Point processes, spatial-temporal

Q1 Q2 A• spatial–temporal• point process (also called space–time or spatio-temporal point process) is a random collection of points, where each point represents the time and location of an event. Examples of events include incidence of disease, sightings or births of a species, or the occurrences of fires, earthquakes, lightning strikes, tsunamis, or volcanic eruptions. Typically the spatial locations are recorded in three spatial coordinates, for example, longitude, latitude, and height or depth, though sometimes only one or two spatial coordinates are available or of interest. Figure 1 is an illustration of a realization of a spatial–temporal point process with one spatial coordinate depicted. Figure 2 displays some point process data consisting of microearthquake origin times and epicenters in Parkfield, CA, between 1988 and 1995, recorded by the US High-Resolution Seismographic Station Network [1]. Figure 3 displays the centroids of wildfires occurring between 1876 and 1996 in Los Angeles County, CA, recorded by the Los Angeles County Department of Public Works (times of the events not shown).

Characterizations

A spatial–temporal point process N is mathematically defined as a random measure on a region $S \subseteq \mathbb{R} \times \mathbb{R}^3$ of space–time, taking values in the nonnegative integers $\mathbb{Z}^+$ (or infinity). In this framework the measure $N(A)$ represents the number of points falling in the subset A of S. For the set A in Figure 1, for example, the value of $N(A)$ is 2. Attention is typically restricted to points in some time interval $[T_0, T_1]$, and to processes with only a finite number of points in any compact subset of S.

Traditionally the points of a point process are thought to be indistinguishable, other than by their times and locations. Often, however, there is other important information to be stored along with each point. For example, one may wish to analyze a list of points in time and space where a member of a certain species was observed, along with the size or age of the organism, or alternatively a catalog of arrival times and locations of hurricanes along with the amounts of damage attributed to each. Such processes may be viewed as *marked* spatial–temporal point processes, that is, random collections of points, where each point has associated with it a further random variable called a *mark*.

Much of the theory of spatial–temporal point processes carries over from that of spatial point processes (*see* **Point processes, spatial**). However, vap021.pub2 the temporal aspect enables a natural ordering of the points that does not generally exist for spatial processes. Indeed, it may often be convenient to view a spatial–temporal point process as a purely temporal point process (*see* **Point processes, temporal**), with vap018.pub2 spatial marks associated with each point. Sometimes investigating the purely temporal (or purely spatial) behavior of the resulting marginalized point process is of interest.

The spatial region of interest is often a rectangular portion of $\mathbb{R}^2$ or $\mathbb{R}^3$, but not always. For the data in Figure 2, for example, the focus is on just one spatial coordinate, and in Figure 3 the region of interest is Los Angeles County, which has an irregular boundary. Cases where the points are spatially distributed in a sphere or an ellipse are investigated by Brillinger [2, 3]. When the domain of possible spatial coordinates is discrete (e.g., a lattice) rather than continuous, it may be convenient to view the spatial–temporal point process as a sequence $\{N_i\}$ of temporal point processes that may interact with one another. For example, one may view the occurrences of cars on a highway as such a collection, where N_i represents observations of cars in lane i.

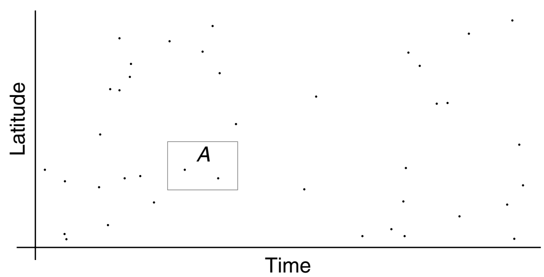

Figure 1 Spatial–temporal point process.

Based in part on the article "Point processes, spatial-temporal" by Frederic Paik Schoenberg, David R. Brillinger, and Peter Guttorp, which appeared in the *Encyclopedia of Environmetrics*.

