Fundamental Concepts in Fluid Mechanics

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3. Concept of a Continuum
4. Dimensions and Units used in Fluid Mechanics
5. Fluid Properties
   - Density and Specific Weight
   - Compressibility
   - Surface tension
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1. DEFINITION OF FLUID MECHANICS

Fluid mechanics is that branch of applied mechanics that is concerned with the statics and dynamics of liquids and gases. The analysis of the behaviour of fluids is based upon the fundamental laws of applied mechanics that relate to the conservation of mass, energy and momentum. The subject branches out into sub-disciplines such as aerodynamics, hydraulics, geophysical fluid dynamics and bio-fluid mechanics.

2. FLUIDS

A fluid is a substance that may flow. That is, the particles making up the fluid continuously change their positions relative to one another. Fluids do not offer any lasting resistance to the displacement of one layer over another when a shear force is applied. This means that if a fluid is at rest, then no shear forces can exist in it, which is different from solids; solids can resist shear forces while at rest. To summarize, if a shear force is applied to a fluid it will cause flow. Recall the example in class when a book was placed between my hands that were previously moving parallel to one another, even in the presence of the fluid, air. The book was somewhat distorted by the shear forces exerted on it by my hands, but eventually adopted a deformed position that resisted the force.

A further difference between solids and fluids is that a solid has a fixed shape whereas a fluid owes its shape at any particular time to that of the vessel containing it.

3. CONTINUUM CONCEPT

The behaviour of individual molecules comprising a fluid determines the observed properties of the fluid and for an absolutely complete analysis, the fluid should be studied at the molecular scale. The behaviour of any one molecule is highly complex, continuously varying and may indeed be very different from neighbouring molecules at any instant of time. The problems normally encountered by engineers do not require knowledge and prediction of behaviour at the molecular level but on the properties of the fluid mass that may result. Thus the interest is more on the average rather than the individual responses of the molecules comprising the fluid. At a microscopic level, a fluid consists of molecules with a lot of space in between. For our analysis, we do not consider the actual conglomeration of separate molecules, but instead assume that the fluid is a continuum, that is a continuous distribution of matter with no empty space. The sketch below illustrates this. Note that the fluid particle consists of an assembly of molecules each having properties such as pressure, temperature, density etc. However, we are interested in the property of the fluid particle at P and therefore we regard P as being a "smear" of matter (represented as a solid filled circle in the figure) with no space.

Recall the example of a crowd in a stadium given in class.

Fluid mass

P

Macroscopic view of a fluid particle P

Individual molecules
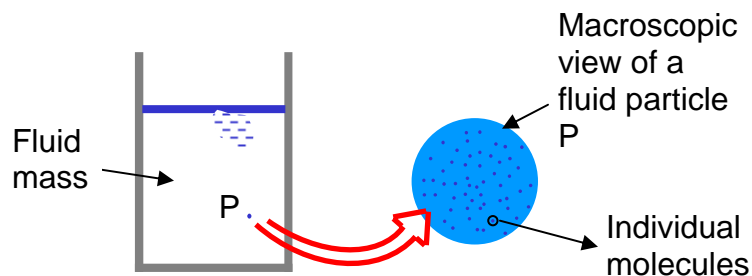
4. DIMENSIONS AND UNITS

Physical quantities require quantitative descriptions when solving engineering problems. Density, which is one such physical quantity, is a measure of the mass contained in unit volume. Density, however, does not represent a fundamental magnitude. There are nine quantities considered to be

fundamental magnitudes, and they are: length, mass, time, temperature, amount of a substance, electric current, luminous intensity, plane angle, and solid angle. The magnitudes of all the quantities can be expressed in terms of the fundamental magnitudes.

To give the magnitude of a quantity a numerical value, a set of units must be selected. Two primary systems of units are commonly used in Fluid Mechanics, namely, the Imperial System (sometimes called the English units) and the International System, which is referred to as SI (Systeme International) units.

The fundamental magnitudes and their units and the factors for conversion from the English unit system to the SI are shown in the two tables on the following pages.

Refer also to the discussion in class on the Bernoulli Equation.

