The New Zealand Curriculum

Mathematics Standards

for years 1–8

Ministry of Education

This resource has been developed to support
The New Zealand Curriculum

The Ministry of Education wishes to acknowledge the lead writer, Vince Wright from The University of Waikato, and the advisers, teachers, students, and others in the mathematics education community who have contributed to the development of the mathematics standards and their accompanying examples.

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INTRODUCTION

The National Standards provide a nationally consistent means for considering, explaining, and responding to students' progress and achievement in years 1–8. They provide reference points, or signposts, that describe the achievement in reading, writing, and mathematics that will enable students to meet the demands of the New Zealand Curriculum. They will help teachers to make judgments about their students' progress so that the students and their teachers, parents, families, and whānau can agree on the next learning goals.

When used in conjunction with effective assessment practices, the National Standards will be a powerful means of informing students, parents, families, whānau, teachers, schools, and the education system about how well things are going and what could be done better to improve learning for all students.

All teachers working in English-medium settings will work with the standards from 2010. From 1 February 2010, consultation and trialling of Ngā Whanaketanga Rūmaki Māori will begin in Māori-medium schools and settings that are working towards implementing Te Marautanga o Aotearoa.

Together, the New Zealand Curriculum, the National Standards, Ka Hikitia, and the Pasifika Education Plan[1] give schools a rich resource to draw on to ensure that all students build strong foundations for a lifetime of learning.

About this book

This book is intended for primary school teachers and leaders. Others, including parents, families, whānau, and students themselves, may also find it useful. It presents the National Standards for mathematics in years 1–8 together with examples of problems and descriptions of students' thinking that illustrate and clarify the standards. The standards and their illustrative examples are set out on the pages following page 15.

The book also:

- explains the thinking behind the mathematics standards (pages 6–7);
- discusses the standards in relation to our understandings about effective mathematics teaching (pages 8–9);
- unpacks the standards in detail (page 10);
- explains the layout of the standards and their illustrations (page 11);
- discusses how the standards will be used, particularly within assessment (pages 12–13);
- shows supporting resources for teaching mathematics and statistics (page 14);
- provides a glossary of terms used in the standards and their illustrations (pages 51–53);
- provides a list of references (page 55).

[1] See the References for publication details for *Ka Hikitia – Managing for Success: The Māori Education Strategy 2008–2012* and *Pasifika Education Plan 2008–2012*.

UNDERSTANDING THE MATHEMATICS STANDARDS

The standards for mathematics are statements about what students should know and be able to do in order to meet the demands of the New Zealand Curriculum. They reflect the complexity and challenge of the problems and contexts that students meet within the mathematics and statistics learning area of the New Zealand Curriculum and within other learning areas that require the use of mathematical and statistical thinking.

The New Zealand Curriculum establishes an expectation of progress through curriculum levels over time. The standards for mathematics set out what can reasonably be expected of most students by the end of each period or year of schooling, from the first year of school through to the end of year 8. However, students start at different points and progress at different rates. That is why, when interpreting achievement, it is important to consider the rate of progress as well as the expected standard.

The New Zealand Curriculum and the standards for mathematics complement each other. The curriculum drives teaching, and the standards support teachers to assess their students' achievement in relation to the curriculum. Together, the curriculum and the standards will play a vital role in the development of students' "ability and inclination to use mathematics effectively – at home, at work, and in the community".[2]

Current data about the numeracy of adults in the workforce gives cause for concern. Significant proportions of New Zealand students in the upper primary years do not currently meet the expectations. Unless this situation is addressed, many of these students will not achieve in mathematics at a level that is adequate to meet the demands of their adult lives.

The purpose of the standards for mathematics is to promote quality teaching and learning in every New Zealand classroom and success for all students. They will assist teachers and schools to monitor student progress and the success of teaching and learning programmes, to decide upon next steps for learning with students, to target students who need extra assistance, and to report to students, their families and whānau, agencies, and the community.

The structure and development of the standards

The mathematics standards are structured according to the strands of the mathematics and statistics learning area of the New Zealand Curriculum. Other options for structuring the standards included mathematical processes,[3] significant mathematical and statistical ideas, and aspects of effective instructional practice.[4]

This decision on how to structure the standards was made for three reasons:

- The standards need to align with mathematics programmes in schools, which are based on the mathematics and statistics learning area of the New Zealand Curriculum.
- The assessment tools commonly used in schools are aligned with the structure of the New Zealand Curriculum.
- Feedback on the draft standards indicated teachers' strong preference for a strand-based structure for the standards.

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2 The Numeracy Development Projects' definition of numeracy (Ministry of Education, 2001, p. 1)

3 See *Mathematics in the New Zealand Curriculum* (1992), pp. 23–29.

4 See *Effective Pedagogy in Mathematics/Pāngarau: Best Evidence Synthesis Iteration [BES]* (2007), pp. 56–88.

Each standard states the level of the New Zealand Curriculum that students will be achieving at and specifies a number of expectations under the strands Number and Algebra, Geometry and Measurement, and Statistics. The development of these expectations drew strongly on the research base underpinning the achievement objectives of the New Zealand Curriculum.

However, differentiating between achievement at each school year required more tightly focused statements than the achievement objectives at curriculum levels 1–4. Developing these statements involved considering:

- research evidence that documents smaller progressions than those defined by the achievement objectives;
- assessment data from sources such as NEMP, *The New Zealand Curriculum Exemplars*, and PATs that highlight task variables that affect the difficulty of mathematical problems;
- evidence from the use of normed assessment tools within the Numeracy Development Projects;
- case studies of successful implementation within the Numeracy Development Projects.

Identifying expectations for each school year also involved examining current levels of achievement by New Zealand students and international expectations as defined by the curriculums and standards of other countries. Research on the numeracy demands of everyday life and the workplace was also taken into account.

THE MATHEMATICS STANDARDS AND EFFECTIVE MATHEMATICS TEACHING

At the heart of effective teaching and learning in mathematics are quality programmes underpinned by the New Zealand Curriculum and based on identified student strengths, interests, and needs. Many research studies, including those informing Ka Hikitia, confirm the importance of quality teaching and show that it is critical to the improvement of student outcomes (for example, Alton-Lee, 2003, and Hattie, 2002). The standards for mathematics have been developed on this basis.

Classrooms are complex social situations in which students' learning is influenced by their interactions with their social and cultural environments and by the degree to which they actively manage their learning. Much is known about the features of teaching that acknowledge this complexity and that improve outcomes for diverse learners. The mathematics Best Evidence Synthesis evaluates, analyses, and synthesises New Zealand and international research on quality teaching in mathematics. Research findings from the Numeracy Development Projects confirm the importance of quality programmes in mathematics and outline their components. These include:[5]

An inclusive classroom climate

Successful teachers create social norms in their classrooms that give students the confidence and ability to take risks, to discuss with others, and to listen actively (Cobb, McClain, & Whitenack, 1995). High expectations for student behaviour and provision of a well-organised environment that maximises students' learning time are critical. The valuing of student diversity – academically, socially, and culturally – is fundamental to the development of positive relationships between teacher and students (Bishop et al., 2003). *Ka Hikitia – Managing for Success: The Māori Education Strategy 2008–2012* and *Pasifika Education Plan 2008–2012* support schools to provide inclusive classroom climates for all students.

Focused planning

Use of a variety of assessment methods, both formal and informal, to identify the needs of students is critical to quality teaching. From this data, successful teachers target concepts and processes to be taught/learned and plan carefully sequenced lessons. They develop learning trajectories that map potential growth paths and can "unpack" these trajectories in detail if needed. Students are aware of (and sometimes set) the learning goals. These goals change and grow as learning occurs.

Problem-centred activities

Cross-national comparisons show that students in high-performing countries spend a large proportion of their class time solving problems (Stigler & Hiebert, 1997). The students do so individually as well as co-operatively. Fundamental to this is a shared belief, between teacher and students, that the responsibility for knowledge creation lies with the students (Clarke & Hoon, 2005).

[5] These seven teacher activities and their descriptions are quoted directly from pages 3–4 in Book 3: *Getting Started* from the Numeracy Development Projects. (Note that the last sentence of the first paragraph is additional to the quotation.)

Responsive lessons

Responsiveness requires teachers to constantly monitor their students' thinking and to react by continually adjusting the tasks, questions, scaffolding, and feedback provided. To this end, quality teachers create a variety of instructional groups to address specific learning needs.

Connections

Askew et al. (1997) report that successful teachers of numeracy are "connectivist". Such teachers use powerful representations of concepts and transparently link mathematical vocabulary and symbols with actions on materials. The use of realistic contexts helps students to connect mathematics with their worlds.

High expectations

Quality teachers ask questions that provoke high-order thinking skills, such as analysing, synthesising, and justifying, and they have high expectations for student achievement. They encourage students to regulate their own learning, make their own learning decisions, and be self-critical. Successful teachers provide incentives, recognition, and support for students to be independent learners.

Equity

Success for all students is a key goal, and quality teachers provide extra time for students with high learning needs. They promote respect and empathy in their students for the needs of others.

To understand equity, the focus needs to be "not only on inequitable social structures and the ideologies that prop them up but also on how such realities play out in the everyday activity within classrooms and other cultural practices" (Anthony and Walshaw, 2007, page 12). In this regard, Te Maro, Higgins, and Averill (2008) found that particular characteristics of the school environment, the orientation of the teacher towards using culturally responsive actions, and certain personal qualities of the teacher (including a focus on developing relationships) produced strong achievement gains for Māori students. It is essential that teachers respect and value each learner for who they are, where they come from, and what they bring with them. The key priority is to ensure that, as part of their professional practice, teachers focus on Māori learners "enjoying education success as Māori" (Ministry of Education, 2009, page 18).

In order to make judgments in relation to the mathematics standards, it is essential that teachers understand the mathematical and statistical content that they are teaching. However, content knowledge is not sufficient in and of itself. Uniformly, the research on quality teaching stresses the importance of teachers' *pedagogical* content knowledge. Shulman (1987) defines pedagogical content knowledge, in part, as a teacher's "understanding of how particular topics, problems, or issues are organized, presented, and adapted to the diverse interests and abilities of learners, and presented for instruction". Teachers must ensure that they understand the conceptual difficulties that students may be having and be able to plan coherent, targeted teaching to address those difficulties.

UNPACKING THE MATHEMATICS STANDARDS

Each mathematics standard is for a designated point in a student's first eight years at school. Recognising that students begin school at different times in the year, the first three standards are described in terms of the time a student has spent at school, that is, "After one year at school", "After two years at school", and "After three years at school". From year 4, the standards are described in terms of the school year the student is in, that is, "By the end of year 4", "By the end of year 5", and so on.

The standards for mathematics build directly on the strands and levels of the mathematics and statistics learning area in the New Zealand Curriculum. Like the curriculum, they place a strong emphasis on students' ability to solve problems and model situations in a range of meaningful contexts by selecting and applying appropriate knowledge, skills, and strategies.

Many learning outcomes from the New Zealand Curriculum are implicit in the standards but not explicitly stated. This is particularly true of basic knowledge across the three strands, including number facts. While knowledge is critically important for mathematical understanding, its primary role is to facilitate the student's solving of problems and modelling of situations. Just demonstrating knowledge – for example, by recalling basic facts – is not sufficient to meet a standard. Rather, the student must use knowledge to think mathematically when solving problems or modelling situations.

Taken as a whole, the standards show how students progress in their learning in mathematics and statistics. This means that the "next step" for a student meeting a particular standard is broadly described by the standard for the next year. Similarly, if a student is not able to meet a standard, their current achievement is likely to be described by a previous standard.

Examples of problems and descriptions of students' thinking in response to the problems accompany each standard. Together, these illustrate and clarify the standard and exemplify the kinds of tasks students should engage with in their learning in mathematics and statistics.

Meeting a standard depends on the nature of a student's responses to given problems, not just their ability to solve the problems. For this reason, the examples give a range of responses to illustrate the types of responses that meet the expectation. In many cases, the examples include responses above and below the expectation in order to show the progression in a student's understanding.

The examples of problems and students' responses are not a definitive collection for use in assessing achievement in relation to the standards. They are illustrative, and they represent only a small sample of possible problems and responses that teachers might draw on in determining whether a student is meeting the standard.

THE LAYOUT OF THE STANDARDS AND THEIR ILLUSTRATIONS

To meet the standard, a student must be able to solve problems and model situations as described in the standard, **independently and most of the time**.

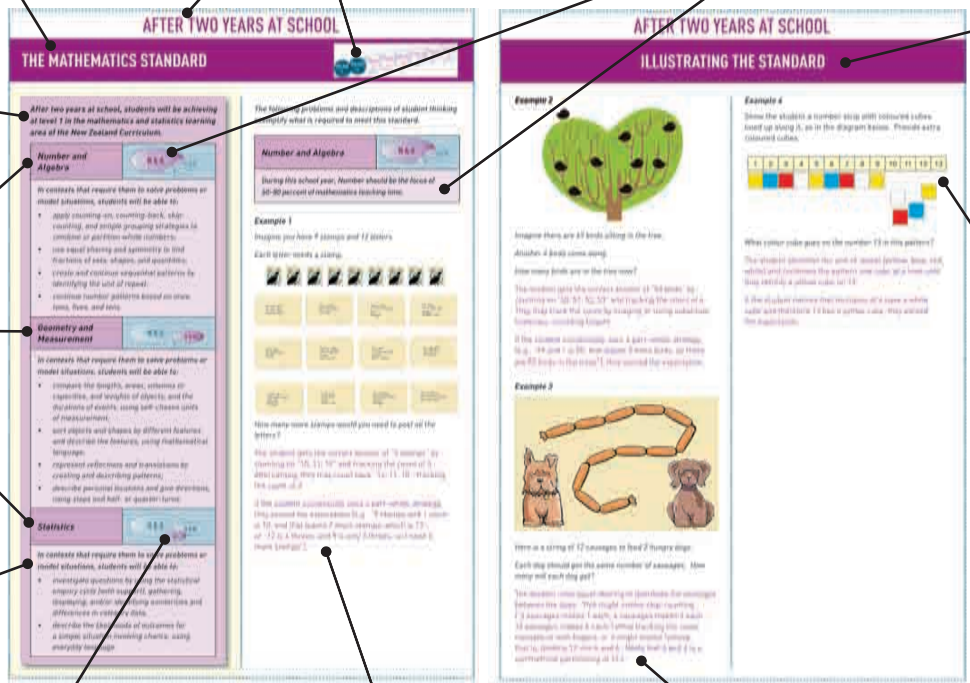
The standard is for a specified point in the first eight years at school.

A strong understanding of Number is vital if students are to succeed in mathematics. The expectations for Number are the **most critical requirement** for meeting the standard.

The standard relates to a level in the New Zealand Curriculum and gives a more tightly focused picture of student achievement at that level.

The standard links to and builds on the strands of the mathematics and statistics learning area in the New Zealand Curriculum.

Students should experience a range of meaningful contexts that engage them in thinking mathematically and statistically.



Examples of problems and students' responses illustrate and clarify the standard.

Examples of problems exemplify the kinds of tasks students should engage with in their learning.

The diagram from *The New Zealand Curriculum* indicates the proportion of time to be spent on each strand.

Meeting a standard depends on the nature of a student's responses to given problems, not just their ability to solve the problems. For this reason, the examples give a range of responses to illustrate the types of responses that meet, exceed, or do not meet the expectation.

Student responses are not the exact responses that a student needs to give but are indicative of what is required to meet the standard.

USING THE MATHEMATICS STANDARDS

Assessment using the standards

The standard for a given year comprises both the description of the curriculum level that students should be achieving at and the individual statements of expectation (expressed as bullet points under the strand headings). When assessing a student's achievement and progress, the teacher needs to make an overall judgment about the student in relation to the whole standard. This judgment needs to be based on evidence collected over a period of time, much of it derived from daily classroom interactions and observations.

Multiple sources of evidence should inform overall teacher judgments about a student's performance. As well as the student's work, sources of evidence may include self- and peer assessments, interviews, observations, and results from assessment tools. A single assessment is insufficient and unacceptable for several reasons:

- While progress over the long term is more stable and predictable, there is considerable variation in student performance in the short term.
- Different types of assessment provide different information; assessments should be chosen for their suitability for purpose and viewed together to arrive at an overall teacher judgment.
- Assessments need to be meaningful for students. For this reason, teachers need to choose a range of assessments that give their students the best opportunity to demonstrate achievement.

The standards and their accompanying examples provide descriptions of how students are expected to solve problems and model situations. The descriptions recognise that students vary in their responses to problems, and they emphasise that how a solution is arrived at is a critical part of the expectation. Teachers should base their decision about a student meeting a given expectation on whether the student solves problems and models situations in the expected way **independently and most of the time**.

A strong understanding of Number is vital if students are to succeed in mathematics. For this reason, the expectations for Number are the **most critical requirement** for meeting a standard. Students' achievement and progress in Number may be assessed using the tools developed in the Numeracy Development Projects.

When assessing achievement and progress in relation to the standards, it is important to remember that students start at different points and progress at different rates. It is therefore necessary to take a long-term view and to work on the basis that students advance in their learning in different ways and at different rates from one another.

