadly consistent with the panel regression of Kaigin data in Tables 1 and 5.[16]

[15] This finding is consistent with Table 1 in Introduction.

[16] Table 7 present panel regression of simulated data from model with the interaction terms with total sales of firms. Although the general features are similar to Table 6, the firm level TFP and firm sales are highly correlated in simulated data and the results are not very stable in the final column.

5 Conclusion

We construct a dynamic general equilibrium model with heterogeneous multiproduct firms to reconcile the aggregate disconnect and firm-level connect between the real exchange rate and real export values. The model is broadly consistent with the other features of aggregate time series and firm-level panel data. The model is tractable and can be used to explain business cycles with intangible capital in both closed and open economies.

We abstract from many important aspects in order to make the framework tractable. We completely ignore monetary aspects of the aggregate economy, including nominal price stickiness and limited exchange rate pass-through on nominal prices. We expect that monetary policy shocks would takeover some of the role of shocks that we attribute to the liquidity service of foreign bonds and foreign demand. We also do not explicitly include tangible capital accumulation by equating total investment in the data with intangible investment in our model. Furthermore, we abstract from other kinds of heterogeneity besides the number of products and the productivity distribution, and thus do not capture all the rich aspects of firm level panel data, including the cross sectional distribution of total sales by export status as in Figure 3. These are topics for future research.

References

[1] Alessandria, G. and H. Choi (2007), "Do Sunk Costs of Export Matter for Net Export Dynamics?" *Quarterly Journal of Economics,* 122(1): 289-336.

[2] Arkolakis, Costas. (2009), "A Unified Theory of Firm Selection and Growth," mimeo, Yale University.

[3] Atkeson, Andrew, and Ariel Burstein (2010), "Innovation, Firm Dynamics, and International Trade," *Journal of Political Economy*, 118(3): 433-484.

[4] Backus, D., P. Kehoe, and F. E. Kydland (1994), "Dynamics of the Trade Balance and the Terms of Trade: The J-curve," *American Economic Review*, 84(1): 84-103.

[5] Berman, Nicolas, Philippe Martin, and Thierry Mayer (2012), "How Do Different Exporters React to Exchange Rate Changes? Theory, Empirics and Aggregate Implications," *Quarterly Journal of Economics,* 127(1): 437-492.

[6] Bernard, A., S. J. Redding and P. Schott (2010), "Multi-Product Firms and Product Switching," *American Economic Review,* 100(1): 70-97.

[7] Bernard, A., S. J. Redding and P. Schott (2011), "Multi-Product Firms and Trade Liberalization," *Quarterly Journal of Economics,* 126: 1271-1318.

[8] Bilbie, F., F. Ghironi, and M. Melitz (2012), "Endogenous Entry, Product Variety, and Business Cycles," *Journal of Political Economy,* 120(2): 304-345.

[9] Chatterjee, A., R. Dix-Carneiro, and J. Vichyanand (2013), "Multi-product Firms and Exchange Rate Fluctuations," *American Economic Journal: Economic Policy*, 5(2).

[10] Das, Sanghamitra, Mark J. Roberts, and James R. Tybout (2007), "Market Entry Costs, Producer Heterogeneity, and Export Dynamics," *Econometrica*, 75(3): 837-873.

[11] Dekle, Robert, Hyeok Jeong, and Heajin Ryoo (2007), "A Re-Examination of the Exchange Rate Disconnect Puzzle: Evidence from Firm Level Data," mimeo.

[12] Dekle, Robert and Heajin Ryoo (2007), "Exchange Rate Fluctuations, Financing Constraints, Hedging, and Exports: Evidence from Firm Level Data," *Journal of International Financial Markets Institutions and Money*, 17(5): 437-451.

[13] Eaton, J., and S. Kortum (2002), "Technology, Geography, and Trade," *Econometrica* 70: 1741-1779.

[14] Fattal Jaef, R. and J. Lopez (2013), "Entry Trade Costs and International Business Cycles," manuscript, World Bank.

[15] Feenstra, K., Russ, K., and M. Obstfeld (2010), "In Search of the Armington Elasticity," manuscript, UC-Berkeley and UC-Davis.

[16] Fitzgerald, D., and S. Haller (2008), "Exchange Rates and Producer Prices: Evidence from Firm Level Data," manuscript, Stanford University.

[17] Ghironi, Fabio, and Marc Melitz (2005), "International Trade and Macroeconomic Dynamics with Heterogenous Firms," *Quarterly Journal of Economics*, 120: 865-915.

[18] Green, Edward (2009), "Heterogeneous producers facing common shocks: An overlapping-generations example," *Journal of Economic Theory,* 144: 2266-2276.

[19] Hayashi, Fumio, and Edward Prescott (2002), "The 1990s in Japan: A Lost Decade," *Review of Economic Dynamics,* 5(1): 206-235.

[20] Hooper, P., K. Johnson, and J. Marquez (2000), "Trade Elasticities for G-7 Countries," *Princeton Studies in International Economics* No. 87, Princeton University.

[21] Kawakami, A. and T. Miyagawa (2013), "Product Switching and Firm Performance in Japan: Empirical Analysis Based on the Census of Manufacturers," *Policy Research Institute, Ministry of Finance, Japan,* (9), 2: 287-314.

[22] Klette, T. J., and S. Kortum (2004), "Innovating Firms and Aggregate Innovation," *Journal of Political Economy,* 112(5): 986-1018.

[23] Melitz, Marc J. (2003), "The Impact of Trade on Intra-industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 71(6): 1695-1725.

[24] Neumeyer, P. and F. Perri (2005), "Business cycles in emerging economics: the role of interest rates," *Journal of Monetary Economics,* 52(2): 345-380.

[25] Obstfeld, M. and K. Rogoff (2000), "The Six Major Puzzles in International Macroeconomics: Is there a Common Cause?," *NBER Macroeconomics Annuals*, 15: 339-390.

[26] Orcutt, G. H. (1950), "Measurement of Price Elasticities in International Trade", *Review of Economics and Statistics* 32: 117-132.

[27] Tybout, J. and M. Roberts (1997), "The Decision to Export in Columbia: An Empirical Model of Exports with Sunk Costs," *American Economic Review.*

[28] Verhoogen, E. (2008), "Trade, Quality Upgrading, and Wage Inequality in the Mexican Manufacturing Sector," *Quarterly Journal of Economics.*

Table 1. Exports regression, Kaigin panel

	(1) All Sample	(2) High Profitability	(3) Low Profitability	(4) Big Employment	(5) Small Employment	(6) Big Sales	(7) Small Sales
log RER	0.374	0.284	0.393	0.338	0.406	0.389	0.369
	(0.049)***	(0.110)**	(0.054)***	(0.061)***	(0.068)***	(0.063)***	(0.065)***
log Y*	0.398	0.315	0.417	0.305	0.424	0.594	0.316
	(0.055)***	(0.125)**	(0.061)***	(0.072)***	(0.076)***	(0.073)***	(0.073)***
log Agg TFP	0.378	1.537	0.106	0.389	0.301	0.588	0.3
	(0.080)***	(0.181)***	-0.089	(0.104)***	(0.111)***	(0.105)***	(0.106)***
log Firm TFP	2.112	2.158	2.091	2.721	1.898	1.726	2.253
	(0.079)***	(0.169)***	(0.089)***	(0.121)***	(0.101)***	(0.111)***	(0.102)***
Cons	6.289	7.744	5.963	10.412	4.803	2.55	7.79
	(1.596)***	(3.611)**	(1.771)***	(2.083)***	(2.190)**	-2.109	(2.107)***
F-stat	325.8	118.6	228.2	250.2	152.5	171.7	196.2
Adj. R-sq.	0.042	0.122	0.029	0.171	0.002	0.126	0.022
# Obs.	9,997	2,034	7,963	3,549	6,448	3,089	6,908

Size differentiation is done by 75th percentile. * $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

