Observations
on
The Application of Chaos Theory
to
Fluid Mechanics

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Overview

Modern Fluid Mechanics is based on the Navier Stokes equations formulated nearly 200 years ago. These are non-linear, tightly-coupled second order partial differential equations forming a deterministic system to which analytical solutions exist in only a few special idealised cases - not in the real world.

The field of Chaos Theory has shown that all physical systems previously thought to be deterministic in fact have unpredictability built into their very nature due to the unavoidable non-linearity they contain. The mathematical models of classical mechanics are idealised approximations. In reality, absolute prediction of the outcome is not possible - only the probability of an outcome.

This article gives a brief overview of some of Chaos Theory and how we see it applies to both Fluid Mechanics and Computational Fluid Mechanics, with a view to how these disciplines could be re-thought to facilitate technical advance in aerospace and related fields.

Introduction

The Navier Stokes Equations are derived from Newton's Laws applied to the motion of a "fluid particle" usually using a control volume approach. They are non-linear, tightly-coupled second order partial differential equations to which no analytical solution is known to exist - in other words, they cannot actually be solved. They are tightly coupled since all the variables are mutually interdependent - any change in a property of the flow field in a part of the domain has an effect on the rest of the domain which in turn affects the initial source and so on: the whole domain is subject to continuous non-linear feedback.

Thinking about it logically, if the effect of a change is to change the very source of that change our current mathematical formulations cannot analyse such a situation. We are in effect trying to analyse $\frac{\partial x}{\partial x}$ since the subsequent change in x is a function of the original change in x.

The non-linearity of the Navier Stokes Equations means they form a fundamentally unpredictable system which can not be predicted with complete certainty.

The current mainstream approach to Fluid Mechanics and CFD (in the practical arena of industrial application) will remain limited until it integrates the findings of Chaos Theory which have been developing rapidly since the 1950s.

Chaos Theory

Chaos Theory can be defined as follows:

"The study of unstable aperiodic behaviour in deterministic non-linear dynamical systems"

"The ability of simple models, without in-built random behaviour, to generate highly irregular behaviour".

This means that a deterministic dynamical system can in fact generate aperiodic disordered behaviour: that is behaviour with a hidden implicit order.

Lack of predictability is inherent in all deterministic feedback systems.

"A dynamicist would believe that to write down a system's equations is to understand the system. But because of the (little bits of) non-linearity in these equations, a dynamicist would find himself helpless to answer the easiest practical questions about the future of the system[1]".

Classical mechanics and dynamics still believes in determinism, expressed most famously by Laplace:

"Give me the past and present co-ordinates of any system and I will tell you its future".

Determinism has at its heart the classical physics idea of a definite trajectory - applied to a particle.

The mathematical model or concept of the definite particle trajectory is in fact of limited usefulness and must be replaced with the broader quantum mechanical concept of *probability*.

A deterministic system is one that is stable, predictable and completely knowable.

However, in reality deterministic systems can give rise to unstable, aperiodic, apparently random behaviour.

Therefore, the "deterministic" system defined by the non-linear Navier Stokes Equations will produce unstable, aperiodic, unpredictable, irregular behaviour.

This fact lies at the heart of the difficulties with modern Fluid Mechanics and CFD.

[1]Chaos, James Gleick, Heinemann 1988, P44

Example of Chaos

The Pendulum

The equation of motion for the pendulum is to be found in any basic mechanics textbook.

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The pendulum is one of the simplest dynamical systems and its properties have been exhaustively investigated Millions of students have learned this equation and the concomitant "fact" stated by Galileo that the period is independent of the angle of swing or amplitude. However, the above equation is incorrect.

The changing angle of motion of the weight causes non-linearity in the equations, meaning the motion cannot be predicted. At small amplitudes, the error is small but still measurable.

As another example, the motion of a swing being pushed in the playground cannot be predicted. It is a damped/ driven oscillator which can give rise to erratic motion that never repeats itself.

The "noise" in the data from a REAL EXPERIMENT done with a pendulum - where the data does not lie perfectly on a straight line or quadratic curve - is **not experimental error**, but the result of the non-linearity and chaos inherent in all real systems.

Galileo's equation is an idealised simplification of reality - an artificial thought experiment which does not in fact reflect reality.

Modern mechanics is still based on these idealised approximations that the early scientists developed between the Renaissance and the twentieth century.

However, even though these systems can give rise to unpredictable behaviour, there is still frequently pattern and order within the unpredictability - such as the vortices in a turbulent flow. The study of these patterns has given rise to the label of *complexity*.

Iteration

"Modern computing allows scientists to perform computations that were unthinkable even 50 years ago. In massive computations, it is often true that a detailed and honest error propagation analysis is beyond current possibilities and this has led to a very dangerous trend. Many scientists exhibit a growing tendency to develop an almost insane amount of confidence in the power and correctness of computers."

