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Unfalsified Control: A Behavioral Approach to Learning and Adaptation[1]

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Abstract

Unfalsified control theory facilitates the representation of adaptive processes of control law discovery from *evolving* information flows and noisy data. In this paper, the theory of unfalsified adaptive control is examined from the behavioral perspective of Willems. An abstract, but parsimonious, min-max optimization problem formulation is developed that descibes and unifies direct adaptive control, learning theory and system identification problems in a common behavioral setting based on the concept of controller/model unfalsification. Thus, adaptive control is seen to be firmly and directly linked to, and to conceptually unified with, the growing body of knowledge on behavioral approaches to model validation and unfalsified system identification. The results elucidate and underscore the fertile conceptual links that exist between adaptive control theory and the rich theory of system identification.

1 Introduction

A fundamental activity in processes of system identification and, more generally, in all processes of scientific discovery is the fitting of models to data. Goals or cost functions are set based on beliefs about instrument accuracy or other model performance criteria, then and scientists attempt experimental validation, or unfalsification, of data against various parameterized classes of plausible models in the hope the one or more of the hypothesized plausible models proves have a superior fit to the data, relative to the specified modeling goals or cost functions. The challenges faced by the to the system identification specialist and the experimental scientist are the same. Moreover, these challenges are not dissimilar to those faced by adaptive control designers seeking to find a faithful mathematical representation of the control-decision-relevant information in evolving observational data.

One interesting development in recent years has been the advent of the unfalsified control paradigm [1]-[14] which has advanced the model validation/unfalsification paradigm of system identification theory to the control validation paradigm for understanding and analyzing adaptive control algorithms. In adaptive control and system identification, as in other scientific endeavors, a parsimonious mathematical representation of the essential issues is preferred. For adaptive control, one such paradigm is provided by unfalsified control theory. The unfalsified control theory views the control problem as an identification problem in which the objective is that of directly identifying a control law or "action rule" that is consistent with traditional control performance goals, prior knowledge, and evolving observational data.

Unfalsified adaptive control software has been developed and design studies have been conducted. For example, the theory has recently been applied to the design of a robust adaptive controller for a two-link robot manipulator arm in [2, 13] and to the design of an adaptive missile autopilot [6]. Unfalsified control theory provided the basis for a general purpose algorithm for automatic tuning of PID controller gains [15]. Recently, the unfalsified control approach has been applied in experimental settings ranging from the ACC benchmark control problem [8] to industrial process control [10, 11]. These design studies seem to confirm theoretical expectations that adaptive controllers optimally designed via unfalsified control theory exhibit a precise, sure-footed response in the face of evolving uncertainties and parameter variations.

In this paper, we formulate the unfalsified adaptive control problem in the behavioral framework of Willems ([16],[17]). Our results are closely related to, but different from, the indirect adaptive method of Polderman [18] in which an unfalsified plant model is identified from within a prescribed model set. Unlike Polderman, we bypass the intermediate step of plant model identification and, also unlike Polderman, we make no

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assumptions about the 'true plant' lying in an assumed model set. We make no assumptions on the plant.

2 Background: Behavioral Theory

At the heart of the behavioral theory of Willems [16, 17] is the definition of a mathematical model. This definition is formulated according to the black box point of view, "in which we focus on how a system behaves, on the way it interacts with its environment, instead of trying to understand, in the tradition of physics, how it is put together and how its components work" ([16],[17]) . This definition of a mathematical model formalizes the black box point of view. Like Willems, we back off "from the usual input/output setting, from the processor point of view, in which systems are seen as influenced by inputs, acting as causes, and producing outputs through these inputs, the internal conditions, and the system dynamics."

Willems begins with the assumption that there is a phenomenon to be modeled. He then "casts the situation in the language of mathematics by assuming that the phenomenon produces elements in a set $\mathbf{Z}$" ([16],[17]), called the *universum.* The elements of $\mathbf{Z}$ are called the outcomes of the phenomenon. "A (deterministic) mathematical model for the phenomenon (viewed purely from the behavioral, the black box point of view) claims that certain outcomes are possible, while others are not. Hence a model recognizes a certain subset $\mathcal{B}$ of $\mathbf{Z}$. This subset will be called the behavior (of the model)." Formally,

Definition 1 *A mathematical model is a pair* $(\mathbf{Z}, \mathcal{B})$*, with* $\mathbf{Z}$ *the universum — its elements are called outcomes — and* $\mathcal{B} \subseteq \mathbf{Z}$ *the behavior.*

Definition 2 *A controller is a mathematical model.*

Regarding data and measurements, Willems [16] says: "We will now cast measurements in this setting. We will assume that we make certain measurements which we will call the data." "... we ... assume that the data consists of observed realizations of the phenomenon itself. Thus, a data set will be a nonempty subset $\mathcal{D}$ of $\mathbf{Z}$. "

Following Safonov and Tsao [1], we will work with data information that can evolve with time. Thus we will have a universum of time signals and a data set contained in a time varying projection of this universum.