Table 2a. Baseline parameterization

β	Discount factor	0.92
θ	Elasticity of substitution between products	4.19
ψ	Frisch elasticity of labor supply	6.02
ψ_0	Labor disutility	12.84
γ_L	Labor share	0.85
α	Productivity distribution shape parameter	3.64
φ	Elasticity of foreign demand	0.75
ϕ	Export cost	3.14
κ	Maintenance cost	16.57
κ_E	Entry cost	89.26
η	Elasticity of entry cost	0.1
δ	Probability of losing product	0.12
λ	Probability of drawing new product for incumbent	0.49
λ_E	Probability of producing new product for entrant	0.41
σ	Std. dev. of noise for sales	1.67
Z	Steady state aggregate productivity	1
Y^*	Steady state foreign demand	10^6
G/C	Steady state govt. expenditure / cons.	0.28
ξ^*	Steady state liquidity shock	0.01
R^*	Steady state foreign interest rate	1.05

Table 2b. Steady state moments (aggregate and cross-sectional)

	Data	Model
C/Y	0.56	0.66
$\epsilon D^*/Y$	0.20	0.19
Exp/Y	0.12	0.12
N_E/N	0.10	0.15
Mean $\log Rev$	17.77	17.77
SD $\log Rev$	1.42	1.84
Mean $\log Dom$	17.65	17.66
SD $\log Dom$	1.41	1.84
Mean $\log Exp$	16.03	15.58
SD $\log Exp$	2.09	1.85
Mean PR	0.03	0.15
SD PR	0.06	0.15
$\#Exp/N$	0.39	1.00
Corr PR, $\log Rev$	0.07	0.71
Corr ES, $\log Rev$	0.17	0.32

Table 3a. Calibration of stochastic processes

	Efficient	Subjective
Standard deviation		
σ_Z (%)	0.87	0.59
σ_{Y^*} (%)	1.35	5.46
σ_G (%)	0.83	0.61
σ_{ξ^*} (%)	22.05	79.16
Autocorrelation		
ρ_Z	0.55	0.73
ρ_{Y^*}	0.94	0.84
ρ_G	0.95	0.95
ρ_{ξ^*}	0.95	0.27

Table 3b. Sample and simulated moments

	Data	Efficient	Subjective
Standard deviation			
SD GDP (%)	0.88	0.93	0.96
	(0.10)		
SD Gov / SD GDP	0.63	0.83	0.59
	(0.11)		
SD Inv / SD GDP	3.13	2.80	2.47
	(0.13)		
SD Exp / SD GDP	4.63	2.41	4.24
	(0.70)		
SD RER (%)	3.52	3.07	3.57
	(0.31)		
Autocorrelation			
AC(1) GDP	0.55	0.34	0.40
	(0.15)		
AC(1) Gov	0.65	0.49	0.49
	(0.07)		
AC(1) Inv	0.58	0.30	0.23
	(0.13)		
AC(1) Exp	0.36	0.45	0.37
	(0.18)		
AC(1) RER	0.49	0.46	0.28
	(0.06)		
Correlation with GDP			
Corr Gov, GDP	0.08	0.12	0.08
	(0.19)		
Corr Inv, GDP	0.96	0.97	0.85
	(0.01)		
Corr Exp, GDP	0.55	0.08	0.53
	(0.19)		
Corr RER, GDP	0.42	-0.04	-0.59
	(0.16)		

Data and output from the model are HP filtered.

HAC robust standard errors are shown in parenthesis.

Table 4. Time series regression on aggregate data

	Data		Model		
	(1) EPI	(2) CPI	(3)	(4)	(5)
log ER	0.24	0.83	0.60	0.74	0.74
	(0.09)***	(0.09)***	(0.16, 1.02)	(0.74, 0.75)	(0.74, 0.75)
log Y*	0.11	0.15		0.99	0.99
	(0.09)	(0.09)*		(0.98, 1)	(0.98, 1)
log TFP					0.01
					(0, 0.02)
Cons	0.00	0.00	10.64	0.00	-0.01
	(0.01)	(0.01)	(1.82, 19.37)	(-0.12, 0.11)	(-0.1, 0.07)
# Obs.	31	31	16	16	16

For the model, 95% bootstrap confidence intervals (with 1,000 simulations) are shown in parenthesis. For data, HAC-robust standard errors in parenthesis.

Table 5. Exports regression with interaction terms, Kaigin panel

	(1) Profitability interaction		(2) Sales interaction
log RER	0.527	log RER	0.242
	(0.065)***		(0.489)
log RER × PR	-1.604	log RER × log Sales	-0.002
	(0.954)*		(0.027)
log Y*	0.383	log Y*	-0.033
	(0.055)***		(0.092)
log Y* × PR	-0.357	log Y* × log Sales	0.037
	(0.155)**		(0.004)***
log Agg TFP	-0.678	log Agg TFP	-3.438
	(0.103)***		(0.718)***
log Agg TFP × PR	21.09	log Agg TFP × log Sales	0.137
	(1.436)***		(0.040)***
log Firm TFP	2.295	log Firm TFP	0.168
	(0.084)***		(0.075)**
Cons	7.573	Cons	-1.159
	(1.626)***		(1.367)
F-stat	233.4	F-stat	744.5
Adj. R-sq.	0.071	Adj. R-sq.	0.303
# Obs.	9,994	# Obs.	9,994

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$.

Table 6. Panel regression on simulated data: Profitability interaction

	Data	Model			
		(1)	(2)	(3)	(4)
log RER	0.527	1.95	1.95	2.00	1.65
	(0.065)***	(-7.4, 11.07)	(-0.02, 3.8)	(0.11, 3.84)	(1.18, 2.21)
log RER × PR	-1.604		-0.25	-1.81	-0.58
	(0.954)*		(-0.3, -0.23)	(-4.11, 0.56)	(-1.39, 0.27)
log Y*	0.383	2.19	2.28	2.36	2.17
	(0.055)***	(-17.9, 23.33)	(-2.06, 6.61)	(-1.88, 6.69)	(0.9, 3.29)
log Y* × PR	-0.357			-2.32	-0.83
	(0.155)**			(-5.72, 1.25)	(-2.03, 0.43)
log Agg TFP	-0.678	-0.83	-2.32	-2.64	-1.16
	(0.103)***	(-29.32, 24.36)	(-8.62, 2.99)	(-8.89, 2.21)	(-2.77, -0.17)
log Agg TFP × PR	21.09			7.76	1.16
	(1.436)***			(0.24, 16.04)	(-1.26, 3.96)
log Firm TFP	2.295				0.98
	(0.084)***				(0.97, 0.99)
Cons	7.573				
	(1.626)***				
# Obs.	9,994	26,752			
		(10536, 38688)			

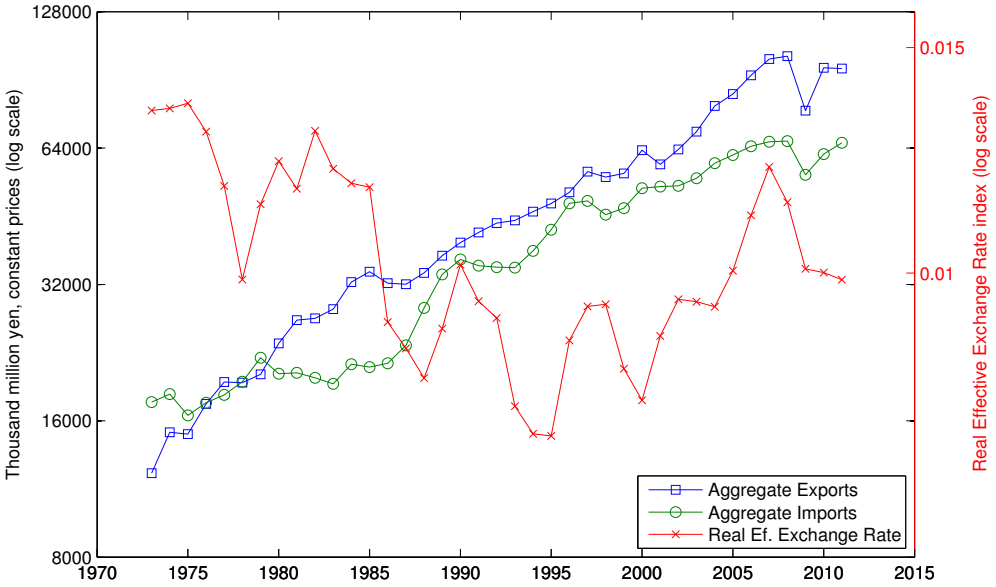
For the model, 95% bootstrap confidence intervals (with 1,000 simulations) are shown in parenthesis.