Chaos and Fractals, Peitgen, Jurgens, Saupe

One of the most important findings of Chaos research is the so called Butterfly Effect or Sensitive Dependence on Initial Conditions. This is accredited to the meteorologist Edward Lorenz[2]. Lorenz discovered that minute differences in the starting or initial conditions of his differential equation model of the atmosphere led to completely different results. The Lorenz Experiment proved that in an iterative computer process, no matter how small a deviation there is in the starting values we choose for a computer simulation, the errors will accumulate so rapidly that after relatively few steps the computer prediction is worthless.

In trying to model real systems, the difficulties are even greater since it means we can never measure reality accurately enough: no matter how precise our observations, the initial difference between those measurements and reality will quickly lead to unpredictable divergence between the model and reality.

Consider for instance the quadratic iterator $p_{n+1} = p_n + rp_n(1-p_n)$ with the constant $r = 3$ and $p_0 = 0.01$. The result of evaluating the expression is then fed back into the expression as p_1.

Peitgen, Jürgens and Saupe[3] shows that with two different calculators, one evaluating to 10 decimal places and one to 12, after 35 iterations a significant difference starts to appear between the two outputs which then rapidly escalates. This is the unavoidable consequence of finite accuracy digital mathematics and computers.

Step	CASIO	HP
1	0.0397	0.0397
2	0.15407173	0.15407173
3	0.5450726260	0.545072626044
4	1.288978001	1.28897800119
5	0.1715191421	0.171519142100
10	0.7229143012	0.722914301711
15	1.270261775	1.27026178116
20	0.5965292447	0.596528770927
30	0.3742092321	0.374647695060

[2] Deterministic Nonperiodic Flow, 1963

[3] Chaos and Fractals, 1992, Springer Verlag, P49

35	0.9233215064	0.908845072341
40	0.0021143643	0.143971503996
50	0.0036616295	0.225758993390

After 35 iterations the two machines diverge and their results bear no resemblance to each other. Comparing the CASIO and the HP, the natural tendency is to believe the HP more because it operates to more decimal places. But if we used another machine, say with 14 or 20 decimal places, the same problem would repeat itself with some delay and after maybe 50 iterations we would see divergence between the HP and the new machine - and so on.

Therefore the addition of more decimal places only delays the onset of "chaos" - or unstable behaviour.

Peitgen, Jürgens and Saupe then go on to show what happens if we change the way the expression is evaluated.

$p+rp(1-p)$ can be rewritten as $(1+r)p -rp^2$.

These two different formulations of the same quadratic expression are not equivalent. On the ***same*** calculator, there is a slight difference after 12 iterations and after 35 iterations again, the differences start to become enormous: it is no longer possible to tell which is the "correct" answer.

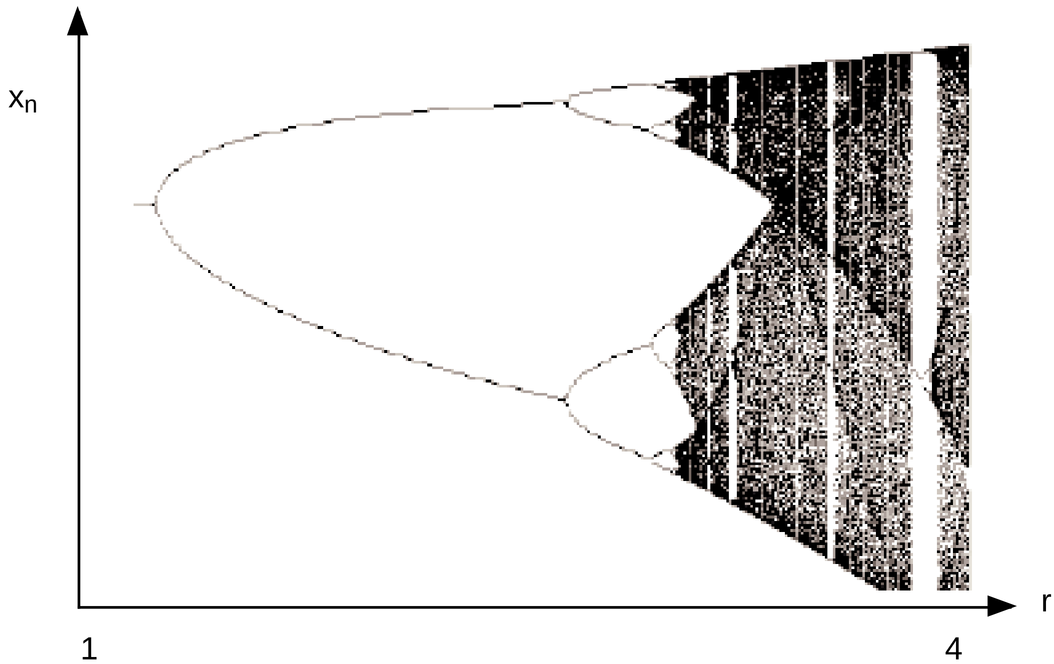
Therefore not only do iterative numerical solutions suffer from the limitations of finite accuracy digital computers but even different mathematical formulations of the same problem are not computationally equivalent and will diverge.

