Redistributive Taxation with Heterogenous Relative Consumption Concerns

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ABSTRACT

This paper examines linear income taxation in a model where preferences over relative consumption are only exhibited by *some* individuals in the population. This heterogeneity in preferences generates several interesting issues in the optimal tax context. We analyze cases where the tax authority (1) uses a non-welfarist objective (one which places variable weight on the welfare of relatively-concerned agents), or (2) remains ignorant of preference heterogeneity and maintains a standard social welfare function. Numerical results are obtained to illustrate 'optimal' tax parameters. A key result is that a government which understands the extent of relative consumption concerns — but places no social weight on individuals with such preferences — nevertheless sets a significantly more progressive tax system than a government ignorant of relative-consumption motivations.

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1 Introduction

The possibility that individuals care not only about their own material well-being — but also about their well-being relative to others in society — has long aroused interest among economists. One familiar area of inquiry has been to investigate the behavioral consequences of agents' concern not just with their own *consumption*, c, but also with this consumption relative to some population *average consumption*, $c/\bar{c}$. Economic models with this feature typically proceed by assuming that all individuals have identical attitudes towards relative standing. By contrast, this paper takes as its starting point a society where *only a subset* of the population is motivated by relative consumption concerns. This assumption is realistic but non-trivial, since although concern with relative standing may be an important characteristic for policy makers to consider, the government of such a society — as well as individuals within it — will likely be unable to directly identify the relatively-concerned. How will tax policy be affected by this heterogeneity, if at all? Extending the general methodology of Boskin and Sheshinski (1978), I study the features of a linear tax system which a government motivated by redistributive goals would impose on such an economy. The objective of this paper is twofold. The first goal is to understand how the presence of preference heterogeneity affects optimal taxation relative to that in homogenous populations. Optimal tax rates are determined numerically. The second (related) goal is to examine the welfare consequences of ignoring preference heterogeneity. To address this objective, we compute the optimal rates which would be set by a government that is unaware of the existence of the relatively-concerned, and thus bases tax policy on an incorrect distribution of underlying abilities.

Recent empirical work has begun to shed light on the existence of relative consumption concerns in real-world situations. Luttmer (2005) and Blanchflower and Oswald (2004) find empirical evidence showing how measures of reported well-being tend to diminish with the earnings and consumption of neighbors.[1] Employing regression analysis, these studies report conditional average effects of relative standing on reported well-being, which by definition vary among the sample. Alpizar et. al (2004) and Johansson-Stenman et. al (2002) use experimental data to measure the importance of relative income position. They find a variety of levels of such importance, including that "a substantial fraction of the respondents are either not positional at all ... or completely po-

[1] Blanchflower and Oswald examine the stylized fact that while incomes and consumption have been increasing over time on average, reported levels of 'happiness' have not. This evidence has been used to support the importance of relative standing to individual well being.

sitional" (Johansson-Stenman et. al 2002, 373). Although these studies substantiate the existence of relative-consumption effects among individuals, they do not concretely pin down the *distribution* of theses attitudes in the population. At very least, casual empiricism suggests that some people care very much about their standing in relation to society, while others significantly less so.

If indeed relative consumption concerns are present socially, there is arguably a scope for governments to improve social welfare through policy. At the heart of the issue is a 'positional externality' (Frank, 2005): individuals that care about relative consumption tend to increase labour supply in an attempt to improve their social position. Of course, others follow suit, so that in equilibrium there is no change in relative standing but all agents are working harder. Tax policy may be used to control this externality. Boskin and Sheshinski (1978) demonstrate that a utilitarian government should implement a more progressive linear tax scheme when individuals have relative consumption as part of their preferences. Oswald (1983) extends this analysis to a non-linear setting, and indicates how marginal tax rates are optimally higher in a 'jealous' economy. These studies use a standard social welfare function that includes 'comparison preferences' as part of the social objective. Avoiding the social welfare approach, Persson (1995) and Corneo (2002) show how increased progressivity in income taxation can lead to Pareto improvements provided that pre-tax earnings are not too widely distributed. Ireland (1998) obtains similar results in a model where individuals expend resources to improve their perceived status in the population. In general, the case for progressive taxation and redistribution is strengthened when relative consumption concerns are present.

The key question of this paper is to what extent increased progressivity is optimal when heterogeneity over both preference types and earning abilities co-exist. A number of new issues are introduced by allowing this kind of heterogeneity. First, it is likely that preference-types are unobservable by the government, owing to different (ability, preference) types earning the same gross income.[2] If so, an attempt to control a relative-consumption externality through income taxation can inadvertently distort the decisions of those who do not make comparisons to others. Since individuals have different underlying preferences, a government faces a tradeoff between mediating the consumption externality and satisfying standard redistributive objectives.[3] Second, it is also

[2]Boadway et. al (2002) consider optimal taxation in a model where individuals differ along two dimensions: ability and preference for leisure. In their paper, the government cannot distinguish the leisure preference by observing earnings alone. The model used here is conceptually different, since agents overwork by choosing labor supply strategically.

[3]O'Donoghue and Rabin (2006) undertake a similar exercise with respect to sin taxes. They allow heterogeneity

possible that the presence of non-concerned individuals in society affects the behavior of concerned individuals. This will be true if those motivated by 'keeping up with the Joneses' make comparisons to others who are not participating in this game. Third, the tax authority must implicitly or explicitly compare the importance of welfare losses caused by relative consumption concerns with the welfare of non-concerned individuals. For example, it may be ethically untenable for a government to place any social weight on the 'enviousness' of certain individuals.