5. FLUID PROPERTIES
i. Density
Density is the ratio of the mass of a given amount of the substance to the volume it occupies.

Mean density is defined as the ratio of a given amount of a substance to the volume that this amount occupies. The density is said to be uniform if the mean density in all parts of the substance is the same. (See sketch in class)

Table 1. Fundamental Magnitudes and Their Units

Quantity	Magnitude	SI unit		English unit	
Length, l	L	metre	m	foot	ft
Mass, m	M	kilogram	kg	slug	slug
Time, t	T	second	s	second	sec
Eelctric current, i		ampere	A	ampere	A
Temperature, T	Θ	kelvin	K	Rankine	°R
Amount of substance	M	kg-mole	kg-mol	lb-mole	lb-mol
Luminous intensity		candela	rd	candela	cd
Plane angle		radian	rad	radian	rad
Solid angle		steradian	sr	steradian	sr

Table 2. Derived Magnitudes

Quantity	Magnitude	SI unit	English unit
Area A	L^2	m^2	ft^2
Volume V	L^3	M^3; L (litre)	ft^3
Veloctiy v	LT^{-1}	m/s	ft/sec
Acceleration a	LT^{-2}	m/s^2	ft/sec^2
Angular velocity ω	T^{-1}	s^{-1}	sec^{-1}
Force F	MLT^{-2}	$kg\ m/s^2$; N (newton)	$slug\text{-}ft/sec^2$ lb (pound)
Density ρ	ML^{-3}	kg/m^3	$slug/ft^3$
Specific weight γ	$ML^{-2}T^{-2}$	N/m^3	lb/ft^3
Frequency f	T^{-1}	s^{-1}	sec^{-1}
Pressure p	$ML^{-1}T^{-2}$	Pa (pascal) N/m^2	lb/ft^2
Stress τ	$ML^{-1}T^{-2}$	N/m^2	lb/ft^2
Surface tension σ	MT^{-2}	N/m	lb/ft
Work W	ML^2T^{-2}	J (joule) N m	ft-lb
Energy E	ML^2T^2	J (joule) N m	ft-lb
Heat rate $\dot{Q}$	ML^2T^{-3}	J/s	Btu/sec
Torque T	ML^2T^{-2}	N m	ft-lb
Power P	ML^2T^{-3}	J/s W (watt)	ft-lb/sec
Viscosity μ	$ML^{-1}T^{-1}$	$N\ s/m^2$	$lb\text{-}sec/ft^2$
Mass flux $\dot{m}$	MT^{-1}	kg/s	Slug/sec
Flow rate Q	L^3T^{-1}	m^3/s	ft^3/sec
Specific heat c	$L^2T^2\Theta^{-1}$	J/(kg K)	Btu/slug-°R
Conductivity K	$MLT^{-3}\Theta^{-1}$	W/(m K)	lb-sec-°R

Density at a point is the limit to which the mean density tends as the volume considered is indefinitely reduced. Expressed mathematically, it is:

$$lim_{V \to \varepsilon} \frac{m}{V}$$

where ε is taken as the minimum volume of a fluid particle below which the continuum assumption fails.

This is illustrated in the sketch below (as completed in class).
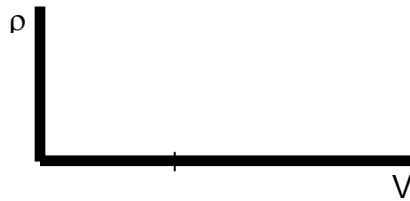
ii. **Compressibility**

The degree of compressibility of a substance is characterized by the bulk modulus of elasticity, K, defined as:

$$K = -\frac{\delta p}{\delta V / V}$$

where δp represents the small increased in pressure applied to the substance that causes a decrease of the volume by δV from its original volume of V.

Note the negative sign in the definition to ensure that the value of K is always positive.

K has the same dimensional formula as pressure, which is: $[ML^{-1}T^{-2}]$

K can also be expressed as a function of the accompanying change in density caused by the pressure increase. Using the definition of density as mass/volume, it can be shown that:

The reciprocal of the bulk modulus is **compressibility**.

