

FLUID MECHANICS

DENSITY, SPECIFIC VOLUME, SPECIFIC WEIGHT, AND SPECIFIC GRAVITY

The definitions of density, specific volume, specific weight, and specific gravity follow:

$$\rho = \lim_{\Delta V \rightarrow 0} \Delta m / \Delta V$$

$$\gamma = \lim_{\Delta V \rightarrow 0} \Delta W / \Delta V$$

$$\gamma = \lim_{\Delta V \rightarrow 0} g \cdot \Delta m / \Delta V = \rho g$$

also $SG = \gamma / \gamma_w = \rho / \rho_w$, where

ρ = density (also mass density),

Δm = mass of infinitesimal volume,

ΔV = volume of infinitesimal object considered,

γ = specific weight,

= ρg ,

ΔW = weight of an infinitesimal volume,

SG = specific gravity,

ρ_w = mass density of water at standard conditions
= 1,000 kg/m³ (62.43 lbf/ft³), and

γ_w = specific weight of water at standard conditions,
= 9,810 N/m³ (62.4 lbf/ft³), and
= 9,810 kg/(m²·s²).

STRESS, PRESSURE, AND VISCOSITY

Stress is defined as

$$\tau(1) = \lim_{\Delta A \rightarrow 0} \Delta F / \Delta A, \text{ where}$$

$\tau(1)$ = surface stress vector at point 1,

ΔF = force acting on infinitesimal area ΔA , and

ΔA = infinitesimal area at point 1.

$$\tau_n = -P$$

$$\tau_t = \mu (dv/dy) \text{ (one-dimensional; i.e., } y), \text{ where}$$

τ_n and τ_t = the normal and tangential stress components at point 1,

P = the pressure at point 1,

μ = absolute dynamic viscosity of the fluid
N·s/m² [lbf/(ft·sec)],

dv = differential velocity,

dy = differential distance, normal to boundary.

v = velocity at boundary condition, and

y = normal distance, measured from boundary.

$$v = \mu / \rho, \text{ where}$$

ν = kinematic viscosity; m²/s (ft²/sec).

For a thin Newtonian fluid film and a linear velocity profile,

$$v(y) = vy/\delta; dv/dy = v/\delta, \text{ where}$$

v = velocity of plate on film and

δ = thickness of fluid film.

For a power law (non-Newtonian) fluid

$$\tau_t = K (dv/dy)^n, \text{ where}$$

K = consistency index, and

n = power law index.

$n < 1 \equiv$ pseudo plastic

$n > 1 \equiv$ dilatant

SURFACE TENSION AND CAPILLARITY

Surface tension σ is the force per unit contact length

$$\sigma = F/L, \text{ where}$$

σ = surface tension, force/length,

F = surface force at the interface, and

L = length of interface.

The capillary rise h is approximated by

$$h = 4\sigma \cos \beta / (\gamma d), \text{ where}$$

h = the height of the liquid in the vertical tube,

σ = the surface tension,

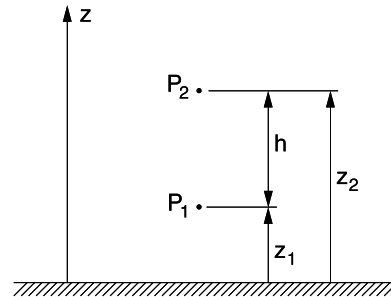
β = the angle made by the liquid with the wetted tube wall,

γ = specific weight of the liquid, and

d = the diameter of the capillary tube.

THE PRESSURE FIELD IN A STATIC LIQUID

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The difference in pressure between two different points is

$$P_2 - P_1 = -\gamma (z_2 - z_1) = -\gamma h = -\rho gh$$

For a simple manometer,

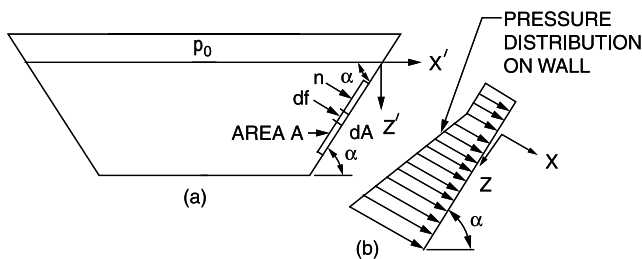
$$P_o = P_2 + \gamma_2 z_2 - \gamma_1 z_1$$

Absolute pressure = atmospheric pressure + gage pressure reading

Absolute pressure = atmospheric pressure – vacuum gage pressure reading

◆ Bober, W. & R.A. Kenyon, *Fluid Mechanics*, Wiley, New York, 1980. Diagrams reprinted by permission of William Bober & Richard A. Kenyon.

FORCES ON SUBMERGED SURFACES AND THE CENTER OF PRESSURE



Forces on a submerged plane wall. (a) Submerged plane surface. (b) Pressure distribution.

The pressure on a point at a distance Z' below the surface is

$$p = p_o + \gamma Z', \text{ for } Z' \geq 0$$

If the tank were open to the atmosphere, the effects of p_o could be ignored.