Making judgments in relation to the standards will indicate whether a student is at, above, below, or well below the expectations for that school year. These judgments will indicate when and where extra support or extension is needed and will assist the teacher to set learning goals with the student and to share them with parents, families, and whānau. Students have capacities for learning that cannot be reliably predicted from achievement at any one point in time and that can be adversely affected by inappropriate categorisation or labelling. Realistic expectations should be conveyed to students in ways that enhance their attitudes to learning and motivate them to take on new challenges.

Research evidence shows a clear association between positive outcomes for students and a detailed knowledge of those students based on quality assessment data. The introduction of National Standards will support teachers as they continue to use assessment to guide instruction (rather than as an end point).

English language learners

English language learners in New Zealand schools are very diverse, and their language learning needs are not always apparent. Students who have good social English language may have had little exposure to the academic language they need for learning.

Students learn mathematics through language and often must demonstrate their knowledge and understanding through language. The mathematics standards set benchmarks for achievement and progress that may be very challenging for students who are new learners of English.

Teaching programmes should address students' learning needs in both English language and mathematics. Teachers need to understand what their English language learners know and are able to do in relation to both the English Language Learning Progressions and mathematics and statistics.

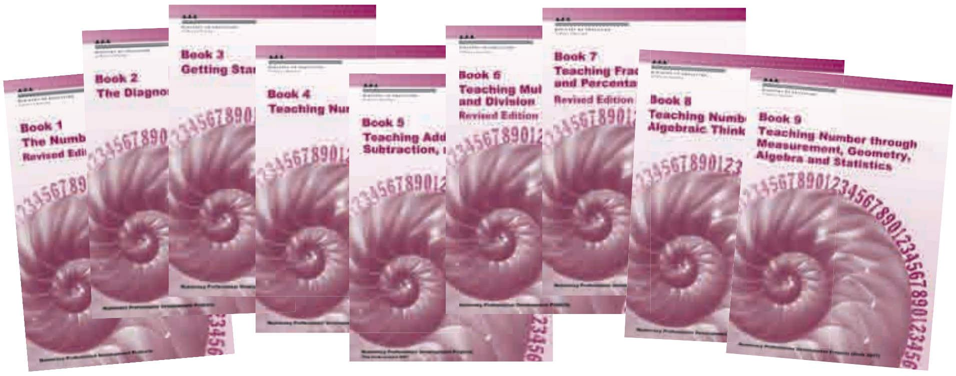
Learners with special needs

Some students face particular challenges in performing numerical calculations, decoding mathematical symbols, visualising spatial relationships, classification, or logic. The mathematics standards will help to identify these students and inform decisions concerning appropriate educational targets, effective instructional practices, and extra support that may be needed.

Due to significant cognitive impairments, a very small number of students work to individual education plans (IEPs) that are developed in consultation with the students' parents, families, and whānau; the students' teachers; and the Ministry of Education. The achievement and progress of these students will be assessed in relation to the standards, as part of their IEPs. Boards will continue to report on these students separately in their annual reports.

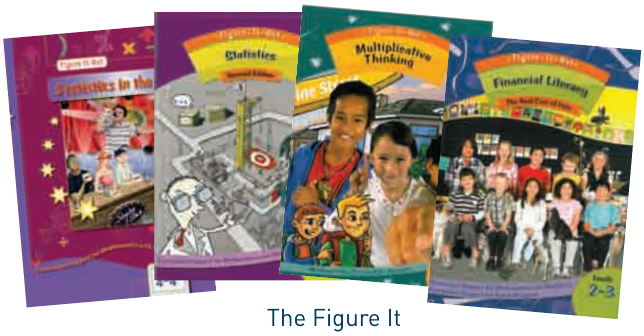
SUPPORTING RESOURCES

Teachers should use a range of resources to support their students' learning in mathematics and statistics. Many of these resources will support the assessment of students' achievement and progress in relation to the mathematics standards.



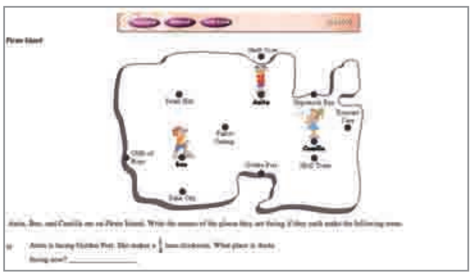
Materials from the Numeracy Development Projects

Effective Pedagogy in Mathematics/ Pāngarau: Best Evidence Synthesis Iteration [BES]



The Figure It Out series

The New Zealand Maths website (www.nzmaths.co.nz), including units of work and digital learning objects



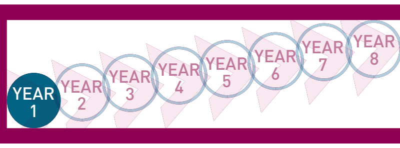
The Assessment Resource Banks (ARBs)

The *Connected* series

National Education Monitoring Project (NEMP) reports

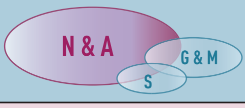
THE MATHEMATICS STANDARDS AND ILLUSTRATIONS

THE MATHEMATICS STANDARD



After one year at school, students will be achieving at early level 1 in the mathematics and statistics learning area of the New Zealand Curriculum.

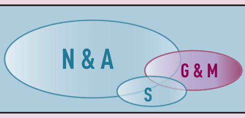
Number and Algebra



In contexts that require them to solve problems or model situations, students will be able to:

- *apply counting-all strategies;*
- *continue sequential patterns and number patterns based on ones.*

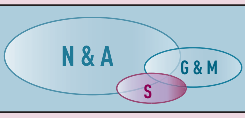
Geometry and Measurement



In contexts that require them to solve problems or model situations, students will be able to:

- *compare the lengths, areas, volumes or capacities, and weights of objects directly;*
- *sort objects and shapes by a single feature and describe the feature, using everyday language;*
- *represent reflections and translations by creating patterns;*
- *describe personal locations and give directions, using everyday language.*

Statistics

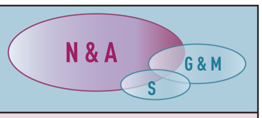


In contexts that require them to solve problems or model situations, students will be able to:

- *investigate questions by using the statistical enquiry cycle (with support), gathering, displaying, and/or counting category data.*

The following problems and descriptions of student thinking exemplify what is required to meet this standard.

Number and Algebra



During this school year, Number should be the focus of 60–80 percent of mathematics teaching time.

Example 1

Imagine you have 4 teddies.

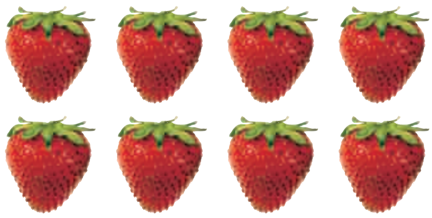
You get 5 more teddies.

How many teddies do you have now?

The student gets the correct answer of 9 teddies by counting all of the objects: 1, 2, 3, 4, 5, 6, 7, 8, 9. They may do so by imaging the teddies, preferably, or by using substituted materials (e.g., fingers or counters). If they successfully use a more sophisticated strategy, such as counting on or doubling, they exceed the expectation.

Example 2

Imagine you have 8 strawberries.



You eat 3. How many strawberries do you have left?

The student gets the correct answer of 5 strawberries by counting all the objects (1, 2, 3, 4, 5, 6, 7, 8) and then counting back (7, 6, 5). They may do so by imaging the strawberries, preferably, or by using substituted materials (e.g., fingers or counters). If they successfully use a more sophisticated strategy, such as immediately counting back from 8 or using known facts, they exceed the expectation.

ILLUSTRATING THE STANDARD

Example 3

Here are 3 kete. There are 3 kūmara in each kete.

How many kūmara are there altogether?

The student gets the correct answer of 9 kūmara by counting all of the objects: 1, 2, 3, 4, 5, 6, 7, 8, 9. They may do so by imaging the kūmara, preferably, or by using substituted materials (e.g., fingers or counters). If they successfully use a more sophisticated strategy, such as skip-counting (3, 6, 9), they exceed the expectation.

Example 4

Build up the pattern below one animal at a time in front of the student.

Copy this pattern with your animal cards.

Which animal comes next in the pattern? How do you know?

The student identifies which animal comes next (the pig) by attending to its relative position in the repeating sequence: cow, pig, sheep.

Geometry and Measurement

Example 5

Provide water in an ungraduated jug or bottle and 3 containers that are similar in capacity.

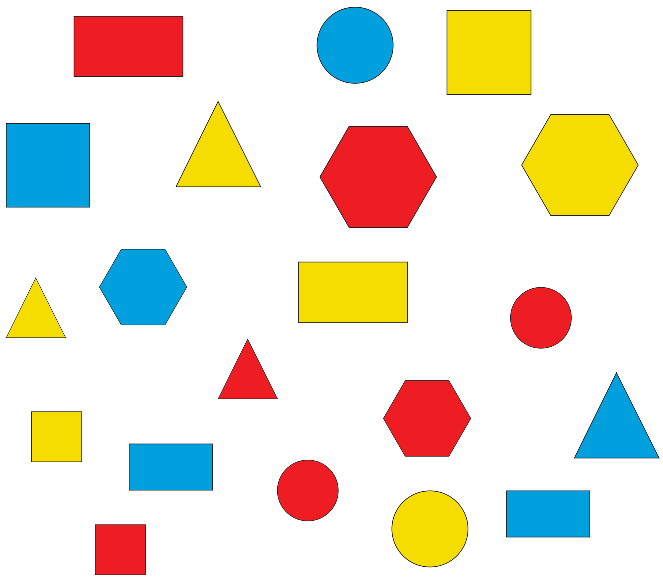
Here are 3 containers. Use water to find out which container holds the most.

The student pours water directly from one container to another to find out which holds the most.

Example 6

Provide the student with a set of attribute blocks.

Here is a set of blocks. Sort the blocks into families.



What is the same about the blocks in each family?

The student sorts the blocks by a feature of their choice and explains their sorting. The feature may be colour, size, shape, thickness, or some other characteristic, such as number of sides, symmetry, "pointiness", or "roundness".

ILLUSTRATING THE STANDARD

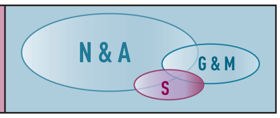
Example 7

Sit with the student at their desk in the classroom.

Imagine I am standing at the door. I need to get to where Rawiri sits. Tell me how to get to his seat.

The student gives clear directions that lead you to Rawiri's seat. They may tell you to move backwards or forwards and to turn right or left. If the student specifies distances in steps or metres or uses half- or quarter-turns, they exceed the expectation.

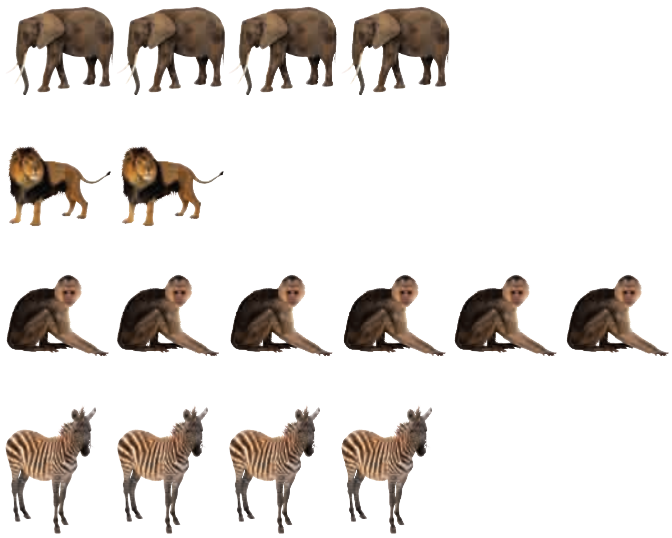
Statistics



Example 8

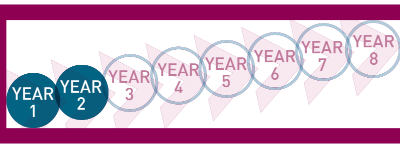
Provide the student with the animal cards shown below, randomly arranged.

Arrange the cards so that someone else can see how many of each animal there are at the zoo. How many zebras are there? Which animal is there most of?



The student sorts the animals into categories and displays the number of animals in each category, using a set grouping or pictograph as above. They correctly answer that there are 4 zebras and more monkeys than any other animal.

THE MATHEMATICS STANDARD



After two years at school, students will be achieving at level 1 in the mathematics and statistics learning area of the New Zealand Curriculum.

Number and Algebra

In contexts that require them to solve problems or model situations, students will be able to:

- *apply counting-on, counting-back, skip-counting, and simple grouping strategies to combine or partition whole numbers;*
- *use equal sharing and symmetry to find fractions of sets, shapes, and quantities;*
- *create and continue sequential patterns by identifying the unit of repeat;*
- *continue number patterns based on ones, twos, fives, and tens.*

Geometry and Measurement

In contexts that require them to solve problems or model situations, students will be able to:

- *compare the lengths, areas, volumes or capacities, and weights of objects and the durations of events, using self-chosen units of measurement;*
- *sort objects and shapes by different features and describe the features, using mathematical language;*
- *represent reflections and translations by creating and describing patterns;*
- *describe personal locations and give directions, using steps and half- or quarter-turns.*

Statistics

In contexts that require them to solve problems or model situations, students will be able to:

- *investigate questions by using the statistical enquiry cycle (with support), gathering, displaying, and/or identifying similarities and differences in category data;*
- *describe the likelihoods of outcomes for a simple situation involving chance, using everyday language.*

The following problems and descriptions of student thinking exemplify what is required to meet this standard.

Number and Algebra

During this school year, Number should be the focus of 60–80 percent of mathematics teaching time.

Example 1

Imagine you have 9 stamps and 12 letters.

Each letter needs a stamp.

How many more stamps would you need to post all the letters?

The student gets the correct answer of 3 stamps by counting on 10, 11, 12 and tracking the count of 3. Alternatively, they may count back 11, 10, 9, tracking the count of 3.

If the student successfully uses a part–whole strategy, they exceed the expectation (e.g., "9 stamps and 1 more is 10, and that leaves 2 more stamps, which is 12", or "12 is 4 threes, and 9 is only 3 threes, so I need 3 more stamps").

ILLUSTRATING THE STANDARD

Example 2

Imagine there are 49 birds sitting in the tree.

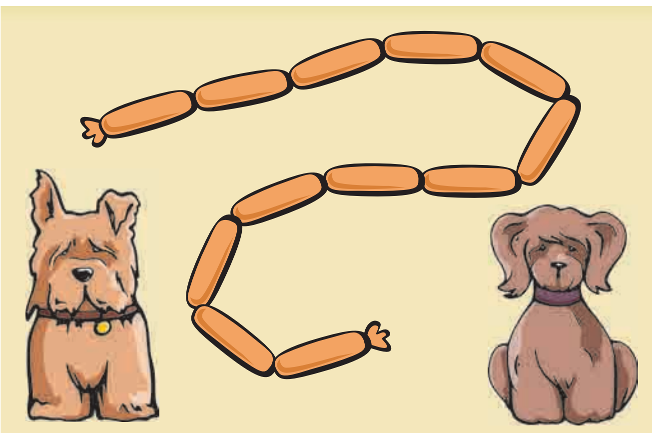
Another 4 birds come along.

How many birds are in the tree now?

The student gets the correct answer of 53 birds by counting on 50, 51, 52, 53 and tracking the count of 4. They may track the count by imaging or using substitute materials, including fingers.

If the student successfully uses a part–whole strategy (e.g., "49 and 1 is 50; that leaves 3 more birds, so there are 53 birds in the tree"), they exceed the expectation.

Example 3



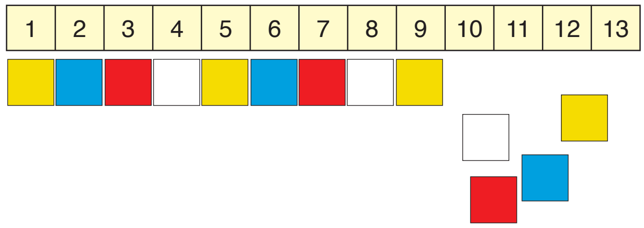
Here is a string of 12 sausages to feed 2 hungry dogs.

Each dog should get the same number of sausages. How many will each dog get?

The student uses equal sharing to distribute the sausages between the dogs. This might involve skip-counting ("2 sausages makes 1 each, 4 sausages makes 2 each … 12 sausages makes 6 each") while tracking the count mentally or with fingers, or it might involve halving, that is, dividing 12 into 6 and 6. (Note that 6 and 6 is a symmetrical partitioning of 12.)

Example 4

Show the student a number strip with coloured cubes lined up along it, as in the diagram below. Provide extra coloured cubes.



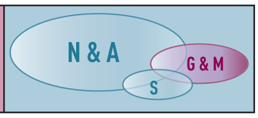
What colour cube goes on the number 13 in this pattern?

The student identifies the unit of repeat (yellow, blue, red, white) and continues the pattern one cube at a time until they identify a yellow cube on 13.

