Tables for Group Theory

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This provides the essential tables (character tables, direct products, descent in symmetry and subgroups) required for those using group theory, together with general formulae, examples, and other relevant information.

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Character Tables

Notes:

(1) Schönflies symbols are given for all point groups. Hermann–Maugin symbols are given for the 32 crystaliographic point groups.

(2) In the groups containing the operation C_5 the following relations are useful:

$$\eta^{+} = \tfrac{1}{2}(1+5^{\frac{1}{2}}) = 1\cdot 61803\mathrm{L} \;\; = -2\cos 144^{\mathrm{o}}$$

$$\eta^{-} = \tfrac{1}{2}(1-5^{\frac{1}{2}}) = -0\cdot 61803\mathrm{L} \;\; = -2\cos 72^{\mathrm{o}}$$

$$\eta^{+}\eta^{+} = 1+\eta^{+} \qquad \eta^{-}\eta^{-} = 1+\eta^{-} \qquad \eta^{+}\eta^{-} = -1$$

$$\eta^{+}+\eta^{-} = 1 \qquad 2\cos 72^{\mathrm{o}} + 2\cos 144^{\mathrm{o}} = -1$$

1. The Groups C_1, C_s, C_i

C_1 (1)	E
A	1

$C_s = C_h$ (m)	E	σ_h		
A′	1	1	x, y, R_z	x^2, y^2, z^2, xy
A″	1	–1	z, R_x, R_y	yz, xz

$C_i = S_2$ ($\bar{1}$)	E	i		
A_g	1	1	R_x, R_y, R_z	$x^2, y^2, z^2,$ xy, xz, yz
A_u	1	–1	x, y, z	

2. The Groups C_n (n = 2, 3,…,8)

C_2 (2)	E	C_2		
A	1	1	z, R_z	x^2, y^2, z^2, xy
B	1	–1	x, y, R_x, R_y	yz, xz

C_3 (3)	E	C_3	C_3^2		$\varepsilon = \exp(2\pi i/3)$
A	1	1	1	z, R_z	$x^2 + y^2, z^2$
E	1	ε	ε^*	$(x, y)(R_x, R_y)$	$(x^2 - y^2, 2xy)(yz, xz)$
	1	ε^*	ε		

C_4 (4)	E	C_4	C_2	C_4^3		
A	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
B	1	–1	1	–1		$x^2 - y^2, 2xy$
E	1	i	–1	–i	$(x, y)(R_x, R_y)$	(yz, xz)
	1	–i	–1	i		

C_5	E	C_5	C_5^2	C_5^3	C_5^4		$\varepsilon = \exp(2\pi i/5)$
A	1	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
E_1	1	ε	ε^2	ε^{*2}	ε^*	$(x,y)(R_x, R_y)$	(yz, xz)
	1	ε^*	ε^{*2}	ε^2	ε		
E_2	1	ε^2	ε^*	ε	ε^{*2}		$(x^2 - y^2, 2xy)$
	1	ε^{*2}	ε	ε^*	ε^2		

C_6 (6)	E	C_6	C_3	C_2	C_3^2	C_6^5		$\varepsilon = \exp(2\pi i/6)$
A	1	1	1	1	1	1	z, R_z	$x^2 + y^2, z^2$
B	1	–1	1	–1	1	–1		
E_1	1	ε	$-\varepsilon^*$	-1	$-\varepsilon$	ε^*	(x, y) (R_x, R_y)	(xy, yz)
	1	ε^*	$-\varepsilon$	-1	$-\varepsilon^*$	ε		
E_2	1	$-\varepsilon^*$	$-\varepsilon$	1	$-\varepsilon^*$	$-\varepsilon$		$(x^2 - y^2, 2xy)$
	1	$-\varepsilon$	$-\varepsilon^*$	1	$-\varepsilon$	$-\varepsilon^*$		

2. The Groups C_n (n = 2, 3,…,8) (cont..)

C_7	E	C_7	C_7^2	C_7^3	C_7^4	C_7^5	C_7^6		$\varepsilon = \exp(2\pi i/7)$
A	1	1	1	1	1	1	1	z, R_z	x^2+y^2, z^2
E_1	1	ε	ε^2	ε^3	ε^{*3}	ε^{*2}	ε^*	(x, y)	(xz, yz)
	1	ε^*	ε^{*2}	ε^{*3}	ε^3	ε^2	ε	(R_x, R_y)	
E_2	1	ε^2	ε^{*3}	ε^*	ε	ε^3	ε^{*2}		$(x^2-y^2, 2xy)$
	1	ε^{*2}	ε^3	ε	ε^*	ε^{*3}	ε^2		
E_3	1	ε^3	ε^*	ε^2	ε^{*2}	ε	ε^{*3}		
	1	ε^{*3}	ε	ε^{*2}	ε^2	ε^*	ε^3		

C_8	E	C_8	C_4	C_2	C_4^3	C_8^3	C_8^5	C_8^7		$\varepsilon = \exp(2\pi i/8)$
A	1	1	1	1	1	1	1	1	z, R_z	x^2+y^2, z^2
B	1	−1	1	1	1	−1	−1	−1		
E_1	1	ε	i	−1	−i	$-\varepsilon^*$	$-\varepsilon$	ε^*	(x, y)	(xz, yz)
	1	ε^*	−i	−1	i	$-\varepsilon$	$-\varepsilon^*$	ε	(R_x, R_y)	
E_2	1	i	−1	1	−1	−i	i	−i		$(x^2-y^2, 2xy)$
	1	−i	−1	1	−1	i	−i	i		
E_3	1	$-\varepsilon$	i	−1	−i	ε^*	ε	$-\varepsilon^*$		
	1	$-\varepsilon^*$	−i	−1	i	ε	ε^*	$-\varepsilon$		

3. The Groups D_n (n = 2, 3, 4, 5, 6)

D_2 (222)	E	$C_2(z)$	$C_2(y)$	$C_2(x)$		
A	1	1	1	1		x^2, y^2, z^2
B_1	1	1	–1	–1	z, R_z	xy
B_2	1	–1	1	–1	y, R_y	xz
B_3	1	–1	–1	1	x, R_x	yz

D_3 (32)	E	$2C_3$	$3C_2$		
A_1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	–1	z, R_z	
E	2	–1	0	$(x, y)(R_x, R_y)$	$(x^2 - y^2, 2xy)$ (xz, yz)

D_4 (422)	E	$2C_4$	$C_2 (= C_4^2)$	$2C_2'$	$2C_2''$		
A_1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	–1	–1	z, R_z	
B_1	1	–1	1	1	–1		$x^2 - y^2$
B_2	1	–1	1	–1	1		xy
E	2	0	–2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)

D_5	E	$2C_5$	$2C_5^2$	$5C_2$		
A_1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	–1	z, R_z	
E_1	2	2 cos 72°	2 cos 144°	0	$(x, y)(R_x, R_y)$	(xz, yz)
E_2	2	2 cos 144°	2 cos 72°	0		$(x^2 - y^2, 2xy)$

D_6 (622)	E	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$		
A_1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	1	–1	–1	z, R_z	
B_1	1	–1	1	–1	1	–1		
B_2	1	–1	1	–1	–1	1		
E_1	2	1	–1	–2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)
E_2	2	–1	–1	2	0	0		$(x^2 - y^2, 2xy)$

4. The Groups C_{nv} (n = 2, 3, 4, 5, 6)

C_{2v} (2*mm*)	E	C_2	$\sigma_v(xz)$	$\sigma'_v(yz)$		
A_1	1	1	1	1	z	x^2, y^2, z^2
A_2	1	1	–1	–1	R_z	xy
B_1	1	–1	1	–1	x, R_y	xz
B_2	1	–1	–1	1	y, R_x	yz

C_{3v} (3*m*)	E	$2C_3$	$3\sigma_v$		
A_1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	–1	R_z	
E	2	–1	0	$(x, y)(R_x, R_y)$	$(x^2 - y^2, 2xy)(xz, yz)$

C_{4v} (4*mm*)	E	$2C_4$	C_2	$2\sigma_v$	$2\sigma_d$		
A_1	1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	–1	–1	R_z	
B_1	1	–1	1	1	–1		$x^2 - y^2$
B_2	1	–1	1	–1	1		xy
E	2	0	–2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)

C_{5v}	E	$2C_5$	$2C_5^2$	$5\sigma_v$		
A_1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	–1	R_z	
E_1	2	2 cos 72°	2 cos 144°	0	$(x, y)(R_x, R_y)$	(xz, yz)
E_2	2	2 cos 144°	2 cos 72°	0		$(x^2 - y^2, 2xy)$

C_{6v} (6*mm*)	E	$2C_6$	$2C_3$	C_2	$3\sigma_v$	$3\sigma_d$		
A_1	1	1	1	1	1	1	z	$x^2 + y^2, z^2$
A_2	1	1	1	1	–1	–1	R_z	
B_1	1	–1	1	–1	1	–1		
B_2	1	–1	1	–1	–1	1		
E_1	2	1	–1	–2	0	0	$(x, y)(R_x, R_y)$	(xz, yz)
E_2	2	–1	–1	2	0	0		$(x^2 - y^2, 2xy)$

5. The Groups C_{nh} (n = 2, 3, 4, 5, 6)

C_{2h} (2/m)	E	C_2	I	σ_h		
A_g	1	1	1	1	R_z	x^2, y^2, z^2, xy
B_g	1	–1	1	–1	R_x, R_y	xz, yz
A_u	1	1	–1	–1	z	
B_u	1	–1	–1	1	x, y	