Another famous example is the iteration of the expression $x_{next} = rx(1-x)$, used by the biologist Robert May to model a fish population.

The parameter r represents the rate of growth of the population.

As the parameter is increased, the final population value converges to a higher value too, reaching a final population of $x = 0.692$ at $r = 2.7$.

As r is increased further, x does not converge to a final value but oscillates between two final values; as r increases still further, this doubles again to 4 values and then doubles again and finally becomes completely chaotic with no convergence - but then new cycles appear again in the midst of this.



Final State Diagram for the Logistic Operator $x_{next} = rx(1-x)$ for $1 < r < 4$

This bifurcation of the long term behaviour of the system to no longer converge on one fixed final state but to oscillate between 2, then 4, then 16 etc final states is called period doubling. This then gives way to oscillation between a myriad of states but with certain order re-appearing within the oscillations.

Therefore, in addition to the inability of the digital computer to carry out an accurate iteration due to the propagation of errors, **unstable behaviour is built into the very nature of the non-linear mathematical model**.

Most forces in real life are non-linear.

In addition, feedback is also common in real life systems, like fluid motion. The use of the description "tightly coupled" for the Navier Stokes equations means a change in any part of the domain propagates and has an effect back on the part that changed. This Non-linear feedback is why no analytical solutions exist.

It has been proved (*Feigenbaum*) that chaos is a **universal** property of non-linear feedback systems.

"Somewhere, the business of writing down partial differential equations is not to have done the work on the problem[4]".

It has been found that unpredictability and uncertainty is the rule in nature while predictability is an idealised over simplification.

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[4]Feigenbaum in Chaos, James Gleick, P187.

The Strange Attractor

The so called Strange Attractor can be said to be the trajectory of the long term behaviour of a dynamical system.

One can imagine that it is a visual representation of the hidden forces that create order in a non-linear system within the unstable, unpredictable disorder.

All non linear systems have these attractors. For a simple pendulum the attractor is a point - where the pendulum comes to rest. For the weather system modelled by Edward Lorenz, the attractor takes the form of the famous "butterfly" of two intersecting loops.

The point we wish to make is simply that in a system of simultaneous differential equations, there is a hidden order with a fractal structure that is an inherent feature of that system of equations, independent of the physical systems they are modelling.

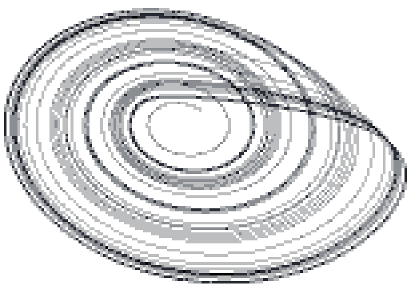
For instance Otto Rössler[5] investigated the following system of differential equations:

$$\frac{\partial x}{\partial t} = -(y+z)$$

$$\frac{\partial y}{\partial t} = x + ay \qquad \text{where a, b, c are constants}$$

$$\frac{\partial z}{\partial t} = b + xz - cz$$

The only non-linearity is the xz term in the third equation: yet a plot of the trajectory (x,y,z) of the numerical integration of this set of equations shows that for certain parameter values, the solution does not converge onto a single final result but onto a complex folded loop.



Rössler Attractor

[5]An Equation for Continuous Chaos, Otto E Rössler, Phys. Lett 57A (1976)

Computational Fluid Dynamics

CFD is concerned with producing a numerical analogue of the defining partial differential equations of fluid motion. This process is called numerical discretisation.

There are 3 major approaches:

1. The Finite Difference Method
2. The Finite Element Method
3. The Finite Volume Method

With the Finite Difference method for instance, we ignore all terms of the third order or higher in the Taylor Series approximation. So right from the beginning, a numerical approximation is introduced and from sensitive dependence on initial conditions, this error can grow as iteration proceeds, producing a different result each time.

Techniques such as von Neumann stability analysis are used to study the stability of these linear difference equations (without taking into account Chaos). But for example even with the Euler Explicit Form of the simplest one dimensional wave equation

$$\left(\frac{\partial u}{\partial t}\right) + c\left(\frac{\partial u}{\partial x}\right) = 0$$

the von Neumann stability analysis shows that this equation leads to an unstable solution no matter what the value of the time step Δt. It is unconditionally unstable.