If the student notices that multiples of 4 have a white cube and therefore 13 has a yellow cube, they exceed the expectation.

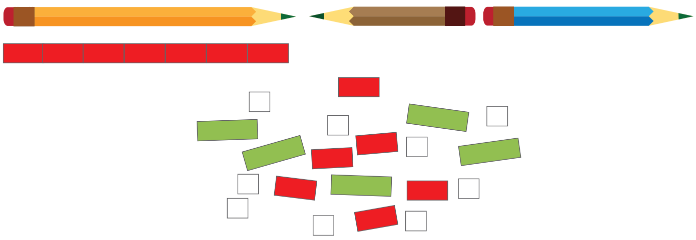
ILLUSTRATING THE STANDARD

Geometry and Measurement



Example 5

Place 3 pencils of different lengths end-on-end with gaps between them, as shown. Give the student a collection of white, red, and light green Cuisenaire™ rods.



Place rods underneath each pencil to show how long it is.

Without moving the pencils, can you tell me how much longer the orange pencil is than the blue pencil?

The student places the same-coloured rods, with no gaps or overlaps, from one end to the other of each pencil.

They count on or back to find the difference in length without needing to directly align the pencils. For example, if the orange pencil is 7 red rods long and the blue pencil is 4 red rods long, the student counts 5, 6, 7 or uses 4 + 3 = 7 to work out that the difference is 3 red rods.

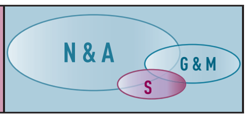
If the student uses different-coloured rods and shows that they understand that, for example, 2 white rods are the same length as 1 red rod, they exceed the expectation.

Example 6

The students work in pairs. One student has a picture of a group of attribute blocks laid out in a certain way. The other student has a set of actual attribute blocks. Without showing their partner the picture or pointing to the blocks, the first student describes to the second how to arrange the group of blocks so that it matches the picture, and the second student follows their instructions.

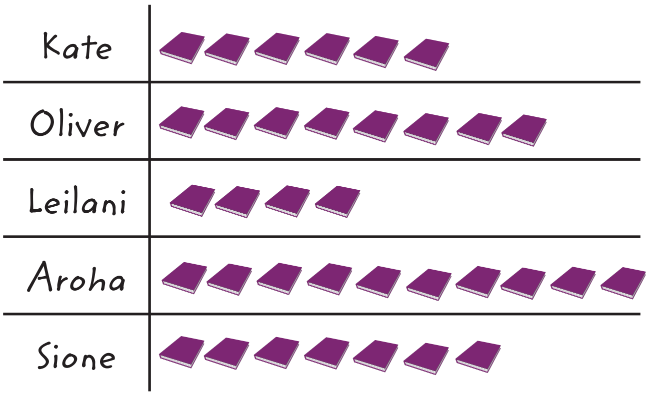
The student giving the instructions uses the appropriate positional language and geometric terms for shapes, and they accurately describe colours and turns (half and quarter). The other student is able to assemble the figure correctly with no errors in position or orientation of shapes.

Statistics



Example 7

Provide the student with the graph below.



It's readathon week. Five students make this graph to show how many books each of them reads during the first day. Each time they finish reading a book, they add a book to the graph.

How many books does each student read in the first day?

How many more books does Aroha read than Leilani?

Do the girls read more books than the boys?

The student is able to say how many books individual students read (e.g., "Kate reads 6 books. Sione reads 7.").

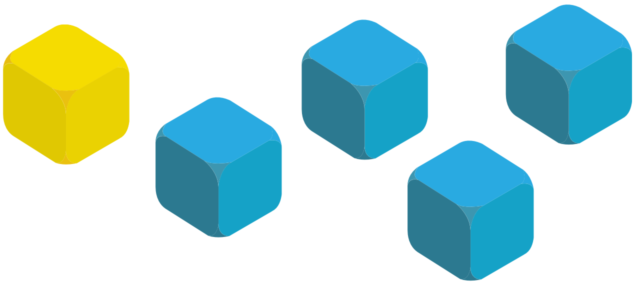
The student finds the difference between the number of books read by Aroha and Leilani by counting on or back (e.g., "Aroha reads 6 more books than Leilani. I just counted the extra ones.").

To compare the total books read by girls and boys, the student needs to recognise which names are girls' names and which are boys'. (You may need to help.) They count up the total for both and compare them. If the student uses additive thinking rather than counting, they exceed the expectation (e.g., "The boys read 8 books and 7 books. 8 + 7 = 15 because 7 + 7 = 14"). If the student realises that the comparison is not representative (or fair) because there are 3 girls and only 2 boys, they exceed the expectation.

The student should be able to ask their own comparison questions about the data, for example, "How many more books does Oliver read than Kate?"

Example 8

Let the student watch as you put 4 blue cubes and 1 yellow cube into a paper bag.



Put your hand in the bag and take out a cube, but don't look at it.

What colour will it be?

The student identifies the two possible outcomes. If they omit one of them (e.g., "It will be blue because there are more of them") or identify an outcome that is not possible (e.g., "It will be green because that is my favourite colour"), they do not meet the expectation.

If the student states that getting a blue cube is more likely than a yellow cube because there are more blue cubes than yellow cubes in the bag, they exceed the expectation.

THE MATHEMATICS STANDARD

After three years at school, students will be achieving at early level 2 in the mathematics and statistics learning area of the New Zealand Curriculum.

Number and Algebra

In contexts that require them to solve problems or model situations, students will be able to:

- *apply basic addition facts and knowledge of place value and symmetry to:*
  - *combine or partition whole numbers*
  - *find fractions of sets, shapes, and quantities;*
- *create and continue sequential patterns with one or two variables by identifying the unit of repeat;*
- *continue spatial patterns and number patterns based on simple addition or subtraction.*

Geometry and Measurement

In contexts that require them to solve problems or model situations, students will be able to:

- *measure the lengths, areas, volumes or capacities, and weights of objects and the duration of events, using linear whole-number scales and applying basic addition facts to standard units;*
- *sort objects and two- and three-dimensional shapes by their features, identifying categories within categories;*
- *represent reflections, translations, and rotations by creating and describing patterns;*
- *describe personal locations and give directions, using whole-number measures and half- or quarter-turns.*

Statistics

In contexts that require them to solve problems or model situations, students will be able to:

- *investigate questions by using the statistical enquiry cycle (with support):*
  - *gather and display category and simple whole-number data*
  - *interpret displays in context;*
- *compare and explain the likelihoods of outcomes for a simple situation involving chance.*

The following problems and descriptions of student thinking exemplify what is required to meet this standard.

Number and Algebra

During this school year, Number should be the focus of 60–80 percent of mathematics teaching time.

Example 1

You have 18 turtles, and you buy another 8 turtles from the pet shop.

How many turtles do you have now?

The student could use "making tens" (e.g., "18 + 2 = 20; that leaves 6 remaining from the 8; 20 + 6 = 26") or apply their knowledge of doubles and place value (e.g., "18 = 10 + 8; first add the 8, then the 10; 8 + 8 = 16, 16 + 10 = 26").

If the student responds very quickly because they know the fact 18 + 8 = 26, this also meets the expectation. If the student counts on, they do not meet the expectation.

Example 2

87 people are at the pōwhiri (welcome). 30 of the people are tangata whenua (locals). The rest of the people are manuhiri (visitors).

How many manuhiri are there?

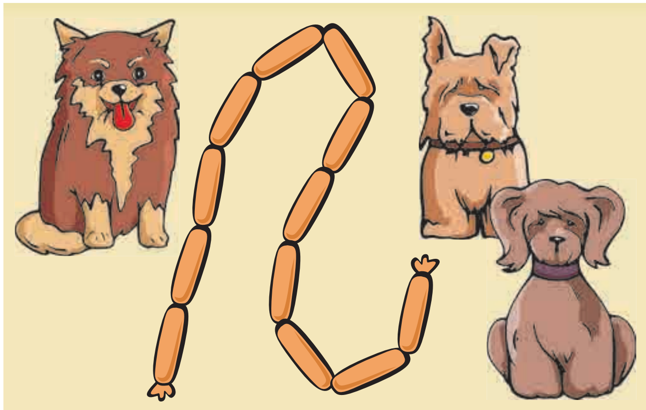
The student uses place value knowledge, combined with either addition or subtraction, to solve the problem. They may add on (30 + 50 = 80, 80 + 7 = 87) or subtract (80 – 30 = 50, so 87 – 30 = 57). If they use counting up or back in tens (e.g., 40, 50, 60, 70, 80, 87), they do not meet the expectation.

If they use a pencil and paper method to subtract 0 from 7 and 3 from 8, this doesn't necessarily demonstrate enough understanding of place value to meet the expectation. If they use this method, they must show that they understand the place value of the digits and that they are not treating them all as ones.

Example 3

Here is a string of 12 sausages to feed 3 hungry dogs.

Each dog should get the same number of sausages. How many will each dog get?

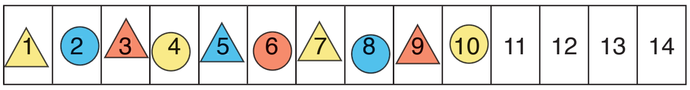


The student applies basic addition facts to share out the sausages equally between the dogs. Their thinking could be based on doubles or equal dealing – for example, 5 + 5 + 2 = 12, so 4 + 4 + 4 = 12 (redistributing 1 from each 5), or 6 + 6 = 12, so 4 + 4 + 4 = 12, or 2 + 2 + 2 = 6, so 4 + 4 + 4 = 12.

If the student solves the problem by one-to-one equal sharing, they do not meet the expectation. If they solve the problem using multiplication facts (3 x 4 = 12 or 12 ÷ 3 = 4), they exceed the expectation.

Example 4

Show the student the illustration below.



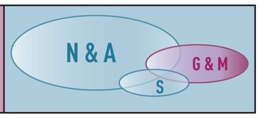
What shape goes on the number 14 in this pattern? What colour will it be?

The student identifies the two variables (shape and colour) in the pattern. They might look at the variables separately and identify the unit of repeat for each ("Yellow, blue, red" and "Triangle, circle"). Or they might look at the variables together to identify the complete unit of repeat ("Yellow triangle, blue circle, red triangle, yellow circle, blue triangle, red circle").

They continue the pattern until they identify that the shape on number 14 is a blue circle. If the student recognises that multiples of 2 in the pattern are circles and multiples of 3 are red and uses this information to solve the problem, they exceed the expectation.

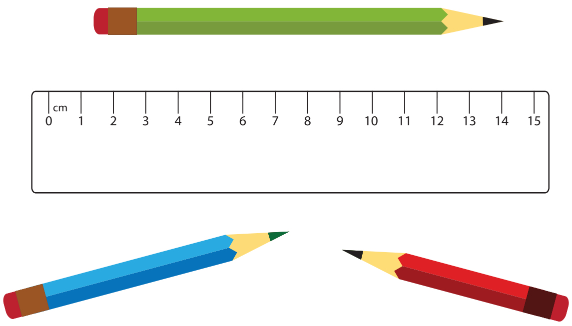
ILLUSTRATING THE STANDARD

Geometry and Measurement



Example 5

Give the student 3 pencils of different lengths and a ruler.



Use the ruler to find the length of each pencil.

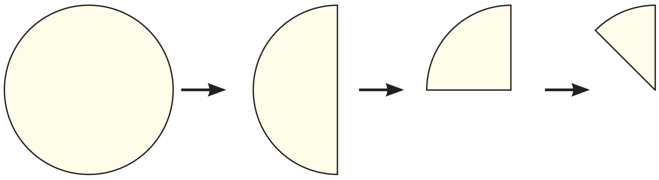
How much longer is the green pencil than the red pencil?

The student correctly measures the length of each pencil to the nearest centimetre: they align the end of the pencil with zero on the scale and read off the measure correctly. They apply basic addition facts to find the difference in length between the green and red pencils (e.g., for 12 centimetres and 9 centimetres: "3 centimetres, because 10 + 2 = 12, so 9 + 3 = 12"; or "3 centimetres, because I know 9 + 3 = 12").

Example 6

Give the student a circle of paper.

Fold this circle into 8 equal-sized pieces.

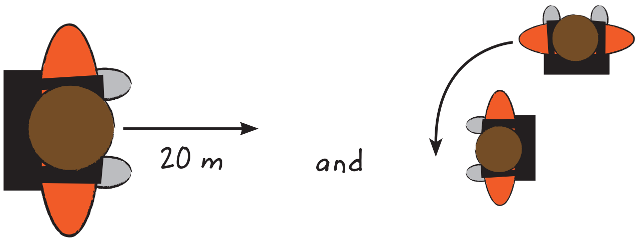


The student uses reflective symmetry through repeated halving to partition the circle into eighths.

Example 7

Give the student a metre ruler or tape measure and show them the illustrations below.

Write a set of instructions to explain to a visitor how to get from the library door to our classroom door. Make sure you include any right or left turns and distances in metres. You can use pictures to give the instructions, like this:

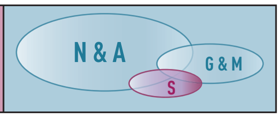


You can also use pictures or descriptions of objects such as buildings or trees.

The student provides a set of instructions that are accurate enough for a visitor to find their way to the classroom door from the library. If the student specifies compass directions or clockwise or anti-clockwise turns, they exceed the expectation.

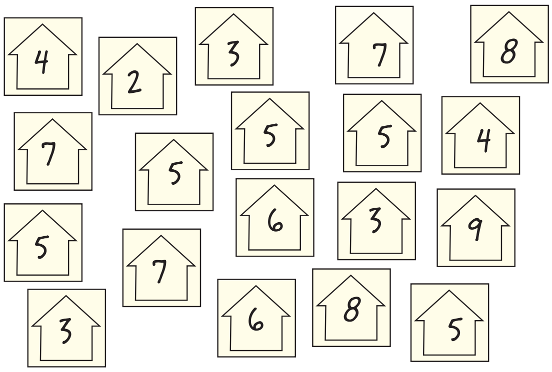
ILLUSTRATING THE STANDARD

Statistics



Example 8

Each student writes the number of people that usually live in their house on a square of paper or a sticker.



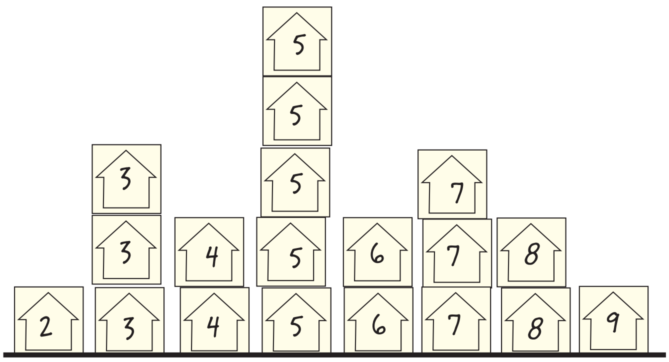
How many people live in the houses of students in our class?

Arrange the squares to find out.

What can you say about your arrangement?

The student sorts the whole-number data into groups.

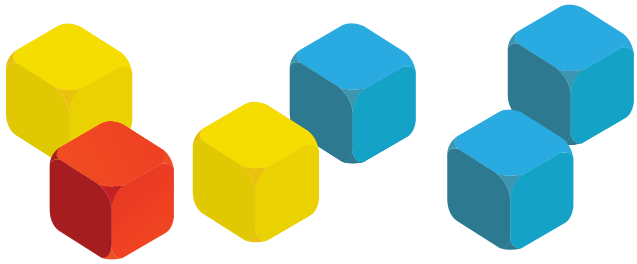
They may display the data in enclosed groupings or in a more organised display, such as a bar graph.



The student makes a statement about the number of people living in students' houses, based on their sorting of the data, for example, "There are lots of different numbers of people living in houses, from 2 to 9" or "5 is the most common number of people".

Example 9

Let the student watch as you put 3 blue cubes, 2 yellow cubes, and a red cube into a paper bag.



Put your hand in the bag and take out a cube, but don't look at it.

What colour is the cube most likely to be?

What colour is it least likely to be?

Explain why.

The student classifies the probability of getting each colour ("Blue is most likely, and red is least likely"). They discuss the numbers and colours of cubes to explain their answer (e.g., "There are 3 blue cubes and only 1 red cube").

If the student gives the probabilities as fractions (e.g., "There is a one-half chance of blue"), they exceed the expectation. If they explain the likelihoods without reference to the number of cubes (e.g., "Yellow is my lucky colour" or "I always get red"), they do not meet the expectation.

BY THE END OF YEAR 4

THE MATHEMATICS STANDARD

By the end of year 4, students will be achieving at level 2 in the mathematics and statistics learning area of the New Zealand Curriculum.