Table 7. Panel regression on simulated data: Sales interaction

	Data	Model			
		(1)	(2)	(3)	(4)
log RER	0.242	1.95	2.01	2.12	3.50
	(0.489)	(-7.4, 11.07)	(1.7, 2.38)	(1.25, 2.91)	(0.47, 6.91)
log RER × log Sales	-0.002		-0.05	-0.07	-0.18
	(0.027)		(-0.05, -0.05)	(-0.16, 0.04)	(-0.59, 0.19)
log Y*	-0.033	2.19	2.00	2.14	-2.13
	(0.092)	(-17.9, 23.33)	(1.18, 2.78)	(0.76, 3.57)	(-8.72, 2.57)
log Y* × log Sales	0.037			-0.02	0.49
	(0.004)***			(-0.16, 0.14)	(-0.03, 1.21)
log Agg TFP	-3.438	-0.83	-2.22	-2.81	-14.78
	(0.718)***	(-29.32, 24.36)	(-3.32, -1.52)	(-5.38, -0.54)	(-28.47, -3.99)
log Agg TFP × log Sales	0.137			0.08	0.23
	(0.040)***			(-0.21, 0.4)	(-0.99, 1.54)
log Firm TFP	0.168				-9.74
	(0.075)**				(-15.35, -4.77)
Cons	-1.159				
	(1.367)				
# Obs.	9,994	26,752			
		(10536, 38688)			

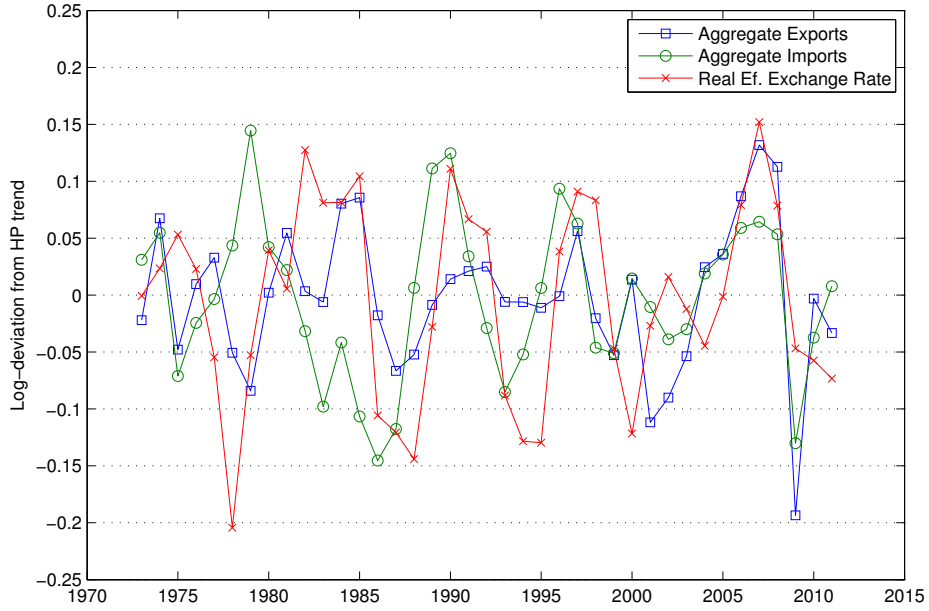
For the model, 95% bootstrap confidence intervals (with 1,000 simulations) are shown in parenthesis.

Figure 1a. Aggregate exchange rate disconnect (levels)



$\text{Corr(Exp, REER)} = \underset{(0.19)}{-0.47^{**}}$ $\quad$ $\text{Corr(Imp, REER)} = \underset{(0.18)}{-0.50^{***}}$

Figure 1b. Aggregate exchange rate disconnect (HP filtered)



$\text{Corr(Exp, REER)} = \underset{(0.10)}{0.51^{***}}$ $\quad$ $\text{Corr(Imp, REER)} = \underset{(0.17)}{0.18}$

Figure 2a. Cross sectional distribution of domestic sales

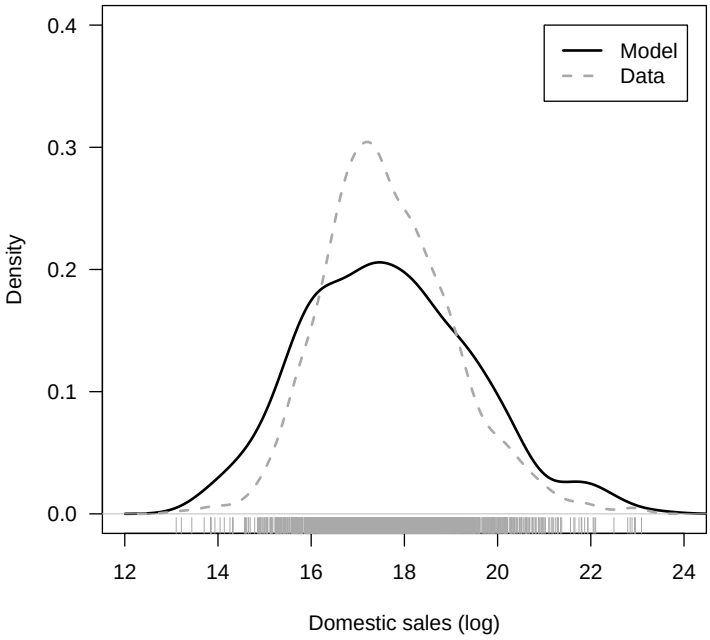


Figure 2b. Cross sectional distribution of exports

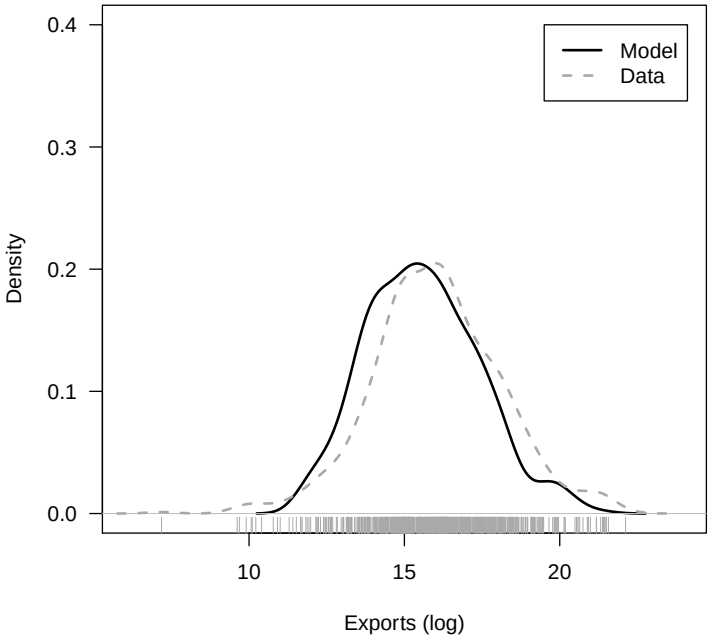


Figure 3a. Cross sectional distribution of total sales by export status: Kaigin data