CFD seems to be largely concerned with the design of mathematical tricks to overcome these computational problems. Anderson[6] shows the Lax method of therefore replacing the time derivative $\frac{\partial u}{\partial t}$ with a first order difference "where u(t) is represented by an average value between grid points i+1 and i-1, i.e.

$$u(t) = \frac{u_{i+1}^{n} - u_{i-1}^{n}}{2}$$

But this is actually a first order spatial difference u(x) calculated from the average value of u at spatial grid points i+1 and i-1 at the same time step n.

The average value of u with respect to time can only be calculated as a first order difference at one spatial grid point at successive time steps n and n+1, i.e.

[6] Computational Fluid Dynamics, JD Anderson, McGraw Hill 1995, P162

$$u(t) = \frac{u_i^{n+1} - u_i^{n-1}}{2}$$

So the Lax method to develop a Courant number is (in this taught example at any rate) based on a dubious foundation.

Error analysis also tends to assume that errors will follow a certain form, normally assumed to be exponential. Not only are we trying to model the underlying physical reality imperfectly with idealised non-linear PDEs, we then try to model the errors in their numerical solution with another mathematical (exponential) model. Exponential error propagation can be considered to indicate sensitive dependence on initial conditions.

Peitgen, Jurgens and Saupe state:[7]

"The relation of the original differential equation to its numerical approximation is very delicate - the stability conditions show that. Changing over to a discrete approximation may change the nature of a problem significantly, a fact which has only entered the consciousness of numerical analysts quite recently. This is another merit of chaos theory".

To start any CFD simulation, the user has to specify the Initial Conditions and the Boundary Conditions. Therefore a major assumption is introduced right at the beginning that we know what those conditions are.

In fact we know from sensitive dependence, that even the smallest variation in the data for the initial conditions will cause huge differences in the predicted results: but it is impossible to specify the initial conditions because infinite accuracy is required and the data is always uncertain.

No matter how fine the grid, uncertainty will always come into play.

Convergence & Stability

Convergence is the ability of a set of numerical equations to represent the analytical solution if such a solution exists.

If the numerical solution tends to the analytical solution as the grid spacing tends to zero, the numerical and analytical solutions converge.

This process is stable if during convergence the errors do not swamp the results.

Now, there are no analytical solutions to the Navier Stokes equations, except for a few idealised situations. Therefore convergence in CFD terms has come to mean whether the iterative solver tends towards a particular value: the user has to decide if that value is a realistic result and a valid solution. Given the

[7] Chaos and Fractals, P683

presence of a Strange Attractor in all non-linear dynamic systems and different final states, bifurcations and period doublings inherent in this type of process, whether the solver converges or not is meaningless. It is itself a process subject to the hidden laws of Chaos.

The fact that Δt in the explicit formulation of the finite difference method has to be sufficiently small to prevent the process becoming unstable - what does that say about the fundamental validity of the approach? It is not a UNIVERSAL application.

In physical terms one can see that there is an inherent timescale in which fluid interactions take place and create their effects on say an immersed body. What are these timescales and where do they come from?

One can use the implicit formulation instead (just as we formulated the quadratic iterator in two different ways). Chaos has shown that the implicit and explicit formulations are not numerically and computationally equivalent. The iterative methods required to solve the numerical equations also require an initial guess to be made to the solution, which is not an independent scientific method.

It is well known that even if a converged numerical solution is found to a Taylor Series approximation of a function, we do not know what function has converged on that point.

All the time, the CFD user has to know what the results should be roughly, to determine if the computed result is realistic or not. This is not real simulation but simply "copying" nature and shows the mathematical models and numerical approximations of them are seriously flawed.

"The Navier Stokes equations are particularly difficult to discretise and solve using numerical techniques". Indeed - because they have no analytical solutions. Because they are complex non-linear **feedback** systems that cannot be solved mathematically and indeed are based on a flawed view of fluids and Newtonian Motion to start with.

Discretisation tries to linearise these non-linear partial differential equations to create simultaneous numerical equations which are hopefully more amenable to solution.

"The non-linearity of the problems forces the use of an iterative solution - we cannot use a direct tridiagonal matrix method for instance. Because we have to then find a solution to those numerical equations that is both converged and resolves the non-linearity." *CT Shaw, Understanding Fluid Mechanics*

Therefore we try to use simplified approximations. It might give us some useful results - or it might not represent what is really going to happen with a real aircraft, ship or car at all. The equations may converge to several oscillating solutions or to a complex aperiodic state - all of these are a correct answer.

Which reflects "reality" - a reality which itself displays the same unpredictable bifurcations and oscillations?

If we completely rely on this approach and the Cartesian mental fixation with finding the "one right answer" - as is increasingly the case in modern engineering design with digital computer modelling tools - a very dangerous trend will develop.