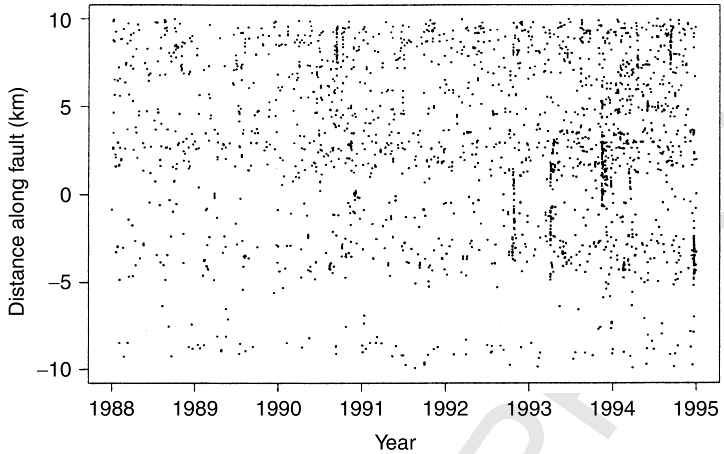

Figure 2 Epicenters and times of Parkfield microearthquakes, 1988–1995.

Any analytical spatial–temporal point process is characterized uniquely by its associated conditional rate process λ [4]. $\lambda(t, x, y, z)$ may be thought of as the frequency with which events are expected to occur around a particular location (t, x, y, z) in space–time, conditional on the prior history, H_t, of the point process up to time t. Note that in the statistical literature (e.g., Refs 5–8), λ is more commonly referred to as the conditional intensity rather than the conditional rate. However, the term *intensity* is also used in various environmental sciences, for example, in describing the size or destructiveness of an earthquake, so to avoid confusion, the term *rate* may be preferred.

Formally, the conditional rate $\lambda(t, x, y, z)$ associated with a spatial–temporal point process N may be defined as a limiting conditional expectation, as follows. Fix any point $p = (t, x, y, z)$ in space–time. Let B_Δ denote the set $(t, t + \Delta t) \times (x, x + \Delta x) \times (y, y + \Delta y) \times (z, z + \Delta z)$, where Δ is the vector $(\Delta t, \Delta x, \Delta y, \Delta z)$. Then

$$\lambda(p) = \lim_{\Delta \to 0} E[N(B_\Delta)|Ht]/|\Delta| \qquad (1)$$

Q3 provided the limit exists•. Some authors instead define $\lambda(p)$ as

$$\lim_{\Delta \to 0} P[N(B_\Delta) > 0|Ht]/|\Delta| \qquad (2)$$

For orderly point processes (processes where $\lim_{|A|\downarrow\emptyset} \Pr\{N(A) > 1\}/|A| = 0$ for interval A), the two definitions are equivalent. λ is a predictable process whose integral C (called the *compensator*) is such that $N - C$ is a martingale. There are different forms of conditioning corresponding to different types of martingales; see Ref. 9, or Ref. 10.

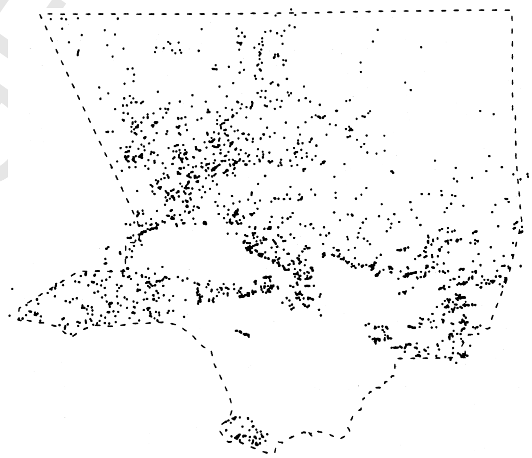

Figure 3 Centroids of recorded Los Angeles County wildfires, 1878–1996.