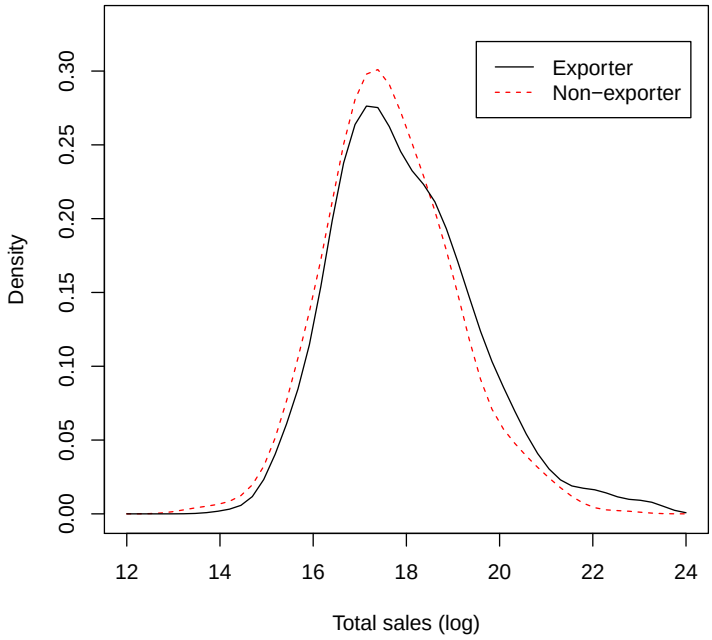


Figure 3b. Cross sectional distribution of total sales by export status: Model

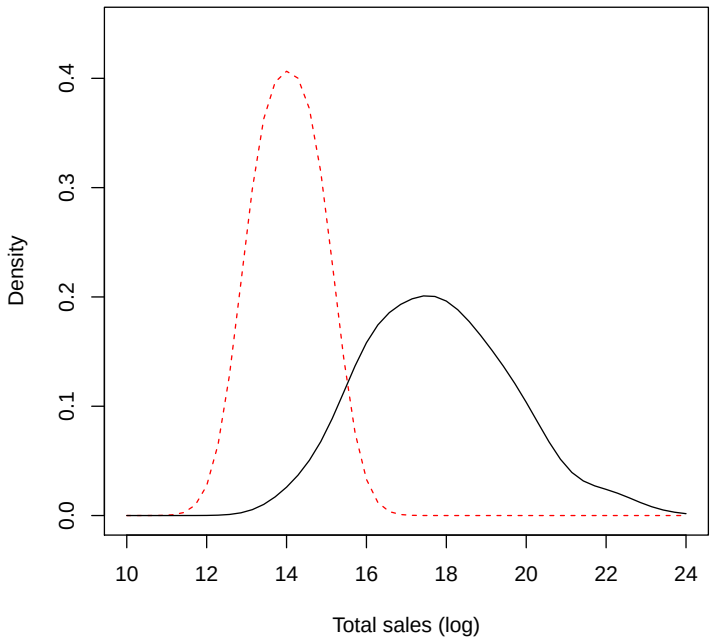


Figure 4. Impulse response to TFP shock Z

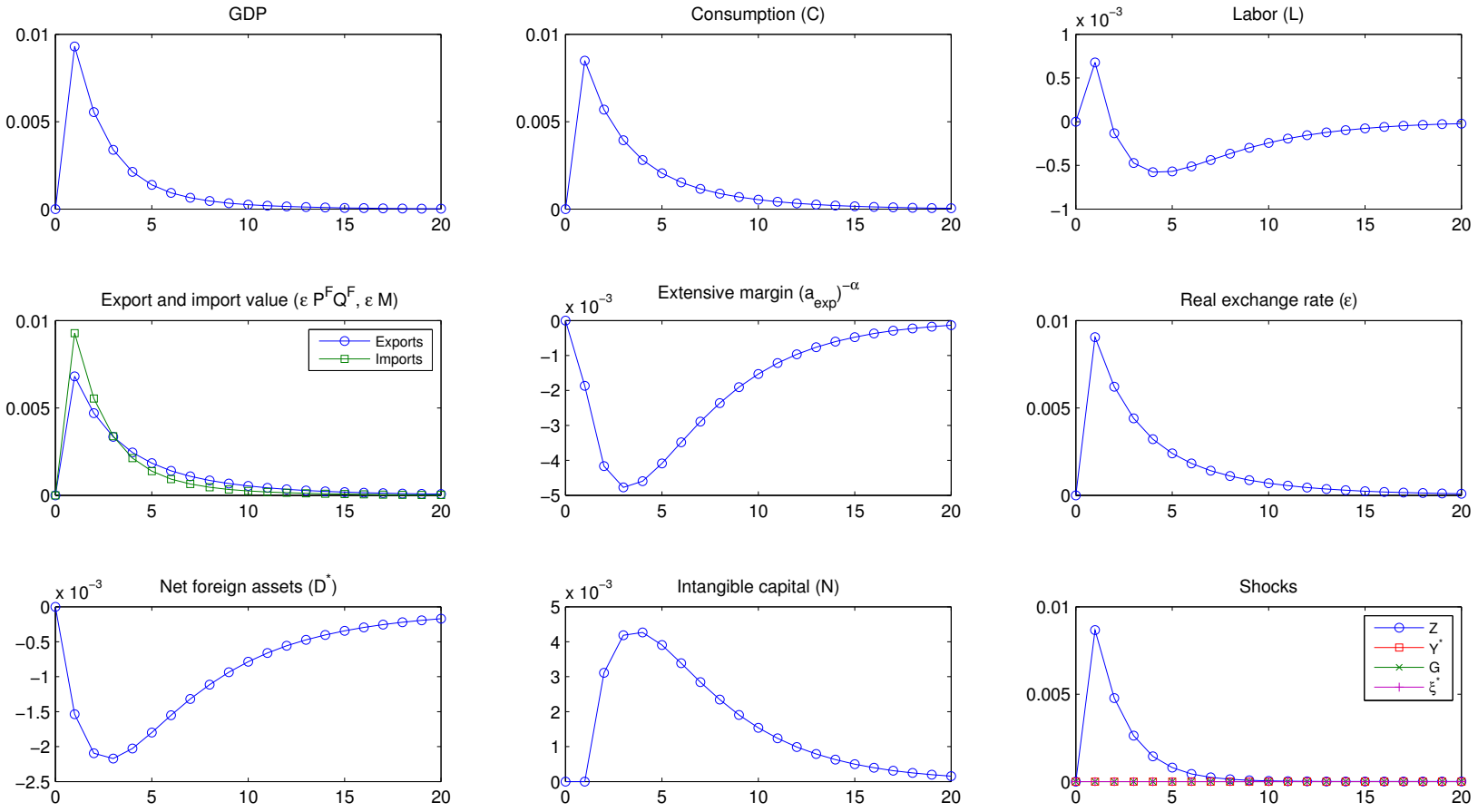


Figure 5. Impulse response to foreign demand shock Y^*

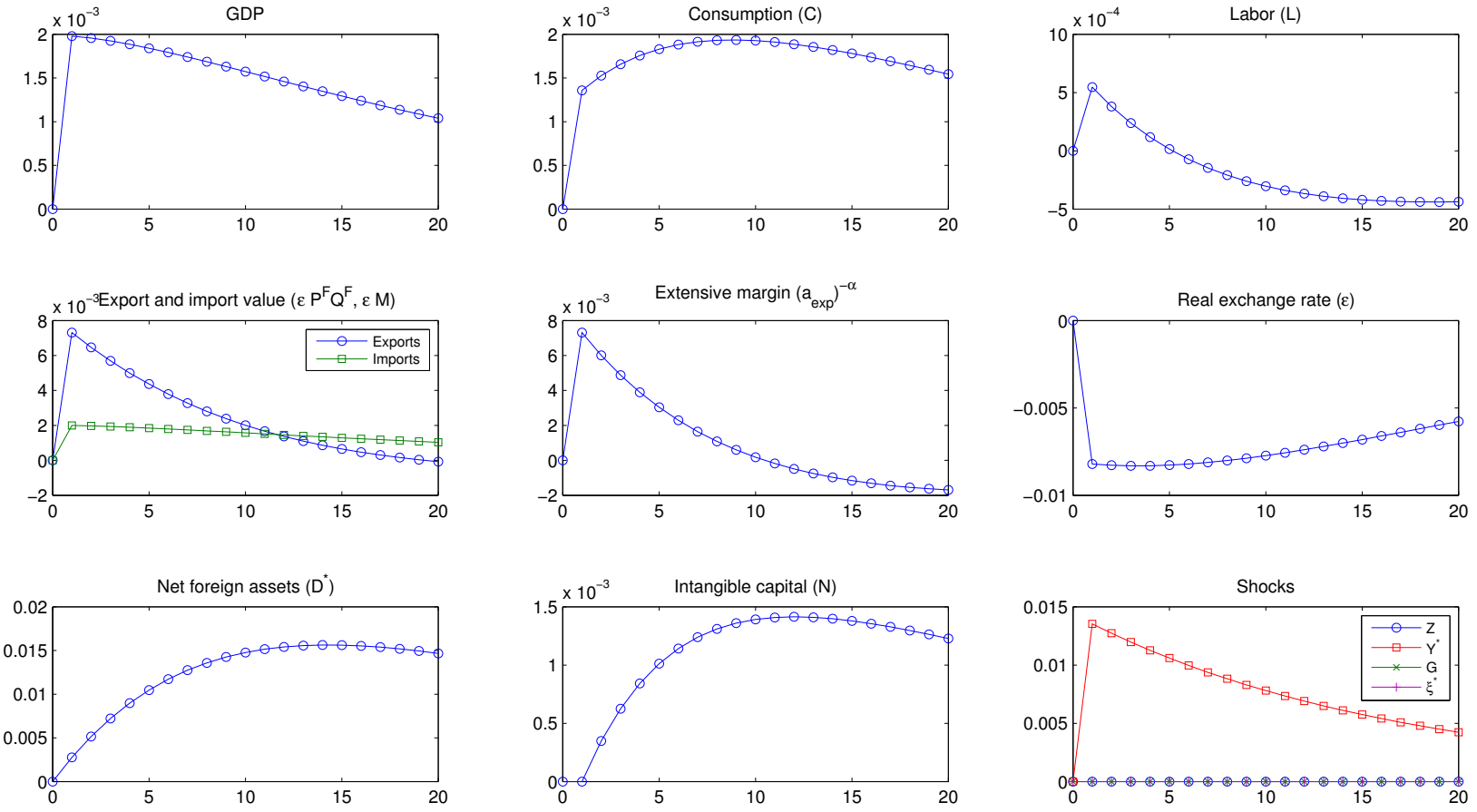


Figure 6. Impulse response to government expenditure shock G

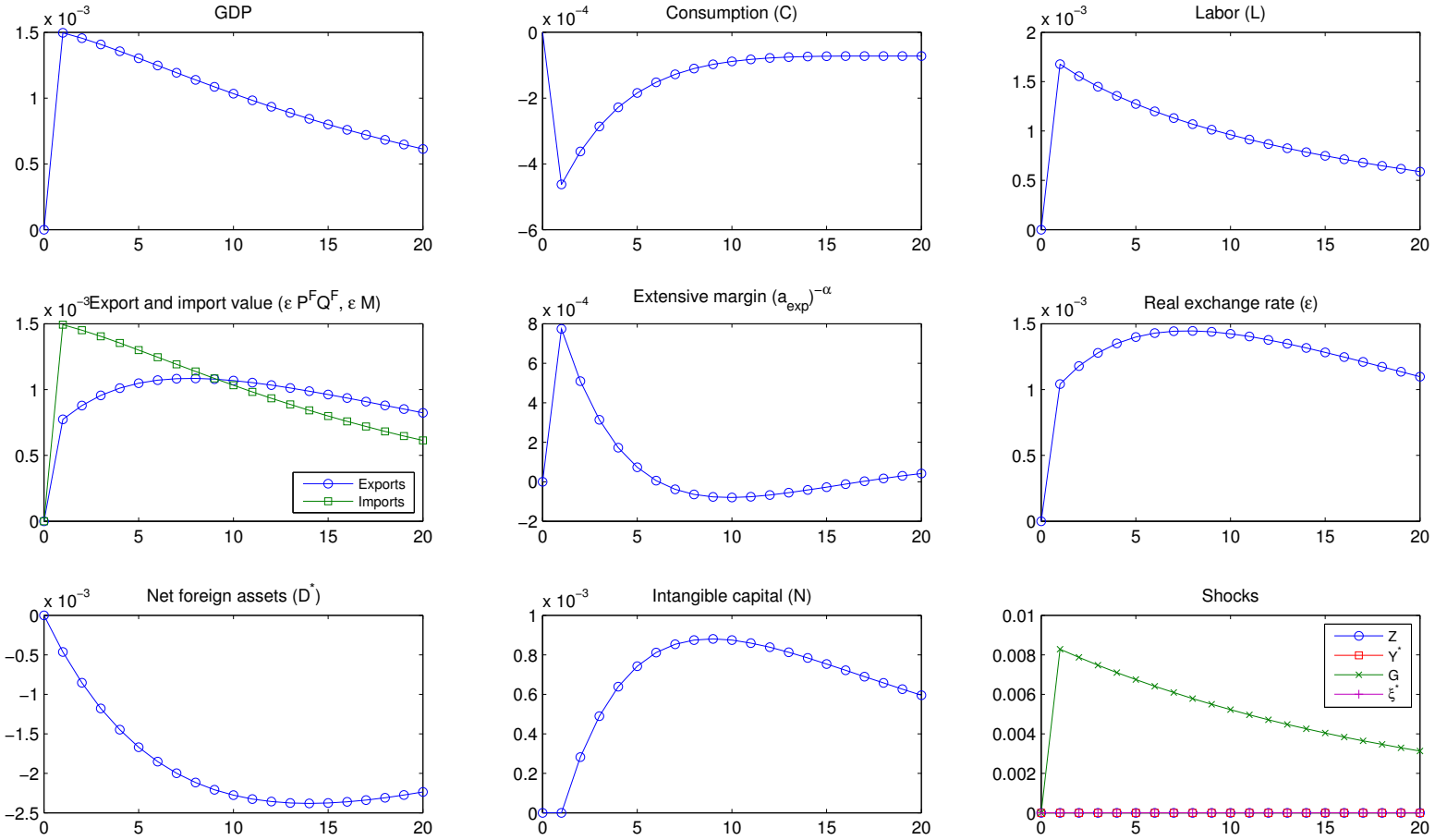


Figure 7. Impulse response to liquidity shock ξ^*

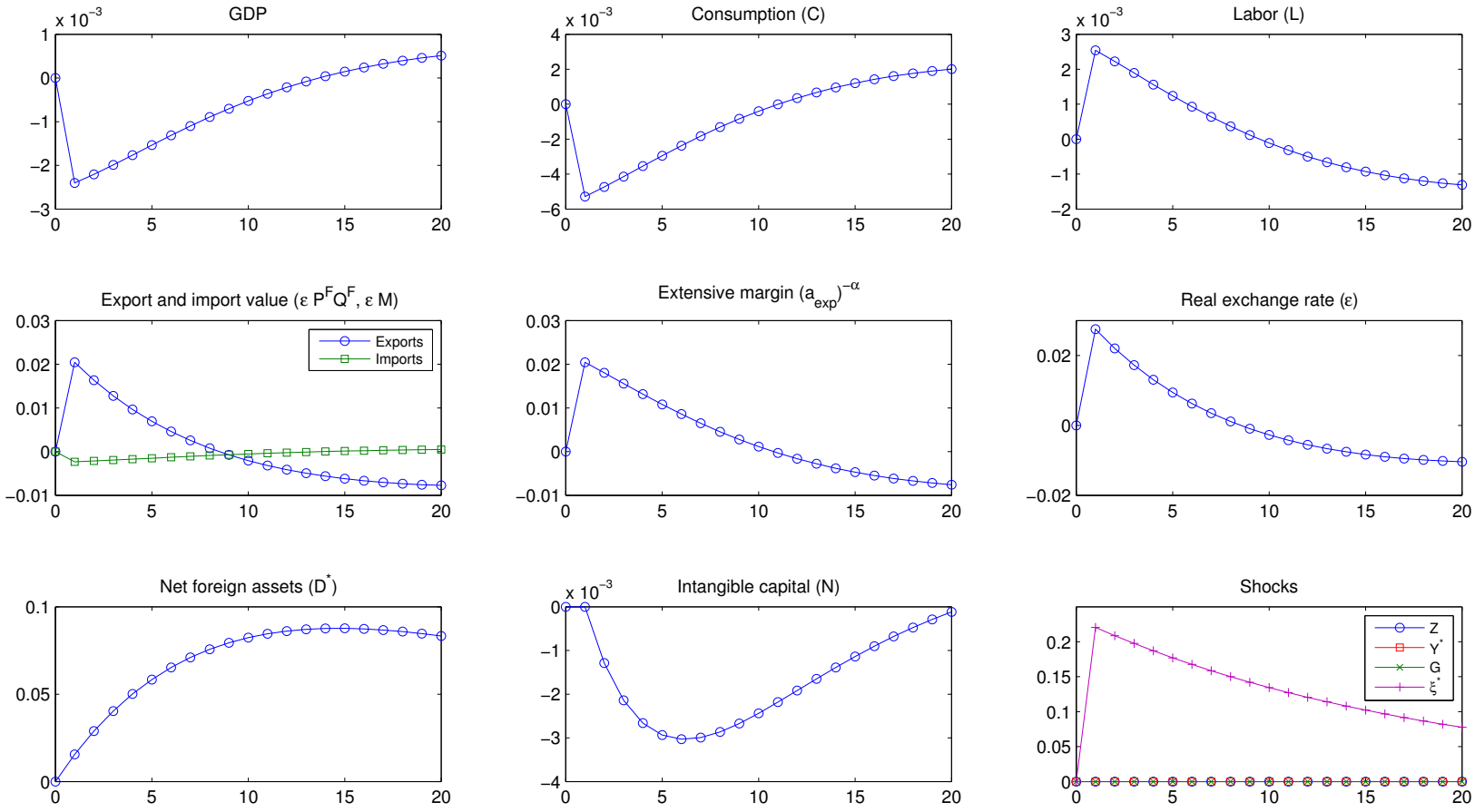
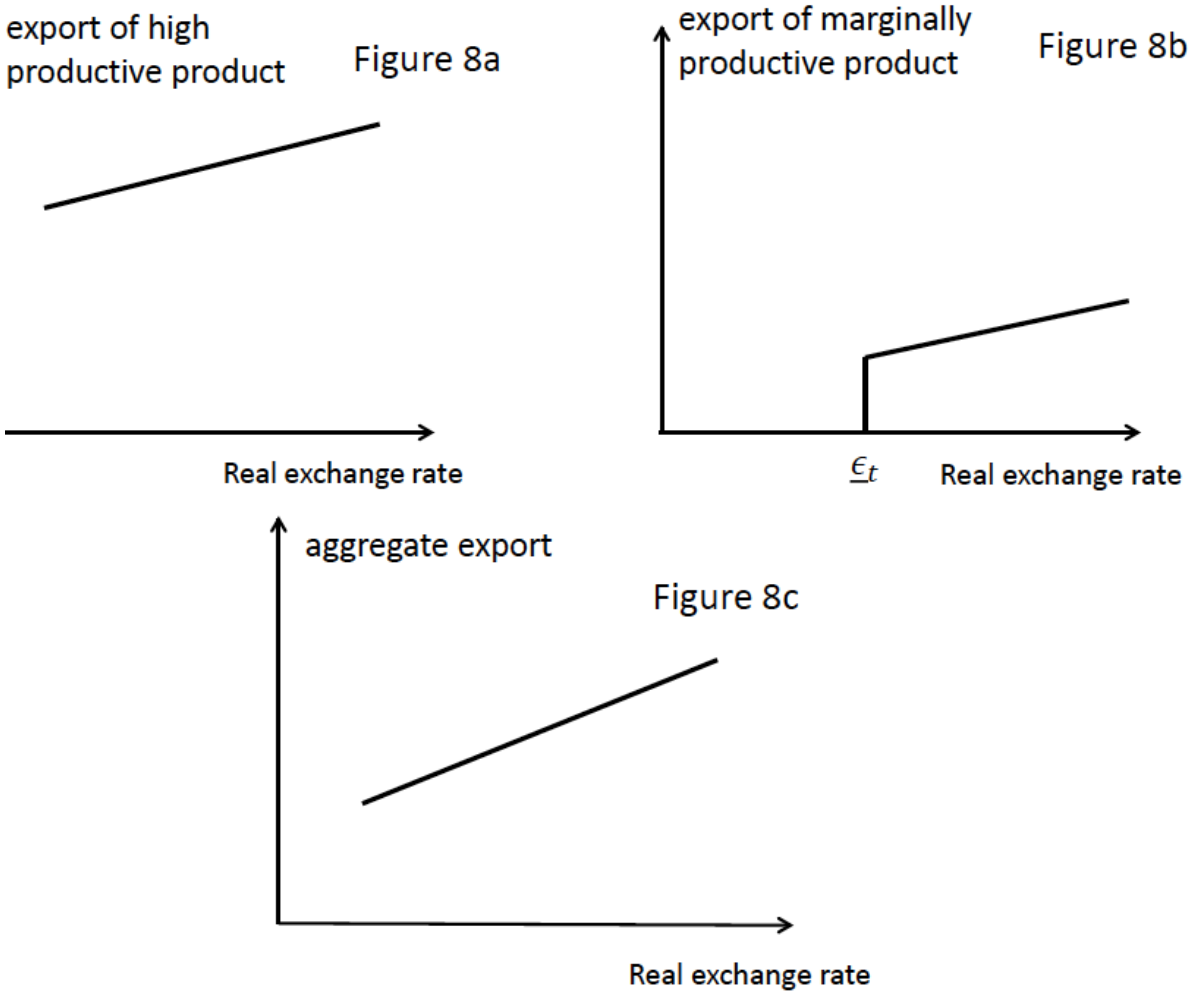


Figure 8. Response of exports to the exchange rate at extensive and intensive margins



A Details of Competitive Equilibrium

Aggregating the product prices for the export market in (12), the aggregate price index of home final goods for the foreign market is

$$p_t^F = \frac{\theta}{\theta-1}\frac{w_t/\epsilon_t}{\overline{a}_t^F N_t^{\frac{1}{\theta-1}} Z_t} = \frac{1}{\epsilon_t}\frac{\overline{a}}{\overline{a}_t^F}, \tag{39}$$

