

MIMO Systems Performance Simulator

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Abstract— Multi Input Multi Output (MIMO) has emerged as a hot topic in wireless communications. This is due to possible dramatic increase in reliability and capacity as compared to single antenna solution. In this project we are developing a MIMO System Performance Simulation (MSPS) tool by using MATLAB for making more realistic studies of MIMO systems. By using this simulator one can see the Bit Error Rate (BER) performance of the system and Minimum Mean Square Error of LMMSE estimator according to given Pilot to Data Power Ratio (P DPR), pilot schemes and based on two types of channel model correlated and non-correlated.

Index Terms— MIMO, Channel estimation, Symbol detection, simulator.

I. INTRODUCTION

MIMO is the use of multiple antennas at both transmitter and receiver to improve communication performance. The main reason of using MIMO technology in wireless communications is that it offers significant increase in data throughput without additional bandwidth or increased transmit power. MIMO exploits the space dimension to improve wireless systems capacity and reliability. MIMO systems offer a promising solution for future generation wireless networks [1]. MIMO channel high data rates are well exploited if the Bit Error Rate (BER) is very low. BER is related to both symbol detection and channel estimation. BER decreases when symbol detection is correctly performed. But the MIMO channel estimation is a requirement for the equalization at the receiver. There are three types of channel estimation techniques; Pilot aided channel estimation, Blind channel estimation, Semi-blind channel estimation. The first one is based on the transmission of known symbols by both transmitters and receivers. These symbols are called pilot symbols [2]. The second one estimates the channel only by using the statistics of the received signal and no pilot symbols are used. The third one is a mix of the first and third one. There are many researches about how many antennas should be used at both transmitter and receiver and which type of estimator should be used at the receiver side [3], [4]. Sometimes the telecommunication systems designers and students need to know how many transmitter and receiver antennas should exist in MIMO system to satisfy a high efficiency and enhance its performance. They may need to know the effect of the pilot-to-data power on the BER. Also, they need to compare some system performance and study the effect of PDPR.

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Two MIMO system simulation tools were developed earlier. One of them the “LabVIEW MIMO-OFDM simulator with variable pilot-to-data power ratio”, in [3], which provides a labview based platform allowing the user to test different MIMO systems BER performance given some control inputs such as number of input and output antennae, pilot symbol model, PDPR, channel length and modulation type. Also MSIM simulator proposed in [4] which give the ability to compare MIMO decoding algorithms complexity. We developed a MIMO system simulator by using MATLAB that aims to provide a tool for MIMO system to evaluate the effect of the pilot-to-data power on the BER. In section II, we present the MIMO system model. In section III, we present some MIMO channel estimation methods for both uncorrelated and correlated channel. Next, we present a symbol detection method and finally we describe our simulation tool and we give some MIMO systems performance comparisons using MSPS.

II. MIMO CHANNEL MODEL

In MIMO systems transmitters send multiple streams by multiple transmit antennas. The transmit streams go through a matrix channel which consists of all $N_t N_r$ paths between the N_t transmit antennas at the transmitter and the N_r receive antennas at the receiver. Then, the receiver gets the received signal vectors by the multiple receive antennas and decodes the received signal vectors into the original information. Flat fading MIMO systems are modeled as:

$$\mathbf{y}(t) = \mathbf{H} \mathbf{s}(t) + \mathbf{n}(t)$$

where $\mathbf{y}(t)$ is the $N_r \times 1$ receive vector at time instant t , \mathbf{H} is the $N_r \times N_t$ channel matrix modeled as

$$\mathbf{H} = \begin{bmatrix} h_{11} & \dots & h_{1N_t} \\ \vdots & \ddots & \vdots \\ h_{N_r 1} & \dots & h_{N_r N_t} \end{bmatrix}$$