C_{3h} $(\bar{6})$	E	C_3	C_3^2	σ_h	S_3	S_3^5		$\varepsilon = \exp(2\pi i/3)$
A'	1	1	1	1	1	1	R_z	$x^2 + y^2, z^2$
E'	$\begin{cases}1\\1\end{cases}$	$\begin{matrix}\varepsilon\\\varepsilon^*\end{matrix}$	$\begin{matrix}\varepsilon^*\\\varepsilon\end{matrix}$	$\begin{matrix}1\\1\end{matrix}$	$\begin{matrix}\varepsilon\\\varepsilon^*\end{matrix}$	$\begin{matrix}\varepsilon^*\\\varepsilon\end{matrix}$	(x, y)	$(x^2 - y^2, 2xy)$
A''	1	1	1	–1	–1	–1	z	
E''	$\begin{cases}1\\1\end{cases}$	$\begin{matrix}\varepsilon\\\varepsilon^*\end{matrix}$	$\begin{matrix}\varepsilon^*\\\varepsilon\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-\varepsilon\\-\varepsilon^*\end{matrix}$	$\begin{matrix}-\varepsilon^*\\-\varepsilon\end{matrix}$	(R_x, R_y)	(xz, yz)

C_{4h} (4/m)	E	C_4	C_2	C_4^3	i	S_4^3	σ_h	S_4		
A_g	1	1	1	1	1	1	1	1	R_z	$x^2 + y^2, z^2$
B_g	1	–1	1	–1	1	–1	1	–1		$(x^2 - y^2, 2xy)$
E_g	$\begin{cases}1\\1\end{cases}$	$\begin{matrix}i\\-i\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-i\\i\end{matrix}$	$\begin{matrix}1\\1\end{matrix}$	$\begin{matrix}i\\-i\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-i\\i\end{matrix}$	(R_x, R_y)	(xz, yz)
A_u	1	1	1	1	–1	–1	–1	–1	z	
B_u	1	–1	1	–1	–1	1	–1	1		
E_u	$\begin{cases}1\\1\end{cases}$	$\begin{matrix}i\\-i\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-i\\i\end{matrix}$	$\begin{matrix}-1\\-1\end{matrix}$	$\begin{matrix}-i\\i\end{matrix}$	$\begin{matrix}1\\1\end{matrix}$	$\begin{matrix}i\\-i\end{matrix}$	(x, y)	

5. The Groups C_{nh} (n = 2, 3, 4, 5, 6) (cont…)

C_{5h}	E	C_5	C_5^2	C_5^3	C_5^4	σ_h	S_5	S_5^7	S_5^3	S_5^9		$\varepsilon = \exp(2\pi i/5)$
A′	1	1	1	1	1	1	1	1	1	1	R_z	x^2+y^2, z^2
E_1'	1	ε	ε^2	ε^{*2}	ε^*	1	ε	ε^2	ε^{*2}	ε^*	(x, y)	
	1	ε^*	ε^{*2}	ε^2	ε	1	ε^*	ε^{*2}	ε^2	ε		
E_2'	1	ε^2	ε^*	ε	ε^{*2}	1	ε^2	ε^*	ε	ε^{*2}	z	$(x^2-y^2, 2xy)$
	1	ε^{*2}	ε	ε^*	ε^2	1	ε^{*2}	ε	ε^*	ε^2		
A″	1	1	1	1	1	−1	−1	−1	−1	−1		
E_1''	1	ε	ε^2	ε^{*2}	ε^*	−1	$-\varepsilon$	$-\varepsilon^2$	$-\varepsilon^{*2}$	$-\varepsilon^*$	(R_x, R_y)	(xz, yz)
	1	ε^*	ε^{*2}	ε^2	ε	−1	$-\varepsilon^*$	$-\varepsilon^{*2}$	$-\varepsilon^2$	$-\varepsilon$		
E_2''	1	ε^2	ε^*	ε	ε^{*2}	−1	$-\varepsilon^2$	$-\varepsilon^*$	$-\varepsilon$	$-\varepsilon^{*2}$		
	1	ε^{*2}	ε	ε^*	ε^2	−1	$-\varepsilon^{*2}$	$-\varepsilon$	$-\varepsilon^*$	$-\varepsilon^2$		

C_{6h} (6/m)	E	C_6	C_3	C_2	C_3^2	C_6^5	i	S_3^5	S_6^5	σ_h	S_6	S_3		$\varepsilon = \exp(2\pi i/6)$
A_g	1	1	1	1	1	1	1	1	1	1	1	1		x^2+y^2, z^2
B_g	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	(R_x, R_y)	(xz, yz)
E_{1g}	1	ε	$-\varepsilon^*$	−1	$-\varepsilon$	ε^*	1	ε	$-\varepsilon^*$	−1	$-\varepsilon$	ε^*		
	1	ε^*	$-\varepsilon$	−1	$-\varepsilon^*$	ε	1	ε^*	$-\varepsilon$	−1	$-\varepsilon^*$	ε		
E_{2g}	1	$-\varepsilon^*$	$-\varepsilon$	1	$-\varepsilon^*$	$-\varepsilon$	1	$-\varepsilon^*$	$-\varepsilon$	1	$-\varepsilon^*$	$-\varepsilon$		$(x^2-y^2, 2xy)$
	1	$-\varepsilon$	$-\varepsilon^*$	1	$-\varepsilon$	$-\varepsilon^*$	1	$-\varepsilon$	$-\varepsilon^*$	1	$-\varepsilon$	$-\varepsilon^*$		
A_u	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	z	
B_u	1	−1	1	−1	1	−1	−1	1	−1	1	−1	1		
E_{1u}	1	ε	$-\varepsilon^*$	−1	$-\varepsilon$	ε^*	−1	$-\varepsilon$	ε^*	1	ε	$-\varepsilon^*$	(x, y)	
	1	ε^*	$-\varepsilon$	−1	$-\varepsilon^*$	ε	−1	$-\varepsilon^*$	ε	1	ε^*	$-\varepsilon$		
E_{2u}	1	$-\varepsilon^*$	$-\varepsilon$	1	$-\varepsilon^*$	$-\varepsilon$	−1	ε^*	ε	−1	ε^*	ε		
	1	$-\varepsilon$	$-\varepsilon^*$	1	$-\varepsilon$	$-\varepsilon^*$	−1	ε	ε^*	−1	ε	ε^*		

6. The Groups D_{nh} (n = 2, 3, 4, 5, 6)

D_{2h} (mmm)	E	$C_2(z)$	$C_2(y)$	$C_2(x)$	i	$\sigma(xy)$	$\sigma(xz)$	$\sigma(yz)$		
A_g	1	1	1	1	1	1	1	1		x^2, y^2, z^2
B_{1g}	1	1	–1	–1	1	1	–1	–1	R_z	xy
B_{2g}	1	–1	1	–1	1	–1	1	–1	R_y	xz
B_{3g}	1	–1	–1	1	1	–1	–1	1	R_x	yz
A_u	1	1	1	1	–1	–1	–1	–1		
B_{1u}	1	1	–1	–1	–1	–1	1	1	z	
B_{2u}	1	–1	1	–1	–1	1	–1	1	y	
B_{3u}	1	–1	–1	1	–1	1	1	–1	x	

D_{3h} $(\bar{6})m2$	E	$2C_3$	$3C_2$	σ_h	$2S_3$	$3\sigma_v$		
A_1'	1	1	1	1	1	1		x^2+y^2, z^2
A_2'	1	1	–1	1	1	–1	R_z	
E'	2	–1	0	2	–1	0	(x, y)	$(x^2-y^2, 2xy)$
A_1''	1	1	1	–1	–1	–1		
A_2''	1	1	–1	–1	–1	1	z	
E''	2	–1	0	–2	1	0	(R_x, R_y)	(xy, yz)

D_{4h} ($4/mmm$)	E	$2C_4$	C_2	$2C_2'$	$2C_2''$	i	$2S_4$	σ_h	$2\sigma_v$	$2\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_{2g}	1	1	1	–1	–1	1	1	1	–1	–1	R_z	
B_{1g}	1	–1	1	1	–1	1	–1	1	1	–1		x^2-y^2
B_{2g}	1	–1	1	–1	1	1	–1	1	–1	1		xy
E_g	2	0	–2	0	0	2	0	–2	0	0	(R_x, R_y)	(xz, yz)
A_{1u}	1	1	1	1	1	–1	–1	–1	–1	–1		
A_{2u}	1	1	1	–1	–1	–1	–1	–1	1	1	z	
B_{1u}	1	–1	1	1	–1	–1	1	–1	–1	1		
B_{2u}	1	–1	1	–1	1	–1	1	–1	1	–1		
E_u	2	0	–2	0	0	–2	0	2	0	0	(x, y)	

6. The Groups D_{nh} (n = 2, 3, 4, 5, 6) (cont…)

D_{5h}	E	$2C_5$	$2C_5^2$	$5C_2$	σ_h	$2S_5$	$2S_5^3$	$5\sigma_v$		
A_1'	1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_2'	1	1	1	–1	1	1	1	–1	R_z	
E_1'	2	2 cos 72°	2 cos 144°	0	2	2 cos 72°	2 cos 144°	0	(x, y)	
E_2'	2	2 cos 144°	2 cos 72°	0	2	2 cos 144°	2 cos 72°	0		$(x^2-y^2, 2xy)$
A_1''	1	1	1	1	–1	–1	–1	–1		
A_2''	1	1	1	–1	–1	–1	–1	1	z	
E_1''	2	2 cos 72°	2 cos 144°	0	–2	–2 cos 72°	–2 cos 144°	0	(R_x, R_y)	(xy, yz)
E_2''	2	2 cos 144°	2 cos 72°	0	–2	–2 cos 144°	–2 cos 72°	0		