Definition 3 *Given a vector space of time signals* $\mathbf{Z}$*, a model* $(\mathbf{Z}, \mathcal{B})$*, a mapping* $P_\tau : \mathbf{Z} \to \mathbf{Z}$ *and a data set* $\mathcal{D}_\tau \triangleq P_\tau \mathcal{D} \subset P_\tau(\mathbf{Z})$*, we say that the model* $(\mathbf{Z}, \mathcal{B})$ *is* unfalsified *by the data set* $\mathcal{D}_\tau$ *if*

$$\mathcal{D}_\tau \subset P_\tau(\mathcal{B}).$$

Typically $P_\tau(x)$ is the experimental observation time sampling operator, which returns values of $x(t)$ only for past time instants (or possibly time intervals) over which experimental observations of $x(t)$ have been recorded. In this setting, definition of controller falsification (cf. [1, 4]) becomes

Definition 4 *Given a vector space of time signals* $\mathbf{Z}$*, a controller* $(\mathbf{Z}, \mathcal{B}_c)$*, a desired closed loop behavior* $(\mathbf{Z}, \mathcal{B}_d)$*, a mapping* $P_\tau : \mathbf{Z} \to \mathbf{Z}$*, and a data set* $\mathcal{D}_\tau \subset P_\tau(\mathbf{Z})$*, we say that a controller* $(\mathbf{Z}, \mathcal{B}_c)$ *is* unfalsified *by the data set* $\mathcal{D}_\tau$ *if*

$$P_\tau((P_\tau^{-1}(\mathcal{D}_\tau)) \cap \mathcal{B}_c) \subset P_\tau(\mathcal{B}_d).$$

The data set $\mathcal{D}_\tau$ is a set of actual experimental observations of the plant behavior as observed through the time-sampler P_τ. Thus, $P_\tau^{-1}(\mathcal{D}_\tau)$ is the set of behaviors that interpolate the observed data. For example, if we have recorded experimental observations of the first component $x_1(t)$ of a vector-valued signal $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in L_2^n[0, \infty)$ during the time interval $t \in [0, 5]$, then $P_\tau^{-1}(x)$ is the set of signals $\{y \in L_2^n[0, \infty) \mid y_1(t) = x_1(t) \forall t \in [0, 5]\}$. The set $\mathcal{B}_c$ is the set of signals which satisfy the constraints imposed by the controller c, so definition 4 says roughly that a controller is defined to be unfalsified if the set of signals x that are consistent with the data and the controller is, at the past observation times, a subset of a given performance target set $P_\tau(\mathcal{B}_d)$.

A particularly useful projection operator for dealing with past time only information is the time truncation operator P_τ defined by

$$P_\tau(x)](t) = \begin{cases} x(t), & \text{if } t \leq \tau \\ 0, & \text{if } t > \tau. \end{cases} \tag{1}$$

As explained by Willems ([16],[17]), the intersection of behaviors is "a way of formalizing that additional laws are imposed on a system." Thus, the role of a controller is to impose constraints on the plant behavior. On the other hand, our goal is to select, based on the data, the constraints imposed by the control law and the performance criterion, the best among the set of given controllers. In order to do that we introduce a *cost function*

$$J(z) : \mathbf{Z} \to \mathbb{R} \tag{2}$$

which may be used to sift controllers and to choose an optimal cost-minimizing controller having the least unfalsified cost based on the experimental evidence $\mathcal{D}_\tau$.

3 Direct Adaptive Control: Behavioral Formulation

We now explain how problems of adaptive control and learning theory may be parsimoniously and precisely embedded within the behavioral framework. At the crux is the observation that most such problems may be faithfully represented in terms of constraints on signals and other variables $z \in \mathbf{Z}$. The set $\mathbf{Z}$ is called the *universum*. Typically, the n-tuple z includes directly observable *manifest variables* z_{manifest} (viz., control and sensor signals), command input signals z_{command}, and possibly additional *latent variables* z_{latent} such as disturbances, noise, state-variable trajectories, error signals and so forth. That is, $z = \{z_{\mathsf{manifest}}, z_{\mathsf{command}}, z_{\mathsf{latent}}\} \in \mathbf{Z}_{\mathsf{manifest}} \times \mathbf{Z}_{\mathsf{command}} \times \mathbf{Z}_{\mathsf{latent}} = \mathbf{Z}$. In stochastic settings, the latent variable n-tuple z_{latent} also includes conditional probability density functions describing some of the other latent variables [4, 5, 12].

