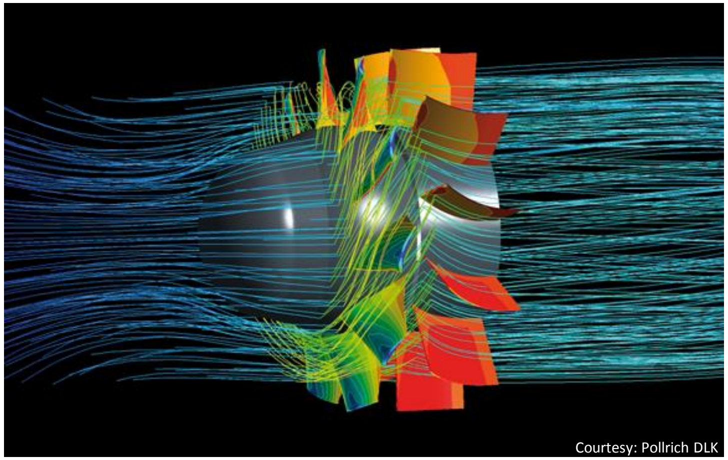
Fluid Mechanics & Aeroacoustics of Fans and Compressors


Courtesy: Pollrich DLK

Day 1: Axial Flow Compressors & Fans

Short Course Offered at BCAM– July 2-4, 2013

Farzad Taghaddosi, Ph.D.

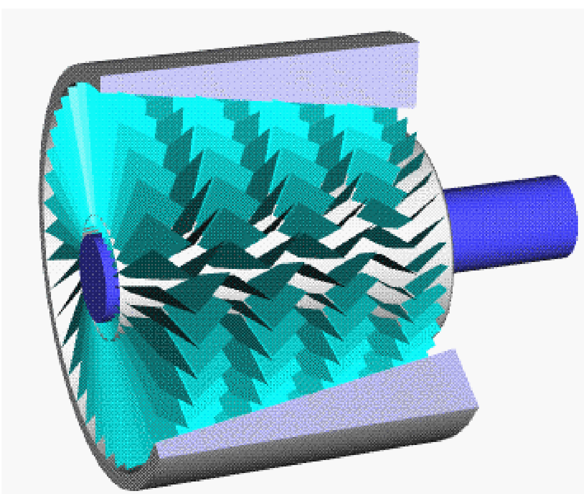
Course Objective

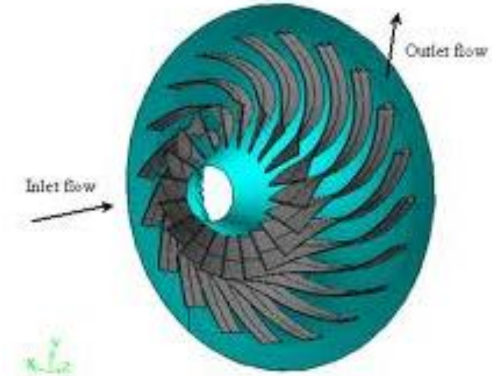
- Provide basic understanding of fluid flow in compressors and fans (axial & centrifugal)

- Understand sources of noise and methods for acoustic analysis

Definition

- Compressors & fans:
  - Use input mechanical energy to increase fluid total pressure
  - Fans ($\Delta p \sim 0.01$ atm), compressors ($\Delta p \geq 1$ atm)
  - Configs: axial, radial/centrifugal, or mixed Flow





Choosing the Right Fan/Compressor

- Performance variables: $w_{stage}, \eta, \Delta p = f(\dot{m}, \rho, N, D, \mu, a)$

- Non-dimensional form: $\psi, \eta, \frac{p_{02}}{p_{01}} = f(\phi, Re, M)$

  – Where $\psi = \dfrac{w_s}{N^2 D^2}$ (loading coefficient) $\qquad \phi = \dfrac{\dot{m}/\rho}{N D^3}$ (flow coefficient)

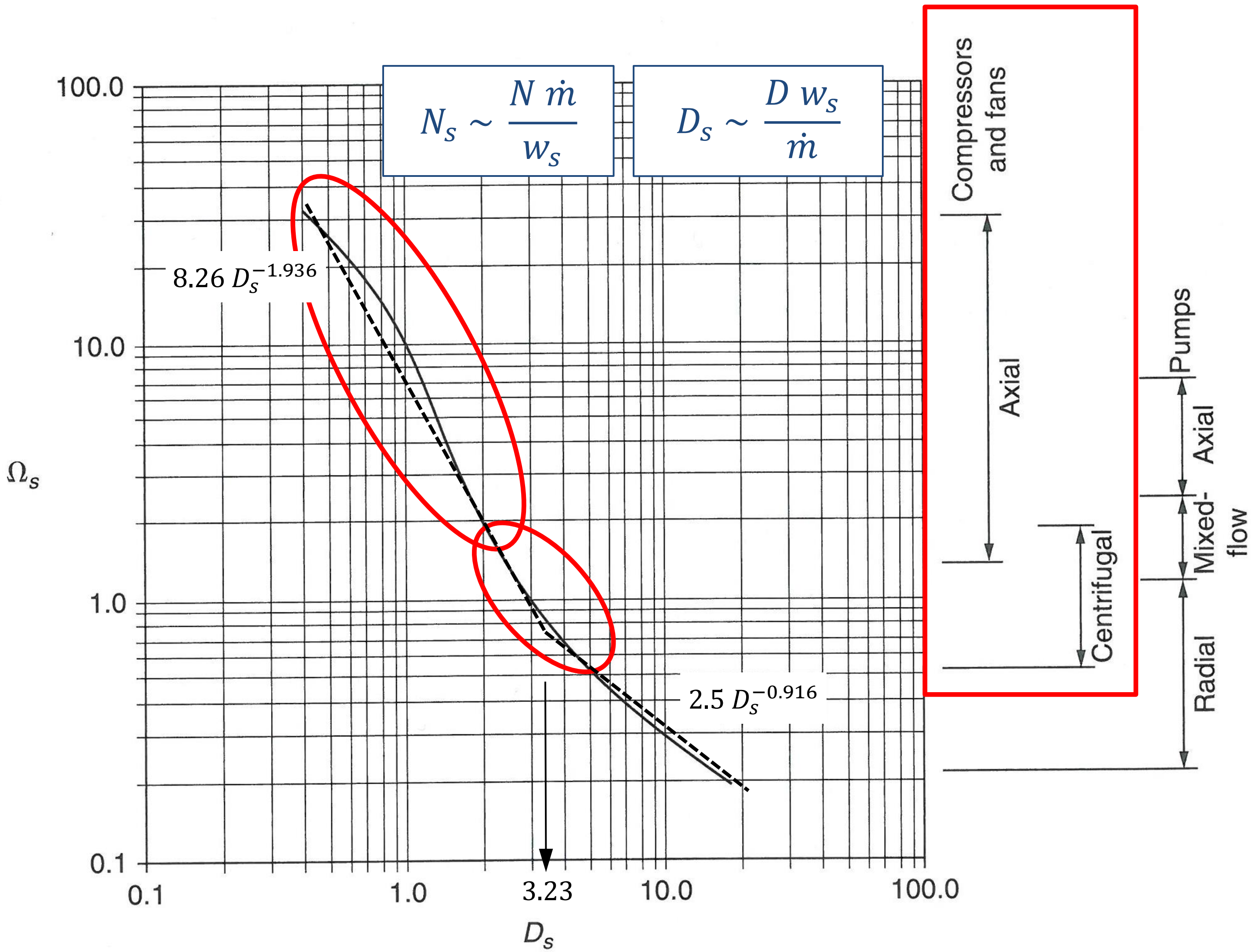
- Specific speed

$$N_s = \frac{\phi^{1/2}}{\psi^{3/4}} = \frac{N(\dot{m}/\rho)^{1/2}}{{w_s}^{3/4}}$$

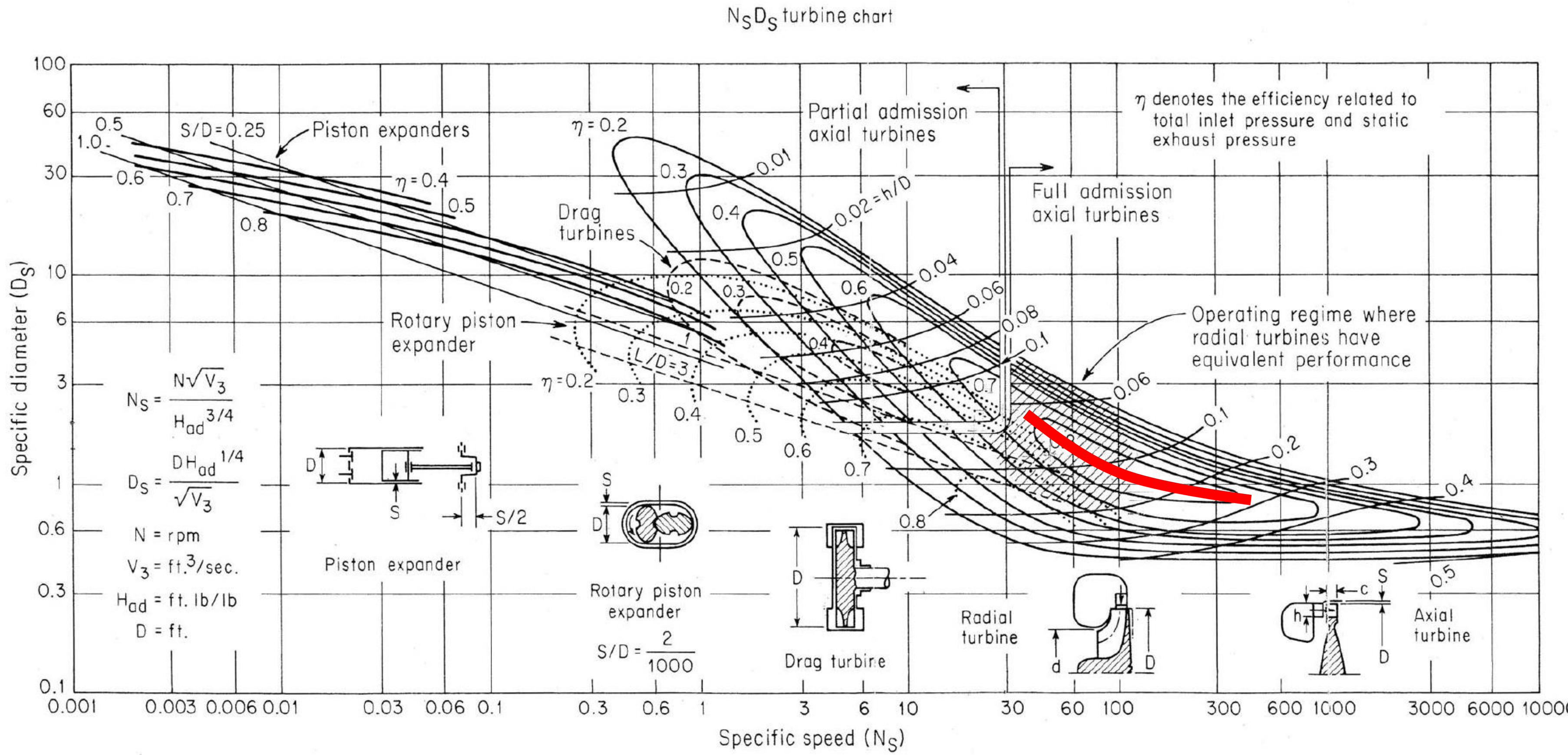
- Specific diameter

$$D_s = \frac{\psi^{1/4}}{\phi^{1/2}} = \frac{D\,{w_s}^{1/4}}{(\dot{m}/\rho)^{1/2}}$$

Cordier Line/Diagram



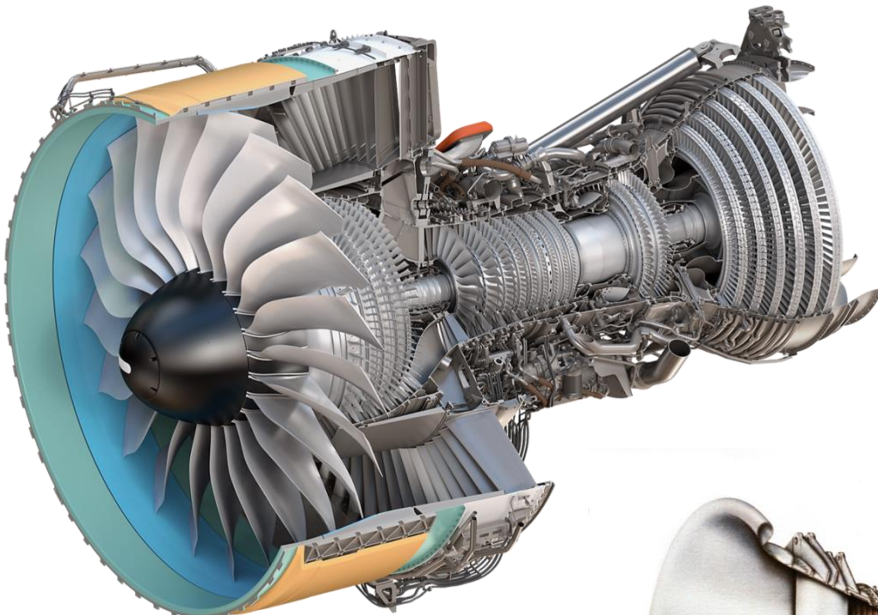
Sample $N_s D_s$ (Balje) Diagram

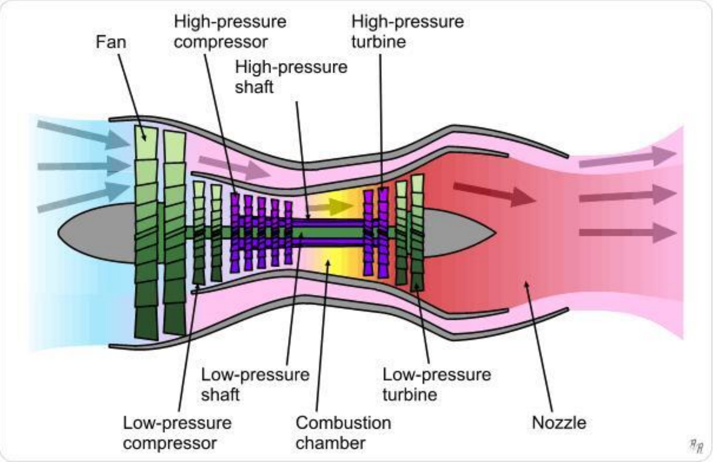


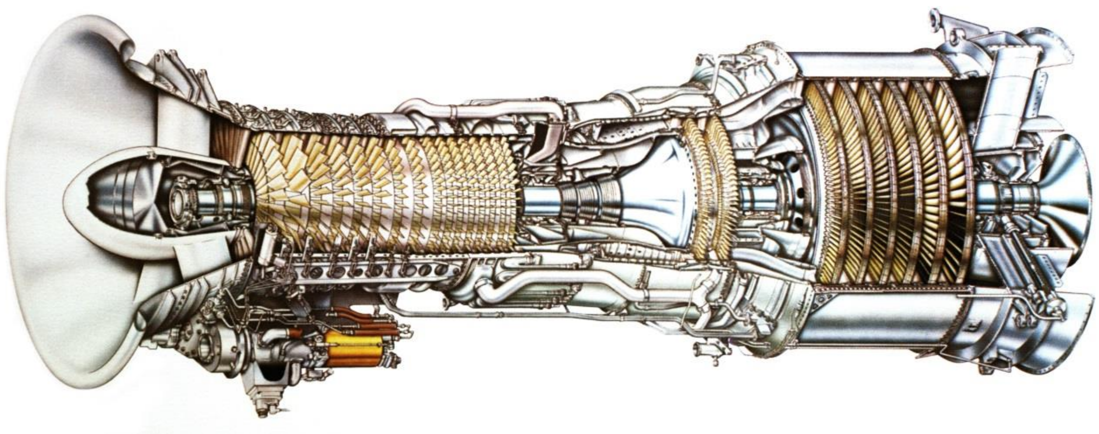
Axial-flow Compressors

Axial-flow Compressors

- Applications: Industrial gas turbines, aircraft engines



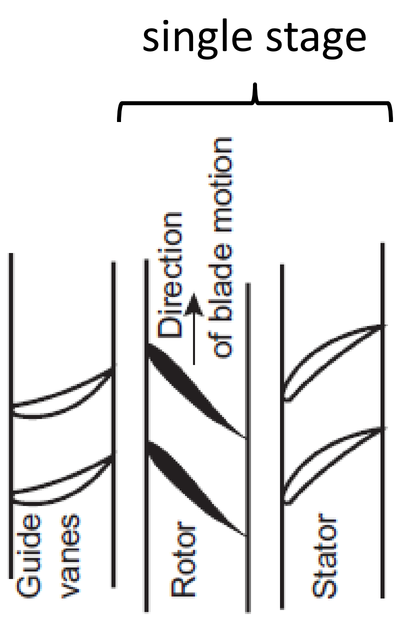


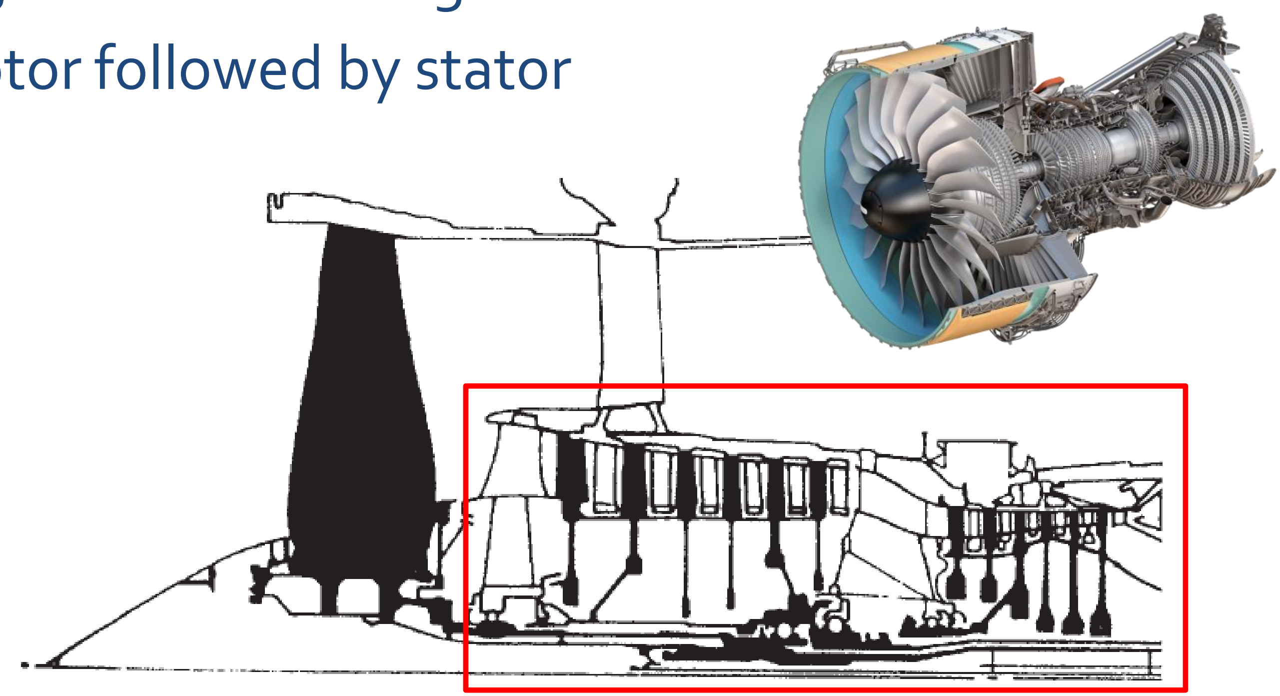


http://www.youtube.com/watch?v=CXSi4GXUojo

Axial-flow Compressors

- Best for applications with high N_s and low D_s
  - High mass flow w/ relatively small Δp per stage
  - Therefore large number of stages are needed
  - Each stage: rotor followed by stator