Models

The behavior of a spatial–temporal point process N is typically modeled by specifying a functional form for $\lambda(t, x, y, z)$, which represents the infinitesimal expected rate of events at time t and location (x, y, z), given all the observations up to time t. Although λ may be estimated nonparametrically [11–17], it is more common to estimate λ via a parametric model.

In general, $\lambda(t, x, y, z)$ depends not only on t, x, y, and z but also on the times and locations of preceding events. When N is a **Poisson process**, however, λ is deterministic, that is, $\lambda(t, x, y, z)$ depends only on t, x, y, and z. The simplest model is the stationary Poisson, where the conditional rate is constant: $\lambda(t, x, y, z) = \alpha$ for all t, x, y, z. In the case of modeling environmental disturbances, this model incorporates the idea that the risk of an event is the same at all times and locations, regardless of where and how frequently such disturbances have occurred previously. Processes that display substantial spatial heterogeneity, such as earthquake epicenters, are sometimes modeled as stationary in time but not space.

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Stationary spatial–temporal point processes are sometimes described by the second-order parameter measure $\rho(t', x', y', z')$, which measures the covariance between the numbers of points in spatial–temporal regions A and B, where region B is A shifted by (t', x', y', z') (*see* **Space–time covariance models**). For example, Kagan and Vere-Jones [18] explore models for ρ in describing spatial–temporal patterns of earthquake hypocenters and times. For a self-exciting (equivalently, clustered) point process, the function ρ is positive for small values of t', x', y', and z'; N is self-correcting (equivalently, inhibitory) if instead the covariance is negative. Thus the occurrence of points in a self-exciting point process is associated with other points occurring nearby in space–time, whereas in a self-correcting process the points have an inhibitory effect.

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Also useful for diagnostic purposes are other second-order statistics, such as interpoint distances or the numbers of points within a distance k of an existing point [19–24]. Typically one compares such properties with those of a stationary Poisson process, though weighted second-order statistics can be used to test against alternative models as well [25–28]. Self-exciting point process models are often used in epidemiology (*see* **Spatial statistics in environmental epidemiology**) and seismology (*see* **Seismological modeling**) to model events that are clustered together in time and space. A commonly used form for such models is a spatial–temporal generalization of the Hawkes model, where $\lambda(t, x, y, z)$ may be written as

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$$\mu(t, x, y, z) + \sum_i \nu(t - t_i, x - x_i, y - y_i, z - z_i) \tag{3}$$

where the sum is over all points (t_i, x_i, y_i, z_i) with $t_i < t$. The functions μ and ν represent the deterministic background rate and clustering density, respectively. Often μ is modeled as merely a function of the spatial coordinates (x, y, z), and may be estimated nonparametrically as in Ref. 29. When observed marks m associated with each point are posited to affect the rate at which future points accumulate, this information is typically incorporated into the function ν, that is, the conditional rate λ is modeled as a background rate plus

$$\sum_i \nu(t - t_i, x - x_i, y - y_i, z - z_i, m - m_i) \tag{4}$$

A variety of forms has been given for the clustering density ν. For instance, in modeling seismological data with two spatial parameters (x and y) and a mark (m) indicating magnitude, Musmeci and Vere-Jones [30] introduced explicit forms for ν, including the diffusion-type model where $\nu(t, x, y, m)$ is given by

$$\frac{C}{2\pi\sigma_x\sigma_y t} \exp\left[\alpha m - \beta t - \frac{\left(\dfrac{x^2}{\sigma_x^2} + \dfrac{y^2}{\sigma_y^2}\right)}{2t}\right] \tag{5}$$

Ogata [29] investigated the case where

$$\nu(t, x, y, m) = \frac{K_0 \exp[\alpha(m - m_0)]}{(t + c)^p (x^2 + y^2 + d)^q} \tag{6}$$

as well as a variety of other models and extensions were proposed in Ref. 31. Several other forms for ν were suggested by Rathbun [32] and Kagan [33]; see Ref. 29 for a review. The clustering density may also be estimated nonparametrically [16, 17, 34].