The coordinates of the *center of pressure* (CP) are

$$y^* = (\gamma I_{y_c} z_c \sin \alpha) / (p_c A) \text{ and}$$

$$z^* = (\gamma I_{y_c} \sin \alpha) / (p_c A), \text{ where}$$

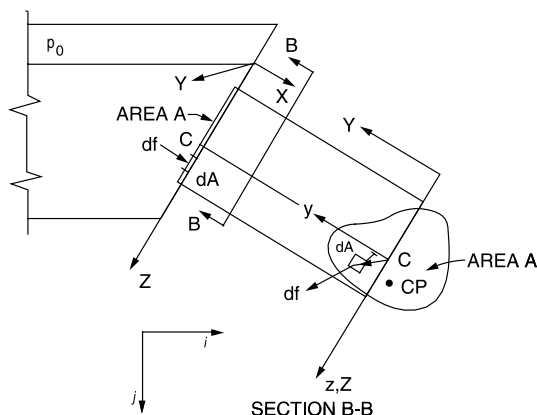
y^* = the y -distance from the centroid (C) of area (A) to the center of pressure,

z^* = the z -distance from the centroid (C) of area (A) to the center of pressure,

I_{y_c} and $I_{y_c z_c}$ = the moment and product of inertia of the area,

p_c = the pressure at the centroid of area (A), and

Z_c = the slant distance from the water surface to the centroid (C) of area (A).



If the free surface is open to the atmosphere, then $p_o = 0$ and $p_c = \gamma Z_c \sin \alpha$.

$$y^* = I_{y_c z_c} / (AZ_c) \text{ and } z^* = I_{y_c} / (AZ_c)$$

The force on a rectangular plate can be computed as

$$\mathbf{F} = [p_1 A_v + (p_2 - p_1) A_v / 2] \mathbf{i} + V_f \gamma_f \mathbf{j}, \text{ where}$$

\mathbf{F} = force on the plate,

p_1 = pressure at the top edge of the plate area,

p_2 = pressure at the bottom edge of the plate area,

A_v = vertical projection of the plate area,

V_f = volume of column of fluid above plate, and

γ_f = specific weight of the fluid.

ARCHIMEDES PRINCIPLE AND BUOYANCY

1. The buoyant force exerted on a submerged or floating body is equal to the weight of the fluid displaced by the body.
2. A floating body displaces a weight of fluid equal to its own weight; i.e., a floating body is in equilibrium.

The *center of buoyancy* is located at the centroid of the displaced fluid volume.

In the case of a body lying at the *interface of two immiscible fluids*, the buoyant force equals the sum of the weights of the fluids displaced by the body.

ONE-DIMENSIONAL FLOWS

The Continuity Equation

So long as the flow Q is continuous, the *continuity equation*, as applied to one-dimensional flows, states that the flow passing two points (1 and 2) in a stream is equal at each point, $A_1 v_1 = A_2 v_2$.

$$Q = Av$$

$$\dot{m} = \rho Q = \rho Av, \text{ where}$$

Q = volumetric flow rate,

\dot{m} = mass flow rate,

A = cross section of area of flow,

v = average flow velocity, and

ρ = the fluid density.

For steady, one-dimensional flow, \dot{m} is a constant. If, in addition, the density is constant, then Q is constant.

♦ Bober, W. & R.A. Kenyon, *Fluid Mechanics*, Wiley, New York, 1980. Diagrams reprinted by permission of William Bober & Richard A. Kenyon.

The Field Equation is derived when the energy equation is applied to one-dimensional flows. Assuming no friction losses and that no pump or turbine exists between sections 1 and 2 in the system,

$$\frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 \text{ or}$$

$$\frac{P_2}{\rho} + \frac{v_2^2}{2} + z_2 g = \frac{P_1}{\rho} + \frac{v_1^2}{2} + z_1 g, \text{ where}$$

P_1, P_2 = pressure at sections 1 and 2,

v_1, v_2 = average velocity of the fluid at the sections,

z_1, z_2 = the vertical distance from a datum to the sections (the potential energy),

γ = the specific weight of the fluid (ρg), and

g = the acceleration of gravity.

FLUID FLOW

The velocity distribution for **laminar flow in circular tubes or between planes** is

$$v(r) = v_{\max} \left[1 - \left(\frac{r}{R} \right)^2 \right], \text{ where}$$

r = the distance (m) from the centerline,

R = the radius (m) of the tube or half the distance between the parallel planes,

v = the local velocity (m/s) at r , and

v_{\max} = the velocity (m/s) at the centerline of the duct.

$v_{\max} = 1.18v$, for fully turbulent flow

$v_{\max} = 2v$, for circular tubes in laminar flow and

$v_{\max} = 1.5v$, for parallel planes in laminar flow, where

\bar{v} = the average velocity (m/s) in the duct.

The shear stress distribution is

$$\frac{\tau}{\tau_w} = \frac{r}{R}, \text{ where}$$

τ and τ_w are the shear stresses at radii r and R respectively.

The **drag force F_D on objects immersed in a large body of flowing fluid or objects moving through a stagnant fluid** is

$$F_D = \frac{C_D \rho v^2 A}{2}, \text{ where}$$

C_D = the **drag coefficient**,

v = the velocity (m/s) of the flowing fluid or moving object, and

A = the **projected area** (m^2) of blunt objects such as spheres, ellipsoids, disks, and plates, cylinders, ellipses, and air foils with axes perpendicular to the flow.

For **flat plates placed parallel with the flow**

$$C_D = 1.33/Re^{0.5} \quad (10^4 < Re < 5 \times 10^5)$$

$$C_D = 0.031/Re^{1/7} \quad (10^6 < Re < 10^9)$$

The characteristic length in the Reynolds Number (Re) is the length of the plate parallel with the flow. For blunt objects, the characteristic length is the largest linear dimension (diameter of cylinder, sphere, disk, etc.) which is perpendicular to the flow.