The model presented attempts to capture the importance of these new issues. A government is restricted to the use of a linear income tax scheme to maximize a potentially non-welfarist social objective. Following Kanbur, Pirttila and Tuomala (2006), the government does not consider the 'jealous' preferences of relative-concerned individuals when choosing policy, but may be aware of the behavioral response of such individuals to policy parameters. If is so aware, it can place a variable weight on the underlying welfare of each preference-type in the economy, but the analysis does not take a stand on what the social weight *should* be given to each group. On the other hand, the government may be unaware of the existence of relative-concerned agents. If so, it perceives higher average earnings as being the result of a different underlying ability distribution, and assumes that preferences are identically unconcerned with position. The government then (mistakenly) uses what amounts to a standard social welfare function in its policy-setting. Regardless of the social objective, agents respond to the tax system according to their underlying preferences. Relative-concerned agents choose labour supply strategically based on the actions of their 'neighbors' in the ability distribution. Agents unconcerned with relative position simply respond to the tax parameters set by the government.

Section 2 of the paper sets up the model of individual and government behavior. Lacking clear analytical results, in Section 3 a quasi-linear form of preferences is chosen to illustrate the model. A numerical simulation is performed to find the tax rates chosen by the government under each objective. Two results from this example are especially noteworthy. First, the chosen tax system tends to be more progressive when relative-concerned individuals compare themselves to unconcerned individuals in the population. Second, the tax system chosen is significantly more progressive when the government acknowledges the relative consumption externality. However, this appears to be true even if the government places *no social weight* on the well-being of relative-

along the dimensions of inherent preference for harmful goods and degree of self-control problem in consuming such goods, and examine optimal taxation in such an environment.

concerned agents. In fact, some agents of this group fare better when the government puts no weight on their welfare, as compared to the case where the government remains ignorant of the relative consumption externality.

Section 4 concludes the paper.

2 A Model with Relative Consumption Externalities

2.1 Basic Structure

The economy of interest is populated by agents who differ along two dimensions: inherent earning ability (indexed by w) and concern with relative consumption levels. Consider first the preference dimension of heterogeneity. An 'ordinary' agent (labelled type o) has preferences over own-consumption (c) and labour-effort (L) represented by:

$$u_o(c, L) = u(c, L) \tag{1}$$

with $u_c > 0$, $u_{cc} \leq 0$, $u_L < 0$, $u_{LL} \geq 0$ and $u_{cL} \leq 0$. Alternatively, an agent may be concerned not only with his own consumption, but also with that of a group of his neighbors. Such an agent is labelled as type r, and his utility function is given by:

$$u_r(c, L, \bar{c}) = u(c, L) + \beta v(c/\bar{c}). \tag{2}$$

$\beta > 0$ parameterizes the magnitude of relative consumption concern among these agents, and $\beta = 0$ would imply that type r's are identical to type o's. $\bar{c}$ represents the average consumption level within type-r's *reference group*, a concept that will be made more precise below. For analytical simplicity, this concern enters additively into the r-agents' preferences, and these agents share identical sub-utility functions over (c, L) with the o-group. I assume $v' > 0$, $v'' \leq 0$ and $v(1) = 0$. The latter assumption serves as a rough benchmark of comparability between preference-groups: namely, an r-agent who consumes more (less) than their neighborhood average fares better (worse) than an o-type with identical (c, L), and utilities are identical across preference groups when $c = \bar{c}$.[4]

[4]With the assumptions made, it can be checked that the marginal rate of substitution between own-consumption and labour for r-types — $MRS_r(c, L)$ — is increasing in $\bar{c}$; moreover, $\partial u_r/\partial \bar{c} < 0$. Dupor and Liu (2003) define the first condition as 'keeping up with the Joneses,' or KUJ (increases in neighborhood average consumption raise the marginal utility of consumption relative to the marginal disutility of work) and 'jealousy' (increases in neighborhood

The size of the population is normalized to 1. Overall, a proportion q_r of the population are of type r, and $q_o = 1 - q_r$ are of type o. I consider a discrete distribution of I positive earning abilities (wages) where an agent of ability type i has wage $w_i \in (w_1, w_2, \ldots, w_I)$. Let $w_1 > 0$ indicate the lowest ability, and w_I the highest. The proportion of each ability in the population is p_i with $\sum_{i=1}^{I} p_i = 1$. An agent's (two-dimensional) type can then be summarized by (θ, i), where $\theta \in \{o, r\}$. Assuming that preferences and abilities are independently distributed, the proportion of each type of agent is $\phi(\theta, i) = q_\theta p_i$, where $\sum_{\theta=r,o} \sum_{i=i}^{I} \phi(\theta, i) = 1$.

2.2 Labour Supply and Consumption: No Taxation

In the laissez-faire (no tax) environment, the optimizing behavior of each agent-type is straightforward. Own consumption is governed by the individual's budget constraint $c_i = w_i L_i$. Thus (dropping subscripts) type o chooses L to maximize $u(wL, L)$, with first-order condition:

$$wu_c + u_L = 0. \tag{3}$$

Denote the solution to (3) as $L^o(w_i)$, which yields the indirect utility function $V^o(w_i) = u(c^o(w_i), L^o(w_i)) = u(w_i L^o(w_i), L^o(w_i))$.

The behavior of individuals concerned with relative consumption is somewhat more complex. An agent of type (r, i) cares about his own-consumption as well as his consumption compared to that of his reference group. Each such agent takes the average consumption level as given. I model the reference group as a neighborhood of individuals with similar abilities and therefore similar earnings. Falk and Knell (2003) demonstrate that it is both theoretically and empirically unrealistic to suppose that individuals make comparisons to the entire population, and instead tend to form a reference group with those with similar characteristics.