D_{6h} (6/*mmm*)	E	$2C_6$	$2C_3$	C_2	$3C_2'$	$3C_2''$	i	$2S_3$	$2S_6$	σ_h	$3\sigma_d$	$3\sigma_v$		
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_{2g}	1	1	1	1	–1	–1	1	1	1	1	–1	–1	R_z	
B_{1g}	1	–1	1	–1	1	–1	1	–1	1	–1	1	–1		
B_{2g}	1	–1	1	–1	–1	1	1	–1	1	–1	–1	1		
E_{1g}	2	1	–1	–2	0	0	2	1	–1	–2	0	0	$(R_x - R_y)$	(xz, yz)
E_{2g}	2	–1	–1	2	0	0	2	–1	–1	2	0	0		$(x^2-y^2, 2xy)$
A_{1u}	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1		
A_{2u}	1	1	1	1	–1	–1	–1	–1	–1	–1	1	1	z	
B_{1u}	1	–1	1	–1	1	–1	–1	1	–1	1	–1	1		
B_{2u}	1	–1	1	–1	–1	1	–1	1	–1	1	1	–1		
E_{1u}	2	1	–1	–2	0	0	–2	–1	1	2	0	0	(x, y)	
E_{2u}	2	–1	–1	2	0	0	–2	1	1	–2	0	0		

7. The Groups D_{nd} (n = 2, 3, 4, 5, 6)

$D_{2d} = V_d$ $(\overline{42})m$	E	$2S_4$	C_2	$2C_2'$	$2\sigma_d$		
A_1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	–1	–1	R_z	
B_1	1	–1	1	1	–1		$x^2 – y^2$
B_2	1	–1	1	–1	1	z	xy
E	2	0	–2	0	0	(x, y) (R_x, R_y)	(xz, yz)

D_{3d} $(\overline{3})m$	E	$2C_3$	$3C_2$	i	$2S_6$	$3\sigma_d$		
A_{1g}	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_{2g}	1	1	–1	1	1	–1	R_z	
E_g	2	–1	0	2	–1	0	(R_x, R_y)	$(x^2 – y^2, 2xy)$ (xz, yz)
A_{1u}	1	1	1	–1	–1	–1		
A_{2u}	1	1	–1	–1	–1	1	z	
E_u	2	–1	0	–2	1	0	(x, y)	

D_{4d}	E	$2S_8$	$2C_4$	$2S_8^3$	C_2	$4C_2'$	$4\sigma_d$		
A_1	1	1	1	1	1	1	1		$x^2 + y^2, z^2$
A_2	1	1	1	1	1	–1	–1	R_z	
B_1	1	–1	1	–1	1	1	–1		
B_2	1	–1	1	–1	1	–1	1	z	
E_1	2	$\sqrt{2}$	0	$-\sqrt{2}$	–2	0	0	(x, y)	
E_2	2	0	–2	0	2	0	0		$(x^2 – y^2, 2xy)$
E_3	2	$-\sqrt{2}$	0	$\sqrt{2}$	–2	0	0	(R_x, R_y)	(xz, yz)

7. The Groups D_{nd} (n = 2, 3, 4, 5, 6) (cont..)

D_{5d}	E	$2C_5$	$2C_5^2$	$5C_2$	i	$2S_{10}^3$	$2S_{10}$	$5\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_{2g}	1	1	1	–1	1	1	1	–1	R_z	
E_{1g}	2	2 cos 72°	2 cos 144°	0	2	2 cos 72°	2 cos 144°	0	(R_x, R_y)	(xy, yz)
E_{2g}	2	2 cos 144°	2 cos 72°	0	2	2 cos 144°	2 cos 72°	0		$(x^2-y^2, 2xy)$
A_{1u}	1	1	1	1	–1	–1	–1	–1		
A_{2u}	1	1	1	–1	–1	–1	–1	1	z	
E_{1u}	2	2 cos 72°	2 cos 144°	0	–2	–2 cos 72°	–2 cos 144°	0	(x, y)	
E_{2u}	2	2 cos 144°	2 cos 72°	0	–2	–2 cos 144°	–2 cos 72°	0		

D_{6d}	E	$2S_{12}$	$2C_6$	$2S_4$	$2C_3$	$2S_{12}^5$	C_2	$6C_2'$	$6\sigma_d$		
A_1	1	1	1	1	1	1	1	1	1		x^2+y^2, z^2
A_2	1	1	1	1	1	1	1	–1	–1	R_z	
B_1	1	–1	1	–1	1	–1	1	1	–1		
B_2	1	–1	1	–1	1	–1	1	–1	1	z	
E_1	2	$\sqrt{3}$	1	0	–1	$-\sqrt{3}$	–2	0	0	(x, y)	
E_2	2	1	–1	–2	–1	1	2	0	0		$(x^2-y^2, 2xy)$
E_3	2	0	–2	0	2	0	–2	0	0		
E_4	2	–1	–1	2	–1	–1	2	0	0		
E_5	2	$-\sqrt{3}$	1	0	–1	$\sqrt{3}$	–2	0	0	(R_x, R_y)	(xy, yz)

8. The Groups S_n (n = 4, 6, 8)

S_4 ($\bar{4}$)	E	S_4	C_2	S_4^3		
A	1	1	1	1	R_z	x^2+y^2, z^2
B	1	−1	1	−1	z	$(x^2-y^2, 2xy)$
E	1	i	−1	−i	(x, y) (R_x, R_y)	(xz, yz)
	1	−i	−1	i		

S_6 ($\bar{3}$)	E	C_3	C_3^2	i	S_6^5	S_6		$\varepsilon = \exp(2\pi i/3)$
A_g	1	1	1	1	1	1	R_z	x^2+y^2, z^2
E_g	1	ε	ε^*	1	ε	ε^*	(R_x, R_y)	$(x^2-y^2, 2xy)$ (xy, yz)
	1	ε^*	ε	1	ε^*	ε		
A_u	1	1	1	−1	−1	−1	z	
E_u	1	ε	ε^*	1	ε	ε^*	(x, y)	
	1	ε^*	ε	1	ε^*	ε		

S_8	E	S_8	C_4	S_8^3	C_2	S_8^5	C_4^3	S_8^7		$\varepsilon = \exp(2\pi i/8)$
A	1	1	1	1	1	1	1	1	R_z	x^2+y^2, z^2
B	1	−1	1	−1	1	−1	1	−1	z	
E_1	1	ε	i	$-\varepsilon^*$	−1	$-\varepsilon$	−i	ε^*	(x, y)	
	1	ε^*	−i	$-\varepsilon$	−1	$-\varepsilon^*$	i	ε		
E_2	1	i	−1	−i	1	i	−1	−i		$(x^2-y^2, 2xy)$
	1	−i	−1	i	1	−i	−1	i		
E_3	1	$-\varepsilon^*$	−i	ε	−1	ε^*	i	$-\varepsilon$	(R_x, R_y)	(xy, yz)
	1	$-\varepsilon$	i	ε^*	−1	ε	−i	$-\varepsilon^*$		

9. The Cubic Groups

T (23)	E	$4C_3$	$4C_3^2$	$3C_2$		$\varepsilon = \exp(2\pi i/3)$
A	1	1	1	1		$x^2+y^2+z^2$
E	$\begin{Bmatrix}1 & \varepsilon & \varepsilon^* & 1 \\ 1 & \varepsilon^* & \varepsilon & 1\end{Bmatrix}$					$(\sqrt{3}\,(x^2-y^2)2z^2-x^2-y^2)$
T	3	0	0	–1	(x, y, z) (R_x, R_y, R_z)	(xy, xz, yz)

T_d ($\bar{4}3m$)	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$		
A_1	1	1	1	1	1		$x^2+y^2+z^2$
A_2	1	1	1	–1	–1		
E	2	–1	2	0	0		$(2z^2-x^2-y^2,\ \sqrt{3}\,(x^2-y^2)$
T_1	3	0	–1	1	–1	(R_x, R_y, R_z)	
T_2	3	0	–1	–1	1	(x, y, z)	(xy, xz, yz)

T_h ($m3$)	E	$4C_3$	$4C_3^2$	$3C_2$	i	$4S_6$	$4S_6^2$	$3\sigma_d$		$\varepsilon = \exp(2\pi i/3)$
A_g	1	1	1	1	1	1	1	1		$x^2+y^2+z^2$
E_g	$\begin{Bmatrix}1 & \varepsilon & \varepsilon^* & 1 & 1 & \varepsilon & \varepsilon^* & 1 \\ 1 & \varepsilon^* & \varepsilon & 1 & 1 & \varepsilon^* & \varepsilon & 1\end{Bmatrix}$									$(2z^2-x^2-y^2,$ $\sqrt{3}\,(x^2-y^2))$
T_g	3	0	0	–1	3	0	0	–1	(R_x, R_y, R_z)	(xy, yz, xz)
A_u	1	1	1	1	–1	–1	–1	–1		
E_u	$\begin{Bmatrix}1 & \varepsilon & \varepsilon^* & 1 & -1 & -\varepsilon & -\varepsilon^* & -1 \\ 1 & \varepsilon^* & \varepsilon & 1 & -1 & -\varepsilon^* & -\varepsilon & -1\end{Bmatrix}$									
T_u	3	0	0	–1	–3	0	0	1	(x, y, z)	

O (432)	E	$8C_3$	$3C_2$	$6C_4$	$6C_2'$		
A_1	1	1	1	1	1		$x^2+y^2+z^2$
A_2	1	1	1	–1	–1		
E	2	–1	2	0	0		$(2z^2-x^2-y^2,$ $\sqrt{3}\,(x^2-y^2))$
T_1	3	0	–1	1	–1	(x, y, z) (R_x, R_y, R_z)	
T_2	3	0	–1	–1	1		(xy, xz, yz)