$\mathbf{s}(t)$ is the $N_t \times 1$ transmit vector $[\mathbf{s}_1(t) \dots \mathbf{s}_{N_t}(t)]^T$ and $\mathbf{n}(t)$ is the $N_r \times 1$ Additive White Gaussian Noise (AWGN) vector at a given instant time. When N samples are received, the received samples can be modeled by:

$$\mathbf{y} = \mathcal{F}(\mathbf{H}) \mathbf{s} + \mathbf{n} = \mathbf{S} \mathbf{h} + \mathbf{n} \quad (1)$$

where $\mathbf{y} = [\mathbf{y}(1)^T \dots \mathbf{y}(N)^T]^T$ is the vector of N samples received at N_r receive antenna, $\mathbf{s} = [\mathbf{s}(1)^T \dots \mathbf{s}(N)^T]^T$ is the vector of transmitted symbols from N_t transmit antenna and $\mathbf{n} = [\mathbf{n}(1)^T \dots \mathbf{n}(N)^T]^T$ is the vector of noise at N_r receive antenna in N samples. $\mathcal{F}(\mathbf{H})$ is a block diagonal matrix

$$\begin{bmatrix} \mathbf{H} & & \\ & \ddots & \\ & & \mathbf{H} \end{bmatrix}$$

also $\mathbf{h} = \text{vec}(\mathbf{H}) = [h_{11} \dots h_{N_r 1} \dots h_{1N_t} \dots h_{N_r N_t}]^T$ and

$$\mathbf{s} = \begin{bmatrix} \mathbf{s}(1)^T \otimes \mathbf{I}_{N_r} \\ \vdots \\ \mathbf{s}(N)^T \otimes \mathbf{I}_{N_r} \end{bmatrix}$$

III. MIMO CHANNEL ESTIMATION

One of the design objectives of receivers is the minimization of bit error detection rate [5]. And one of the most popular and widely used approaches to the MIMO channel estimation is the use of pilot symbols (also referred to as training sequences) where the channel is estimated based on the received data and the knowledge of training symbols [2]. Known symbols can be time-multiplexed with data symbols (see Figure 1: Time Multiplexed Pilot and Data) or embedded (see Figure 2: Embedded Pilot) to them. In this section, we consider two types of channel estimators.

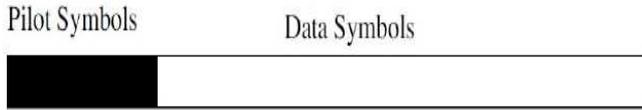


Figure 1: Time Multiplexed Pilot and Data

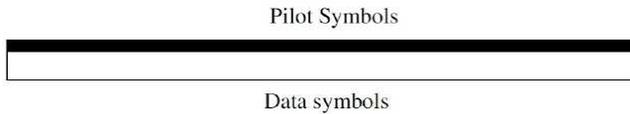


Figure 2: Embedded Pilot

When pilot symbols are used, the MIMO channel model in (1) could be written as:

$$\mathbf{y} = \mathcal{T}(\mathbf{H})(\mathbf{s}_p + \mathbf{s}_d) + \mathbf{n} = (\mathbf{S}_p + \mathbf{S}_d)\mathbf{h} + \mathbf{n}$$

Data symbols are i.i.d with mean zero and variance σ_d^2 and pilot symbols known symbols with allocated power σ_p^2 .

A. Least Squares Estimator [6]:

Least Square Estimation (LSE). ss based on the minimization of the cost function which is the following:

$$L(\mathbf{y}; \mathbf{h}) = (\mathbf{y} - \mathbf{S}_p \mathbf{h})^H (\mathbf{y} - \mathbf{S}_p \mathbf{h})$$

The minimization of the last function gives the following least square estimator

$$\hat{\mathbf{h}} = (\mathbf{S}_p^H \mathbf{S}_p)^{-1} \mathbf{S}_p^H \mathbf{y} \quad (2)$$

In the training based estimation, \mathbf{S}_p represents the Pilot symbols matrix. $(\mathbf{S}_p^H \mathbf{S}_p)^{-1}$ is called a pseudo inverse matrix. This matrix exist only if \mathbf{S}_p is full column rank. The main property of LSE is its boarder range of application because of its simple implementation.