Thus, consider the labour-choice problem of a type (r, i) individual who compares himself to a community consisting entirely of other (r, i) individuals. His choice of L is made to maximize (2)

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average consumption make others feel worse). They show that the presence of jealousy is sufficient to stimulate extra labour supply, and that this effect is strengthened when individuals also attempt to keep up with the Joneses. For example, a Cobb-Douglas utility function with arguments $(c, L, \bar{c})$ exhibits jealousy but not (KUJ). Type r individuals would still work more than their type o counterparts, but the extent of this extra work would be unaffected by the magnitude of average consumption in the neighborhood.

subject to $c = wL$ and given $\bar{c}$:

$$wu_c + u_L + \beta v'(c/\bar{c})(w/\bar{c}) = 0. \tag{4}$$

(4) can be solved implicity for $L(w, \bar{c})$: this is a reaction function stipulating individual labour supply given a level of average consumption in the reference group.[5] In a symmetric Nash equilibrium, $L(w, \bar{c})$ is the same for all r-agents with ability w, thus average consumption in this group is $\bar{c} = wL(w, wL)$. Substituting this expression into (4) yields:

$$wu_c + u_L + \beta v'(1)(1/L) = 0. \tag{5}$$

The implicit solution to (5) is the equilibrium labour supply of type (r, i), denoted $L^r(w_i)$. Since $v'(\cdot) > 0$, it can be seen that a type (r, i) agent supplies more work effort in equilibrium than his (o, i) counterpart.[6] Indirect utility is given by $V^r(w_i) = u_r(w_i L^r(w_i), L^r(w_i))$. Recall that $v(1) = 0$, implying that the r-type individual 'overworks' relative to a similarly-abled type o. In other words, since additional labour supply has no effect on relative consumption in equilibrium, type r is engaged in a positional battle where the extra labour supply of neighbors forms a negative externality (see e.g. Frank, 2005). The degree of additional labour supply on the part of r-agents varies according to ability, although the magnitude of this difference may increase or decrease with ability, in general.

2.3 Labour Supply and Consumption: Linear Income Taxation

It is straightforward to extend the preceding model of labour supply to one in which the government imposes a linear income tax of the form $T(Y) = tY - k$, where $t > 0$ is the marginal tax rate on earned income and k is a lump sum transfer (or tax) given to all taxpayers. A linear tax is clearly a simplification, but allows us to focus on the distributional impacts of taxation across preference types without the complication of incentive compatibility constraints and a possible multidimensional screening problem. Consumption for each agent becomes $c = w(1 - t)L + k$.

[5] Persson (1995) follows a similar game-theoretic approach to determine labour supply. His model involves a two-agent case where each agent has a different ability. By contrast, agents here are identical within a reference group, but there are I reference groups, each corresponding to a different ability. (4) is akin to equation (2) in Dupor and Liu (2003), except there are I such conditions in this model: one for each ability level.

[6] i.e. $dL/d\beta > 0$ for all $\beta \geq 0$, thus $L^r(w_i) > L^o(w_i)$.

Thus, labour supply for type o's must satisfy the first order condition:

$$w(1-t)u_c + u_L = 0 \tag{6}$$

with the solution $L^o(w_i, t, k)$. Indirect utility is given by $V_i^o(w_i, t, k) = u(w_i(1-t)L^o(w_i, t, k) + k, L^o(w_i, t, k))$. For type r's, the first order condition for labour supply becomes:

$$w(1-t)u_c + u_L + \beta v'((w(1-t)L + k)/\bar{c})(w(1-t)/\bar{c}) = 0. \tag{7}$$

At this point, I introduce the possibility that the reference group for an individual of type (r, i) may also include some individuals of type (o, j) with $i < j$; that is, 'ordinary' types of higher ability who earn the same (pre-tax) incomes as a given set of r types with lower abilities. Recall that in equilibrium, $\bar{c}_i = w_i L^r(w_i)$, where $L^r(w_i)$ solves (5). In the laissez-faire environment, relative consumption (and hence labour supplies of type (r, i)) are unchanged should an individual of type (o, j) enter the (r, i) neighborhood; i.e.

$$w_j L^o(w_j) = w_i L^r(w_i) \tag{8}$$

Call the w_j satisfying (8) $\hat{w}_i$: this is the o-type ability which may pool with type (r, i) when reference groups are characterized by the same pre-tax earned income. One might imagine an original state without taxation in which this pooling occurs over time through some form of migration (either physical, or through social networks).

In equilibrium, the relative proportions of o to r types in such reference groups is indeterminate. Let ρ_i be the fraction of type r's in a reference group defined by abilities (i, j) as above. Then average consumption in this group is:

$$\bar{c}_i = \rho_i w_i L^r(w_i) + (1-\rho_i)\hat{w}_i L^o(\hat{w}_i). \tag{9}$$

I assume that the imposition of redistributive taxation in this economy does not immediately alter the reference-group.[7] If so, average consumption in each reference group is a linear combination

[7] The model presented is static, but it is possible to imagine how reference groups might change dynamically with heterogeneity in preference groups. Post-tax, type (r, i) might respond to the presence of type (o, j)'s in the neighborhood by supplying more labour, such that the net income (i.e. consumption) of (r, i) now exceeds that of type (o, j). Despite an immediate utility gain from these events (because $c_i^r > \bar{c}_i$), r-types may subsequently compare

of the post-tax incomes of types (r, i) and (o, j). The equilibrium labour supply of the r-types in each group is the L which solves (7), with average consumption given by

$$\bar{c}_i = \rho_i w_i (1-t) L + (1-\rho_i) \hat{w}_i (1-t) L^o(\hat{w}_i, t, k) + k. \tag{10}$$