For example: a dangerous resonant swing was engendered in the new Millennium Footbridge over the River Thames in London opened in 2000. The bridge had to be closed and modified, because the "random" motion of pedestrians set up a resonance which then in turn forced their walking into a pattern in phase with that resonance which then amplified the effect and so on: a complex non-linear feedback system that could have destroyed the bridge. The bridge was designed ***exclusively*** with computer modelling techniques which did not predict this. Clearly there was more "order" in the initial "random" walking of the pedestrians than the model took into account: something someone familiar with Chaos Theory would have foreseen since hidden order is the rule in Nature.

CFD - Conclusion

The current approach to CFD has its uses. It has been refined empirically to a point where it can now produce models that are a reasonable and useful reflection of reality for well known and accepted geometries and flow regimes.

However, this is the fundamental drawback: it can only predict what is already known. If the user does not know what result is "reasonable" or "what to expect" he does not know whether the result is useful or not. To get the solving process to converge the user often has to input the likely end result beforehand. This can not be described as a rigorous or really even an acceptable scientific method. It is more akin to following a kitchen recipe than carrying out a scientific experimental procedure.

Within the domain of what we already know about fluid behaviour - experimentally - it provides a useful tool for technicians to apply current known techniques. It may not provide accurate guidance if asked to explore outside the current known parameters and worse could lead to our knowledge and investigative spirit stagnating.

To quote Professor Charles L Fefferman, Princeton University, Dept of Mathematics:

"There are many fascinating problems and conjectures about the behaviour of solutions of the Euler and Navier Stokes equations. Since we do not know whether these solutions exist, our understanding is at a very primitive level. Standard methods from PDE appear inadequate to settle the problem. Instead, we probably need some deep new ideas".

Libchaber's Experiment - Helium in A Small Box

In 1977 the French physicist Albert Libchaber set out to design an experiment to investigate the onset of turbulence.

The apparatus consisted of a cuboid reservoir $1mm^3$ machined out of stainless steel, filled with liquid helium. Convection was produced in the helium by heating the bottom of the box by one thousandth of a degree C. This is the classic system known as Rayleigh - Bénard convection. The dimensions of the cell were chosen to allow only two convection rolls to form. The fluid was supposed to rise in the middle, flow out to left and right and descend on the outer surfaces of the cell.

However even this simplest of fluid dynamics experiments demonstrates the key characteristics of chaos: bifurcations and period doubling.

If even the most tightly controlled experiment of this nature still demonstrates chaotic behaviour - that is complex unpredictable disorder with elements of order - can current simulation approaches ever model complex real flows without taking chaos into account?

"Computer simulations break reality into chunks: as many as possible but always too few. No computer today can completely simulate even so simple a system as Libchaber's liquid helium cell. A real world fluid, even in a stripped down millimeter cell, has the potential for all the free motion of natural disorder".

Libchaber has stated:

"Computer simulations help to build intuition or refine calculations, but they ***do not give birth to genuine discovery****."*[8]

[8]Chaos, James Gleick, P210

Viscosity and Friction

In Mechanics and Dynamics, friction (of a motor car for instance driving on a particular surface) cannot be assigned a constant since it depends on the speed of the vehicle. The friction is non-linear. Classical mechanics does use a "co-efficient of friction" but his is again a linear classical approximation.

Viscosity is friction for a fluid. Viscosity is independent of the density of the fluid. The logical inconsistency of this fact that the viscosity of a particular gas is independent of its density - i.e. of a variation in the number of molecules of gas per unit volume when it is supposed to be the effect of those molecules sticking onto an immersed body which creates friction drag - is not explained. The fact that viscosity is not independent of temperature is a further paradox.

The viscous drag on a body moving in a fluid depends on the square of its velocity, where the viscosity or friction of the fluid is assumed to be a constant (at a given temperature) in classical fluid mechanics. However, in physical terms, the friction or viscous drag exerted by the fluid on the body is due to some inherent property of the fluid acting on the body, which is independent of the velocity of that body: it is simply that we can easily observe the velocity of the body and the friction upon it and create a mathematical relationship, e.g. the following well known sequence of equations for laminar shear stress, coefficient of friction and friction drag:

$$\tau_0 = k\mu \frac{U_0}{x}\sqrt{\mathrm{Re}_x}$$ k=0.332 for the Blasius solution

$$C_f = \frac{\tau_0}{\rho U_0^2 / 2}$$

$$F_s = C_f B L \rho U_0^2$$

However, relying on these mathematical *models* creates a tendency to forget the physical mechanisms at work: the friction drag observed is a non-linear complex resultant interaction: while it is understandable how the idea of a constant viscosity was derived it is clear that the inherent property of a fluid which creates friction drag is not a linear constant.

Viscosity is measured over a "linear" regime in a rotary viscometer for most fluids. It is defined as shear stress (of the fluid on a reference area) divided by rate of shear strain.