Note that the value of K depends on the relation between pressure and density under which the compression occurs. The isothermal bulk modulus is the value when compression occurs while the temperature is held constant. The isentropic bulk modulus is the value when compression occurs under adiabatic conditions.

For liquids, K is very high (2.05 GPa for water at moderate pressure) and so there is very little change of density with pressure. For this reason, the density of liquids can be assumed to be constant without any serious loss in accuracy.

On the other hand, gases are very compressible.

iii. Surface Tension

Surface tension is the surface force that develops at the interface between two immiscible liquids or between liquid and gas or at the interface between a liquid and a solid surface. Because of surface tension, small water droplets, gas bubbles and drops of mercury tend to maintain spherical shapes.

The presence of surface tension and its dynamics are due to complex interactions at the molecular level along interfaces. Away from interfaces, molecules are surrounded by like molecules on all sides and so intermolecular force interactions result in a zero net force. At interfaces, molecules interact with molecules of the same fluid on only one side. The molecules at the interfaces experience a net force that puts the interface under tension.

The ultimate magnitude and direction of this tension force is determined not only by what happens on either side of the interface, but by the way molecules of the two fluids interact with each other. Surface tension, therefore, is specific to the participating fluids. Surface tension forces are also sensitive to the physical and chemical condition of the solid surface in contact, such as its roughness, cleanliness, or temperature.

If a line is imagined drawn in a liquid surface, then the liquid on one side of the line pulls that on the other side. The magnitude of surface tension is defined as that of the tensile force acting across and perpendicular to a short, straight element of the line drawn in the surface divided by the length of that line.

Dimensional Formula: $[MLT^{-2}]/[L] = [MT^{-2}]$

A common symbol for surface tension is σ.

The forces of attraction binding molecules to one another give rise to <u>cohesion</u>, the tendency of the liquid to remain as one assemblage of particles rather than to behave as a gas and fill the entire space within which it is confined. On the other hand, forces between the molecules of a fluid and the molecules of a solid boundary give rise to <u>adhesion</u> between the fluid and the boundary.   It is the interplay of these two forces that determine whether the liquid will "wet" the solid surface of the container. If the adhesive forces are greater than the cohesive forces, then the liquid will wet the surface; if vice versa, then the liquid will not. It is rare that the attraction between molecules of the liquid exactly equals that between molecules of the liquid and molecules of the solid and so the liquid surface near the boundary becomes curved.

For a curved surface, the resultant surface tension forces is towards the concave side. For equilibrium, the pressure on the concave side must be greater than that on the convex side by an amount equal to

$$\sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

where, R_1 and R_2 are the surface radii of curvature in two perpendicular directions.

The capillarity phenomenon is due to the rise or depression of the meniscus of the liquid due to the action of surface tension forces.

The water column in the sketch below rises to a height h such that the weight of the column is balanced by the resultant surface tension forces acting at θ to the vertical at the contact with the tube.
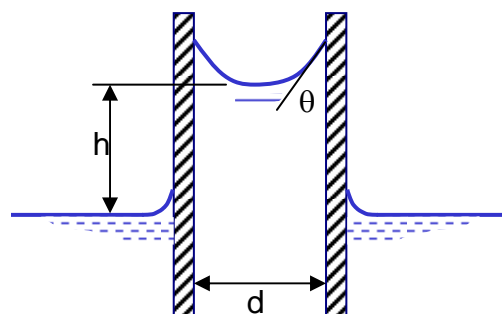
And from equilibrium of forces,

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

iv. Vapour Pressure

At the surface of a liquid, molecules are leaving and re-entering the liquid mass. The activity of the molecules at the surface creates a vapour pressure, which is a measure of the rate at which the molecules leave the surface. When the vapour pressure of the liquid is equal to the partial pressure of the molecules from the liquid which are in the gas above the surface, the number of molecules leaving is equal to the number entering. At this equilibrium condition, the vapour pressure is known as the saturation pressure.

The vapour pressure depends on the temperature, because molecular activity depends upon heat content. As the temperature increases, the vapour pressure increases until boiling is reached for the particular ambient atmospheric pressure.