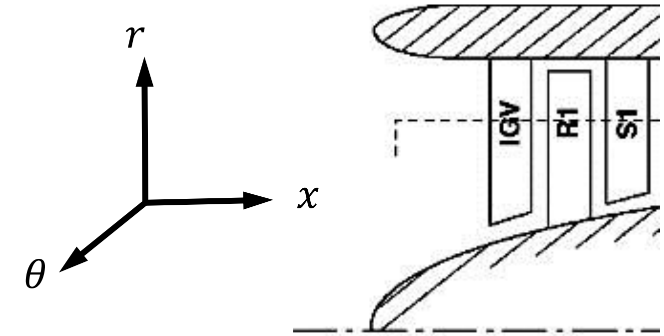


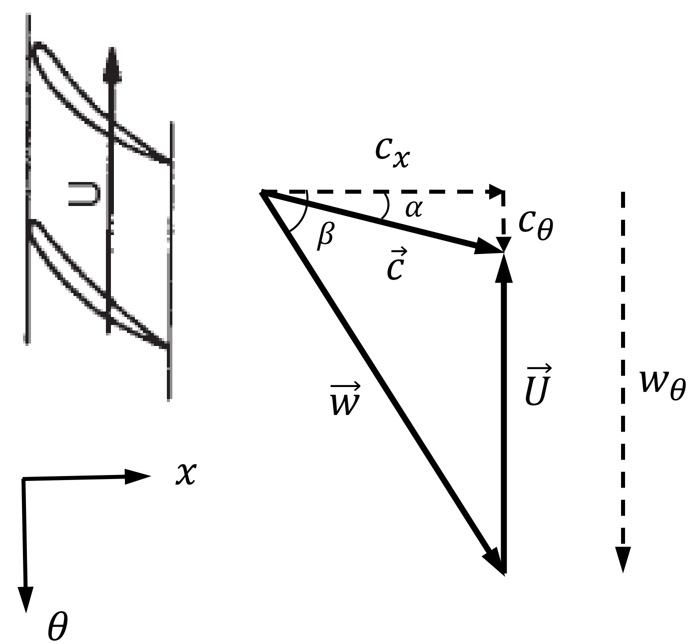


compressor section of a turbofan engine

Terminology

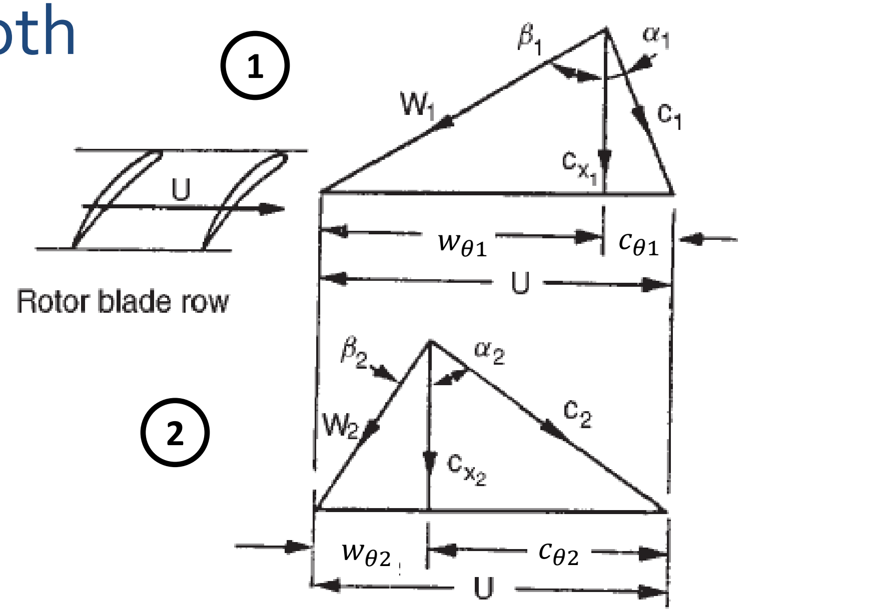
- Cylindrical coordinate system
- Velocity components
  - » c_x: axial
  - » c_r: radial
  - » c_θ: tangential/circumferential
  - » $c_m = \sqrt{c_x^2 + c_r^2}$: meridional
- Relative frame of reference
  - » $\vec{U}$: blade tangential velocity $= r\vec{\omega}$
  - » $\vec{w}$: relative velocity
  - » $\vec{c}$: absolute velocity $= \vec{w} + \vec{U}$
- Flow angles
  - » α: absolute
  - » β: relative

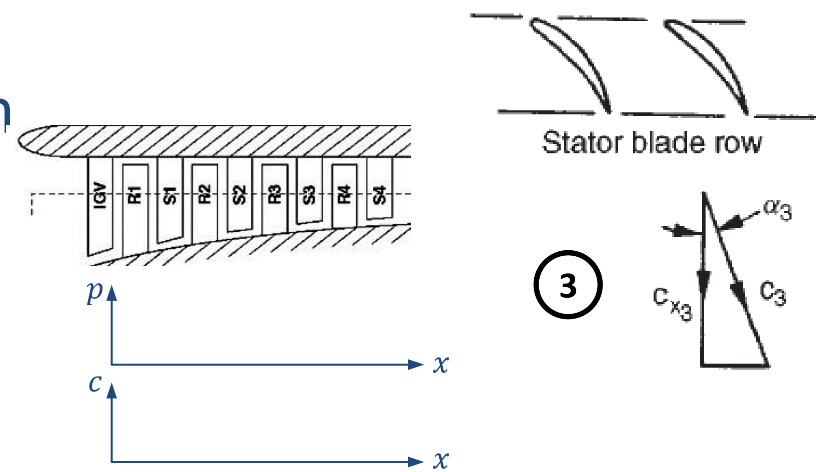




Flow through Compressor Stage

- Pressure increase through both rotor and stator
- Moderate pressure rise (flow deceleration) due to adverse pressure gradient
- As a result, blades have small curvature/camber; are very thin
- Need multiple stages to create large pressure increase





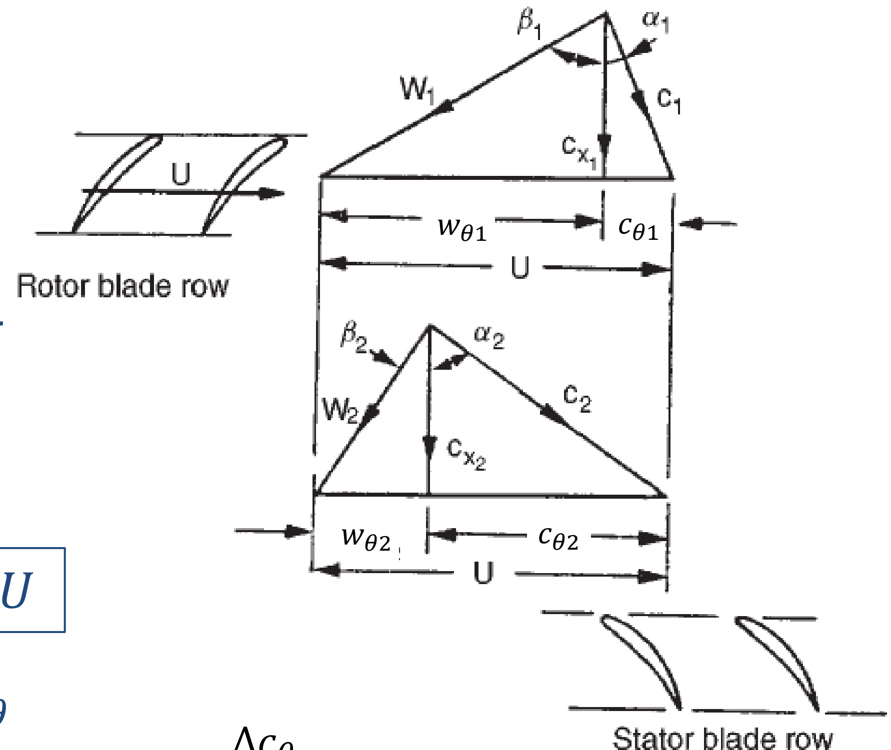
Stage Work (Loading)

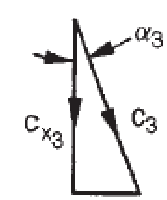
- Euler's equation:

$$w_s = U_2 c_{\theta 2} - U_1 c_{\theta 1}$$

$$\boxed{w_s = U(c_{\theta 2} - c_{\theta 1})}$$ at mean radius

  - » All work done in rotor, none in stator

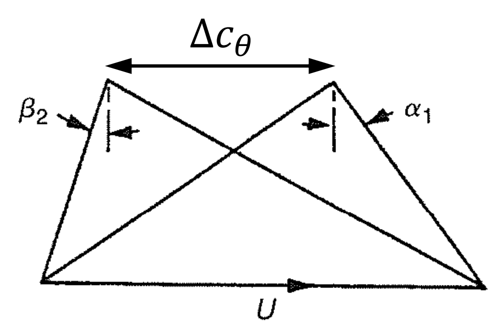




- Loading coefficient

$$\boxed{\psi = w_s/U^2 = (c_{\theta 2} - c_{\theta 1})/U = \Delta c_\theta/U}$$

  - » ψ directly related to flow turning Δc_θ
  - » Higher $\psi \rightarrow$ reduced no. of stages
  - » Reducing inlet swirl $\rightarrow$ higher ψ
  - » $\psi_{design} \sim 0.4$



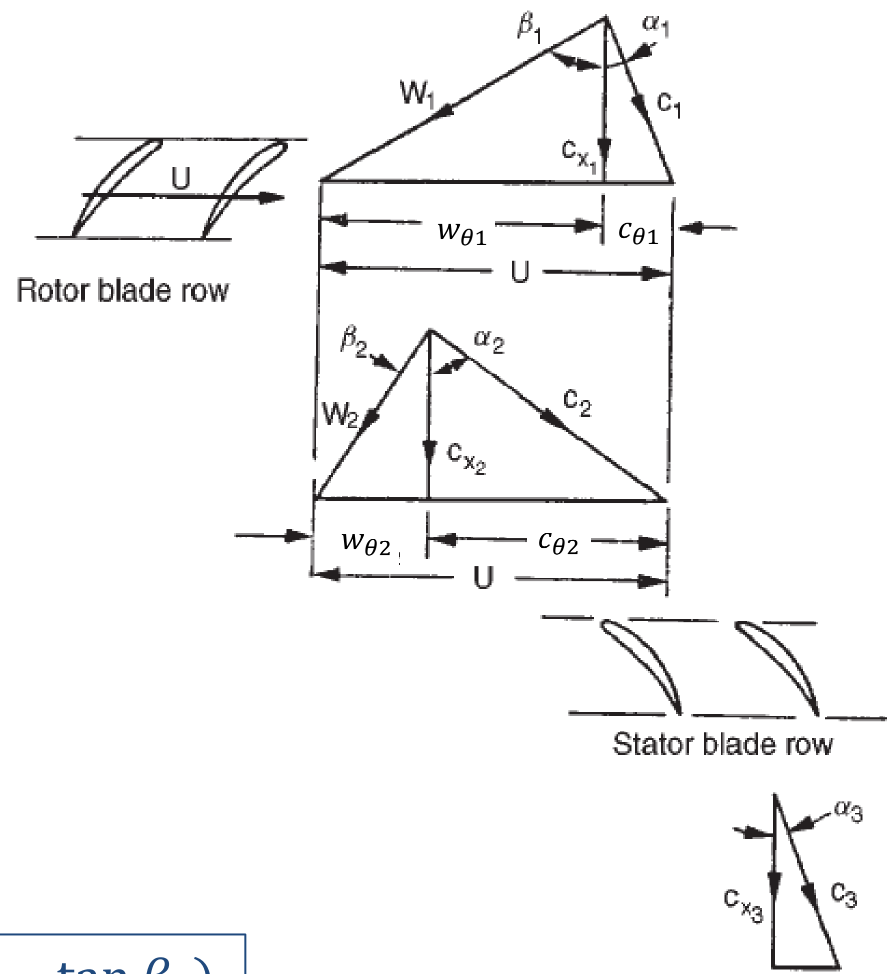
Flow Coefficient

- Definition: $\phi = c_x/U$
  - Determines change in flow angles
  - Typically c_x const. thru compressor
  - Higher ϕ → reduced flow turning
  - $\phi_{design} \sim$ 0.4-0.8
  - ψ and ϕ are directly related:

$$\psi = (c_{\theta 2} - c_{\theta 1})/U$$

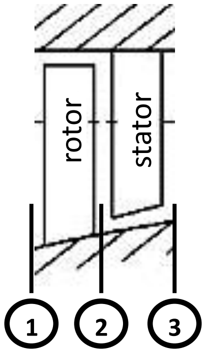
⋮

$$\psi = \phi(\tan\alpha_2 - \tan\alpha_1) = \phi(\tan\beta_1 - \tan\beta_2)$$

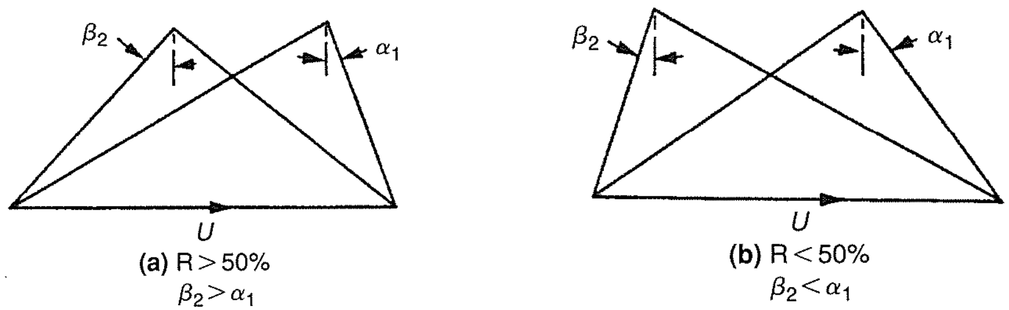


Stage Reaction

$$R = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = \frac{h_2 - h_1}{h_3 - h_1} \approx \frac{(\Delta p)_{rotor}}{(\Delta p)_{stage}}, \qquad 0 \le R \le 1$$



- Impacts asymmetry of velocity triangles hence blade shapes



- Typical range: $R_{design} \approx$ 0.5-0.8

- Relationship with ψ and ϕ: $\psi = 2(1 - R - \phi \tan \alpha_1)$
  - » Higher reaction tends to reduce stage loading

Stage Thermodynamics

- Work:

$$w_s = U_2 c_{\theta 2} - U_1 c_{\theta 1} = h_{02} - h_{01}$$

$$\rightarrow \; h_{01} - U_1 c_{\theta 1} = h_{02} - U_2 c_{\theta 2}$$

$$I \equiv h_0 - U c_\theta \quad \textbf{rothalpy}$$

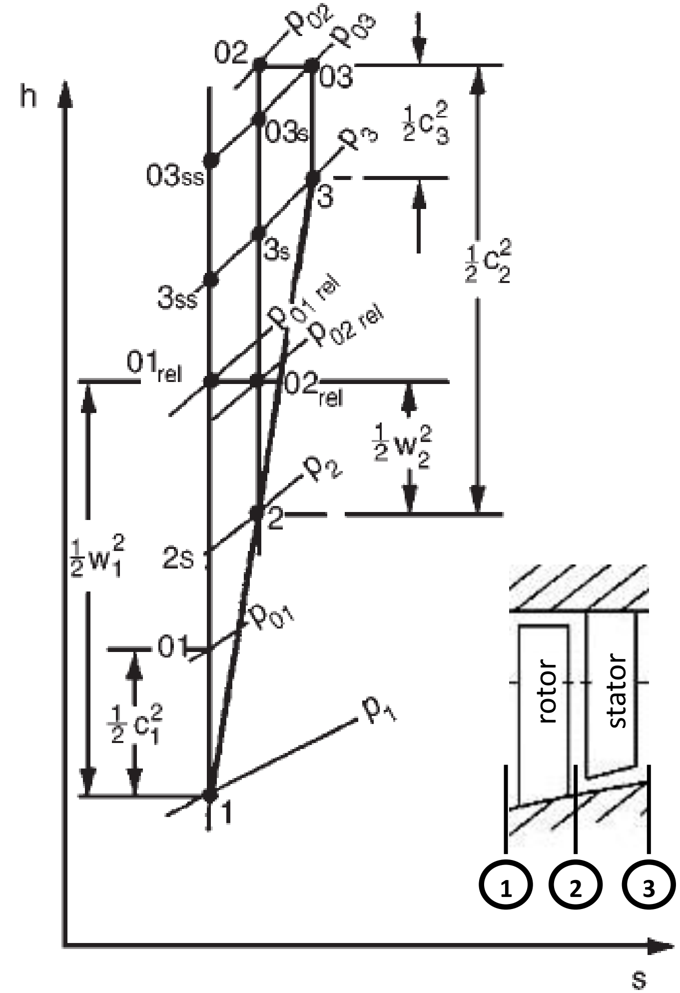
$$I \equiv h + \frac{w^2}{2} - \frac{U^2}{2} \rightarrow \boxed{I \equiv h_{0,rel} - U^2/2}$$

- Enthalpy change across rotor:
  - » $I_1 = I_2$ or at mean radius ($U_1 = U_2$):

$$\boxed{h_{01,rel} = h_{02,rel}} \qquad h_{0,rel} = h + w^2/2$$

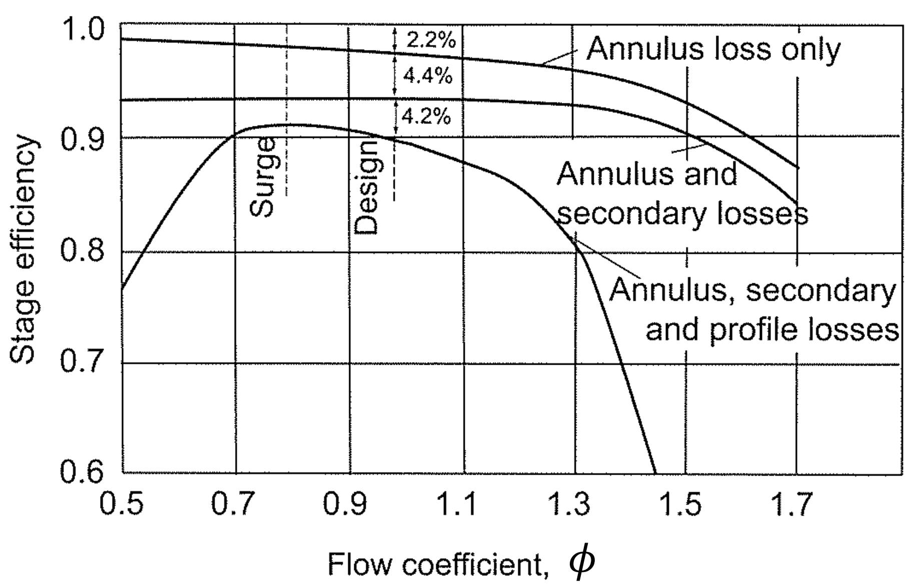
- Enthalpy change across stator ($U = 0$):

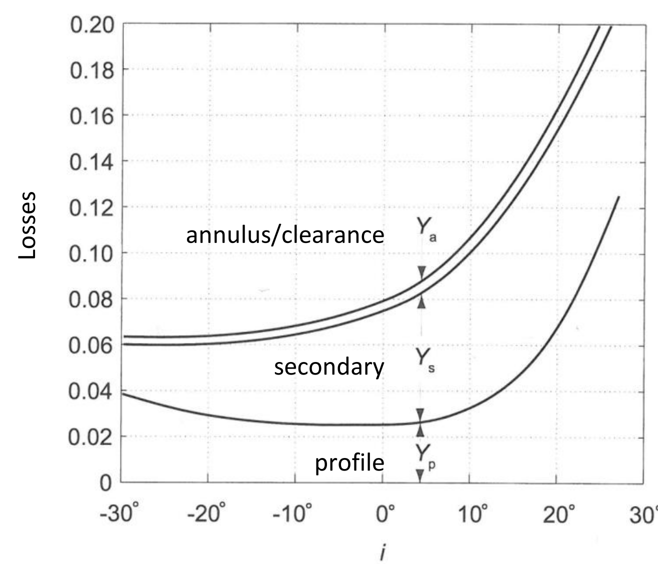
$$\boxed{h_{02} = h_{03}} \qquad h_0 = h + c^2/2$$