Number and Algebra

In contexts that require them to solve problems or model situations, students will be able to:

- *apply basic addition and subtraction facts, simple multiplication facts, and knowledge of place value and symmetry to:*
  - *combine or partition whole numbers*
  - *find fractions of sets, shapes, and quantities;*
- *create, continue, and give the rule for sequential patterns with two variables;*
- *create and continue spatial patterns and number patterns based on repeated addition or subtraction.*

Geometry and Measurement

In contexts that require them to solve problems or model situations, students will be able to:

- *measure the lengths, areas, volumes or capacities, weights, and temperatures of objects and the duration of events, reading scales to the nearest whole number and applying addition, subtraction, and simple multiplication to standard units;*
- *sort objects and two- and three-dimensional shapes by two features simultaneously;*
- *represent and describe the symmetries of a shape;*
- *create nets for cubes;*
- *describe personal locations and give directions, using simple maps.*

Statistics

In contexts that require them to solve problems or model situations, students will be able to:

- *investigate questions by using the statistical enquiry cycle independently:*
  - *gather and display category and simple whole-number data*
  - *interpret displays in context;*
- *compare and explain the likelihoods of outcomes for a simple situation involving chance, acknowledging uncertainty.*

The following problems and descriptions of student thinking exemplify what is required to meet this standard.

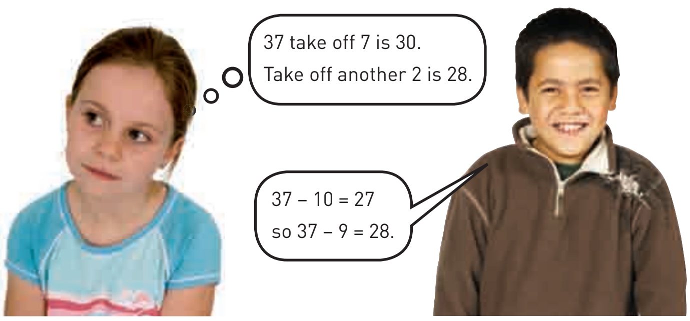
Number and Algebra

During this school year, Number should be the focus of 60–80 percent of mathematics teaching time.

Example 1

Imagine you have 37 lollies and you eat 9 of them.

How many lollies would you have left?



The student gets to the answer 28 by mentally partitioning numbers (e.g., 9 = 7 + 2 in the first calculation) and by using tidy numbers (e.g., 10 in the second calculation).

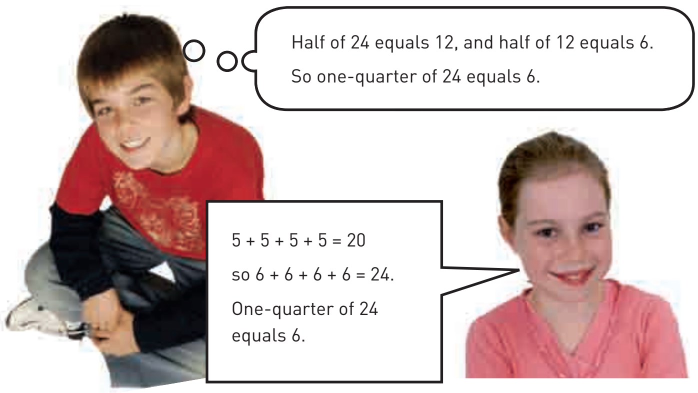
Source: NumPA, Numeracy Development Projects, Book 2: *The Diagnostic Interview*, page 8.

ILLUSTRATING THE STANDARD

Example 2

If there are 24 marbles in the bag, how many should each student get?

The student applies their knowledge of symmetry or number facts to partition the set of 24 – for example, by using repeated halving or by using trial and improvement with addition facts.



If the student knows or derives the fact 4 x 6 = 24, they exceed the expectation.

Because 6×4=24 & there is 4 children So they get 6 each.

Source: Figure It Out, *Multiplicative Thinking*, Levels 2–3, page 4.

Example 3

Here is a 3-section matchstick fence.

How many matchsticks would it take to make an 8-section fence?

The student continues the number pattern by using repeated addition, possibly in conjunction with written recording.

Sections	1	2	3	4	5	6	7	8
Matches	4	7	10	13	16	19	22	25
			+3	+3	+3	+3	+3	

10 ← 3 sections
+3
13 ← 4 sections
+3
16 ← 5 sections
+3
19 ← 6 sections
+3
22 ← 7 sections
+3
25 ← 8 sections

If the student draws an 8-section fence and then counts the matchsticks, they do not meet the expectation.

Using a multiplicative strategy (e.g., (7 x 3) + 4 = 25 or (8 x 3) + 1 = 25) exceeds the expectation.

Source: adapted from Figure It Out, *Algebra*, Level 3, page 2.

BY THE END OF YEAR 4

ILLUSTRATING THE STANDARD

Geometry and Measurement

Example 4

Give the student the 3 items shown and the torn measuring tape.

Measure the lengths of the bookmark, snake, and ribbon, using the tape measure. The piece of measuring tape has been torn, but it can still be used for measuring.

The student understands that any point on a whole-number scale can be used as an arbitrary zero. They calculate the difference between 2 points on the measuring tape to find the length of an item, giving the correct number and unit of measurement (e.g., 13 cm).

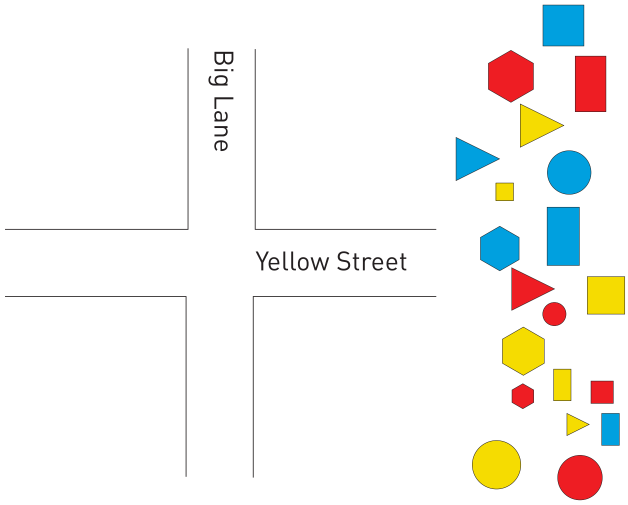
The student must provide an accurate measurement for the length of the ribbon, which is longer than the tape measure. They might take 2 measurements and add the results, or they might fold the ribbon in 2 and double the measurement of that length.

Source: NEMP's 2005 report on mathematics, page 40.

Example 5

Give the student the diagram and attribute blocks as per the illustration.

Put all the yellow blocks on Yellow Street. Put all the big blocks on Big Lane. Which blocks should go in the intersection?

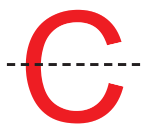


The student simultaneously sorts the blocks by 2 features, size and colour, in order to place the blocks that are both big and yellow in the intersection.

Example 6

Give the student cards with the letters shown below on them.

The letter C has one line of reflective symmetry.



The letter S has half-turn symmetry.

What reflective and turn symmetry do these letters have?

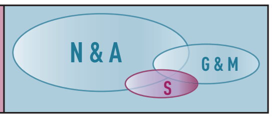
H R Z

The student identifies the symmetries of each letter as follows:

- H has two lines of reflective symmetry (vertical and horizontal) and half-turn symmetry.
- R has neither reflective nor turn symmetry.
- Z has half-turn symmetry.

ILLUSTRATING THE STANDARD

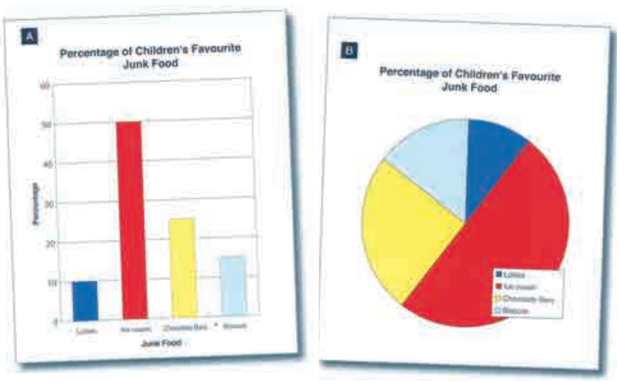
Statistics



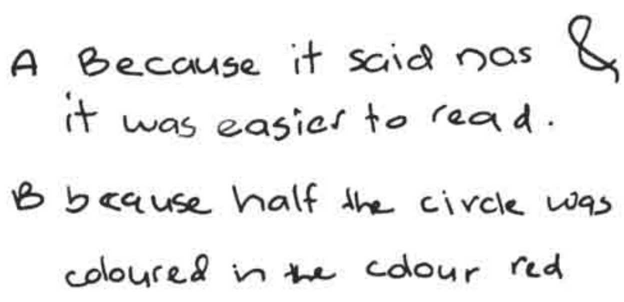
Example 7

Here are 2 graphs showing information on a group of children's favourite junk foods.

1. *What percentage of children said biscuits were their favourite junk food? Which graph did you use to work this out? Why did you use that graph?*
2. *Which junk food did half the children say was their favourite junk food? Which graph did you use to work this out? Why did you use that graph?*



The student answers questions 1 and 2 correctly by reading from one of the graphs. They justify their choice of graph by explaining how it provides the required information.



For question 1, they will use the bar graph, which gives number information. The pie chart shows proportions and is therefore the easier display to use in answering question 2. However, the student may also use the bar graph, noting that 50 percent is the same as a half.

Source: adapted from NEMP's 2007 report on graphs, tables, and maps, page 17.

Example 8

Ask the student to play the game Will Your Parents Let You? Give them 3 different-coloured dice with different mixes of yes and no faces. The red dice has 5 yes faces and 1 no face, blue has 3 yes faces and 3 no faces, and green has 1 yes face and 5 no faces.

Show the student a number of coloured cards with illustrated scenarios, as in the following examples. The red cards show scenarios that parents are likely to say yes to; blue cards show scenarios that parents may or may not agree to; and green cards show scenarios that are unlikely to be allowed.

The student chooses one card at a time. They roll a red dice if they have chosen a red card, a blue dice if they chose a blue card, and a green dice after choosing a green card. Once the dice gives them an answer, they put that card in a yes pile or a no pile.

1. *What do you notice about the colours of the cards in the yes pile and in the no pile? Can you explain this by looking at the 3 dice?*
2. *Imagine there's something you really want to do. Which dice would you use to find out whether you can do it or not? Will you get a yes when you roll that dice?*

The student should notice that the yes pile contains lots of red cards and the no pile contains lots of green cards. They should be able to explain that this is because the red dice has more yes faces than the green one.

In answer to question 2, the student should reply that the red dice would be best because it gives the best chance of getting a yes. They should acknowledge that a no answer is still possible with the red dice, even though a yes answer is more likely.

Source: V. Wright (1997). *Topic Based Mathematics 1*. Hamilton: Mathematics Advisory Service.

BY THE END OF YEAR 5

THE MATHEMATICS STANDARD

YEAR 1 YEAR 2 YEAR 3 YEAR 4 YEAR 5 YEAR 6 YEAR 7 YEAR 8

By the end of year 5, students will be achieving at early level 3 in the mathematics and statistics learning area of the New Zealand Curriculum.

Number and Algebra

N & A G & M S

In contexts that require them to solve problems or model situations, students will be able to:

- *apply additive and simple multiplicative strategies and knowledge of symmetry to:*
  - *combine or partition whole numbers*
  - *find fractions of sets, shapes, and quantities;*
- *create, continue, and predict further members of sequential patterns with two variables;*
- *describe spatial and number patterns, using rules that involve spatial features, repeated addition or subtraction, and simple multiplication.*

Geometry and Measurement

N & A G & M S

In contexts that require them to solve problems or model situations, students will be able to:

- *measure time and the attributes of objects, choosing appropriate standard units and working with them to the nearest tenth;*
- *sort two- and three-dimensional shapes, considering the presence and/or absence of features simultaneously and justifying the decisions made;*
- *represent and describe the results of reflection, rotation, and translation on shapes;*
- *create nets for rectangular prisms;*
- *draw plan, front, and side views of objects;*
- *describe locations and give directions, using grid references and points of the compass.*

Statistics

N & A G & M S

In contexts that require them to solve problems or model situations, students will be able to:

- *investigate summary and comparison questions by using the statistical enquiry cycle:*
  - *gather, display, and identify patterns in category and whole-number data*
  - *interpret results in context;*
- *order the likelihoods of outcomes for simple situations involving chance, experimenting or listing all possible outcomes.*

The following problems and descriptions of student thinking exemplify what is required to meet this standard.

Number and Algebra

N & A G & M S

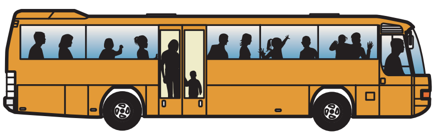
During this school year, Number should be the focus of 50–70 percent of mathematics teaching time.

Example 1

There are 53 people on the bus.

26 people get off.

How many people are left on the bus?



The student uses an efficient part–whole strategy for subtraction, such as subtracting in parts (e.g., 53 – 6 = 47, 47 – 20 = 27; or 53 – 20 = 33, 33 – 6 = 27) or subtracting a tidy number (e.g., 53 – 30 = 23, 23 + 4 = 27).

If they count back or use repeated subtraction (e.g., 53 – 10 = 43, 43 – 10 = 33 ...), they do not meet the expectation. If they use inverse relationships between subtraction and addition, such as adding on (e.g., 26 + 4 = 30, 30 + 23 = 53, 4 + 23 = 27, so 26 + 27 = 53) or doubling (e.g., 26 + 26 = 52, so 26 + 27 = 53), they exceed the expectation.

If the student uses a written algorithm to solve the problem, they must explain the place value partitioning involved.

Source: NumPA, Numeracy Development Projects, Book 2: *The Diagnostic Interview*, page 8.

ILLUSTRATING THE STANDARD

Example 2

Socks cost $6 per pair. *Hankies cost $3 each.*

How much does it cost to buy 4 pairs of socks and 8 hankies?

The student will generally use some form of written recording when working through this problem. Solving the problem using only mental calculations is also acceptable.

The student uses multiplication facts and addition to correctly solve the problem. They may do so in any order and may work out the multiplication facts if they do not know them (e.g., by calculating 4 x 6 as double 2 x 6 or 8 x 3 as 10 x 3 – 6). The addition should make use of part–whole strategies (e.g., 24 + 24 = 40 + 8 = 48).

48. I added 6x4
then 8X3.

Vertical algorithms should not be needed for this problem. If the student uses repeated addition (e.g., 6 + 6 + 6 + 6 + 3 + 3 ...), they do not meet the expectation. If they use only multiplication (e.g., "4 x 6 = 8 x 3, so the total cost is 8 x 6 = 48"), they exceed the expectation.

Example 3

Show the student the following illustration.

The factory orders 48 wheels.

How many of each toy can they make with the 48 wheels?

The student uses known multiplication facts or builds up answers with addition and multiplication. For example, to find how many twos are in 48 (for scooters), they may use doubles knowledge (24 + 24 = 48). To find how many threes are in 48 (for tricycles), they may use addition and multiplication (e.g., 12 x 3 = 36, so 13 x 3 = 36 + 3 = 39, 14 x 3 = ...).

If they use properties of multiplication efficiently, they exceed the expectation (e.g., 48 ÷ 3 is the same as 30 ÷ 3 = 10 plus 18 ÷ 3 = 6, so 48 ÷ 3 = 16; or 48 ÷ 6 = 8 (known fact), so 48 ÷ 3 = 16).

Source: Figure It Out, *Number*, Levels 2–3, page 15.

Example 4

Show the student the following patterns.

How many tiles will be in pattern 4? How do you know?

How many tiles will be in pattern 6? Explain how you know.

The student identifies the rule for the pattern – that it is growing by four tiles each time because one tile is added to each arm. They use either addition (e.g., 5 + 4 = 9, 9 + 4 = 13) or multiplication (e.g., 4 x 3 = 12, 12 + 1 = 13) to find the number of tiles in pattern 4.

To find the number of tiles in pattern 6, they may use repeated addition (e.g., 13 + 4 = 17, 17 + 4 = 21) or multiplication (e.g., 4 x 5 = 20, 20 + 1 = 21). If they use counting on combined with drawing, they do not meet the expectation.

ILLUSTRATING THE STANDARD

Geometry and Measurement

Example 5

Give the student access to water, a capacity measure (e.g., a marked jug), a funnel, a 3 litre bottle, an unmarked 250 millilitre plastic cup, and kitchen scales.

1. *Find out how much water the plastic cup holds.*
2. *Without using the bottle, estimate how many cups you could pour from a:*

   a) 1 litre bottle *b) 3 litre bottle* *c) 1.5 litre bottle.*
3. *Use the scales to find out the weight of the 3 litre bottle when it is full of water.*
4. *How much would a full 1.5 litre bottle weigh? Use the scales to check your answer.*

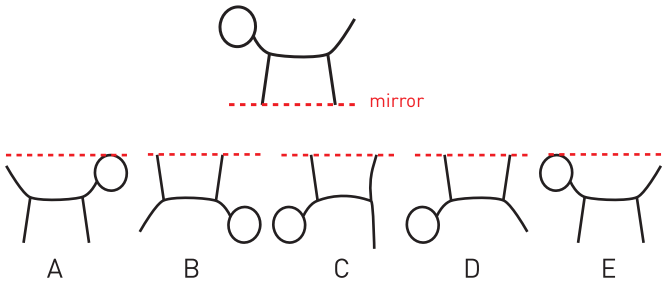
The student correctly reads the scales on the capacity measure and the kitchen scales to the nearest whole number (e.g., "The full 3 litre bottle weighs 3 kilograms") or the nearest tenth (e.g., when weighing a half-full 3 litre bottle). They use their knowledge of place value and multiplication to connect results (e.g., "A 1 litre bottle holds 4 cups because 4 x 250 = 1000 mL" and "A 3 litre bottle holds 12 cups because 3 x 4 = 12").