9. The Cubic Groups (cont…)

O_h ($m3m$)	E	$8C_3$	$6C_2$	$6C_4$	$3C_2$ ($=C_4^2$)	i	$6S_4$	$8S_6$	$3\sigma_h$	$6\sigma_d$		
A_{1g}	1	1	1	1	1	1	1	1	1	1		$x^2+y^2+z^2$
A_{2g}	1	1	–1	–1	1	1	–1	1	1	–1		
E_g	2	–1	0	0	2	2	0	–1	2	0		$(2z^2-x^2-y^2, \sqrt{3}\,(x^2-y^2))$
T_{1g}	3	0	–1	1	–1	3	1	0	–1	–1	(R_x, R_y, R_z)	
T_{2g}	3	0	1	–1	–1	3	–1	0	–1	1		(xy, xz, yz)
A_{1u}	1	1	1	1	1	–1	–1	–1	–1	–1		
A_{2u}	1	1	–1	–1	1	–1	1	–1	–1	1		
E_u	2	–1	0	0	2	–2	0	1	–2	0		
T_{1u}	3	0	–1	1	–1	–3	–1	0	1	1	(x, y, z)	
T_{2u}	3	0	1	–1	–1	–3	1	0	1	–1		

10. The Groups I, I_h

I	E	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$		$\eta^{\pm} = \frac{1}{2}\left(1 \pm 5^{\frac{1}{2}}\right)$
A	1	1	1	1	1		$x^2+y^2+z^2$
T_1	3	η^+	η^-	0	–1	(x, y, z) (R_x, R_y, R_z)	
T_2	3	η^-	η^+	0	–1		
G	4	–1	–1	1	0		
H	5	0	0	–1	1		$(2z^2 - x^2 - y^2,$ $\sqrt{3}\ (x^2 - y^2)$ $xy, yz, zx)$

I_h	E	$12C_5$	$12C_5^2$	$20C_3$	$15C_2$	i	$12S_{10}$	$12S_{10}^3$	$20S_6$	15σ		$\eta^{\pm} = \frac{1}{2}\left(1 \pm 5^{\frac{1}{2}}\right)$
A_g	1	1	1	1	1	1	1	1	1	1		$x^2 + y^2 + z^2$
T_{1g}	3	η^+	η^-	0	–1	3	η^-	η^+	–1	–1	(R_x,R_y,R_z)	
T_{2g}	3	η^-	η^+	0	–1	3	η^+	η^-	0	–1		
G_g	4	–1	–1	1	0	4	–1	–1	1	0		
H_g	5	0	0	–1	1	5	0	0	–1	1		$(2z^2 - x^2 - y^2,$ $\sqrt{3}\ (x^2 - y^2))$ (xy, yz, zx)
A_u	1	1	1	1	1	–1	–1	–1	–1	–1		
T_{1u}	3	η^+	η^-	0	–1	–3	η^-	η^+	0	1	(x, y, z)	
T_{2u}	3	η^-	η^+	0	–1	–3	η^+	η^-	0	1		
G_u	4	–1	–1	1	0	–4	1	1	–1	0		
H_u	5	0	0	–1	1	–5	0	0	1	–1		

11. The Groups $C_{\infty v}$ and $D_{\infty h}$

$C_{\infty v}$	E	C_2	$2C_\infty^\phi$	…	$\infty\sigma_v$		
$A_1 \equiv \Sigma^+$	1	1	1	…	1	z	x^2+y^2, z^2
$A_2 \equiv \Sigma^-$	1	1	1	…	−1	R_z	
$E_1 \equiv \Pi$	2	−2	$2\cos\phi$	…	0	(x,y) (R_x,R_y)	(xz, yz)
$E_2 \equiv \Delta$	2	2	$2\cos 2\phi$	…	0		$(x^2-y^2, 2xy)$
$E_3 \equiv \Phi$	2	−2	$2\cos 3\phi$	…	0		
…	…	…	…	…	…		
…	…	…	…	…	…		

$D_{\infty h}$	E	$2C_\infty^\phi$	…	$\infty\sigma_v$	i	$2S_\infty^\phi$	…	∞C_2		
Σ_g^+	1	1	…	1	1	1	…	1		x^2+y^2, z^2
Σ_g^-	1	1	…	−1	1	1	…	−1	R_z	
Π_g	2	$2\cos\phi$	…	0	2	$-2\cos\phi$	…	0	(R_x, R_y)	(xz, yz)
Δ_g	2	$2\cos 2\phi$	…	0	2	$2\cos 2\phi$	…	0		$(x^2-y^2, 2xy)$
…	…	…	…	…	…	…	…	…		
Σ_u^+	1	1	…	1	−1	−1	…	−1	z	
Σ_u^-	1	1	…	−1	−1	−1	…	1		
Π_u	2	$2\cos\phi$	…	0	−2	$2\cos\phi$	…	0	(x,y)	
Δ_u	2	$2\cos 2\phi$	…	0	−2	$-2\cos 2\phi$	…	0		
…	…	…	…	…	…	…	…	…		

12. The Full Rotation Group (SU_2 and R_3)

$$\chi^{(j)}(\phi) = \begin{cases} \dfrac{\sin\left(j+\frac{1}{2}\right)\phi}{\sin\frac{1}{2}\phi} & \phi \neq 0 \\ 2j+1 & \phi = 0 \end{cases}$$

Notation : Representation labelled $\Gamma^{(j)}$ with j = 0,1/2, 1, 3/2,…∞, for R_3 j is confined to integral values (and written l or L) and the labels $S \equiv \Gamma^{(0)}$, $P \equiv \Gamma^{(1)}$, $D \equiv \Gamma^{(2)}$, etc. are used.

Direct Products

1. General rules

(*a*) For point groups in the lists below that have representations *A, B, E, T* without subscripts, read $A_1 = A_2 = A$, etc.

(*b*)

	g	u
g	g	u
u		g

	′	″
′	′	″
″		′

(*c*) Square brackets [ ] are used to indicate the representation spanned by the antisymmetrized product of a degenerate representation with itself.

Examples

For D_{3h} $E' \times E''\ \ A_1'' + A_2'' + E$

For D_{6h} $E_{1g} \times E_{2g} = 2B_g + E_{1g}$.

2. For C_2, C_3, C_6, D_3, D_6, C_{2v}, C_{3v}, C_{6v}, C_{2h}, C_{3h}, C_{6h}, D_{3h}, D_{6h}, D_{3d}, S_6

	A_1	A_2	B_1	B_2	E_1	E_2
A_1	A_1	A_2	B_1	B_2	E_1	E_2
A_2		A_1	B_2	B_1	E_1	E_2
B_1			A_1	A_2	E_2	E_1
B_2				A_1	E_2	E_1
E_1					$A_1 + [A_2] + E_2$	$B_1 + B_2 + E_1$
E_2						$A_1 + [A_2] + E_2$

3. For D_2, D_{2h}

	A	B_1	B_2	B_3
A	A	B_1	B_2	B_3
B_1		A	B_3	B_2
B_2			A	B_1
B_3				A

4. For C_4, D_4, C_{4v}, C_{4h}, D_{4h}, D_{2d}, S_4

	A_1	A_2	B_1	B_2	E
A_1	A_1	A_2	B_1	B_2	E
A_2		A_1	B_2	B_1	E
B_1			A_1	A_2	E
B_2				A_1	E
E					$A_1 + [A_2] + B_1 + B_2$

5. For C_5, D_5, C_{5v}, C_{5h}, D_{5h}, D_{5d}

	A_1	A_2	E_1	E_2
A_1	A_1	A_2	E_1	E_2
A_2		A_1	E_1	E_2
E_1			$A_1 + [A_2] + E_2$	$E_1 + E_2$
E_2				$A_1 + [A_2] + E_1$

6. For D_{4d}, S_8

	A_1	A_2	B_1	B_2	E_1	E_2	E_3
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3
A_2		A_1	B_2	B_1	E_1	E_2	E_3
B_1			A_1	A_2	E_3	E_2	E_1
B_2				A_1	E_3	E_2	E_1
E_1					$A_1 + [A_2] + E_2$	$E_1 + E_2$	$B_1 + B_2 + E_2$
E_2						$A_1 + [A_2] + B_1 + B_2$	$E_1 + E_3$
E_3							$A_1 + [A_2] + E_2$

7. For T, O, T_h, O_h, T_d

	A_1	A_2	E	T_1	T_2
A_1	A_1	A_2	E	T_1	T_2
A_2		A_1	E	T_2	T_1
E			$A_1 + [A_2] + E$	$T_1 + T_2$	$T_1 + T_2$
T_1				$A_1 + E + [T_1] + T_2$	$A_2 + E + T_1 + T_2$
T_2					$A_1 + E + [T_1] + T_2$

8. *For D_{6d}*

	A_1	A_2	B_1	B_2	E_1	E_2	E_3	E_4	E_5
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3	E_4	E_5
A_2		A_1	B_2	B_1	E_1	E_2	E_3	E_4	E_5
B_1			A_1	A_2	E_5	E_4	E_3	E_2	E_1
B_2				A_1	E_5	E_4	E_3	E_2	E_1
E_1					$A_1 + [A_2] + E_2$	$E_1 + E_3$	$E_2 + E_4$	$E_3 + E_5$	$B_1 + B_2 + E_4$
E_2						$A_1 + [A_2] + E_4$	$E_1 + E_5$	$B_1 + B_2 + E_2$	$E_3 + E_5$
E_3							$A_1 + [A_2] + B_1 + B_2$	$E_1 + E_5$	$E_2 + E_4$
E_4								$A_1 + [A_2] + E_4$	$E_1 + E_3$
E_5									$A_1 + [A_2] + E_2$

9. For *I*, *I_h*

	A	T_1	T_2	G	H
A	A	T_1	T_2	G	H
T_1		$A + [T_1] + H$	G + H	$T_2 + G + H$	$T_1 + T_2 + G + H$
T_2			$A + [T_2] + H$	$T_1 + G + H$	$T_1 + T_2 + G + H$
G				$A + [T_1 + T_2] + G + H$	$T_1 + T_2 + G + 2H$
H					$A_1 + [T_1 + T_2 + G] + G + 2H$

10. For $C_{\infty v}$, $D_{\infty h}$

	Σ^+	Σ^-	Π	Δ
Σ^+	Σ^+	Σ^-	Π	Δ
Σ^-		Σ^+	Π	Δ
Π			$\Sigma^+ + [\Sigma^-] + \Delta$	$\Pi + \Phi$
Δ :				$\Sigma^+ + [\Sigma^-] + \Gamma$

Notation

Σ	Π	Δ	Φ	Γ	…
$\Lambda = 0$	1	2	3	4	…

$\Lambda_1 \times \Lambda_2 = |\Lambda_1 - \Lambda_2| + (\Lambda_1 + \Lambda_2)$

$\Lambda \times \Lambda = \Sigma^+ + [\Sigma^-] + (2\Lambda)$.