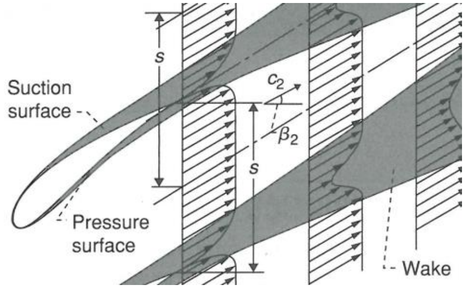
Stage Losses & Efficiency

- Efficiency of the compressor is impacted by the losses in each stage (rotor+stator)
- Losses are typically quantified using correlations obtained from experimental tests

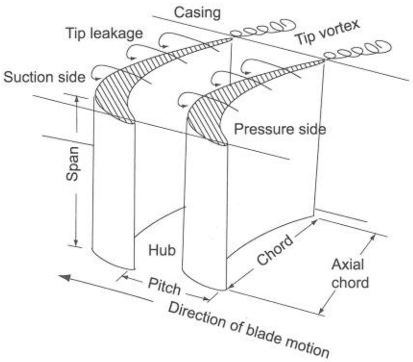




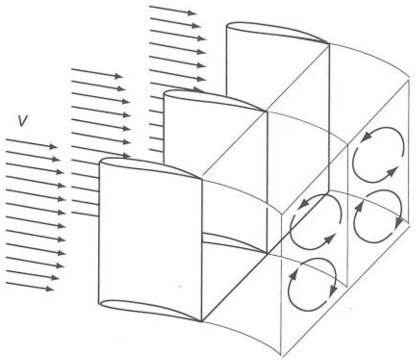
- Profile/annulus losses
  - BL drag & wake mixing



- Tip leakage losses
  - » Tip vortex mixing



- Secondary flow losses
  - Corner stalls, 3D effects



- Shock-induced losses

Stage Loss Metrics

- Enthalpy loss coefficients:

rotor $$\zeta_R = \frac{h_2 - h_{2s}}{w_2^2/2}$$

stator $$\zeta_N = \frac{h_3 - h_{3s}}{c_3^2/2}$$

- Stagnation pressure loss coefficients:

$$Y_R = \frac{p_{01,rel} - p_{02,rel}}{p_{01,rel} - p_1}$$

$$Y_N = \frac{p_{02} - p_{03}}{p_{02} - p_2}$$

- Loss coefficients are related:
  - At low Mach numbers: $Y \approx \zeta$
  - But at higher Mach numbers: $Y > \zeta$

Stage Efficiency

- Efficiency

$$\eta_{tt} \cong 1 - \frac{\gamma - 1}{\gamma} \frac{\left[Y_R\left(1 - p_1/p_{01,rel}\right) + Y_N(1 - p_2/p_{02})\right]}{1 - T_1/T_{03}}$$

Or

$$\eta_{tt} \cong \left[1 - \frac{\zeta_R w_2^2 (T_3/T_2) + \zeta_N c_3^2}{2(h_{03} - h_{01})}\right]$$

» For low-speed/incompressible machines:

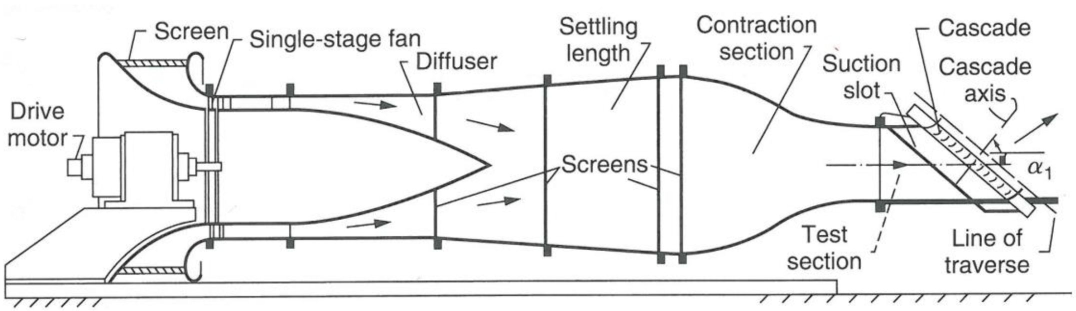
$$\eta_{tt} \cong 1 - \frac{T_{03}\Delta s_{stage}}{h_{03} - h_{01}} = 1 - \frac{\Delta p_{0,R} + \Delta p_{0,N}}{\rho(h_{03} - h_{01})}$$

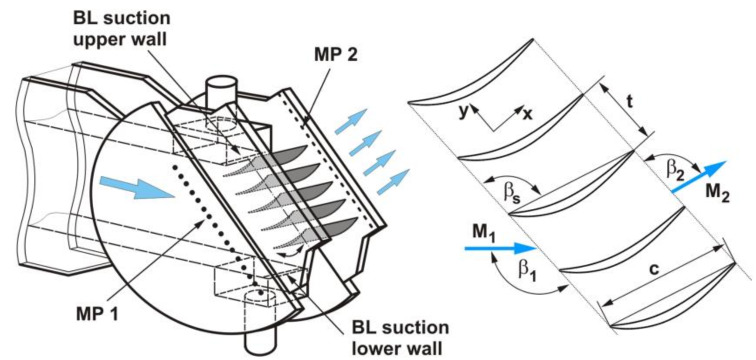
Or

$$\eta_{tt} \cong 1 - \frac{(w_1^2 Y_R + c_2^2 Y_N)}{2(h_{03} - h_{01})}$$

Measuring Losses: Cascade Flow Analysis

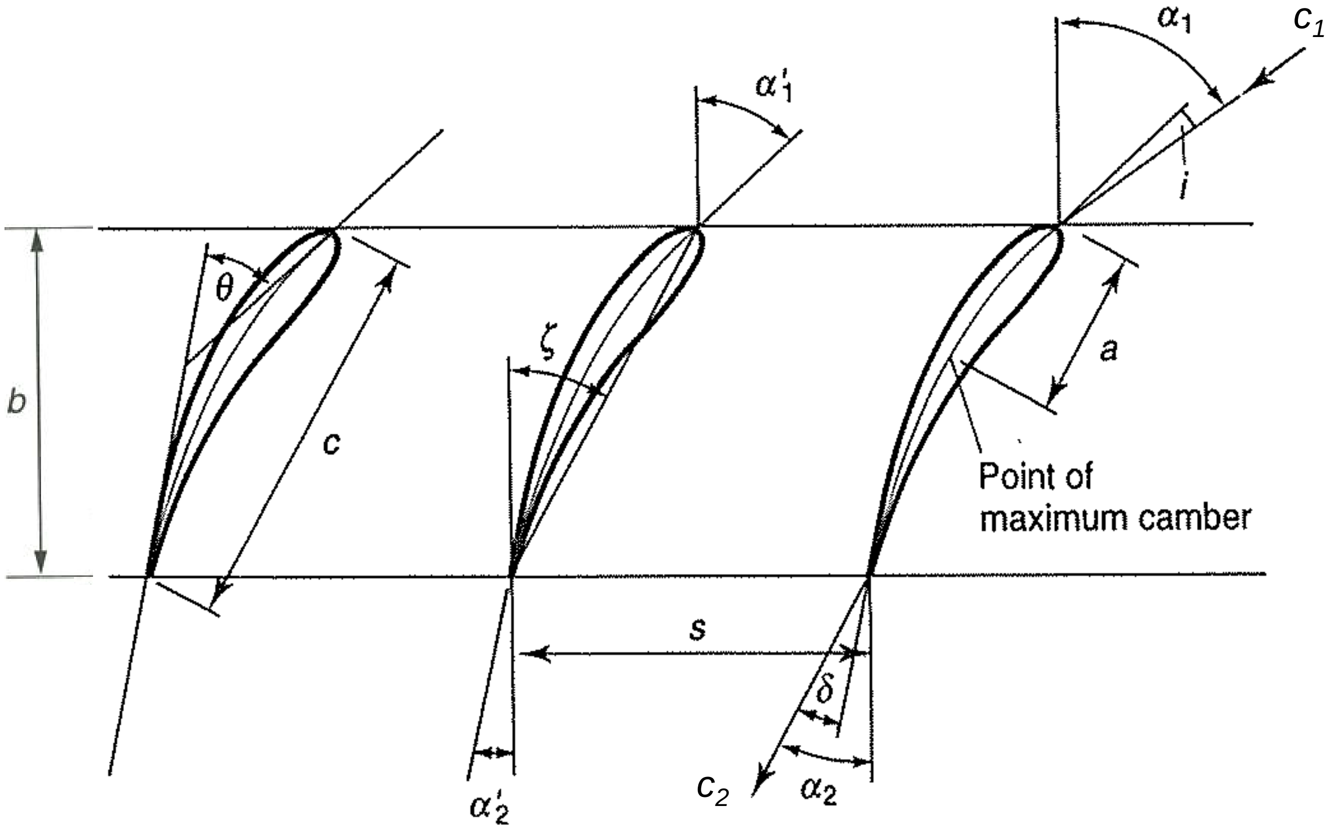
- Sample Cascade Tunnel





- Main objective:
  - Characterize losses
  - Measure exit flow angle
- Based on simplified 2D, steady flow
- Both design and off-design conditions are tested

Cascade Nomenclature



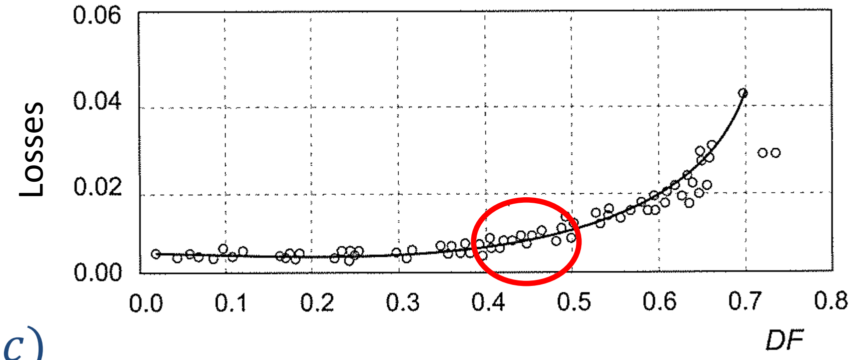
α'_1 = blade inlet angle
α'_2 = blade outlet angle
θ = blade camber angle
$= \alpha'_1 - \alpha'_2$
ζ = setting or stagger angle
s = pitch (or space)
ε = deflection
$= \alpha_1 - \alpha_2$
α_1 = air inlet angle
α_2 = air outlet angle
c_1 = air inlet velocity
c_2 = air outlet velocity
i = incidence angle
$= \alpha_1 - \alpha'_1$
δ = deviation angle
$= \alpha_2 - \alpha'_2$
c = chord
b = axial chord
c/s = solidity

Some Design Criteria

- Diffusion factor

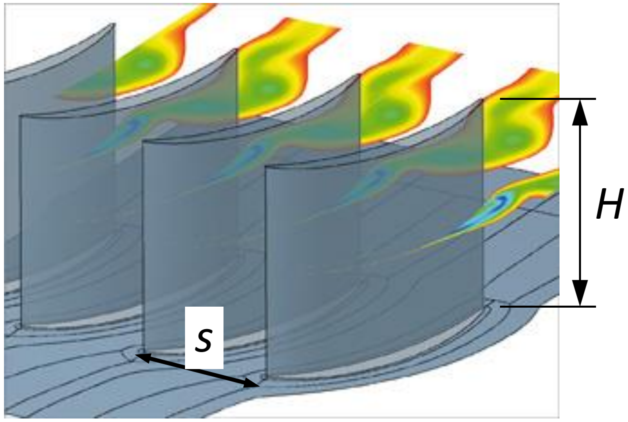
$$DF = \left(\frac{w_1 - w_2}{w_1}\right) + \left(\frac{\Delta c_\theta}{2w_1}\right)\left(\frac{s}{c}\right) \approx 0.45$$

deceleration turning



  - » Helps determine space-chord ratio (s/c)
  - » For given DF, higher turning requires reduced blade spacing to avoid separation
  - » Typical values: $s/c \approx 0.8 - 1.2$

- Inlet swirl angle: $\alpha_1 \approx 20° - 30°$
  - » Helps reduce relative inlet Mach number
  - » Reduces flow turning hence stage loading



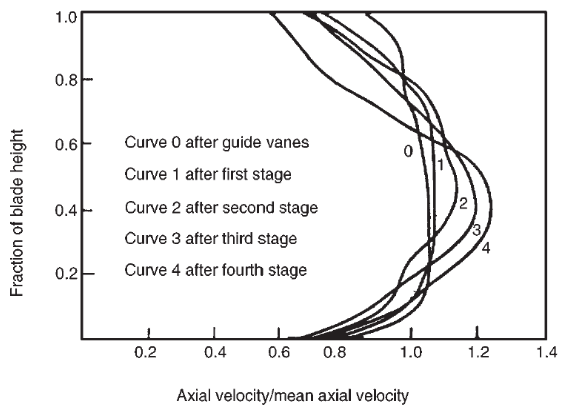
- Blade aspect ratio: $H/c \approx 1 - 2$

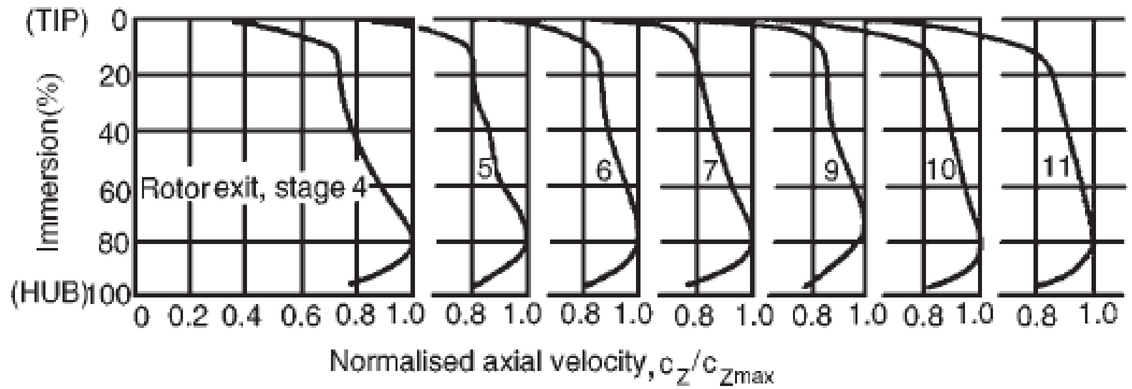
- Blade spacing: $s/b \approx 0.5$

Multi-stage Compressors

- Effective annulus area is reduced because of BL growth
  - Axial velocity is adversely impacted

Impact reduced after ~ 4th stage

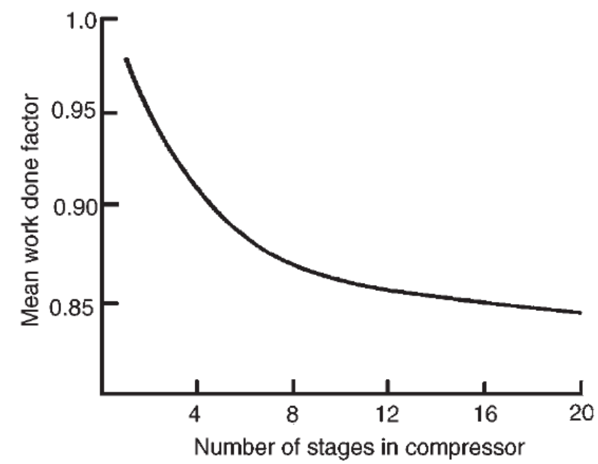




  - The effect is taken into account by introducing work-done factor (λ):

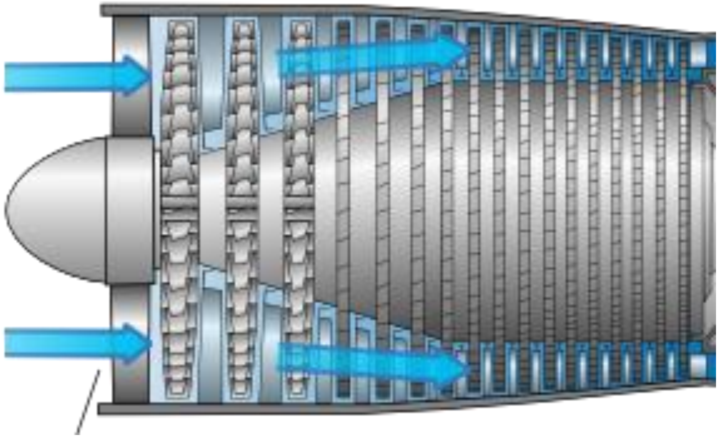
$$w_s = \lambda U(c_{\theta 2} - c_{\theta 1})$$

  - American design practice: apply blockage factor to account for reduced annulus area



Radial Flow Variations

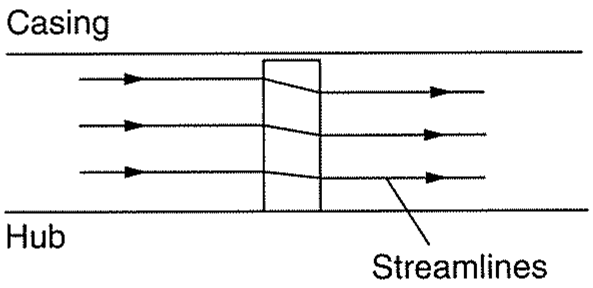
- 2D flow assumption only valid when r_{hub}/r_{tip} is large ($\geq$ 0.8) – typically last stages blades
- For $r_{hub}/r_{tip} \approx$ 0.4-0.8, blade speed (U) & flow angles will significantly vary from hub to tip
  - » Blades require significant twist



Radial Flow Variations

- Change in annulus shape means c_r cannot be ignored, although still smaller than c_x and c_θ

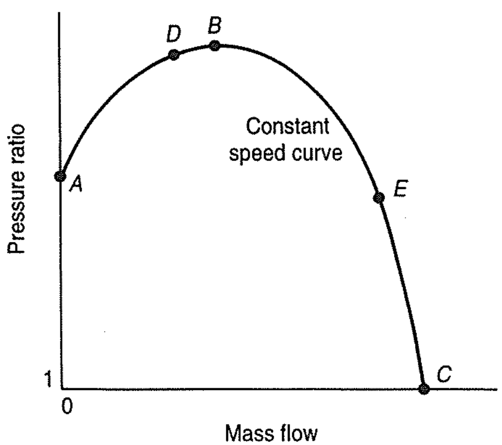
- Pressure increase from hub to tip to counter centrifugal forces acting on the fluid will cause slight variation in the radial direction

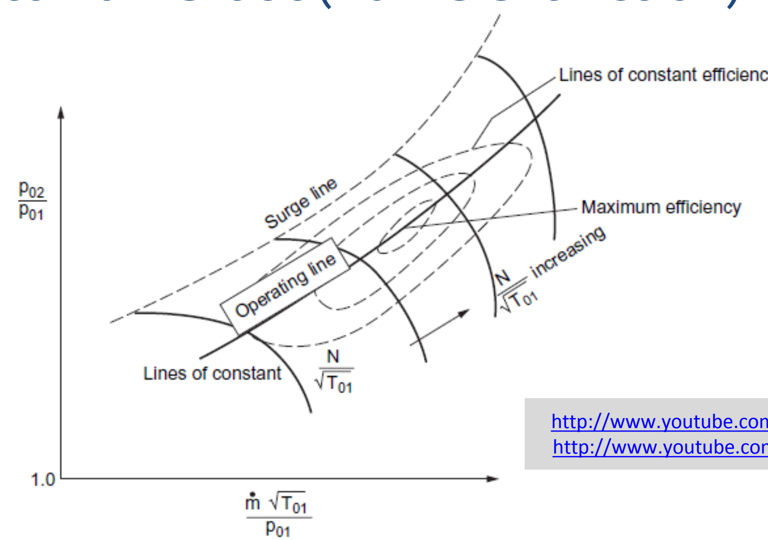


- Radial flow variation is taken into account by solving "radial equilibrium equation"