Call this solution $L^r(w_i, \rho_i, t, k)$. How is ρ_i determined? A natural possibility is that it reflects the relative proportions of abilities (r, i) and (o, j) in the population. In following sections, it will be convenient to define p_i in this sense:

$$\rho_i \equiv \frac{\phi(r, i)}{\phi(r, i) + \phi(o, j)} = \frac{q_r p_i}{q_r p_i + q_o p_j}. \tag{11}$$

As is well known, the labour supply response of type o individuals to tax parameters is a combination of income and substitution effects. I do not elaborate upon these here. For type r, however, there is an additional channel by which changes in tax parameters affect labour supply. Recall that these individuals respond strategically to the labour supplies of others in the reference group, through (7). I term this the *reference-group effect.*[8] The overall direction of this effect is ambiguous in equilibrium. The reaction function of a type r is $L^r(w, t, k, \bar{c})$. Then the reference-group effect considers:

$$\frac{\partial L^r(w, t, k, \bar{c})}{\partial \bar{c}} \frac{\partial \bar{c}}{\partial t} = \rho w \frac{\partial L^r}{\partial t} + (1-\rho) \hat{w} \frac{\partial L^o}{\partial t} \tag{12}$$

To illustrate this ambiguity, suppose $\rho < 1$, so that the decisions of (o, j) types affect the labour supply of type (r, i)'s in the neighborhood via $\bar{c}_i$. An increase in t may be expected to decrease the labour supply of type (o, j), but average consumption in the group may tend rise or fall *overall* depending on the response of type r's to this change. The optimal response of type (r, i) to a decrease in $L^o(\hat{w}_i, t, k)$ may actually be an *increase* in labour supply.

Since the direction of $\partial L^r / \bar{c}$ is itself ambiguous, it similarly impossible to determine the sign $\partial L^r / \partial p$. An increase in the fraction of ordinary types in a reference group may either increase or decrease the work incentive of relative consumption concerned types in the same group. Figure 1 depicts the variability in labour supply due to changes in the form of the reference group. Two

themselves with a group with the same (higher) net incomes. This group could include a set of type (o, m), where $w_m > w_j$, given that net incomes increase monotonically with ability.

[8]This effect is absent in the estimates of Boskin and Sheshinski (1978), who employ a logarithmic utility function over consumption and relative consumption. Their form of utility ensures that there is no marginal response of labour supply via changes in average consumption. i.e. the 'keeping up with the Joneses' feature is absent.

scenarios are shown (corresponding to the numerical example in Section 3): one for a high marginal tax rate, and one for a lower rate. In each case, k is set at zero. For the case shown, labour supply decreases monotonically in p. That is, a type r with a given wage (here, $w = 1.4$) works harder in the presence of more type o's.

2.4 'Optimal' Linear Taxation

As described above, attention is restricted to a linear tax system that a government with a possibly non-welfarist objective would impose in this economy. The government can always observe incomes but not necessarily preference types. If preferences are unobservable, each agent must face the same marginal tax rate and receive the same transfer, obviating the need to be concerned with incentive compatibility constraints as in a non-linear optimal tax problem. However, lacking unambiguous analytical results for the optimal tax system chosen under objectives described below, I do not explicitly present a solution to the optimization problem here. The next section includes numerical results to provide some results for the model.

The policy problem of setting a linear tax system is investigated under two sets of assumptions about the government's capabilities. In the first case, labelled 'perceptive,' the government understands the existence of preference heterogeneity and knows the proportions of each type in the population. In the second case, labelled 'ignorant,' the government assumes that all preferences are of the ordinary variety. Observing (pre-tax) incomes, it misperceives the true distribution of underlying abilities, and believes higher incomes on average to be the result of higher abilities than actually exist. In this sense it unconsciously ignores the relative consumption externality among r-types.

The social objective of the perceptive government is given by:

$$S = \sum_{\theta=r,p} \sum_{i=1}^{I} S_\theta(V^\theta(w_i))\phi(\theta, i). \tag{13}$$

Following the descriptions of O'Donoghue and Rabin (2006) and Kanbur, Pirttila and Tuomala (2006), the function $S_\theta(V^\theta(w_i))$ is non-welfarist in the sense that it may not correspond to individual utilities. I adopt this convention in the following sense: if an individual is of type o, then $S_o(V^o(w_i)) = a\omega(V^o(w_i))$, where $\omega(\cdot)$ is an increasing concave function that respects the 'ordinary' preferences of these individuals, and $a \in [1/2, 1]$ is the social weight placed on this type of individual by the government (as in Boadway et. al, 2002). If an individual is of type r, then $S_r(V^r(w_i)) = (1-a)\omega(V_*^r(w_i))$, where

$$V_*^r(w_i) = u_o(c^r(w_i), L^r(w_i)). \tag{14}$$

The perceptive government chooses t and k to maximize (13) subject to budget balance

$$k + R = t\bigg(\sum_{i=1}^{I}\phi(o,i)w_iL^o(w_i,t,k) + \sum_{i=1}^{I}\phi(r,i)w_iL^r(w_i,\rho_i,t,k)\bigg) \tag{15}$$

where $R \geq 0$ is a revenue requirement. The expression in the round brackets on the right hand side of (15) is simply the average gross earnings of individuals under some (t,k) and potentially heterogenous reference groups (as given by ρ_i).