where $\overline{a}_t^F$ is the average productivity of the exported products given by

$$\overline{a}_t^F \equiv \left[\int_{\underline{a}_t}^{\infty} a^{\theta-1} f(a) da\right]^{\frac{1}{\theta-1}} = \left[\frac{\alpha}{\alpha+1-\theta}\left(\underline{a}_t\right)^{\theta-\alpha-1}\right]^{\frac{1}{\theta-1}} = \overline{a}\cdot\left(\underline{a}_t\right)^{-\frac{\alpha+1-\theta}{\theta-1}}.$$

The zero profit condition for export implies

$$q_t^F\left(\underline{a}_t\right) = (\theta-1)\,\phi\underline{a}_t Z_t.$$

Using the property $q_t^F(a)/q_t^F(\underline{a}_t) = \left(p_t^F(a)/p_t^F(\underline{a}_t)\right)^{-\theta} = \left(a/\underline{a}_t\right)^{\theta}$ for $a > \underline{a}_t$ from (12) and (14), we have the aggregate supply of home export as

$$\begin{aligned} Q_t^F &= \left[\int_{\underline{a}_t}^{\infty} q_t^F(a)^{\frac{\theta-1}{\theta}} f(a) N_t da\right]^{\frac{\theta}{\theta-1}} \\ &= q_t^F\left(\underline{a}_t\right) N_t^{\frac{\theta}{\theta-1}} \left[\int_{\underline{a}_t}^{\infty}\left(\frac{a}{\underline{a}_t}\right)^{\theta-1} f(a) da\right]^{\frac{\theta}{\theta-1}} \\ &= (\theta-1)\,\phi Z_t N_t^{\frac{\theta}{\theta-1}} \overline{a}^{\theta}\left(\underline{a}_t\right)^{-\frac{\alpha\theta+1-\theta}{\theta-1}}. \end{aligned}$$

Substituting the export price index p_t^F in (39) into the the export demand equation in (4), the aggregate demand for export is given by

$$Q_t^F = \left(\underline{a}_t\right)^{-\frac{\alpha+1-\theta}{\theta-1}\varphi}\epsilon_t^{\varphi} Y_t^*. \tag{40}$$

Then, the export market clearing condition solves for the cutoff productivity $\underline{a}_t$ such that

$$\begin{aligned} \underline{a}_t &= \left[\frac{(\theta-1)\,\phi\overline{a}^{\theta} Z_t N_t^{\frac{\theta}{\theta-1}}}{\epsilon_t^{\varphi} Y_t^*}\right]^{\frac{\theta-1}{\alpha(\theta-1)+(\alpha+1-\theta)(1-\varphi)}} \\ &= \left[\frac{\alpha(\theta-1)\,\phi}{\alpha+1-\theta}\frac{A_t^H N_t}{\epsilon_t^{\varphi} Y_t^*}\right]^{\frac{\theta-1}{\alpha(\theta-1)+(\alpha+1-\theta)(1-\varphi)}}. \end{aligned}$$

This is (19) in the text.