Flow Instability - Surge

- It is caused by drop in delivery pressure due to reduction in $\dot{m}$
- If p_{exit} does not drop fast enough, air will reverse direction and flow upstream due to resulting pressure gradient. This will cause sudden drop of compressor exit pressure, reversing air flow direction…
- The cycle can then continue at high frequency
- Surge characterized by vibration in "axial" direction, causes excessive blade vibration, and can lead to flame-out (flame extinction)

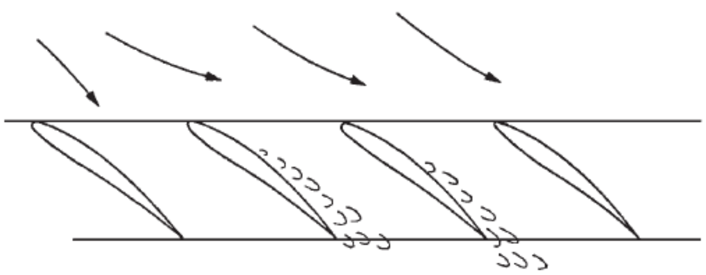


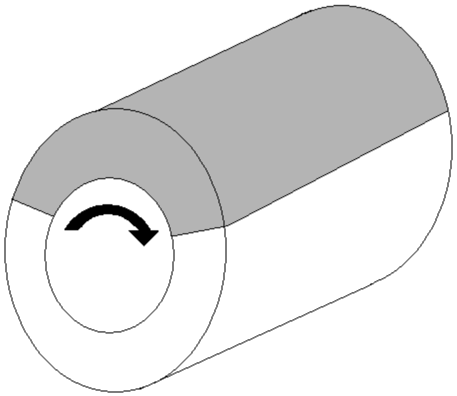


http://www.youtube.com/watch?v=osAT6mwkr94
http://www.youtube.com/watch?v=9KhZwsYtNDE

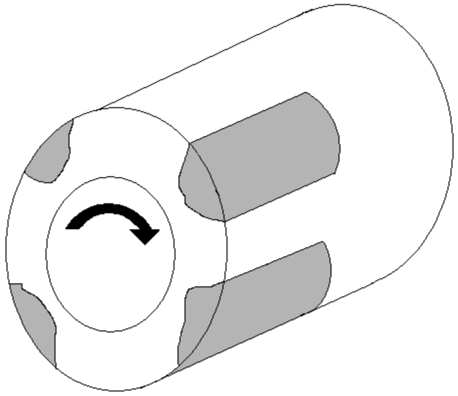
Flow Instability – Rotating Stall

- An instability usually observed at low operating speed (N)
- Is caused by blade stall (due to increased loading, tip vortex or corner stall), leading to flow blockage and change in angle-of-attack of neighboring blades (increase on one side and decrease on another side)
- This causes neighboring blade stall an recover creating stall patches that will travel around compressor annulus
- Rotating stall can exist in normal operating conditions; both part-span and full-span stall has been observed
- It causes vibration in *circumferential* direction





Full-span stall



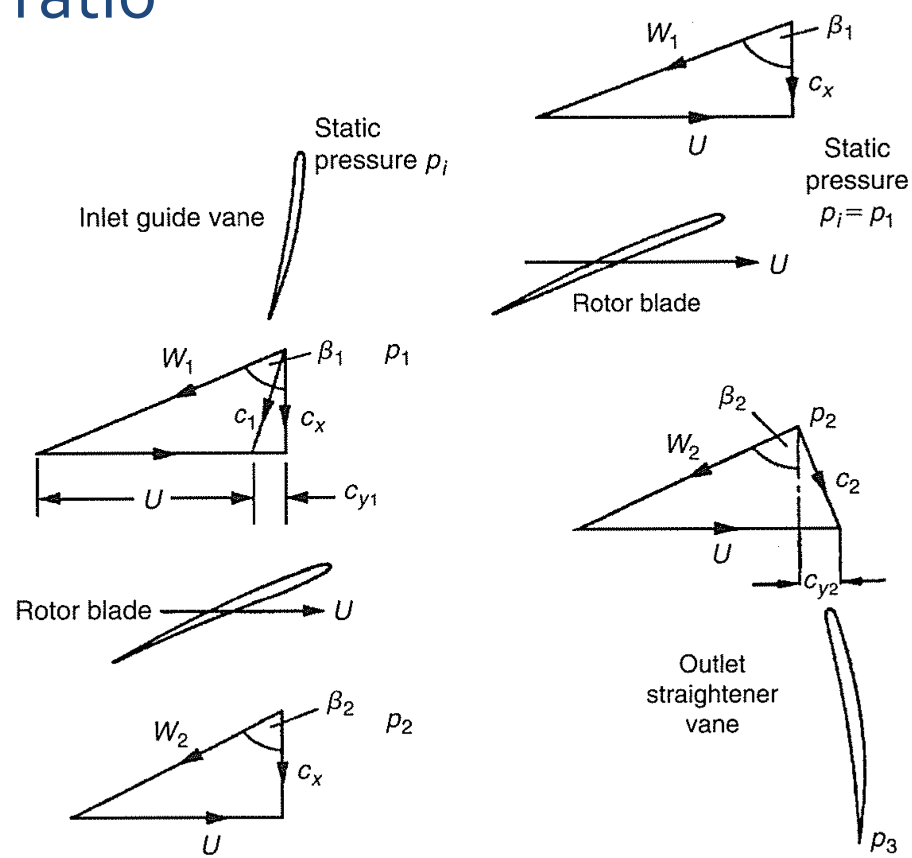
Part-span stall

Low-speed Ducted Fans

Introduction

- Ducted fans are essentially single-stage compressors but with low pressure ratio

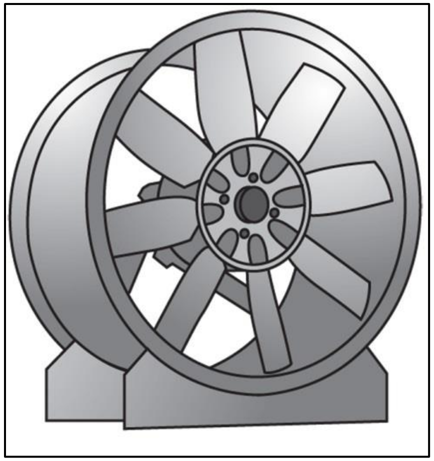
- Two configurations may be used:
  a) IGV – Rotor
  b) Rotor – OGV



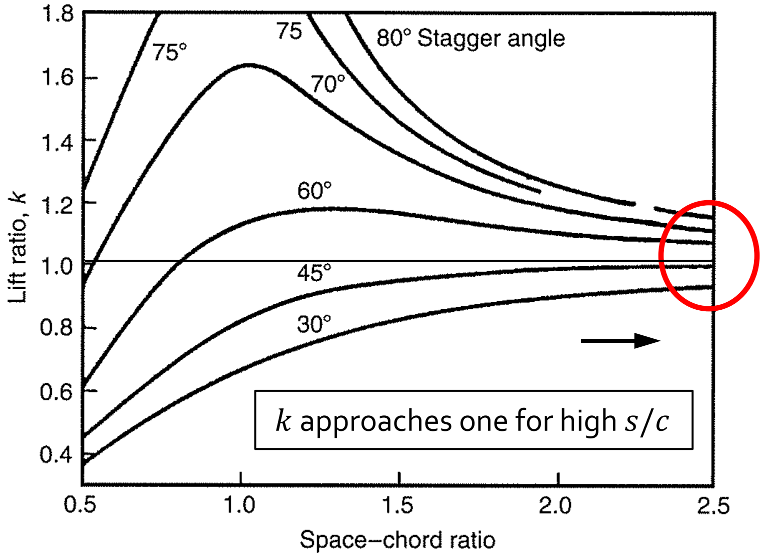
(a) Fan with inlet guide vanes **(b)** Fan with outlet guide vanes

Introduction

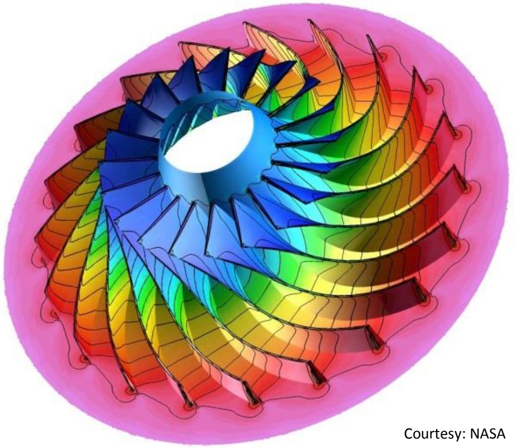
- Ducted fans have typically higher space-chord ratio (low solidity) compared to compressors
- Isolated airfoil theory is often used since the influence of neighboring blades is small



s/c is much greater



Fluid Mechanics & Aeroacoustics of Fans and Compressors



Courtesy: NASA

Day 2: Centrifugal Compressors & Fans

Short course offered at BCAM– July 2-4, 2013

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COLORADO MESA UNIVERSITY

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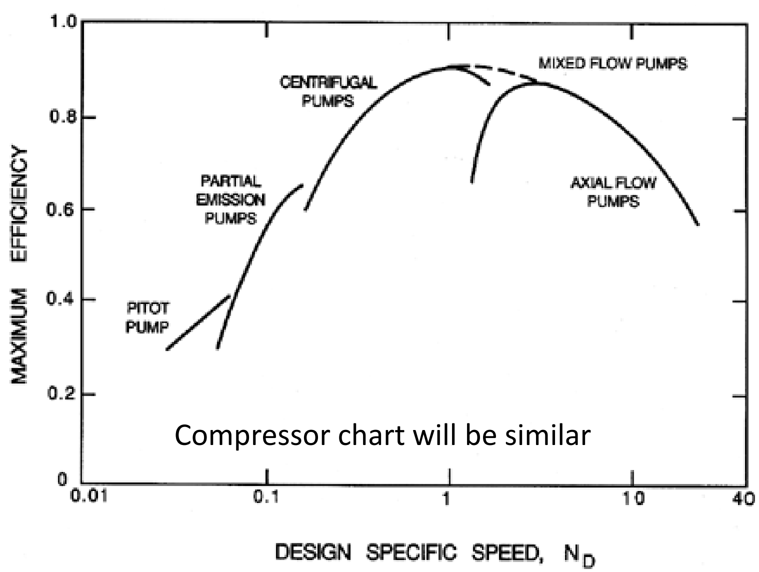
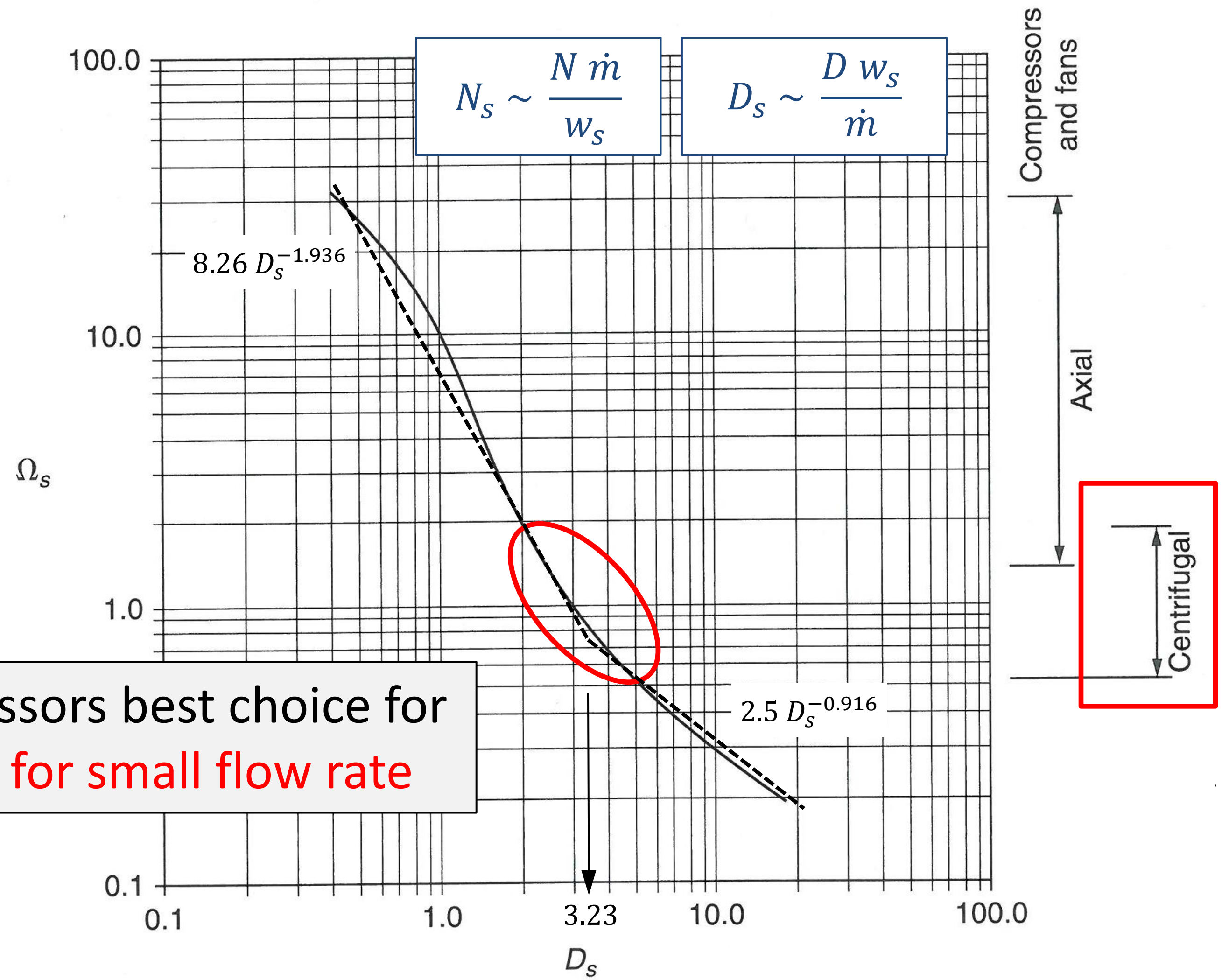
Baltogar

Introduction

- Efficiency of axial flow compressors sharply drops at low flow rates
  - » Increased losses due to larger surface/volume ratio of annulus
  - » Manufacturing of small parts, high maintenance cost, etc.



Cordier Diagram



Centrifugal compressors best choice for
high pressure rise for small flow rate

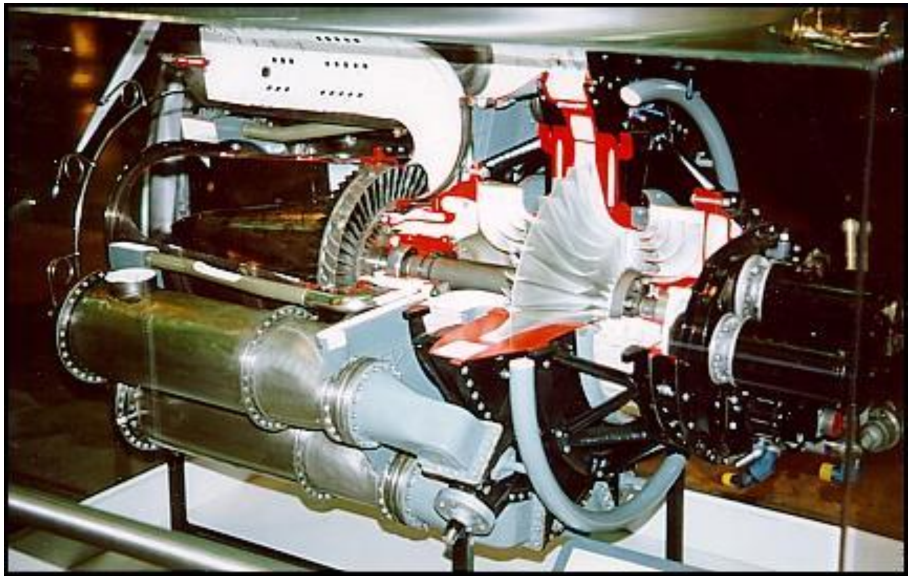
Introduction

- Centrifugal compressors
  - » Smaller number of components
  - » More compact design
  - » Pressure ratio's as high as 8:1

- Centrifugal fans/blowers: Δp small, about a few inches of water ($\rho_e / \rho_i \leq 1.05$)
  - » Usually treated as incompressible

First Jet Engine (Frank Whittle) - 1930

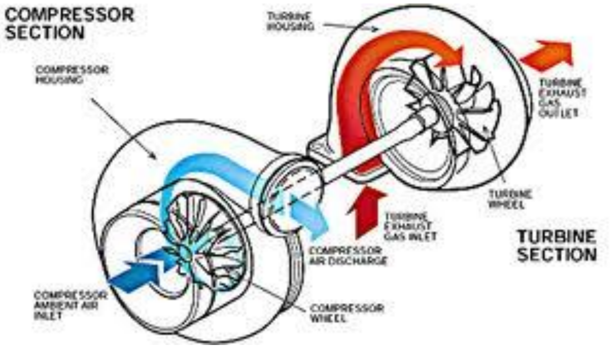
- Used a centrifugal compressor



- Soon became apparent that they are not suitable for higher mass flows
  - Larger frontal area, lower efficiency, etc.)