Sometimes λ is modeled as a product of marginal conditional intensities

$$\lambda(t, x, y, z) = \lambda_1(t)\lambda_2(x, y, z) \tag{7}$$

or even

$$\lambda(t, x, y, z) = \lambda_1(t)\lambda_2(x)\lambda_2(y)\lambda_4(z) \tag{8}$$

These forms embody the notion that the temporal behavior of the process is independent of the spatial behavior, and in the latter case that furthermore the behavior along each of the spatial coordinates can be estimated separably; see Refs 32, 35–37. Occasionally one subdivides the spatial region into a finite number of subregions and fits temporal point process models to the data within each subregion. In such a case the conditional intensity may be written

$$\lambda(t, x, y, z) = \sum_i \lambda_1(t)1_i(x, y, z) \tag{9}$$

where the 1_i are indicator functions denoting the relevant region. An example is given in Ref. 38. The introduction of interactions between different subregions is incorporated into this model by Lu *et al.* [39].

Point process models may also depend on spatial covariates; see, for example, Refs 40–42. For further remarks on modeling and examples, see Refs 43–45.

Estimation and Inference

The parameter vector θ for a model with conditional rate $\lambda(t, x, y, z; \theta)$ is usually estimated by maximizing the log-likelihood function

$$L(\theta) = \int_{T_0}^{T_1} \int_x \int_y \int_z \log[\lambda(t, x, y, z; \theta)]\, \mathrm{d}N(t, x, y, z) - \int_{T_0}^{T_1} \int_x \int_y \int_z \lambda(t, x, y, z; \theta)\, \mathrm{d}z\, \mathrm{d}y\, \mathrm{d}x\, \mathrm{d}t \tag{10}$$

vam009.pub2 Asymptotic properties of the **maximum likelihood estimator** $\hat{\theta}$ have been derived under various conditions, along with formulas for **standard errors**; vas054.pub2 see, for example, Refs 46, 47. Alternatively, simulations [48] may be useful for obtaining approximate standard errors and for other types of inference (*see* var038.pub2 **Resampling methods**). Point processes may also be estimated via EM-type methods [49] or via partial likelihoods [50], or in certain cases via regression and minimum contrast estimation methods [51].

The estimated conditional rate $\lambda(t, x, y, z; \hat{\theta})$ can be used directly for prediction and risk assessment (*see* **Risk assessment, seismological**). See Refs 4, 52, for example. vas011m.pub2

Spatial–temporal point processes may be evaluated using various types of residual analysis. For instance, one may compare standardized versions of the difference between the observed and expected numbers of points across pixels, as described in Ref. 53 or Ref. 54. Alternatively, one may obtain rescaled residuals, by selecting a spatial coordinate and rescales the point process in that direction. If the z coordinate is chosen, for example, then each point (t_i, x_i, y_i, z_i) of the observed point process is moved to a new point $(t_i, x_i, y_i, \int_{z_0}^{z_i} \lambda(t_i, x_i, y_i, z; \hat{\theta})\, \mathrm{d}z)$, where z_0 is the lower boundary in the z direction of the spatial region being considered. The resulting rescaled process is stationary Poisson if and only if the model is correctly specified [55, 56]. Hence a useful method for assessing the fit of a point process model is to examine whether the rescaled point process looks like a Poisson process with unit rate. Several tests exist for this purpose; see, for example, Ref. 57 or Ref. 58. An alternative residual method based on random thinning, where each point is kept with a probability inversely proportional to its associated conditional intensity, was proposed in Ref. 59. The thinned residual method is similar to the stochastic declustering technique of [60], in which points are randomly removing according to their conditional intensities, in order to separate mainshocks from aftershocks.

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Further Reading

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(*See also* **Point processes, dynamic**; **Stochastic model**; **Stochastic process**) vap019.pub2 vas058.pub2 vas060.pub2

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