If the student uses their knowledge of conversions between units (e.g., "1 litre of water weighs 1 kilogram, so 1.5 litres weighs 1.5 kilograms"), they exceed the expectation.

Source: adapted from Figure It Out, *Measurement*, Levels 2–3, page 6.

Example 6

Show the student the following illustration.



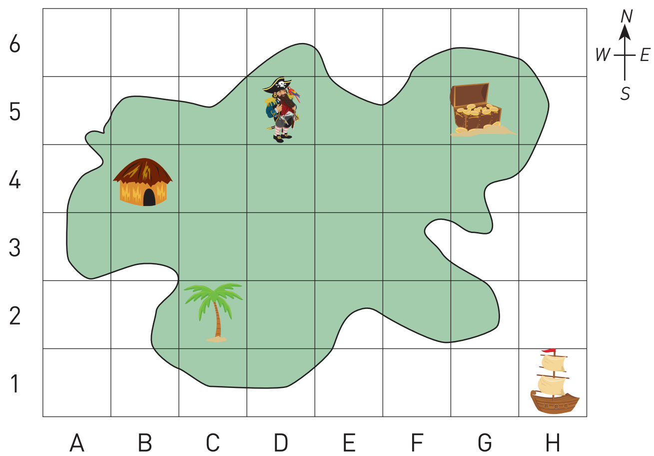
Will the drawing look like A, B, C, D, or E when it is reflected in the mirror? Why?

The student correctly identifies D as the answer. They explain their choice by referring to features that change or do not change, for example, "The dog has to be upside down", "It has to be facing the same way", "It must still have straight legs and a bent tail".

Source: adapted from PAT Test 2, Item 31, © NZCER 2007.

Example 7

Show the student the following illustration.



What things are at B4 and C2 on the map?

What is the location of the treasure?

The pirate wants to use his compass to get back to his ship. In what direction should he go?

The student correctly names the objects at B4 (a hut) and C2 (a tree) and gives the location of the treasure as G5. They state that the pirate must travel south-east to get to his ship, and they can trace his path.

Source: adapted from PAT Test 2, Item 12, © NZCER 2007.

BY THE END OF YEAR 5

ILLUSTRATING THE STANDARD

Statistics

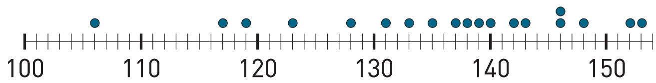
Example 8

Ask each student in the class to measure their height to the nearest centimetre and to record it on a sticker. Put the stickers onto a board or photocopy them as data cards.

Sort and display the heights of the students in our class. What patterns can you find in the data?

The student sorts the heights from shortest to tallest. They are able to group the measurements into intervals and use displays for comparison, with or without the use of computer technology. For example:

15	2	3				
14	0	2	3	6	6	8
13	1	3	5	7	8	9
12	3	8				
11	7	9				
10	6					



The student makes statements about the data based on the ideas of middle, spread, and clustering, for example, "The middle height is about 133 centimetres", "We are between 105 and 155 centimetres tall", "Most people are between 130 and 150 centimetres tall".

Example 9

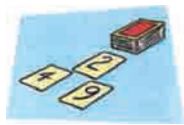
Students play the following game with a pack containing 10 digit cards (0, 1, 2 ... 9).

1. Shuffle the digit cards.

2. Place them face down in a pile.

3. Pick up the top 2 cards.

4. Now pick up a third card.



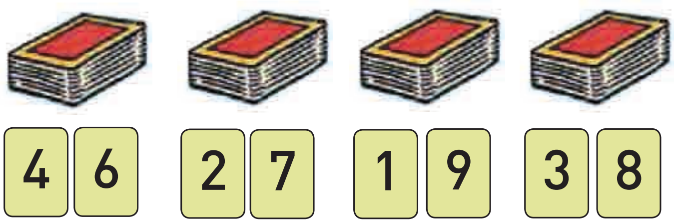
5. If the number on that card is not between the other 2 numbers, you get nothing.

6. If it is, take a counter.

7. Put your 3 cards back on the pack and shuffle it. Now it's your classmate's turn.

8. When there are no more counters to take, the person who has the most counters wins.

Give the student the following four scenarios and ask them to compare the chances of winning.



The student compares the probabilities of winning in the various scenarios by assessing the likelihood of getting a number between the two that are exposed. They may list the possibilities: the number 5 for between 4 and 6; 3, 4, 5, 6 for between 2 and 7; and so on. To meet the expectation, the student orders the probabilities correctly, noting that 2–7 and 3–8 have equal likelihood. 4–6 is the least likely to win and 1–9 the most likely.

If the student uses fractions to order the probabilities, they exceed the expectation (e.g., "There is a one-half (4 out of 8) chance of getting a card between 2 and 7").

Source: Figure It Out, *Statistics* (revised), Levels 2–3, page 24.

THE MATHEMATICS STANDARD

By the end of year 6, students will be achieving at level 3 in the mathematics and statistics learning area of the New Zealand Curriculum.

Number and Algebra

In contexts that require them to solve problems or model situations, students will be able to:

- *apply additive and simple multiplicative strategies flexibly to:*
  - *combine or partition whole numbers, including performing mixed operations and using addition and subtraction as inverse operations*
  - *find fractions of sets, shapes, and quantities;*
- *determine members of sequential patterns, given their ordinal positions;*
- *describe spatial and number patterns, using:*
  - *tables and graphs*
  - *rules that involve spatial features, repeated addition or subtraction, and simple multiplication.*

Geometry and Measurement

In contexts that require them to solve problems or model situations, students will be able to:

- *measure time and the attributes of objects, choosing appropriate standard units;*
- *use arrays to find the areas of rectangles and the volumes of cuboids, given whole-number dimensions;*
- *sort two- and three-dimensional shapes (including prisms), considering given properties simultaneously and justifying the decisions made;*
- *represent and describe the results of reflection, rotation, and translation on shapes or patterns;*
- *identify nets for rectangular prisms;*
- *draw or make objects, given their plan, front, and side views;*
- *describe locations and give directions, using grid references, turns, and points of the compass.*

Statistics

In contexts that require them to solve problems or model situations, students will be able to:

- *investigate summary and comparison questions by using the statistical enquiry cycle:*
  - *gather or access multivariate category and whole-number data*
  - *sort data into categories or intervals, display it in different ways, and identify patterns*
  - *interpret results in context, accepting that samples vary;*
- *order the likelihoods of outcomes for situations involving chance, considering experimental results and models of all possible outcomes.*

BY THE END OF YEAR 6

ILLUSTRATING THE STANDARD

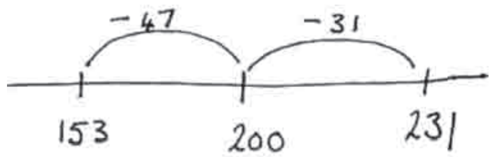
The following problems and descriptions of student thinking exemplify what is required to meet this standard.

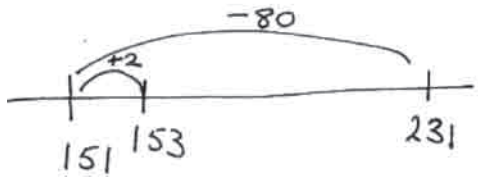
Number and Algebra	N & A / G & M / S
During this school year, Number should be the focus of 50–70 percent of mathematics teaching time.	

Example 1

Mitchell had 231 toy sports cars. He sold 78 of them. How many cars did he have left?

The student solves the problem by using an efficient strategy that involves mental calculation and place value understanding. They may draw on the inverse relationship of addition and subtraction, as illustrated in the speech bubble below. Use of recording is acceptable. If the student uses a vertical algorithm to solve the problem, they must explain the place value partitioning involved.



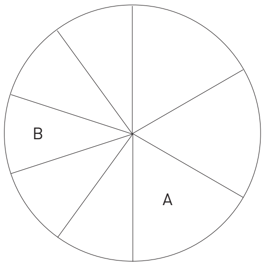


78 plus 22 is 100. 131 more is 231. So the difference between 78 and 231 is 22 plus 131, which is 153.

Source: GloSS Assessment K, Task 6.

Example 2

1. *What fractions of the whole birthday cake are pieces A and B? Explain your answer.*
2. *You have 60 jelly beans to decorate the top of the cake. If the jelly beans are spread evenly, how many of them will be on $\frac{4}{10}$ of the cake?*



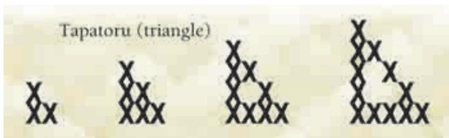
Double 3 which is 6. A is in one column so that makes $\frac{1}{6}$.

The student uses either rotational symmetry, mapping how many of A or B will fit into a full turn, or multiplication to correctly name the fractions (e.g., "B is $\frac{1}{5}$ of $\frac{1}{2}$, so it is $\frac{1}{10}$").

They use division and multiplication to find the number of jelly beans on four-tenths of the cake (e.g., "60 ÷ 10 = 6 jelly beans on $\frac{1}{10}$, 4 x 6 = 24 jelly beans").

Example 3

This is how the tapatoru pattern grows.



How many crosses will be in the 20th tapatoru pattern? Show how you worked out your answer.

The student uses repeated addition or a multiplication rule in conjunction with a recording strategy. Alternatively, they might use spatial features of the pattern to solve the problem (e.g., by noting there's an extra cross on each side as the pattern grows).

Tapatoru No.	1	2	3	4	20
Crosses	3	6	9	12	60

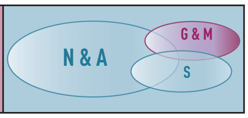
It's times three

The 20th pattern will have 21 crosses on each side. So that will be 3 x 21 = 63 crosses, less 3 so the corners are not counted twice.

Source: adapted from Figure It Out, *Algebra*, Level 3, page 3.

ILLUSTRATING THE STANDARD

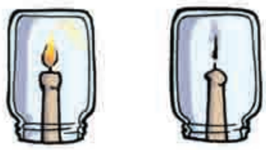
Geometry and Measurement



Example 4

When you put a jar over a burning candle, the flame will soon go out.

This is because the flame uses up the oxygen in the jar.



Do this activity with a classmate. Before you start, draw up a table like this:

Candle Trials					
Time for flame to go out					Capacity in mL
Trial	1	2	3	Middle	
Jar 1					
Jar 2					
Jar 3					
Jar 4					
Jar 5					

1. *Get five jars of different sizes.*

   Using a stopwatch, time how long the candle flame takes to go out after you put each jar over it.

   Do this 3 times for each jar and then record the middle time on your table.

2. *Measure each jar's capacity by filling it with water and pouring the water into a measuring jug.*

   Record the measurements in your table.

3. *Can you predict how long the flame will take to go out if you know the capacity of the jar?*

4. *Stick the candle in the base of an ice cream container. Put about 2 centimetres of water in the container. Put the jar over the lighted candle.*

   Water rises up into the jar as the oxygen is used up.

   Estimate what fraction of the air in the jar was oxygen.

The student carries out the investigation in an organised manner. They accurately measure both time and capacity, using appropriate units and devices. They use their measurement data to "generalise" the time required for a candle flame to go out (about 3 seconds per 100mL of air). They estimate the fraction of the air in the jar that was oxygen as approximately one-fifth or 20 percent. (Note that the rise in water is due to a change in pressure, but it gives a reasonable estimate for the fraction of the air that was oxygen.)

Source: adapted from Figure It Out, *Measurement*, Levels 3–4, page 11.

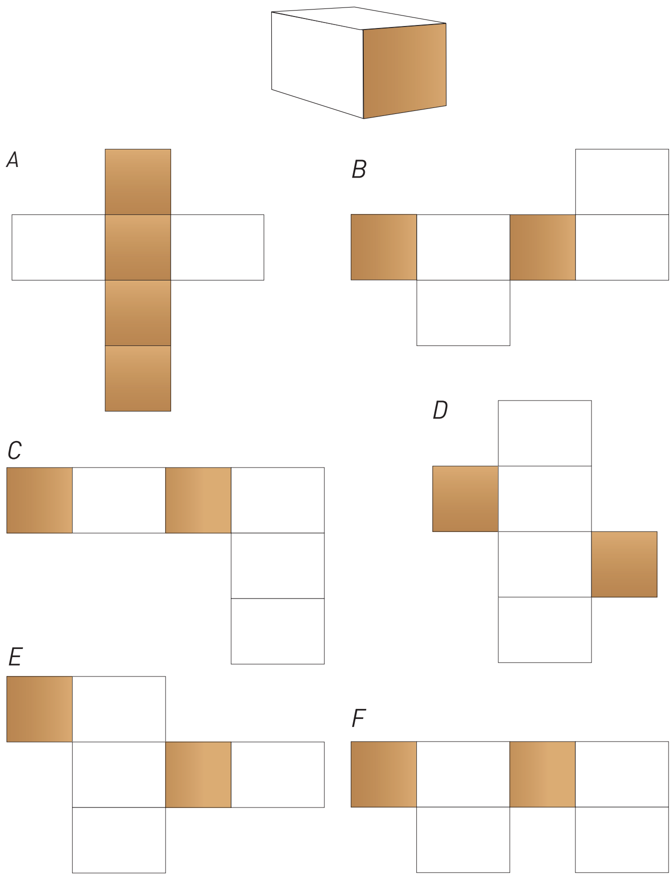
BY THE END OF YEAR 6

ILLUSTRATING THE STANDARD

Example 5

Show the student the following illustration. Explain that they have to answer the question without physically cutting or folding the paper.

How many of these nets will fold up to make the box? Which ones are they?



The student correctly identifies that three nets – B, D, and E – will fold to make the cuboid model (a rectangular prism). They understand that the model must have four rectangular faces and two square faces, and they can visualise whether the faces overlap when folded and how the connected faces form parts of the model.

Source: adapted from NEMP's 2005 report on mathematics, page 46.

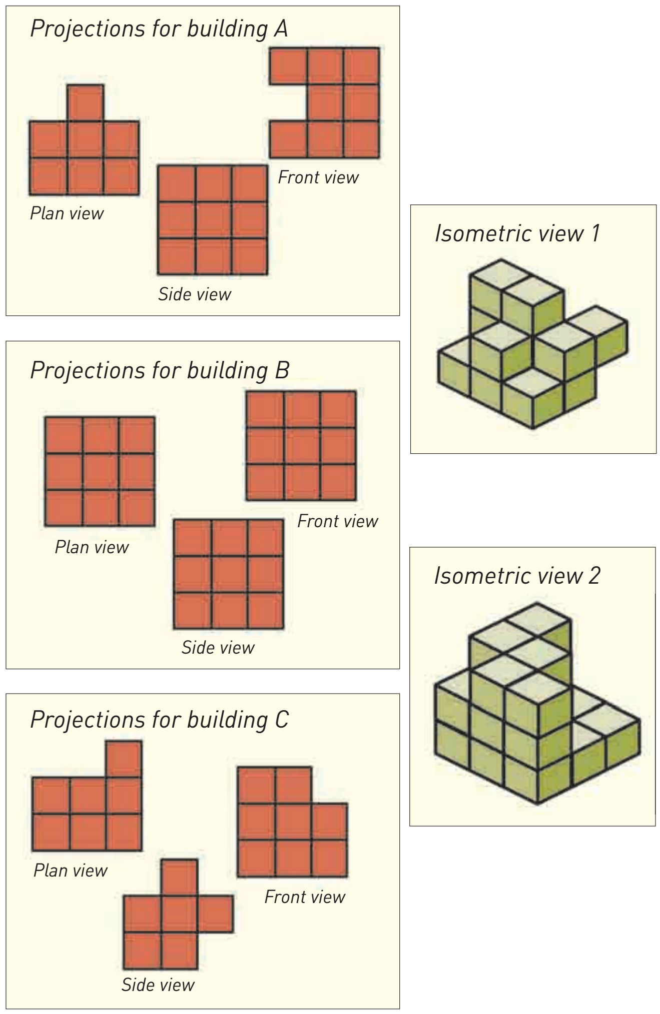
Example 6

Provide the student with interlocking cubes and the following illustration.

Here are drawings for 3 buildings. The projections (plan, front, and side views) and isometric views have been mixed up, and one of the isometric drawings is missing.

Match the projections with the isometric views for 2 of the buildings.

Then use the projections of the third building to assemble it, using interlocking cubes. If you can, draw an isometric view of this building.