Some Applications

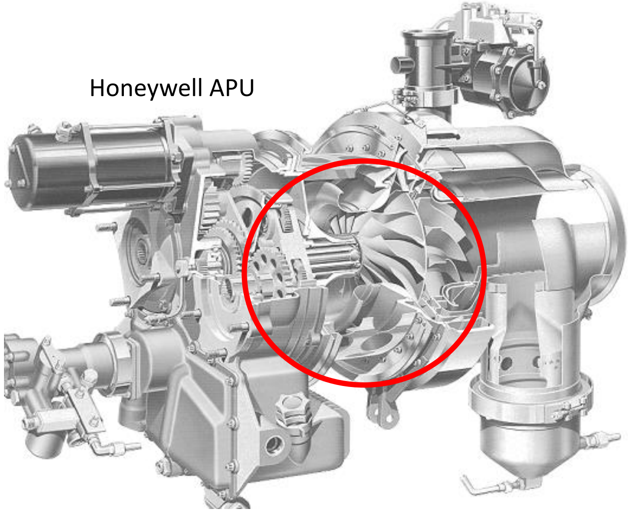
- Automobile turbochargers



- Auxiliary Power Units (APU's)

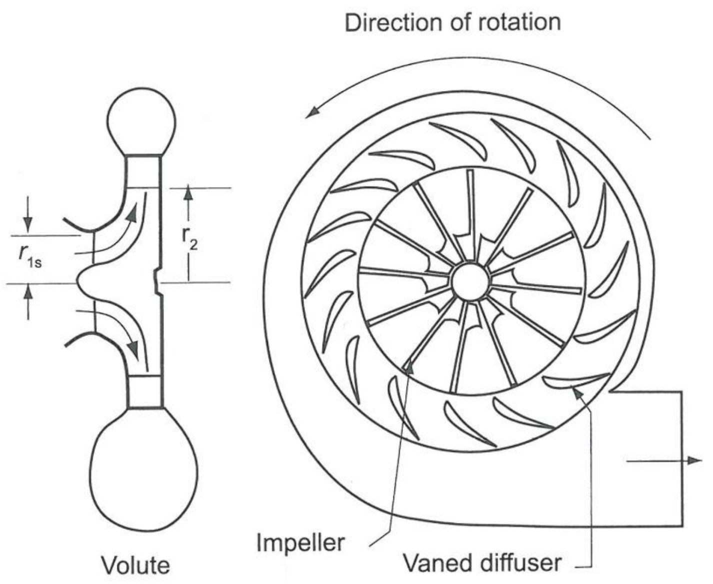
- Gas pipeline, refrigeration, process plants

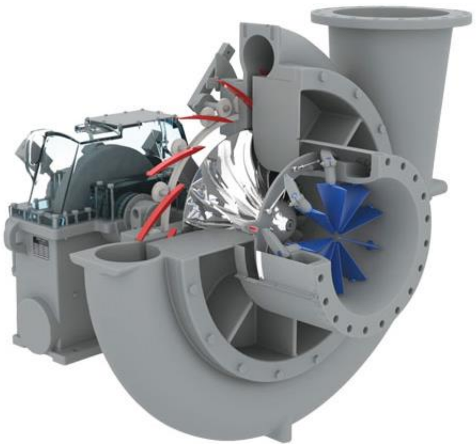
Honeywell APU



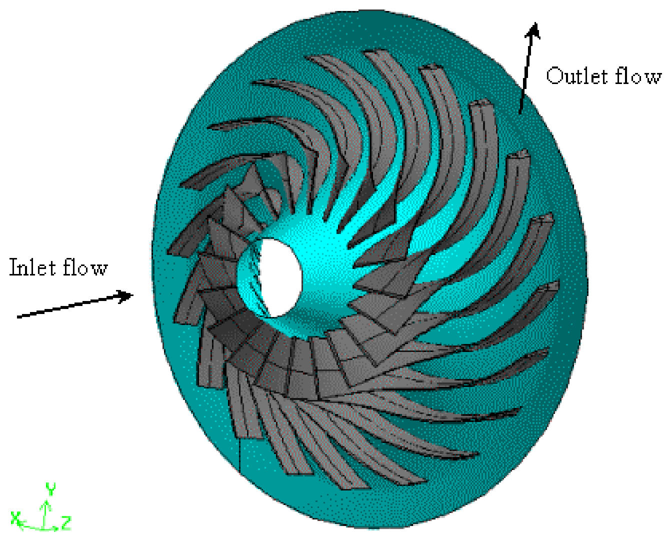
Components & Operation

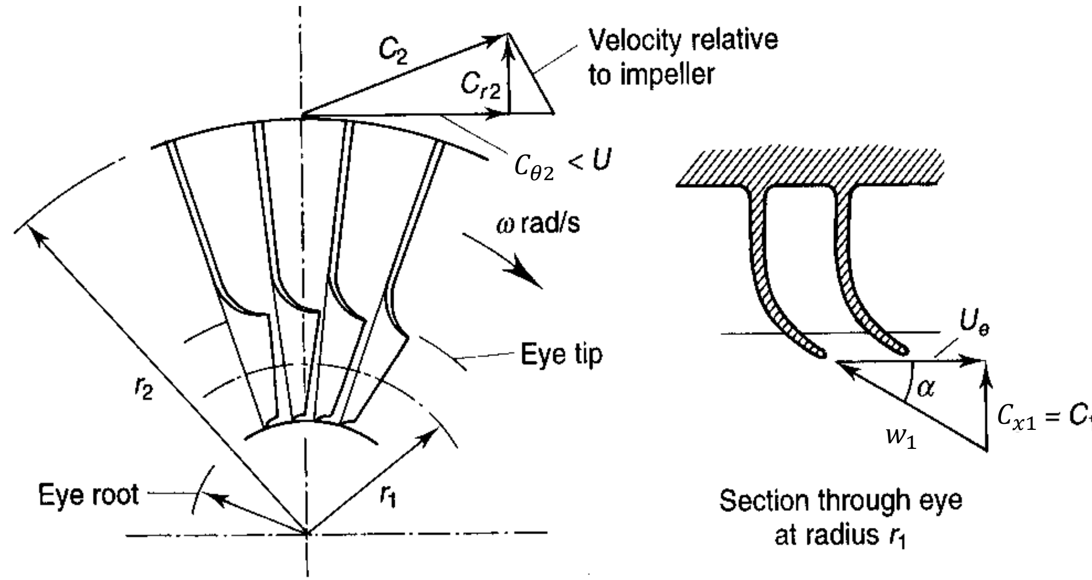
- Impeller: pressure rise due to centrifugal action & diffusion
- Diffuser (vaned / vaneless): pressure rise by diffusion (velocity almost reduced to inlet value)
- Design practice: 50-50 pressure rise across impeller & diffuser
- Scroll or Volute: collects and delivers the air





Flow Path





- Air enters through impeller eye in axial direction
- Unless inlet guide-vanes (IGV's) are used, vanes must be curved to allow smooth inflow
- Air leaves impeller tip with absolute velocity c_2
- Some impellers have shroud to reduce leakage & losses

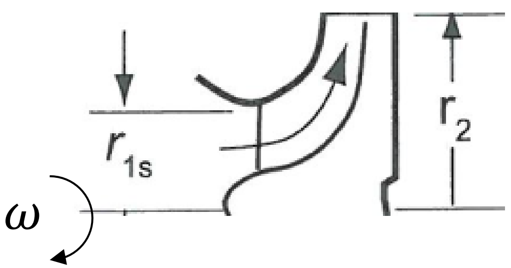
Stage Work

- Impeller: $I_1 = I_2$ (constant rothalpy)

$$I \equiv h + \frac{w^2}{2} - \frac{U^2}{2}$$

$$\rightarrow \; h_2 - h_1 = \underbrace{\frac{1}{2}(U_2^2 - U_1^2)}_{\text{centrifugal action} \sim 75\%} + \underbrace{\frac{1}{2}(w_1^2 - w_2^2)}_{\text{flow deceleration} \sim 25\%}$$

$U_2 > U_1$



» Δh directly related to Δp

- Diffuser: $h_{02} = h_{03}$ (constant stagnation enthalpy)

$$h_0 \equiv h + \frac{c^2}{2} \qquad \rightarrow h_3 - h_2 = \frac{1}{2}(c_2^2 - c_3^2)$$

Δp due to flow deceleration

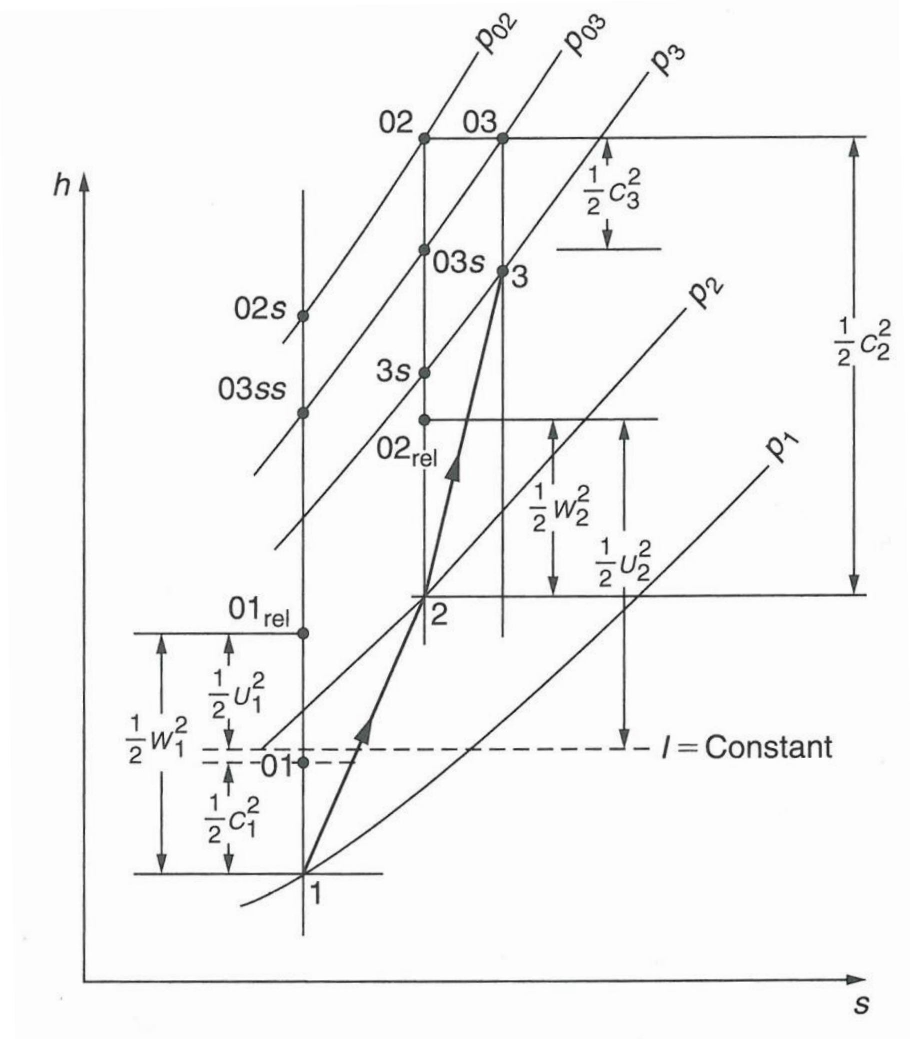
Stage Thermodynamics

- Impeller
  - Rothalpy:

$$I \equiv h + \frac{w^2}{2} - \frac{U^2}{2}$$

- Diffuser
  - Stagnation enthalpy:

$$h_0 \equiv h + \frac{c^2}{2}$$



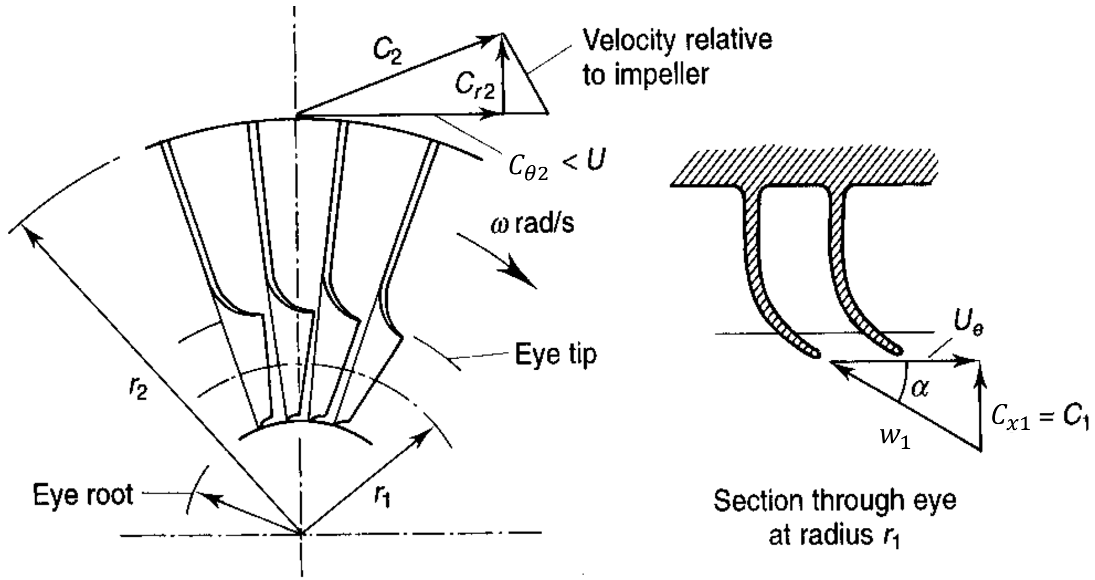
Stage Thermodynamics

- Work:

$$w = U_2 c_{\theta 2} - U_1 c_{\theta 1}$$

$c_{\theta 1} = 0$ for axial inflow

$$\rightarrow w = U_2 c_{\theta 2} = h_{02} - h_{01}$$



- Slip factor
  - » Ideally: $c_{\theta 2} = U_2$, but in reality: $c_{\theta 2} < U_2$ due to less than perfect guidance received because of finite no. of vanes
  - » Define $\sigma_s = c_{\theta 2}/U_2$ as slip factor:

$$\sigma_s \approx 1 - \frac{0.63\pi}{N_{vane}} \approx 1 - \frac{2}{N_{vane}}$$

Stanitz formula

Stage Thermodynamic

- Power input factor (λ)
  - » Correction factor to account for losses in the impeller only
  - » $\lambda \approx 1.035 - 1.04$

- Overall stagnation pressure ratio:

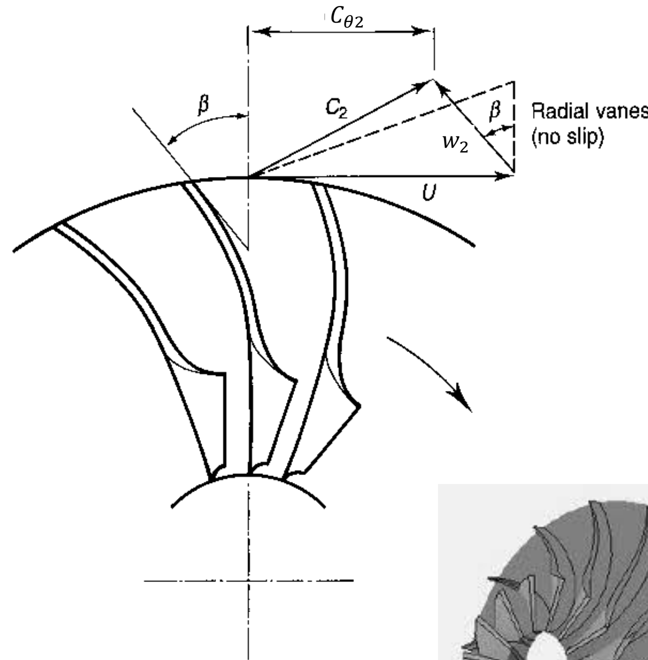
$$\frac{p_{03}}{p_{01}} = \left[1 + \frac{\eta_c \lambda (\sigma_s U_2^2 - U_1 c_{\theta 1})}{c_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

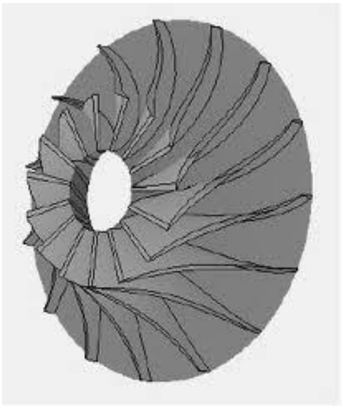
η_c : isentropic efficiency
c_p : specific heat
T_{01}: inlet stagnation temperature

Impeller Design Considerations

Backward Swept Vanes

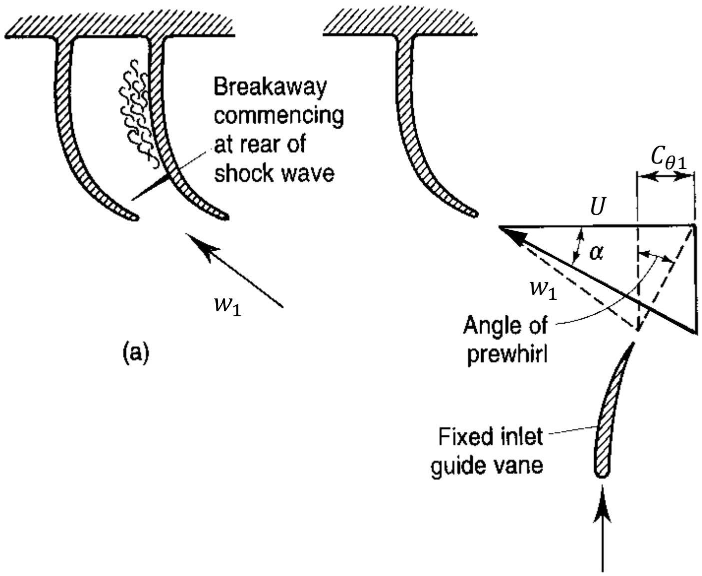
- Radial impeller designs lead to high exit velocity c_2, which may lead to flow separation in diffuser
- Backward swept vanes will reduce (increase w_2) hence reduce diffusion in both impeller & diffuser
- Because of more controlled diffusion in impeller & diffuser both overall efficiency and operating margin improve
- To maintain pressure ratio, however, rpm has to be increased. Therefore, centrifugal stresses will be higher
- Swept vanes will also experience bending stresses
- Typical bend angles: $\beta = 30°\text{-}40°$





Impeller Design Considerations

- Inlet pre-whirl
  - » Without pre-swirling inflow (using inlet guide vanes), and hence relative Mach no. will be very high
  - » The flow can become supersonic, creating shock waves, which in interaction with the BL may cause flow separation
  - » Adding pre-whirl will help reduce inlet Mach number
  - » But, as a result, $c_{\theta 1}$ will no longer be zero, which mean more work is needed to create the same pressure ratio:



$$\frac{p_{03}}{p_{01}} = \left[1 + \frac{\eta_c \lambda (\sigma_s U_2^2 - U_1 c_{\theta 1})}{c_p T_{01}}\right]^{\gamma/(\gamma-1)}$$

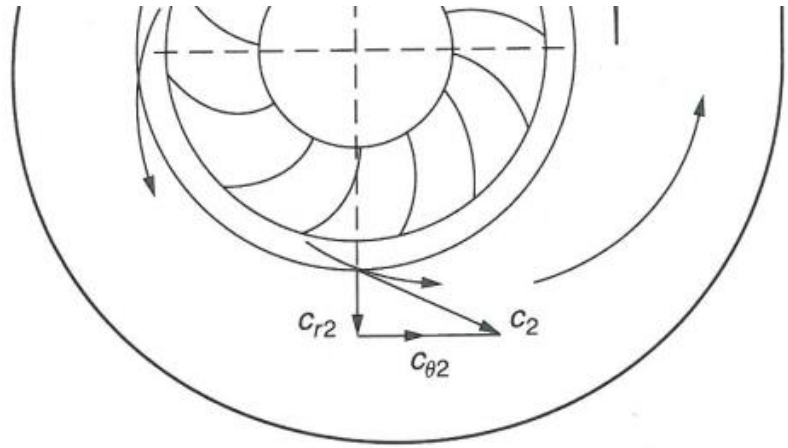
  - » Since w_1 will be highest at the tip of the eye (highest U_1), one can minimize the impact by adding pre-whirl near tip of the eye only

Diffuser Design Considerations

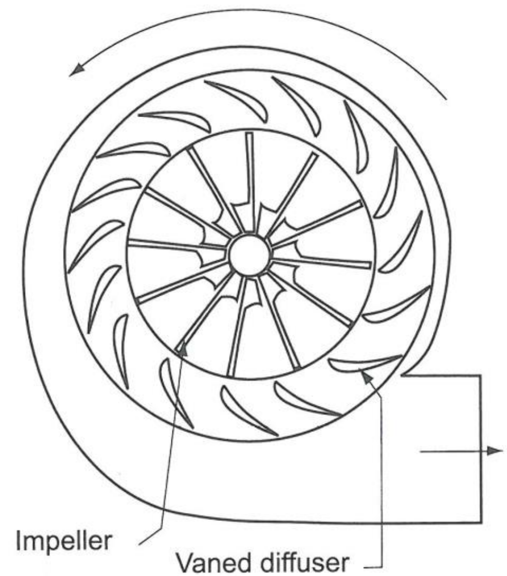
- Can further increase pressure in diffuser by reducing c_2 (or $c_{\theta 2}$)
  - Since past impeller exit, angular momentum stays constant:

$$rc_\theta = \text{const.}$$

increasing radius will achieve this

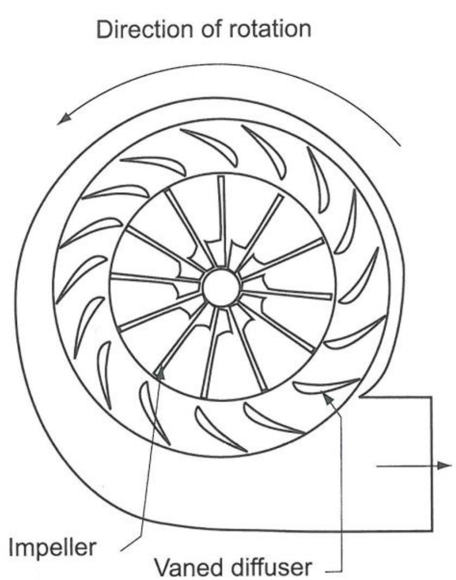


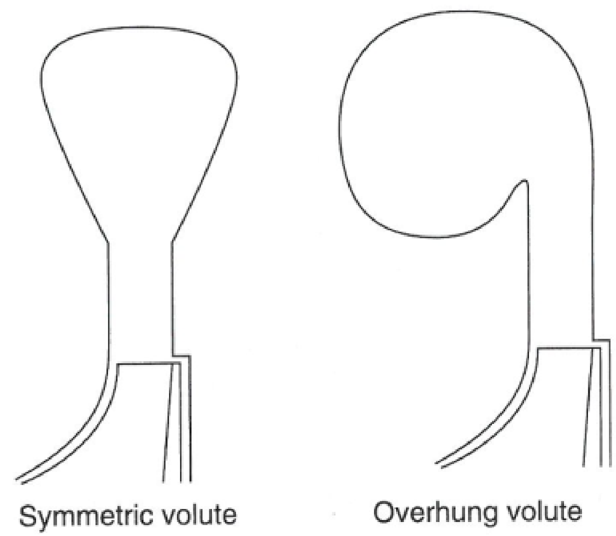
- Vaneless diffuser: reduce c_θ by increasing radius
- Vaned diffuser: use vanes to reduce c_θ faster