Equation (14) implies that the government sets $\beta = 0$ when socially evaluating the welfare of type r. It therefore disregards any utility individuals receive from consuming more or less than reference group average. By construction, this assumption is unimportant if $\rho_i = 1$, since $c_i = \bar{c}_i$ in this case. If $\rho_i < 1$ (there are type o's in the reference group), the government doesn't care about utility which r's may receive from 'beating' average consumption within their neighborhood. It does, however, care about the extent of the relative consumption externality, since overwork will reduce an r-type's equilibrium well-being. $(1-a)$ is a measure of the government's concern for this reduced welfare. If $a = 1/2$, equal weight is placed on each type's (non-envious) well-being in the social objective; if $a = 1$, the government does not try to correct the relative consumption externality since it places no weight on the welfare loss of type r's.

An *ignorant* government observes $\tilde{I} \neq I$ income levels, and assuming ordinary preferences for all agents deduces a population distribution of abilities $\tilde{p}_1$ type w_1, $\tilde{p}_2$ type w_2, etc. The social objective of the ignorant government is given by:

$$\tilde{S} = \sum_{i=1}^{\tilde{I}}\omega(V^o(w_i))\tilde{p}_i. \tag{16}$$

This policymaker places identical social weight on all agents since it does not consider preference heterogeneity. Instead, it chooses (t,k) to maximize (16) subject to what it believes to be the budget constraint:[9]

$$k + R = t\left(\sum_{i=1}^{\tilde{I}} \tilde{p}_i w_i L^o(w_i, t, k)\right). \tag{17}$$

Lastly, I consider a hypothetical situation where the government is still restricted to linear tax systems, but can observe preference types and therefore condition the tax system on this variable. That is, type o's face the tax system defined by (t_o, k_o) and type r's by (t_r, k_r). Knowing the behavioral response of each type to the policy parameters, the government chooses (t_o, k_o, t_r, k_r) to maximize (13) subject to

$$q_o k_o + q_r k_r + R = t_o\left(\sum_{i=1}^{I} \phi(o,i) w_i L^o(w_i, t_o, k_o)\right) + t_r\left(\sum_{i=1}^{I} \phi(r,i) w_i L^r(w_i, \rho_i, t_r, k_r)\right). \tag{18}$$

(18) implies that individuals with identical incomes may nevertheless face different tax burdens based on their underlying preferences. This is an issue considered in more depth by Fleurbaey and Maniquet (2006).

3 Example: Quasi-Linear Preferences

In this section, I illustrate the model with individual preferences that take a specific quasi-linear in consumption form, and provide a numeric calculation of linear tax and transfer rates under different government objectives. With quasi-linear in consumption preferences, closed-form solutions for equilibrium labour supplies can be obtained. The marginal utility of own-consumption is constant within preference groups (r and o), which serves to weaken the traditional marginal utility of consumption argument for redistribution.[10] Moreover, there is no income effect on labour supply. This clarifies the reference-group effect of changes in the tax parameters (particularly, k) for type r's. I show, for example, that L^r may depend upon k via this effect.

For type-o agents, the utility function is simply[11]

$$u_o(c, L) = c - \alpha L^2 \tag{19}$$

[9] It is likely that this budget constraint would not be balanced after-tax. I account for this in numerical simulations by reducing or increasing k correspondingly.

[10] If the government were utilitarian and the population entirely composed of type o's, there would be no social desire for redistribution.

[11] The compensated and uncompensated elasticities of labour supply are identical in this case and equal unity when the exponent on labour is quadratic.

and for type r:

$$u_r(c, L, \bar{c}) = c - \alpha L^2 + \beta\left(\frac{c}{\bar{c}} - 1\right). \tag{20}$$

Note also $MRS^o(c, L) = 1/2L$ and $MRS^r(c, L) = \beta/(2\alpha L\bar{c})$: for type r's, the marginal utility of own consumption decreases in average consumption.

Without taxation, $L^o(w) = w/2\alpha$. Equilibrium labour supply for type r's without taxation ($t = k = 0$ and $p = 1$) solves (5):

$$L^r(w) = \frac{w + (w^2 + 8\alpha\beta)^{1/2}}{4\alpha} \tag{21}$$

Clearly, $L^r(w) > L^o(w)$. The 'overwork' of type r is given by:

$$L^r(w) - L^o(w) = \frac{(w^2 + 8\alpha\beta)^{1/2} - w}{4\alpha}. \tag{22}$$

This difference is increasing in β (relative consumption concern), decreasing in α (labour disutility), and ambiguous in w (ability). Similar results are obtainable for consumption differences. Intuitively, an increase in concern for relative consumption worsens the level of over-consumption by type r's.

With linear taxation in place, the labour supply of type o's becomes $L^o(w, t) = w(1 - t)/2\alpha$. Assuming that there is pre-tax migration (so that the reference group for ability type (r, i) includes a fraction $(1 - \rho_i)$ of type o's with ability $\hat{w}_i$ from (8)), we have (dropping subscript i):

$$\begin{aligned} L^r(w, \rho, t, k) \quad = \quad & \frac{1}{2\alpha w(1-t)\rho}\Big\{\rho(w(1-t))^2 - 2\alpha(A+k) + \\ & \Big(\big[\rho(w(1-t))^2 + 2\alpha(A+k)\big]^2 + 8\alpha\beta\rho(w(1-t))^2\Big)^{1/2}\Big\} \end{aligned} \tag{23}$$

where $A \equiv (1 - \rho)\hat{w}(1 - t)L^o(\hat{w}, t)$ are the post-tax earnings of the type o's in the reference group. If no migration occurs (or type r's can identify type o's and do not compare themselves to them), $\rho = 1$ and L^r simplifies to:

$$\begin{aligned} L^r(w, 1, t, k) \quad = \quad & \frac{1}{2\alpha w(1-t)}\Big\{(w(1-t))^2 - 2\alpha k + \\ & \Big(\big[(w(1-t))^2 + 2\alpha k\big]^2 + 8\alpha\beta(w(1-t))^2\Big)^{1/2}\Big\}. \end{aligned} \tag{24}$$