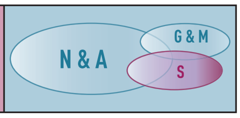


The student correctly matches the projections and isometric views for two buildings (building B with isometric view 2; building C with isometric view 1). They then accurately assemble a model of a building that agrees with the projections for building A. If they draw an accurate isometric view of their building, they exceed the expectation.

Source: adapted from Figure It Out, *Geometry*, Levels 3–4, page 15.

ILLUSTRATING THE STANDARD

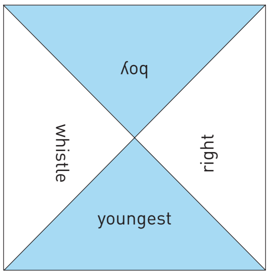
Statistics



Example 7

Have each student in the class create a data card with answers to the following questions:

- Are you a boy or a girl?
- Can you whistle?
- Are you the oldest, the youngest, the only, or a middle child in your family?
- Which hand do you usually write with?

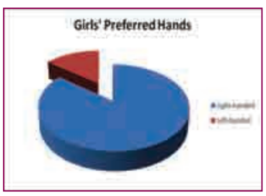


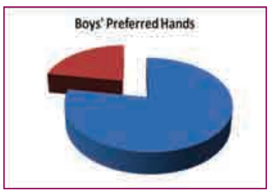
Photocopy all the data cards onto A4 paper. Organise the students into pairs, hand out a set of data cards to each pair, and have them cut out all the data cards.

Suggest some different types of questions that could be answered from the data – for example, summary questions like "Can more people whistle than can't whistle?" or comparison questions like "Are more boys or girls left-handed?"

Sort the class data to find the answers to your questions and display the results using graphs so that your classmates can clearly see the answers.

The student asks summary and comparison questions that can be answered using the information provided by the data cards. They sort and present the data in ways that clearly answer their questions and communicate their findings. To highlight differences, they use pictographs or bar graphs (made from the data cards). To highlight proportions, they might use strip graphs or pie charts.

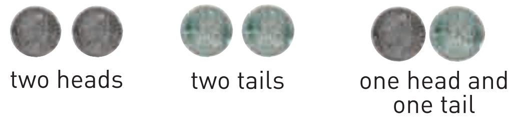




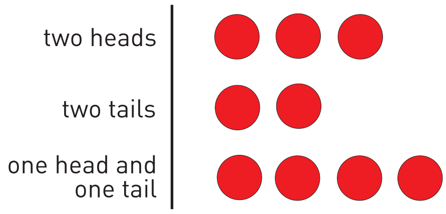
Source: nzmaths website, Statistics, Level 3, Data Squares: www.nzmaths.co.nz/node/160

Example 8

When you toss two coins together, you could get these results:



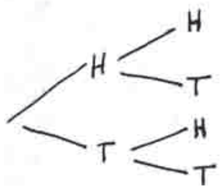
Toss two coins 24 times.
Each time you toss, put a new counter on a graph to show what you got, like this:

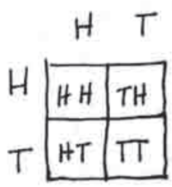


What does the graph show?

Draw a diagram to explain why this happens.

The student's results will almost certainly suggest that the likelihood of heads-heads or tails-tails is less than that of one head and one tail. To explain their results, they should develop a model of all possible outcomes. Suitable models include:





From the model, they should explain that there is only one way of getting heads-heads or tails-tails but two ways of getting one head and one tail. If the student expresses the likelihoods as fractions, they exceed the expectation.

Source: adapted from Figure It Out, *Statistics* (1999), Levels 2–3, page 22.

BY THE END OF YEAR 7

THE MATHEMATICS STANDARD

YEAR 1 YEAR 2 YEAR 3 YEAR 4 YEAR 5 YEAR 6 YEAR 7 YEAR 8

By the end of year 7, students will be achieving at early level 4 in the mathematics and statistics learning area of the New Zealand Curriculum.

Number and Algebra

N & A | G & M | S

In contexts that require them to solve problems or model situations, students will be able to:

- *apply additive and multiplicative strategies flexibly to whole numbers, ratios, and equivalent fractions (including percentages);*
- *apply additive strategies to decimals;*
- *balance positive and negative amounts;*
- *find and represent relationships in spatial and number patterns, using:*
  - *tables and graphs*
  - *general rules for linear relationships.*

Geometry and Measurement

N & A | G & M | S

In contexts that require them to solve problems or model situations, students will be able to:

- *measure time and the attributes of objects, using metric and other standard measures;*
- *make simple conversions between units, using whole numbers;*
- *use side or edge lengths to find the perimeters and areas of rectangles and parallelograms and the volumes of cuboids, given whole-number dimensions;*
- *sort two- and three-dimensional shapes into classes, defining properties and justifying the decisions made;*
- *identify and describe the transformations that have produced given shapes or patterns;*
- *create or identify nets for rectangular prisms and other simple solids;*
- *draw plan, front, side, and perspective views of objects;*
- *describe locations and give directions, using grid references, simple scales, turns, and points of the compass.*

Statistics

N & A | G & M | S

In contexts that require them to solve problems or model situations, students will be able to:

- *investigate summary, comparison, and relationship questions by using the statistical enquiry cycle:*
  - *gather or access multivariate category and measurement data*
  - *sort data and display it in multiple ways, identifying patterns and variations*
  - *interpret results in context, accepting that samples vary and have no effect on one another;*
- *order the likelihoods of outcomes for situations involving chance, checking for consistency between experimental results and models of all possible outcomes.*

BY THE END OF YEAR 7

ILLUSTRATING THE STANDARD

The following problems and descriptions of student thinking exemplify what is required to meet this standard.

Number and Algebra

N & A G & M S

During this school year, Number should be the focus of 40–60 percent of mathematics teaching time.

Example 1

There are 6 baskets and 24 muffins in each basket. How many muffins are there altogether?

The student uses an efficient multiplicative strategy to solve the problem mentally. This might involve drawing on their knowledge of place value (e.g., 6 x 20 + 6 x 4), working with tidy numbers (e.g., 6 x 25 – 6 x 1), or doubling and halving (e.g., 6 x 24 = 12 x 12).

If the student uses repeated addition or doubling (e.g., 24 + 24 = 48, 48 + 24 = 72 ...), they do not meet the expectation. If they use a vertical algorithm to solve the problem, they must explain the place value partitioning involved.

Source: NumPA, Numeracy Development Projects, Book 2: *The Diagnostic Interview*, page 41.

Example 2

Tama has 4.95 litres of petrol in one can and 7.5 litres in the other can.

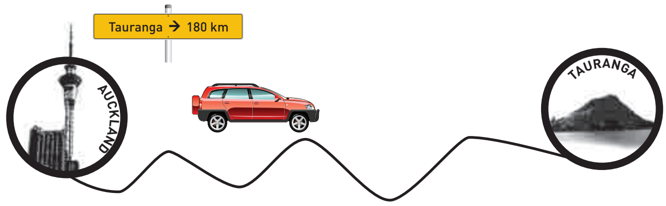
How much petrol does he have altogether?

The student demonstrates their understanding of decimal place value when combining the amounts. Appropriate strategies include using compensation (e.g., 4.95 + 7.5 = 4.45 + 8 = 12.45), working with tidy numbers (e.g., 5 + 7.5 = 12.5, so 4.95 + 7.5 = 12.45), or drawing on knowledge of place value (e.g., 4 + 7 = 11 and 0.9 + 0.5 = 1.4, so 4.95 + 7.5 = 12.45).

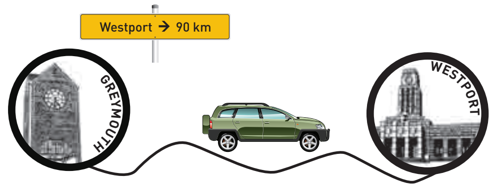
If the student combines place values inappropriately (e.g., 4.95 + 7.5 = 11.100 or 4.95 + 7.5 = 11.145), they do not meet the expectation. If they use a vertical algorithm to solve the problem, they must explain the place value partitioning involved.

Example 3

Show the student the following illustration.



The Smith family are $\frac{1}{3}$ of the way home.



The Hohepa family are $\frac{4}{6}$ of the way home.

The Smith family and the Hohepa family are both driving home from their holidays. Which family has travelled the greatest distance?

The student shows that they understand that the value of a fraction of an amount depends on both the fraction and the amount. They do so by calculating the distance each family has travelled, using multiplication and division (e.g., $\frac{1}{3}$ of 180 = 180 ÷ 3 = 60). If the student recognises that $\frac{4}{6}$ is equivalent to $\frac{2}{3}$, the second calculation is considerably simplified ($\frac{2}{3}$ of 90 = 90 ÷ 3 x 2 = 60).

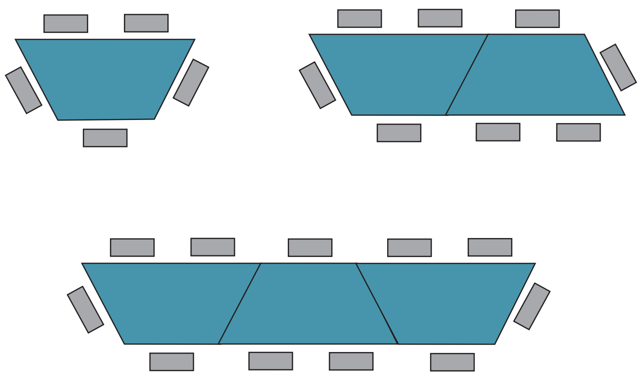
If the student bases their answer on just the amounts (e.g., "The Smiths because 180 is greater than 90") or just the fractions (e.g., "The Hohepas because $\frac{4}{6}$ is greater than $\frac{1}{3}$"), they do not meet the expectation. If they notice and use the doubling and halving relationship ($\frac{1}{3}$ of 180 = $\frac{4}{6}$ of 90 because $\frac{4}{6}$ = 2 x $\frac{1}{3}$), they exceed the expectation.

ILLUSTRATING THE STANDARD

Example 4

Show the student the illustration below.

Funky Furniture sells tables that can be joined together for large meetings. Tables and chairs are set up this way.



If a line of 24 tables is set out like this, how many chairs will be needed? Can you give a rule for the number of chairs needed for any given number of tables?

The student recognises that 3 extra chairs are needed for each extra table. They apply multiplicative thinking to calculate the number of chairs needed for 24 tables (e.g., "21 more tables x 3 = 63 extra chairs, 11 + 63 = 74 chairs altogether" or "5 chairs for table one + 23 tables x 3 = 74 chairs altogether").

The student devises a general rule for any number of tables (e.g., "Multiply the number of tables by 3 and add 2"). If they give an algebraic equation (e.g., "If x = tables and y = chairs, then y = 3x + 2"), they exceed the expectation.

Source: adapted from PAT, Test 4, Item 30, © NZCER 2007.

Geometry and Measurement

N & A G & M S

Example 5

Provide the students with coins and kitchen scales, as required for b) below.

The students at Springfield School made a coin trail using 20-cent coins to raise money for Daffodil Day.

a) The length of the coin trail was 21 000 millimetres. What was its length in centimetres? What was it in metres?

b) Here are 100 twenty-cent coins. Use the kitchen scales to find their combined weight. Using your answer, what would 1000 twenty-cent coins weigh? What would 10 twenty-cent coins weigh?

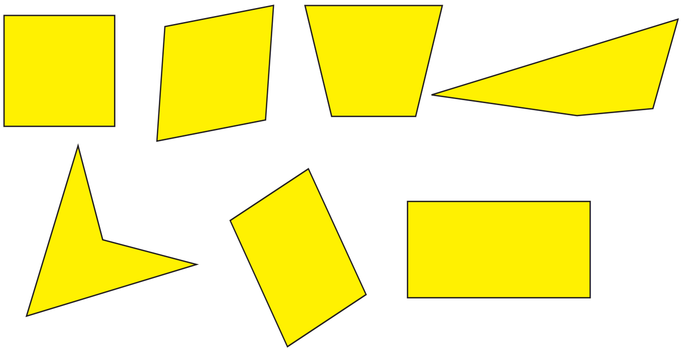
For b), the student reads the scales accurately to give the combined weight as 400 grams. They use their knowledge of place value, metric measures, and multiplicative strategies to correctly answer all other questions – for example, for a), "There are 10 millimetres in a centimetre, so 21 000 mm = 2100 cm; there are 1000 millimetres in a metre, so 21 000 mm = 21 m"; for b), "1000 coins must weigh 10 times 400 grams, which is 4000 grams or 4 kilograms; 10 coins must weigh one-tenth of 400 grams, which is 40 grams."

Source: adapted from "Coin trail" (MS2161) in the Assessment Resource Banks (ARBs) at http://arb.nzcer.org.nz

ILLUSTRATING THE STANDARD

Example 6

Give the student the following collection of shapes.



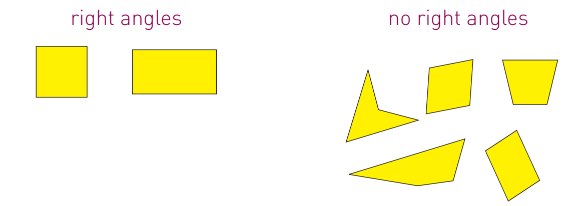
a) What is a common property of all these shapes?

b) Identify a property that some of the shapes have and sort all the shapes into groups by that property.

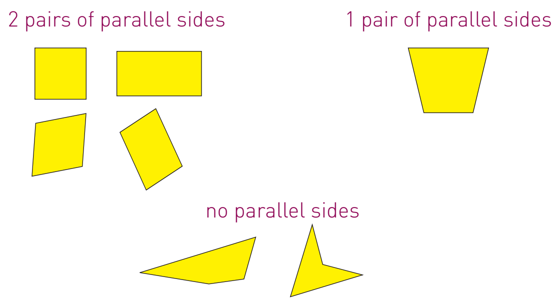
For a), the student identifies at least one property that is common to all the shapes – for example, they all have 4 sides, 4 corners (vertices), or straight sides (that is, they are all polygons).

For b), the student identifies an appropriate property and sorts the shapes into classes by that property – for example, whether each shape has:

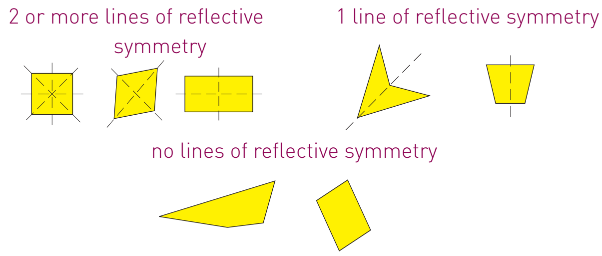
- right angles – shapes with:



- pairs of parallel sides – shapes with:

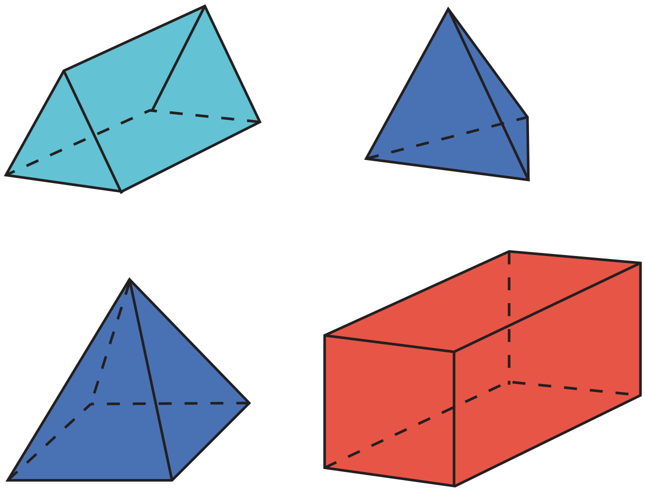


- lines of reflective symmetry – shapes with:



Example 7

Provide square grid paper, a ruler, and a protractor. Show the student the following illustration.



Draw a net for each of these solids. You may try each net by cutting it out and folding it to make the solid. It may take several attempts to get it right.

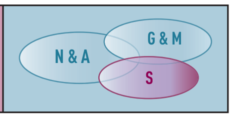
The student creates nets for the four solids by visualising the shape and size of each face and how the faces fit together. They describe the similarities and differences between the solids (e.g., rectangular faces, triangular versus square ends) and use this information to help construct the nets.

The student must precisely measure the dimensions of the faces and orient them so that, when brought together, they form an accurate model of the original solid. It is acceptable to support the student by suggesting that, when drawing a net, they orient the solid's faces on horizontal and vertical axes.

Source: adapted from Figure It Out, *Geometry*, Level 3, page 11.

ILLUSTRATING THE STANDARD

Statistics



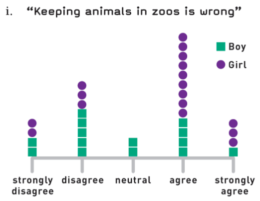
Example 8

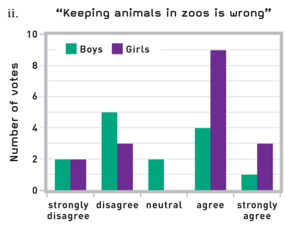
Show the student the illustrations below.