Dimensional Formula: $[ML^{-1}T^{-2}]$

v. Viscosity

Viscosity can be thought of as the internal "stickiness" of a fluid. It is one of the properties that controls the amount of fluid that can be transported in a pipeline during a specific period of time. It accounts for the energy losses associated with the transport of fluids in ducts, channels and pipes. Further, viscosity plays an important role in the generation of turbulence. Needless to say, viscosity is an extremely important fluid property in our study of fluid flows.

All real fluids resist any force tending to cause one layer to move over another, but the resistance occurs only when the movement is taking place. On removal of the external force, flow subsides because of the resisting forces. But unlike solids that may return to their original position, the fluid particles stay in the position they have reached and have no tendency to

return to their original positions. The resistance to the movement of one layer of fluid over an adjoining one is due to the viscosity of the fluid.

<u>Causes of Viscosity</u>
To understand the causes of viscosity of a fluid, consider the observed effects of temperature on the viscosity of a gas and a liquid. It has been noted that for gases, viscosity increases with increasing temperature and for liquids, viscosity decreases with increasing temperature. The reason for this is that viscosity appears to depend on two phenomena, namely the transfer of momentum between molecules and the intermolecular (cohesive) forces between molecules of the fluid.

Consider a fluid consisting of two layers aa and bb as shown below, with the layer aa moving more rapidly than bb. Some molecules in aa owing to their thermal agitation will migrate to bb and take with them the momentum they have as a result of the overall velocity of aa. These molecules on colliding with molecules in the bb layer transfer their momentum resulting in an overall increase in the velocity of bb. In turn, molecules from bb, also owing to thermal agitation cross over to layer aa and collide with molecules there. The net effect of the crossings is that the relative motion between the two layers is reduced: layer aa is slowed down because of the collision with the slower molecules; layer bb is accelerated because of collision with faster molecules.

Now use this to explain why it is observed that viscosity of a gas increases with increasing temperature.

With a liquid, transfer of momentum between layers also occurs as molecules move between the two layers. However, what is different from the gas is the strong intermolecular forces in the liquid. Relative movement of layers in a liquid modifies these intermolecular forces, thereby causing a net shear force that resists the relative movement. The effect of increasing the temperature is to reduce the cohesive forces while simultaneously increasing the rate of molecular interchange. The net effect of these two in liquids is a decrease in viscosity.
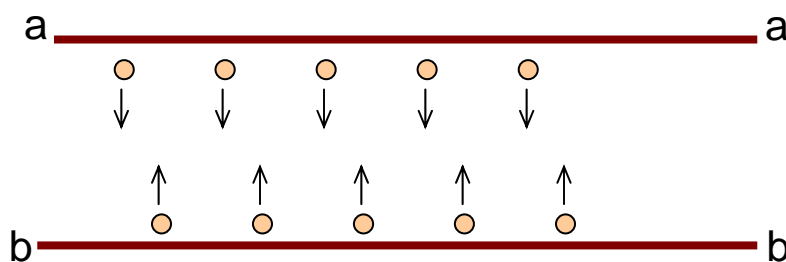
Quantitative Definition of Viscosity

The experiment described in class for defining a fluid showed the deformation of a fluid with constant pressure. See the first figure below. Here the top boundary was moving with constant velocity and due to the NO SLIP condition, the velocity profile was as shown on the right of the figure. Other flow conditions are given below.

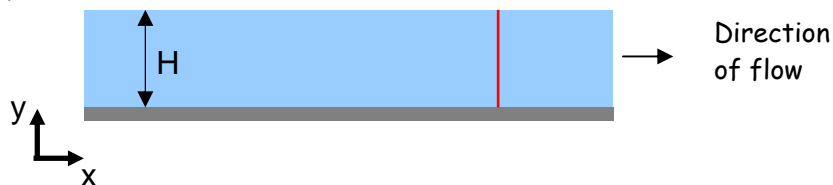*Insert the velocity profile next to each figure.*