In either case, the labour supply of type r's depends on the income guarantee, k, despite the absence of direct income effects in labour supply. It can be shown for this functional form that $\partial L^r/\partial k < 0$ with or without $\rho = 1$. Intuitively, an increase in k (which is received by all consumers) weakens the incentive to work harder, since the marginal return to relative consumption from work is smaller. It becomes more difficult to outperform the reference group at the margin. Note also that the sign of $\partial L^r/\partial t < 0$ is ambiguous for type r, although increases in t unambiguously lead to labour supply reductions for type o via the substitution effect. Suppose $\rho = 1$. Then an increase in t has two effects for type r's: first, it encourages them to work *less* (through the standard substitution effect); second, it encourages them to work *more* (since it dampens the work effort of others in the reference group and hence their consumption as well). The latter effect is strongest when $\rho \to 0$ — the reference group is almost entirely o-types — in which case $\bar{c}$ falls unambiguously and $MRS^r(c, L)$ rises. It is possible that an increase in the marginal tax rate, if not accompanied by a sufficient increase in transfers, could actually worsen the relative consumption externality in this case.

3.1 Numerical Results

Numerical simulations of the model are undertaken with $I = 3$ underlying ability types. Individual labour supply functions are given from the preceding section. The results are obtained with preference parameters $\alpha = 2$ (for all agents), and $\beta = 1/2$ (for type r). There are three underlying ability levels: $w_1 = 1$, $w_2 = 1.41$ and $w_3 = 1.79$. These levels are chosen intentionally. With $\beta = 1/2$ and $\alpha = 2$, $\hat{w}_1 = w_2$ (type $(r, 1)$ and type $(o, 2)$ belong to reference group 1) and $\hat{w}_2 = w_3$ (type $(r, 2)$ and type $(o, 3)$ belong to reference group 2). Type $(r, 3)$'s form a reference group unto themselves. I assume that abilities are independently distributed across preference types with $p_1 = 0.3$, $p_2 = 0.5$ and $p_3 = 0.2$. Thus, the underlying ability distribution is roughly skewed right for both preference groups. The proportion of r types is set at $q_r = 0.5$ (so 50% of agents are of each type).

Recall that a perceptive government understands the existence and extent of each type of preference in the population, and therefore also the relative consumption externality among the type r's. An ignorant government believes the pre-tax distribution to be generated by a different distribution of earning abilities and 'ordinary' preferences. Figure 2 illustrates both cases of government perception for the present example. Note also the size of pooling which occurs given migration between type o's and r's pre-tax. Specifically,

$$\rho_1 = \frac{q_r p_1}{q_r p_1 + q_o p_2} = 3/8 \qquad \rho_2 = \frac{q_r p_2}{q_r p_2 + q_o p_3} = 5/7 \qquad \rho_3 = \frac{q_r p_3}{q_r p_3} = 1 \ ; \tag{25}$$

5/8 of type $(r, 1)$'s neighborhood are actually type $(o, 2)$, and 2/7 of type $(r, 2)$'s neighborhood are type $(o, 3)$.

For the simulations, the function $\omega = (V)^{1/2}$ is used. Thus, the social objective of a perceptive government is:

$$S = a \sum_{i=1}^{3} \phi(o, i)(V^o(w_i))^{1/2} + (1 - a) \sum_{i=1}^{3} \phi(r, i)(V_*^r(w_i))^{1/2} \tag{26}$$

and that of an ignorant government is:

$$\tilde{S} = \sum_{i=1}^{4} \tilde{p}_i (V^o(w_i))^{1/2} \tag{27}$$

where (as in Figure 2) $\tilde{p}_1 = 0.15$, $\tilde{p}_2 = 0.3$, $\tilde{p}_4 = 0.35$ and $\tilde{p}_4 = 0.10$. In all cases, outside revenue requirements are nil ($R = 0$).

Table 1 presents the chosen tax parameters under various objectives and cognitive assumptions for the government when preference types are unobservable. Consider first a perceptive government. If it places equal social weight on both types of agent, it sets a 30.3% marginal rate and returns 13.8% of average income to each taxpayer when there is are mixed reference groups (ρ variable). This is understandably a much more progressive system than that set by a government which ignores relative consumption effects (i.e. $t = 7.2\%$, $k = 4.1\%$). The ignorant government imposes a tax system not much different than that in which there are no type r's in the population (the final row of Table 1).

	ρ variable		$\rho = 1$	
	t	k	t	k
Perceptive ($a = 1/2$)	0.303	0.138	0.301	0.137
Perceptive ($a = 1$)	0.264	0.125	0.256	0.122
Ignorant	0.072	0.041	0.072	0.041
All Type o	0.066	0.030	0.066	0.030

Table 1: Optimal Linear Tax Parameters (Type Unobservable)

Note, however, the system chosen by a perceptive government which places *no weight* on the welfare of type r's ($a = 1$). Here, the optimal marginal tax rate is much larger than that which would be imposed in the absence of r-types (26.4% versus 6.6%) even though no social weight is given to type r. This result points to the fiscal benefits of raising taxes from r types to fund transfers to type o for redistributive purposes. In other words, the government which recognizes relative consumption effects trades off the distortion created on type o's via a higher marginal rate with the greater equity permitted by taxing type r's at this same rate. This observation is brought into stark relief by considering the hypothetical case where preference types are observable and $a = 1$ (Table 2). In this case, the optimal r-tax system imposes a high marginal rate 73.5%, compared with the o-system rate of 4.1%. Revenues are transferred from r's to o's via the lump sum transfer: o's would receive $k_o = 0.175$, and r's $k_r = 0.097$.