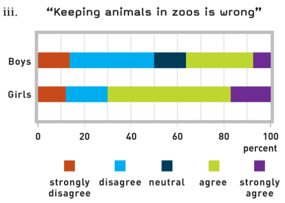
Here are the results from a class opinion poll, recorded on a tally chart and displayed in three different graphs.

"Keeping animals in zoos is wrong"

	Boys	Girls
Strongly disagree	||	|||
Disagree	𝍸	|||
Neutral	||	
Agree	||||	𝍸 ||||
Strongly agree	|	|||

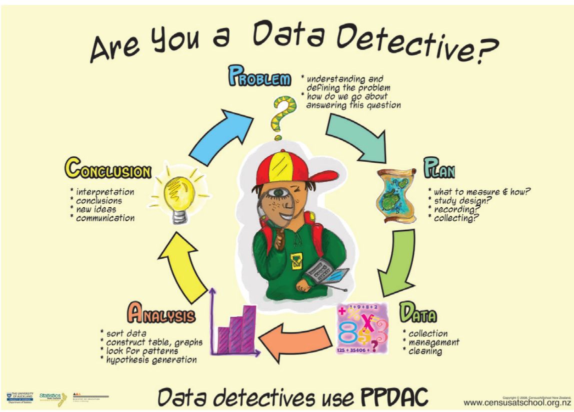






Look at the data gathered in the poll. Suggest some different types of questions that could be answered from the data, for example, summary questions like "How many girls disagree that keeping animals in zoos is wrong?" or comparison questions like "Do more boys or girls agree that keeping animals in zoos is wrong?"

Now write down some "I wonder" questions about people's opinions on topics of interest to you, your friends, or your family. Work with one or two other students to use the statistical enquiry cycle to investigate one or more of your questions.



Make sure your records of your investigation clearly show how you gathered, sorted, and displayed your data and what you interpreted from it.

The student should move through all stages of the enquiry cycle. (Note that at times their findings may require them to go back to previous stages.) They may phrase the problem as a summary question (e.g., "What do people feel about banning fireworks?"), a comparison question (e.g., "Do boys and girls feel differently about banning fireworks?"), or a relationship question (e.g., "Is there a link between people's ages and how they feel about banning fireworks?").

In planning the investigation, the student should consider what data they need to answer the question and how they will collect and manage this data. This may involve finding a way of "measuring" aspects such as people's feelings or attitudes.

Having collected the data, the student should consider how they will sort and display it to provide answers to their question. They should use tables and graphs and may access a computer program to create their displays. Their displays should be appropriate for the type of data, for example, bar or pie charts for category data, stem-and-leaf plots or dot plots for whole-number data, histograms for measurement data, and line graphs for time-series data. The student may use multiple displays to identify patterns and variations in the data.

The student should interpret and report their results in context, using features of their displays to support their findings and acknowledging that different samples might give different results.

Sources: adapted from Figure It Out, *Statistics*, Levels 3–4 (revised edition), page 6. Poster from www.censusatschool.org.nz/resources/poster/

ILLUSTRATING THE STANDARD

Example 9

Show the student a bucket containing 2 red balls and 2 blue balls.

What are all the possible outcomes when you randomly draw 2 balls from the bucket?

What is the probability of getting 2 red balls?

How many times would you expect to get 2 red balls in 60 draws?

Now trial the situation by drawing 2 balls 60 times and recording your results on a tally chart. Then summarise your results on a frequency table, like this:

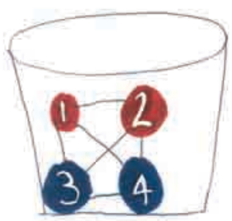
Red-red	Red-blue	Blue-blue
12	41	7

How do your results compare with your prediction of how often you'd draw 2 red balls?

Do the results make you change your prediction?

If you repeated the trial with 60 draws, how many times would you get 2 red balls?

The student creates a model of all the possible outcomes when 2 balls are removed from the bucket. From this, they identify that 2 red balls is one of 6 possible outcomes, and they predict that this outcome should occur about 10 times in 60 draws.



You could get 3-2 or 4-1 or 3-4 or 1-2 or 3-1 or 2-4.

Red 1	Red 2
Red 1	Blue 1
Red 1	Blue 2
Red 2	Blue 2
Red 2	Blue 1
Blue 1	Blue 2

The student accepts that the experimental results are consistent with their prediction despite any variation from 10 red-red occurrences. They predict that in a second trial, 2 red balls will be drawn approximately 10 times. They also recognise that the results are unlikely to be identical to those in the first trial, that is, they accept the variability and independence of samples.

Source: adapted from *The New Zealand Curriculum Exemplars*, Statistics: Probability, Level 3ii: www.tki.org.nz/r/assessment/exemplars/maths/stats_probability/sp_3b_e.php

THE MATHEMATICS STANDARD

By the end of year 8, students will be achieving at level 4 in the mathematics and statistics learning area of the New Zealand Curriculum.

Number and Algebra

In contexts that require them to solve problems or model situations, students will be able to:

- *apply multiplicative strategies flexibly to whole numbers, ratios, and equivalent fractions (including decimals and percentages);*
- *use multiplication and division as inverse operations on whole numbers;*
- *apply additive strategies flexibly to decimals and integers;*
- *find and represent relationships in spatial and number patterns, using:*
  - *tables and graphs*
  - *equations for linear relationships*
  - *recursive rules for non-linear relationships;*
- *apply inverse operations to simple linear relationships.*

Geometry and Measurement

In contexts that require them to solve problems or model situations, students will be able to:

- *use metric and other standard measures;*
- *make simple conversions between units, using decimals;*
- *use side or edge lengths to find the perimeters and areas of rectangles, parallelograms, and triangles and the volumes of cuboids;*
- *sort two- and three-dimensional shapes into classes, considering the relationships between the classes and justifying the decisions made;*
- *identify and describe the features of shapes or patterns that change or do not change under transformation;*
- *create or identify nets for rectangular prisms and other simple solids, given particular requirements;*
- *draw or make objects, given their plan, front, and side views or their perspective views;*
- *describe locations and give directions, using scales, bearings, and co-ordinates.*

Statistics

In contexts that require them to solve problems or model situations, students will be able to:

- *investigate summary, comparison, and relationship questions by using the statistical enquiry cycle:*
  - *gather or access multivariate category, measurement, and time-series data*
  - *sort data and display it in multiple ways, identifying patterns, variations, relationships, and trends and using ideas about middle and spread where appropriate*
  - *interpret results in context, identifying factors that produce uncertainty;*
- *express as fractions the likelihoods of outcomes for situations involving chance, checking for consistency between experimental results and models of all possible outcomes.*

BY THE END OF YEAR 8

ILLUSTRATING THE STANDARD

The following problems and descriptions of student thinking exemplify what is required to meet this standard.

Number and Algebra	N & A / G & M / S
During this school year, Number should be the focus of 40–60 percent of mathematics teaching time.	

Example 1

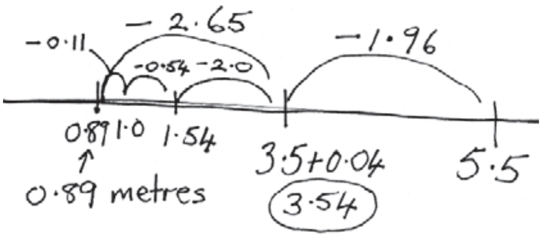
Mani competed in the hop, step, and jump at the athletics sports. Her jump was 2.65 metres, and her step was 1.96 metres. The total of her triple jump was 5.5 metres.

How long was her hop?

The student applies their knowledge of decimal place value to correctly calculate the answer. They use a combination of mental and written strategies, which may include equations, vertical algorithms, or empty number lines.

2·65 + 1·96 =
2·60 + 2·01 =
2·00 + 2·61 = 4·61

5.50 − 4·61 = ·89



89cm

550 − 461
550 − 50 = 500
500 − 411
500 − 400 = 100
100 − 11 = 89

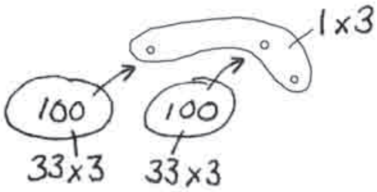
2·65
1 9 6
4·6 1
8 9
5.50

Example 2

Andre has ordered 201 tennis balls. They are sold in cans of 3 balls.

How many cans should he receive?

201÷3
67
3)201
67×3 67
×3
201
67cans



33+33+1 = 67 cans.

67 cans
70 cans = 210 balls
(70x3) − 9(= 3cans)
70 − 3 = 67

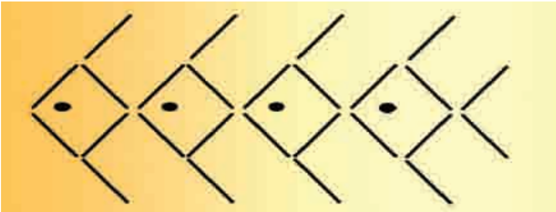
The student gets the correct answer of 67 and, when explaining their strategy, demonstrates understanding of division and place value. Their strategy might involve partitioning numbers into hundreds, tens, and ones, using tidy numbers (e.g., 210) and compensating, or using divisibility rules (e.g., "There are 33 threes in 100 with one left over").

Source: adapted from GloSS, Assessment I, Task 10.

ILLUSTRATING THE STANDARD

Example 3

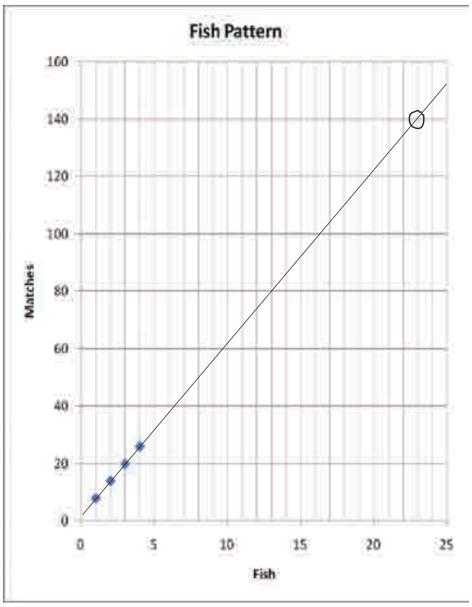
With 26 matchsticks, you can make 4 fish in this pattern.



How many fish can you make with 140 matchsticks?

Write an equation that gives the rule for the number of matchsticks you need for a given number of fish.

The student finds a linear relationship between the number of fish and the number of matchsticks, and they write an equation that expresses that relationship (e.g., $y = 6x + 2$). To solve the problem, they use a graph or apply inverse operations to their rule or equation, for example, "undoing" or "reversing" the "six times the number of fish plus two" rule (140 – 2 = 138, 138 ÷ 6 = 23). If they simply continue a table to solve the problem (1 fish, 8 matches; 2 fish, 14 matches; 3 fish, 20 matches ...), they do not meet the expectation.

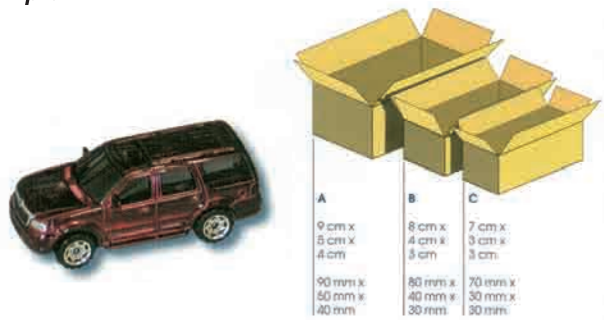


8 matches for fish one
6 matches for each fish after that.
140 − 8 = 132
$6\overline{)132}$ = 22
22 + 1 = 23 fish

Geometry and Measurement

N & A
G & M
S

Example 4



Give the student a ruler, a toy car to measure, and the illustration of boxes shown above.

Use the ruler to measure as accurately as possible how long, how wide, and how high this car is. Give your answer firstly in millimetres and then in centimetres.

That's 75 mm long, 32 mm wide, and 28 mm high.

That's 7.5 cm long, 3.2 cm wide, and 2.8 cm high.

Which of the three boxes would best fit the car?

The car won't fit into box C.
Box B is best because the car fits into it with the least room to spare.

What is the volume of that box?

The box is 8 cm long, 4 cm wide, and 3 cm deep. 8 x 4 = 32, 32 x 3 = 96, so the volume is 96 cubic cm.

Using the ruler, the student accurately measures the length, width, and height of the toy car to the nearest millimetre, and they are able to convert between millimetres and centimetres. They choose the most suitable box – that is, the one with dimensions that exceed the dimensions of the car by the least possible amount.

Source: adapted from NEMP's 2005 report on mathematics, page 40.

ILLUSTRATING THE STANDARD

Example 5

Is there a family that all 3 of these solid shapes belong to? Why?

Is there another family of solid shapes that the Rolo packet could belong to?

The student states that all three solids are prisms. They explain that a prism has a uniform cross-section and that this gives the prism its name (e.g., a "triangular prism").

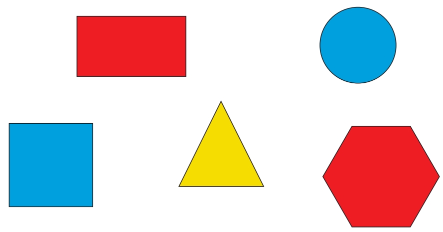
There is debate about the definition of a prism and whether a cylinder is a prism. If the student rejects the cylinder as a prism, explaining that it does not have rectangular faces like other prisms, they still meet the expectation.

In answer to the second question, the student could place the cylinder in the family of curved solids that includes spheres and cones. Any other plausible possibility for an alternative family of solids is also acceptable (e.g., solids with circular faces).

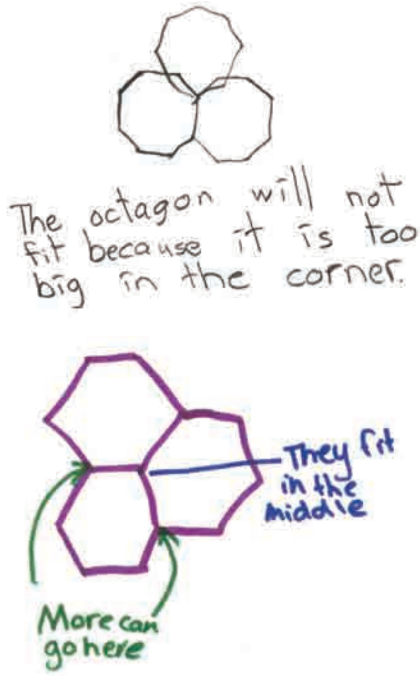
Source: adapted from NEMP's 2005 report on mathematics, page 44.

Example 6

Provide the student with a selection of shapes including squares, rectangles, diamonds, regular hexagons, regular octagons, circles, and equilateral, right-angled, and scalene triangles.



Which of these shapes will tessellate? Why?



The student explains that shapes that tessellate must fit together around a point and that therefore, for a regular shape, its interior angle must divide evenly into 360. For each shape, they refer to an angular measure to justify their conclusion as to whether it will tessellate or not (e.g., "An equilateral triangle tessellates because 6 x 60° = 360°, so 6 triangles will surround a point").

Source: *The New Zealand Curriculum Exemplars,* Geometry: Tessellations, Level 4: www.tki.org.nz/r/assessment/exemplars/maths/geometry/ge_4a_e.php

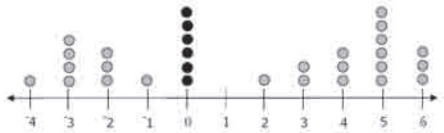
BY THE END OF YEAR 8

ILLUSTRATING THE STANDARD

Statistics

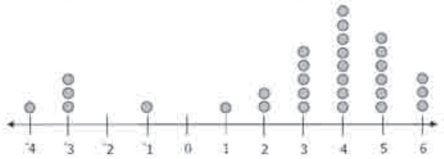
Example 7

Jane's class was doing a unit on healthy eating. Jane wanted to see if the unit would make any difference to her classmates' eating habits, so she developed a scale to measure the healthiness of the lunches they were eating. She applied the scale before and after the unit and created two dot plots to display the results.



Total healthy point scores of lunches **before** unit

● No lunch



Total healthy point scores of lunches **after** unit

Jane concluded that because of the unit, her classmates were now eating healthily. Do you agree? Why or why not?

Yes This class has 30 students. I counted the amount of lunches under 0 and above 0. I halved the numbers on the 0. In the first test there's 15 healthy and 12 unhealthy. 2nd test there's 25 healthy ~~$~~ & 5 unhealthy.

The student uses data from the graphs to support and/or argue against Jane's conclusion. For example, they should identify that more students are now eating healthier lunches and that all students are now bringing or buying a lunch. With prompting, they should be able to identify that although the spread of unhealthy to healthy lunches has not changed, the clustering of lunch scores has shifted to more above zero than below, and therefore the "middle healthiness" has increased.

The student should point out that Jane's conclusion that "her classmates were now eating healthily" is not supported by the data, which shows that a small group of students continue to eat unhealthy lunches. They should also recognise that without additional data (such as a larger sample across different days or information from interviews), the improvement in lunch healthiness is not necessarily due to the class unit. For example, the tuck shop may have changed its menu while the class was doing the unit.