***11. The Full Rotation Group* (*SU*$_2$ and *R*$_3$)**

$$\Gamma^{(j)} \times \Gamma^{(j')} = \Gamma^{(j+j')} + \Gamma^{(j+j'-1)} + \ldots + \Gamma^{(|j-j'|)}$$
$$\Gamma^{(j)} \times \Gamma^{(j)} = \Gamma^{(2j)} + \Gamma^{(2j-2)} + \ldots + \Gamma^{(0)} + [\Gamma^{(2j-1)} + \ldots + \Gamma^{(1)}]$$

Extended rotation groups (double groups):

Character tables and direct product tables

D_2^*	E	R	$2C_2(z)$	$2C_2(y)$	$2C_2(x)$
$E_{1/2}$	2	–2	0	0	0

D_3^*	E	R	$2C_3$	$2C_3R$	$3C_2$	$3C_2R$
$E_{1/2}$	2	–2	1	–1	0	0
$E_{3/2}$	{ 1	–1	–1	1	i	–i }
	{ 1	–1	–1	1	–i	i }

D_4	E	R	$2C_4$	$2C_4R$	$2C_2$	$4C_2'$	$4C_2''$
$E_{1/2}$	2	–2	$\sqrt{2}$	$-\sqrt{2}$	0	0	0
$E_{3/2}$	2	–2	$-\sqrt{2}$	$\sqrt{2}$	0	0	0

D_6^*	E	R	$2C_6$	$2C_6R$	$2C_3$	$2C_3R$	$2C_2$	$6C_2'$	$6C_2''$
$E_{1/2}$	2	–2	$\sqrt{3}$	$-\sqrt{3}$	1	–1	0	0	0
$E_{3/2}$	2	–2	$-\sqrt{3}$	$\sqrt{3}$	–1	1	0	0	0
$E_{5/2}$	2	–2	0	0	–2	2	0	0	0

T_d^*	E	R	$8C_3$	$8C_3R$	$6C_2$	$6S_4$	$6S_4R$	$12\sigma_d$
O^*	E	R	$8C_3$	$8C_3R$	$6C_2$	$6C_4$	$6S_4R$	$12C_2'$
$E_{1/2}$	2	–2	1	–1	0	$\sqrt{2}$	$-\sqrt{2}$	0
$E_{5/2}$	2	–2	1	–1	0	$-\sqrt{2}$	$\sqrt{2}$	0
$G_{3/2}$	4	–4	–1	1	0	0	0	0

$E_{1/2} \times E_{1/2} = [A] + B_1 + B_2 + B_3$

	$E_{1/2}$	$E_{3/2}$
$E_{1/2}$	$[A_1] + A_2 + E$	$2E$
$E_{3/2}$		$[A_1] + A_1 + 2A_2$

	$E_{1/2}$	$E_{3/2}$
$E_{1/2}$	$[A_1] + A_2 + E$	$B_1 + B_2 + E$
$E_{3/2}$		$[A_1] + A_2 + E$

	$E_{1/2}$	$E_{3/2}$	$E_{5/2}$
$E_{1/2}$	$[A_1] + A_2 + E_1$	$B_1 + B_2 + E_2$	$E_1 + E_2$
$E_{3/2}$		$[A_1] + A_2 + E_1$	$E_1 + E_2$
$E_{5/2}$			$[A_1] + A_2 + B_1 + B_2$

	$E_{1/2}$	$E_{5/2}$	$E_{3/2}$
$E_{1/2}$	$[A_1] + T_1$	$A_2 + T_2$	$E + T_1 + T_2$
$E_{5/2}$		$[A_1] + T_1$	$E + T_1 + T_2$
$G_{3/2}$			$[A_1 + E + T_2] + A_2 + 2T_1 + T_2]$

Direct products of ordinary and extended representations for T_d^* and O*

	A_1	A_2	E	T_1	T_2
$E_{1/2}$	$E_{1/2}$	$E_{5/2}$	$G_{3/2}$	$E_{1/2} + G_{3/2}$	$E_{5/2} + G_{3/2}$
$E_{5/2}$	$E_{5/2}$	$E_{1/2}$	$G_{3/2}$	$E_{5/2} + G_{3/2}$	$E_{1/2} + G_{3/2}$
$G_{3/2}$	$G_{3/2}$	$G_{3/2}$	$E_{1/2} + E_{5/2} + G_{3/2}$	$E_{1/2} + E_{5/2} + 2G_{3/2}$	$E_{1/2} + E_{5/2} + 2G_{3/2}$

Descent in symmetry and subgroups

The following tables show the correlation between the irreducible representations of a group and those of some of its subgroups. In a number of cases more than one correlation exists between groups. In C_s the σ of the heading indicates which of the planes in the parent group becomes the sole plane of C_s; in C_{2v} it becomes must be set by a convention); where there are various possibilities for the correlation of C_2 axes and σ planes in D_{4h} and D_{6h} with their subgroups, the column is headed by the symmetry operation of the parent group that is preserved in the descent.

C_{2v}	C_2	C_s $\sigma(zx)$	C_s $\sigma(yz)$
A_1	A	A′	A′
A_2	A	A″	A″
B_1	B	A′	A′
B_2	B	A″	A″

C_{3v}	C_3	C_s
A_1	A	A′
A_2	A	A″
E	E	A′ + A″

C_{4v}	C_{2v} σ_v	C_{2v} σ_d
A_1	A_1	A_1
A_2	A_2	A_2
B_1	A_1	A_2
B_2	A_2	A_1
E	$B_1 + B_2$	$B_1 + B_2$

[Other subgroups: C_4, C_2, C_6]

D_{3h}	C_{3h}	C_{3v}	C_{2v} $\sigma_h \rightarrow \sigma_v$	C_s σ_h	C_s σ_v
A'_1	A′	A_1	A_1	A′	A′
A'_2	A′	A_2	B_2	A′	A″
E′	E′	E	$A_1 + B_2$	2A′	A′ + A″
A''_1	A″	A_2	A_2	A″	A″
A''_2	A″	A_1	B_1	A″	A′
E″	E″	E	$A_2 + B_1$	2A″	A′ + A″

[Other subgroups: D_3, C_3, C_2]

D_{4h}	D_{2d} C_2' ($\rightarrow C_2'$)	D_{2d} C_2'' ($\rightarrow C_2'$)	D_{2h} C_2'	D_{2h} C_2''	D_2 C_2'	D_2 C_2''	C_{4h}	C_{4v}	C_{2v} C_2, σ_v	C_{2v} C_2, σ_d
A_{1g}	A_1	A_1	A_g	A_g	A	A	A_g	A_1	A_1	A_1
A_{2g}	A_2	A_2	B_{1g}	B_{1g}	B_1	B_1	A_g	A_2	A_2	A_2
B_{1g}	B_1	B_2	A_g	B_{1g}	A	B_1	B_g	B_1	A_1	A_2
B_{2g}	B_2	B_1	B_{1g}	A_g	B_1	A	B_g	B_2	A_2	A_1
E_g	E	E	$B_{2g} + B_{3g}$	$B_{2g} + B_{3g}$	$B_2 + B_3$	$B_2 + B_3$	E_g	E	$B_1 + B_2$	$B_1 + B_2$
A_{1u}	B_1	B_1	A_u	A_u	A	A	A_u	A_2	A_2	A_2
A_{2u}	B_2	B_2	B_{1u}	B_{1u}	B_1	B_1	A_u	A_1	A_1	A_1
B_{1u}	A_1	A_2	A_u	B_{1u}	A	B_1	B_u	B_2	A_2	A_1
B_{2u}	A_2	A_1	B_{1u}	A_u	B_1	A	B_u	B_1	A_1	A_2
E_u	E	E	$B_{2u} + B_{3u}$	$B_{2u} + B_{3u}$	$B_2 + B_3$	$B_2 + B_3$	E_u	E	$B_1 + B_2$	$B_1 + B_2$

Other subgroups: D_4, C_4, S_4, $3C_{2h}$, $3C_s$, $3C_2$, C_i, $(2C_{2v})$

D_6	$D_{3d}C_2''$	$D_{3d}C_2'$	D_{2h} $\sigma_h \rightarrow \sigma(xy)$ $\sigma_v \rightarrow \sigma(yz)$	C_{6v}	C_{3v} σ_v	C_{2v} C_2'	C_{2v} C_2''	C_{2h} C_2	C_{2h} C_2'	C_{2h} C_2''
A_{1g}	A_{1g}	A_{1g}	A_g	A_1	A_1	A_1	A_1	A_g	A_g	A_g
A_{2g}	A_{2g}	A_{2g}	B_{1g}	A_2	A_2	B_1	B_1	A_g	B_g	B_g
B_{1g}	A_{2g}	A_{1g}	B_{2g}	B_2	A_2	A_2	B_2	B_g	A_g	B_g
B_{2g}	A_{1g}	A_{2g}	B_{3g}	B_1	A_1	B_2	A_2	B_g	B_g	A_g
E_{1g}	E_g	E_g	$B_{2g} + B_{3g}$	E_1	E	$A_2 + B_2$	$A_2 + B_2$	$2B_g$	$A_g + B_g$	$A_g + B_g$
E_{2g}	E_g	E_g	$A_g + B_{1g}$	E_2	E	$A_1 + B_1$	$A_1 + B_1$	$2A_g$	$A_g + B_g$	$A_g + B_g$
A_{1u}	A_{1u}	A_{1g}	A_u	A_2	A_2	A_2	A_2	A_u	A_u	A_u
A_{2u}	A_{2u}	A_{2g}	B_{1u}	A_1	A_1	B_2	B_2	A_u	B_u	B_u
B_{1u}	A_{2u}	A_{1u}	B_{2u}	B_1	A_1	B_1	B_1	B_u	A_u	B_u
B_{2u}	A_{1u}	A_{2u}	B_{3u}	B_2	A_2	A_1	A_1	B_u	B_u	A_u
E_{1u}	E_u	E_u	$B_{2u} + B_{3u}$	E_1	E	$A_1 + B_1$	$A_1 + B_1$	$2B_u$	$A_u + B_u$	$A_u + B_u$
E_{2u}	E_u	E_u	$A_u + B_{1u}$	E_2	E	$A_2 + B_2$	$A_2 + B_2$	$2A_u$	$A_u + B_u$	$A_u + B_u$