B. Linear Minimum Mean Square Error Estimator [6]:

To develop the Linear Minimum Mean Square (LMMSE) estimator for the channel estimation, a set of coefficients which minimizes the mean square error (MSE), should be computed. The cost function of MMSE is given by:

$$E[\|\hat{\mathbf{h}} - \mathbf{h}\|^2]$$

This computation takes its name from the linear representation of the estimator as a linear function of \mathbf{y} :

$$\hat{\mathbf{h}} = \mathbf{W} \mathbf{y}$$

The minimization of the cost function under the linearity constraint of the estimator with respect to the received signal gives the following estimator expression:

$$\hat{\mathbf{h}} = \mathbf{W} \mathbf{y} = (\mathbf{S}_p^H \mathbf{S}_p + \sigma_n^2 \mathbf{R}_h^{-1})^{-1} \mathbf{S}_p^H \mathbf{y} \quad (3)$$

Where \mathbf{R}_h is the channel auto-correlation function which could be also unknown. This is why some assumptions should be added.

1) Non-correlated channel [7]:

If the channel is non-correlated equation (3) becomes:

$$\hat{\mathbf{h}} = (\mathbf{S}_p^H \mathbf{S}_p + \sigma_n^2 \mathbf{I})^{-1} \mathbf{S}_p^H \mathbf{y}$$

Where \mathbf{R}_h is replaced by \mathbf{I} and $\sigma_n^2 = 1$.

2) Correlated channel [7]:

The channel estimation will be done in three steps :

- a) Estimate the channel by LS.
- b) Find the estimate of the channel auto-correlation matrix.

$$\mathbf{R}_h = \frac{\sum_{l=1}^Q \hat{\mathbf{h}}_l \hat{\mathbf{h}}_l^H}{Q}$$

Where Q is the number of channel estimator.

- c) Re-estimate the channel by LMMSE given the found auto-correlation matrix (see equation (3)).

IV. SYMBOL DETECTION

Detection theory provides a set of rules for a decision-making method, which is used to observe the received signals and predict the transmitted signals. The results of detection are subject to transmission errors. However, the goal of the detection theory is to deal with transmission error such that an acceptable quality of performance can be obtained at the receiver, thereby leading to an optimum receiver [5]. Minimum Mean Square Error (MMSE) detector estimates the transmitted vector \mathbf{s} by applying the linear transformation to the received vector \mathbf{y} . It finds out the estimate $\hat{\mathbf{s}}$ of the transmitted symbol vector \mathbf{s} as:

$$\hat{\mathbf{s}} = \mathbf{Q}_z(\mathbf{s}_{LMMSE}) = (\mathcal{T}(\mathbf{H})^H \mathcal{T}(\mathbf{H}) + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I})^{-1} \mathcal{T}(\mathbf{H})^H \mathbf{y}$$

MMSE detectors balance the noise enhancement and multi-stream interference by minimizing the total error.

V. MIMO SIMULATOR DESIGN

We designed MSPS as two subsystems which are channel estimation subsystem which estimates the channel using LMMSE estimator to get the channel taps estimates and LS estimator to estimate the channel correlation matrix in case the channel is correlated. The second stage is responsible of symbol detection.

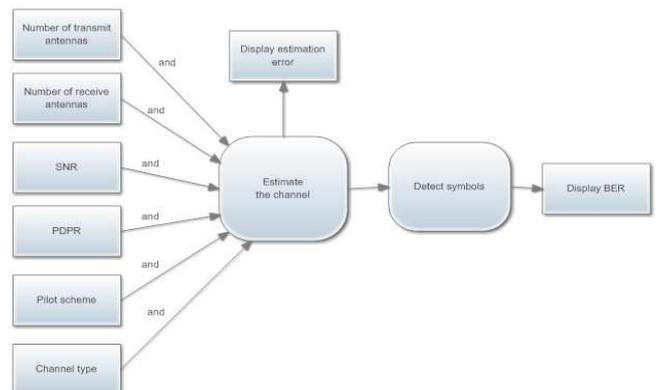


Figure 3: MSPS Design

Moreover, we designed one interface for the MSPS which is composed of two panels; one control panel receiving the user inputs and a display panel showing both channel estimation error as function of SNR and BER as function of SNR.