Source: Second Tier Support Material for the revised New Zealand Curriculum.

Example 8

This is a game you might use at the school gala.

Put 2 red balls and 2 blue balls in a bucket. Without looking, a player takes out 2 balls. If the balls are the same colour, they win. If the balls are different, they lose.

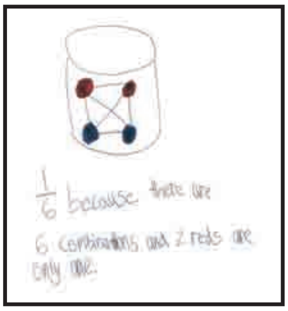
Carry out an experiment by playing 30 games and recording how often the player wins and loses. Draw a diagram to show all the possible outcomes when you draw 2 balls from the bucket. Does this help explain your results? How?

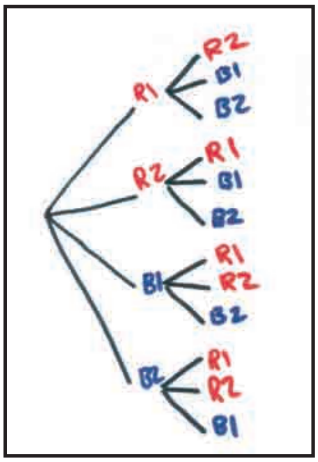
If you played 30 more games, would the results be the same as or different from your first experiment? If they would be different, how?

The student plays 30 games and organises their results systematically, for example, by using a table or tally chart. They notice that there are more losses than wins. (The results will generally be around 10 wins and 20 losses.)

Win: Blue-Blue, Red-Red	Loss: Red-Blue
卌 卌 || 12	卌 卌 卌 ||| 18

The student creates a model of all possible outcomes, for example, a network or tree diagram.





From the model, the student concludes that the chances of winning and losing are one-third and two-thirds respectively. They accept that their results may not exactly reflect these likelihoods (e.g., 12 wins from 30 games is slightly more than one-third).

The student understands that the first experiment does not influence the second. They explain that the results are likely to be around 10 wins and 20 losses but unlikely to be identical to the results from the first experiment – that is, they accept the variability and independence of samples. (In this case, the sample consists of 30 games.)

Source: adapted from *The New Zealand Curriculum Exemplars*, Statistics: Probability, Level 4: www.tki.org.nz/r/assessment/exemplars/maths/stats_probability/sp_4a_e.php

GLOSSARY: USING A SHARED LANGUAGE

All professions use a particular language to convey specific meanings. As educational professionals, school leaders and teachers use certain words and terms with specific and precise meanings to ensure shared understandings. This glossary explains terms used within the mathematics standards and their illustrations.

Additive: a term used to describe a range of problem-solving strategies that use the properties of addition and subtraction to partition (split) and combine numbers

Addition facts: the basic facts of addition; equations in which two single-digit numbers are combined by addition to give a sum. For each basic addition fact (e.g., 7 + 9 = 16), there are one or two related basic subtraction facts (e.g., 16 – 7 = 9 and 16 – 9 = 7).

Algebraic equation: a statement of equality between variables that uses letters, the equals sign, and operation symbols to describe the relationship (e.g., $y = 3x + 2$)

Algorithm: a series of steps that can be followed mechanically to find a solution. There are standard written algorithms used for addition, subtraction, multiplication, and division.

Array: an arrangement or tessellation of objects in rows and columns, for example, an arrangement of unit squares to measure the area of a rectangle. Such an arrangement is a useful representation for illustrating the properties of multiplication and division.

Attribute blocks: plastic blocks that have two sizes, three colours, five geometric shapes, and two thicknesses and that are useful for developing ideas about classification

Axes: the horizontal and vertical reference lines for a geometric figure; the x and y axes of a number plane

Bar graph: a chart that displays the frequency of category or number variables as bars of varying height (but equal width)

Bearings: the direction or position of something relative to a fixed point, usually measured in degrees, clockwise from north

Capacity: fluid volume, that is, how much liquid a container can hold

Category data: non-numeric data (e.g., colours, genders, flavours) that can be organised into distinct groups

Cluster: a grouping or concentration of data points in a display (e.g., of pulse rates between 70 and 90 beats per minute)

Comparison question: a question that compares two or more subsets of data (e.g., male and female) in relation to a common variable (e.g., age)

Compensation: rounding to tidy numbers (e.g., 5, 10, 20, 100) to perform an operation and then compensating for the rounding by adding or subtracting so that an exact answer or a better estimate is obtained

Co-ordinates: the position of a point in relation to a set of axes or grid lines and usually described using ordered pairs (e.g., 3,10)

Counting all: solving simple addition or subtraction problems by counting all of a set of objects from one (the first object)

Counting back: an early subtraction strategy in which students solve problems by counting back by ones (e.g., for 8 – 3, counting back from 8: 7, 6, 5)

Counting on: an early addition strategy in which students solve problems by counting up from a number (e.g., for 5 + 3, counting on from 5: 6, 7, 8)

Cuboid: a rectangular prism, that is, a 3-dimensional solid with six rectangular faces. Note that a cube is a special case of a cuboid.

Data cards: cards used to collect data, usually with several variables to a card. Data cards can be sorted to reveal patterns and differences.

Dimensions: the measures of the length, width, and height of an object

Dot plot: a graph in which each value of a numeric variable is represented by a dot

Doubling and halving: a multiplication strategy in which one number is halved and the other is doubled (e.g., 50 x 40 = 100 x 20). Note that the answer remains unchanged.

Equal dealing: equal sharing (see below) in which an equal number of objects is given to each subset over a number of cycles

Equal sharing: dividing a set of objects into equal subsets

Equilateral triangle: a triangle with all sides equal

Equivalent fractions: fractions that represent the same value (e.g., $\frac{1}{2}$ and $\frac{2}{4}$)

Frequency table: a table that uses numbers to show how many items are in several categories or how often several things occur (e.g., the number of students with black, brown, red, and blonde hair)

General rule: a generalisation that describes how two variables are related and that may be expressed in words or as an equation (e.g., "You get the next number by doubling the first number" or $y = 2x$)

Half-turn symmetry: rotational symmetry of 180 degrees: if you turn the object halfway, it maps onto itself

Histogram: a type of bar graph used to display the distribution of measurement data in equal intervals across a continuous range

Imaging: visualising physical objects and the movement of the objects mentally

Interval: a section of the range of measurement data (e.g., heights grouped into intervals of 150–160 cm, 160–170 cm, and so on)

Inverse relationship: the relationship between two operations in which the effects of one reverse those of the other and vice versa (e.g., multiplication and division)

Isometric view: a 2-dimensional drawing of a 3-dimensional object in which the three axes are at an angle of 120 degrees to each other

Knowledge: information that a person can recall or access without significant mental effort (e.g., basic addition or multiplication facts)

Line graph: a graph in which data points are joined by straight lines, typically used to represent time-series data, with the horizontal axis representing time

Linear relationship: a relationship between two variables in which the value of one variable is always the value of the other multiplied by a fixed number plus or minus a constant (e.g., $y = 2x + 3$). The graph of a linear relationship is a straight line.

Making tens: a mental calculation strategy that uses combinations of numbers that add up to 10 (e.g., 53 + 7 = 60 because 3 + 7 = 10)

Mixed operations: problems that involve more than one arithmetic operation (+, –, x, ÷), for example, 4 x 7 – 3

Multiplication facts: the basic facts of multiplication: the products of single-digit numbers (e.g., 8 x 9 = 72). For each basic multiplication fact, there are one or two related basic division facts (e.g., 72 ÷ 9 = 8 and 72 ÷ 8 = 9).

Multiplicative strategies: a range of problem-solving strategies that use the properties of multiplication and division

Multivariate category data: a data set in which data for two or more variables is collected for each item or person (e.g., gender, eye colour, and favourite television programme)

Net: a 2-dimensional figure that can be cut and folded to form a 3-dimensional object

The Number Framework: a set of progressions, in both number strategies and knowledge, that outline a growth path for students over time

Ordinal position: the place of a number in the whole-number counting sequence from 1 (e.g., ninth, twenty-fifth)

Outcome: the result of a trial in a probability activity or a situation that involves an element of chance

Parallelogram: a 4-sided polygon in which both pairs of opposite sides are parallel

Partitioning: the breaking up of numbers, shapes, or objects into parts (e.g., by place value) to facilitate understanding, calculation, or problem solving

Part–whole strategy: a strategy for solving difficult problems that involves partitioning (splitting) numbers into manageable components and then recombining them to get a final result (e.g., 38 + 46 might be calculated as (30+ 40) + (8 + 6))

Perimeter: the distance around the outside of a shape

Perspective view: a 2-dimensional drawing that gives a 3-dimensional view of an object

Pictograph: a graph of category or whole-number data in which pictures or symbols represent the number in each category or grouping

Pie chart: a graph that represents category data by splitting a circle into sectors, the size of each sector being proportional to the frequency of the data it represents

Place value: the value of a digit in a number, as determined by its place (e.g., the 4 in 47 is in the tens place, giving it a total value of 4 x 10 or 40)

Place value partitioning: splitting numbers by the place values of the digits that compose them to perform calculations (e.g., 234 – 70 can be thought of as 23 tens minus 7 tens plus 4)

Plan view: a 2-dimensional drawing (projection) of an object that shows how it would look from above

Polygon: a closed 2-dimensional figure with three or more straight sides

Prism: a 3-dimensional solid that has a regular cross-section (e.g., a triangular prism has a triangular cross section)

Probability: the likely occurrence of an event (a particular outcome), which can be represented by a number from 0 (impossible) to 1 (certain)

Projections: 2-dimensional drawings of a 3-dimensional object that show how the object looks from different directions, for example, from the top (the plan view), the front, or the side

Proportional thinking: using equivalent fractions to solve problems that involve fractions, percentages, decimals, or ratios

Rectangular prism: a solid that has a regular rectangular cross-section

Recursive rule: a rule that describes the connection between a term and the next term in a number sequence (e.g., + 4 for the sequence 6, 10, 14, …)

Reflective symmetry: the regularity of a pattern or shape in which one half of the figure is a mirror image of the other half

Relationship question: a question about the link between two variables in a set of data

Right-angled triangle: a triangle with one interior angle of 90 degrees

Rotation: a turn about a point

Rotational symmetry: the regularity of a pattern or shape that allows the figure to be mapped onto itself by a less than 360 degree turn about a point

Scalene triangle: a triangle that has no equal sides

Skip-counting: counting in amounts other than ones (e.g., 2s, 5s, or 10s)

Solids: 3-dimensional objects

Spatial patterns: patterns in which figures or objects are arranged in consistent ways in relation to position or distance

Spread: the difference between the lowest and highest values in a data set. Spread may be measured by calculating the range.

Standard units: units of measurement that are universally accepted, such as the units of the metric system, as opposed to informal, non-standard units, such as hand spans

Statistical enquiry cycle: an investigative cycle for data-based enquiry

Stem-and-leaf graph: a graph used to display discrete number data for the purpose of a summary (a single graph) or a comparison (a back-to-back graph). The "stem" is created from a common place value in the numbers (e.g., tens), and the "leaves" represent the remaining place value, typically, ones.

Strategy: a way of working out the answer to a problem

Subtraction facts: the basic facts of subtraction: equations in which two numbers are combined by subtraction. Subtraction facts are related to the addition of single-digit numbers (e.g., 8 + 6 = 14 leads to the subtraction facts 14 – 6 = 8 and 14 – 8 = 6).

Summary question: an investigative question about the distribution of a single variable (e.g., "How long does it typically take a year 6 student to run 100 metres?")

Symmetry: in geometry, the regularity of a figure or object that maps onto itself by reflection or rotation. Using symmetry is important in Number, for example, when sharing by equal dealing or when finding doubles and halves by equal partitioning (e.g., 4 + 4 is a symmetrical partitioning of 8).

Tally chart: a method of recording frequency of data using stroke marks (e.g., 𝍸)

Tessellate: cover a surface with repeats of the same shape without gaps or overlaps

Tidy numbers: numbers that are easy to work with for a given calculation (e.g., 5 instead of 4.95 or 20 instead of 19)

Time-series data: a set of data gathered over time to investigate time-related trends

Transformation: the mapping in space of every point in a figure or object onto a new location. Common transformations are reflection, rotation, translation, and enlargement.

Translation: a transformation that moves every point in a figure a given distance in the same direction

Trial: one instance of carrying out an experiment (e.g., flipping a coin ten times)

Unit conversion: changing from one unit of measurement to another unit for the same attribute so that both represent the same quantity (e.g., from 1.5 litres (L) to 1500 millilitres (mL))

Variable: a numerical value that can vary, for example, across data or in describing a mathematical relationship (e.g., in the equation $y = 4x$, x and y are variables)

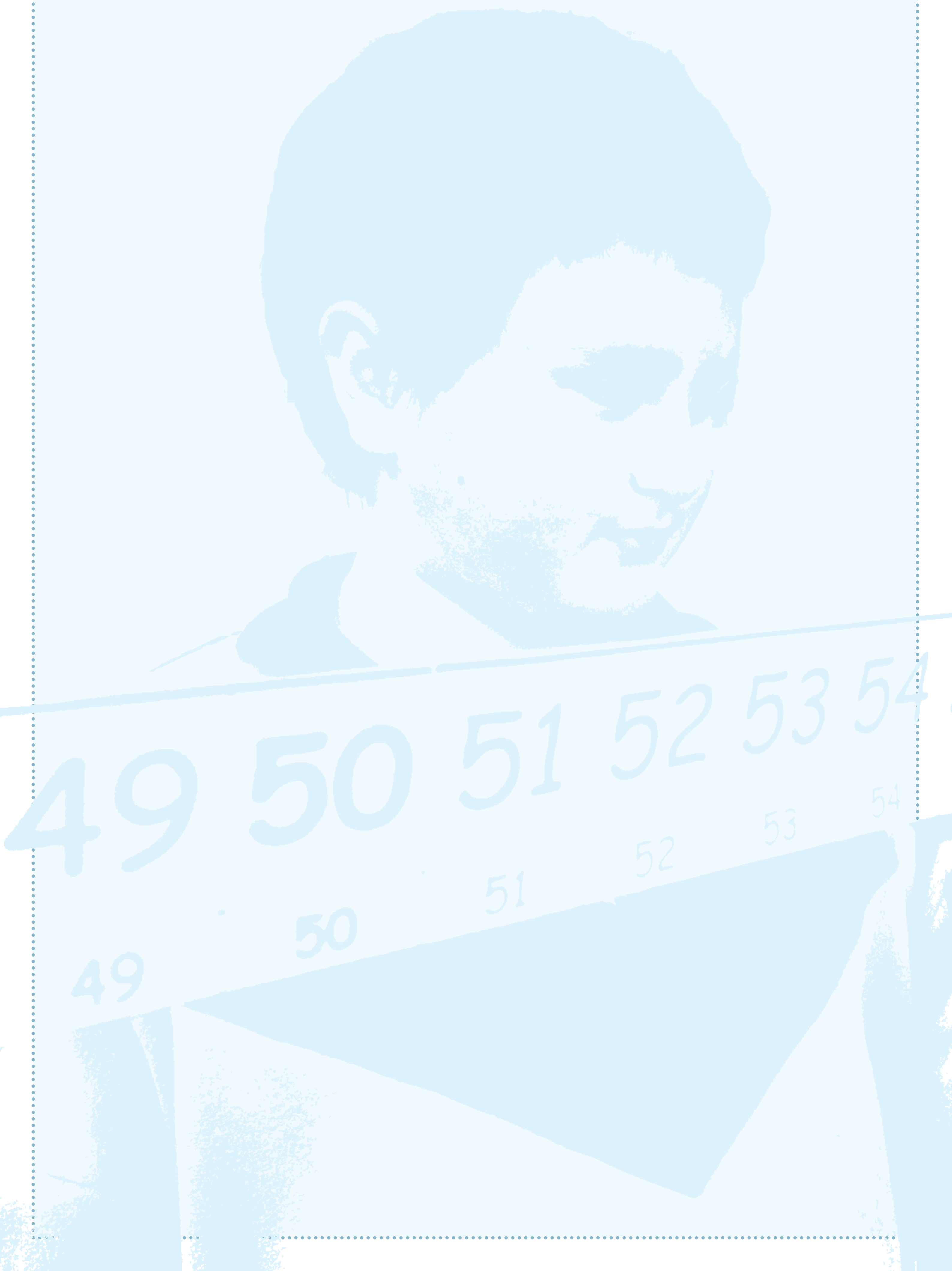
Vertical algorithm: a standard written working form that organises calculations into columns to make use of place value

Volume: the amount of space occupied by a 3-dimensional object

Weight: the measure of the heaviness of an object (in scientific terms, the force of gravity acting on a mass)

Whole numbers: all numbers in the set that includes zero and the counting numbers (e.g., 0, 1, 2, 3 ...)

Written algorithm: *see* algorithm; the written recording of a series of steps for calculating an answer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49 50 51 52 53 54

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