Other subgroups: D_6, $2D_{3h}$, C_{6h}, C_6, C_{3h}, $2D_3$, S_6, D_2, C_3, $3C_2$, $3C_g$, C_i

T_d	T	D_{2d}	C_{3v}	C_{2v}
A_1	A	A_1	A_1	A_1
A_2	A	B_1	A_2	A_2
E	E	$A_1 + B_1$	E	$A_1 + A_2$
T_1	T	$A_2 + E$	$A_2 + E$	$A_2 + B_1 + B_2$
T_2	T	$B_2 + E$	$A_1 + E$	$A_1 + B_2 + B_1$

Other subgroups: S_4, D_2, C_3, C_2, C_s.

O_h	O	T_d	T_h	D_{4h}	D_{3d}
A_{1g}	A_1	A_1	A_g	A_{1g}	A_{1g}
A_{2g}	A_2	A_2	A_g	B_{1g}	A_{2g}
E_g	E	E	E_g	$A_{1g} + B_{1g}$	E_g
T_{1g}	T_1	T_1	T_g	$A_{2g} + E_g$	$A_{2g} + E_g$
T_{2g}	T_2	T_2	T_g	$B_{2g} + E_g$	$A_{1g} + E_g$
A_{1u}	A_1	A_2	A_u	A_{1u}	A_{1u}
A_{2u}	A_2	A_1	A_u	B_{1u}	B_{1u}
E_u	E	E	E_u	$A_{1u} + B_{1u}$	E_u
T_{1u}	T_1	T_2	T_u	$A_{2u} + E_u$	$A_{2u} + E_u$
T_{2u}	T_2	T_1	T_u	$B_{2u} + E_u$	$A_{1u} + E_u$

Other subgroups: T, D_4, D_{2d}, C_{4h}, C_{4v}, $2D_{2h}$, D_3, C_{3v}, S_6, C_4, S_4, $3C_{2v}$, $2D_2$, $2C_{2h}$, C_3, $2C_2$, S_2, C_s

R_3	O	D_4	D_3
S	A_1	A_1	A_1
P	T_1	$A_2 + E$	$A_2 + E$
D	$E + T_2$	$A_1 + B_1 + B_2 + E$	$A_1 + 2E$
F	$A_2 + T_1 + T_2$	$A_2 + B_1 + B_2 + 2E$	$A_1 + 2A_2 + 2E$
G	$A_1 + E + T_1 + T_2$	$2A_1 + A_2 + B_1 + B_2 + 2E$	$2A_1 + A_2 + 3E$
H	$E + 2T_1 + T_2$	$A_1 + 2A_2 + B_1 + B_2 + 3E$	$A_1 + 2A_2 + 4E$

Notes and Illustrations

General Formulae

(a) Notation

h	the *order* (the number of elements) of the group.
$\Gamma^{(i)}$	labels the *irreducible representation.*
$X^{(i)}(R)$	the *character* of the operation R in $\Gamma^{(i)}$.
$D^{(i)}_{\mu\nu}(R)$	the $\mu\nu$ element of the *representative matrix* of the operation R in the irreducible representation $\Gamma^{(i)}$.
l_i	the *dimension* of $\Gamma^{(i)}$ (the number of rows or columns in the matrices $\mathbf{D}^{(i)}$)

(b) Formulae

(i) *Number of irreducible representations of a group = number of classes.*

(ii) $$\sum_i l_i^2 = h$$

(iii) $$\chi^{(i)}(R) = \sum_{\mu} D^{(i)}_{\mu\mu}(R)$$

(iv) Orthogonality of representations:

$$\sum D^{(i)}_{\mu\nu}(R)^* D^{(i')}_{\mu'\nu'}(R) = (h/l_i)\delta_{ii'}\delta_{\mu\mu'}\delta_{\nu\nu'}$$

($\delta_{ij}=1$ if $i=j$ and $\delta_{ij}=0$ if $i \neq j$

(v) Orthogonality of characters:

$$\sum_R \chi^{(i)}(R)^* \chi^{(i)}(R) = h\delta_{ii'}$$

(vi) Decomposition of a direct product, reduction of a representation: If

$$\Gamma = \sum_i a_i \Gamma^{(i)}$$

and the character of the operation R in the reducible representation is $\chi(R)$, then the coefficients a_t are given by

$$a_i = (l/h)\sum_R \chi^{(i)}(R)^* \chi(R).$$

(vii) Projection operators:

The projection operator

$$P^{(i)} = (l_i / h)\sum_R \chi^{(i)}(R)^* R$$

when applied to a function f, generates a sum of functions that constitute a component of a basis for the representation $\Gamma^{(i)}$; in order to generate the complete basis $P^{(i)}$ must be applied to l_i distinct functions f. The resulting functions may be made mutually orthogonal. When $l_i = 1$ the function generated is a basis for $\Gamma^{(i)}$ without ambiguity:

$$P^{(i)} f = f^{(i)}$$

(viii) Selection rules:

If $f^{(i)}$ is a member of the basis set for the irreducible representation $\Gamma^{(i)}$, $f^{(k)}$ a member of that for $\Gamma^{(k)}$, and $\hat{\Omega}^{(j)}$ an operator that is a basis for $\Gamma^{(j)}$, then the integral

$$\int d\tau f^{(i)*} \hat{\Omega}^{(j)} f^{(k)}$$

is zero unless $\Gamma^{(i)}$ occurs in the decomposition of the direct product $\Gamma^{(j)} \times \Gamma^{(k)}$

(ix) The *symmetrized* direct product is written $\Gamma^{(i)} \overset{s}{\times} \Gamma^{(i)}$, and its characters are given by

$$\chi^{(i)}(R) \overset{s}{\times} \chi^{(i)}(R) = \tfrac{1}{2}\chi^{(i)}(R)^2 + \tfrac{1}{2}\chi^{(i)}(R^2)$$

The *antisymmetrized* direct product is written $\Gamma^{(i)} \overset{a}{\times} \Gamma^{(i)}$ and its characters are given by

$$\chi^{(i)}(R) \overset{a}{\times} \chi^{(i)}(R) = \tfrac{1}{2}\chi^{(i)}(R)^2 + \tfrac{1}{2}\chi^{(i)}(R^2)$$

Worked examples

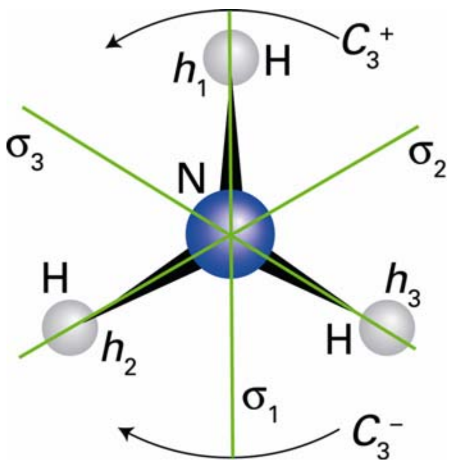
1. *To show that the representation* Γ *based on the hydrogen* 1*s-orbitals in* NH_3 (C_{3v}) *contains* A_1 *and* E, *and to generate appropriate symmetry adapted combinations.*

A table in which symmetry elements in the same class are distinguished will be employed:

C_{3v}	E	C_3^+	C_3^-	σ_1	σ_2	σ_3
A_1	1	1	1	1	1	1
A_2	1	1	1	–1	–1	–1
E	2	–1	–1	0	0	0
$D(R)$	$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$	$\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$	$\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$	$\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$	$\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$	$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$
$x(R)$	3	0	0	1	1	1
Rh_1	h_1	h_2	h_3	h_1	h_3	h_2
Rh_2	h_2	h_3	h_1	h_3	h_2	h_1

The *representative matrices* are derived from the effect of the operation R on the basis *(h_1, h_2, h_3);* see the figure below. For example

$$C_3^+(h_1,h_2,h_3)=(h_2,h_3,h_1)=(h_1,h_2,h_3)\begin{pmatrix}0&0&1\\1&0&0\\0&1&0\end{pmatrix}$$



According to the general formula (b)(iii) the *character* $\chi(R)$ is the sum of the diagonal elements of $\mathbf{D}(R)$. For example, $\chi(\sigma_2) = 0 + 1 + 0 = 1$. The *decomposition* of Γ follows from the formula (b)(vi):

$$\Gamma = a_1A_1 + a_2A_2 + a_EE$$

where

$$a_1 = \tfrac{1}{6}\{1\times3+2\times1\times0+3\times1\times1\}=1$$
$$a_2 = \tfrac{1}{6}\{1\times3+2\times1\times0+3\times1\times(-1)\}=0$$
$$a_E = \tfrac{1}{6}\{2\times3+2\times(-1)\times0+3\times0\times1\}=1$$

Therefore

$$\Gamma = A_1 + E$$

Symmetry adapted combinations are generated by the projection operator in (b)(vii). Using the last two rows of the table,

$$\phi(A_1) = \wp^{(A_1)} h_1 = \tfrac{1}{6}(1 \times h_1 + 1 \times h_2 + 1 \times h_3 + 1 \times h_1 + 1 \times h_3 + 1 \times h_2) = \tfrac{1}{3}(h_1 + h_2 + h_3)$$

$$\begin{cases} \phi(E) = \wp^{(E)} h_1 = \tfrac{2}{6}(2 \times h_1 - 1 \times h_2 - 1 \times h_3 + 0 \times h_1 + 0 \times h_3 + 0 \times h_2) = \tfrac{1}{3}(2h_1 - h_2 - h_3) \\ \phi'(E) = \wp^{(E)} h_2 = \tfrac{2}{6}(2 \times h_2 - 1 \times h_3 - 1 \times h_1 + 0 \times h_3 + 0 \times h_2 + 0 \times h_1) = \tfrac{1}{3}(-h_1 + 2h_2 - h_3) \end{cases}$$

$\phi(E)$ and $\phi'(E)$ provide a valid basis for the E representation, but the orthogonal combinations

$$\phi_a(E) = (1/6)^{\frac{1}{2}}(2h_1 - h_2 - h_3) = (3/2)^{\frac{1}{2}}\phi(E)$$
$$\phi_b(E) = (1/2)^{\frac{1}{2}}(h_2 - h_3) = (1/2)^{\frac{1}{2}}\{\phi(E) + 2\phi'(E)\}$$

would be a more useful basis in most applications.