(a)
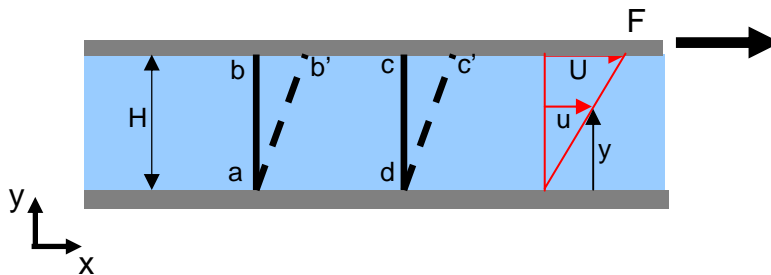
(b)
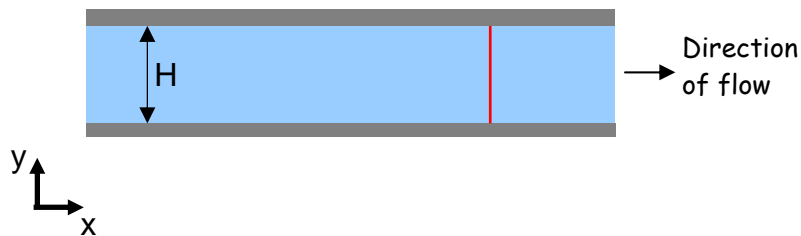
(c)

From experiments with various fluids, Sir Isaac Newton postulated that for the <u>straight and parallel motion</u> of a given fluid, the tangential stress between two adjoining fluid layers is proportional to the velocity gradient in a direction perpendicular to the layers. That is:

$$\tau = \mu \frac{\partial u}{\partial y} \tag{1}$$

where μ is a constant for a particular fluid at a particular temperature. The coefficient of proportionality is the **absolute viscosity** (sometimes referred to as the coefficient of viscosity). Note that μ is a scalar quantity, while the other terms are vector quantities. Note also that the surface over which the stress acts is perpendicular to the velocity gradient. If the velocity u increases with y, then the velocity gradient is positive and so τ also must be positive. So the positive sense of the shear stress is defined as being the same as the positive sense of the velocity.

From these definitions, the dimensional formula for viscosity is:

Dimensional Formula:

Kinematic Viscosity:
The **kinematic viscosity**, ν, is defined as the ratio of absolute viscosity to density:

$$\upsilon = \frac{\mu}{\rho} \tag{2}$$

Dimensional formula: $[L^{-2}T^{-1}]$

The interest in expressing this ratio will become clearer in discussion on Reynolds number and its use in turbulent and laminar flows where the ratio of viscous forces, (which is proportional to μ), to the inertial forces (which is proportional to ρ) is involved.
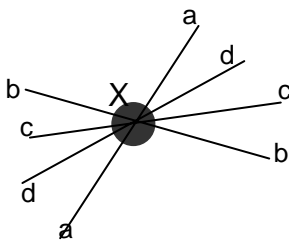
vi. Pressure
To define pressure, consider some imaginary surface of area A at an arbitrary part of a fluid. This surface must experience forces, say of magnitude F, due to a very large number of molecular collisions from the fluid adjoining it. Pressure, which is a scalar quantity, is defined as the ratio of the force and the area, that is F/A.

Dimensional Formula is: $[ML^{-1}T^{-2}]$

The units are: the pascal (Pa) N/m². Sometimes pressures of large magnitude are expressed in atmospheres (atm). One atmosphere is taken as 1.03125×10^5 Pa. A pressure of 10^5 is called a bar. For pressures less than that of the atmosphere, the units are normally expressed as millimetres of mercury vacuum.

Pascal's Law
It is important to realize that for a fluid having no shear forces, the direction of the plane over which the force due to pressure acts has no effect on the magnitude of the pressure at a point. This result is known as the Pascal's Law and its derivation is given below.

Point P in a fluid having pressure p. The pressure has the same magnitude regardless of which plane the force due to the pressure acts.