AERODYNAMICS

Airfoil Theory

The lift force on an airfoil is given by

$$F_L = \frac{C_L \rho v^2 A_p}{2}$$

C_L = the lift coefficient

v = velocity (m/s) of the undisturbed fluid and

A_p = the projected area of the airfoil as seen from above (plan area). This same area is used in defining the drag coefficient for an airfoil.

The lift coefficient can be approximated by the equation

$$C_L = 2\pi k_1 \sin(\alpha + \beta) \text{ which is valid for small values of } \alpha \text{ and } \beta.$$

k_1 = a constant of proportionality

α = angle of attack (angle between chord of airfoil and direction of flow)

β = negative of angle of attack for zero lift.

The drag coefficient may be approximated by

$$C_D = C_{D\infty} + \frac{C_L^2}{\pi AR}$$

$C_{D\infty}$ = infinite span drag coefficient

$$AR = \frac{b^2}{A_p} = \frac{A_p}{c^2}$$

The aerodynamic moment is given by

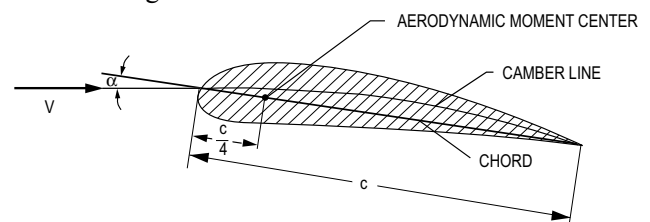
$$M = \frac{C_M \rho v^2 A_p c}{2}$$

where the moment is taken about the front quarter point of the airfoil.

C_M = moment coefficient

A_p = plan area

c = chord length



Reynolds Number

$$Re = vD\rho/\mu = vD/\nu$$

$$Re' = \frac{v^{(2-n)} D^n \rho}{K \left(\frac{3n+1}{4n} \right)^n 8^{(n-1)}}, \text{ where}$$

ρ = the mass density,

D = the diameter of the pipe, dimension of the fluid streamline, or characteristic length.

μ = the dynamic viscosity,

ν = the kinematic viscosity,

Re = the Reynolds number (Newtonian fluid),

Re' = the Reynolds number (Power law fluid), and

K and n are defined in the Stress, Pressure, and Viscosity section.

The critical Reynolds number $(Re)_c$ is defined to be the minimum Reynolds number at which a flow will turn turbulent.

Flow through a pipe is generally characterized as laminar for $Re < 2,100$ and fully turbulent for $Re > 10,000$, and transitional flow for $2,100 < Re < 10,000$.

Hydraulic Gradient (Grade Line)

The hydraulic gradient (grade line) is defined as an imaginary line above a pipe so that the vertical distance from the pipe axis to the line represents the *pressure head* at that point. If a row of piezometers were placed at intervals along the pipe, the grade line would join the water levels in the piezometer water columns.

Energy Line (Bernoulli Equation)

The Bernoulli equation states that the sum of the pressure, velocity, and elevation heads is constant. The energy line is this sum or the “total head line” above a horizontal datum. The difference between the hydraulic grade line and the energy line is the $v^2/2g$ term.

STEADY, INCOMPRESSIBLE FLOW IN CONDUITS AND PIPES

The energy equation for incompressible flow is

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f \text{ or}$$

$$\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_f$$

h_f = the head loss, considered a friction effect, and all remaining terms are defined above.

If the cross-sectional area and the elevation of the pipe are the same at both sections (1 and 2), then $z_1 = z_2$ and $v_1 = v_2$.

The pressure drop $p_1 - p_2$ is given by the following:

$$p_1 - p_2 = \gamma h_f = \rho g h_f$$

COMPRESSIBLE FLOW

See **MECHANICAL ENGINEERING** section.

The *Darcy-Weisbach equation* is

$$h_f = f \frac{L}{D} \frac{v^2}{2g}, \text{ where}$$

f = $f(Re, e/D)$, the Moody or Darcy friction factor,

D = diameter of the pipe,

L = length over which the pressure drop occurs,

e = roughness factor for the pipe, and all other symbols are defined as before.

An alternative formulation employed by chemical engineers is

$$h_f = (4f_{\text{Fanning}}) \frac{Lv^2}{D2g} = \frac{2f_{\text{Fanning}}Lv^2}{Dg}$$

$$\text{Fanning friction factor, } f_{\text{Fanning}} = \frac{f}{4}$$

A chart that gives f versus Re for various values of e/D , known as a *Moody* or *Stanton diagram*, is available at the end of this section.

Friction Factor for Laminar Flow

The equation for Q in terms of the pressure drop Δp_f is the Hagen-Poiseuille equation. This relation is valid only for flow in the laminar region.

$$Q = \frac{\pi R^4 \Delta p_f}{8\mu L} = \frac{\pi D^4 \Delta p_f}{128\mu L}$$

Flow in Noncircular Conduits

Analysis of flow in conduits having a noncircular cross section uses the *hydraulic diameter* D_H , or the *hydraulic radius* R_H , as follows

$$R_H = \frac{\text{cross-sectional area}}{\text{wetted perimeter}} = \frac{D_H}{4}$$

Minor Losses in Pipe Fittings, Contractions, and Expansions

Head losses also occur as the fluid flows through pipe fittings (i.e., elbows, valves, couplings, etc.) and sudden pipe contractions and expansions.