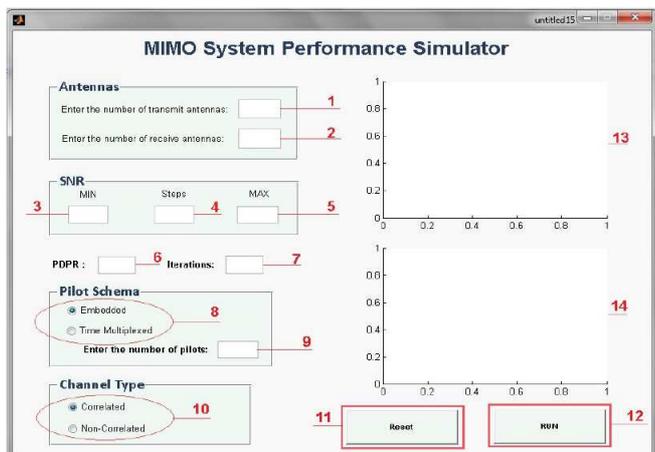


Figure 4: MSPS User Interface

More details about the system interface are in table

| No | Description | No. | Description |
|----|--------------------------------------|-----|---|
| 1 | Enter number of transmit antennas. | 8 | Choose one of pilot schema. |
| 2 | enter number of receive antennas. | 9 | enter number of pilots if MX. |
| 3 | enter SNR lower bound value. | 10 | Correlated or non-correlated channel |
| 4 | enter SNR simulation steps. | 11 | Reset button to reset all fields to be empty. |
| 5 | enter SNR upper bound value. | 12 | Run button |
| 6 | enter pilot to data power ratio. | 13 | Figure for channel estimation error. |
| 7 | enter number of simulation iteration | 14 | Figure for BER. |

Table 1: MSPS Interface Description

VI. MSPS TESTING

We implemented MSPS with Matlab for the facilities it provides for signal processing algorithms.

A. Exceptions Handling

We tested that it is able to provide error message whenever the user input is not correct or not complete Figure 5 shows the system output when a field is not provided.

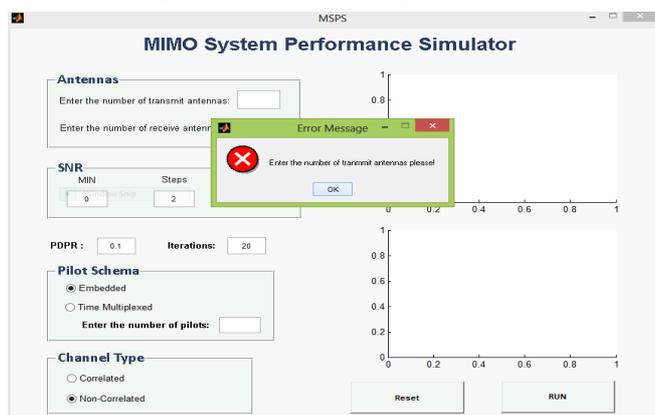


Figure 5: MSPS Exceptions Handling

B. Channel Estimation Error and BER Function of SNR

We also tested that our system is given correct and coherent results regarding channel estimation error and BER. In instance, the channel estimation error and BER are decreasing when SNR is increasing. In Figure 6, we simulate a 2X4 MIMO system with Pilot to Data Power Ratio PDPR=0.1 and 20 pilot symbols in a time multiplexed scheme. We notice that both channel estimation error and BER are decreasing when SNR is increasing.