2. *To determine the symmetries of the states arising from the electronic configurations e^2 and $e^1t_2^1$ for a tetrahedral complex (T_d), and to determine the group theoretical selection rules for electric dipole transitions between them.*

The spatial symmetries of the required states are given by the direct products in Table 7.

$$E \times E = A_1 + [A_2] + E \qquad E \times T_2 = T_1 + T_2$$

Combination of the electron spins yields both singlet and triplet states, but for the e^2 configuration some possibilities are excluded. Since the total (spin and orbital) state must be antisymmetric under electron interchange, the antisymmetrized spatial combination [A_2] must be a triplet, and the symmetrized combinations A_1 and E are singlets. For the $e^1t_2^1$ configuration there are no exclusions. The required terms are therefore

$$e^2 \rightarrow {}^1A_1 + {}^3A_2 + {}^1E$$

$$e^1t_2^1 \rightarrow {}^1T_1 + {}^1T_2 + {}^3T_1 + {}^3T_2$$

The selection rules are obtained from formula (b)(viii). For electric dipole transitions the operator $\Omega^{(j)}$ has the symmetry of a vector (x, y, z), which from the character table for T_d transforms as T_2. From the table of direct products, Table 7,

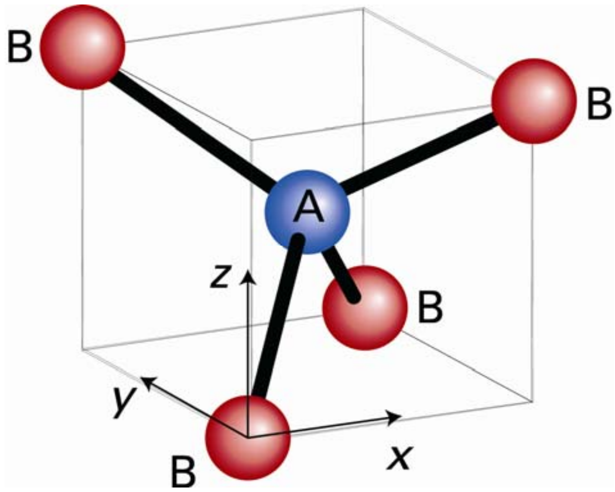
$$A_1 \times T_2 = T_2 \qquad A_2 \times T_1 = T_2 \qquad E \times T_2 = E \times T_1 = T_1 + T_2$$

Assuming the spin selection rule $\Delta S = 0$, the allowed transitions are

$$e^2\ {}^1A_1 \leftrightarrow e^1t_2^1\ {}^1T_2 \qquad e^2\ {}^3A_2 \leftrightarrow e^1t_2^1\ {}^3T_1 \qquad e^2\ {}^1E \leftrightarrow e^1t_2^1\ {}^1T_1, {}^1T_2$$

3. *To determine the symmetries of the vibrations of a tetrahedral molecule* AB_4, *and to predict the appearance of its infrared and Raman spectra.*

The molecule is depicted in the figure below and the character table for the point group T_d is given on page 15.



The representations spanned by the vibrational coordinates are based on the 5 × 3 cartesian displacements less the representations T_1 and T_2, which are accounted for by the rotations (R_x, R_y, R_z) and the translations (x, y, z). The stretching vibrations are the subset based on the 4 bonds of the molecule. For a particular symmetry operation, only atoms (or bonds) that remain invariant can contribute to the character of the cartesian displacement representation, $\Gamma^{(all)}$ (or the stretching representation, $\Gamma^{(stretch)}$).

C_3: Two atoms invariant, x, y, z, interchanged $\quad \chi^{(all)}(C_3) = 0$
One bond invariant $\quad \chi^{(stretch)}(C_3) = 1$

$C_2(z)$: Central atom invariant; x, y, sign reversed, z invariant $\quad \chi^{(all)}(C_3) = 0$
No bonds invariant $\quad \chi^{(stretch)}(C_2) = 0$

$S_4(z)$: Central atom invariant; x, y, interchanged, z sign reversed $\quad x^{(all)}(S_4) = -1$
No bonds invariant $\quad \chi^{(stretch)}(S_4) = 0$

$\sigma_d(z)$: Three atoms invariant; x, y, interchanged, z invariant $\quad x^{(all)}(\sigma_d) = 3$
Two bonds invariant $\quad \chi^{(stretch)}(\sigma_d) = 2$

The characters of the representations $\Gamma^{(all)}$ and $\Gamma^{(stretch)}$ are therefore

	E	$8C_3$	$3C_2$	$6S_4$	$6\sigma_d$	
$\Gamma^{(all)}$	15	0	–1	–1	3	$= A_1 + E + T_1 + 3T_2$
$\Gamma^{(stretch)}$	4	1	0	0	2	$= A_1 + T_2$

$\Gamma^{(all)}$ and $\Gamma^{(stretch)}$ have been decomposed with the help of formula (b)(vi) (compare Example 1). Allowing for the rotations and translations contained in $\Gamma^{(all)}$ there are therefore four fundamental vibrations, conventionally labelled ν_1 (A_1), ν_2(E), ν_3(T_2), and ν_4(T_2). ν_1 and ν_2 are stretching and bending vibrations respectively, ν_3 and ν_4 involve both stretching and bending motions.

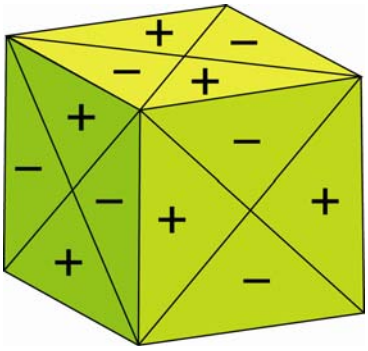
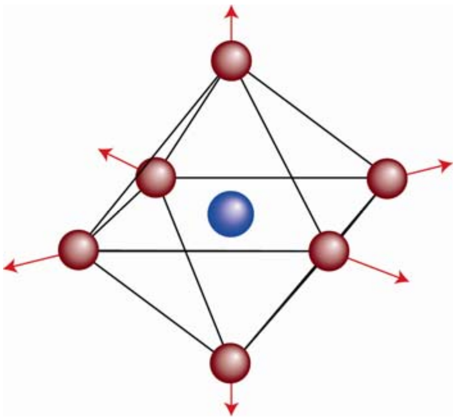
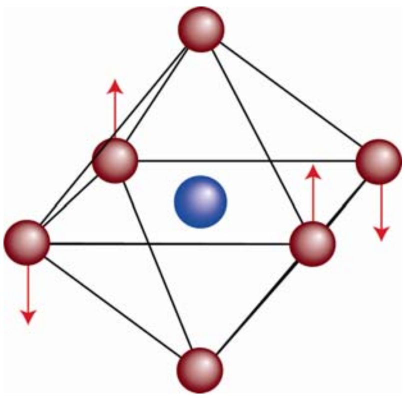
The selection rule (b)(viii) gives the spectroscopic properties of the vibrations. Infrared activity is induced by the dipole moment (a vector with symmetry T_2, according to the character table for T_d) as the operator $\hat{\Omega}^{(j)}$ In the case of the Raman effect, $\hat{\Omega}^{(j)}$ is the component of the polarizability tensor ($A_1 + E + T_2$). $f^{(i)}$ is the ground vibrational state (A_1), and $f^{(k)}$ is the excited state (with the same symmetry as the vibration in the case of the fundamental; as the direct product of the appropriate representations in the case of an overtone or a combination band). $\nu_1(A_1)$and $\nu_2(E)$ are therefore Raman active and $\nu_3(T_2)$ and $\nu_4(T_2)$ are infrared and Raman active. The following overtone and combination bands are allowed in the infrared spectrum:

$\nu_1 + \nu_3$, $\nu_1 + \nu_4$, $\nu_2 + \nu_3$, $\nu_2 + \nu_4$, $2\nu_3$, $\nu_3 + \nu_4$, $2\nu_4$

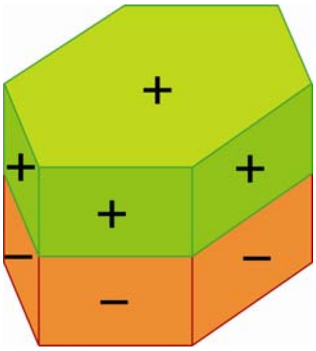
Examples of bases for some representations

The customary bases—polar vector (e.g. translation *x*), axial vector (e.g. rotation R_x), and tensor (e.g. *xy*)—are given in the character tables.

It may be of some assistance to consider other types of bases and a few examples are given here.