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} + h_f + h_{f,\text{fitting}}$$

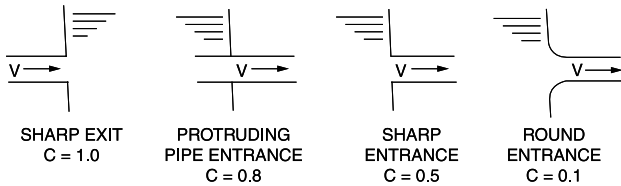
$$\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_f + h_{f,\text{fitting}}, \text{ where}$$

$$h_{f,\text{fitting}} = C \frac{v^2}{2g}, \text{ and } \frac{v^2}{2g} = 1 \text{ velocity head}$$

Specific fittings have characteristic values of C , which will be provided in the problem statement. A generally accepted *nominal value* for head loss in *well-streamlined gradual contractions* is

$$h_{f,\text{fitting}} = 0.04 \frac{v^2}{2g}$$

The head loss at either an *entrance* or *exit* of a pipe from or to a reservoir is also given by the $h_{f, \text{fitting}}$ equation. Values for C for various cases are shown as follows.



PUMP POWER EQUATION

$$\dot{W} = Q\gamma h/\eta = Q\rho gh/\eta, \text{ where}$$

Q = volumetric flow (m^3/s or cfs),

h = head (m or ft) the fluid has to be lifted,

η = efficiency, and

\dot{W} = power (watts or ft-lbf/sec).

For additional information on pumps refer to the **MECHANICAL ENGINEERING** section of this handbook.

COMPRESSIBLE FLOW

See the **MECHANICAL ENGINEERING** section for compressible flow and machinery associated with compressible flow (compressors, turbines, fans).

THE IMPULSE-MOMENTUM PRINCIPLE

The resultant force in a given direction acting on the fluid equals the rate of change of momentum of the fluid.

$$\Sigma F = Q_2\rho_2v_2 - Q_1\rho_1v_1, \text{ where}$$

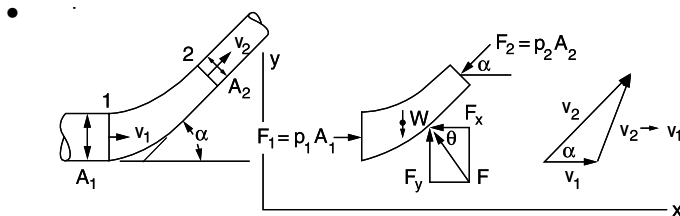
ΣF = the resultant of all external forces acting on the control volume,

$Q_1\rho_1v_1$ = the rate of momentum of the fluid flow entering the control volume in the same direction of the force, and

$Q_2\rho_2v_2$ = the rate of momentum of the fluid flow leaving the control volume in the same direction of the force.

Pipe Bends, Enlargements, and Contractions

The force exerted by a flowing fluid on a bend, enlargement, or contraction in a pipe line may be computed using the impulse-momentum principle.



$$p_1A_1 - p_2A_2\cos\alpha - F_x = Q\rho(v_2\cos\alpha - v_1)$$

$$F_y - W - p_2A_2\sin\alpha = Q\rho(v_2\sin\alpha - 0), \text{ where}$$

F = the force exerted by the bend on the fluid (the force exerted by the fluid on the bend is equal in magnitude and opposite in sign), F_x and F_y are the x -component and y -component of the force,

p = the internal pressure in the pipe line,

A = the cross-sectional area of the pipe line,

W = the weight of the fluid,

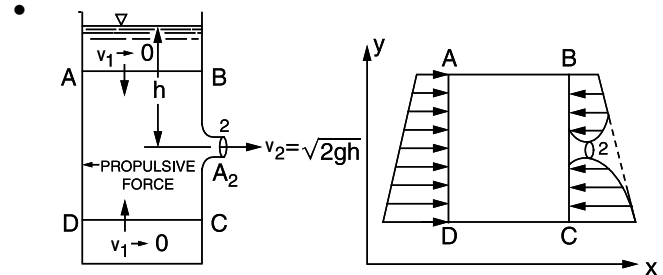
v = the velocity of the fluid flow,

α = the angle the pipe bend makes with the horizontal,

ρ = the density of the fluid, and

Q = the quantity of fluid flow.

Jet Propulsion



$$F = Q\rho(v_2 - 0)$$

$$F = 2\gamma hA_2, \text{ where}$$

F = the propulsive force,

γ = the specific weight of the fluid,

h = the height of the fluid above the outlet,

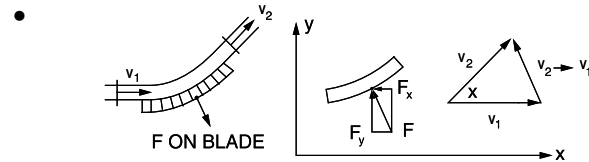
A_2 = the area of the nozzle tip,

$Q = A_2\sqrt{2gh}$, and

$v_2 = \sqrt{2gh}$.