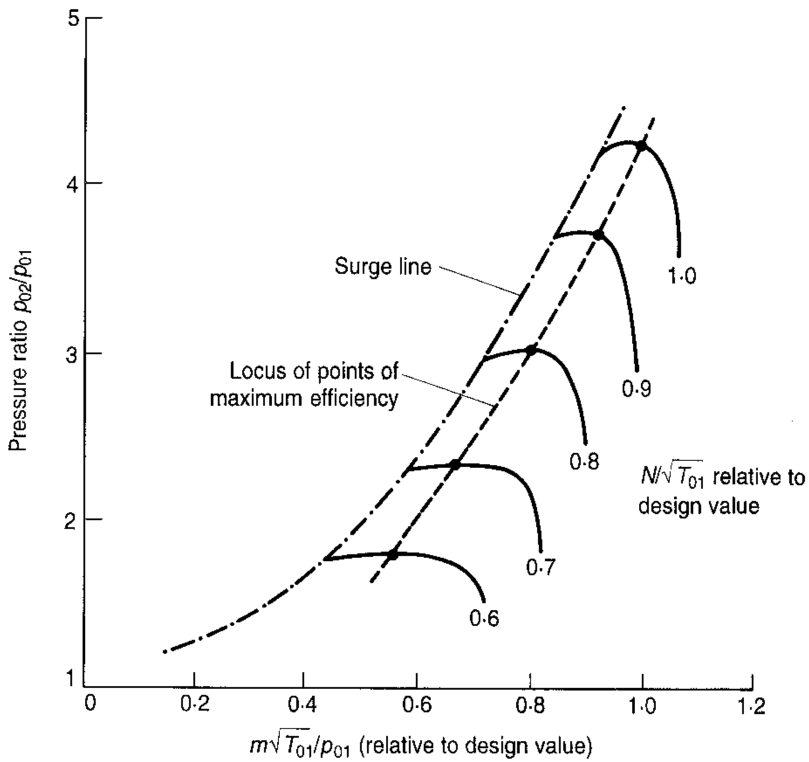
Diffuser Design Considerations

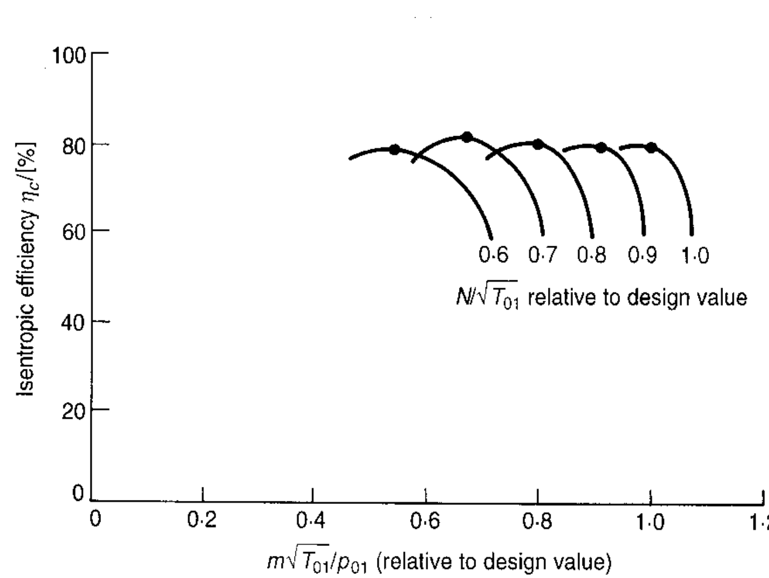
- Volute or Scroll
  - » Collects and delivers the flow
  - » Spiral-shaped channel of increasing cross-sectional area





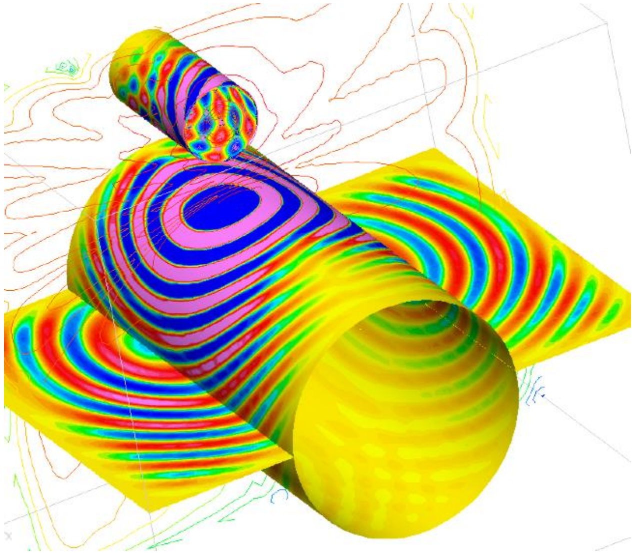
Performance Characteristics





- Centrifugal compressors can also suffer from instabilities such as rotating stall & surge

Fluid Mechanics & Aeroacoustics of Fans and Compressors



Day 3: Introduction to Aeroacoustics

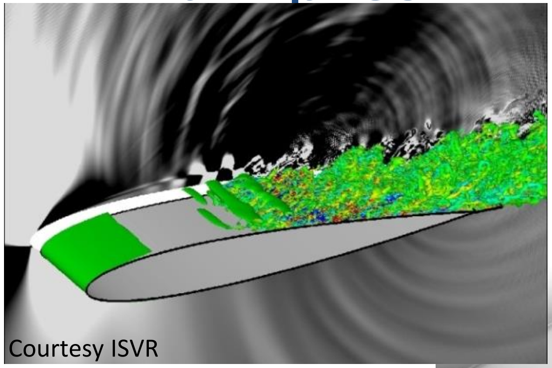
Short course offered at BCAM– July 2-4, 2013

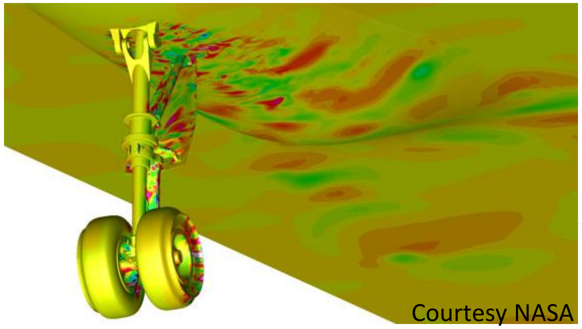
Farzad Taghaddosi, Ph.D.

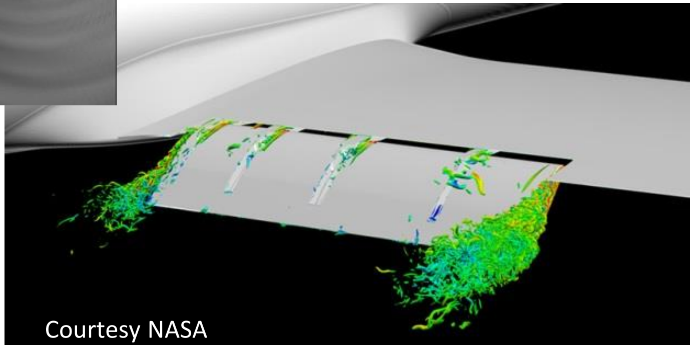
COLORADO MESA UNIVERSITY bcam basque center for applied mathematics BFA DFB Bizkaiko Foru Aldundia Diputación Foral de Bizkaia Baltogar

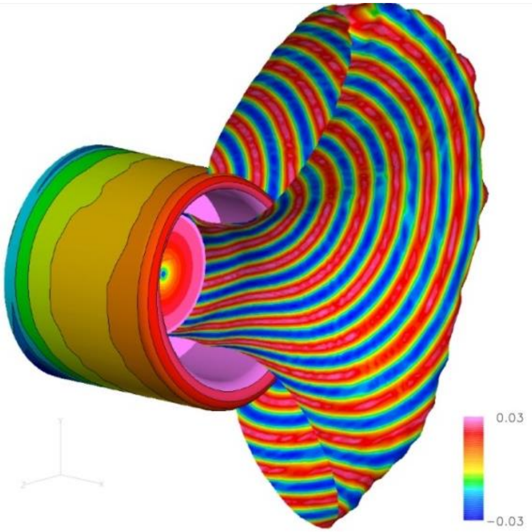
Introduction

- What is aero-acoustics?
  - » Study of sound generated by aerodynamic sources
- Examples:


Courtesy ISVR

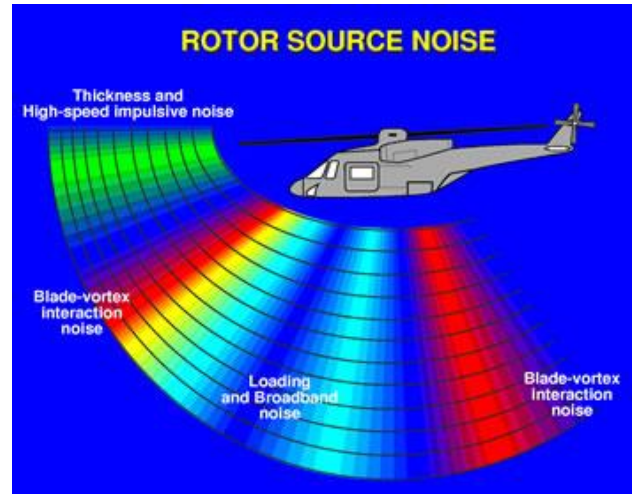

Courtesy NASA

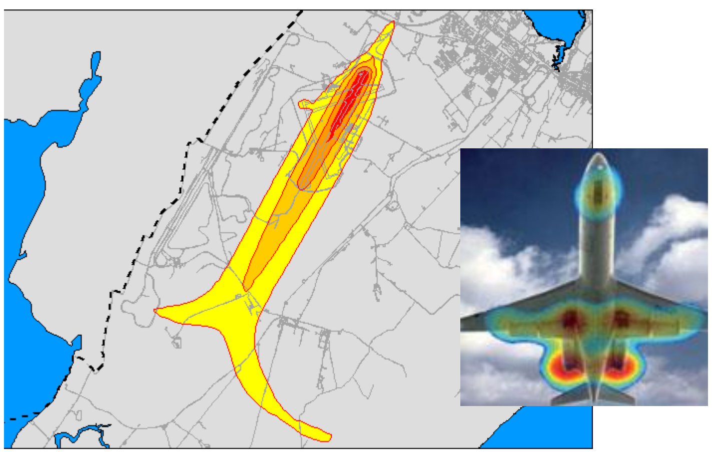

Courtesy NASA

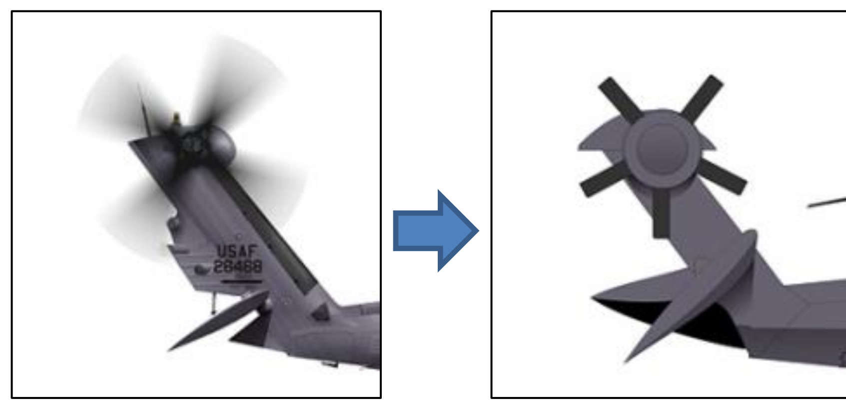


Introduction

- There is obviously a need to reduce man-made noise







Physical Nature of Sound

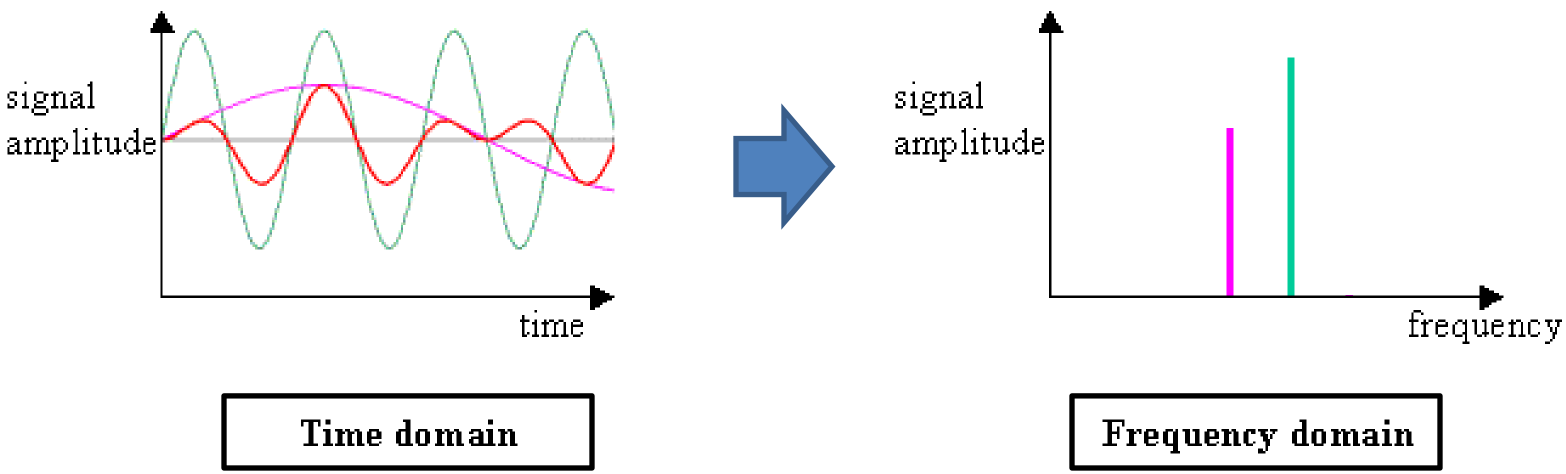
- Sound: pressure disturbances/fluctuations of very small amplitude ($p' := p - \bar{p}$)
  » Sound waves require a medium to travel

- For any p', there is an associated fluctuations of velocity particles (v')

- Speed of sound: speed of sound propagation in a medium; in undisturbed medium $c_0 = (\partial p' / \partial \rho')_s$
  – Note that v' and c_0 are not the same

Noise Signal Analysis

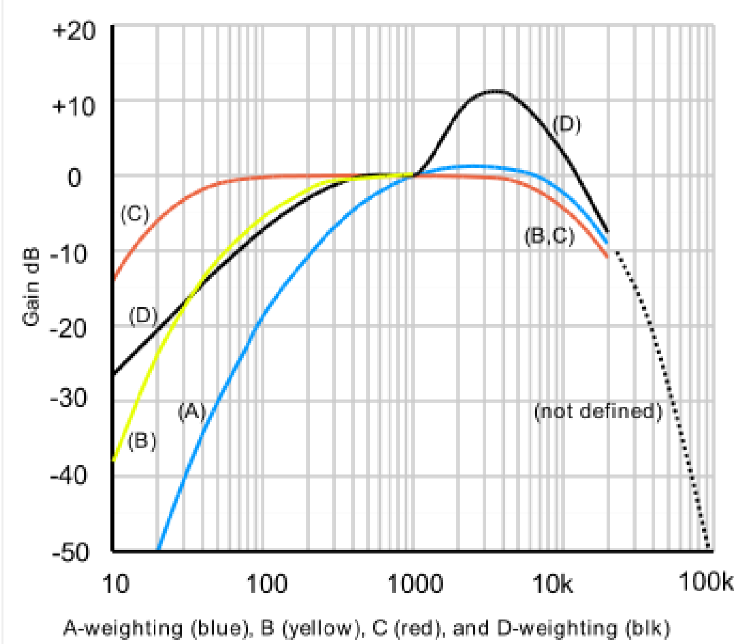
- Noise signals are measured in the “time domain” but are analyzed in the “frequency domain” using Fourier transform
- Any complex signal can be decomposed this way



Noise Signal Analysis

- Human ear can hear a sound, if the frequency content of the signal is in the range: **20 Hz – 20 kHz,** provided the signal amplitude is higher than threshold of hearing. The amplitude is measured using sound pressure level (SPL)

- Amplitude is usually weighted within above freq. range to replicate human ear sensitivity
  - » A-weighting (dBA) is most commonly used



Metrics

- Strength of acoustic signal is measured using rms (root-mean-square) value, defined as: $p'_{rms} = \sqrt{\overline{(p')^2}}$
  - » Threshold of hearing: $p'_{rms} \approx 10^{-5}$ Pa
  - » Threshold of pain: $p'_{rms} \approx 10^{2}$ Pa
  - » Because of large range of values, logarithmic scale is used
  - » Acoustic signal strength is measured using sound pressure level (SPL)

- Sound pressure level (SPL or L_p):

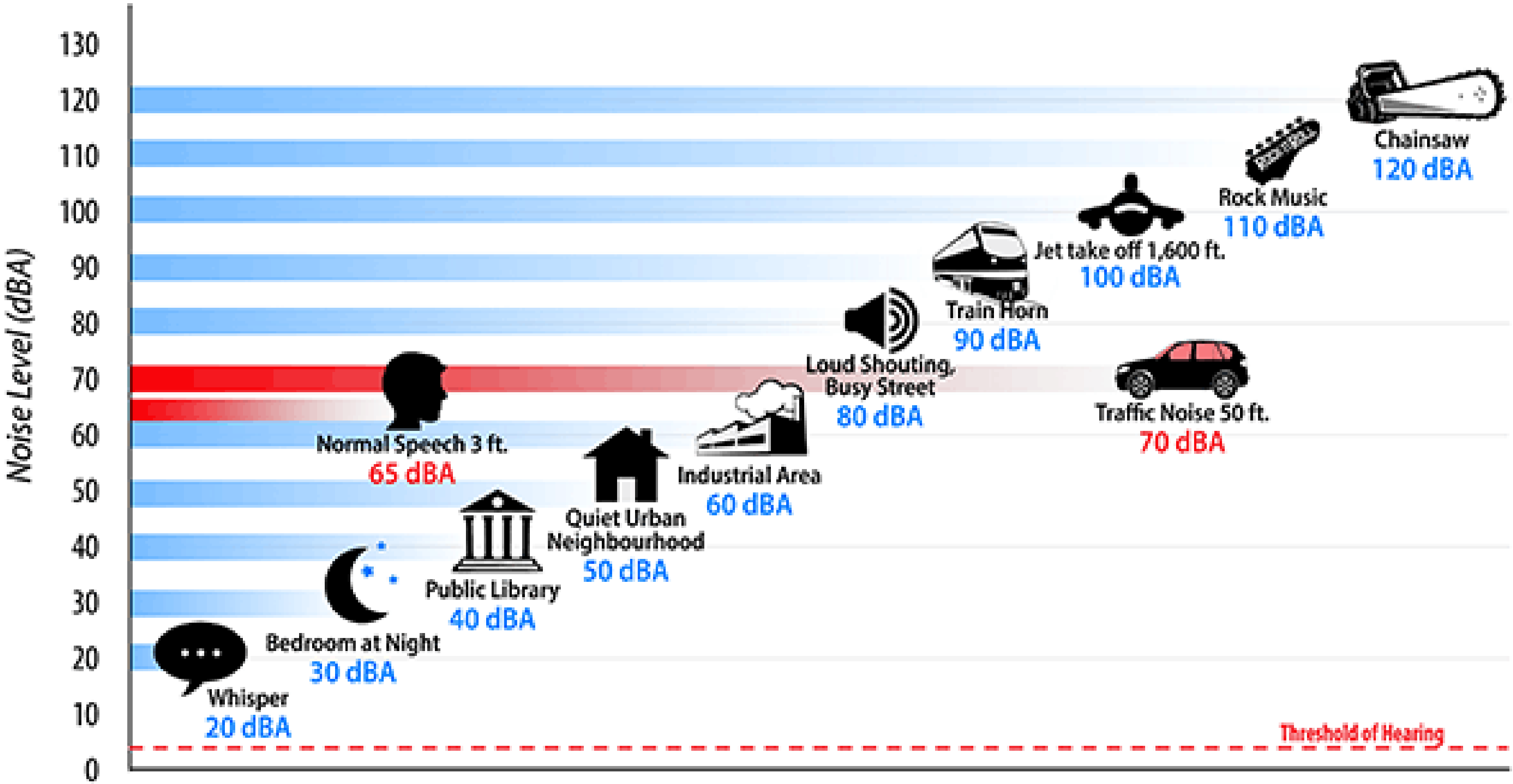
$$SPL = 10\ \mathrm{Log}\,(p'^2_{rms}/p^2_{ref}) = 20\ \mathrm{Log}(p'_{rms}/p_{ref}) \quad \text{(dB)}$$

where $p_{ref} = 2 \times 10^{-5}$ Pa for air and 10^{-6} Pa in other media.