	ρ variable				$\rho = 1$			
	t_o	k_o	t_r	k_r	t_o	k_o	t_r	k_r
$a = 1/2$	0.064	0.036	0.524	0.240	0.064	0.037	0.516	0.236
$a = 1$	0.041	0.175	0.735	0.097	0.041	0.178	0.687	0.085

Table 2: Optimal Linear Tax Parameters (Type Observable)

Table 3 presents agents' welfare for the laissez-faire scenario ('LF') as well as under tax schemes when ρ is variable. The first three rows are of greatest interest. Redistributive taxation by a perceptive government benefits the lowest two ability types for both types, relative to laissez-faire. Notably, this result holds for type r's when $a = 1$ as well: i.e. relative to no redistribution, a majority of r types fare better when redistribution is imposed even when their group is given no social consideration. Indeed, $(r, 2)$ and $(r, 3)$ derive greater benefits when their welfare is *not* explicitly considered in the social objective.

	Type o			*Type r*		
	$V^o(w_1)$	$V^o(w_2)$	$V^o(w_3)$	$V^r_*(w_1)$	$V^r_*(w_2)$	$V^r_*(w_3)$
LF	0.125	0.250	0.404	0.000	0.154	0.329
Perceptive ($a = 1/2$)	0.199	0.259	0.335	0.099	0.166	0.254
Perceptive ($a = 1$)	0.193	0.261	0.345	0.090	0.167	0.265
Observable ($a = 1/2$)	0.146	0.256	0.391	0.212	0.229	0.261
Observable ($a = 1$)	0.292	0.407	0.549	0.001	0.001	0.012

Table 3: Agent Welfare (ρ variable)

The results also indicate the reference group effect on tax schedules. For example, Table 1 illustrates differences in the progressivity of taxation when type o's are pooled in the reference groups of type r's. Figure 1 suggests that the equilibrium labour supply of type r's tends to decrease in ρ for this example. Thus, as ρ approaches 1, optimal tax rates fall. A similar result holds for the observable-type scenario shown in Table 2.

4 Conclusion

This paper has investigated the properties of a linear taxation system when only a subset of agents in the economy have relative consumption concerns. Previous models of taxation with relative consumption effects (Boskin and Sheshinski , 1978; Oswald, 1983; Persson, 1995; Ireland, 1998; Corneo, 2002) have typically avoided this issue by assuming the same population-wide preferences over relative consumption. In those environments, increases in the progressivity of the tax system may be recommended, since the relative consumption externality is then controlled to a greater extent. The goal here has been to examine whether this result continues to hold when there is heterogeneity in preferences with respect to relative consumption and the government can place variable weights on each type of individual. This question has been examined in a model where relative-consumption concerned individuals strategically choose labour supply in a neighborhood that includes individuals of similar earning abilities.

Using numerical simulations with quasi-linear preferences, I demonstrate three key findings. First, the unobservability of preference types may generate high optimal marginal tax rates, even in the case where no social weight is placed on individual welfare losses from the relative consumption externality. This leads to an unlikely result: by concerning itself only with the welfare of 'ordinary' types, the government unintentionally mediates the relative consumption externality among type r's as well. Second, the optimal progressivity of the tax system depends on the nature of individuals' reference groups. The 'reference group effect' (where changes in tax parameters influence labour supply through average consumption in the neighborhood) can be magnified when r-types compare themselves to o-types in the population. Thus, different tax schedules are optimal (given the objective of the government) depending on the extent of pooling between type o's and r's. Finally, a tax authority which is ignorant of the relative consumption externality may mistakenly set a rather non-progressive system, which achieves neither the 'best' redistributive policy nor mediates the relative consumption externality. Ironically, social welfare could be improved by acknowledging the existence of relative consumption effects, even if no weight were actually placed on the welfare of individuals with this behavioral feature.

The analysis presented is incomplete and dependent on specific functional forms of preferences. It would be useful to more carefully characterize the optimal linear tax policies under more general specifications. Lastly, the assumption of linearity of the tax system is unrealistic and limits the instruments available for policy. Expanding the analysis to account for more general tax functions should uncover possibilities for welfare improvements among both ordinary and relative-consumption concerned individuals.

References

1. Alpizar, F. Carlsson, F., and O. Johansson-Stenman (2004). How Much do we Care About Absolute versus Relative Income and Consumption? *Journal of Economic Behavior and Organization*, 56: 405-421.

2. Blanchflower, D. G. and A. J. Oswald (2004). Well Being Over Time in Britain and the U.S.A. *Journal of Public Economics*, 88: 1359-1386.

3. Boadway, R., Marchand, M., Pestiau, P. and M. del Mar Racionero (2002). Optimal Redistribution with Heterogenous Preferences for Leisure. *Journal of Public Economic Theory*, 4: 475-498.

4. Boskin, M. J. and E. Sheshinski (1978). Optimal Redistributive Taxation When Individual Welfare Depends on Relative Income. *Quarterly Journal of Economics*, 92: 589-601.

5. Corneo, G. (2002). The Efficient Side of Progressive Income Taxation. *European Economic Review* 46: 1359-1368.

6. Dupor, B. and W. Liu (2003). Jealousy and Equilibrium Overconsumption. *American Economic Review*, 93: 423-428.

7. Falk, A. and M. Knell (2004). Choosing the Joneses: On the Endogeneity of Reference Groups. *Scandinavian Journal of Economics*, 106: 417-435.