Deflectors and Blades

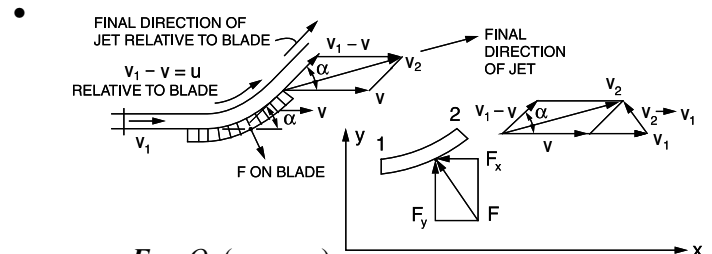
Fixed Blade



$$-F_x = Q\rho(v_2\cos\alpha - v_1)$$

$$F_y = Q\rho(v_2\sin\alpha - 0)$$

Moving Blade



$$-F_x = Q\rho(v_{2x} - v_{1x}) = -Q\rho(v_1 - v)(1 - \cos\alpha)$$

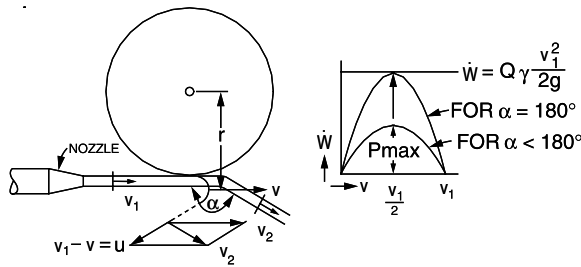
$$F_y = Q\rho(v_{2y} - v_{1y}) = +Q\rho(v_1 - v)\sin\alpha, \text{ where}$$

v = the velocity of the blade.

♦ Bober, W. & R.A. Kenyon, *Fluid Mechanics*, Wiley, New York, 1980. Diagram reprinted by permission of William Bober & Richard A. Kenyon.

• Vennard, J.K., *Elementary Fluid Mechanics*, 6th ed., J.K. Vennard, 1954.

Impulse Turbine



$$\dot{W} = Q\rho(v_1 - v)(1 - \cos \alpha)v, \text{ where}$$

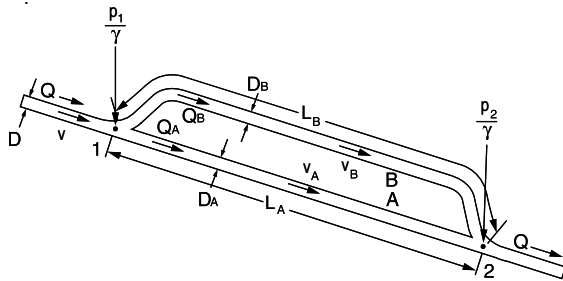
\dot{W} = power of the turbine.

$$\dot{W}_{\max} = Q\rho(v_1^2/4)(1 - \cos \alpha)$$

When $\alpha = 180^\circ$,

$$\dot{W}_{\max} = (Q\rho v_1^2)/2 = (Q\gamma v_1^2)/2g$$

MULTIPATH PIPELINE PROBLEMS



The same head loss occurs in each branch as in the combination of the two. The following equations may be solved simultaneously for v_A and v_B :

$$h_L = f_A \frac{L_A}{D_A} \frac{v_A^2}{2g} = f_B \frac{L_B}{D_B} \frac{v_B^2}{2g}$$

$$(\pi D^2/4)v = (\pi D_A^2/4)v_A + (\pi D_B^2/4)v_B$$

The flow Q can be divided into Q_A and Q_B when the pipe characteristics are known.

OPEN-CHANNEL FLOW AND/OR PIPE FLOW

Manning's Equation

$$v = (k/n)R^{2/3}S^{1/2}, \text{ where}$$

k = 1 for SI units,

k = 1.486 for USCS units,

v = velocity (m/s, ft/sec),

n = roughness coefficient,

R = hydraulic radius (m, ft), and

S = slope of energy grade line (m/m, ft/ft).

Also see Hydraulic Elements Graph for Circular Sewers in the **CIVIL ENGINEERING** section.

Hazen-Williams Equation

$$v = k_1 C R^{0.63} S^{0.54}, \text{ where}$$

C = roughness coefficient,

k_1 = 0.849 for SI units, and

k_1 = 1.318 for USCS units.

Other terms defined as above.

WEIR FORMULAS

See the **CIVIL ENGINEERING** section.

FLOW THROUGH A PACKED BED

A porous, fixed bed of solid particles can be characterized by

L = length of particle bed (m)

D_p = average particle diameter (m)

Φ_s = sphericity of particles, dimensionless (0–1)

ϵ = porosity or void fraction of the particle bed, dimensionless (0–1)

The Ergun equation can be used to estimate pressure loss through a packed bed under laminar and turbulent flow conditions.

$$\frac{\Delta p}{L} = \frac{150 v_o \mu (1 - \epsilon)^2}{\Phi_s^2 D_p^2 \epsilon^3} + \frac{1.75 \rho v_o^2 (1 - \epsilon)}{\Phi_s D_p \epsilon^3}$$

Δp = pressure loss across packed bed (Pa)

v_o = superficial (flow through empty vessel) fluid velocity $\left(\frac{\text{m}}{\text{s}}\right)$