- » Doubling p'_{rms} will increase the noise by only ~ 6dB (= 20 Log 2)

Typical Noise Levels

Loud noise of short duration is less annoying to human ear than a persistent noise of lower amplitude



Metrics

- Sound intensity Level (IL)
  - Sound intensity (energy flux): $\vec{I}(\boldsymbol{x}) = p'\overrightarrow{v'}$; or time-averaged: $I = \overline{p'v'}$
  - The direction of the intensity is the average direction in which the acoustic energy is flowing

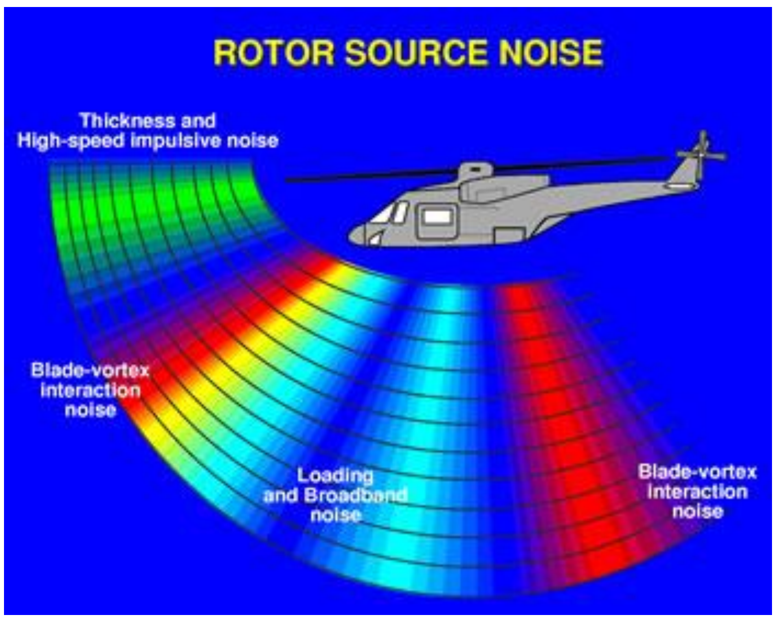
$$IL = 10\ \mathrm{Log}(I/I_{ref}) \quad \text{(dB)} \qquad \text{where } I_{ref} = 10^{-12}\ \mathrm{W/m^2}$$

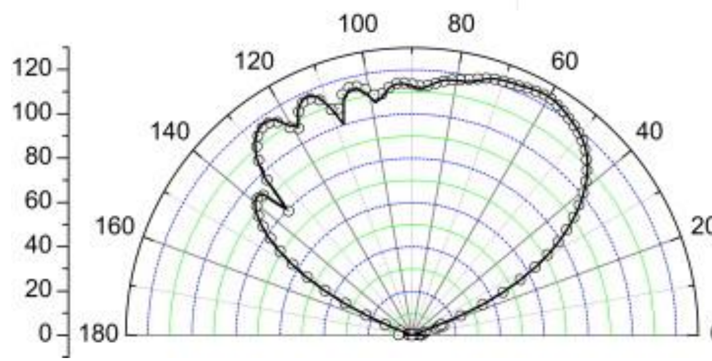
- Sound power level (L_W)
  - Is the power of sound sources enclosed within an area, A
  - Sound power is thus obtained by integrating intensity over the area
  - It is independent of integration area as long as A encloses all sources

$$L_W = 10\ \mathrm{Log}(P/P_{ref}) \quad \text{(dB)} \qquad \text{where } P_{ref} = 10^{-12}\ \mathrm{W}$$

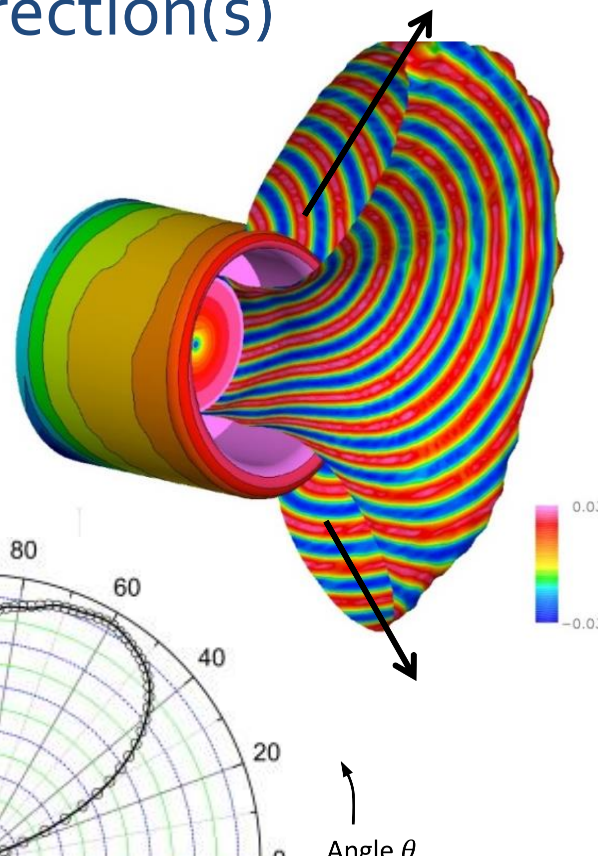
Directivity

- In general, noise is a directional phenomena, i.e., it radiates more intensely in certain direction(s)
- Examples:





Typical directivity plot



Wave Equation

- General form of governing equations:

continuity $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

momentum $$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \nabla \cdot \vec{\tau}_{ij} + \vec{f}$$

- Assumptions:
  - » Neglect body ($\vec{f}$) and viscous forces ($\vec{\tau}_{ij}$)
  - » Small perturbations : $\rho = \rho_0 + \rho'$, $p = p_0 + p'$, etc.
  - » Stagnant fluid ($\vec{v}_0 = 0$) with uniform properties ($\rho_0 =$ const) at observer

- Combine continuity & momentum equations :

$$\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x^2} = 0 \quad \text{or} \quad \frac{\partial^2 p'}{\partial t^2} - c_0^2 \frac{\partial^2 p'}{\partial x^2} = 0$$

  - » This is the homogenous wave equation

More on Wave Equation

$$\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x^2} = 0$$

or

$$\frac{\partial^2 p'}{\partial t^2} - c_0^2 \frac{\partial^2 p'}{\partial x^2} = 0$$

- It is both linear and homogeneous
- Solutions can be sought using Green's function
- Only governs sound propagation without any references to sound sources

Lighthill's Equation

- General form of governing equations:

continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

momentum

$$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -\nabla p + \nabla \cdot \vec{\tau}_{ij} + \vec{f}$$

- Assumptions:
  - » **DO NOT** neglect body ($\vec{f}$) and viscous forces ($\vec{\tau}_{ij}$)
  - » Small perturbations : $\rho = \rho_0 + \rho'$, $p = p_0 + p'$, etc.
  - » Stagnant fluid ($\vec{v}_0 = 0$) with uniform properties ($\rho_0 =$ const) at observer

- Combine continuity & momentum equations:

$$\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i x_j} - \frac{\partial f_i}{\partial x_i}$$

$$T_{ij} = \rho u_i u_j - \tau_{ij} + (p' - c_0^2 \rho')\delta_{ij}$$

Lighthill stress tensor

  - » This is called **Lighthill's equation** (1952)

Lighthill's Acoustic Analogy

$$\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i x_j} - \frac{\partial f_i}{\partial x_i}$$

$$T_{ij} = \rho u_i u_j - \tau_{ij} + (p' - c_0^2 \rho')\delta_{ij}$$

Lighthill stress tensor

- Lighthill's equation is exact (based on Navier-Stokes eqs)
- The word "analogy" refers to the fact that we can determine the sound field of a complex noise generating phenomena by treating it as source terms of the wave eq.
- It is a non-homogeneous equation where the right-hand side represents aeroacoustic sources
- Solution can be sought using Green's function, if the source terms can be suitably modeled

Sources of Aerodynamic Sound

$$\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \frac{\partial^2 \rho'}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i x_j} - \frac{\partial f_i}{\partial x_i}$$

$$T_{ij} = \rho u_i u_j - \tau_{ij} + (p' - c_0^2 \rho')\delta_{ij}$$

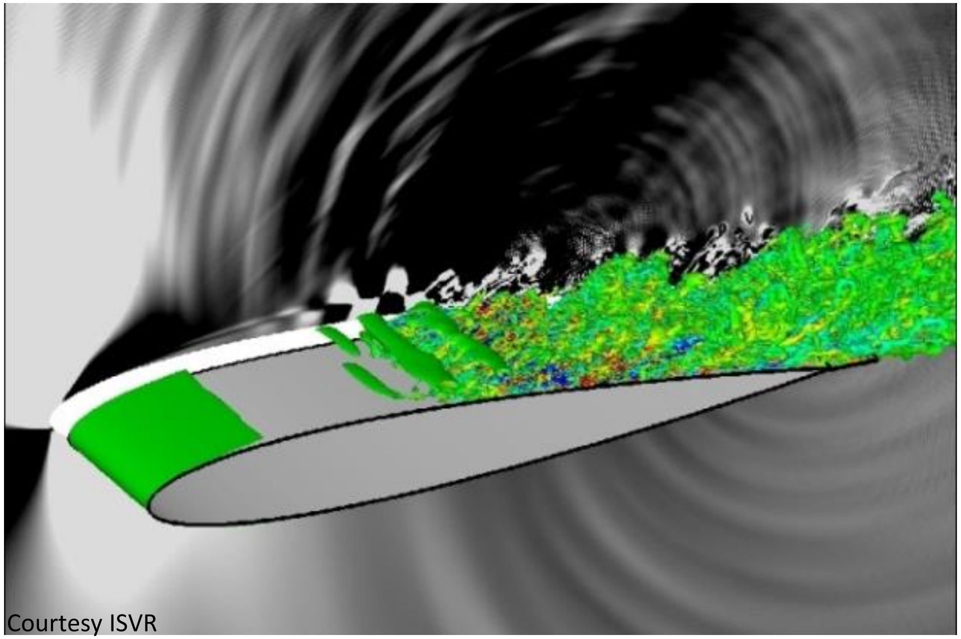
Lighthill stress tensor

The right-hand side represents the sources:

- Monopole:
  - » Any changes in the entropy (where $s' = p' - c_0^2 \rho'$ will be non-zero) or deviation from uniform speed of sound (c_0)
- Dipole:
  - » Acoustic field due to external forces exerted on the flow ($\partial f_i / \partial x_i$)
- Quadrupole:
  - » Induced by non-linear convective forces represented by the Reynolds stress tensor ($\rho u_i u_j$), such as turbulence
  - » Due to viscous forces (τ_{ij})

Modeling Acoustic Sources

- Monopole
  - Thickness noise
- Dipole
  - Loading noise
- Quadrupole
  - BL/viscous effects



http://www.acs.psu.edu/drussell/demos/rad2/mdq.html

FW-H Equation

- Ffowcs Williams–Hawkings equation is a generalization of the Lighthill analogy to add sound field associated with sources in arbitrary motion
- Like Lighthill's equation, FW-H equation is derived using full Navier-Stokes equations w/o simplifying assumptions
- FW-H vs. Lighthill:

FW-H Equation

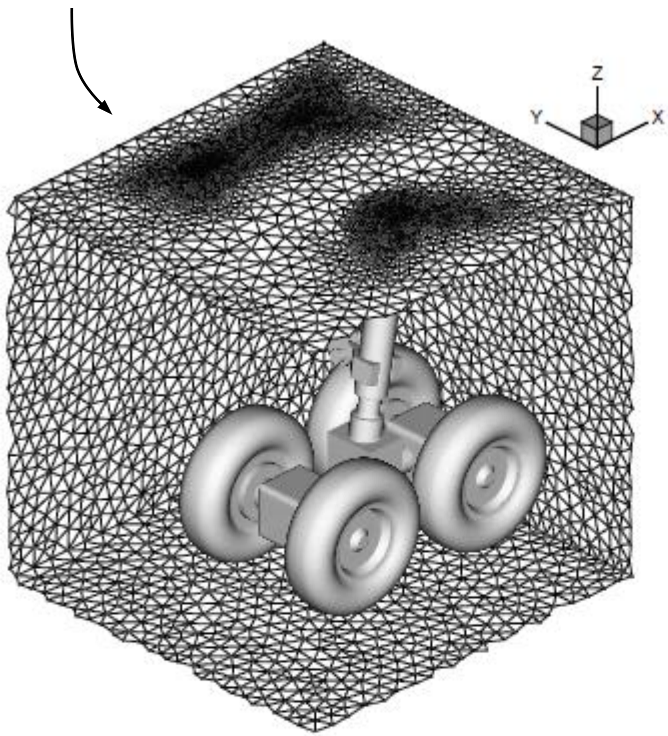
$$
\begin{aligned}
4\pi a_\infty^2 H \rho'(\boldsymbol{x},t) \;&=\; \boldsymbol{\nabla}_x\cdot\boldsymbol{\nabla}_x\cdot \int\limits_{V_\infty} \frac{JH\boldsymbol{T}}{r|1-M_r|}\, dV(\boldsymbol{\eta}) - \\
&- \;\; \boldsymbol{\nabla}_x\cdot \int\limits_{\partial V_H} \frac{[\rho\boldsymbol{v}(\boldsymbol{v}-\boldsymbol{v}_H)-\boldsymbol{\tau}+p\boldsymbol{I}]\boldsymbol{n}_\xi}{r|1-M_r|}\, \frac{|\boldsymbol{\nabla}_\xi f|}{|\boldsymbol{\nabla}_\eta f|} J\, dS(\boldsymbol{\eta}) + \\
&+ \;\; \frac{\partial}{\partial t} \int\limits_{\partial V_H} \frac{[\rho(\boldsymbol{v}-\boldsymbol{v}_H)+\rho_\infty \boldsymbol{v}_H]\cdot\boldsymbol{n}_\xi}{r|1-M_r|}\, \frac{|\boldsymbol{\nabla}_\xi f|}{|\boldsymbol{\nabla}_\eta f|} J\, dS(\boldsymbol{\eta})
\end{aligned}
$$

- The first term on the RHS (volume integral) is the Lighthill tensor.
- Surface integrals are associated with moving source assumption. So, in absence of moving sources, FW-H reduces to Lighthill eqn
- If the source term represented by T_{ij} is moved inside the control surface ∂V_H, volume integral will vanish b/c of Heaviside function. This has very important practical implications

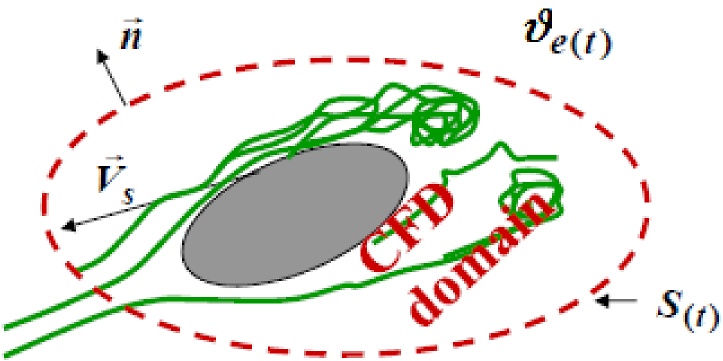
FW-H Equation

- Practical applications:

FW-H surface



Courtesy: Souliez et al.