A viscometer, or Taylor Couette Apparatus, consists of a fixed outer cylinder with a rotating inner cylinder between which is a thin layer of fluid. At low rotation speeds, the fluid tracks the rotation and moves in a circle around the axis of rotation. This is the regime used to determine the coefficient of viscosity for fluids.

However, this velocity gradient is not linear. The standard parabolic velocity gradient of the boundary layer for laminar flow is of course not linear and shows that the kinetic energy dissipated by the fluid is non-linear. Viscosity is of course defined in terms of the shear stress divided by rate of shear strain at the boundary wall but this is a classical linear approximation. The gradient is in fact not linear and therefor the classical definition of viscosity as a "constant" for a fluid is an approximation.

The current model of classifying flow into "different regimes" is scientifically limiting. Chaos Theory has shown that the onset of turbulence, instability and complex motion in fluid flow cannot be predicted: certain patterns appear but they are not completely quantifiable or predictable. Only probabilities can be assigned.

Flow is a continuum, from the lowest to the highest velocities, with certain general patterns mostly observed in certain general "speed regimes" but there is no fundamental distinction between "laminar" and "turbulent" flow as taught by classical fluid mechanics. Therefore models which try to distinguish between them are missing the point: whatever it is that "tells" the fluid to change from a highly ordered flow to a much more complex disordered flow, but which still contains elements of aperiodic order, lies beyond the realm of mathematics. It lies in the implicit order behind the phenomena we observe. This is where we need to look to find solutions to our engineering problems.

The Problem with Calculus

Differential Calculus is now 300 years old. It is based on the concept of taking smaller and smaller "steps" or changes to model in a linear way systems represented by differential equations.

We are all familiar with the expression "as dx tends to 0".

Basic calculus at school and undergraduate level usually involves idealistic, simplified mathematical models that differentiate or integrate nicely to definable results.

However, the geometry of nature is fractal. In fact, the universe itself is fractal across all scales.

A well known example is the famous question: "How Long is the Coastline of Britain?"

The answer is: it depends on your scale of measurement. The smaller your scale of measurement, i.e. the smaller your dx, the longer the coastline. Due to its fractal nature, the length of the coastline of Britain is in fact mathematically **infinite**. dx can never be made small enough to capture the "real" length of the coastline.

Therefore, in dealing with the study of nature, which is what science and engineering is, we are dealing with fractal phenomena. Calculus cannot handle this: it can handle only standard finite definite geometry, not the real fractal geometry of nature. It is again an approximation, and that gap in knowledge of the "initial conditions" or uncertainty will always lead to unpredictability in our mathematical models.

Turbulence

It is well known now that turbulent flow is described not by superposition of many modes or the buildup of frequencies (as postulated by Lev D. Landau in the 1930s) but by Strange Attractors. Turbulence arrives in a sudden transition, not in the continuos piling up of different frequencies.

Therefore there are inherent structures that distinguish turbulence from true randomness: turbulent flow is not completely random but subject to a force of hidden implicit order which we have yet to fully identify.

Turbulence modelling in CFD takes no account of this hidden implicit order in Nature which gives rise to ordered vortices, vortex rings and vortex streets within even the most disordered turbulent systems.

The mathematician Benoit Mandelbrot who first brought the fractal characteristics of nature to public attention argues that turbulence has fractal geometry, i.e. is self similar across different scales and could be modelled with a fractal approach.

The famous paper published by Ruelle and Takens (On The Nature of Turbulence, 1971) showed that the trajectories of fluid particles at the onset of turbulent flow can be described by strange attractors which themselves have fractal form. Rather than using the NS Equations to model turbulence, they proposed that just three independent motions cause all the complexities of turbulence where x describes a fluid in turbulent motion as quasi-periodic functions of time:

$$x(t) = f(\omega_1 t, \ldots., \omega_k t) \qquad y(t) = f(\omega_1 t, \ldots.., \omega_k t) \qquad z(t) = f(\omega_1 t, \ldots.., \omega_k t)$$

Much of the mathematics in this landmark paper is incorrect but its postulation of the strange attractor was a watershed.

Uncertainty

Science is now at last admitting that the last 300 years of classical physics is the idealised approximation that it clearly is and not a determined set of absolute Laws.

"We need a new formulation of the fundamental laws of physics. Probability plays a role in most sciences. Still, the idea that probability is merely a state of mind has survived. We now have to go a step further and show how probability enters the fundamental laws of physics, whether classical or quantum".[9]

The deterministic view of classical physics is that once the initial conditions are known, everything that follows is automatically determined. Nature is an automaton. All processes are time reversible.

This view is still the basis of Computational Fluid Dynamics. CFD takes the deterministic Navier Stokes Equations, discretises them and specifies the initial and boundary conditions at all points in the domain of interest.