From $(39, 40)$, the home export value in terms of home final goods is

$$S_t^F \equiv \epsilon_t p_t^F Q_t^F = (\underline{a}_t)^{\frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}} \epsilon_t^{\varphi} Y_t^*.$$

This is (20) in the text.

The labor supply condition (23)together with the composite input price equation (8) can be written as

$$L_t = \frac{1}{(\psi_0 C_t)^{\psi}} \left(\frac{w_t}{\epsilon_t^{1-\gamma_L}} \right)^{\frac{\psi}{\gamma_L}}.$$

Similarly, the labor demand equation (9) together with the equation (8) can be written as

$$X_t = \left(\frac{w_t}{\epsilon_t} \right)^{\frac{1-\gamma_L}{\gamma_L}} \frac{L_t}{\gamma_L}.$$

Then from labor market clearing condition), we have the aggregate composite input as

$$\begin{aligned} X_t &= \frac{1}{\gamma_L (\psi_0 C_t)^{\psi}} \left[\frac{{w_t}^{1-\gamma_L+\psi}}{\epsilon_t^{(1-\gamma_L)(1+\psi)}} \right]^{\frac{1}{\gamma_L}} \\ &= X_t^F + X_t^H, \end{aligned}$$

where X_t^F and X_t^H denote the aggregate composite input use for export market and for the home market. Using $(11, 11, 13, 14)$, we have

$$\begin{aligned} X_t^F &= \int_{\underline{a}_t}^{\infty} \left[\frac{q_t^F(a)}{a Z_t} + \phi \right] f(a) N_t da \\ &= \int_{\underline{a}_t}^{\infty} \phi \left[\left(\frac{a}{\underline{a}_t} \right)^{\theta-1} (\theta - 1) + 1 \right] f(a) N_t da \\ &= \phi \frac{\theta\alpha + 1 - \theta}{\alpha + 1 - \theta} (\underline{a}_t)^{-\alpha} N_t, \end{aligned}$$

$$\begin{aligned} X_t^H &= \int_1^{\infty} \frac{q_t^H(a)}{a Z_t} f(a) N_t da \\ &= \frac{q_t^H(1)}{Z_t} \int_1^{\infty} a^{\theta-1} f(a) N_t da \\ &= \frac{Q_t^H}{A_t^H}. \end{aligned}$$

Together with (1) , we have $(24, 25, 26)$ in the text.

The profit arising from selling a product with productivity $a_{hit} = a$ in the home market is

$$\begin{aligned} \pi_t^H(a) &\equiv p_t^H(a)q_t^H(a) - w_t x_t^H(a), \\ &= \frac{1}{\theta - 1} w_t x_t^H(a). \end{aligned}$$

The profits from exporting a product with productivity $a_{hit} = a \geq \underline{a}_t$ to foreign market is

$$\begin{aligned} \pi_t^F(a) &\equiv \epsilon_t p_t^F(a)q_t^F(a) - w_t x_t^F(a), \\ &= w_t \left[\frac{1}{\theta - 1} x_t^F(a) - \frac{\theta}{\theta - 1} \phi \right]. \end{aligned}$$

Thus we have the average profit as

$$\begin{aligned} \overline{\pi}_t &= \int_1^\infty \left\{ \pi_t^H(a) + \pi_t^F(a) \right\} f(a) da \\ &= w_t \left[\frac{X_t}{(\theta - 1)N_t} - \frac{\theta}{\theta - 1} \phi \cdot (\underline{a}_t)^{-\alpha} \right]. \end{aligned}$$

This is (31) in the text.

Combining the free entry condition and the average value function in (28, 29), we have

$$\overline{V}_t = \overline{\pi}_t - \kappa + (1 - \delta + \delta\lambda) \frac{\kappa_{Et}}{\lambda_E}.$$

Substituting this of date t+1 into (28) , we we have (30) in the text.

The necessary and sufficient condition that the firm strictly prefers to maintain a product with the lowest productivity by paying the fixed cost is

$$\begin{aligned} 0 &< V_t(1) = \pi_t^H(1) - \kappa + E_t\{\Lambda_{t,t+1} \left[(1 - \delta) V_{t+1}(1) + \delta\lambda \overline{V}_{t+1} \right]\} \\ &= \pi_t^H(1) - \kappa + \delta\lambda \frac{\kappa_{Et}}{\lambda_E} + (1 - \delta) E_t[\Lambda_{t,t+1} V_{t+1}(1)], \text{ for all } t \end{aligned}$$

Thus a sufficient condition is $(Condition 2)$ in the text.

B Steady State

B.1 Steady State of the Aggregate Variables

In steady state, $\Lambda_{t,t+1} = \beta$ and $R_t = 1/\beta$. The free entry condition (28) and the average value function in (29) imply that the steady state average profit is given by the following constant:

$$\overline{\pi} = \kappa + \frac{\kappa_E}{\lambda_E}\left(\frac{1}{\beta} - 1 + \delta - \delta\lambda\right). \tag{41}$$

Or directly from the average profit equation (31), the average profit is related with other equilibrium aggregates such that

$$\overline{\pi} = \frac{w}{\theta - 1}\left[\frac{X}{N} - \theta\phi\underline{a}^{-\alpha}\right]. \tag{42}$$

From the foreign bond holding equation (22), we have

$$\epsilon D^* = \frac{\xi^* C}{1 - \beta R^*}, \tag{43}$$

which, combined with the current account balance equation (5) together with (10) and (20), implies

$$(1 - \gamma_L)\frac{wX}{N} = \underline{a}^{\frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}}\frac{\epsilon^\varphi Y^*}{N} + \frac{\xi^*\left(R^* - 1\right)}{1 - \beta R^*}\frac{C}{N}. \tag{44}$$

The export cut-off productivity equation (19), combined with (17), implies

$$\underline{a}^{\alpha + \frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}} = \frac{\alpha\phi\theta}{\alpha + 1 - \theta}\frac{wN}{\epsilon^\varphi Y^*}. \tag{45}$$

Combining the above two equilibrium relationships (44) and (45) regarding foreign assets and export markets with the average profit equation (42), we have

$$\begin{aligned}
\frac{\alpha\phi\theta w\underline{a}^{-\alpha}}{\alpha + 1 - \theta} &= \underline{a}^{\frac{(\alpha+1-\theta)(1-\varphi)}{\theta-1}}\frac{\epsilon^\varphi Y^*}{N} \\
&= (1 - \gamma_L)\frac{wX}{N} - \frac{\xi^*\left(R^* - 1\right)}{1 - \beta R^*}\frac{C}{N} \\
&= \frac{\alpha}{\alpha + 1 - \theta}\left[\frac{wX}{N} - (\theta - 1)\overline{\pi}\right],
\end{aligned} \tag{46}$$

which can be rearranged into

$$\left(\frac{\theta - 1}{\alpha + 1 - \theta} + \gamma_L\right)\frac{wX}{N} + \frac{\xi^*\left(R^* - 1\right)}{1 - \beta R^*}\frac{C}{N} = \frac{\alpha(\theta - 1)}{\alpha + 1 - \theta}\overline{\pi}. \tag{47}$$