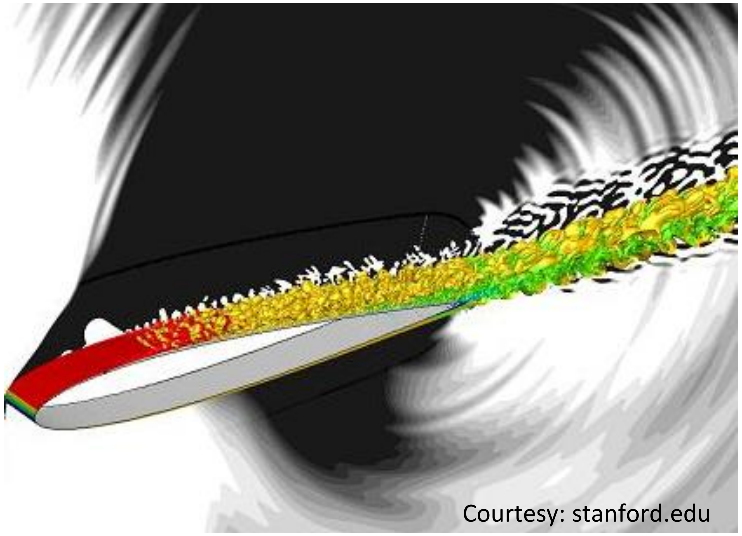


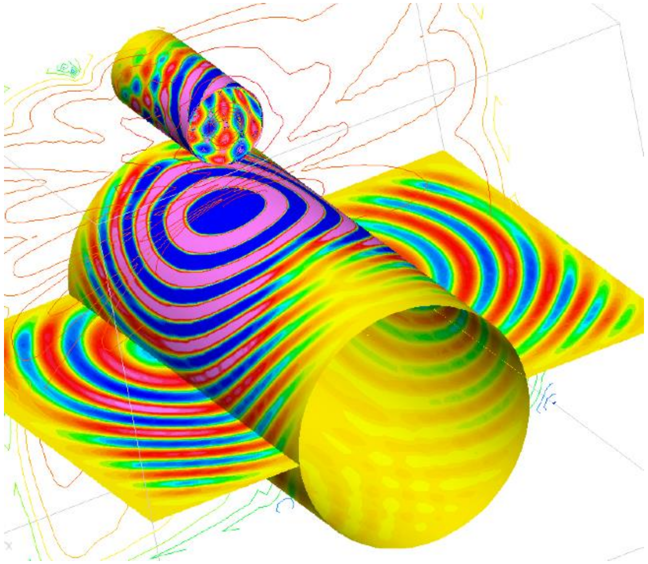
Kirchhoff Integral

- Based on solution of the homogeneous wave equation using the free-space Green's function
- Is equivalent to the FW-H integral, if integration surface is placed in the linear region of the flow
- FW-H is superior because is valid in both linear and nonlinear flow regions
- It can yield wrong answers if homogenous wave equation not satisfied on the control surface
- Linear assumption usually valid in a region far enough from the sources

Noise Prediction Using CAA

- Computational Aero-Acoustics (CAA) refers to the numerical simulation of sound propagation/radiation, with noise sources either modeled or resolved as part of the simulation
- Although CAA relies on existing CFD methodology, it requires special treatment in certain aspects of simulation


Courtesy: stanford.edu



Computational Aero-Acoustics

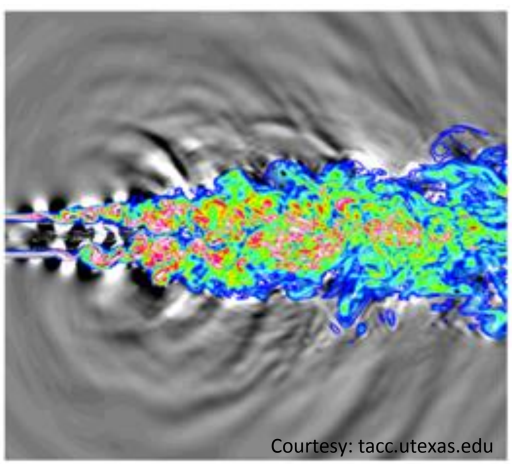
- Two main issues arise in acoustic simulation:

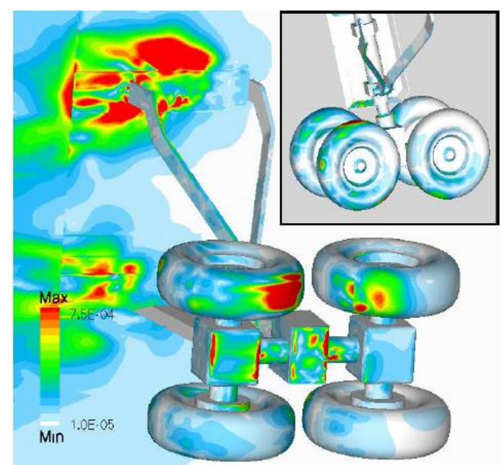
1. Acoustic perturbations ($p', \rho', \overrightarrow{u'}$) are usually several orders of magnitude ($\sim 10^{-6}$) smaller than background flow variables ($p, \rho, \vec{u}$)

   Therefore, discretization method used should be able to resolve such disparity, while maintaining amplitude & phase characteristics of the waves

   This requires the use of high-order methods, which are computationally expensive (compact FD schemes, DRP schemes, spectral methods)

2. Boundary conditions should be non-reflective to avoid contamination of interior solution



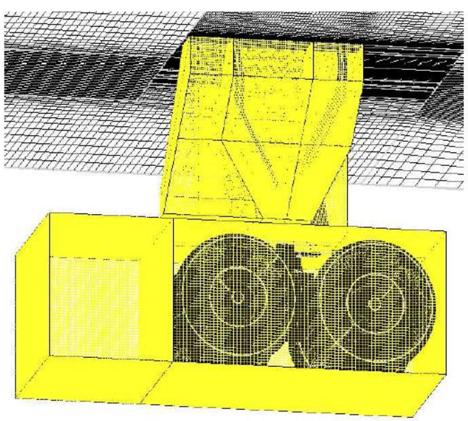


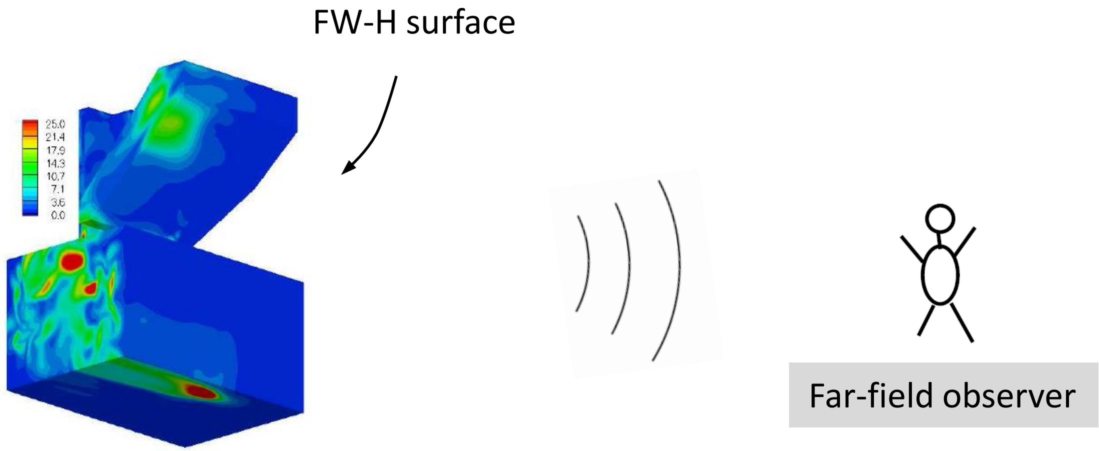
Courtesy: NASA (Lockard & Khorrami)

Noise Prediction – Hybrid Methods

- Typically, it is desired to calculate the noise at the far-field
- Using CAA, it is generally impractical or impossible to extend the computational domain to the far-field
- A hybrid approach is therefore the best (often only) choice:
  - Use CAA in the near-field
  - Use FW-H equation for far-field noise propagation. Note that accuracy of far-filed predictions will heavily depend on accuracy of predicted sources on the FW-H surface

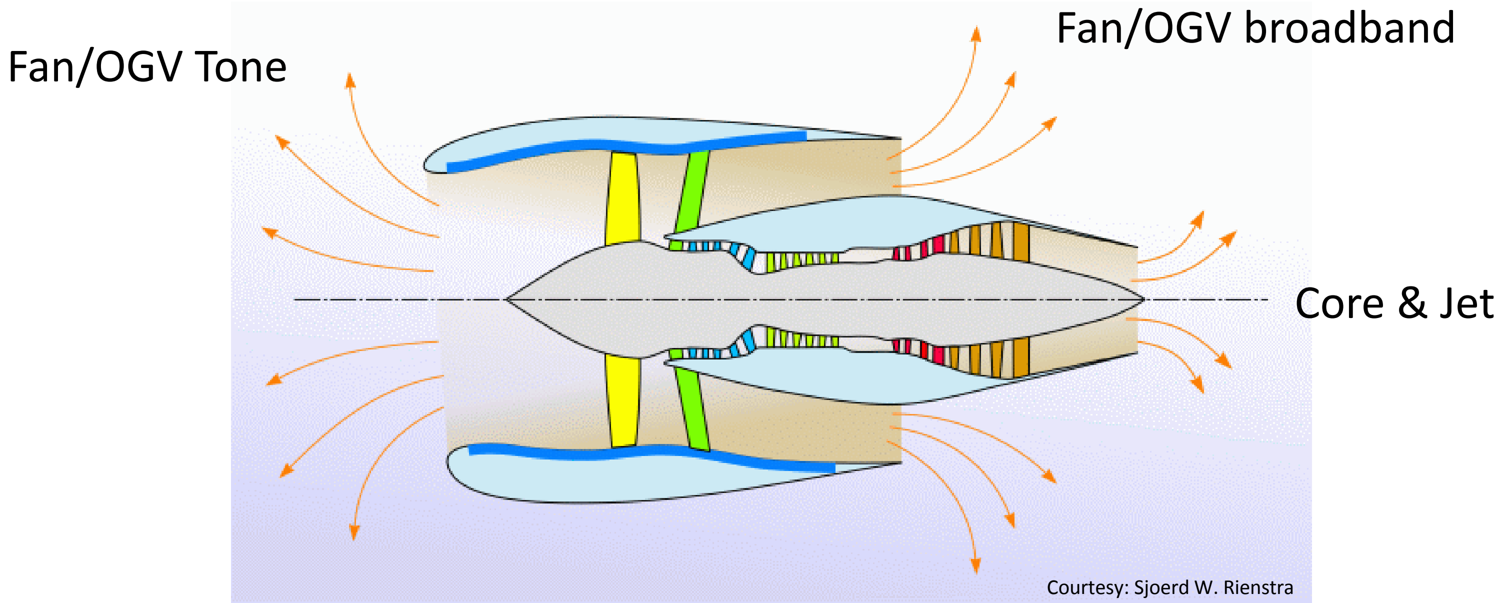
Courtesy: NASA (Lockard & Khorrami)





Turbofan Engine Noise

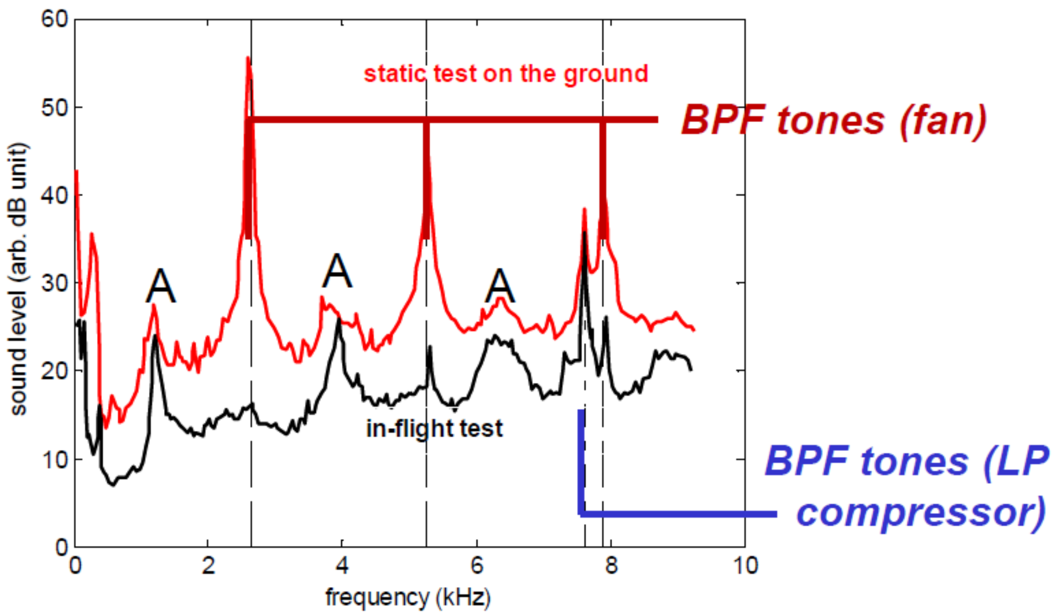
- Different sources:
  - Fan/OGV interaction (tone & broadband)
  - Core noise (broadband)
  - Jet noise (broadband)



Fan Noise Mechanism

Caused by interaction of rotor with downstream stator/OGV. It consists of:

- Tone noise associated with periodic aerodynamic interactions
  - Tonal noise is radiated at multiples of blade-passage frequency (BPF)
- Broadband noise associated with turbulence



Fan Noise Mechanism

- It creates a pressure field, locked to the rotor, which is made of m-lobe patterns each rotating at the speed $nB\Omega/m$:
  - n: harmonics of the blade-passing frequency
  - B and V are the number of rotor and stator blades, respectively
  - Ω is the rotor's angular speed
  - $m = nB \pm kV$, where k is a positive integer
- According to Tyler-Sofrin theory, the pressure field at the fan face for a circular duct is then given by:

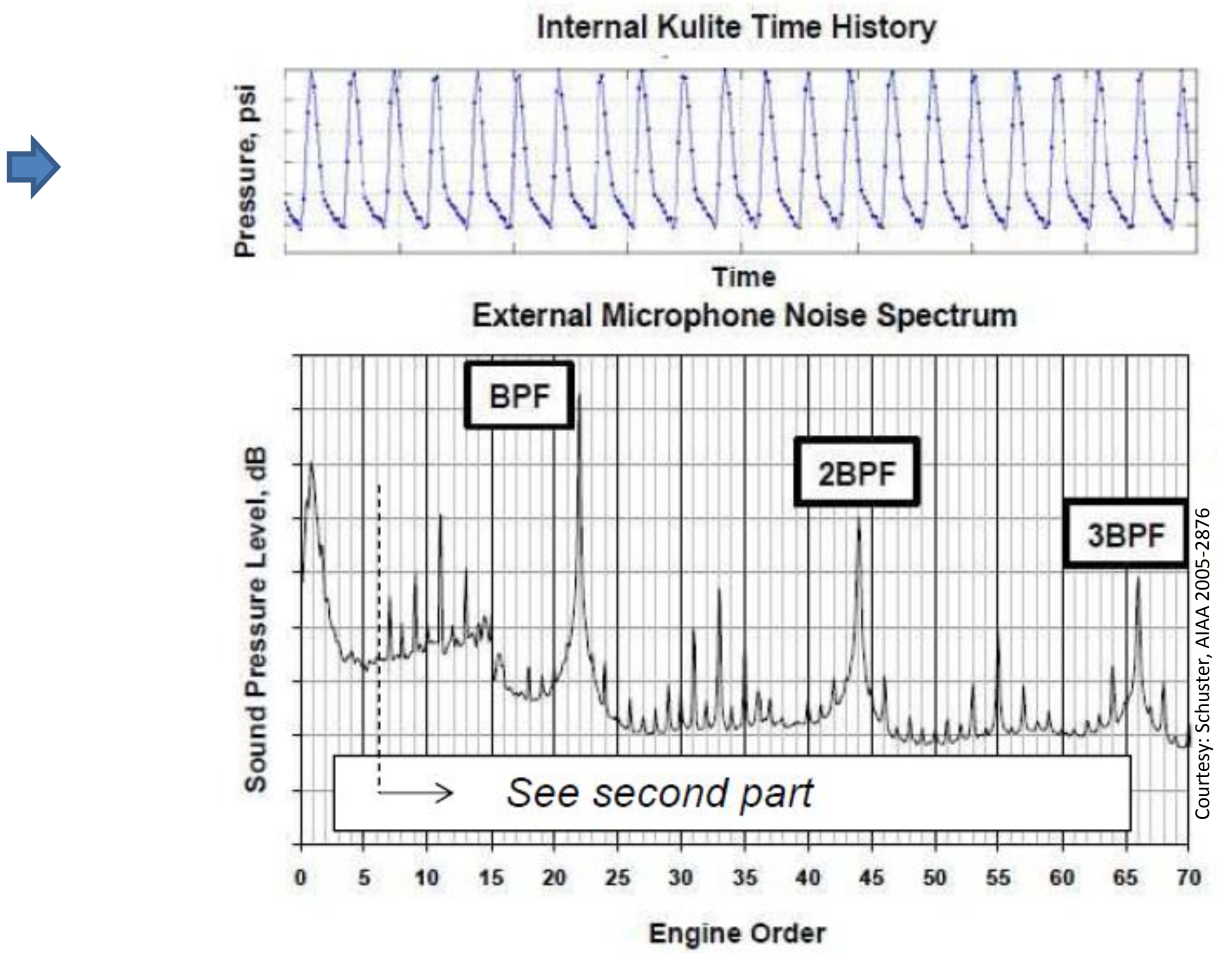
$$p'_{ms}(x,r,\theta,t) = \sum_{s} A_{ms}\, J_m(k_{ms} r)\, e^{i(m\theta + k_x x - \omega t)}$$

  - J_m : Bessel function of the first kind and order m
  - k_{ms} : eigenvalues defined by $J'_m(k_{ms}R) = 0$;
  - s : radial mode number
  - k_x : axial wave number
  - $\omega = \Omega R/c_0$: non-dimensional frequency with R being the duct radius

Fan Noise Mechanism

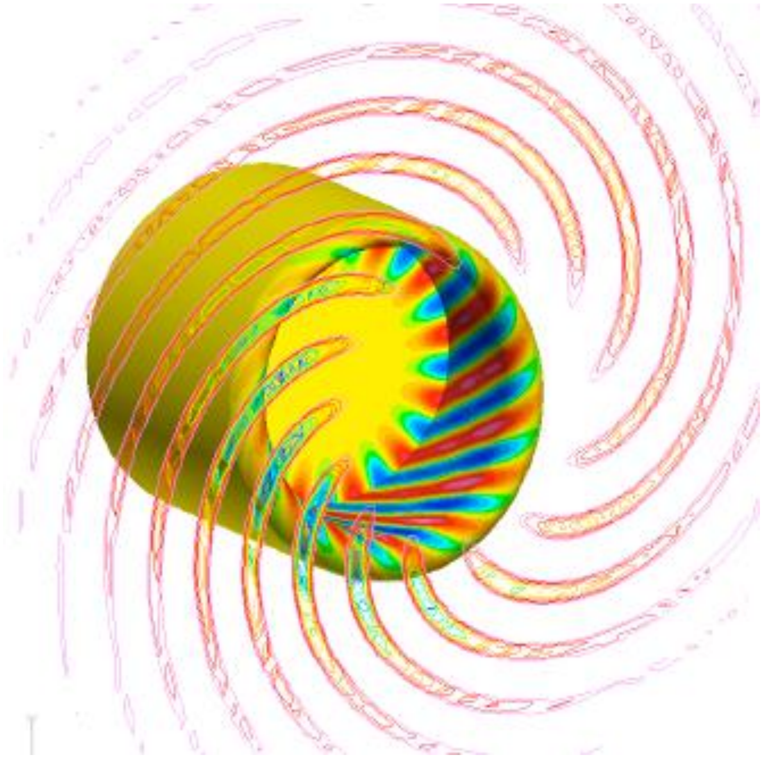
- m is also called engine order

rotor-locked pressure fluctuations



Fan Noise Mechanism

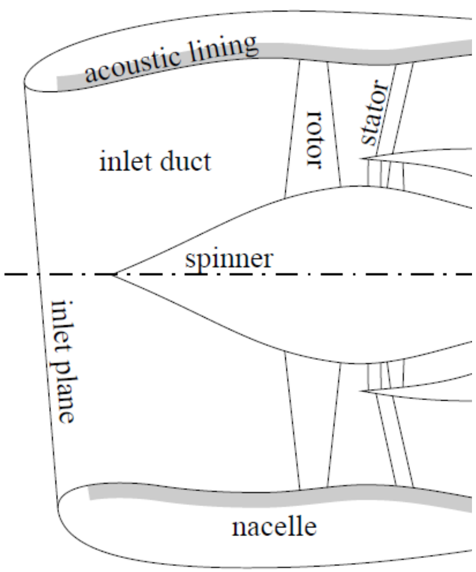
- Acoustic field is made up of m-lobe patterns



- For rotors with spinner, radial variation is given by:

$$A_{ms}[J_m(k_{ms}r) + Y_m(k_{ms}r)]$$

Y_m : Bessel function of second kind



Courtesy: Sjoerd W. Rienstra

Fan Noise Propagation

- The acoustic waves given by

$$p'_{ms}(x,r,\theta,t) = \sum_s A_{ms}\, J_m(k_{ms}r)\, e^{i(m\theta + k_x x - \omega t)}$$

  will propagate down the duct only if k_x is real-valued, which happens if $\omega / k_{ms} > 1$ (assuming no mean flow). Otherwise, k_x will be complex and the corresponding mode will be damped and not propagate (cut-off mode)

- The pressure field given by the above equation could alternately be obtained by direct simulation of rotor/stator flow interaction

Fan Noise Propagation

- Typically, a hybrid approach is used to determine far-field noise of the fan. CAA is used (LEE, potential flow) to simulate flow propagation inside the duct and in a small region surrounding duct exit, where FW-H is located.

- More complex CAA analysis should include the effect of duct boundary layer and sound refraction

- The effect of liners on duct wall can be simulated by defining impedance boundary conditions