ρ = fluid density $\left(\frac{\text{kg}}{\text{m}^3}\right)$

μ = fluid viscosity $\left(\frac{\text{kg}}{\text{m} \cdot \text{s}}\right)$

FLUID MEASUREMENTS

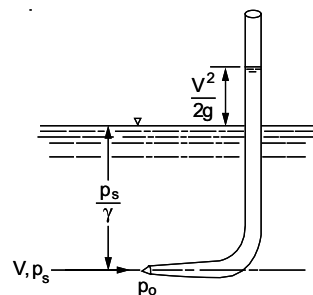
The Pitot Tube – From the stagnation pressure equation for an *incompressible fluid*,

$$v = \sqrt{(2/\rho)(p_0 - p_s)} = \sqrt{2g(p_0 - p_s)/\gamma}, \text{ where}$$

v = the velocity of the fluid,

p_0 = the stagnation pressure, and

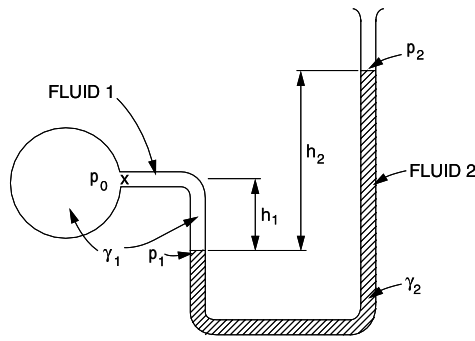
p_s = the static pressure of the fluid at the elevation where the measurement is taken.



For a *compressible fluid*, use the above incompressible fluid equation if the Mach number ≤ 0.3 .

• Vennard, J.K., *Elementary Fluid Mechanics*, 6th ed., J.K. Vennard, 1954.

MANOMETERS



For a simple manometer,

$$p_0 = p_2 + \gamma_2 h_2 - \gamma_1 h_1 = p_2 + g(\rho_2 h_2 - \rho_1 h_1)$$

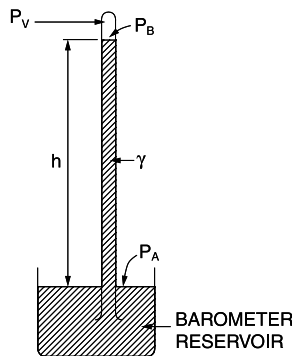
$$\text{If } h_1 = h_2 = h$$

$$p_0 = p_2 + (\gamma_2 - \gamma_1)h = p_2 + (\rho_2 - \rho_1)gh$$

Note that the difference between the two densities is used.

Another device that works on the same principle as the manometer is the simple barometer.

$$p_{\text{atm}} = p_A = p_v + \gamma h = p_B + \gamma h = p_B + \rho g h$$



p_v = vapor pressure of the barometer fluid

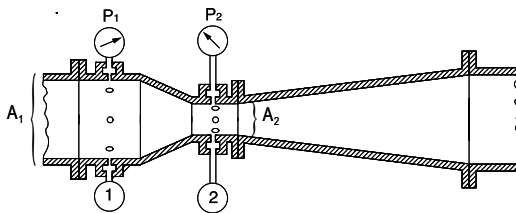
Venturi Meters

$$Q = \frac{C_v A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{2g \left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right)}, \text{ where}$$

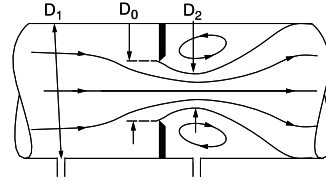
C_v = the coefficient of velocity, and

$$\gamma = \rho g.$$

The above equation is for *incompressible fluids*.



Orifices The cross-sectional area at the vena contracta A_2 is characterized by a *coefficient of contraction* C_c and given by $C_c A$.



$$Q = C A_0 \sqrt{2g \left(\frac{p_1}{\gamma} + z_1 - \frac{p_2}{\gamma} - z_2 \right)}$$

where C , the *coefficient of the meter (orifice coefficient)*, is given by

$$C = \frac{C_v C_c}{\sqrt{1 - C_c^2 (A_0/A_1)^2}}$$

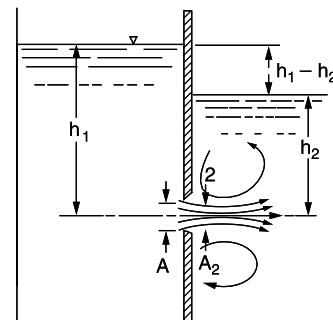
ORIFICES AND THEIR NOMINAL COEFFICIENTS				
	SHARP EDGED	ROUNDED	SHORT TUBE	BORDA
C	0.61	0.98	0.80	0.51
C_c	0.62	1.00	1.00	0.52
C_v	0.98	0.98	0.80	0.98

For incompressible flow through a horizontal orifice meter installation

$$Q = C A_0 \sqrt{2\rho(p_1 - p_2)}$$

Submerged Orifice operating under steady-flow conditions:

• Q

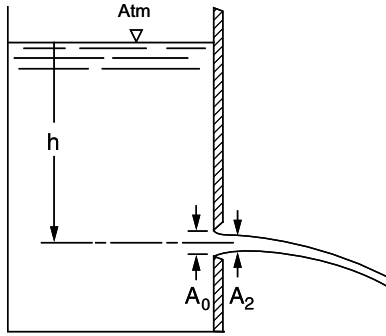


$$Q = A_2 v_2 = C_c C_v A \sqrt{2g(h_1 - h_2)} \\ = CA \sqrt{2g(h_1 - h_2)}$$

in which the product of C_c and C_v is defined as the *coefficient of discharge* of the orifice.

- Bober, W. & R.A. Kenyon, *Fluid Mechanics*, Wiley, New York, 1980. Diagram reprinted by permission of William Bober & Richard A. Kenyon.
- Vennard, J.K., *Elementary Fluid Mechanics*, 6th ed., J.K. Vennard, 1954.