The whole finite difference/ finite element/ finite volume approach believes that the state of any position in the domain, both in space and in time, then follows automatically from this specified initial state of all the positions in the domain.

We know that this is not true. Therefore, we know that CFD is a highly simplified and unrealistic approach.

Nature is Non-linear, complex, disordered and uncertain, giving rise to self organisation. Most of the "differential equations" used to mathematically model nature do not have solutions and in any case are modelling the wrong thing anyway. They are idealised approximations, that are analytically unsolvable, are themselves chaotic systems and are "solved" on computers using digital techniques that are also subject to chaos! Chaos upon chaos upon chaos!

There is no such thing as determinism. There is only probability, uncertainty and hidden order.

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[9] The End of Certainty, Ilya Prigogine, Éditions Odile Jacob, 1996, P16

Reversible vs Irreversible Processes

This section quotes extensively from "The End of Certainty" by the Nobel Laureate Ilya Prigogine.

The 19th century left us with a dual heritage - that of a time reversible deterministic universe and that of an evolutionary universe associated with entropy.

Newton's Laws describe a time reversible universe. According to classical dynamics and mechanics, all processes are time reversible; it does not matter if we change t for -t in the equations.

Thermodynamics deals specifically with irreversible time oriented processes, such as radioactive decay or the effect of viscosity. They have a direction in time and are irreversible dissipative processes that are said to increase "entropy" while reversible processes such as the motion of a frictionless pendulum are the same in past and present: they are time symmetrical.

Nature involves both Time Reversible and Irreversible processes but irreversible ones are the rule and reversible ones the exception: reversible ones are idealisations.

The distinction between time reversible and irreversible processes was introduced through the concept of entropy associated with the so-called Second Law of Thermodynamics. According to this "law", irreversible processes produce entropy while reversible ones do not.

However, according to the "fundamental laws of physics" there should be no irreversible processes. So we have two conflicting views of nature from the 19th century.

Boltzmann's Probability based interpretation makes the macroscopic nature of our observations responsible for the irreversibility we observe in reality.

He gave the example of two boxes connected by a valve, one at high and the other at low pressure. When the valve is opened, the pressure equalises, irreversibly. We do not see the pressure in one box or the other ever increase and the other decrease again spontaneously.

Boltzmann said "If we could follow the individual motion of the molecules, we would see a time reversible system in which each molecule follows the laws of physics".

"Because we can only describe the number of molecules in each compartment, we conclude the system evolves towards equilibrium - so irreversibility is not a basic law of nature but merely a consequence of the approximate macroscopic nature of our observations".

Prigogine than also quotes from "The Quark and the Jaguar", in which it is argued that because there are statistically so many more ways for the gas molecules to order themselves in equilibrium rather than in a state of low and high pressure, that is what we will tend to see. It is argued that (theoretically) if you continue to watch long enough, the two boxes will return to their initial state spontaneously.

Therefore the explanation for irreversibility is that there are more ways for disorder to occur than for higher degrees of order. This implies that it is our own ignorance, our "coarse graining" that leads to the Second Law of Thermodynamics. For Laplace's Demon, a well informed observer, the world would appear totally time reversible.

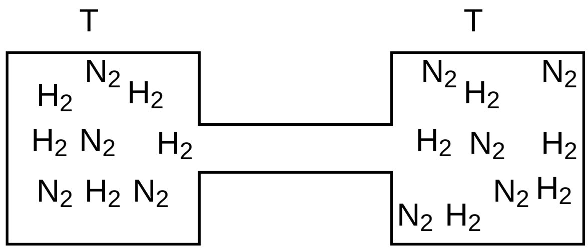
Max Planck disagreed: in his "Treatise on Thermodynamics" he wrote:

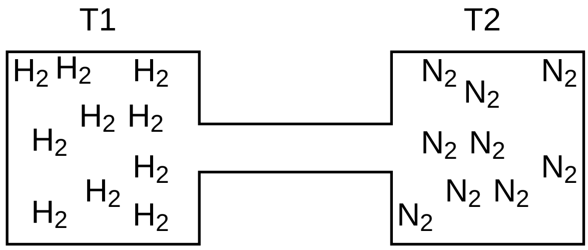
"It is absurd to assume the validity of the Second Law depends on the skill of the physicist or chemist in observing or experimenting. The law has nothing to do with experiment: it asserts that there exists in nature a quantity which always changes in the same way in all natural processes. The limitation of the law, if any, must lie in the same province as its essential idea, in the Observed Nature and not in the Observer."

The Laws of Physics as formulated in the traditional way describe an idealised, stable world quite different from the unstable evolving world in which we live. ***The main reason to discard the banalisation of irreversibility is that we can no longer associate the arrow of time only with an increase in disorder***. Recent developments in non-equilibrium physics and chemistry show that the arrow of time is a ***source of order***.