It is convenient to view the constraints on the n-tuple of signals $z \in \mathbf{Z}$ as arising from four distinct types of information, each possibly evolving with time:

1. Goal (cost function and/or design specification)
2. Belief (assumptions, prior knowledge, noise models, plant parameterizations, etc.)
3. Hypothesis (candidate control law, candidate plant/noise model)
4. Data (observations, samples of the signal z_{manifest} available at current time τ)

Each of these four types of information is representable as a mathematical constraint on z:

$\forall z_{\mathsf{command}},\ J(z) \le \gamma,$	(cost $J(z) \ge 0$ no bigger than γ for any command input)	(3)
$K(z) = 0,$	(hypothetical controller and/or model $K \in \mathbf{K}$)	(4)
$B(z) \le 0,$	(fixed beliefs, assumptions & prior knowledge)	(5)
$P_\tau(z_{\mathsf{manifest}}) = z_{\mathsf{data}},$	(z_{manifest} must interpolate observed data z_{data}) .	(6)

In turn, the four constraints (3)-(6) define, respectively, four subsets of the universum $\mathbf{Z}$, viz.

$$\mathbf{Z}_{\mathsf{goal}}(\gamma), \mathbf{Z}_{\mathsf{hypothesis}}(K), \mathbf{Z}_{\mathsf{belief}}, \mathbf{Z}_{\mathsf{data}} \subset \mathbf{Z}. \tag{7}$$

Thus, the problem of direct adaptive control (or, controller identification from data), can be formulated in the Willems behavioral framework as follows.

Problem 1 (Behavioral Adaptive Control) *Given a class of controllers $\mathcal{K} \triangleq \{(\mathbf{Z}, \mathcal{B}_c(\theta)) \mid \theta \in \Theta\}$, where Θ is a set of parameter vectors, the performance (cost) index $J(z)$ the time truncation operator P_τ, τ, and a data set $\mathcal{D}_\tau \subset P_\tau \mathbf{Z}$, find the set of parameters Θ^* such that $K \in \mathbf{K}$ that minimizes the cost γ subject to the constraint (cf. [4, 5, 12]) that, for each*

$$\xi \in \mathbf{Z}_{\mathsf{hypothesis}}(K) \cap \mathbf{Z}_{\mathsf{belief}} \cap \mathbf{Z}_{\mathsf{data}}, \tag{8}$$

there is at least one z such that

$$z_{\mathsf{command}} = \xi_{\mathsf{command}} \tag{9}$$

and

$$z \in \mathbf{Z}_{\mathsf{goal}}(\gamma) \cap \mathbf{Z}_{\mathsf{hypothesis}}(K(\theta)) \cap \mathbf{Z}_{\mathsf{belief}} \cap \mathbf{Z}_{\mathsf{data}}. \tag{10}$$

4 Discussion

If the set (8) is empty for some K, then the currently available data z_{data} provides no information on this K, which is therefore trivially optimal with cost $\gamma = 0$; otherwise, the adaptive feedback control problem emerges as the following optimization: At each time τ, find a control law K which solves

$$\gamma_{\mathsf{opt}} := \min_K \max_\xi \min_z \gamma \tag{11}$$

subject to (8)-(10). In many practical cases (e.g., [1, 2, 5, 6, 13]), the cost γ can be expressed directly in terms of z_{data} and K in which case (8)-(11) simplify to $\gamma_{\mathsf{opt}} := \min_K \gamma(z_{\mathsf{data}}, K)$.

Noteworthy are the symmetries revealed in the condition (10) with respect the information content of goal, belief, hypothesis and data. Set intersection is a commutative and associative operation; so all four types of information are logically equivalent in (10). For example, this means that the prejudice inherent in viewing one's data through a prism of belief $\mathbf{Z}_{\mathsf{belief}}$ is logically equivalent to assuming additional data "interpolation" constraints ($\mathbf{Z}_{\mathsf{data}} \leftarrow \mathbf{Z}_{\mathsf{data}} \cap \mathbf{Z}_{\mathsf{belief}}$). The standard *unfalsified control* problem considered in [1, 3, 4, 6, 13, 14] corresponds to the limiting case in which "the prism of belief" $\mathbf{Z}_{\mathsf{belief}}$ is the unconstraining "all-pass" filter $\mathbf{Z}$ (i.e., the universum).

Equations (8)–(10) underscore fertile conceptual links between adaptive control theory and the rich theory of system identification: The chief differences between identification and adaptive control arise from the precise forms of the cost functions $J(z)$ and of the admissible hypotheses $K(z)$. In system identification the admissible $K(z)$'s are typically noisy open-loop plant models and the cost function $J(z)$ measures probable modeling error deduced via a prism of beliefs about noise statistics. In adaptive control on the other hand, the admissible $K(z)$'s might typically be candidate controllers and the cost $J(z)$ could be a weighted sum of the sizes of tracking error and control signals.

5 Conclusions

The main goal of unfalsified control theory has been to close the loop on the adaptive and robust control design processes by developing data-driven methods to complement traditional model-based methods for the design of robust control systems. The crux of the unfalsified control theory is the observation that adaptive control is from a behavioral theory perspective essentially equivalent to system identification. In this paper, we have developed a behavioral formulation of the problem of direct adaptive control, viz., the problem of identifying an optimal controller that is unfalsified by data available at each time τ with respect to the least value of a cost function. The main result is the formulation of direct adaptive control problems provided by Problem 1. This result establishes a firm theoretical link between Willems' behavioral framework and direct adaptive control theory, expanding known links to model validation, unfalsified system identification theory, and behavioral indirect adaptive control approach of Polderman [18].

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