Orifice Discharging Freely into Atmosphere



$$Q = CA_0 \sqrt{2gh}$$

in which h is measured from the liquid surface to the centroid of the orifice opening.

DIMENSIONAL HOMOGENEITY AND DIMENSIONAL ANALYSIS

Equations that are in a form that do not depend on the fundamental units of measurement are called *dimensionally homogeneous* equations. A special form of the dimensionally homogeneous equation is one that involves only *dimensionless groups* of terms.

Buckingham's Theorem: The *number of independent dimensionless groups* that may be employed to describe a phenomenon known to involve n variables is equal to the number $(n - \bar{r})$, where \bar{r} is the number of basic dimensions (i.e., M, L, T) needed to express the variables dimensionally.

- Vennard, J.K., *Elementary Fluid Mechanics*, 6th ed., J.K. Vennard, 1954.

SIMILITUDE

In order to use a model to simulate the conditions of the prototype, the model must be *geometrically*, *kinematically*, and *dynamically similar* to the prototype system.

To obtain dynamic similarity between two flow pictures, all independent force ratios that can be written must be the same in both the model and the prototype. Thus, dynamic similarity between two flow pictures (when all possible forces are acting) is expressed in the five simultaneous equations below.

$$\left[\frac{F_I}{F_p} \right]_p = \left[\frac{F_I}{F_p} \right]_m = \left[\frac{\rho v^2}{p} \right]_p = \left[\frac{\rho v^2}{p} \right]_m$$

$$\left[\frac{F_I}{F_v} \right]_p = \left[\frac{F_I}{F_v} \right]_m = \left[\frac{vl\rho}{\mu} \right]_p = \left[\frac{vl\rho}{\mu} \right]_m = [\text{Re}]_p = [\text{Re}]_m$$

$$\left[\frac{F_I}{F_G} \right]_p = \left[\frac{F_I}{F_G} \right]_m = \left[\frac{v^2}{lg} \right]_p = \left[\frac{v^2}{lg} \right]_m = [\text{Fr}]_p = [\text{Fr}]_m$$

$$\left[\frac{F_I}{F_E} \right]_p = \left[\frac{F_I}{F_E} \right]_m = \left[\frac{\rho v^2}{E_v} \right]_p = \left[\frac{\rho v^2}{E_v} \right]_m = [\text{Ca}]_p = [\text{Ca}]_m$$

$$\left[\frac{F_I}{F_T} \right]_p = \left[\frac{F_I}{F_T} \right]_m = \left[\frac{\rho l v^2}{\sigma} \right]_p = \left[\frac{\rho l v^2}{\sigma} \right]_m = [\text{We}]_p = [\text{We}]_m$$

where

the subscripts p and m stand for *prototype* and *model* respectively, and

F_I = inertia force,

F_p = pressure force,

F_v = viscous force,

F_G = gravity force,

F_E = elastic force,

F_T = surface tension force,

Re = Reynolds number,

We = Weber number,

Ca = Cauchy number,

Fr = Froude number,

l = characteristic length,

v = velocity,

ρ = density,

σ = surface tension,

E_v = bulk modulus,

μ = dynamic viscosity,

p = pressure, and

g = acceleration of gravity.

♦ **PROPERTIES OF WATER^f (SI METRIC UNITS)**

Temperature °C	Specific Weight ^a , γ , kN/m ³	Density ^a , ρ , kg/m ³	Absolute Dynamic Viscosity ^a , μ Pa·s	Kinematic Viscosity ^a , ν m ² /s	Vapor Pressure ^e , p_v , kPa
0	9.805	999.8	0.001781	0.000001785	0.61
5	9.807	1000.0	0.001518	0.000001518	0.87
10	9.804	999.7	0.001307	0.000001306	1.23
15	9.798	999.1	0.001139	0.000001139	1.70
20	9.789	998.2	0.001002	0.000001003	2.34
25	9.777	997.0	0.000890	0.000000893	3.17
30	9.764	995.7	0.000798	0.000000800	4.24
40	9.730	992.2	0.000653	0.000000658	7.38
50	9.689	988.0	0.000547	0.000000553	12.33
60	9.642	983.2	0.000466	0.000000474	19.92
70	9.589	977.8	0.000404	0.000000413	31.16
80	9.530	971.8	0.000354	0.000000364	47.34
90	9.466	965.3	0.000315	0.000000326	70.10
100	9.399	958.4	0.000282	0.000000294	101.33

♦ **PROPERTIES OF WATER (ENGLISH UNITS)**

Temperature (°F)	Specific Weight γ (lb/ft ³)	Mass Density ρ (lb • sec ² /ft ⁴)	Absolute Dynamic Viscosity μ ($\times 10^{-5}$ lb • sec/ft ²)	Kinematic Viscosity ν ($\times 10^{-5}$ ft ² /sec)	Vapor Pressure p_v (psi)
32	62.42	1.940	3.746	1.931	0.09
40	62.43	1.940	3.229	1.664	0.12
50	62.41	1.940	2.735	1.410	0.18
60	62.37	1.938	2.359	1.217	0.26
70	62.30	1.936	2.050	1.059	0.36
80	62.22	1.934	1.799	0.930	0.51
90	62.11	1.931	1.595	0.826	0.70
100	62.00	1.927	1.424	0.739	0.95
110	61.86	1.923	1.284	0.667	1.24
120	61.71	1.918	1.168	0.609	1.69
130	61.55	1.913	1.069	0.558	2.22
140	61.38	1.908	0.981	0.514	2.89
150	61.20	1.902	0.905	0.476	3.72
160	61.00	1.896	0.838	0.442	4.74
170	60.80	1.890	0.780	0.413	5.99
180	60.58	1.883	0.726	0.385	7.51
190	60.36	1.876	0.678	0.362	9.34
200	60.12	1.868	0.637	0.341	11.52
212	59.83	1.860	0.593	0.319	14.70