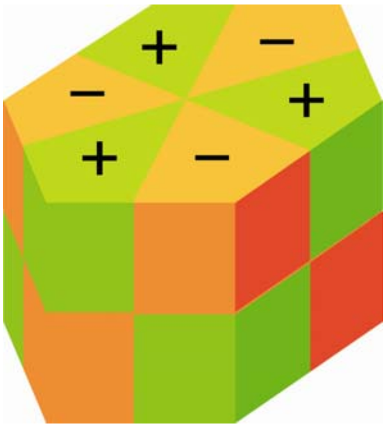
	Base	Irreducible Representation
1		A_2 in T_d
2	$x(1)y(2) - x(2)y(1)$	A_2 in C_{4v}
3	The normal vibration of an octahedral molecule represented by 	A_{1g} in O_h
4	The three equivalent normal vibrations of an octahedral molecule, one of which is represented by 	T_{2u} in O_h

5 The π-orbital of the benzene molecule represented by



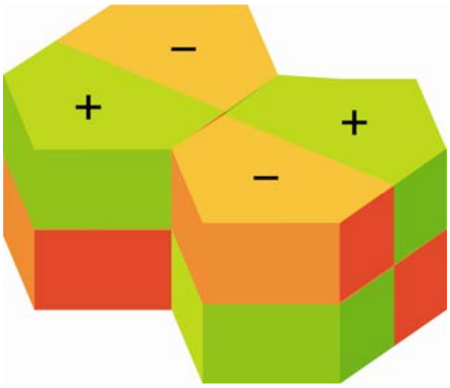
A_{2u} in D_{6h}

6 The π-orbital of the benzene molecule represented by



B_{2g} in D_{6h}

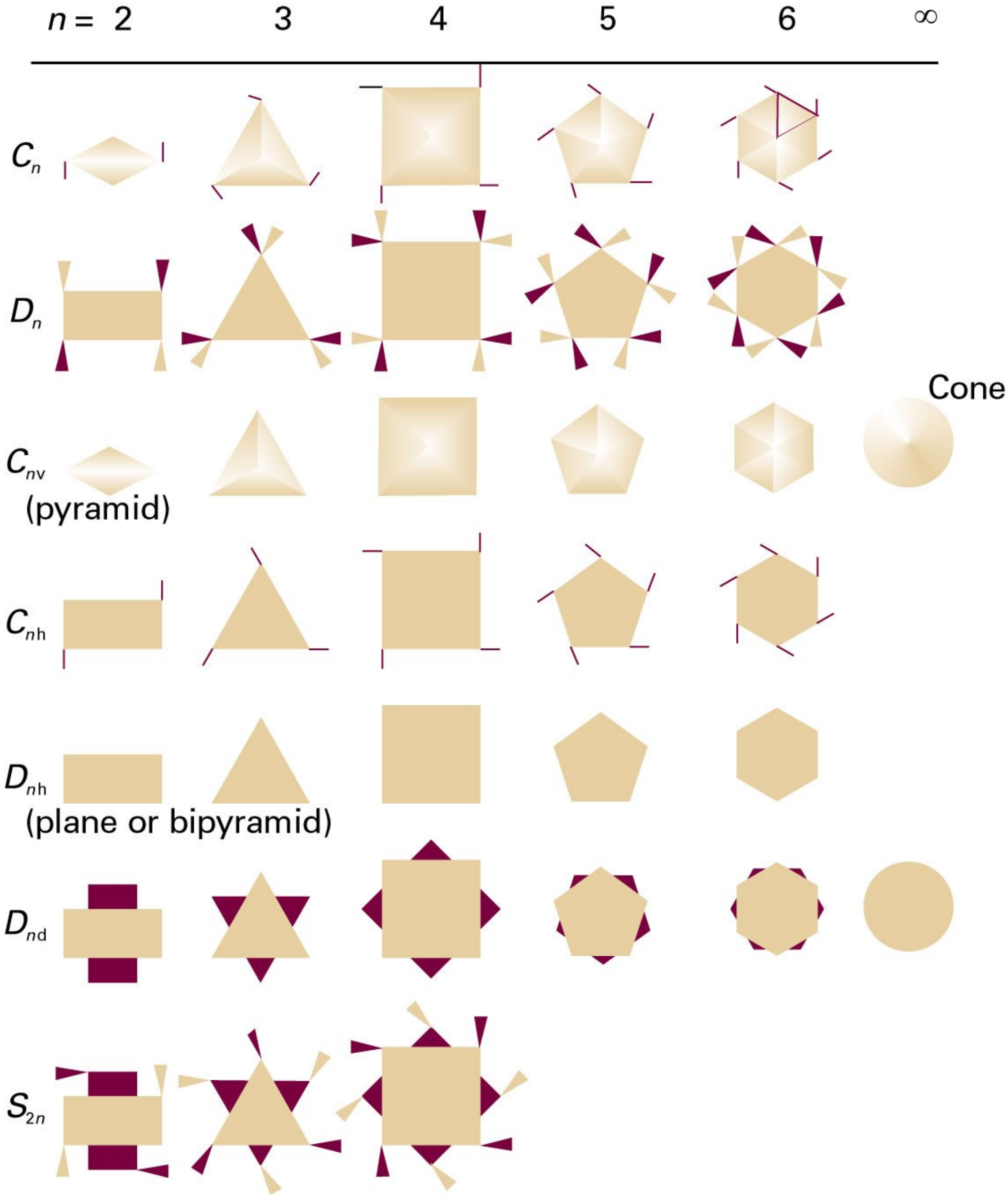
7 The π-orbital of the naphthalene molecule represented by



A_u in D_{2h}

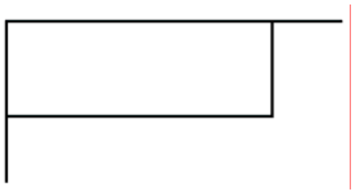
Illustrative Examples of Point Groups

I ***Shapes***

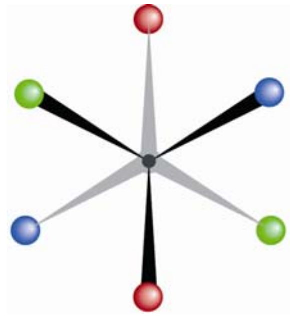


The character tables for (a), C_n, are on page 4; for (b), D_n, on page 6; for (c), C_{nv}, on page 7; for (d), C_{nh}, on page 8; for (e), D_{nh}, on page 10; for (f), D_{nd}, on page 12; and for (g), S_{2n}, on page 14.

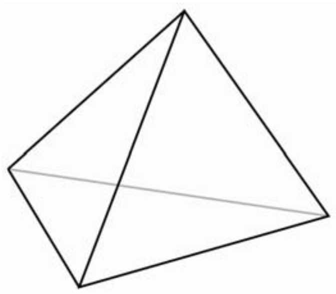
C_s



C_i

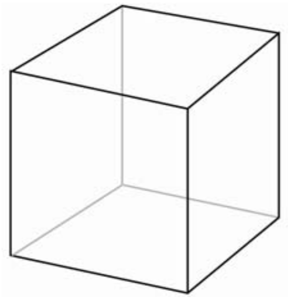


T_d



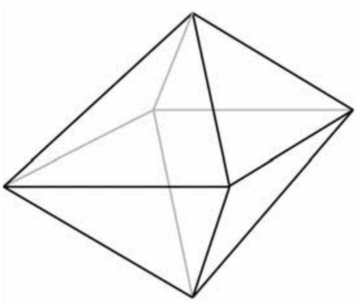
tetrahedron

O_h



cube

O_h



octahedron

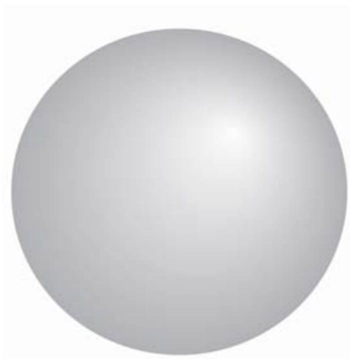
I_h

dodecahedron

I_h

icosahedron

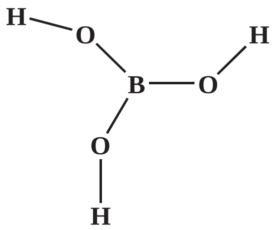
R_3



sphere

The character table for C_s is on page 3, for C_i on page 3, for T_d on page 15, for O_h on page 16, for I_h on page 17, and for R_3 on page 19.

II *Molecules*

Point group	Example	Page number for character table
C_1	CHFClBr	3
C_s	BFClBr (planar), quinoline	3
C_i	meso-tartaric acid	3
C_2	H_2O_2, S_2C1_2 (skew)	4
C_{2v}	H_2O, HCHO, C_6H_5C1	7
C_{3v}	NH_3 (pyramidal), $POC1_3$	7
C_{4v}	SF_5Cl, $XeOF_4$	7
C_{2h}	*trans*-dichloroethylene	8
C_{3h}	 (in planar configuration)	8
D_{2h}	*trans*-PtX_2Y_2, C_2H_4	10
D_{3h}	BF_3 (planar), $PC1_5$ (trigonal bipyramid), 1:3: 5–trichlorobenzene	10
D_{4h}	$AuCl_4^-$ (square plane)	10
D_{5h}	ruthenocene (pentagonal prism), IF_7 (pentagonal bipyramid)	11
D_{6h}	benzene	11
D_{2d}	CH_2=C=CH_2	12
D_{4d}	S_8 (puckered ring)	12
D_{5d}	ferrocene (pentagonal antiprism)	13
S_4	tetraphenylmethane	14
T_d	CCl_4	15
O_h	SF_6, FeF_6^{3-}	16
I_h	$B_{12}H_{12}^{2-}$	17
$C_{\infty v}$	HCN, COS	18
$D_{\infty h}$	CO_2, C_2H_2	18
R_3	any atom (sphere)	19