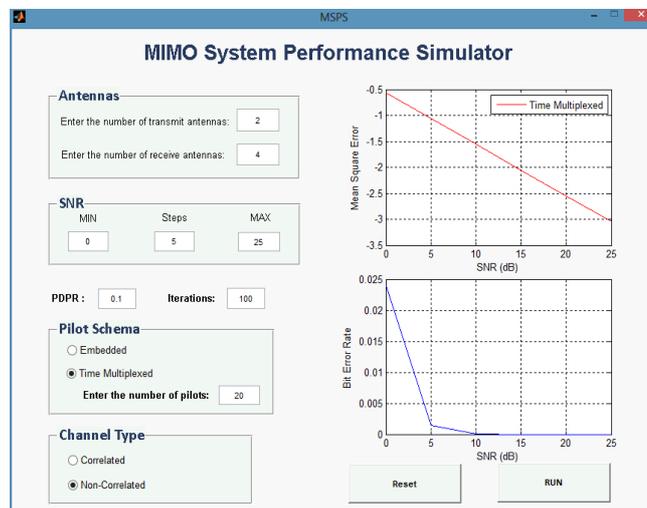


Figure 6: 2X4 MIMO with Time Multiplexed Pilot

C. Comparison of Multiplexed and Embedded Schemes

In Table 2 to Table 7 we tested different MIMO systems performances with both multiplexed and embedded pilot scheme. We made our simulations for both 2X2 and 2X4 MIMO systems and for two different PDPR. First, for PDPR=0.1 the channel MSE for the multiplexed scheme is better than that of the embedded scheme. However, the channel MSE for the embedded scheme is very close to the multiplexed scheme with P=10 at high SNR. Also, multiplexed scheme gives better BER. Second, when PDPR=0.5, the embedded scheme and the multiplexed scheme where P=10 give almost same channel MSE and very closed BER at low SNR. Whereas BER of the multiplexed scheme outperforms that of the embedded scheme at high SNR mainly the multiplexed scheme of P=20. This may be due to the fact that in the embedded scheme data symbols are received simultaneously with pilot and this will make data symbol detection exposed to more errors. But even though embedded scheme gives higher BER the system throughput of the embedded scheme is better than that of the multiplexed. Also, we noticed that the 2x4 case gives worst channel the best BER and this due to MIMO space diversity which is increased when receive antenna number increased. Also, increasing the pilot symbols number for the multiplexed scheme decreases the BER mainly at high SNR.

| PDPR | Number of antenna | SNR=0d B | SNR=10d B | SNR=20d B |
|----------|--------------------|----------|-----------|-----------|
| PDPR=0.1 | $N_t=2$ $N_r=2$ | -0.435 | -1.4 | -2.394 |
| | $N_t=2$ $N_r=4$ | -0.135 | -1.1 | -2.1 |
| PDPR=0.5 | $N_t=2$ $N_r=2$ | -1.1 | -2.1 | -3.1 |
| | $N_t=2$ $N_r=4$ | -0.8 | -1.8 | -2.79 |

Table 2: Channel Estimation Error in Embedded Scheme

| PDPR | Number of antenna | SNR=0d B | SNR=10d B | SNR=20d B |
|----------|--------------------|----------|-----------|-------------|
| PDPR=0.1 | $N_t=2$ $N_r=2$ | 0.135 | 0.0323 | 0.014 |
| | $N_t=2$ $N_r=4$ | 0.067 | 0.0126 | 0.0122 |
| PDPR=0.5 | $N_t=2$ $N_r=2$ | 0.092 | 0.0174 | 0.01 |
| | $N_t=2$ $N_r=4$ | 0.023 | 0.001 | $5*10^{-4}$ |

Table 3: BER in Embedded Scheme

| PDPR | Number of Antenna | SNR=0d B | SNR=10d B | SNR=20d B |
|----------|--------------------|----------|-----------|-----------|
| PDPR=0.1 | $N_t=2$ $N_r=2$ | -0.667 | -1.574 | -2.2 |
| | $N_t=2$ $N_r=4$ | -0.36 | -1.2 | -2.35 |
| PDPR=0.5 | $N_t=2$ $N_r=2$ | -1.1 | -2.12 | -3.1 |
| | $N_t=2$ $N_r=4$ | -0.8332 | -1.6 | -2.8 |

Table 4: Channel Estimation Error in Multiplexed Scheme (P=10)

| PDPR | Number of Antenna | SNR=0d B | SNR=10d B | SNR=20d B |
|----------|--------------------|----------|-----------|-----------|
| PDPR=0.1 | $N_t=2$ $N_r=2$ | -0.865 | -1.85 | -2.85 |
| | $N_t=2$ $N_r=4$ | -0.566 | -1.55 | -2.5 |
| PDPR=0.5 | $N_t=2$ $N_r=2$ | -1.226 | -2.2 | -3.22 |
| | $N_t=2$ $N_r=4$ | -0.928 | -1.9 | -2.916 |