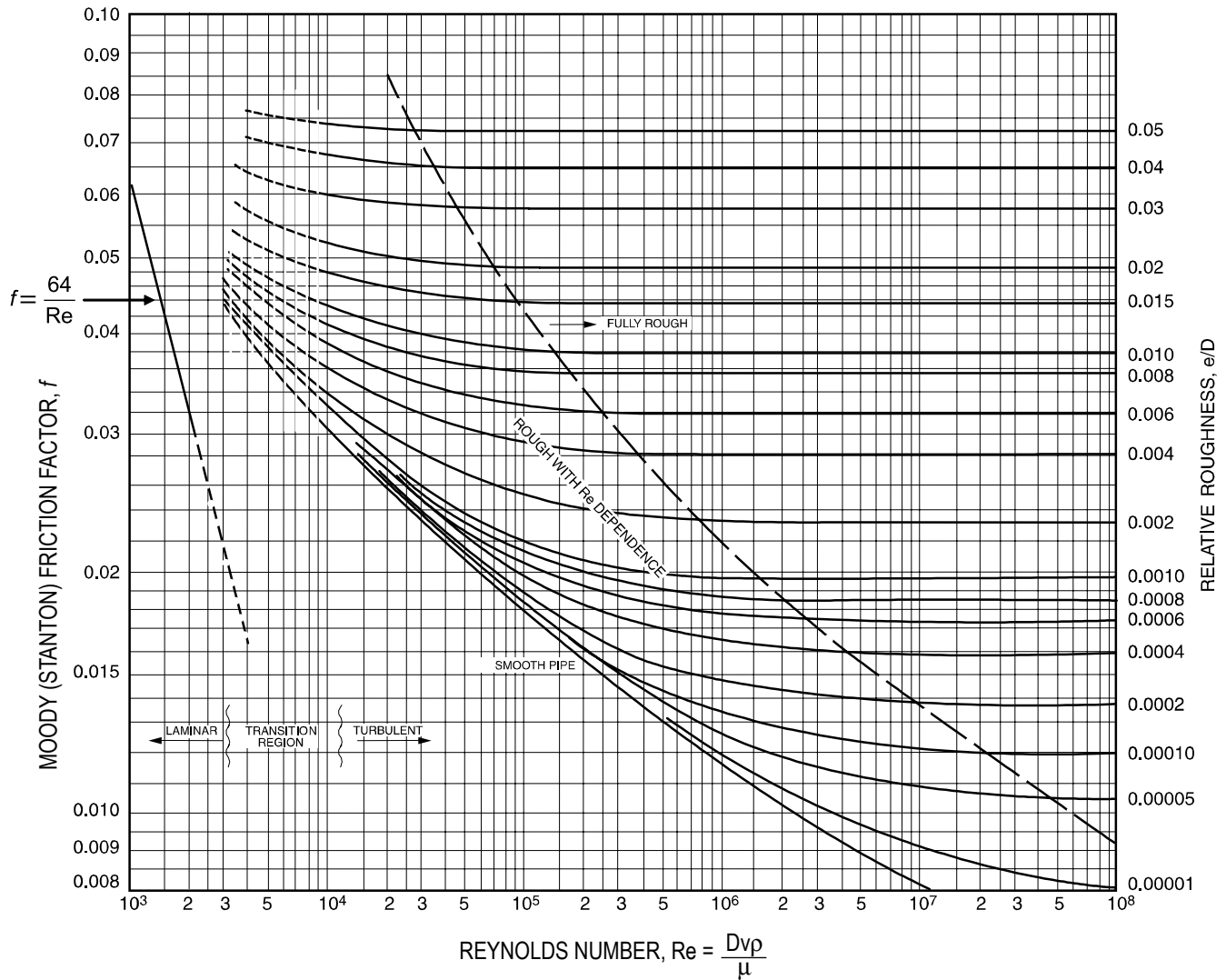
♦ ^aFrom "Hydraulic Models," *ASCE Manual of Engineering Practice*, No. 25, ASCE, 1942.

^eFrom J.H. Keenan and F.G. Keyes, *Thermodynamic Properties of Steam*, John Wiley & Sons, 1936.

^fCompiled from many sources including those indicated: *Handbook of Chemistry and Physics*, 54th ed., The CRC Press, 1973, and *Handbook of Tables for Applied Engineering Science*, The Chemical Rubber Co., 1970. Vennard, J.K. and Robert L. Street, *Elementary Fluid Mechanics*, 6th ed., Wiley, New York, 1982.

MOODY (STANTON) DIAGRAM

Material	e (ft)	e (mm)
Riveted steel	10.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Cast iron	0.00085	0.25
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015



From ASHRAE (The American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.)

DRAG COEFFICIENTS FOR SPHERES, DISKS, AND CYLINDERS