From the total measure of products evolution equation (15),

$$\frac{N_E}{N} = \frac{\delta}{\lambda_E}(1-\lambda). \tag{48}$$

The final goods market clearing condition (34), we get

$$\frac{C}{N} + \frac{G}{N} + \kappa_E \frac{\delta}{\lambda_E}(1-\lambda) + \kappa = A^H \left(\frac{X}{N} - \phi \underline{a}^{-\alpha} \frac{\theta\alpha + 1 - \theta}{\alpha + 1 - \theta} \right),$$

which, using (17), (41), and (42), implies

$$\frac{\theta - 1}{\alpha + 1 - \theta} \frac{wX}{N} + (1+g)\frac{C}{N} = \frac{\alpha(\theta-1)}{\alpha+1-\theta}\overline{\pi} + \frac{\kappa_E}{\lambda_E}\left(\frac{1}{\beta} - 1\right), \tag{49}$$

where $g = G/C$. Using these equilibrium conditions (47) and (49) for current account and domestic final goods market, we can solve for $\frac{C}{N}$ and $\frac{wX}{N}$ simultaneously as functions of parameters and exogenous variables, such that

$$\begin{aligned} \frac{C}{N} &= c, \\ \frac{wX}{N} &= x. \end{aligned}$$

From the composite input price equation (17) together with (16) and (19), we have

$$w = \frac{\theta - 1}{\theta} \left(\frac{\alpha}{\alpha + 1 - \theta} \right)^{\frac{1}{\theta - 1}} Z N^{\frac{1}{\theta - 1}} \equiv w(N). \tag{50}$$

Given c and x, combining the equilibrium aggregate quantity of composite input in (24) with (50), the real exchange rate is

$$\epsilon = \left\{ \frac{w(N)}{N^{\gamma_L}} \left[\gamma_L (\psi_0 c)^{\psi} x \right]^{-\frac{\gamma_L}{1+\psi}} \right\}^{\frac{1}{1-\gamma_L}} \equiv \epsilon(N). \tag{51}$$

Given $w(N)$ and $\epsilon(N)$, from (19), the export cutoff productivity is

$$\underline{a} = \left[\frac{\alpha\theta\phi}{\alpha + 1 - \theta} \frac{w(N) N}{\epsilon(N)^{\varphi} Y^*} \right]^{\frac{\theta - 1}{\alpha(\theta-1) + (\alpha+1-\theta)(1-\varphi)}} \equiv \underline{a}(N). \tag{52}$$

Given c and $\epsilon(N)$, the steady state foreign bond holding can be found from (43) such that

$$D^* = \frac{\xi^*}{1 - \beta R^*} \frac{cN}{\epsilon(N)} \equiv D^*(N). \tag{53}$$

Now, to solve for the steady state values, it is enough to solve for the steady state N, which can be found by plugging the steady state cutoff productivity $\underline{a}(N)$ into the steady state current account balance equation (44) such that

$$(1-\gamma_L)x - \frac{\xi^*(R^*-1)}{1-\beta R^*}c = \left[\frac{\alpha\theta\phi w(N)}{\alpha+1-\theta}\right]^{\frac{(\alpha+1-\theta)(1-\varphi)}{\alpha(\theta-1)+(\alpha+1-\theta)(1-\varphi)}} \left[\frac{\epsilon(N)^{\varphi}Y^*}{N}\right]^{\frac{\alpha(\theta-1)}{\alpha(\theta-1)+(\alpha+1-\theta)(1-\varphi)}}. \quad (54)$$

B.2 Steady State of the Cross Section

In order to derive the steady state distribution of domestic and export sales and profit rate across firms, we need to derive the steady state distribution of the number of products across firms. We follow Klette and Kortum (2004) to use a continuous time approximation for the evolution of number of products by each firms. This is, for a time interval of infinitesimal length Δt, the firm who has j number of products looses one existing product with probability $j\delta \cdot \Delta t$, gains one new product with probability $j\lambda\delta \cdot \Delta t$, gains or looses more than one number of products with negligible probability $o(\Delta t)$ where $\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} \to 0$. Let M_j be the steady state measure of firms with j number of products. In the steady state, the rate of entering firms to acquire one product is equal to the rate of firms with one number of products loosing their product as

$$\lambda_E N_E = \delta M_1.$$

Similarly, the rate of firms with one number of product gaining a new product is equal to the rate of firms with two number of product loosing one product as

$$\lambda\delta M_1 = 2\delta M_2.$$

More generally the rate of firms with $j-1$ number of product gaining a new product is equal to the rate of firms with j number of product loosing one product as

$$(j-1)\lambda\delta M_{j-1} = j\delta M_j,$$

or

$$M_j = \frac{j-1}{j}\lambda M_{j-1} = \frac{\lambda^{j-1}}{j}M_1.$$

Because we assumed $\lambda < 1$ by (Assumption 2), the total measure of firms with some products is

$$M \equiv \sum_{j=1}^{\infty} M_j = \frac{M_1}{\lambda}\left(\lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{3} + ...\right) = \frac{M_1}{\lambda}\left[-\ln(1-\lambda)\right].$$

Then the fraction of firms with j number of products in total firms is

$$m_j \equiv \frac{M_j}{M} = \frac{\lambda^j}{j \cdot [-\ln(1-\lambda)]}.$$

Once we know the fraction of firms with j number of products m_j, we can compute numerically the distribution of domestic sales, export sales, total sales, and profit rate across firms, because we know the distribution of productivity of each product a is independent and distributed according to the Pareto distribution as $F(a) = 1 - a^{-\alpha}$ for $a \in [1, \infty)$ by (Assumption 1). We also know domestic sales, export sales, and profit of a product with productivity a as

$$s^H(a) = p^H(a)q^H(a) = \left(p^H(a)\right)^{1-\theta} Q^H = \left(\frac{a}{\underline{a}}\right)^{\theta-1} \frac{Q^H}{N} \tag{55}$$

$$s^F(a) = \epsilon p^F(a)q^F(a) = I(a-\underline{a}) \cdot \left(\frac{a}{\underline{a}}\right)^{\theta-1} s^F(\underline{a}) = I(a-\underline{a}) \cdot \left(\frac{a}{\underline{a}}\right)^{\theta-1} \theta w \phi \tag{56}$$

$$\begin{aligned} \pi(a) &= p^H(a)q^H(a) + \epsilon p^F(a)q^F(a) - w\left[\frac{q^H(a)+q^F(a)}{aZ} + I(a-\underline{a})\phi\right] \\ &= \left(\frac{a}{\underline{a}}\right)^{\theta-1} \frac{Q^H}{\theta N} + I(a-\underline{a}) \cdot \left[\left(\frac{a}{\underline{a}}\right)^{\theta-1} - 1\right] w\phi. \end{aligned} \tag{57}$$

C Calibration for Panel Data

For calibration for firm level panel data, we start with the steady state distribution of number of products across firms. Then we conduct $1,000$ simulations of length $T = 16$ and $M = 100,000$ firms. In order to make the calibration comparable to Kaigin data, for each of 1,000 simulations, we build a balanced panel of large exporters by first selecting the largest firms in terms of the total sales on the initial year until we accumulate 60% of total sales. Then, we select the subsample of the firms that exported for the 16 years without interruption. Finally, we re-sampled

the observations with less than 500 firms in the final panel of exporters. The medium number of firms in the panel simulation is 1,672.

Consider a firm h with I_{ht} with the portfolio of productivity distribution as $(a_{h1t}, a_{h2t}, ..., a_{hI_{ht}t})$. For the panel calibration, we can trace the life and death of each product (amoeba) with productivity a_{hit} at date t. Between date t and date $t+1$, $\{(a_{hit})\}$ evolves as

(i) stays the same as $\{(a_{hit})\}$, with probability $(1-\lambda\delta)(1-\delta)$

(ii) adds a new product $\{(a_{hit}), (\widetilde{a})\}$, with probability $\lambda\delta(1-\delta)$

(iii) replaced by a new product $\{(\widetilde{a})\}$, with probability $\lambda\delta^2$

(iv) the product dies without replacement $\varnothing$, with probability $\delta(1-\lambda\delta)$

Here $\widetilde{a}$ is the productivity of a new product which is distributed according to the identical Pareto distribution of $F(a) = 1 - a^{-\alpha}$ for $a \in [1, \infty)$.

Once we characterize the evolution of productivity portfolio $(a_{h1t}, a_{h2t}, ..., a_{hI_{ht}t})$ of each firms, we can compute the evolution of firm level domestic and export sales and profit, by using these functions of product a at date t which are the function of the aggregate conditions as $(55, 56, 57)$.