8. Fleurbaey, M. and F. Maniquet (2006). Fair Income Tax. *Review of Economic Studies* 73: 55-83.

9. Frank, R. (2005). Positional Externalities Cause Large and Preventable Welfare Losses. *American Economic Review*, 95: 137-141.

10. Ireland, N. (1998) Status-seeking, Income Taxation, and Efficiency. *Journal of Public Economics*, 70: 99-113.

11. Johansson-Stenman, O., Carlsson, F. and D. Daruvala (2002). Measuring Future Grandparents' Preferences for Equality and Relative Standing. *Economic Journal*, 112: 362-383.

12. Kanbur, R., Pirttila, J. and M. Tuomala (2006). Non-welfarist Optimal Taxation and Behavioural Public Economics. *Journal of Economic Surveys*, 20: 849-868.

13. Luttmer, E. (2005). Neighbors as Negatives: Relative Earnings and Well-Being. *Quarterly Journal of Economics*, 120: 963-1002.

14. O'Donoghue, T. and M. Rabin (2003). Studying Optimal Paternalism, Illustrated by a Model of Sin Taxes. *American Economic Review*, 93: 186-191.

15. O'Donoghue, T. and M. Rabin (2006). Optimal Sin Taxes. *Journal of Public Economics*, 90: 1825-1849.

16. Oswald, A. J. (1983). Altruism, Jealousy and the Theory of Optimal Non-Linear Taxation. *Journal of Public Economics*, 20: 77-87.

17. Persson, M. (1995). Why are Taxes so High in Egalitarian Societies? *Scandinavian Journal of Economics*, 97: 569-580.

18. Racionero, M. del Mar (2000). Optimal Redistribution with Unobservable Preferences for an Observable Merit Good. *International Tax and Public Finance*, 7:479-501.

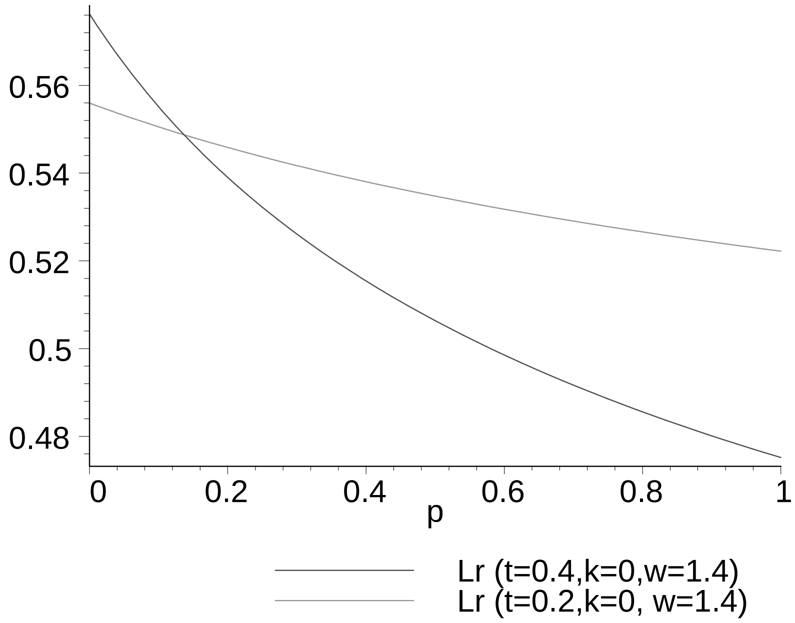

Figure 1: Sensitivity of r-type labour supply to presence of o-types.

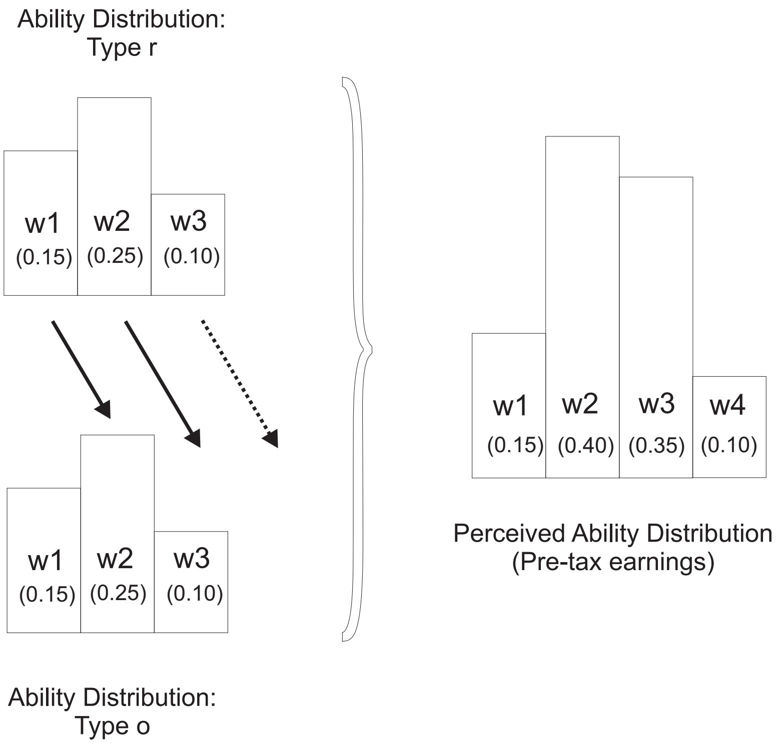

Figure 2: Perceived versus Actual Ability Distributions