Table 5: Channel MSE in Multiplexed Scheme (P=20)

| PDPR | Number of Antenna | SNR=0d B | SNR=10d B | SNR=20d B |
|----------|--------------------|----------|---------------|----------------|
| PDPR=0.1 | $N_t=2$ $N_r=2$ | 0.11 | 0.0133 | 0.001 |
| | $N_t=2$ $N_r=4$ | 0.028 | $1.3*10^{-3}$ | $0.1*10^{-3}$ |
| PDPR=0.5 | $N_t=2$ $N_r=2$ | 0.0965 | 0.0145 | $1.63*10^{-3}$ |
| | $N_t=2$ $N_r=4$ | 0.02 | 0.0012 | $0.3*10^{-3}$ |

Table 6: BER in Multiplexed Scheme (P=10)

| PDPR | Number of Antenna | SNR=0d B | SNR=10d B | SNR=20d B |
|----------|--------------------|----------|-------------|---------------|
| PDPR=0.1 | $N_t=2$ $N_r=2$ | 0.1 | 0.015 | 0.0028 |
| | $N_t=2$ $N_r=4$ | 0.025 | $2*10^{-4}$ | $0.5*10^{-5}$ |
| PDPR=0.5 | $N_t=2$ $N_r=2$ | 0.095 | 0.0123 | $1.6*10^{-3}$ |
| | $N_t=2$ $N_r=4$ | 0.02 | $2*10^{-4}$ | $2.5*10^{-7}$ |

Table 7: BER in Multiplexed Scheme (P=20)

D. Comparison of Correlated and Non-Correlated Channel

In this part we compared correlated MIMO channel results to that of the non-correlated one. In Table 8 and Table 9, we noticed that the correlation deteriorates both embedded and time-multiplexed schemes performances.

| | Number of antenna | SNR=0d B | SNR=10d B | SNR=20d B |
|--------------|--------------------|----------|-----------|-----------|
| Channel MMSE | $N_t=2$ $N_r=2$ | 0.3 | -0.435 | -1.4 |
| | $N_t=2$ $N_r=4$ | 0.61 | -0.135 | -1.1 |
| BER | $N_t=2$ $N_r=2$ | 0.2368 | 0.15 | 0.0765 |
| | $N_t=2$ $N_r=4$ | 0.1367 | 0.052 | 0.017 |

Table 8: MMSE and BER for Correlated MIMO Channel with Embedded Pilot (PDPR=0.1)

| PDPR | Number of Antenna | SNR=0d B | SNR=10d B | SNR=20d B |
|--------------|--------------------|----------|-----------|----------------|
| Channel MMSE | $N_t=2$ $N_r=2$ | -0.87 | -1.85 | -2.85 |
| | $N_t=2$ $N_r=4$ | -0.56 | -1.5 | -2.55 |
| BER | $N_t=2$ $N_r=2$ | 0.19 | 0.105 | 0.06 |
| | $N_t=2$ $N_r=4$ | 0.0715 | 0.03 | $9.27*10^{-3}$ |

Table 9: MMSE and BER for Correlated MIMO Channel with Time-Multiplexed Pilot (PDPR=0.1 and P=20)

VII. CONCLUSION

MIMO system is a promising technology in wireless communications. In this work, we implemented MIMO system performance simulation (MSPS) tool with two stages: the first stage role is to estimate channel by using LMMSE estimator and the second stage detects symbols by using MMSE detector. We developed an interface by using MATLAB to show different results of estimation and detection errors by changing some inputs like number of antennas, SNR and PDPR. We noticed that when PDPR increases MMSE and BER decrease for both pilot time multiplexed scheme and embedded scheme. Also, multiplexed scheme gives better performance than that of embedded mainly when the number of pilot symbols is increased. But we should point that increasing number of pilots decreases the system throughput. We noticed also that as the number of receive antennas increases BER decreases

for both multiplexed and embedded and this is due to the spatial diversity benefit provided by MIMO systems. In the future, we propose to add other estimation and detection techniques, other channel modes and other symbol modulations and encoding techniques.

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