Take the following experiment, well known in the 19th century.

A simple Thermal Diffusion Experiment. Two boxes are connected by a tube, forming one interconnected volume. The box contains a mixture of Hydrogen and Nitrogen gas at the same temperature T. One box is heated to temperature T1 >T and the other cooled to temperature T2 <T.





The system evolves to a steady state in which the Hydrogen is concentrated in one box and the Nitrogen in the other.

So the *entropy* produced by the *irreversible heat flow* from one box to the other leads to an *ordering* process.

(Therefore the Second "Law" of Thermodynamics is incorrect, since the natural process has created increased order .)

In the world of Deterministic Chaos, Laplace's Demon can no longer predict the future unless he knows the initial conditions with infinite precision. Only then can he continue to use a trajectory description. But there is an even more powerful instability that leads to the destruction of trajectories, <u>**whatever the precision of the initial description**</u>. This form of instability applies to both classical and quantum mechanics.

This instability is DIFFUSION.

For integrable systems where diffusive contributions are absent, we can come back to a trajectory description but in general, **the Laws of Dynamics have to be formulated at the level of Probability Distributions**.

"The basic question is therefore: in which situations can we expect the diffusive terms to be observable? When this is so, probability becomes a basic property of nature. This question, which involves defining the limits of the validity of Newtonian Dynamics is nothing short of revolutionary. For centuries, trajectories have been considered the basic, primitive object of classical physics. In contrast, we now consider them to be of limited validity for resonant systems."

(A resonant system is any system in classical mechanics of the periodic motions of different bodies. In other words, most of classical mechanics is resonant).

For *transient* interactions (e.g. a beam of particles collides with an obstacle and escapes) the diffusive terms are negligible. But for *persistent* interactions (e.g. a steady flow of particles falls onto an obstacle) they become dominant.

The appearance of the diffusive terms for persistent interactions means the breakdown of the Newtonian as well as the orthodox quantum mechanical descriptions.

Every theory that we have today is based on physical concepts that are then expressed through mathematical idealisations.

We discover fluctuations, bifurcations and instabilities at all levels. Stable systems leading to certitudes are only idealisations. The world is NOT made of stable dynamical systems.

Conclusion

Current approaches to numerical and analytical modelling of fluid flow are seriously deficient. They can only predict what is already known and cannot model the real disordered but still ordered complexity of real processes.

It is clear that both real physical processes and the current digital computer systems used to model them are both subject to hidden laws of nature which cannot be expressed by Newtonian Dynamics.

Indeed, Newton's Second Law F=ma is defined for acceleration of a particle by a force in a straight line. However, there is no such thing as straight line motion anywhere in the known universe. Therefore F=ma is itself an idealised classical approximation which from sensitive dependence on initial conditions will inexorably lead to divergence between models based on this foundation and reality.

Therefore we need a way of formulating fluid motion that is more realistic, which captures the natural chaotic order and influences which affect all natural non-linear processes.

Is such a formulation possible?

We believe that it is and here present our views on how to achieve the above breakthrough.

Current mechanics models everything in terms of particles. Most of fluid mechanics is based on treatment of a fluid "particle" or element, though how such a thing can be defined is never explained. The very idea of taking the solid matter concept of Shear Modulus and applying it to a fluid, where in fact a linear "rate of shear strain" cannot exist and a fluid "particle" cannot be defined to deform like the classical solid block, is logically flawed. Quantum Theory proved decades ago that tangible matter does not exist but is composed of wave fields, not particles.

All of motion is in fact wave motion. Matter is waves. Motion is waves. Thought is waves. All is waves.

The universe is in fact holographic. The appearance of matter is created by the interference of waves: matter is an interference pattern. Hence the fractal nature of Nature. A holographic interference pattern is fractal. Waves are fractal because they are harmonic. If the frequency of a wave is doubled the scale has changed but the pattern is the same. That is why the octave of a note sounds the same as its subharmonic. It has the same fractal pattern, just a different scale. Our ear even emits sound which interferes with sound waves we hear - i.e. hearing is a holographic interference process. Artificial Holophonic Sound recordings sound like a real life event happening right there by the listener, not a recording coming out of the speakers.

Most people are familiar with the three dimensional reality of holographic images.

Therefore to create faithful engineering models - we need to create a dynamic holographic wave modelling system that uses WAVES. Not numerical equations and artificial analogues of waves - that is like trying to smell a cup of coffee from its photograph. We need to use a harmonically scaled analogue of the real wave system (the engineering artifact we want to build in its natural environment where it will operate) using a wave modelling system that uses the same waves on a smaller scale: it will have to be on the same fractal scaling as the "real" system, i.e. a subharmonic. Otherwise, it will not give the same results. And even so, there will still be uncertainty in the results because that is the nature of Nature. But we can get as close as it is possible to get.