Imagine a small prism with plane faces and triangular section, surrounding the point, P, in question. The rectangular face ABB'A is assumed vertical and the rectangular face BB'C'C is horizontal, and the face AA'C'C slants at an arbitrarily defined angle to the horizontal. Assume the mean density to be ρ. The most general case is for a fluid accelerating with an acceleration component in the x and y direction being a_x and a_y respectively. Note the fluid accelerates as a whole body with no relative motion between its layers. That is to say that no shear forces are acting.
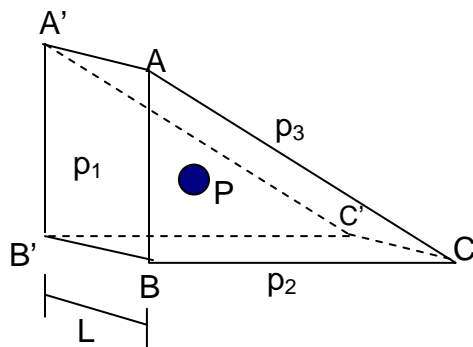
This means that the forces on the two end faces, ABC and A'B'C' are acting only perpendicular to these faces.

Resolving in the horizontal direction, we get:

$$p_1 ABL - p_3 ACL \cos A = (p_1 - p_3)ABL \qquad (3)$$

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because AC cosA = AB.

From Newton's Second Law, the net force is equal to the product of the mass of the fluid and the mean acceleration in the horizontal direction.

Therefore,

$$(p_1 - p_3)ABL = \left(\frac{1}{2}BCABL\right)\rho a_x \qquad (4)$$

That is,

$$p_1 - p_3 = \left(\frac{1}{2}BC\right)\rho a_x \qquad (5)$$

If the size of the prism is reduced so that it converges on the then its dimensions approach zero. Therefore, the right hand side of the above equation tends to zero.

So,
$$p_1 = p_3 \qquad (6)$$

Now consider the forces acting on the prism in the y (vertical) direction, and they are due to the weight and pressure. The resultant of these forces is the product of the mass of the prism and its acceleration in the y direction. So,

$$p_3 ACL\cos C + \left(\frac{1}{2}BCABL\right)\rho g - p_2 BCL = \left(\frac{1}{2}BCABL\right)\rho a_y \qquad (7)$$

which becomes after rearrangement:

$$p_3 - p_2 = \frac{1}{2}AB\rho\left(a_y - g\right) \qquad (8)$$

In the limit as the prism converges to the point P, the length AB approaches zero, and hence the right side of the above equation approaches zero. So from this,

$$p_3 = p_2 \qquad (9)$$

So from the above,

$p_1 = p_2 = p_3$ (10)

Now recall that the direction of the sloping face, AA'C'C was arbitrarily chosen. Therefore, the results above will be valid for any value of the angle ACB.

Also, the plane ABB'A' may face any point of the compass and so we may conclude that:

The pressure is independent of the direction of the surface used to define it.

6. EQUATION OF STATE

A perfect gas is one in which its molecules behave like tiny, perfectly elastic spheres in random motion, and would influence each other only when collided.

The kinetic theory of gases, which is based on perfect gases, states that for equilibrium conditions, the absolute pressure, p, the volume V occupied by mass m, and the absolute temperature T would be related as follows:

$pV = mRT$ (11)

or

$p = \rho RT$ (12)

where ρ is the density and R the gas constant whose value depends on the gas concerned.

Any equation relating p, ρ and T is known as the equation of state. Note that the equation of state is valid even when the gas is not in mechanical or thermal equilibrium.

The dimensional formula for R can be derived as follows:

$$\frac{p}{\rho T} : \frac{\left[F/L^2 \right]}{\left[\dfrac{M}{L^3} \theta \right]} = \frac{[FL]}{[M\theta]}$$

where [F] is the dimensional symbol for force and [θ] is the for temperature. The units are in J/kg K.

Universal gas constant   The product of the relative molecular mass, M and the gas constant R. This value is constant for all perfect gases.

Isothermal process: change of density of a gas occurring such that the temperature remains constant.

Adiabatic process: change of density of a gas occurring with no heat transfer to or from the gas.

Isentropic process: If in addition to the adiabatic process, no heat is generated within the gas, say, by friction, then the process is isentropic. The absolute pressure and density of a perfect gas are related by the additional expression:

$$\frac{p}{\rho^{\gamma}} = constant \tag{13}$$

where $\gamma = c_p/c_v$, c_p and c_v being the specific heat capacities at constant pressure and constant volume respectively.